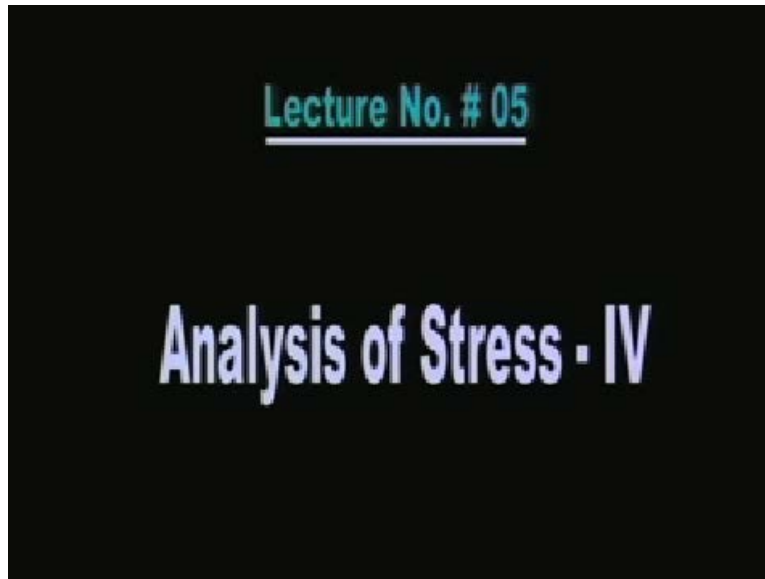



Strength of Materials
Prof S. K. Bhattacharya
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Lecture - 5
Analysis of Stress - IV

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Welcome to the 5th lesson of Strength of Materials. We will be discussing certain aspects of analysis of stress in this particular lesson.

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Specific Instructional Objectives

- After completing this lesson one will be able to:
- Evaluate stresses on any plane through stress-transformation equations and using Mohr's circle.
- Evaluate Principal stresses & locate principal planes.
- Evaluate octahedral stresses.

Now, it is expected that once this particular lesson is completed, one should be able to evaluate stresses on any plane through stress transformation equations using Mohr's circle. We will be looking into some more aspects of it then evaluate principal stresses and locate principal planes at a particular point on the stress body. Also, one should be able to evaluate octahedral stresses which we will look into in this lesson.

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Scope

- This lesson includes:
 - Recapitulation of transformation equations.
 - Recapitulation of evaluation of stresses using Mohr's circle.
 - Octahedral Planes & Stresses on such planes
 - Examples for evaluation of stresses at different planes.

The scope of this particular lesson includes recapitulation of transformation of equations which we have derived earlier, recapitulation of evaluation of stresses using Mohr's circle. We will be looking into some aspects of octahedral stresses; we will define

octahedral plane and stresses acting on such planes and then we will be looking into some examples of how to evaluate the stresses at a particular point at a stress body.

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Stresses on inclined plane

- $\sigma_{x'} = (\sigma_x + \sigma_y)/2 + \cos 2\theta(\sigma_x - \sigma_y)/2 + \tau_{xy} \sin 2\theta$
- $\tau_{x'y'} = -\sin 2\theta(\sigma_x - \sigma_y)/2 + \tau_{xy} \cos 2\theta$
- $\sigma_{y'} = (\sigma_x + \sigma_y)/2 - \cos 2\theta(\sigma_x - \sigma_y)/2 - \tau_{xy} \sin 2\theta$

We have discussed how to evaluate the stress on a particular plane which is inclined at angle of θ with respect to x-plane and these we have termed as transformation equations. If we have a body which is acted on by the rectangular stress components, which are σ_x σ_y in the y-direction and the shearing stresses. We can compute stresses on any plane, the normal to which it is making an angle θ with the x-plane. This is what we have defined as the normal stress on this plane as, σ_x prime and the shearing stress as $\tau_{x'y'}$.

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Stresses on inclined plane

- $\sigma_{x'} = (\sigma_x + \sigma_y)/2 + \cos 2\theta(\sigma_x - \sigma_y)/2 + \tau_{xy} \sin 2\theta$
- $\tau_{x'y'} = -\sin 2\theta(\sigma_x - \sigma_y)/2 + \tau_{xy} \cos 2\theta$
- $\sigma_{y'} = (\sigma_x + \sigma_y)/2 - \cos 2\theta(\sigma_x - \sigma_y)/2 - \tau_{xy} \sin 2\theta$

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The $\sigma_{x'}$ is given in terms of σ_x , σ_y and τ_{xy} as $(\sigma_x + \sigma_y)/2 + (\sigma_x - \sigma_y)/2 \cos 2\theta + \tau_{xy} \sin 2\theta$. The normal stress on that x prime plane and $\tau_{x'y'}$ - the shearing stresses in that particular plane is equal to $(\sigma_x - \sigma_y)/2 \sin 2\theta + \tau_{xy} \cos 2\theta$. And also the normal stress to the perpendicular plane in the x prime can be written as $(\sigma_x + \sigma_y)/2 - (\sigma_x - \sigma_y)/2 \cos 2\theta - \tau_{xy} \sin 2\theta$. And eventually we have seen that the $\sigma_{x'}$ plus $\sigma_{y'}$ is equal to $\sigma_x + \sigma_y$ which gives us the first stress invariant.

