Strength of Materials

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Lecture#40

Springs-II

Welcome to the second lesson of the tenth module which is on springs part 2.

In the previous lesson of this particular module, we have looked into the aspects of different kinds of springs.

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We have discussed a special type of spring which is called as helical spring and we have seen that helical spring is of two types, one is called as close-coiled helical spring and another one is called open-coiled helical springs.

In the previous lesson we have discussed about the close-coiled helical springs and we have looked into how the stresses and the deflections are induced in the springs when they are loaded.

In this particular lesson we are going to discuss about the open-coiled helical springs.

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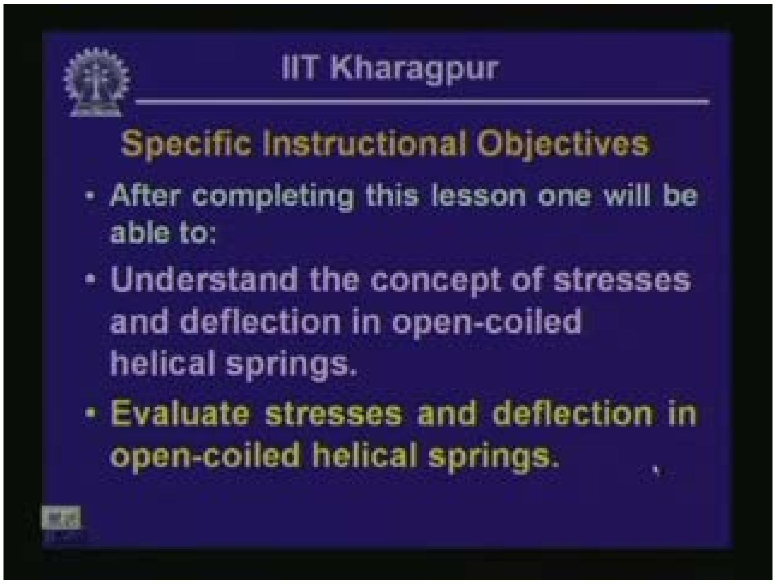

Hence it is expected that once this particular lesson gets completed, one should be in a position to understand the concept of stresses and deflection in open-coiled helical springs and also one should be in a position to evaluate stresses and deflection in open-coiled helical springs which are subjected to loads.

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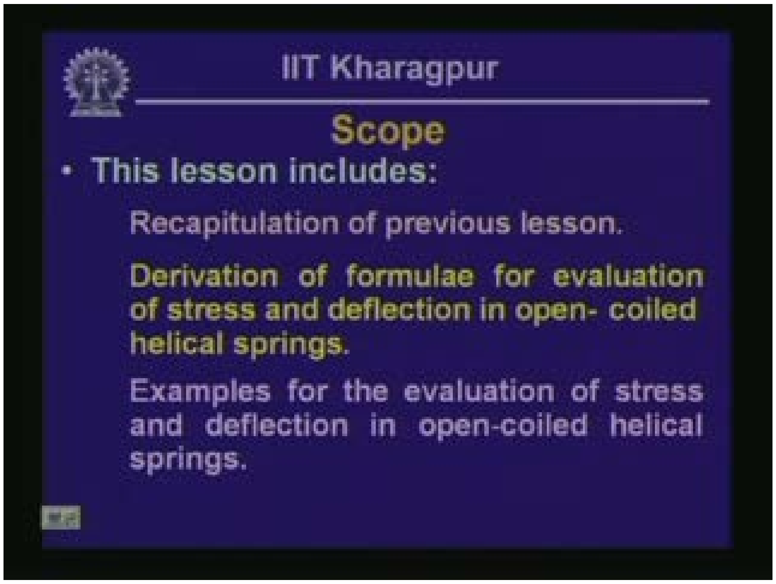

The scope of this particular lesson therefore includes,

- Recapitulation of previous lesson, certain aspects of the previous lesson through the answers to the questions posted last time.
- We will be deriving a formula for evaluating stresses and deflection in open-coiled helical springs.

