Strength of Materials
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Lecture - 9
Analysis of Strain - III

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Welcome to the 3rd lesson on module 2 on analysis of strain.

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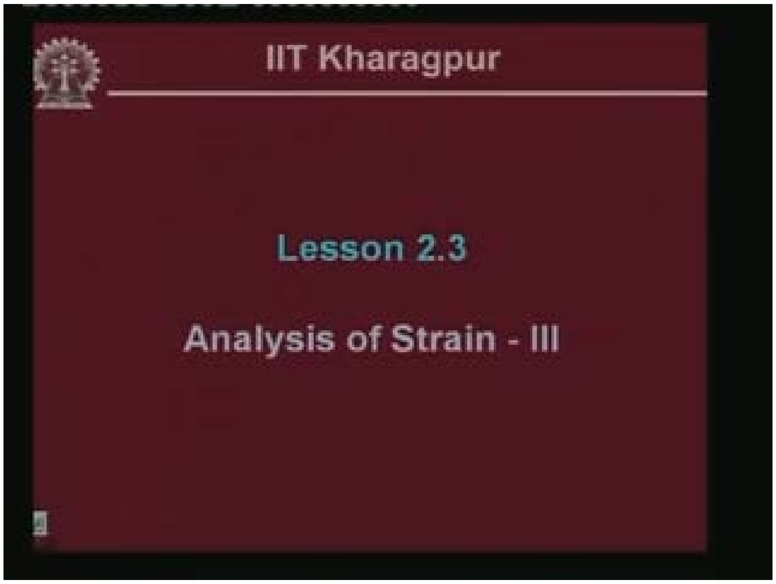

This particular lesson we have designated as analysis of strain 3. In the last lesson, we have discussed certain aspects of strain. In this particular lesson we will be discussing some more aspects of strain.

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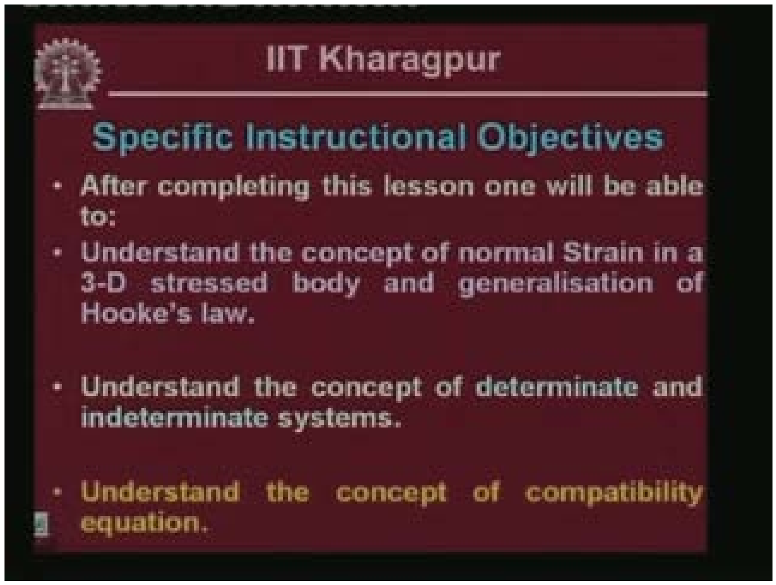

It is expected that after this particular lesson is completed, one will be able to understand the concept of normal strain in a three dimensional stress body and thereby we can go for the generalization of Hooke's law. One should be able to understand the concept of determinate and indeterminate systems, also one should be able to understand the concept of compatibility which will eventually arise out of this indeterminate system which we are going to discuss subsequently.

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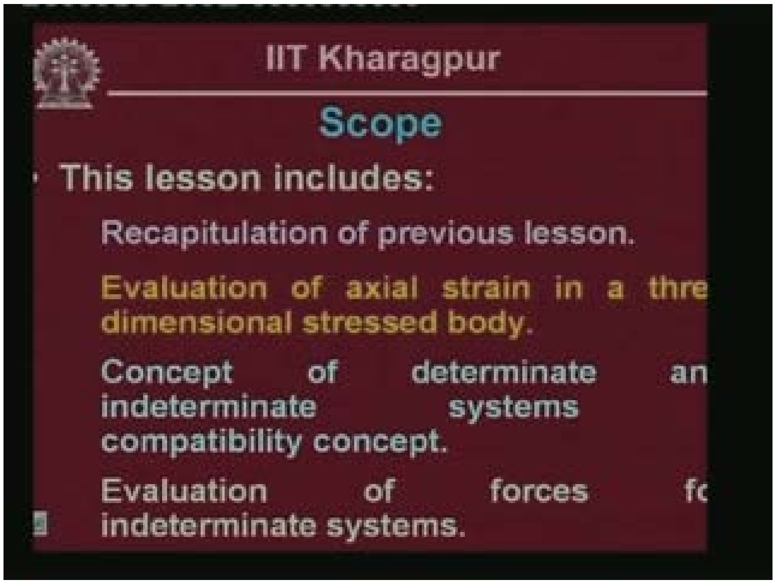

This particular lesson includes the recapitulation of previous lesson which we will be doing through the questions which we posed last time. We will be discussing those questions and eventually it will give you the recapitulation of the previous lesson, evaluation of axial strain in a three dimensional stress body, concept of determinate and indeterminate systems and thereby the compatibility criteria, how it will arise out from that particular system and finally the evaluation of forces for indeterminate systems.

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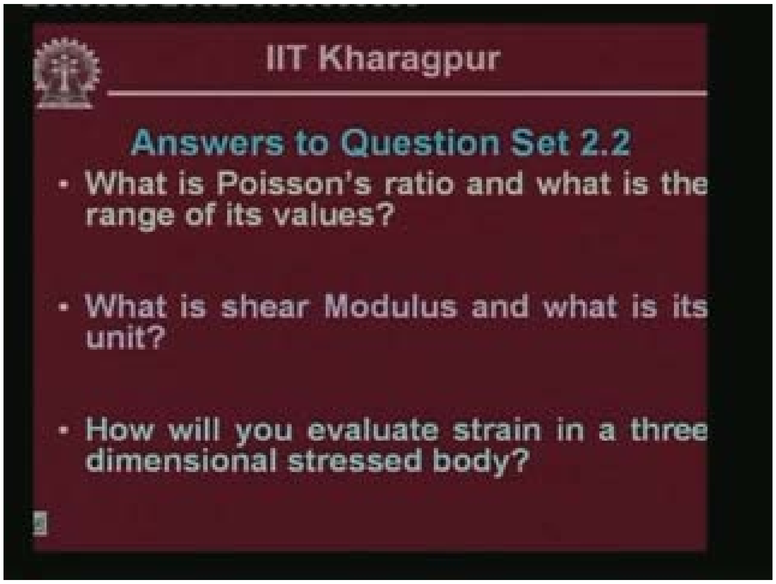

Here are some questions:
The first question is: What is Poisson's ratio and what is the range of its values?

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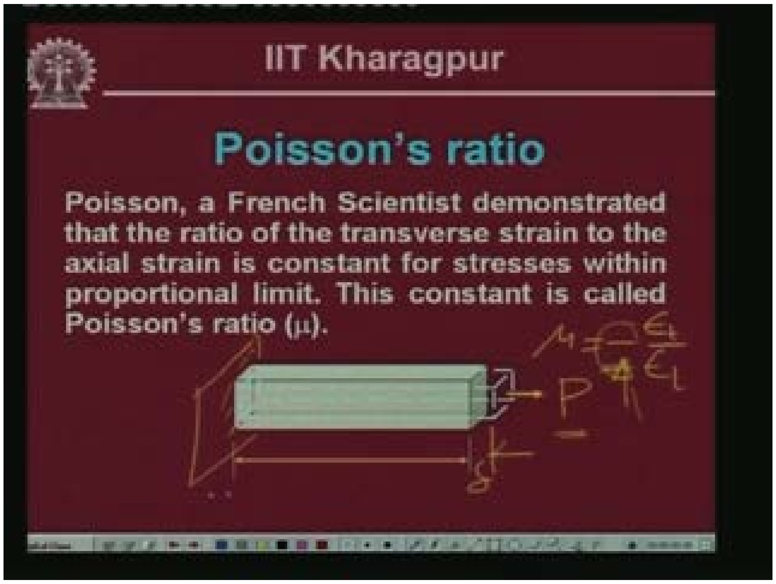

Now, if we remember, last time we discussed that if a body which is subjected to axial pull (is held up at this end and is being pulled by axial load p) eventually, this bar extends or elongates and this elongation we had indicated by delta and thereby there is contraction on the other sides. The French scientist Poisson demonstrated that the ratio of this strain which is the transverse strain to the axial strain is constant for stresses within the proportional limit and this particular constant is designated as Poisson's ratio.

So far as the definition of the Poisson ratio is concerned, the ratio of the lateral strain to the longitudinal strain and if you remember we had designated this M as the transverse strain to the longitudinal strain and remember that we had put this particular sign as negative because where this is being pulled by an axially tensile pull, then there is contraction on other sides and this negative sign indicates that there is contraction. Similarly, if this particular bar is subjected to an axial compressive force where the length in the axial direction is going to shorten thereby the other two directions is going to increase and eventually that shortening of the axial length is being designated as negative to the expansion of the other two sides and that is why this negative sign comes in.