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Principal Stresses

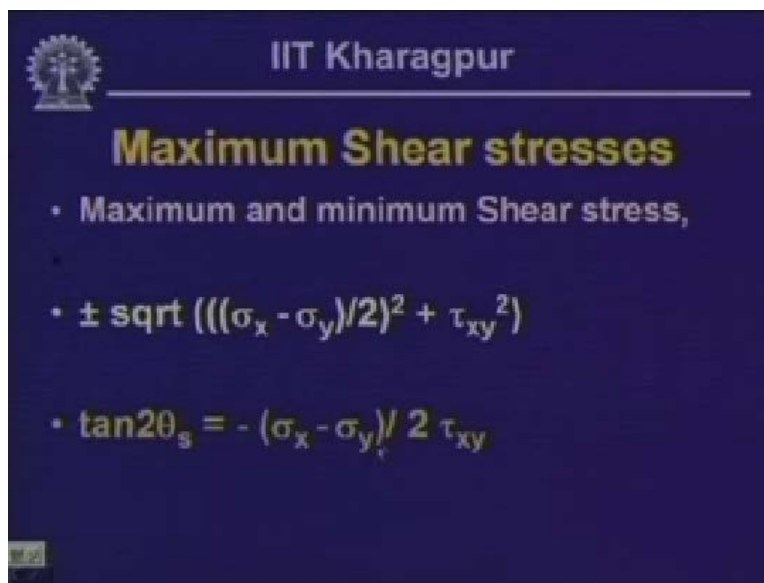
- Values of Principal Stresses are:
- $\sigma_1 = (\sigma_x + \sigma_y)/2 + \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$
- $\sigma_2 = (\sigma_x + \sigma_y)/2 - \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$

$$\tan 2\theta_p = 2 \tau_{xy} / (\sigma_x - \sigma_y)$$

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Thereby we had calculated the principal stresses which are the maximum normal stress, the maximum and minimum in two different planes it acts, and the magnitude of those principal stresses are, σ_1 is equal to $(\sigma_x + \sigma_y) / 2 + \sqrt{((\sigma_x - \sigma_y) / 2)^2 + \tau_{xy}^2}$. The σ_2 is the minimum principal stress is equal to $(\sigma_x + \sigma_y) / 2 - \sqrt{((\sigma_x - \sigma_y) / 2)^2 + \tau_{xy}^2}$. And the plane which gives us the maximum normal stress or the principal stresses can be evaluated through this equation, where $\tan 2\theta_p$ is equal to $2\tau_{xy} / (\sigma_x - \sigma_y)$. And in the physical plane this is denoted by θ_p .

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Also, we have evaluated the maximum shear stresses on planes and we had observed that the maximum shear stress occurs in planes which are at an angle of 45 degrees with the difference to the principal planes. And the magnitude of the maximum and minimum shear stresses are plus square root of $(\sigma_x - \sigma_y) / 2$ whole square plus τ_{xy} square and the minimum one is minus square root of $(\sigma_x - \sigma_y) / 2$ whole square plus τ_{xy} square). This we had elaborated last time, τ_{max} is equal to square root of $(\sigma_x - \sigma_y) / 2$ whole square plus τ_{xy} square and τ_{min} equals to minus of that quantity and the angle which can be evaluated from the normal stresses and the shear stresses and this gives the plane shear stresses are maximum and minimum.

Also, we had looked in to how to compute the stresses at any plane, using the concept of Mohr's circle apart from the equations of equilibrium. We had used the equations of equilibrium to evaluate stress at a particular point; now we will look in to what we had looked into - how to evaluate any point using Mohr's circle of stress.

Let us assume that, at a particular point in the stress body, the normal components of stress which are acting, the rectangular stresses are: σ_x , σ_y the normal stress, τ_{xy} the shearing stress. Now if we like to represent them in the Mohr's circle, last time we had seen that this is the sigma axis, this is the τ axis. And we can denote the centre of the Mohr's circle at a distance from the origin which is at $(\sigma_x \text{ plus } \sigma_y)$ by 2, which was A, and we have denoted this particular quantity as A. This particular point on the Mohr's circle denotes this plane in the physical space which is A, and there by this particular point represents the normal stress σ_x and the shearing stress τ_{xy} . This particular point represents this particular plane B, where the normal stress is the σ_y , and this is τ_{xy} .

Now if we join these two lines which eventually pass through the center and in this Mohr's circle, the angle between these two planes is 180 degrees, which is 2θ and in the physical space between these two plates is 90 degrees; half of this particular angle. There by this particular point is the maximum normal stress, this is the plane where maximum normal stress acts; and this is the plane where the minimum normal stress acts.

From this particular diagram, we can observe that the maximum normal stress which we generally denotes as σ_1 , this is nothing but equals to the distance from the origin to the centre plus the radius. So this is $(\sigma_x \text{ plus } \sigma_y)$ by 2 and this is the radius which is equals to this hypotenuse which is this square plus this square (Refer Slide Time:). And this distance is $(\sigma_x \text{ plus } \sigma_x \text{ minus } \sigma_y)$ by 2.

And eventually this distance comes as $(\sigma_x \text{ minus } \sigma_y)$ by 2 and this is τ_{xy} and hence σ_1 is equal to $(\sigma_x \text{ plus } \sigma_y)$ by 2 plus square root of $((\sigma_x \text{ minus } \sigma_y)$ by 2) whole square plus τ_{xy} square. And this gives you the minimum principal stress which is this distance minus the radius, which will be $(\sigma_x \text{ plus } \sigma_y)$ by 2 minus square root of $((\sigma_x \text{ minus } \sigma_y)$ by 2) whole square plus τ_{xy} square.

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Now let us look into the given state of stress at a particular point as to how you evaluate this stress at a particular plane by constructing the Mohr's circle. Let us assume that we have a stress body where the stresses at a particular point are: σ_x is the normal stresses in the x-plane and σ_y is the normal stress in the normal plane, τ_{xy} gives the shear stress, which has positive shear this is on the positive y-axis in this plane; and this is on the positive x-direction in this particular plane and these are these components σ_x , σ_y and τ_{xy} . As we have defined last time that, this axis represents σ the normal stress axis, and the y-axis represents the shearing stress axis.

