


**Strength of Materials**  
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**Lecture - 6**  
**Analysis of Stress - V**

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Welcome to the Lesson 6 of the course on Strength of Materials. In this particular lesson we are going to discuss certain aspects of analysis of stress.

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### Specific Instructional Objectives

- After completing this lesson one will be able to:
- Understand the concept of Stresses in Polar co-ordinate system.
- Understand the concept of stress for axi-symmetric bodies.

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Once this particular lesson is completed one should be able to understand the concept of the stresses in polar coordinate system, you will be able to understand the concept of the stress for **axi-symmetric** bodies which eventually can be derived from this polar coordinate system of stresses. We will also look into how to evaluate stresses at different points.

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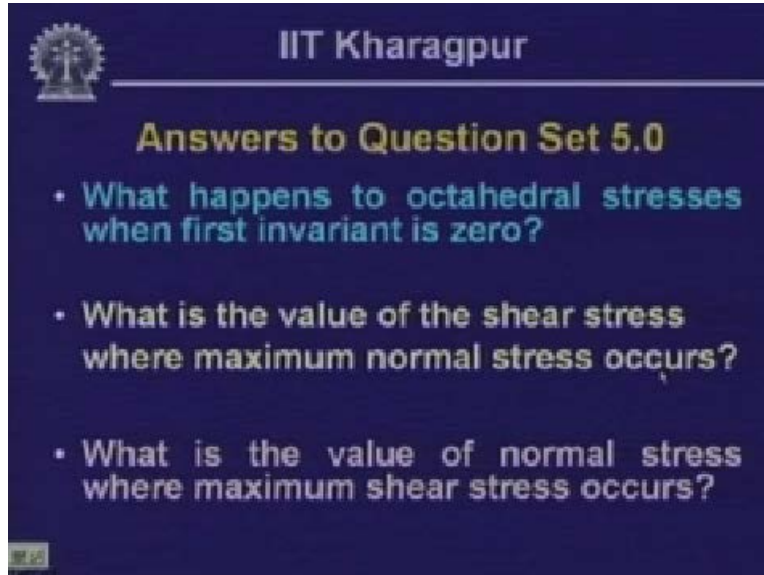
### Scope

- This lesson includes:
  - Recapitulation of previous lesson.
  - Evaluation of stresses in Polar co-ordinate system.
  - Examples for evaluation of stresses.

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This particular lesson includes the recapitulation of the lessons we discussed already such as evaluation of stresses in polar coordinate system and examples for evaluation of stresses at particular point in the stress body.

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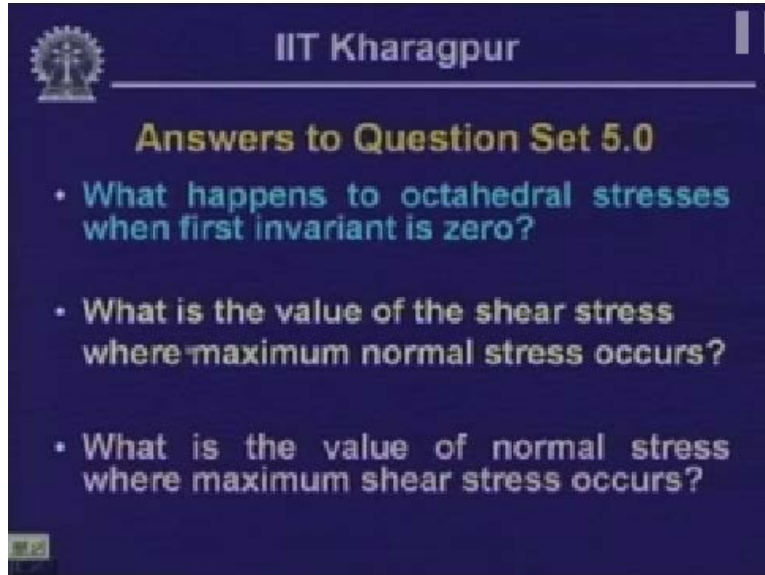
Some questions to be answered:

What happens to octahedral stresses when first invariant is 0?

Now let us look into octahedral stresses. The normal stresses on the octahedral planes which we had calculated  $\sigma_{oct}$  is equal to  $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ . We had defined the octahedral planes as the planes which are equally inclined with the principal axis reference system. And thereby the stresses which are acting  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  in the rectangular stress system and the octahedral stress as defined is  $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$  which is the summation of the normal stresses in three dimensional stress system, this is called as first invariant.

If you remember  $\tau_{oct}^2$  is equal to  $\frac{2}{9}(\sigma_1 + \sigma_2 + \sigma_3)^2 - \frac{2}{9}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$ . Hence as it has been asked if  $(\sigma_1 + \sigma_2 + \sigma_3)$  is equal to 0 then eventually the normal stress on octahedral plane,  $\sigma_{oct}$  is equal to 0. So, if the first invariant is 0 then the octahedral normal stress is equal to 0, only shear stress will exist on the octahedral plane.

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Now the second question which was posed was:

What is the value of the shear stress where maximum normal stress occurs?

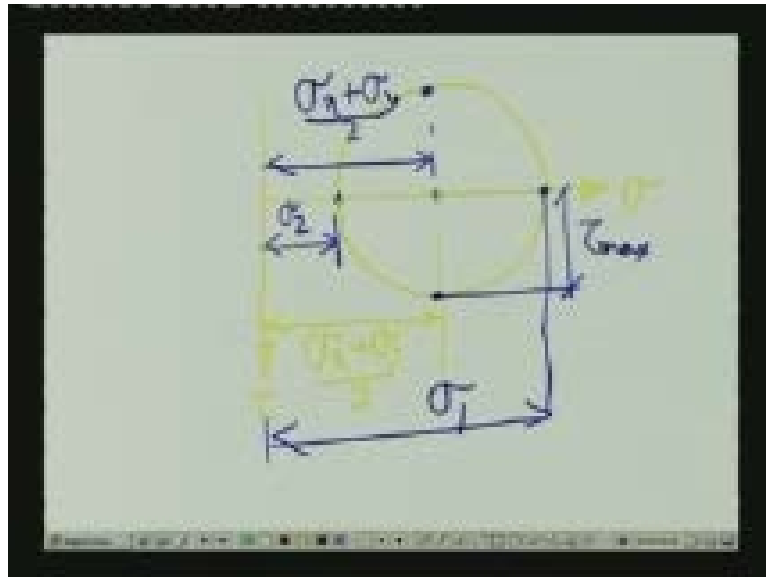
The third question is:

What the value of normal stress is where maximum shear stress occurs?

Probably these two questions can be answered through the same diagram.

If you remember, last time we had shown how to plot the Mohr's circle. This is the sigma-axis and this is the  $\tau$ -axis. Now, if we draw the Mohr's circle of stress, the centre of the Mohr's circle from  $\tau$ -axis is given as  $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$  and the maximum value of the normal stress at this point in this particular plane we normally designate as  $\sigma_1$ , and the minimum value of the normal stress is  $\sigma_2$ . So, if you note in these two planes where the maximum and minimum normal stress acts the value of the shear stresses are 0. So the plane where the maximum normal stress acts there the value of the shear stresses are 0 and these planes are called as principal planes.