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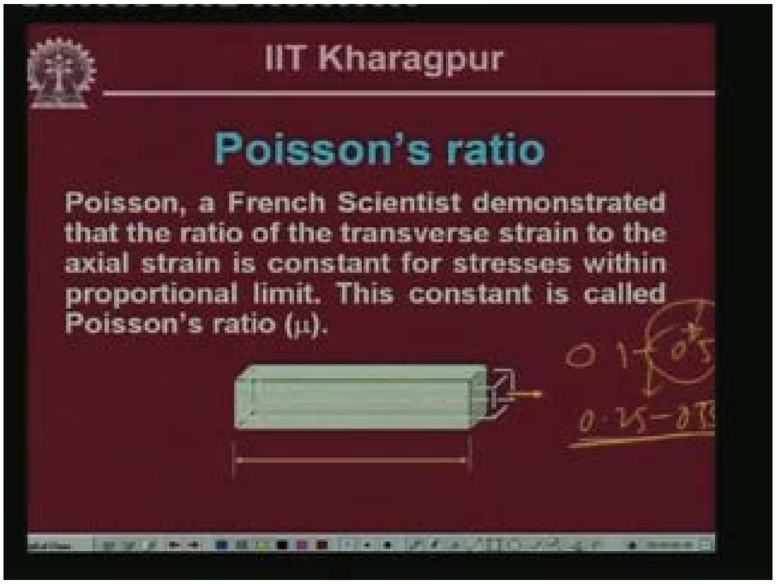

We said that what is the range of its values; now is what is its ratio for different material, where there is very small range of values, generally the values vary between 0.25 to 0.35 for some limiting materials, we get values in the range of around 0.1 or for some material we get in the range of 0.5 which is that of rubber. And in fact 0.5 is the limiting value at the other side in the sense that the range of the values in general we can say between 0.1 to 0.5 and most of the material lies between 0.25 to 0.35.5 is the maximum value of the Poisson's ratio.

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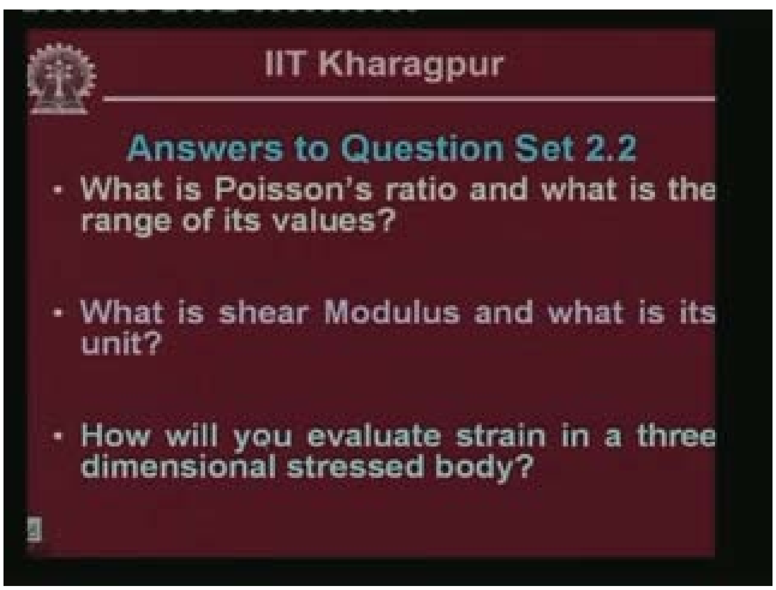

What is shear modulus and what is its unit?

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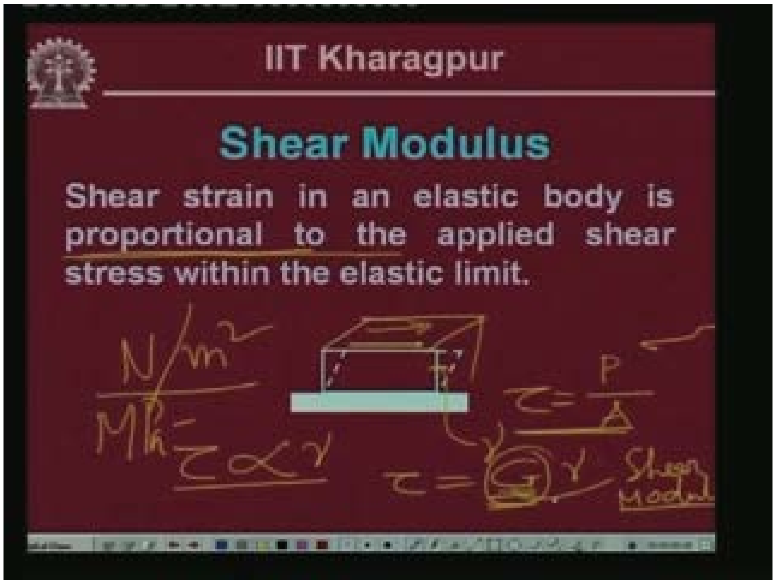

The shear strain in an elastic body is proportional to the applied shear stress. Now this is the body which is acted on by this is subjected to the surface force and there by the shear stress tau equal to the applied load by the cross-sectional area is given as the shear stress and within the limit of the proportionality, the shearing strain is proportional to the shearing strain which we have designated as gamma. So tau is proportional to gamma and removing this proportionality sign, we say tau is equal to G into gamma where G is termed as shear modulus this is called as shear modulus and this has the unit similar to the unit of modulus of elasticity. Now which is in Pa or in N by m square or MPa which is 10 to the power 6 into N by m square or GPa, thus the unit of the shear module G.

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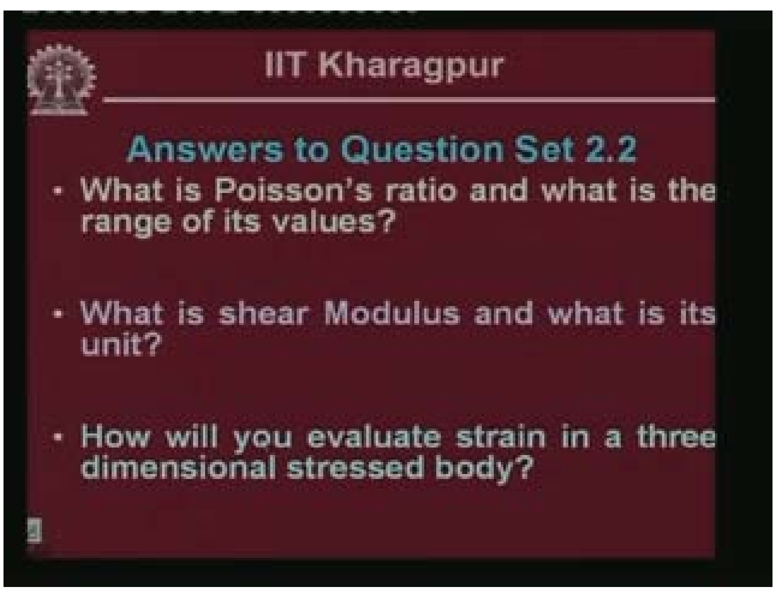

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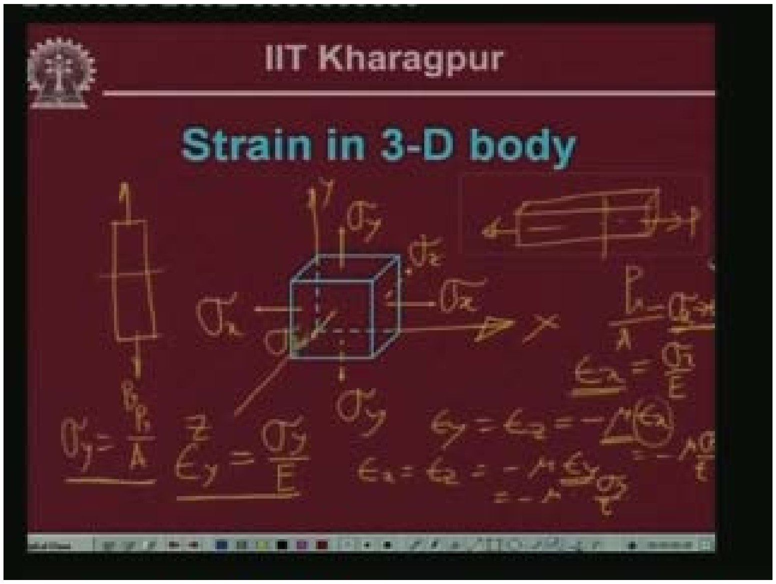

The 3rd question is; how you will evaluate stain in a three dimensional stress body?
Let us assume that these are the three directions this is x, this is y and this is z and correspondingly as we have defined earlier the stress components sigma$_x$, sigma$_y$ and sigma$_z$. Now as we have seen earlier that when a bar is subjected to axial pull, the stress at any cross section is P by a the cross sectional area which we have designated as normal stress sigma$_x$ and correspondingly the strain in the axial direction is epsilon$_x$ and as per Hooke's law epsilon$_x$ is equal to sigma$_x$ by e. Also, we have noted from the concept of Poisson's ratio, that if we have axial load acting in the x direction only and thereby there is strain in the x direction as epsilon$_x$ the strain in the other two direction y and z direction epsilon$_y$ is equal to epsilon$_z$ is equal to -M into epsilon$_x$ is as per the definition of Poisson's ratio.

Also, if we consider that the body subjected to the axial load in the y direction instead of in the x direction, that means we have a bar which is subjected to load in the y direction it is p there by the stress again at any cross-section is p by a and let us call that stress as sigma$_y$ which is p is acting in the y direction so this is p by a. Let us call this as p$_y$ this is p$_x$. Now because of this application of the load, there is strain in the y direction which we designate as epsilon$_y$ and that is going to be equal to according to the Hooke's law is sigma$_y$ by e. Now due to Poisson's effect, then the strain in the x and z direction will be epsilon$_x$ is equal to epsilon$_z$ is equal to minus M epsilon$_y$ and if we substitute the values of epsilon$_x$ and epsilon$_y$ in this corresponding places this is M sigma$_x$ by e and this is minus M sigma $_y$ by e.