We take the direction of the shear stress downwards, to have the compatibility of the rotational angle which is in an anticlockwise direction, which will be represented in the anticlockwise direction in Mohr's circle as well. Thereby the normal positive stress and the normal positive shear stress on this plane, σ_x and τ_{xy} , if we represent on the Mohr's plane, let us assume that, $\sigma_x > \sigma_y$, then we have the plane which is represented as σ_x and τ_{xy} as this particular point where normal stress is σ_x and shear stress is τ_{xy} . We can represent this particular plane on this Mohr's plane wherein the stresses are σ_x and τ_{xy} .

Now please note here that the shear which is acting in the positive y-direction on x-plane we have considered this as the positive shear. This particular shear along this complimentary stress is causing the rotation in the anticlockwise direction which we denoted as positive shear in the Mohr's plane. Whereas the shear which is acting in the y-plane, this along the complimentary shear on the other side is causing rotation in the clockwise direction. Based on this we are calling this as a negative shear on this Mohr's plane. If we try to represent the normal and shear stress in the Mohr's plane, we have

positive σ_y and the negative τ_{xy} , which represents point B, which is for this particular plane.

Now if we join these two lines as we have seen earlier, eventually this will pass through the centre of the Mohr's circle. And we have seen that the distance of the centre of the Mohr's circle from the τ axis is equal to $(\sigma_x + \sigma_y) / 2$, which is the average stress. Now taking this as centre and OB or OA as radius if we plot the circle, eventually we are going to get the Mohr's circle.

Please note that the Mohr's circle need not be constructed geometrically, if we can represent them in this particular form we can compute the other stresses directly from this diagram itself.

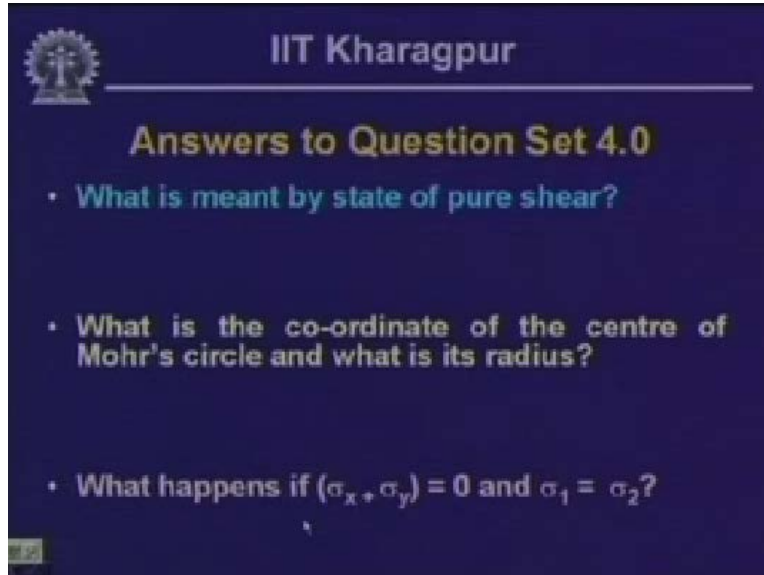
In this particular diagram, the maximum normal stress occurs at this particular plane and the minimum normal stress occurs at this particular plane. And this normal stress we denote as σ_1 . This particular normal stress we denote as σ_2 . From plane A if we move by angle $2\theta_P$ we come to the plane where the maximum principle stress is located. So, in the physical plane if we rotate by angle θ_P in the anticlockwise direction this gives the normal and perpendicular to the plane where the σ_1 acts. That is how we decide about the planes for the maximum principal stresses and correspondingly the maximum shear stress.

Likewise the maximum shear stresses, will occur at this particular point which is the highest point in this circle. And this is τ_{max} which is eventually equal to the radius of the circle. This is the radius so this is the positive maximum shear and this is the minimum shear which is negative of this radius. This gives us the values of maximum shear stresses.

Please note that, at these points where the maximum shear stresses occur there we will have some values of normal stress which is in contrast to the one where we have the maximum normal stresses where shear stresses are 0.

Now if we like to evaluate the stress at a particular plane from this Mohr's diagram. This is the plane, the normal to which is oriented at angle θ with x-axis. Since this is the reference plane, from this particular plane we move in the anticlockwise direction as that of the physical plane by an angle of 2θ ; this is 2θ then the point which we get on the periphery of the circle represents this particular plane. The corresponding stresses represent the normal stress and this represents the shearing stress. That is how we compute stresses at any plane through Mohr's circle.

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Here is a question:

What is meant by state of pure shear? Now at a point in stress body as we have seen that it is acted on by the normal stress and the shearing stresses components, now in this particular body or the stress body if we do not have any normal stress but we have only the shear stresses then we say that this particular point of the stress body is subjected to pure state of shear.

Now if we look into Mohr's diagram corresponding to this particular state of stress, we have a stress body in which the state of stress is in the form shear alone, and this if we try to represent in terms of the Mohr's circle then this is our reference axis σ and τ on this particular plane where normal stress is 0, we have only τ positive so normal stresses is 0 τ is positive here, this is the point which represents this particular plane.

The perpendicular plane which is at an angle of 90 degrees in the Mohr's circle will be 180 where normal stresses is 0 and we have negative shear of equal amount of magnitude so these are the two points on the Mohr's circle. Since σ_x and σ_y both are 0 so eventually this will be the centre of Mohr's circle. If we draw a circle with centre as this particular point and this radius from O to A, we get the Mohr's circle for this kind of stress which is acting. Thereby this gives us the maximum value of normal stress which is σ_1 and this is the minimum value of normal stress which is σ_2 .