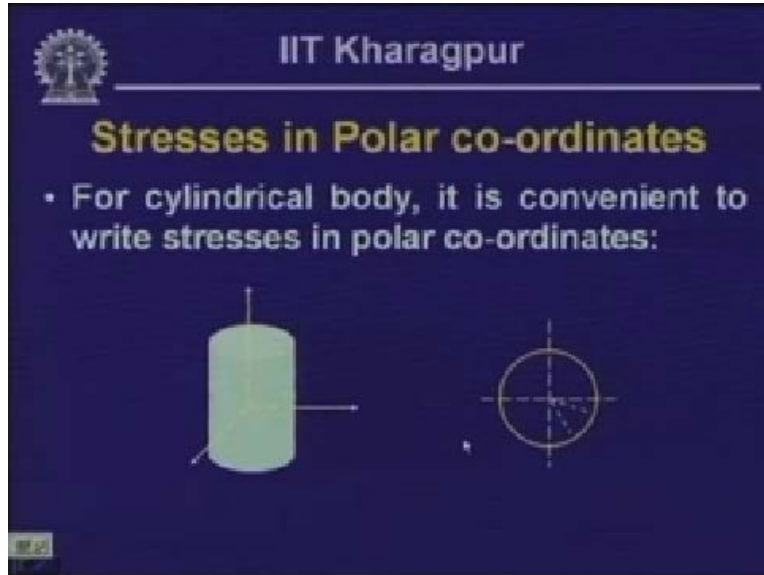
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The third question was:

What is the value of the normal stress in the plane where the maximum shear stress acts? These are the planes where the maximum shear stress acts. This is the maximum positive shear and this is the maximum negative shear. If you note, the maximum shear the value of the shear stress is that of the radius of the Mohr's circle is  $\tau_{max}$ . If you note here, in this plane we have the normal stress which is equal to this particular magnitude  $(\sigma_x \text{ plus } \sigma_y)$  by 2. So, from this diagram it self you can answer both the questions. The planes where the normal stresses are at maximum the shear stresses are 0 and the planes where shear stresses are maximum there normal stresses exist and the value of normal stresses are  $(\sigma_x \text{ plus } \sigma_y)$  by 2.

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Let us look into aspects of how to evaluate stresses in polar coordinates?

So far we have discussed about the rectangular stresses in a body where we have assumed that the boundaries are straight boundaries. Now there are several cases where other than the straight boundaries we get problems, we get structural elements where the surfaces are curved and to represent the stresses on those curved surfaces.

It is ideal to represent them in terms of a coordinate system which can be expressed in terms of radius and the rotational angle  $\theta$  which we call as cylindrical axis or polar reference axis. If we have cylindrical body of this particular form, in this we have earlier seen the reference axis system as x, y, and z.

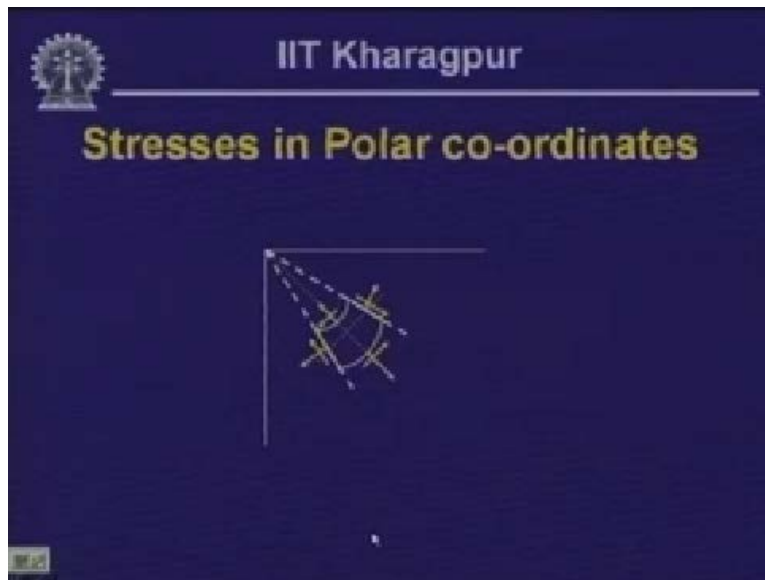
Now if we select a point here on this body, let us say this is P, the coordinate of this particular point can be described by these coordinates x, y and z. Also, this particular point can be represented through another reference system which if we project this point on this xz-plane and draw a line over here and if we define this particular angle as  $\theta$  and its projected length as vector r and this distance as y then the coordinate of this particular point can be expressed as a function of r,  $\theta$ , and y. This is with reference to the Cartesian system x, y, and z and this particular reference is with reference to the polar coordinate system which is r,  $\theta$  and y.

Now, if we look into the plan of this or the cross section of this, then if we draw two radial lines from the centre, let us say that this particular radial line is at an angle  $\theta$  then these two radial lines make a small angle of  $d\theta$ . Now if we take a small element over here and try to look into the stresses that will act then we will have two planes, this particular plane normal to this plane is in the direction of the radius; we call this as the r-plane. Over these we will have the stresses which are acting as the normal stress and so is this which we call as  $\sigma_r$ .

The stress normal to this surface acting along the circumferential direction is called as  $\sigma_\theta$ . Also, we will have the tangential stress on this plane as well as on this plane and we will have tangential stress in the radial direction as well. This tangential stress which we defined as the shearing stresses is tau on the r-plane acting in the direction of  $\theta$ , we call this as  $\tau_{r\theta}$ .

The other tangential stress on the  $\theta$ -plane is  $\tau_{\theta r}$ , eventually  $\tau_{r\theta}$  is equal to  $\tau_{\theta r}$ . So we define the state of stress on this particular body at a particular point is equal to  $\sigma_r$ , the radial stress, the tangential stress is  $\sigma_\theta$ , and the shearing stresses  $\tau_{r\theta}$ . If a particular body at a point is subjected to these stresses then how we arrive at the equations for equilibrium.

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Here I have tried to represent state of stress at a particular point in a stress body, and as we have designated that this particular stress which is acting normal to the r-plane, let us call that as  $\sigma_r$ . Let us assume that this particular distance is dr. The first radial line let us call, this is at distance of  $\theta$ , and the small angle made by these two radial line is  $d\theta$ . The radial distance from the origin to the first part of the element, let us call that as r.

Hence the stresses which are acting  $\sigma_r$  is the normal stress, the tangential stress is  $\tau_{r\theta}$ , and the circumferential normal stress as  $\sigma_\theta$ . So the stress which is acting on this surface, since it is at a distance dr, while deriving the equilibrium equations at the particular point with reference to the rectangular Cartesian axis system if you try to find out the stress at two different points then there is incremental stresses which is  $\sigma_r$  plus  $(\partial\sigma_r$  by  $\partial r)dr$ . Likewise, the tangential stress which is the shearing stress is equal to  $\tau_{r\theta}$  plus  $(\partial\tau_{r\theta}$  by

$\partial r)dr$ . On this we have  $\sigma_\theta$ , so on this particular surface the normal circumferential stress is equal to  $\sigma_\theta$  plus  $(\partial \sigma_\theta \text{ by } \partial \theta)d\theta$ .