- We will also be looking into some examples for the evaluation of stress and the deflection in the open-coiled helical springs.

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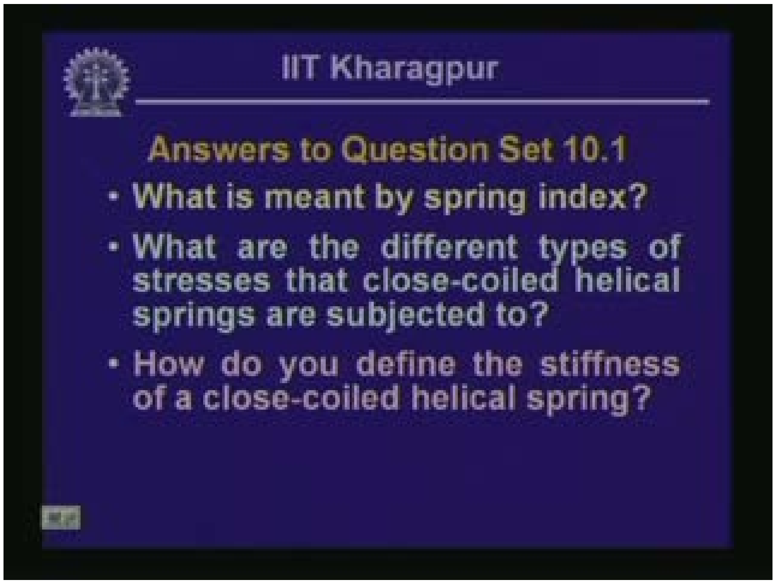

Let us look into the answers of the questions which were posted last time.

The first question was what is meant by spring index?

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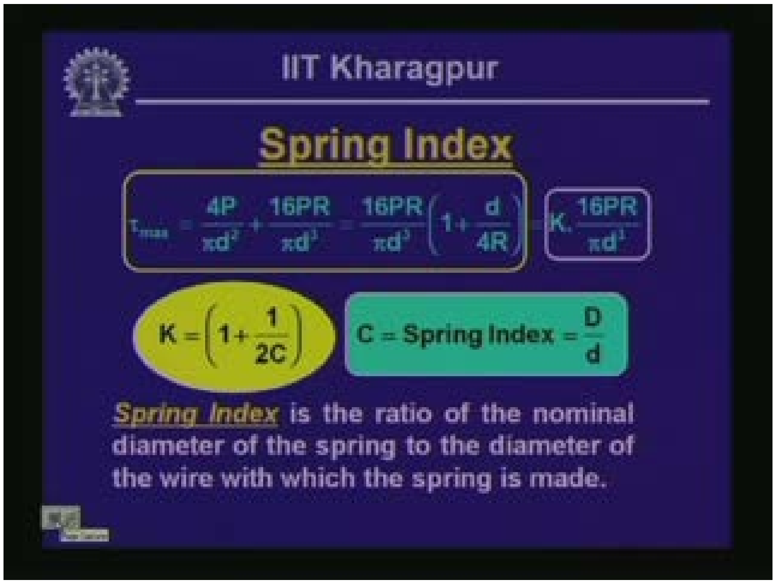

If you remember while discussing about the close-coiled helical springs, we had discussed, how the stresses are induced.

We have seen that primarily the stress which acts on the springs is the direct shearing stress and the twisting moment. Both the forces induce the shearing stress in the member.

The direct shear force induce the shear stress and the twisting moment also induce the shear stress and thereby we get the total shear stresses which we have designated as tau max ($\tau_{\max}$) which is equal to $\frac{16PR}{\pi d^3}$.

You know these terms, P is axial load that is applied in the spring, R is the helix radius or the mean radius of the spring and d is the diameter of the wire with which the spring is manufactured.

Then we came across a term which we have designated as K and if you look into this particular parameter K, it equals to $1+\frac{1}{2C}$.