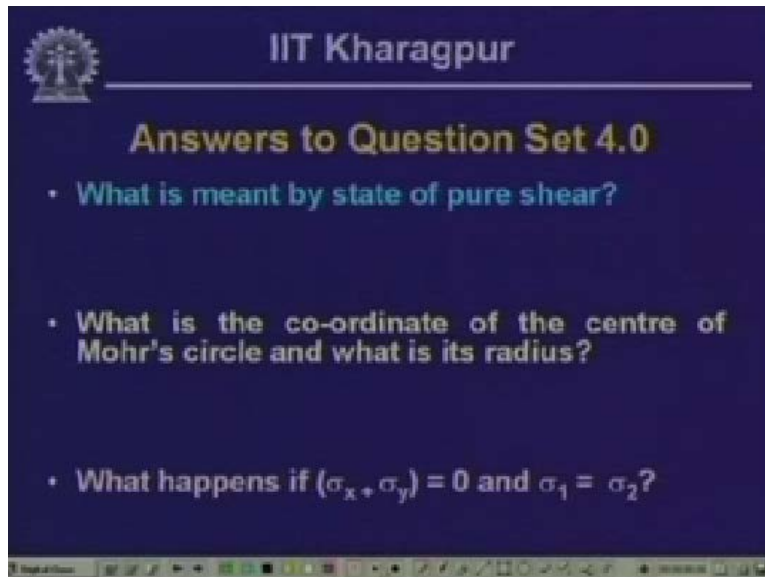
Please note that here the values of σ_1 and σ_2 , which is in the opposite direction is equals to the shearing stress τ . So the maximum normal stress is τ and the minimum normal stress also is τ , but it is the negative τ .

And if we look into the plane, now this is the plane which is representing this particular face which is face A, so this is the plane which is representing the face A and from here the plane on which the maximum normal stress acts is at angle of 90 degrees, which is $2\theta_P$. So θ_P in the physical plane, it will be 45 degrees.

With respect to this if we draw a normal which is at angle of 45 degrees, the plane on which maximum normal stress acts is this. Maximum normal stress is equal to σ_1 , which is equals to τ . The other normal stress σ_2 will be acting on the plane which is perpendicular to this but in the opposite direction which is σ_2 , which is negative τ .

So, if we have a stress representation which is like you have the stress τ here, the normal stress which is acting in this plane, which is τ here. This also represents the state of pure shear. Mind that this particular plane is at an angle of 45 degrees with reference to the x-plane. So either we represent that at a point in stress body, the stresses are purely in the form of shear or in the form of normal stresses, having the magnitude as that of a shear which is normal tensile in the maximum normal stress direction and the compressive τ in the other normal direction. So this is state of pure shear.

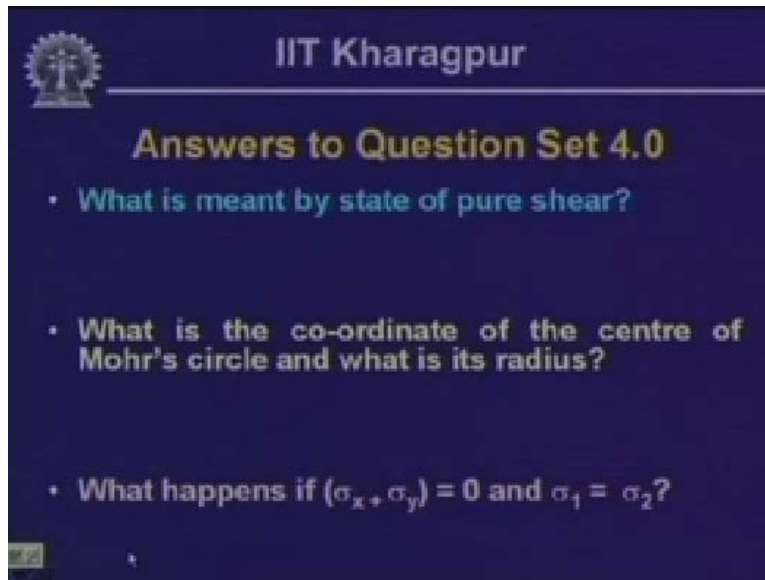
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Now what is the co-ordinate of the centre of Mohr's circle and what is its radius? By this time you have observed that the values of the radius and the position of the centre in the Mohr's circle which we have represented now, this is σ and this is τ . The centre is located at the distance of $(\sigma_x \text{ plus } \sigma_y)$ by 2. Based on these we have drawn the Mohr's circle. The plane which is representing is having σ_x and τ_{xy} . There by this particular distance is $(\sigma_x \text{ minus } \sigma_y)$ by. So the radius R is equals square root of $(\sigma_x \text{ minus } \sigma_y)$ by 2


square plus τ_{xy} square. The centre is at the distance of $(\sigma_x \text{ plus } \sigma_y)$ by 2 and the radius of the circle is square root of $(\sigma_x \text{ minus } \sigma_y)$ by 2 square plus τ_{xy} square.

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Now what happens if σ_x plus σ_y is equal to 0 and σ_1 is equal to σ_2 ? This also we observed that, the maximum, the Mohr's plane again, this is σ and this is τ , and if we draw the Mohr's circle, the centre is $(\sigma_x \text{ plus } \sigma_y)$ by 2 and this is σ_1 , this is σ_2 . If σ_x plus σ_y is equal to 0, this particular point moves to this for the state of pure shear. If σ_1 is equal to σ_2 , this reduces, means the circle reduces to point and there by there are no shear in the particular plane.

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Octahedral stresses


- A plane, which is equally inclined to all the three Principal axes of reference is termed as Octahedral plane. For such planes:
- $n_x = n_y = n_z$
- Since $n_x^2 + n_y^2 + n_z^2 = 1$
- Hence $n_x = n_y = n_z = \pm 1/\sqrt{3}$

Having looked into the aspects and how to evaluate the stresses through equations of transformations and through the Mohr's circle let us look into some more aspects of stresses which is known as Octahedral stress. To find the octahedral stress, in fact the octahedral stresses are the stresses which act on the octahedral planes. We will have to know what we really mean by octahedral planes. The octahedral plane is the plane which is equally inclined to all the three principal axes of reference. This is generally termed as octahedral plane.