The shearing stress which is acting on this surface is  $\tau_{r\theta}$  is equal to  $\tau_{r\theta}$  plus  $(\partial \tau_{r\theta} \text{ by } \partial \theta)d\theta$ . Also, if we draw a tangent at this particular point please note that normal stress on this surface makes an angle of  $d\theta$  by 2. If we take the equilibrium of forces in the radial direction and in the tangential direction we can get the equations of equilibrium. Now on this particular plane, the area on which this particular stress acts is  $r$  plus  $dr$   $d\theta$ ;  $d\theta$  being small this particular length you can write as  $r$  plus  $dr$   $d\theta$  and this particular length as equal to  $rd\theta$ .

If we assume the thickness of this particular element perpendicular to the plane of this board as unit then area of this particular surface is equals to  $(r \text{ plus } dr) d\theta$  into 1 and this multiplied by the stress will give the force in this particular plane. Similarly the force on this particular plane is  $\sigma_r$  into  $rd\theta$  into 1.

Also, we have the tangential stresses on this surface which are in the radial direction and  $\sigma_\theta$  will have the component in the tangential direction and also in the radial direction and the component in the radial direction will be  $\sigma_\theta(\sin d\theta \text{ by } 2)$  and  $d\theta$  being small we can approximate  $\sin d\theta \text{ by } 2$  is equal to  $d\theta \text{ by } 2$ . So, if we write down the equations of equilibrium in the radial direction then the equations we get are:  $\sigma_r$  plus  $(\partial \sigma_r \text{ by } \partial r)$  into  $dr$  is the normal stress on the outer plane multiplied by area which is  $(r \text{ plus } dr) d\theta$  minus  $\sigma_r rd\theta$  (the normal stress which is acting on the first plane where  $\sigma_r$  acts is equal to the  $\sigma_r$  the stress times  $rd\theta$  into 1) is the area so this is the force.

Then the shearing forces which are acting in the  $\theta$  plane are: plus  $\tau_{r\theta}$  plus  $(\partial \tau_{r\theta} \text{ by } \partial \theta)$  acting over the length  $d\theta$  and the area is  $dr$  minus  $\tau_{r\theta} \theta$  over the area  $dr(1)$  minus corresponding to the  $\sigma_\theta$  we have  $\sigma_\theta$  plus  $\partial \sigma_\theta$  over a length  $d\theta$  acting over the area  $dr$  minus into  $d\theta \text{ by } 2$ , that sine component of that and this is minus so this is plus or  $\sigma_\theta$  is acting hence this is plus and this is minus so this is  $\sigma_\theta$  or this component called sine component is acting in the reverse direction of the radial direction so this is also negative and this is also negative so  $\sigma_\theta dr d\theta \text{ by } 2$  is equal to 0.



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$$\begin{aligned} & \left( \tau_r + \frac{\partial \tau_r}{\partial r} dr \right) (r + dr) d\theta - \sigma_\theta r d\theta \\ & + \left( \tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta \right) dr - \tau_{r\theta} dr \\ & - \left( \sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta \right) dr \left( \frac{d\theta}{2} \right) \end{aligned}$$

Hence on simplification this gives  $\sigma_r r d\theta$  plus  $\partial \sigma_r r$  by  $dr$  into  $r d\theta$ , when we multiply with  $dr$  we have plus  $\sigma_r dr d\theta$ , now this particular term when multiplied by  $dr d\theta$  since we will have term  $dr$  square. And since this being small we are neglecting that particular term, minus  $\sigma_r r d\theta$  plus  $\tau_{r\theta} dr$  plus  $\partial \tau_{r\theta}$  by  $\partial \theta d\theta dr$  minus  $\tau_{r\theta} dr$  minus  $\sigma_\theta dr d\theta$  by 2 minus  $d\theta$  square. We neglect that minus  $\sigma_\theta dr d\theta$  by 2. Now from this we find that  $\sigma_r r d\theta$  and minus  $\sigma_r r d\theta$  these two terms cancel out;  $\tau_{r\theta} d\theta$  and minus  $\tau_{r\theta} d\theta$ .

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$$\begin{aligned} & \left( \tau_r + \frac{\partial \tau_r}{\partial r} dr \right) (r + dr) d\theta - \sigma_\theta r d\theta \\ & + \left( \tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta \right) dr - \tau_{r\theta} dr \\ & - \left( \sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta \right) dr \left( \frac{d\theta}{2} \right) \\ & - \sigma_\theta dr \frac{d\theta}{2} = 0 \end{aligned}$$

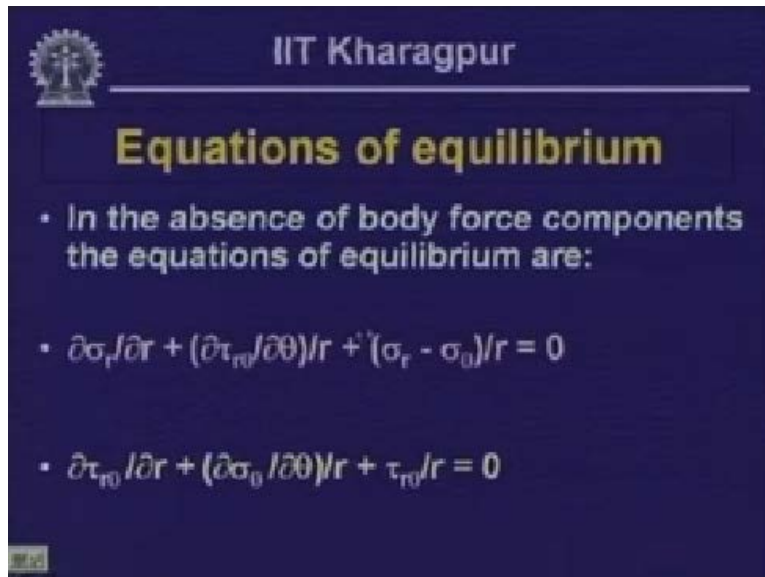
~~$\sigma_r r d\theta + \sigma_r dr d\theta + \sigma_r r d\theta + \sigma_r dr d\theta$~~   
 ~~$- \sigma_r r d\theta - \sigma_r r d\theta - \sigma_r dr d\theta - \sigma_r dr d\theta$~~   
 ~~$- \tau_{r\theta} d\theta + \tau_{r\theta} d\theta - \sigma_\theta dr d\theta - \sigma_\theta dr d\theta$~~

So we have  $\partial \sigma_r$  by  $\partial r$   $dr$   $rd\theta$  plus  $\sigma_r$   $dr$   $d\theta$  plus  $\partial \tau_{r\theta}$   $d\theta$   $dr$  and minus  $\sigma_\theta$  by 2 and  $\sigma_\theta$  by 2 if we combine them together this  $dr$   $d\theta$  and this is equal to 0 for the equilibrium of the forces in the radial direction.

If we write it down and divide the whole by  $dr$   $d\theta$  then what is left out is,  $\partial \sigma_r$  by  $\partial r$  plus 1 by  $r$   $\partial \tau_{r\theta}$  by  $\partial \theta$  plus  $(\sigma_r$  minus  $\sigma_\theta)$  by  $r$  is equal to 0. This is the equilibrium equation in the radial direction. Similarly, if we take, equilibrium of the forces in the circumferential direction then we can write down the equation as  $(\sigma_\theta$  plus  $\partial \sigma_\theta$  by  $\partial \theta$   $d\theta$ )  $dr$  minus  $\sigma_\theta$   $dr$  plus  $(\tau_{r\theta}$  plus  $\partial \tau_{r\theta}$  by  $\partial r$ )  $(r$  plus  $dr)$   $d\theta$  minus  $\tau_{r\theta}$   $rd\theta$  is equal to 0.