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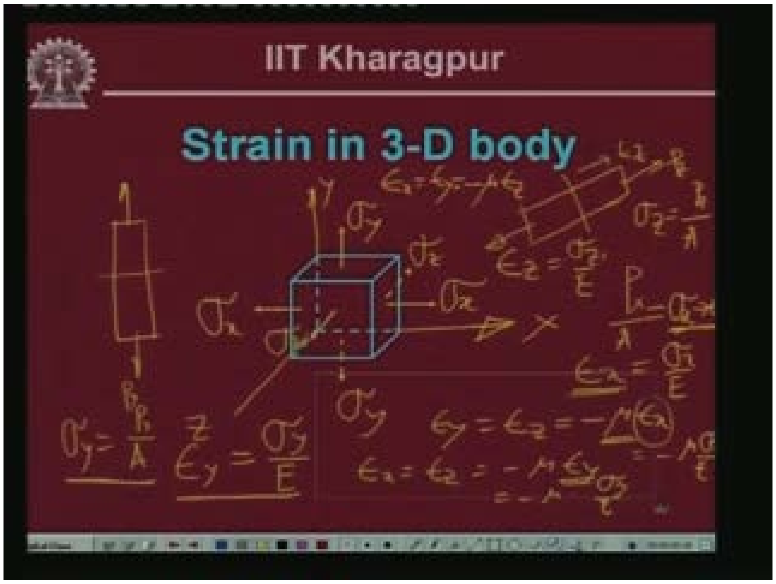

Likewise, if we assume that the body is acted on by force in the z direction p then the stress at any cross section, we call that as $sigma_z$ is equal to p_z by a. Also the strain corresponding to that in the z direction is $epsilon_z$ is equal to $sigma_z$ by e $epsilon_z$ is equal to $sigma_z$ by e. As we have seen in the previous cases that when it is acted on by the load p either in the x direction or in the y direction, we could compute the strains in the other two directions in the present case also when it is subjected to axial pull in the z direction and is the strain which is acting in the z direction is $epsilon_z$ the corresponding strain in the epsilon in the x and y directions are $epsilon_x$ and $epsilon_y$ is equal to minus M into $epsilon_z$. So in this particular case $epsilon_x$ is equal to $epsilon_y$ is is equal to minus M into $epsilon_z$ which is M into $sigma_z$ by e.

Hence, if we combine all three cases together, if we propose them since we are considering the body with in the elastic limit. Hence if this proposition is valid we can combine them together with the individual results. If we join them together, we will get the same effect when all these are acting simultaneously so in that particular case when all the three are acting we can write then the strain in the x direction $epsilon_x$ is equal to $sigma_x$ by e because of the load which is acting in directly that and the strain which we are getting in the x direction because of y and z direction is equal to M into $sigma_y$ by e minus M $sigma_z$ by e or this is equal to $sigma_x$ by e minus $Msigma_y$ plus $sigma_z$ by e. Likewise, the strain in the y direction $epsilon_y$ can be written as $sigma_y$ by e minus M into $sigma_x$ plus $sigma_z$ by e and $epsilon_z$ is equal to $sigma_z$ by e minus M into $sigma_x$ plus $sigma_y$ by e.

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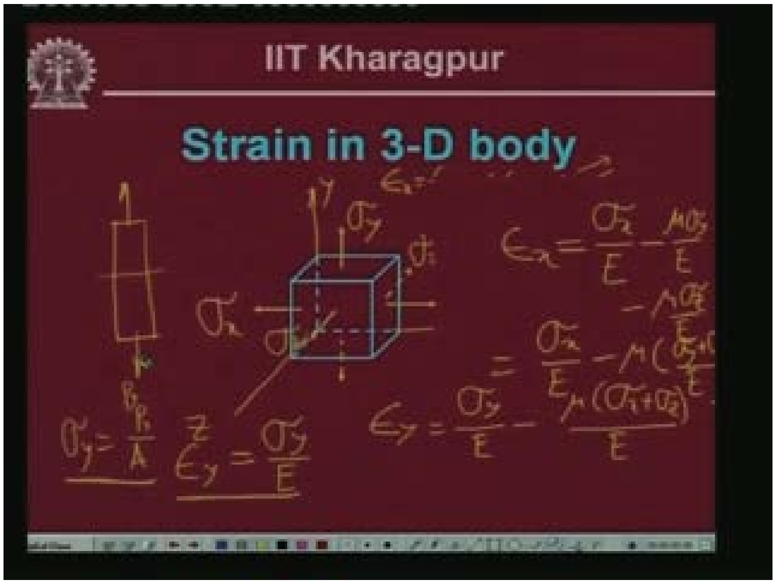

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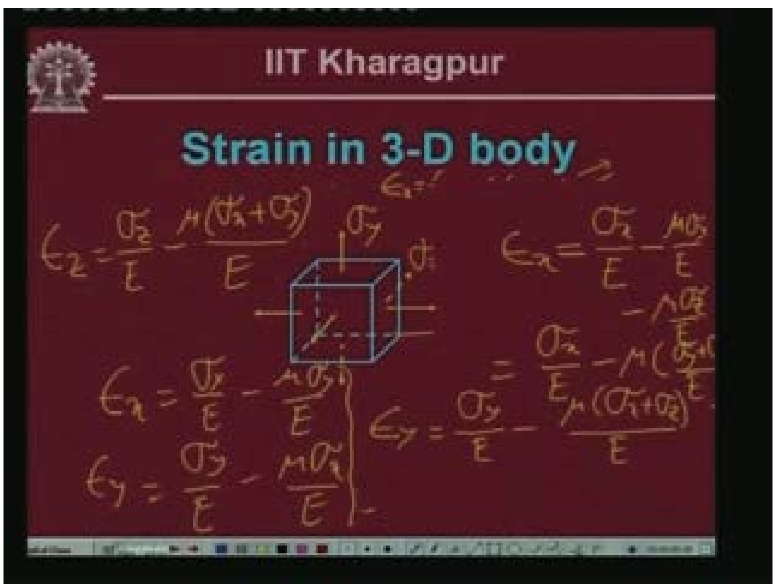

Now if we reduce this three dimensional form to a two dimensional one giving the x and y plane, then the stresses which will be acting sigma_x and sigma_y and eventually sigma_z is equal to 0. In that particular case, we will get epsilon_x is equal to sigma_x by e minus M sigma_y by e and epsilon_y is equal to sigma_y by e minus M sigma_x by e. This results we have seen it earlier so this is in fact in a generalized form of Hooke’s law which is applicable in case of a three dimensional body in three direction epsilon_x epsilon_y epsilon_z. The strain in the three directions can be represented in terms of the stresses sigma_x sigma_y and sigma_z and in terms of the modulus of velocity e.

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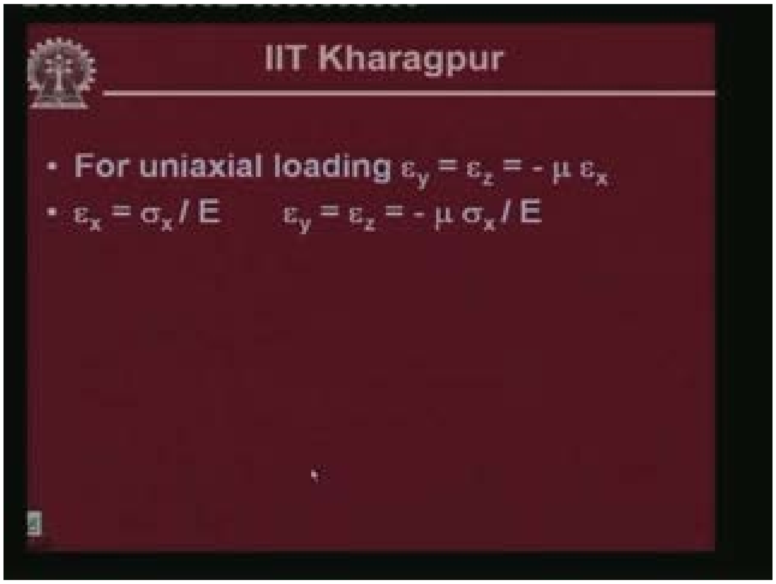

These are the aspects we discussed.

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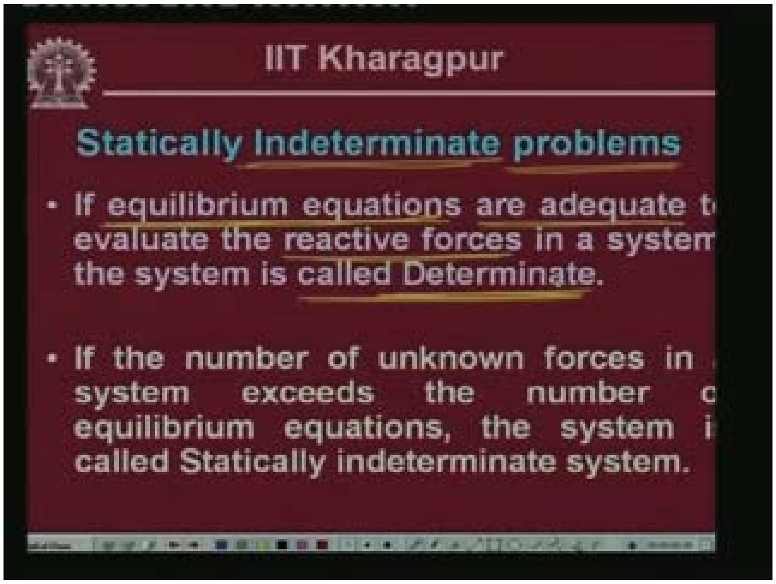

Now let us look into the important aspects which we do come across in many problems related to different fields which we designate as statically indeterminate problems. Now when we talk about statically indeterminate problem naturally we will first know what we really mean by a determinate system we have seen earlier that we have three states of equilibrium equations in statics there is summation of horizontal forces are 0 summation vertical forces are 0, summation moments are 0. These are the equations of equilibrium. If we consider any system from which we can evaluate the reactive forces by using these equilibrium equations we call those systems as statically determinate you see if equilibrium equations are adequate this is important that equilibrium equations are adequate to evaluate the reactive forces then the system is called as determinate. Let us look into some example.