Now, if we look in to the reference plane, it is said that the, normally we represent the stresses in terms of rectangular stress component which are σ_x , σ_y and τ_{xy} with respect to x, y and z axis.

Instead of representing in x, y and z form if we represent reference axis system which is denoted by principal stresses σ_1 , σ_2 and σ_3 then if we have plane which is equally inclined to these three reference axis then we call this particular plane as octahedral plane. And since this particular plane is equally inclined to these reference planes, the direction cosines of this particular plane on these planes, n_x , n_y , n_z they are going to be same.

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Octahedral stresses

- A plane, which is equally inclined to all the three Principal axes of reference is termed as Octahedral plane. For such planes:
- $n_x = n_y = n_z$
- Since $n_x^2 + n_y^2 + n_z^2 = 1$
- Hence $n_x = n_y = n_z = \pm 1/\sqrt{3}$

So this is what is indicated here: For octahedral planes which are equally inclined to the reference principal axes, they have the direction cosine of n_x , n_y and n_z all equal. We have seen earlier that the (n_x) whole square plus (n_y) whole square plus (n_z) whole square the direction cosines of three planes is equal to 1. So there by it leads us to the value of n_x is equal to n_y is equal to n_z as plus or minus square root of 3 of the octahedral planes.

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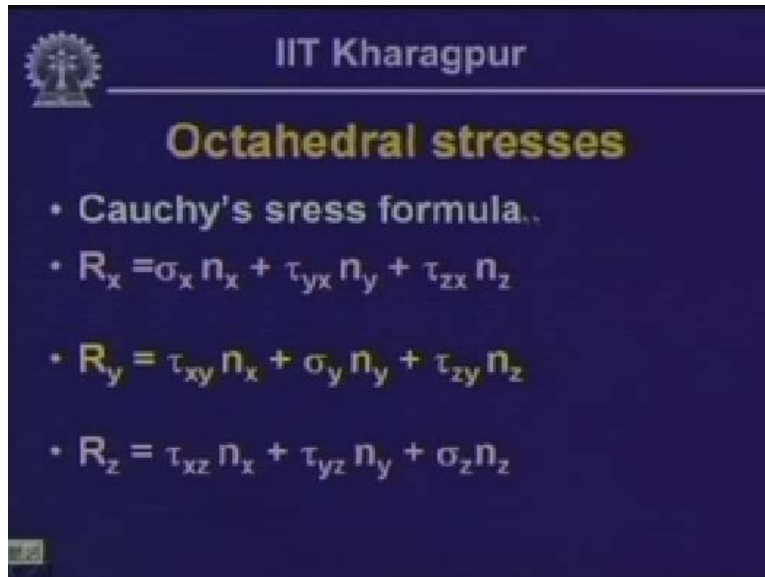
Octahedral Planes



So this is the representation of the octahedral planes, in fact the planes which are equally inclined with reference to the principal axes system, we get eight such planes. This represents σ_1 , this is represents σ_2 and this is represents σ_3 . Now we will get eight such

planes which are equally inclined to these three reference axis and three reference planes. That is why these particular planes are called octahedral planes. So the stresses which act, the normal stress, the shearing stresses which act on these octahedral planes, we call those stresses as octahedral stresses.

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Octahedral stresses

- Cauchy's stress formula.
- $R_x = \sigma_x n_x + \tau_{yx} n_y + \tau_{zx} n_z$
- $R_y = \tau_{xy} n_x + \sigma_y n_y + \tau_{zy} n_z$
- $R_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z$

Now for the computation of the octahedral stresses on the octahedral plane we use Cauchy's stress formula which we have derived already. Where in at any plane which is having outward normal n the resulting stress in the x-direction, y-direction and in the z-direction, can be represented in terms of the normal rectangular stress components: σ_x , σ_y , σ_z and the corresponding shear stresses: τ_{xy} , τ_{yx} , and τ_{zx} along with the direction cosines n_x , n_y , and n_z .

Now we have already seen that the octahedral planes are the planes which can equally inclined to the three principal reference axes σ_1 , σ_2 , and σ_3 . Thereby if we take the stress in the σ_1 direction, we will have only σ_1 and shearing stresses will be absent; because principal stress, principal plane does not have the shearing stresses.

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- Choosing the Principal axes as coordinate axes, the rectangular stress components at a point may be written as:
 - $\sigma_x = \sigma_1$ $\sigma_y = \sigma_2$ $\sigma_z = \sigma_3$
 - And $\tau_{xy} = \tau_{yz} = \tau_{xz} = 0$
 - Hence, on any plane, with normal \mathbf{n}
 - $R_x = \sigma_1 n_x$; $R_y = \sigma_2 n_y$; $R_z = \sigma_3 n_z$

So if we take those components as, σ_x is equal to σ_1 , σ_y is σ_2 and σ_z as σ_3 . The shearing stress components on this are zero because those are the principal planes.

Now if we choose any plane having outward normal \mathbf{n} on which the rectangular stress components are σ_1 , σ_2 , and σ_3 then correspondingly the components of the resultant stress could be R_x , R_y , and R_z is equal to $\sigma_1 n_x$ from Cauchy's stress formula $\sigma_2 n_y$ and $\sigma_3 n_z$. So in the Cauchy's stress formula we are replacing σ_x by σ_1 , σ_y by σ_2 , and σ_z by σ_3 the shearing stress components as 0. Hence R_x is equal to $\sigma_1 n_x$, R_y is equal to $\sigma_2 n_y$ and R_z is equal to $\sigma_3 n_z$.