This gives the expression finally after simplification as,  $\partial \tau_{r\theta}$  by  $\partial r$  plus  $(1$  by  $r)$   $\partial \sigma_\theta$  by  $\partial \theta$  plus  $\tau_{r\theta}$  by  $r$  is equal to 0. So these are the two equations, the equation  $\partial \tau_{r\theta}$  by  $\partial r$  plus  $(1$  by  $r)$   $\partial \sigma_\theta$  by  $\partial \theta$  plus  $\tau_{r\theta}$  by  $r$  is equal to 0 and  $\partial \sigma_r$  by  $\partial r$  plus 1 by  $r$   $\partial \tau_{r\theta}$  by  $\partial \theta$  plus  $(\sigma_r$  minus  $\sigma_\theta)$  by  $r$  is equal to 0. These are represented in terms of the stresses as  $\sigma_r$ ,  $\tau_{r\theta}$ , and  $\sigma_\theta$ . They are written in terms of the polar reference axis as  $\theta$  and  $r$ .

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These are the equations of equilibrium and in this particular case we have not accounted for the components of the body forces. Both in the radial and circumferential direction we have neglected the body forces. Hence we have  $\partial \sigma_r$   $\partial r$  plus 1 by  $r$   $(\tau_{r\theta}$  by  $\partial \theta$   $\sigma_r)$  plus  $(\sigma_r$  minus  $\sigma_\theta)$  by  $r$  is equal to 0; this is the first equation of the equilibrium.  $\partial \tau_{r\theta}$  by  $\partial r$  plus 1 by  $r$   $(\partial \sigma_\theta$  by  $\partial \theta)$  plus  $\tau_{r\theta}$  by  $r$  is equal to 0 is another equation of equilibrium. These are the two equations of equilibrium which are explained in reference to the polar coordinate system.

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### Stresses for axi-symmetric bodies

- If an axi-symmetric body is loaded symmetrically, the stress components do not depend on  $\theta$ . The shear stresses do not exist since the deformations are symmetric. Hence the equilibrium equations reduce to:

$$\frac{\partial \sigma_r}{\partial r} + (\sigma_r - \sigma_\theta)/r = 0$$

Thereby from this we can find out the stresses that are referred in axi-symmetric body. We encounter several kinds of structural elements where the stresses or the boundaries may not be perfectly straight; you can have curved boundary over which there could be stress which can be radial stress or which can be described by the stress  $\sigma_r$  and  $\sigma_\theta$  and if the loading on such body is symmetrical then we have a perfectly symmetrical body or loading is perfectly symmetrical in its vertical direction.

Then if we take any cross section or any longitudinal section for that matter, if we take section through the diameter at every section the level of the stress will be the same. Hence it shows that the stresses at any of these sections are independent on wherever we take the section, so the stresses are independent of  $\theta$  and these kinds of bodies are called as axi-symmetric bodies. That means these bodies are perfectly symmetrical with reference to the vertical axis. For such bodies if the loading also is vertical and symmetrical then any cross section we take, at each section the same state of stress exists. This kind of stress and the body we call as axi-symmetry bodies.


The bodies which are perfectly symmetrical referring to its vertical axis we call them as axi-symmetry bodies and for axi-symmetry bodies if they are loaded symmetrically then the stress components do not depend on  $\theta$ . Therefore, any longitudinal section we take the shear stress components are absent because we have the symmetric deformation and thereby the shear stress components do not exist. If we take the absence of the shearing stresses then the equilibrium equation reduces to  $\frac{\partial \sigma_r}{\partial r}$  plus  $(\sigma_r$  minus  $\sigma_\theta)$  by  $r$  is equal to 0 where only the normal stresses exist and the shearing stresses are absent.

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### Example Problem - 1

- The state of stress at a point is shown in the figure. Determine the normal and shear stresses acting on the vertical plane using transformation equations.



The diagram shows a stress element with a horizontal top surface and a vertical right surface. The top surface is subjected to a normal stress of 60 MPa (tension). The right surface is subjected to a normal stress of 40 MPa (tension) and a shear stress of 30 MPa (shear). An inclined plane is shown at a 45-degree angle to the horizontal, with a normal stress of 50 MPa (tension) and a shear stress of 40 MPa (shear).

We have tried to give an outline of the state of stress if we refer them in terms of the polar coordinate system. Earlier in a stress body we have looked into that, if we have the rectangular components of the stresses  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  we looked into how to evaluate the stresses at different points and on different planes.

Now, if we try to represent the stresses on any plane in a polar coordinate system where the normal stresses  $\sigma_r$ ,  $\sigma_\theta$ , and shearing stress  $\tau_{r\theta}$  exist we have seen how to write down the equations of equilibrium.

Here if you look in this particular point in the stress body the stresses given are; on a horizontal plane the normal stress is 60 MPa, on a particular plane which is inclined with reference to this horizontal plane is 45 degrees, the normal stresses are 50 MPa; the shearing stress is 40 MPa; and on this horizontal plane we have shearing stress as 30 MPa. What we will have to compute is the normal and shear stresses which are acting on the vertical plane using transformation equations.

Here the given values are  $\sigma_y$  which equals to 60 MPa which is positive;  $\tau_{xy}$  is given as 30 MPa; and on this particular plane on which we have defined  $\sigma_x$  prime the normal plane is equal to 50 MPa and the shearing stress  $\tau_{x'y'}$  is equal to 40 MPa. We will have to compute what is the value of  $\sigma_x$  which is acting on the vertical plane and correspondingly what is the shearing stress tau. These are the two values we have to evaluate.

If you remember the transformation equations on any plane which is  $\sigma_x$  prime is equal to  $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ the normal stresses plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \text{ into } \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$ . Now in this particular problem the stress on the inclined plane is given as  $\sigma_x$  prime

and  $\tau_{x'y'}$ , and normal to this particular plane is at an angle of 45 degrees. So  $\theta$  here is 45 degrees thereby  $2\theta$  is 90 degrees. Now  $\sigma_x$  prime is given as 50, so 50 is equal to  $(\sigma_x + \sigma_y)$  by 2, as  $\sigma_y$  is equal to 60, so  $(\sigma_x + 60)$  by 2 plus  $(\sigma_x - 60)$  by 2 into  $\cos$  of 90 is (0) plus  $\tau_{xy}$   $\sin$  90 which is (1).

The second equation is;  $\tau_{x'y'}$  is equal to 40 is equal to minus  $(\sigma_x - \sigma_y)$  by 2  $\cos 2\theta$ , where  $(\sigma_x - \sigma_y)$  by 2  $\sin 2\theta$  (which is 1) plus  $\tau_{xy}$   $\cos 2\theta$  (which is equals to 0). So from this we get minus  $\sigma_x$  plus 60 is equal to 80 or  $\sigma_x$  is equal to minus 20 MPa. Now if  $\sigma_x$  is minus 20 if we substitute in this equation one for  $\sigma_x$  value minus 20 plus 60 is equal to 40, 40 by 2 is equal to 20 so 50 is equal to 20 plus  $\tau_{xy}$  and thereby this gives you  $\tau_{xy}$  is 30 MPa, which is the shear stress component in the horizontal plane. So, if we draw the element now on which the stresses act we have on this as  $\sigma_y$ , now we have evaluated the  $\sigma_x$  and also  $\tau_{xy}$  on this plane which is at an angle of 45 degrees with the x-plane on which the  $\sigma_x$  prime and  $\tau_{x'y'}$  acts. These are the values of  $\sigma_x$ ;  $\sigma_x$  gives you minus 20.