Here in the slide if you look into the formula: $1+\frac{d}{4R}$, 4R is nothing but 2 into twice R and twice R can be written as D, the mean diameter of the spring helix and thereby we can write this particular parameter as $1+\frac{d}{2D}$.

As you can see here the formula for K has twice C. And C therefore stands as D by d and this particular ratio is called as spring index. Therefore we define the spring index as the ratio of the nominal diameter of the spring or the helix diameter, to the diameter of the wire with which the spring is made. This is called as spring index.

Let us look into the second question.

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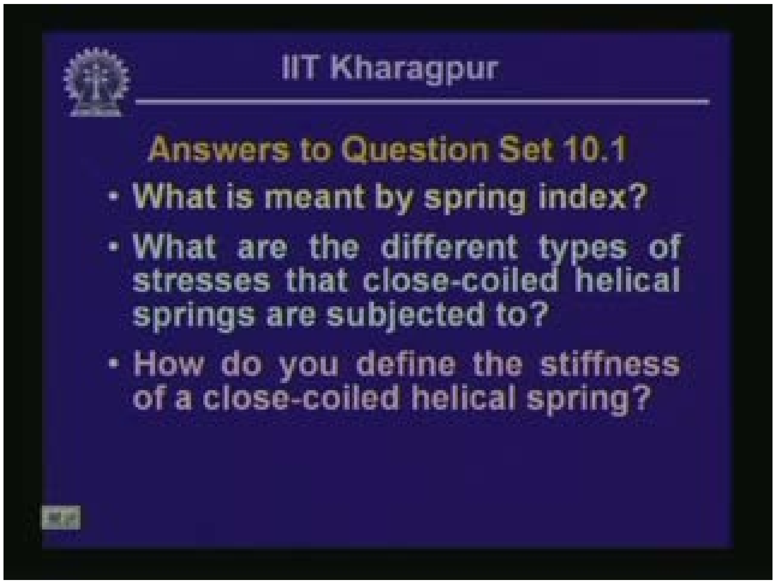

What are the different types of stresses that close-coiled helical springs are subjected to?

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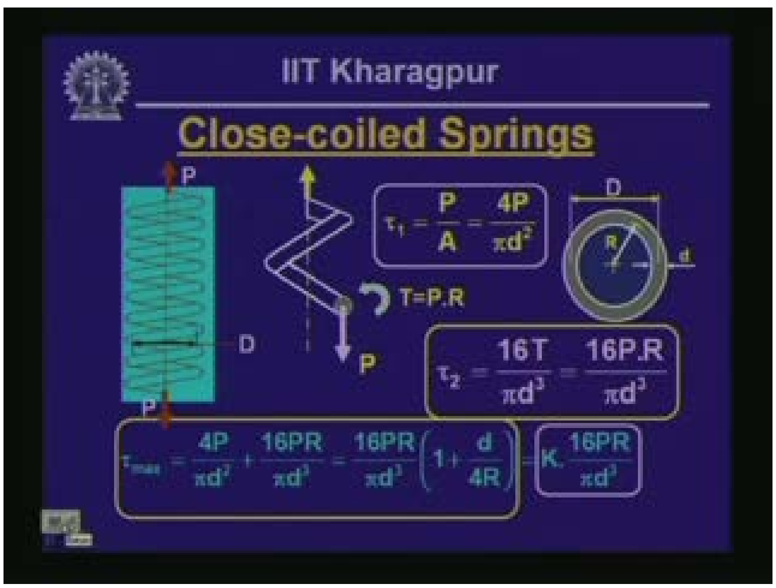

As we have seen that the close-coiled helical springs are subjected to two kinds of forces. One is the direct force P and another is the twisting moment T. Incase of close-coiled helical springs they are wound in such a way that each helix is lying in one plane and if that happens then the force transfer to the wire is the axial force P which is transferred to the central part of the wire along with the twisting moment T which is equal to P times R.