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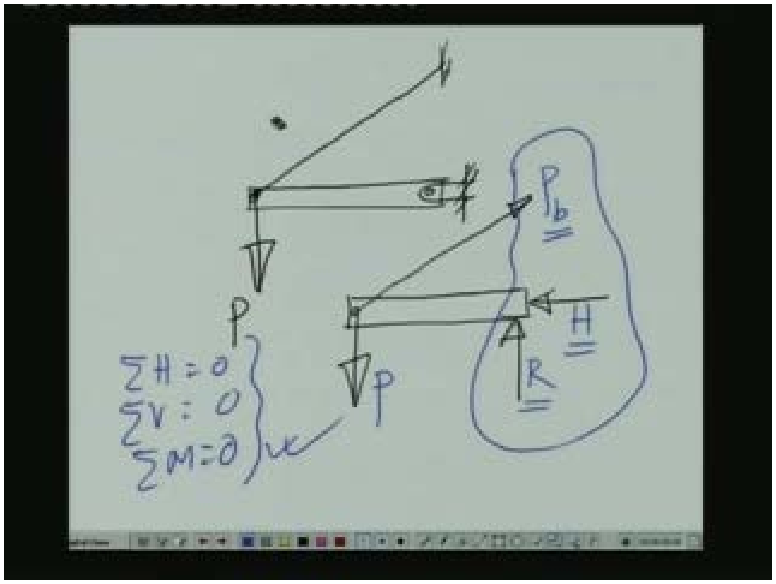

We have a rigid bar which is pinned at this particular end and is held up by a rod and is subjected to a vertical load p, now if we draw the free body diagram of this particular configuration removing this supports the free body diagram it will look like this. This bar being pin connected is subjected to axial loads only hence there is axial pull P_b and these are the reactive forces R and H and this the external load P. This is the free body diagram of this particular system.

Now in this particular free body diagram the unknown forces are the reactive forces R H and the axial pull in the member P_b. So these three parameters can be evaluated from the equations of static that means if we take summation horizontal forces is equal to 0 summation vertical force is equal to 0, summation moment is equal to 0 and if we apply these three equations for this; we can evaluate R H and P radically from these equations. Hence this particular system is statically determinate. Now let us change the configurations of this particular system later.

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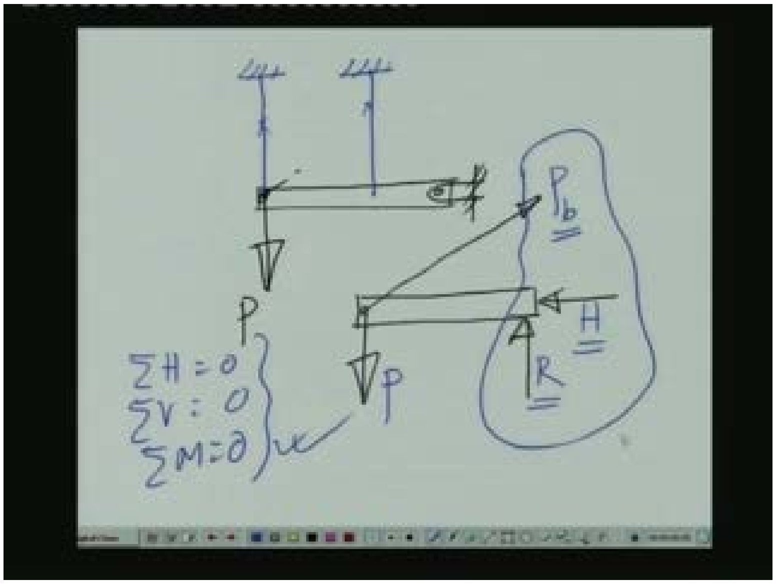

This is connected by two wires of this particular form. Now these rods will undergo extension because they are subjected to axial pull when this particular load will be acting.

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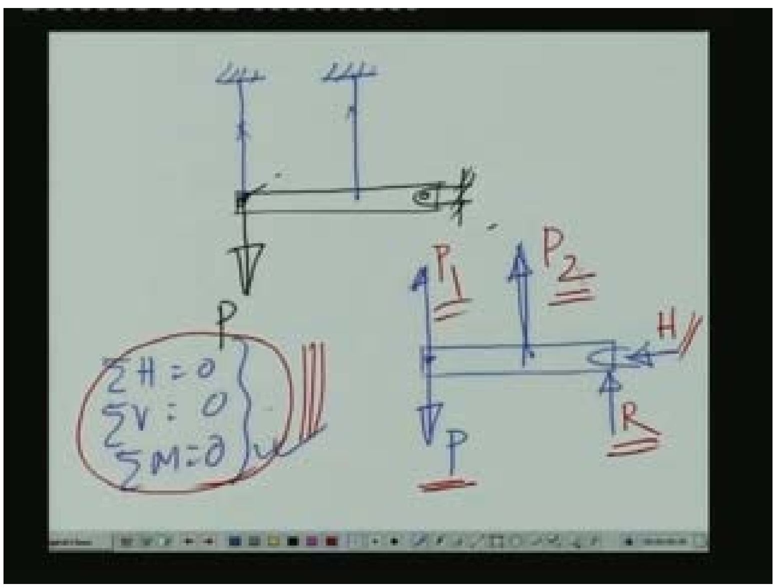

Now if we try to plot the free body diagram of this particular system, this is the rigid bar pinned over here, this is subjected to the reactive force vertical and horizontal as before here we have the applied load P, now two rods which are supporting this bar will have axial pull in the rods. So there are forces which are reactive forces R, H and let us call this as P_1 and P_2 which are under the action of the axial load P or the transverse load P. Now in this particular case, if we look into we have 4 unknown reactive forces R, H, P_1 and P_2 and since we have only three equations of statics this particular system cannot be solved using these equations of equilibrium alone. Hence there are systems like this where you cannot solve or evaluate the unknown forces based on the equations of equilibrium alone or equations of statics alone. Now you need an additional equation here over and above these three equations to solve these unknown reactive forces.

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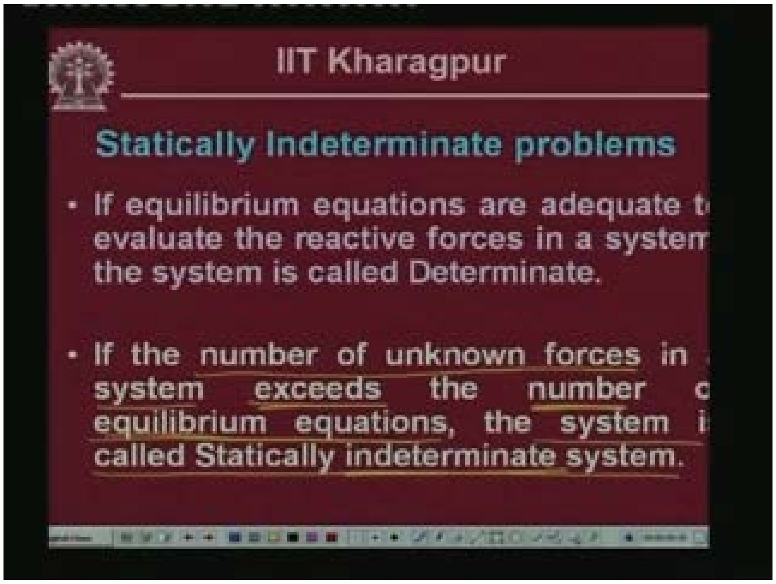

This is what is indicated here; if the number of unknown forces in a system exceeds the number of equilibrium equations then the system is called statically indeterminate system. The system is indeterminate because we cannot solve the unknown forces using the equations of equilibrium but then it does not mean we cannot evaluate the unknown forces for those indeterminate systems so for the solution of this indeterminate systems, will have to find out ways by which we can generate additional equations from which we can solve this particular unknown reactive forces.

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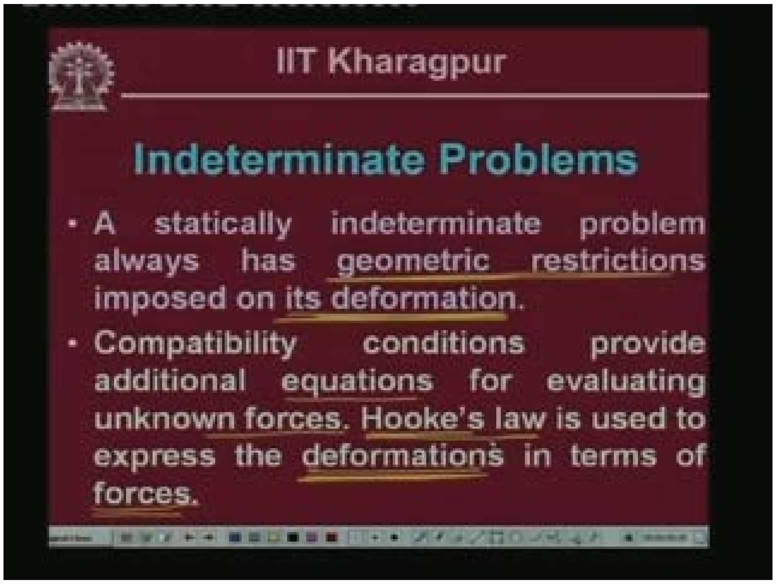

So the indeterminate problems are the ones in which a statically indeterminate problem always has geometric restrictions imposed on its deformation. In fact from this, the compatibility criteria comes in, the indeterminate system always gives rise to geometric restrictions and these geometric restrictions when expressed mathematically gives rise to the equation of compatibility. We have seen that we have three equations of equilibrium which has statical equations of equilibrium, over and above if we write down the equations of compatibility which do arise from this geometric restrictions for the deformation, this equation of compatibility along with the equations of equilibrium will lead us to the system of equations from which you can solve the unknown reactive forces in the indeterminate system.