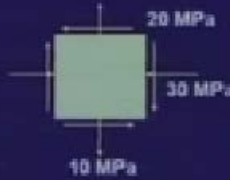
Here if you see I have made a mistake that the stress is minus 20 so that indicates that the normal stress will be acting in the opposite direction, it will be a compressive stress whereas  $\sigma_y$  is acting in the positive direction. Here  $\tau_{xy}$  is positive so the direction of  $\tau_{xy}$  is in the positive direction of y. That is the solution for this particular problem.

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### Example Problem - 2

- The state of stress at a point in a stressed body is as shown in figure. Evaluate Principal stresses, maximum shear stresses with the associated normal stresses using Mohr's circle.

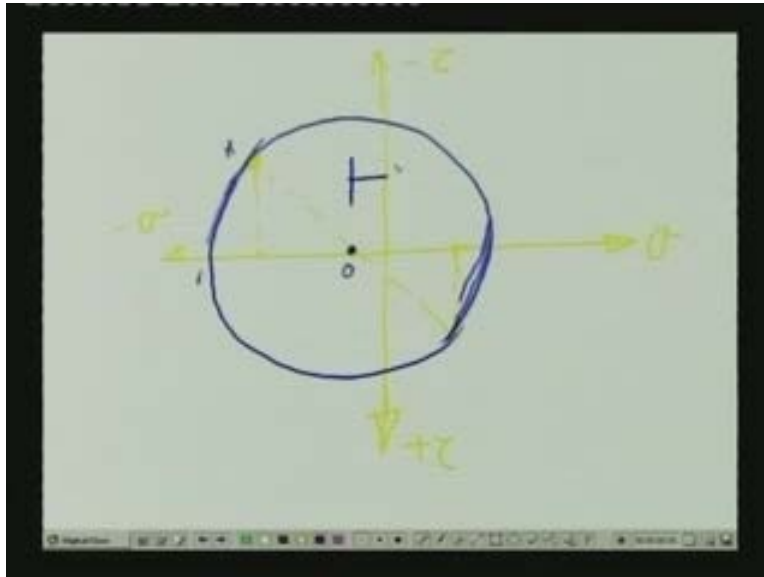


Here is another problem. This particular problem states that these are the stresses acting with the rectangular axis system which are  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  and we will have to evaluate the principal stresses, maximum shear stresses along with the normal stresses. We will have to use Mohr's circle to evaluate these quantities. Here if you look into the normal stresses which is acting in the x-plane is compressive in nature having magnitude of 30 MPa. So  $\sigma_x$  is equal to minus 30 MPa,  $\sigma_y$ , the normal stress on the y-plane is 10 MPa, so  $\sigma_y$  is equal to plus 10 MPa. Then we have the shearing stress.

If you note the direction of the shearing stress it is opposite to the positive y-direction. Also, this particular shear along with the complimentary shear on the other face is causing rotation in the clockwise direction which according to our sign convention is negative.

If you remember in the Mohr's circle, we said that the shearing stresses which are causing anticlockwise rotation is positive, since here this is the clockwise rotation so these are negative shear. So, on this particular surface, since this is causing anticlockwise, this particular shear is a positive shear. Now if we try to represent these in the Mohr's circle then let us see how it looks like.

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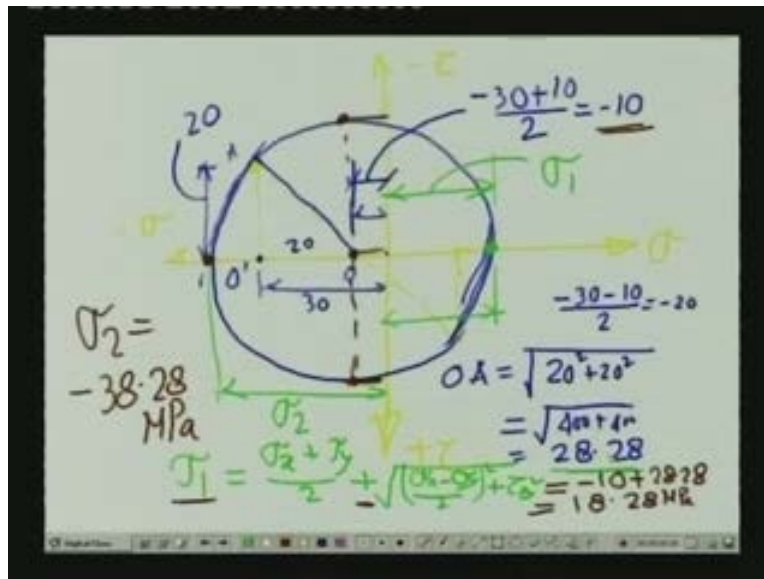
This is the reference axis system and this is the sigma plus and this is minus sigma, this is plus tau and this direction is minus  $\tau$ . Now on the x-plane we have  $\sigma_x$  is equal to minus 30MPa which is this direction and  $\tau_{xy}$  is equal to minus 20 so this is the shear, so we get the point somewhere here, which is  $\sigma_x$  is equal to minus 30 and  $\tau$  is equal to minus 20. Then we have in the perpendicular plane the y-plane which is 90 degrees with reference to the physical plane, here in the Mohr's plane it will be 180 degrees and we have plus  $\sigma_y$  is equal to 10 MPa, and we have plus tau is equal to 20 MPa.

Now if we join these two points this is where it crosses the sigma line, we get the centre of the Mohr's circle. So with this as centre O as the centre and OA as the radius we draw the circle. This gives us the Mohr's circle of which the centre is this particular point and as you know the distance of the centre from the tau -axis ( $\sigma_x$  plus  $\sigma_y$ ) by 2.

Now here  $\sigma_x$  is equal to minus 30,  $\sigma_y$  is equal to plus 10 that divided by 2, that gives you minus 10. So this distance is minus 10 MPa, this particular point refers to  $\sigma_x$  which is from here is minus 30, so the distance from here to here is 20 which is ( $\sigma_x$  minus  $\sigma_y$ ) by 2. As we have evaluated earlier this particular distance OO' is equal to ( $\sigma_x$  minus  $\sigma_y$ ) by 2 and  $\sigma_x$  is equal to minus 30  $\sigma_y$  is equal to minus 10 by 2 is equal to minus 20. So from here to here is 20, and this is  $\tau_{xy}$  which is also 20. So the radius or this particular distance OA is equal to  $\sqrt{20^2 + 20^2}$  is equal to  $\sqrt{400 + 400}$  and this gives us the value of 28.28; 28.28 is the distance OA. Now, we are going to evaluate the principal stresses. The principal stresses are: this is the maximum principal stress which is  $\sigma_1$ , and this is the minimum principal stress, which is  $\sigma_2$ . What will be the value of maximum principal

stress, which is acting on this particular plane? Is the distance from here to here which is in terms of the Mohr's circle, is the radius minus this. So here the radius is 28.28 MPa and this is the 10 MPa. So this is going to be equal to 18.2 MPa. So  $\sigma_1$  is equal to  $(\sigma_x \text{ plus } \sigma_y)$  by 2 plus the radius which is square root of  $(\sigma_x \text{ minus } \sigma_y)$  by 2) whole square plus  $\tau_{xy}$  square.