These two forces P and this twisting moment, introduce stresses in the spring wire and this stresses are tau1 ($\tau 1$) which is P by A, the direct shearing stress because of the shearing force P which is equal to $\frac{4}{\pi d^2}$ and because of this twisting moment T which is equals to P times R, we get a shearing stress tau which is equals to $\frac{16T}{\pi d^3}$.

As you know that T by J is equals to tau(τ) by rho(ρ). Tau is therefore $\frac{T\rho}{J}$ and ρ is $\frac{d}{2}$ and J is $\frac{\pi d^4}{32}$.

If we substitute the value of rho (ρ) and J we get this as $\frac{16T}{\pi d^3}$ and T being equal to P times R. The shearing stress tau is equal to $\frac{16PR}{\pi d^3}$.

These are the two stresses that a close-coiled helical spring is subjected to, which is $\frac{4P}{\pi d^2}$ is the direct shear and $\frac{16PR}{\pi d^3}$ is the shearing stress which is generating from the twisting moment T.

Therefore the maximum shearing stress that a wire up the close-coiled helical spring is subjected to is equal to $\frac{16PR}{\pi d^3}\left[1+\frac{d}{4R}\right]$ which we have designated as K. This is the value of the stress that a close-coiled helical spring is subjected to.

The last question which we had is, how do you define the stiffness of a close-coiled helical spring?

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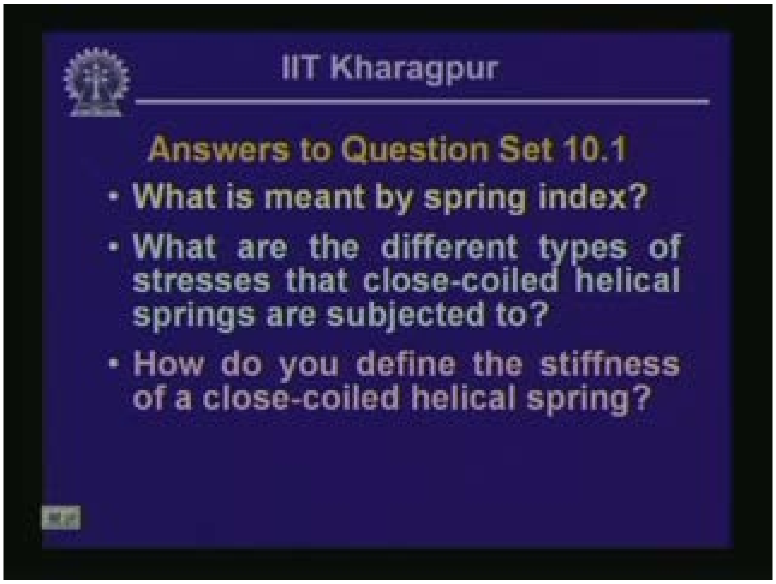

To answer this question let us look into the expression for the deflection that we had derived.

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If you remember for the close-coiled helical spring we had the value of the deflection delta because of the action of the axial load P was $\frac{64PR^3N}{Gd^4}$, where these terms again;

P is the axial force.

R is the radius of the spring,

N is the number of turns in the spring coil.

G is the shear modules and d is the diameter of the wire.

If we write this particular expression in a little different way, if we represent P as a function of the displacement delta, then we get the expression P is equal to $\frac{Gd^4}{64PR^3N}\Delta$.

Generally we define stiffness as the force requires producing unit displacement.

Here, if we replace delta as unit, the force require producing this unit displacement, we define the coefficient as the stiffness.

So if we write down this expression as P equals to K_s times Δ, when delta is equals to 1, the force which we get is equal to the stiffness of the spring. Thereby the spring stiffness K_s defined as $\frac{Gd^4}{64PR^3N}$.

In fact we had talked about the spring index which was also of the same form that P by delta which was equals to $\frac{Gd^4}{64R^3N}$.

This is what we define as the spring stiffness.

Hence these were the answers of the questions which were posted last time.