Now you can distinguish between a determinate system and indeterminate system. Once again a determinate system is the one for which the unknown reactive forces can be evaluated based on the equations of equilibrium. And for indeterminate systems we cannot evaluate the unknown reactive forces based on the equations of equilibrium alone and for that we need to have additional equations depending on the number of unknown reactive forces you have and those additional equations do generate from the equations of compatibility. And the equations of compatibility can be written down in terms of the geometrical constraints of the restrictions which we impose in terms of deformation.

So, compatibility conditions provide additional equations for evaluating unknown forces and thereafter we can adopt this Hooke's law to express the deformation in terms of forces. Because we first write down the compatibility criteria based on the deformation and then we adopt Hooke's law to relate the deformation to the forces and finally we get equations from these compatibility criteria which help us to solve the unknown reactive forces.

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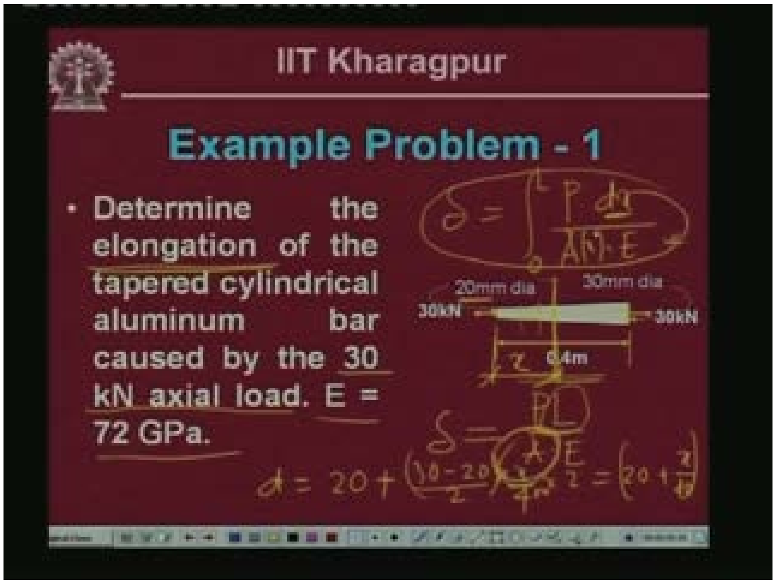

Now let us look into the problems where such indeterminacy comes in but before that let us look into some example problems which we already discussed. Determine the elongation of the tapered cylindrical aluminum bar caused by the 30 kilo Newton axial load and the value of e is given over here and in this particular problem, it is given that the diameter of this bar at this place is 20 mm and the diameter at this end is 30 mm and over the length of 400 mm it is varying gradually.

Now our job is to find out the deformation to find out the elongation of the gradually varying member. And if you remember we have discussed how to compute the elongation in such members since the cross section is varying at every position. Hence the cross sectional area is not constant and we cannot compute directly delta from the expression P_l by ae, where a is no longer a constant parameter over the length l. Hence what we need to do is that we got to compute delta is equal to $\int$over the length of the member as P_dx over the small segment. We say the area which we write as a function of x into e and from this expression we compute the value of delta.

Now if we take a section which is at a distance of x from the left end, then the diameter at this particular section d is equal to 20 plus 30 so 30 minus 20 is the value at the end. So at this particular point it will be in the ratio of x to 400 so this is 30 minus 20 into x by 40 and 30 minus 20 by 2 and twice of that since on this side we have an extra and on this side we have an extra about 20. This is 2, so 20 plus 30 minus 20 into x by 400. This is nothing but is equal to 20 plus x by 40. This is the diameter at this cross section which is at a distance of x from the left end. Let us compute delta for this particular case.

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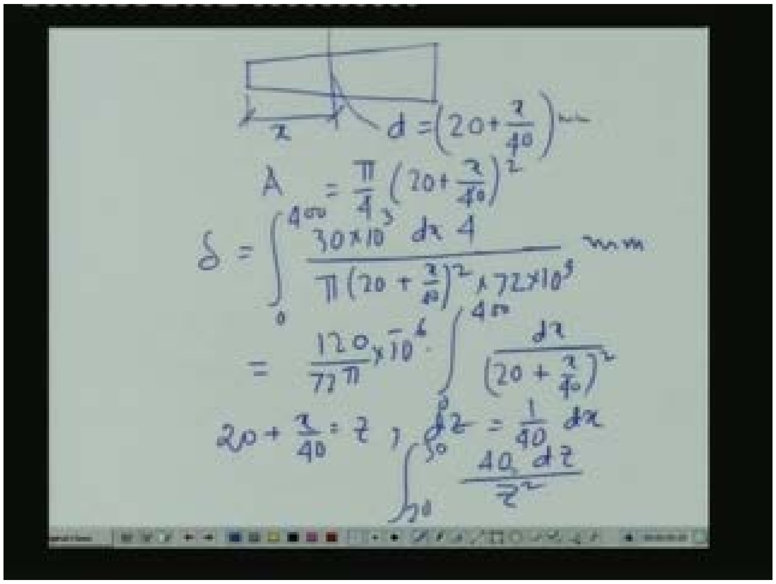

So we have the bar which is tapered and we have just seen that at a cross section which is at a distance of x, the diameter here d is equal to 20 plus x by 40 mm hence the cross sectional area a is equal to pi by 4 into 20 plus x by 40 square and delta is equal to ∫02400 the load is 30 kilo Newton. So 30 into 10 cube so much of Newton dx by a which is pi 20 plus x by 40 square into 4pi by 2 is equal to 4 into e which is 72 Gpa so 70 into 10 to the power 9 so much of mm and this if we compute this will give rise to is equal to 120 by 72pi(10 to the power minus 6) ∫0 to 400. This is dx by 20 plus x by 40 square. If we substitute 20 plus x by 40 as I said, we get a dz as 1 by 40 dx and when x is 0, z is equal to 20 and when x is equal to 400 z is equal to 30 so it varies from 20 to 30. This ∫gets transformed to this particular part as ∫20 to 30 dz 40 into dz by z square. This is nothing but minus 1 by z∫ which is 20 to 30 eventually. Then this comes as is equal to delta.

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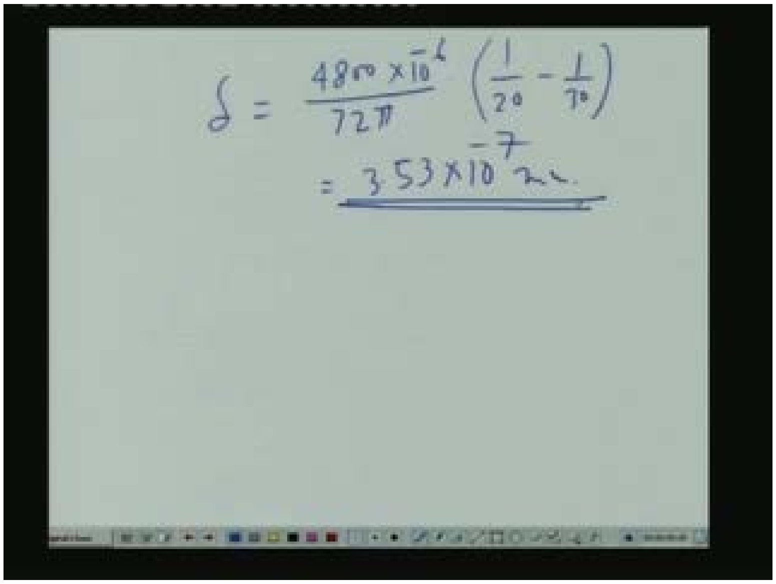

If we compute the whole thing it is 4800(10 to the power minus 6) by 72pi (1 by 20 minus 1 by 30 is equal to 3.53(10 to the power minus 7). This is the value of the delta which we get from the competition for the value of the delta for the varying diameter.

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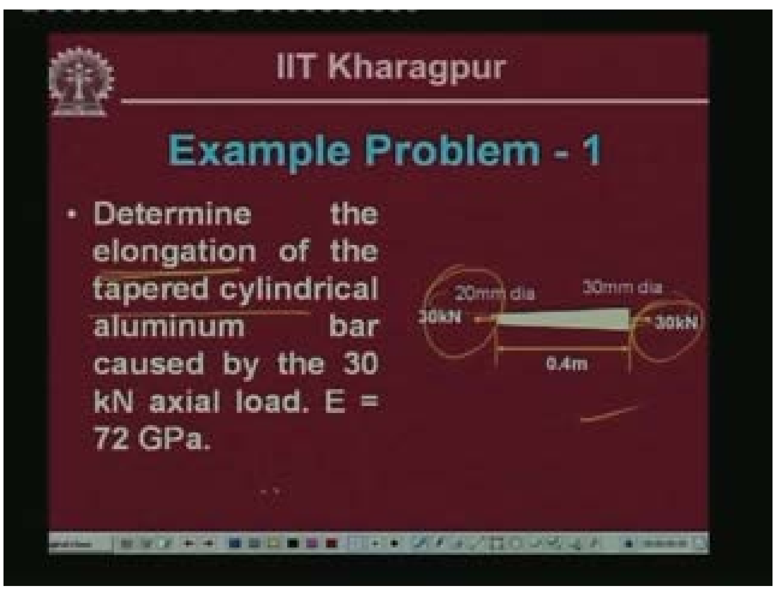

So, for a member which is tapered and if we know the diameters at its two ends then we can compute the amount of elongation the member will be undergoing.