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This particular quantity is nothing but the radius, which is equals to 28.28. So the value of  $\sigma_1$  is equal to  $(\sigma_x \text{ plus } \sigma_y)$  by 2 is equal to minus 10, and this is 28.28 is equal to minus 10 plus this is 28.28. Therefore, Stress is equal to 18.28 MPa. This is the maximum principal stress. What will be the minimum principal stress? Minimum principal stress will act on this particular plane, which is  $\sigma_2$ , which is  $(\sigma_x \text{ plus } \sigma_y)$  by 2 minus square root of  $((\sigma_x \text{ minus } \sigma_y)$  by 2) whole square plus  $\tau_{xy}$  square.

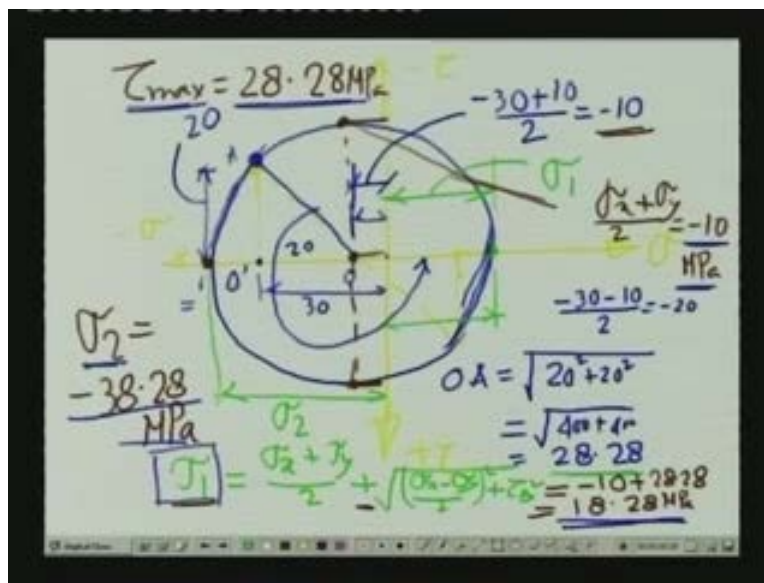
In terms of the Mohr's circle, if you look into, the distance is this **radius plus this distance and this radius we have got as 28.28 plus we have 10.** So the distance from here to here is 38.28 and in terms of this equation,  $(\sigma_x \text{ plus } \sigma_y)$  by 2 is equal to minus 10 and this radius is 28.28 which is negative. So in a combined form  $\sigma_2$  is equal to minimum principal stress is equal to minus 38.28 MPa.

From this particular diagram, if we look into that, this is the value of the radius and this is the plane, where we get the value of maximum value of the shear stress and this is the plane where you get the minimum value of the shear stress. Now at this particular point, the value of the normal stress is this which is  $(\sigma_x \text{ plus } \sigma_y)$  by 2 is equal to minus 10 MPa in this particular problem.



Therefore the maximum value of shear stress is equal to radius is equal to 28.28 MPa. So  $\tau_{\max}$  is equal to 28.28 MPa and the corresponding normal stress on these particular planes where the maximum and minimum share stress occurs, that is equals to the  $(\sigma_x \text{ plus } \sigma_y)$  by 2 is equal to minus 10. So the normal stress on this plane at is equals to  $(\sigma_x \text{ plus } \sigma_y)$  by 2 is equal to minus 10 MPa. So these are the values which we get, they are: the maximum principal stress  $\sigma_1$  is equal to 28.28 MPa; the minimum principal stress  $\sigma_2$  is is equal to minus 38.28 MPa; the maximum shear stresses  $\tau_{\max}$  is equal to 28.28 MPa; the normal stress which acts on the maximum and the minimum shear stress is equals to minus 10 MPa.


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We can evaluate the position of these planes, the maximum, and minimum normal stresses with reference to the plane. Now this is the plane which is representing the vertical plane, normal to the plane which coincides with the x-axis, which you call as x-plane.

Now, with reference to this particular plane, if we go in the anticlockwise direction this particular angle will give us the value half of which is the orientation in the physical plane which locates the maximum principal stress and perpendicular to that is the plane where in this minimum normal stress acts. These are the values and that is how we can compute the stresses using Mohr's circle.

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### Example Problem - 3

- The Principal stresses at a point are  $\sigma_1 = (-2 + \sqrt{10})$  MPa,  $\sigma_2 = 1$  MPa and  $\sigma_3 = (-2 - \sqrt{10})$  MPa. Evaluate the Octahedral stresses.

Here is another problem.

We already know that the octahedral stresses are the stresses which acts on the octahedral plane and octahedral plane are the planes are equally inclined with respect to the principal axes,  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  axes system. Now, if we know the values of principal stresses at a point, we can compute the values of octahedral stresses. Now,  $\sigma_1$ , the maximum principal stresses is given as minus 2 plus  $\sqrt{10}$  MPa,  $\sigma_2$  is equal to 1 and  $\sigma_3$  is equal to minus 2 minus  $\sqrt{10}$  MPa. And we will have to evaluate, the values of octahedral stresses.

So let us compute these values. So we have  $\sigma_1$  is equal to minus 2 plus  $\sqrt{10}$  MPa, as the maximum principal stress. Then we have  $\sigma_2$ , the second principal stress as 1 MPa  $\sigma_3$  as equals to minus 2 minus  $\sqrt{10}$  MPa. Now as we have seen in the beginning itself, that the value of the sigma octahedral is equal to 1 by 3 ( $\sigma_1$  plus  $\sigma_2$  plus  $\sigma_3$ ).

So here the values of  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are given, this is equal to 1 by 3(minus 2 plus  $\sqrt{10}$  + 1 minus 2 minus  $\sqrt{10}$ ),  $\sqrt{10}$  - these get cancelled so minus 4 plus 1 is equal to minus 3 is equal to minus 1 MPa. This is the value of  $\sigma$  of the normal octahedral stress.

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The image shows a handwritten derivation on a whiteboard. The equations are as follows:

$$\sigma_1 = -2 + \sqrt{10} \text{ MPa}$$

$$\sigma_2 = 1 \text{ MPa} \quad \sigma_3 = -2 - \sqrt{10} \text{ MPa}$$

$$\sigma_{\text{Oct}} = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$

$$= \frac{1}{3} (-2 + \sqrt{10} + 1 - 2 - \sqrt{10})$$

$$= -1 \text{ MPa}$$

$$\tau_{\text{Oct}}^2 = \frac{2}{9} (\sigma_1 + \sigma_2 + \sigma_3)^2 - 6 (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)$$

$$= + \frac{2}{9} - 6 (-2 + \sqrt{10} - 2 - \sqrt{10} + 4 - 10)$$


For tau octahedral, we will compute in this form, this is (tau octahedral) whole square is equal to 2 by 9 ( $\sigma_1$  plus  $\sigma_2$  plus  $\sigma_3$ ) whole square minus 6 ( $\sigma_1 \sigma_2$  plus  $\sigma_2 \sigma_3$  plus  $\sigma_3 \sigma_1$ ). So here we have computed  $\sigma_1$  plus  $\sigma_2$  plus  $\sigma_3$  as minus 1, so this is minus 1<sup>2</sup> that is 2 by 9 (minus 6) of substitute the values of  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  and this gives you a as 6 into  $\sigma_1$  is minus 2 plus  $\sqrt{10}$   $\sigma_2$   $\sigma_3$  is minus 2 minus  $\sqrt{10}$  n and  $\sigma_1$  and  $\sigma_3$  is equal to plus 4 minus 10. This gives you minus 10; so the  $\tau$  octahedral is equal to  $\sqrt{\frac{78}{3}}$  is equal to 2.944 MPa. So this is the value of sigma octahedral and tau octahedral.