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- Resultant stress has a magnitude
- $R^2 = \sigma_1^2 n_x^2 + \sigma_2^2 n_y^2 + \sigma_3^2 n_z^2$
- If σ is the normal and τ is the shearing stress on this plane
- $\sigma = \sigma_1 n_x^2 + \sigma_2 n_y^2 + \sigma_3 n_z^2$
- And $\tau^2 = R^2 - \sigma^2 = n_x^2 n_y^2 (\sigma_1 - \sigma_2)^2 + n_y^2 n_z^2 (\sigma_2 - \sigma_3)^2 + n_z^2 n_x^2 (\sigma_3 - \sigma_1)^2$


So the resultant stress R square in terms of σ_1 , σ_2 and σ_3 is equal to R_x square plus R_y square plus R_z square is equal to σ_1 square n_x square plus σ_2 square n_y square plus σ_3 square n_z square.

Now if σ is the normal stress and τ is the shearing stress on this particular plane, which we are defining as octahedral plane, then the normal stress sigma can be represented in terms of σ_1 , σ_2 and σ_3 as $\sigma_1 n_x$ square, $\sigma_2 n_y$ square, and σ_3 square n_z . These we compute by writing the equations of equilibrium in the normal direction as we have done in the previous cases.

The τ square shearing stress can be represented as R square minus σ square R square being this value substituted as σ square if we take and substitute here and if we simplify we get an expression which is like this: n_x square n_y square $(\sigma_1$ minus $\sigma_2)$ whole square plus n_y square n_z square $(\sigma_2$ minus $\sigma_3)$ whole square plus n_z square n_x square $(\sigma_3$ minus $\sigma_1)$ whole square. These two values σ and τ are the stress on any plane, which has a normal n . Now for octahedral planes, we have noted earlier that, n_x , n_y , n_z are of the equal magnitude, n_x is equal to n_y is equal to n_z is equal to plus or minus 1 by square root of 3.

Now if we substitute the values of n_x , n_y , and n_z in this expression we can get the normal stress on the octahedral plane which is equal to $(\sigma_1$ plus σ_2 plus $\sigma_3)$ by 3. Likewise the τ square is equal to 1 by 9 $(\sigma_1$ minus $\sigma_2)$ whole square plus 1 by 9 $(\sigma_2$ minus $\sigma_3)$ whole square plus 1 by 9 $(\sigma_3$ minus $\sigma_1)$ whole square.

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Octahedral Stresses

- $\sigma_{\text{oct}} = (\sigma_1 + \sigma_2 + \sigma_3)/3 = I_1/3$
- $\tau_{\text{oct}}^2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 9$
- $9 \tau_{\text{oct}}^2 = 2(\sigma_1 + \sigma_2 + \sigma_3)^2 - 6(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)$
- Hence, $\tau_{\text{oct}} = \sqrt{2 [(I_1^2 - 3I_2)^{1/2}] / 3}$

The values of normal stress, σ octahedral on the σ plane having n_x , n_y and n_z as 1 by square root of 3, is equals to $(\sigma_1$ plus σ_2 plus $\sigma_3)$ by 3. If you remember we have defined the values of σ_1 plus σ_2 plus σ_3 as the sum of the normal stress component as equals to the first invariant. So σ_1 plus σ_2 plus σ_3 is equal to I_1 the first invariant, (τ octahedral) whole square is equal to $[(\sigma_1$ minus $\sigma_2)$ whole square plus $(\sigma_2$ minus $\sigma_3)$ whole square plus $(\sigma_3$ minus $\sigma_1)$ whole square by 9] or 9 into (τ octahedral) whole square if we expand these and simplify we get in this particular form which is $2(\sigma_1$ plus σ_2 plus $\sigma_3)$ whole square minus $6(\sigma_1 \sigma_2$ plus $\sigma_2 \sigma_3$ plus $\sigma_3 \sigma_1)$.

Now σ_1 plus σ_2 plus σ_3 we have noted as the first invariant I_1 . This particular expression $\sigma_1 \sigma_2$ plus $\sigma_2 \sigma_3$ plus $\sigma_3 \sigma_1$ is known as the second invariant I_2 . So we can write down the expression for τ octahedral in terms of these two invariants I_1 and I_2 which is square root of $2[I_1$ square minus $3I_2)$ to the power $1/2]$ by 3.


So we get the normal stress and the shearing stress on the octahedral plane. Now, if σ_1 plus σ_2 plus σ_3 is equal to 0 for a particular stress condition then on octahedral plane we will not get any normal stress and thereby octahedral plane will be subjected to only shearing stress and that is what is represented through this expression.

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Example Problem - 1

- The state of stress at a point in a body is as shown in the figure. Find the Normal and shear stresses acting on plane a-a using transformation equations.



Having known these transformation equations, the concept of the Mohr's circle, and the concept of the octahedral stresses acting on the octahedral planes let us look into some examples on how we compute stresses on any plane if we know the stress at a particular point on a body through the rectangular stress components like σ_x , σ_y and τ_{xy} in two dimensional plane.

The problem is, the state of stress at a point in a body is represented here where we have σ_x as 20MPa, σ_y as 30 MPa and the shearing stress as 20MPa. Now what we will have to do is to find the normal and shear stresses acting on plane aa using transformation equations. So we have to find out the stress on this particular plane. This particular plane is inclined to the vertical at an angle of 30 degrees. So, if we take the normal to this particular plane the normal makes an angle of 30 degrees with x-axis.

So, if we write down the transformation equations for the evaluation of the stresses the 2θ is equal to 60 degrees. We have the stresses acting on a particular body at a point; σ_x as 20, σ_y as 30, and τ_{xy} as 20MPa. We are interested to find stress at a particular plane which is at an angle of 30 degrees with x-axis. We have σ_x is equal to plus 20, σ_y is equal to plus 30, τ_{xy} is equal to plus 20 and 2θ is equal to 60 degrees. The transformation equation as we have observed is equal to σ_x prime which is angle θ is equal to 30 degrees is equal to $(\sigma_x + \sigma_y) \cos^2 \theta + (\sigma_x - \sigma_y) \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta$.