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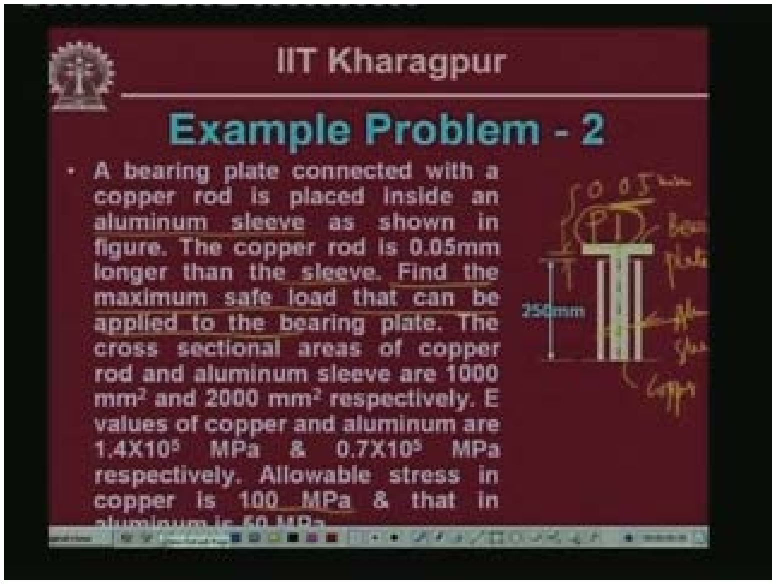

The next example is in terms of the indeterminate problem. Now here a bearing plate connected with a copper rod is placed inside an aluminum sleeve as shown in figure so here this is the bearing plate and this bearing plate is connected with a copper rod going inside so this is the copper rod and this is the aluminum sleeve. This is placed inside an aluminum sleeve as shown in figure the copper rod is 0.05 mm longer than the sleeve so this gap is 0.05 mm so the copper rod is longer than this sleeve by 0.05 mm.

Now we will have to find the maximum safe load that can be applied to the bearing plate. On this plate the maximum load P that can be applied has to be computed. The cross-sectional areas of copper rod and aluminum sleeve are 1000 mm square and 2000 mm square e values of copper and aluminum are 1.4(10 to the power 5) MPa and 0.7(10 to the power 5) MPa. Allowable stress in copper is 100 MPa and that in aluminum is 50 MPa. The maximum permissible stress in aluminum and copper are given. The cross-sectional area for the copper and the aluminum sleeve are given. Now our job is to compute that what will be the value of P which can be safely transferred on top of this bearing plate. Let us look into the free body diagram of this particular system.

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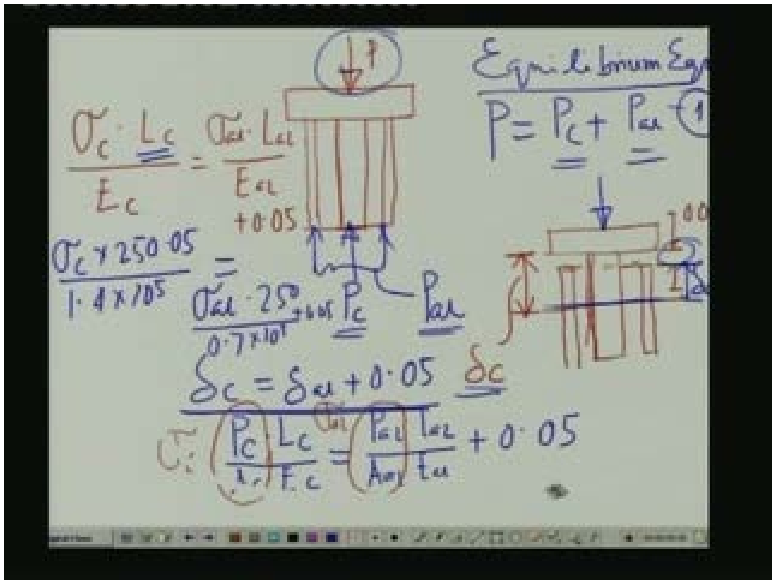

If we cut off the sleeve at some distance from the top, then this is the bearing plate the copper rod is attached with this. And this is acted on by load p now when the rod will get compressed and cover up the distance 0.05 mm. Eventually it will touch the sleeve so this bearing plate along with the copper rod it has compressed cover up covered up that 0.05 mm gap between the sleeve and the copper rod. That much of compression as or the deformation has undergone into the copper plate and eventually the bearing plate is going to touch on the sleeve and when the bearing plate will be touching the sleeve, then the load which is acting on the bearing plate will be transmitted to the sleeve.

So at this free body diagram, if we now transfer the reactive forces, the reactive forces go as acting on this. They are the force in this copper bar and the forces in the combined form in the sleeve. These two together, let us call that the load in the aluminum. We have these two forces P_c and $P_{aluminum}$ which are being generated because of the externally applied load P. So if we write down the equilibrium equation so equilibrium of this particular system is that P is equal to P_c plus $P_{aluminum}$. This is equation 1.

In this particular system we cannot have any other equilibrium equation. So we have only one equation of equilibrium and there are two unknown forces and they are P_c and P_a the two unknown forces and one single equation. Hence we cannot solve this particular system unless, we have an additional equation and this additional equation can be generated from the comparative criteria in terms of the deformation and then we can solve for these two unknown forces. So now let us look into what is going to be the comparative equation in this particular case. Let me look into another free body diagram of the top part of the system.

This is the bearing plate along with a copper rod and the sleeve is in this location and this is the gap since copper rod is 0.05 mm greater than the sleeve. This is an excess rated from and let say after deformation, the bearing comes in this particular position hence the deformation that copper rod undergoes is this much this we call as $delta_c$ and in the process when copper rod deform to this extend and if this is the position where the bearing plate has come, then the deformation which is existing in the or which has occurred in the aluminum is this much this we call as $delta_{aluminum}$.

From this particular configuration, we can write $delta_c$ is equal to $delta_{aluminum}$ plus 0.05 mm. So this is the criterion which is getting developed from the compatibility of the two systems that initially when the load is applied on this bearing plate, first the bearing plate along with the copper rod has to be deformed to the extent of 0.05 mm. Then the load gets transferred on to this sleeve. Then the sleeve deforms and this is the final level of the deformation and if these total deformation we call as $delta_c$ then $delta_c$ consist of two parts; one is this 0.05 mm gap which has to be made up by the copper tube at the rod along with the deformation of this sleeve. This is the compatibility equation.

Now if we apply Hooke's law to this deformation equation we can get additional equation from which you can evaluate as P_c and $P_{aluminum}$. Now $delta_c$, we can write in terms of the load and cross sectional area which is PL by ae. Let us call the P_c is the force which acting in the copper rod times length of the copper is L_c by area ac into ec is equal to P in the aluminum times l of aluminum by a of aluminum into e of aluminum plus 0.05. Now in this particular case since the values of limiting stresses are given, let us compute the values in terms of stresses now as we know P by the cross sectional area is the stress. This particular part P_c by a c we can write as the $sigma_c$ the normal stress in the copper rod and P_l by a_l, we can write as $sigma_{aluminum}$ which is the normal stress in the aluminum sleeve.

Hence this compatibility equation in terms of stresses we can write as $sigma_c$ into L_c by e_c, this is equal to sigma $_{aluminum}$ into length of the aluminum by e of aluminum plus 0.05. Now if we substitute the values the value of L_c the length of the copper tube the length of the aluminum sleeve is given as 250 mm and since the copper rod is 0.05 mm longer than the aluminum sleeve, the length of the copper rod is 250 plus 0.05 so 250.05. $sigma_c$(250.05) is the length of the copper rod by e of copper rod which is 1.4(10 to the power 5); this is equal to $sigma_{aluminum}$ into the length of the aluminum is 250 by e which 0.7 into 10 to the power 5 plus 0.05.