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### Example Problem - 4

- A billboard supporting steel truss is shown in the figure. The cross sectional area of all members is  $100 \text{ mm}^2$ . Calculate the stress in each member due to horizontal forces as indicated. The truss joints are pinned.



Here is another problem, this is the supporting structure which supports a billboard, many a times we use boards, sign boards on which the advertisements are put and these boards are supported by some steel structures and these boards are subjected to the wind pressure.

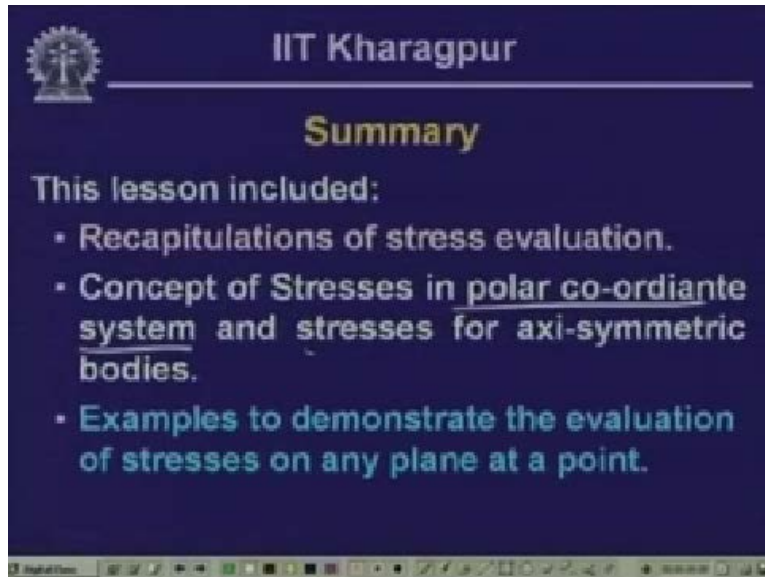
The wind pressure when it comes on the board eventually it transfers the load on to the supporting structure, hence this is one of the supporting structures in which we have framework and all the members are connected in the pin joint and this is the force which is acting on this member. Now our job is to find out the stress in each of these members from this particular force. What we need to do is, first we evaluate forces in each of the members which are arising from these external forces, and thereby we can compute the stress since the cross sectional area of the member is given, force divided by the area will give us the stress.

If we assume this angle as  $\theta$ , then this being 6 from the similar triangle we get this as 3, so eventually this is also  $\theta$ , and if we drop a perpendicular this is also  $\theta$ , and this is also  $\theta$  and the values of  $\cos \theta$  is 4 by 5. This particular hypotenuse, this being 3, and this being 4, is 5 so is this, this is also five, so the values of  $\cos \theta$  is equal to 4 by 5 and value of  $\sin \theta$  is equal to 3 by 5. Now what we need to do is to draw free body diagram and evaluate the forces.

One section we can take here and draw the free body diagram, and another section we can take here and draw the free body diagram and we can compute the forces. Once we compute the forces, we can find out the stresses. You try to compute the stresses for this particular problem; In this particular lesson we tried to look into the stresses which we have evaluated earlier with reference to Cartesian system  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$ . Also, we have tried to look into that if a body which is does not have the straight boundary and there are

curved boundary, how to represent the stresses  $\sigma_r$ ,  $\sigma_\theta$  and  $\tau_{r\theta}$  which are in terms of polar coordinate systems.

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The slide features the IIT Kharagpur logo in the top left corner. The title "IIT Kharagpur" is centered at the top. Below it, the word "Summary" is written in a large, bold, yellow font. The main content is a list of bullet points under the heading "This lesson included:". The first bullet point is "Recapitulations of stress evaluation." The second bullet point is "Concept of Stresses in polar co-ordinate system and stresses for axi-symmetric bodies." The third bullet point is "Examples to demonstrate the evaluation of stresses on any plane at a point." The slide has a dark blue background with white and yellow text.

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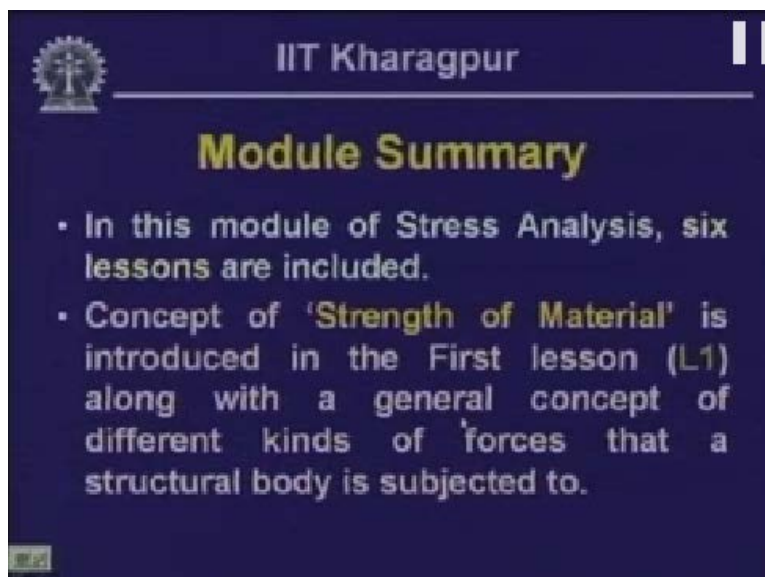
## Summary

This lesson included:

- Recapitulations of stress evaluation.
- Concept of Stresses in polar co-ordinate system and stresses for axi-symmetric bodies.
- Examples to demonstrate the evaluation of stresses on any plane at a point.

And also from there we have seen how to evaluate stresses for the axi-symmetric bodies and also we looked in to some examples to demonstrate how we can evaluate stresses at any point on the stress body either using transformation equations or by the use of the Mohr's circle. We also tried to see how to compute octahedral stresses on a particular, on a particular stress body; keep in mind this octahedral stress is useful when we talk about the evaluation of the stress in the inelastic when we go beyond the elastic strain.

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The slide features the IIT Kharagpur logo in the top left corner. The title "IIT Kharagpur" is centered at the top. Below it, the words "Module Summary" are written in a large, bold, yellow font. The main content is a list of bullet points. The first bullet point is "In this module of Stress Analysis, six lessons are included." The second bullet point is "Concept of 'Strength of Material' is introduced in the First lesson (L1) along with a general concept of different kinds of forces that a structural body is subjected to." The slide has a dark blue background with white and yellow text.