Now if we substitute the values of rectangular stress components then this is equal to $(\sigma_x + \sigma_y) \cos^2 \theta + (\sigma_x - \sigma_y) \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta$ is equal to $(20 + 30) \cos^2 30 + (20 - 30) \sin^2 30 + 2 \times 20 \times \sin 30 \times \cos 30$ that is 25; plus $(\sigma_x - \sigma_y) \sin^2 \theta$ by 2 is equal to $(20 - 30) \sin^2 30$ by 2 is equal to $(-10) \times \frac{9}{16} = -5.625$; plus $2\tau_{xy} \sin \theta \cos \theta$ is equal to $2 \times 20 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = 10\sqrt{3} = 17.32$.

20 minus 30 by 2 is equal to (minus 10 by 2) cos 60 degrees; τ_{xy} is equal to 20 sin 60 degrees. So this is equal to 25 minus 2.5 20 into square root of 3 by 2 so this gives us the value of 22.5 plus 17.32 is equal to 39.82 MPa. This is the value of normal stress on this particular plane which is inclined at an angle of 30 degrees with the vertical.

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The image shows a whiteboard with handwritten mathematical work and a diagram. The diagram depicts a square stress element with a diagonal line representing a plane inclined at 30 degrees to the vertical. The original principal stresses are $\sigma_x = +20$ and $\sigma_y = +30$. The angle between the vertical and the inclined plane is $\theta = 30^\circ$, so the angle between the horizontal and the inclined plane is $2\theta = 60^\circ$. The transformed normal stress $\sigma_{x'}$ is calculated as follows:

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= 25 + \left(\frac{-10}{2}\right) \cos 60^\circ + 20 \sin 60^\circ \\ &= 25 - 2.5 + 20 \frac{\sqrt{3}}{2} \\ &= 22.5 + 17.32 = 39.82 \text{ MPa} \end{aligned}$$

Similarly, we can compute the value of shear stress on that plane which is $\tau_{x'y'}$ is equal to σ_x minus σ_y by 2 sin 2θ plus τ_{xy} cos 2θ . This is equal to minus σ_x is equal to 20, σ_y is equal to minus 30 by 2 sin 60 degrees plus τ_{xy} is equal to 20 cos 60 degrees.

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The image shows a handwritten derivation for the shear stress $\tau_{x'y'}$ on a rotated plane. The steps are as follows:

$$\begin{aligned}\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{20 - 30}{2} \sin 60^\circ + 20 \cos 60^\circ \\ &= 5 \times \frac{\sqrt{3}}{2} + 20 \times \frac{1}{2} \\ &= 4.33 + 10 \\ &= \boxed{14.33 \text{ MPa}}\end{aligned}$$

So this equals to 5 into square root of 3 by 2 plus 20 into 1 by 2 is equal to 4.33 plus 10 plus 14.33 MPa so the value of the normal stress, σ_x prime is equal to 39.82 MPa and the value of shear stress is equals to 14.33 MPa. So this is the solution of this particular problem where we are computing the normal stress and the shearing stresses.

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The slide features the IIT Kharagpur logo and title. The text reads:

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Example Problem - 2

- Solve the Example Problem-1 using Mohr's circle of stress.

The diagram shows a square element with a side length of 30 MPa. The top and bottom faces are subjected to normal stresses of 20 MPa (tension). The left and right faces are subjected to normal stresses of 30 MPa (compression). Shear stresses of 20 MPa are shown acting on all four faces, with arrows pointing towards the center of the element.

Let us look into this example 2 where we will have to solve this particular problem or rather we have to solve this which is represented in 1 using Mohr's circle of stress. In the previous problem we used the transformation **equations for the solution** of the stresses at

any plane. Now we are going to evaluate the stresses on any plane using the Mohr's circle of the stress.

If we look into the state of stress which we had we represent that as σ_x is equal to 20 MPa, σ_y is equal to 30 MPa and τ_{xy} is equal to 20MPa. As we had represented in a Mohr's plane, here this is σ axis, this is τ axis.

First of all if we locate the plane, where the normal stress is 20 and the shearing stress is 20 then we go in this direction as 20 and the positive shear stress as 20. Now this particular shear stress is positive because the shear stress on this plane and the complimentary shear they are making a rotation which is anticlockwise in nature which we call as positive, so this is 20 and this is 20.

Let us represent this plane which is 90 degrees with this x plane. Eventually in Mohr's circle it is 180 degrees so we represent σ_y and the τ as negative, τ is negative because the shearing stress on this plane is causing a rotation which is clockwise in nature so this is negative τ , this is σ_y and this is τ_{xy} . If we join these two points, this particular point is eventually the centre of the Mohr's circle which is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$.

Now if we plot Mohr's circle, this is the point representing x-plane and this is the point which is representing y-plane. This is the maximum normal stress which is σ_1 and this is the minimum normal stress which is σ_2 . Now we are interested to evaluate the stress at a plane, which is making an angle of 30 degrees with x-plane.

Eventually that will make 60 degrees in the Mohr's circle with respect to this. So with reference to this line, say line OA if we represent that plane which is represented by this point say C, this particular angle is 60 degrees. Here we know this magnitude, we can compute this magnitude. This is σ_x OO prime is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$, which is equals to $(20 \text{ plus } 30) \text{ by } 2$, which is equal to 25 MPa.