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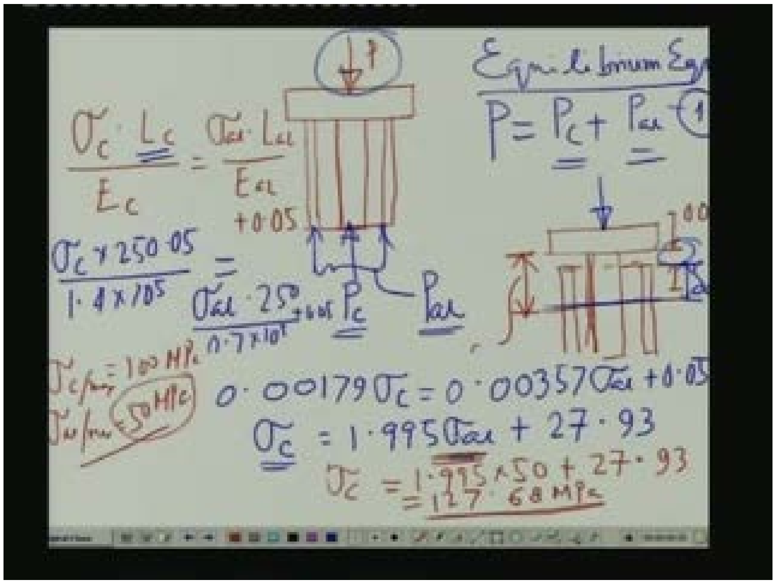

Now this if we compute this comes as 0.00179 into sigma_{c} is equal to 0.00357 $\text{sigma}_{\text{aluminum}}$ plus 0.05 or sigma_{c} is equal to 1.995 into $\text{sigma}_{\text{aluminum}}$ plus 27.93. Now this is the relationship between the stresses in the copper rod to the stress in the aluminum rod. Here the limiting values of the stresses in copper and aluminum is given now the maximum allowable stress in copper rod is 100 MPa $\text{sigma}_{\text{cmax}}$ is 100 MPa as $\text{sigma}_{\text{aluminum}}$ allowable or max as we call it is equal to 15 MPa. Hence if we take the value of allowable stress in aluminum as 50 MPa and if you substitute it here then we get the value of sigma_{c} is equal to 1.99 into 50 plus 27.93 eventually and the value of sigma_{c} comes as 127.68 MPa. Since the maximum allowable stress in copper is 100 MPa, we cannot afford to go for stress up to this level.

We cannot apply the stress in the aluminum up to 50 MPa, because if we go up to a stress level of 50 MPa in aluminum and the stress level that is expected in the copper rod is going beyond the allowable stress of 100 MPa. Hence we cannot allow the stress in aluminum sleeve to go up to its limiting value which is 50 MPa. That means we will have to limit the stress in the copper rod up to 100 MPa and check on how much stress it can generate in the aluminum sleeve from which we can evaluate the safe load. Hence if we go the other way limiting the value of sigma_{c} to 100 MPa we need to see how much we can get as the value of $\text{sigma}_{\text{aluminum.}}$

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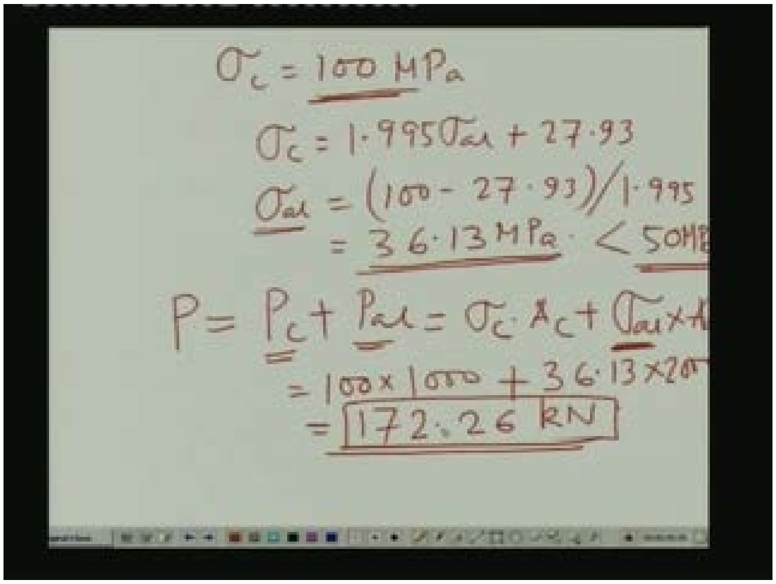

So we limit the value of sigma$_c$ as is equal to 100 MPa and from the relationship which we have sigma$_c$ is equal to 1.995 sigma$_{aluminum}$ plus 27.93. The value of sigma$_{aluminum}$ comes as 100 minus 27.93 by 1.995 and this is is equal to 36.13 MPa. So by allowing the copper rod to go up to its limiting value of 100 MPa, the maximum stress that can be generated in the aluminum sleeve is 36 MPa which is well within the allowable limit of the 50 MPa stress. Hence the safe load that can be transfer on the bearing without causing any distress to the member is equal to P$_c$ plus P$_{aluminum}$ safe load in the copper and the aluminum copper rod and the aluminum sleeve, this is equal to sigma$_c$ into area of the copper rod plus sigma$_{aluminum}$ which is the maximum we can allow multiplied by the area of the aluminum. And this is is equal to 100 multiplied by the cross sectional area of the copper which is 1000 mm square plus sigma allowable is 36.13 for aluminum into 2000 mm square and these eventually gives 172.26 kilo Newton. So the load which can be applied on the bearing must be lesser than or is equal to 172.26 kilo Newton and thereby the maximum stress that can be generated in the copper rod is 100 MPa but the aluminum sleeve will not go up to 50 MPa. But it will be well within 50 MPa and thereby the whole system will be safe.

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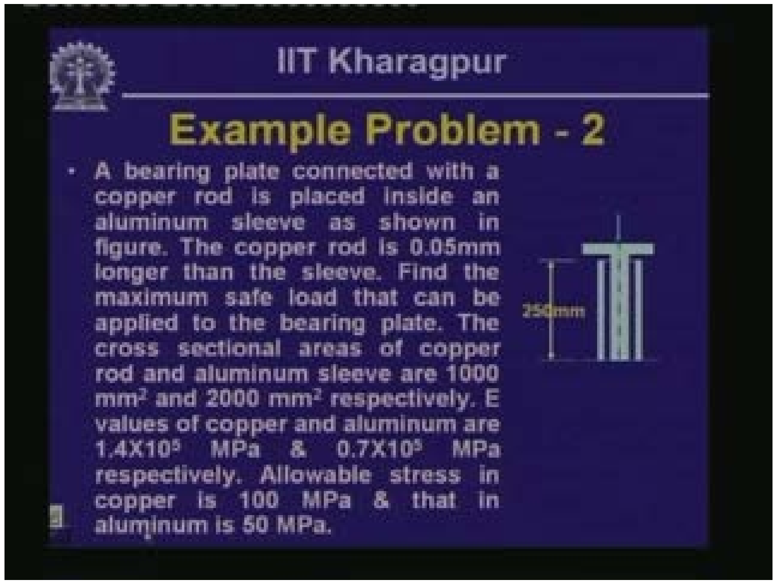

Let us look into another example of such indeterminate system.

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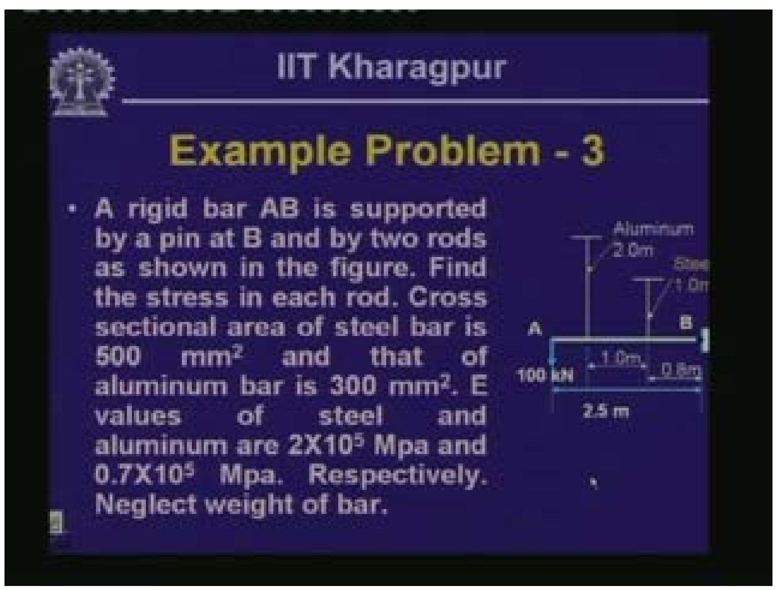

Here AB is a rigid bar, AB is supported by A pinned at B and by two rods. These bars is pinned at B is supported by two rods which are at a distance of 0.8 meter and 1.8 meter from B. This rod is a steel rod of length 1m and this is aluminum rod of length 2 meter and this bar is subjected to a load of 100 kilo Newton at the point A. Now we will have to find out the stress in each bar or each rod and the cross sectional area of those bars are given steel bar is 500 mm square and that of aluminum bar is 300 mm square and the value of e are given as 2(10 to the power 5) and as 0.7(10 to the power 7) MPa for the steel and the aluminum and let us assume that the weight of this bar is negligible. Let us write down the free body of this system and write down the equations of equilibrium and absorb what happens.

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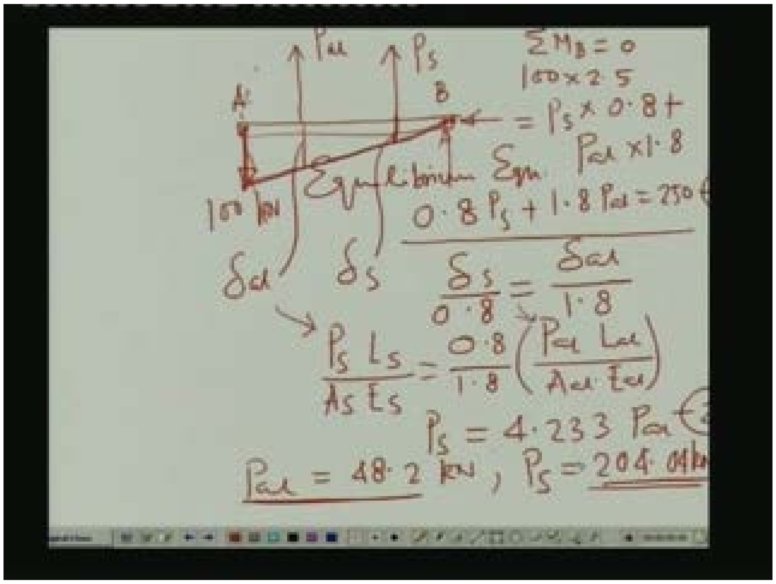

If we take the free body this is the rigid bar pinned at this end acted on by two rods they are steel and aluminum and at these end we have the reactive forces for the pin and at these end the load 100 kilo Newton is acting. Now if we like to find out the forces let us take the moment of now here we have 1, 2, 3, 4 unknown parameters and we have three equations of equilibrium. Hence we cannot solve it means statically determinate one or using the equations of equilibrium. Hence we will have to have an additional equation which are generating from the equation of compatibility and the equations of equilibrium if we take the moment about B, then we have 100 moment about B is equal to 0 so we have 100(2.5) is equal to P_s(0.8) plus $P_{aluminum}$ (1.8). So the equilibrium equation is 0.8P_s plus 1.8 $P_{aluminum}$ is equal to 250. So this is the equilibrium equation.