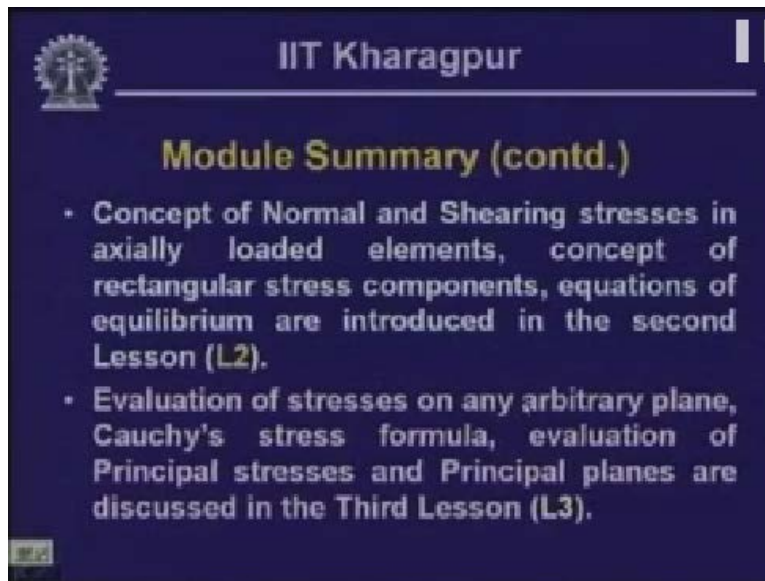
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## Module Summary

- In this module of Stress Analysis, six lessons are included.
- Concept of 'Strength of Material' is introduced in the First lesson (L1) along with a general concept of different kinds of forces that a structural body is subjected to.

This particular lesson was last in the series of the module stress analysis. We have computed or we have looked into the six lessons in the particular module stress analysis. These six lessons if we look into chronologically, in the first lesson I tried to give you the general concept of kinds of forces and what really is the meaning of subject strength of material, so it was introduced to you.

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Subsequently the second lesson we had, I tried to give you the concept of normal and shearing stresses and you know how to evaluate the equations of the equilibrium from the stresses that are acting in the Cartesian system,  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$ .

In the third lesson we have tried to evaluate stresses on any arbitrary plane, if we have any plane which is oriented with the value of  $\theta$  with reference to x-plane and thereby we arrived at values of the stresses which we have defined as Cauchy's stress formula.

We tried to evaluate the maximum normal stresses which we defined it as principal stresses, and these principal stresses which we are acting in the principal planes, we have tried to locate them in this particular lesson. We have demonstrated that through few examples.

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**Module Summary (contd.)**

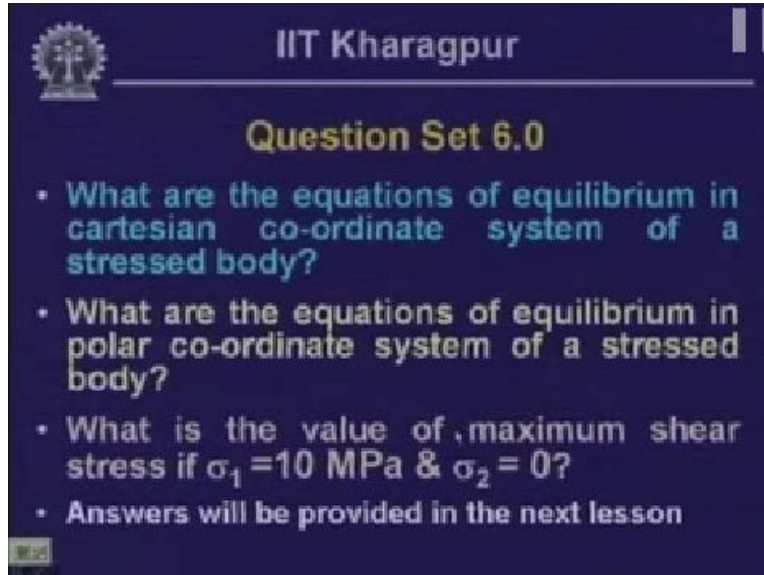
- Evaluation of stresses on any plane using stress transformation equations or using Mohr's circle, construction of Mohr's circle have been discussed in the Fourth Lesson (L4).
- Concept of Octahedral stresses along with evaluation of stresses at a point in different planes is introduced in the Fifth Lesson (L5).

In the fourth lesson we have evaluated the stress on any plane using transformation equations, in fact we have discussed this lesson as well. How to evaluate stress any plane using transformation equations or using Mohr's circle?

In the fourth lesson we have discussed in detail how to construct a Mohr's circle if we know the stresses at a particular point in a body. In the fifth lesson we tried to give you the concept of octahedral stresses and also we looked into how to evaluate the stress at a particular point in the stress body at different planes.

As you know the concept of the octahedral stress, that this particular stress or the stresses which act on the octahedral planes which are inclined equally with the principal axes system and these stresses are useful when we talk about the evaluation of stress in the inelastic stage.

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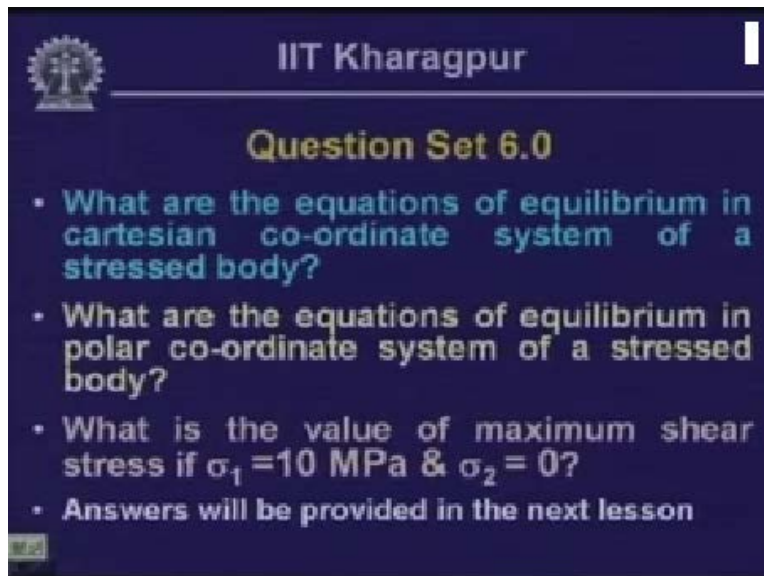
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**Question Set 6.0**

- What are the equations of equilibrium in cartesian co-ordinate system of a stressed body?
- What are the equations of equilibrium in polar co-ordinate system of a stressed body?
- What is the value of maximum shear stress if  $\sigma_1 = 10 \text{ MPa}$  &  $\sigma_2 = 0$ ?
- Answers will be provided in the next lesson

In this particular lesson, we tried to give you the concept of stresses in the polar coordinate systems that you have already looked into, now having looked into these aspects of stresses here are some questions to answer.

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This slide is identical to the one above, showing the IIT Kharagpur logo, the title 'Question Set 6.0', and the same four bullet points regarding equilibrium equations and maximum shear stress.

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**Question Set 6.0**

- What are the equations of equilibrium in cartesian co-ordinate system of a stressed body?
- What are the equations of equilibrium in polar co-ordinate system of a stressed body?
- What is the value of maximum shear stress if  $\sigma_1 = 10 \text{ MPa}$  &  $\sigma_2 = 0$ ?
- Answers will be provided in the next lesson

What are the equations of equilibrium in Cartesian coordinate system of a stressed body?  
What are the equations of equilibrium in polar coordinate system of a stressed body?  
What is the value of maximum shear stress if  $\sigma_1$  is equal to 10MPa, the maximum principal stress, and the minimum principal stress  $\sigma_2$  is equal to 0?