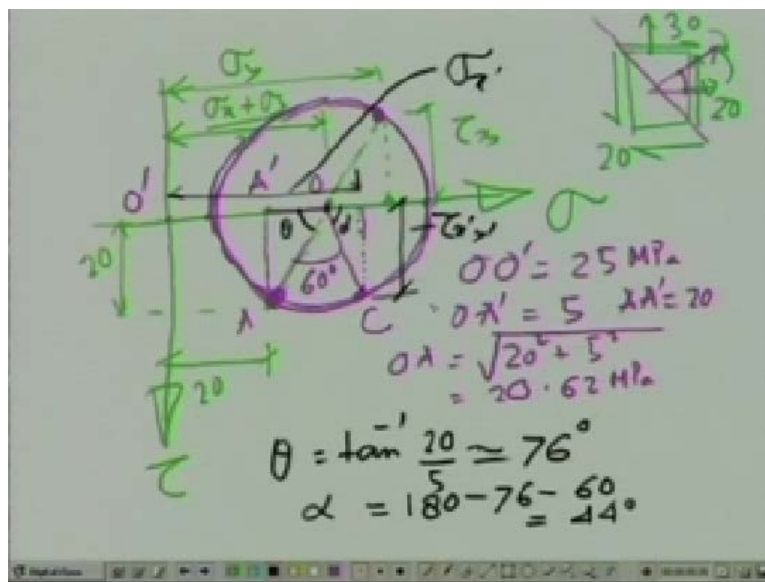
Let us say this is A prime, which is A. OA prime eventually is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ minus σ_x , which is σ_x is 20 here. Eventually OA prime is equal to 5. And AA prime is equal to 20. So the radius OA is equal to square root of 20 square plus 5 square is equal to 20.62 MPa. Now we are interested to find out stress at this particular plane, if we drop a perpendicular to the σ axis, this will gives us the value of σ_x on this plane, that is the value of normal stress, let us call as σ_x prime, if we call this as x prime plane, and this magnitude will gives us the magnitude τ, x prime y prime.

Now if we represent this angle by θ , then θ is equals to $\tan^{-1} \frac{AA \text{ prime}}{OA \text{ prime}}$ and AA prime being 20 and OA prime being 5, so this is $\tan^{-1} \frac{20}{5} \approx 76$ degrees. So if we represent this angle by α , then α is equal to $180 \text{ minus } 76 \text{ minus}$

60 is equal to 44 degrees. OC being the radius, then normal stress σ_x prime is equal to the distance OO prime plus radius $\cos \alpha$.

So the σ_x prime is equal to the central distance; σ_x is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } R \cos \alpha$. R is equal to square root of $[(\sigma_x \text{ minus } \sigma_y) \text{ by } 2]^2 \text{ plus } \tau_{xy}^2$ square 20.62, as we have seen. So σ_x prime is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ is equal to 25 plus $20.62 \cdot 2 \text{ into } \cos \alpha$, $\cos 44$ degrees is 0.72. This is going to give us a value of 39.85 MPa, which is similar to which we have computed through the transformation equation. The shearing stress on this plane, $\tau_x \text{ prime } y \text{ prime}$ from the Mohr's circle, if you look into $\tau_x \text{ prime } y \text{ prime}$ is equal to $R \sin \alpha$ is equal to $20.62 \text{ into } 0.7$ is equal to 14.43 MPa. This is similar to the one which we have computed using the transformation equation.

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From this you can compute the value of normal stress represented through this point which is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ is equal to R. Now $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ is equal to 25, and R is 20.62, that will give the value as 45.62. σ_1 is equal to 45.62. σ_2 is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$, this distance minus the radius which is 25 minus 20.62 which is equal to 4.38. We get the values of maximum and minimum normal stress or the maximum and the minimum principal stresses. The value of the radius gives the maximum value of the positive and negative shear, which is equals to 20.62 MPa.


So from the Mohr's circle we can compute the stress at any plane and the maximum principal stress and the minimum shear stress without even using the transformation equations and without going for the geometrical construction of the Mohr's circle.

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Example Problem - 3

- The state of stress at a point is shown in the figure. Determine the normal and shear stresses acting on the vertical plane using transformation equations.



60 MPa
45°
50 MPa
40 MPa
30 MPa


The state of a stress at a particular point is shown in the figure. Determine the normal and the shear stresses acting on the vertical plane using transformation equations.

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Example Problem - 4


- The state of stress at a point in a stressed body is as shown in figure. Evaluate Principal stresses, maximum shear stresses with the associated normal stresses using Mohr's circle.



20 MPa
30 MPa
10 MPa

This is the state of a point in a stress at a body, you have to evaluate the principal stress and the maximum shear stresses and the associated normal stress using the Mohr's circle.

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Example Problem - 5

- The Principal stresses at a point are $\sigma_1 = (-2 + \sqrt{10})$ Mpa, $\sigma_2 = 1$ Mpa and $\sigma_3 = (-2 - \sqrt{10})$ Mpa. Evaluate the Octahedral stresses.

These are the values of the principal stresses at a particular point; you have to evaluate the octahedral stresses.

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Summary

This lesson included:

- Recapitulations of stress evaluation using transformation equations & Mohr's circle.
- Evaluation of Octahedral stresses
- Examples to demonstrate the evaluation of stresses on any plane at a point.

So in this particular lesson, what we have done is we have recapitulated the aspects which we have discussed in the particular last lesson, and we have evaluated the octahedral stresses and also solved some examples to demonstrate the evaluation of stresses on any plane.

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Question Set 5.0

- What happens to octahedral stresses when first invariant is zero?
- What is the value of the shear stress where maximum normal stress occurs?
- What is the value of normal stress where maximum shear stress occurs?
- Answers will be provided in the next lesson

Here are some more questions:

What happens to octahedral stresses when first invariant is zero?

What is the value of the shear stress where maximum normal stress occurs? What is the value of normal stress where maximum shear stress occurs?