Now from the compatibility if this load is acting at this end and this is the pinned end this will undergo; the bar will move in this particular form and this will move in the form of a circle with the center of this particular point. Now this being a smaller deformation, we consider this arc as a straight length; hence this is a triangular form. At these locations this bar will undergo deformation and let us call this deformation as $delta_s$ and the deformation here as $delta_{aluminum}$. Hence the compatibility is that delta from this particular triangular configuration.

We can write deltas by 0.8 is equal to $delta_{aluminum}$ by 1.8. So this is the compatibility relation and delta if we write in terms of P in terms of using the Hooke's law which is P_l by a e. So this is P_s L_s by A_s e_s. That is for steel is equal to 0.8 by 1.8 into $P_{aluminum}$ length aluminum divided by $A_{aluminum}$ into $e_{aluminum}$ and if we substitute this values we get the relationship between the load in P_s and the force in $P_{aluminum.}$ Eventually we get P_s is equal to 4.233 $P_{aluminum}$, this is the second equation. Now from these two equations one and two we can solve for P_s and $P_{aluminum}$ and if we substitute the values of P_s over here, we get the values of $P_{aluminum}$ as is equal to 48.2 kilo

Newton and thereby P_s is equal to 204.04 kilo Newton. These are the forces acting in these two rods and once we know the forces that are acting then the stresses can be known.

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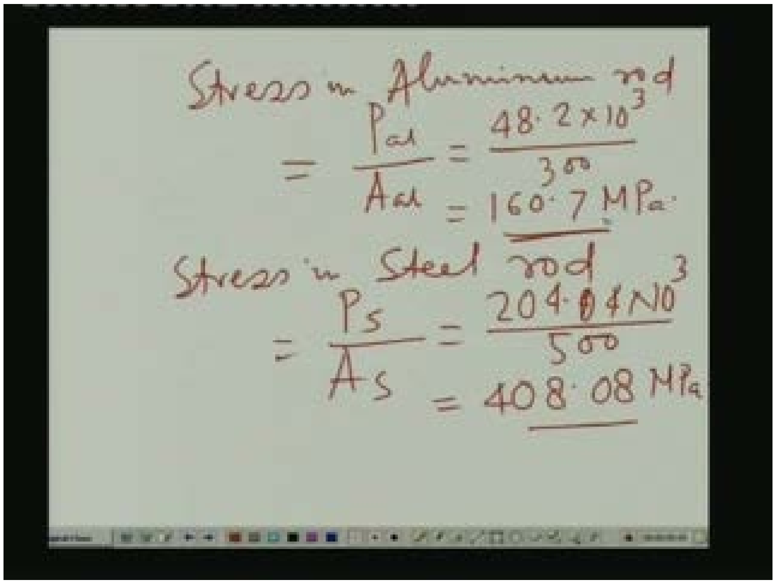

The stress in the aluminum bar stress in aluminum rod is equal to $P_{aluminum}$ by $A_{aluminum}$ which is cross sectional area of aluminum which is equal to 48.2 is the load 10 cube so much of Newton by 300 is the cross sectional area and these eventually comes as 160.7 MPa. And stress in steel bar or steel rod this is equal to P_{steel} by A_{steel} is equal to 204.40(10 cube) so much of Newton by 500 and this comes as 408.08 MPa. Here, though the stresses level of stresses appear to be little higher may be beyond the limiting capacity of this material. However, theoretically we are computing what are the values of the stresses if the limiting values are given, we can always compare and we can say whether this is safe or unsafe, whether the system will be able to withstand this load or not that we can always compute.

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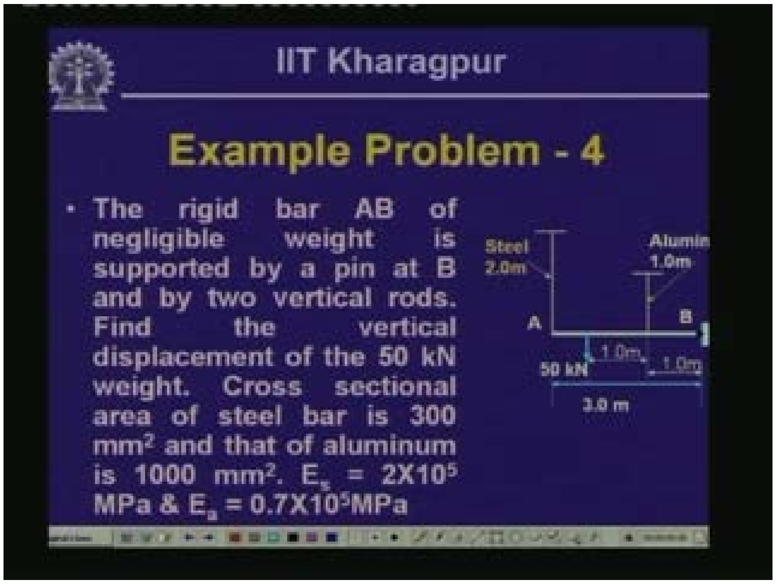

Now we have another problem which is a similar type but of little variation that the rigid bar AB of negligible weight is supported by a pin at B and by two vertical rods of aluminum and steel; we will have to find out the vertical displacement of the 50 kilo Newton weight at this location, how much displacement it undergoes etc, the parameters of these members are given.

To summarize the whole thing we can say; in this particular lesson we included the concept of strains in a three dimensional stress body and concept of determinate and indeterminate systems. Now, we clearly know what we really mean by determinate system and indeterminate system and thereby how to arrive at the compatibility criteria to solve the unknown reactions in indeterminate systems. Also, we know some examples to demonstrate the evaluation of forces in indeterminate system.

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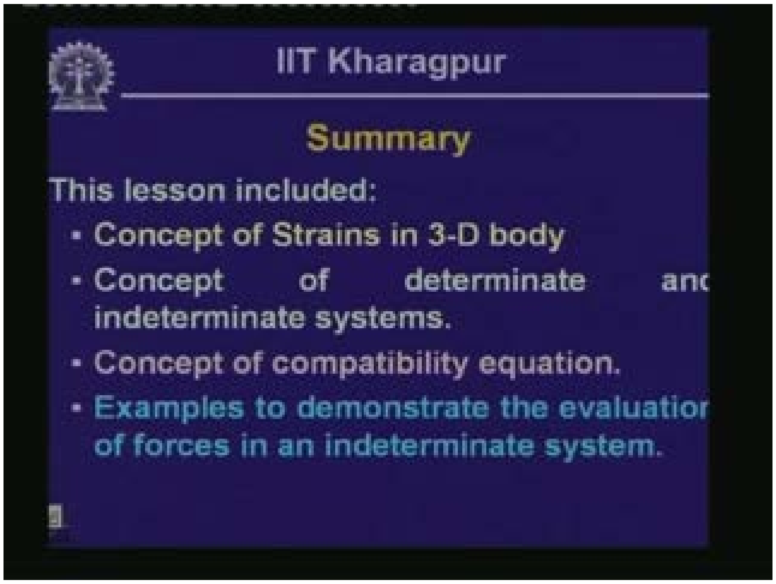

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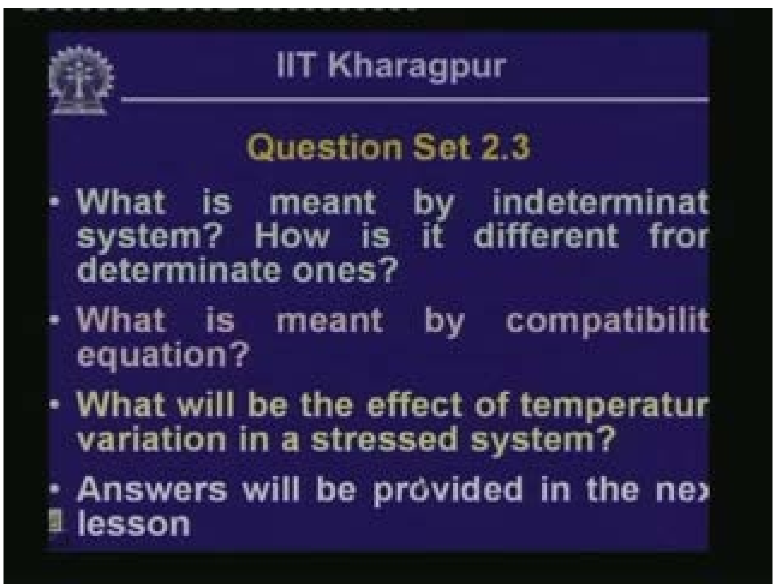

Here are some questions to be answered:

What is meant by indeterminate system and how is it different from determinate ones? What is meant by compatibility equation and what will be the effect of temperature variation in a stressed system?