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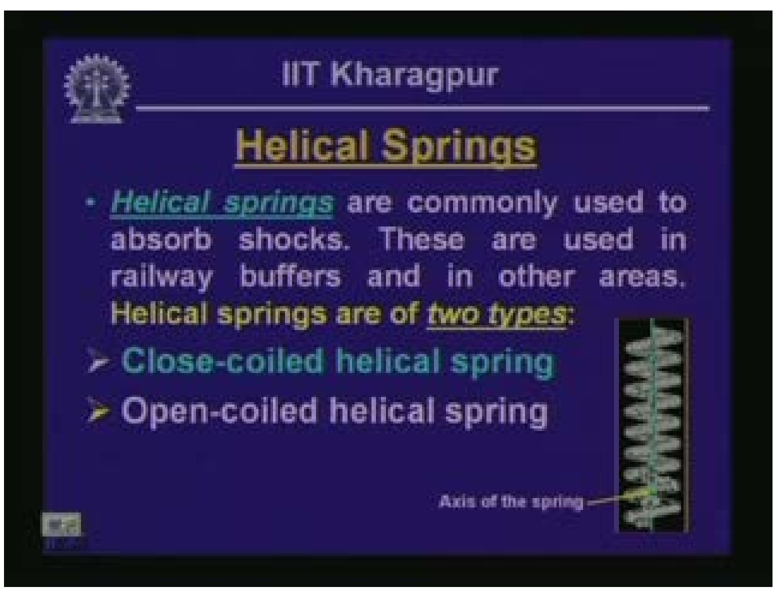

Let us look into the aspects of the open-coiled helical spring. As we know that the helical springs are commonly used to absorb shocks and these are used in railway buffers and many other areas.

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Take a look into this springs shown in the above slide are having two different wounds.

In one of the spring the wire is wounded in such a way that they are at a very close distance, the pitch between the two wounds are very close. This resembles the great extend to the close-coiled helical spring. In fact in reality the pitch are much less and they are almost in the same horizontal plane.

Whereas if you look into the other spring where the pitch between the two helix is substantially large in comparison to the previous one and here they do not lie in the same plane.

If we consider one such helix from one point to the other point, they do not lie in the same plane. The plane is inclined with respect to the horizontal one. This is basically the difference between the close-coiled helical spring and the open-coiled helical spring and thereby there is difference in the load transfer mechanism as well.

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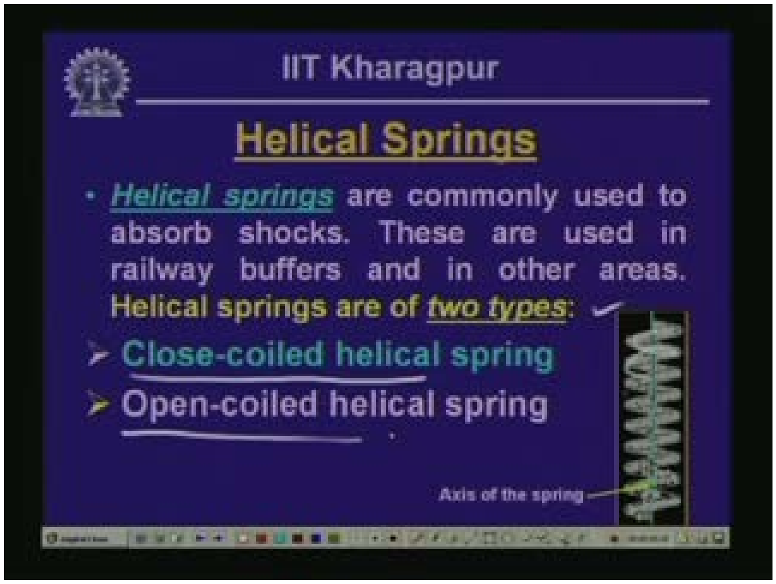

We have seen the forces that the close-coiled springs are subjected to.

Now we will look into what are the forces these open-coiled springs will be subjected to.

Here is the definition of the open-coiled helical spring.

The wires, with which open-coiled helical springs are formed, are wound so that each turn of the spring is not lying in one plane as the pitch of the coils is larger.

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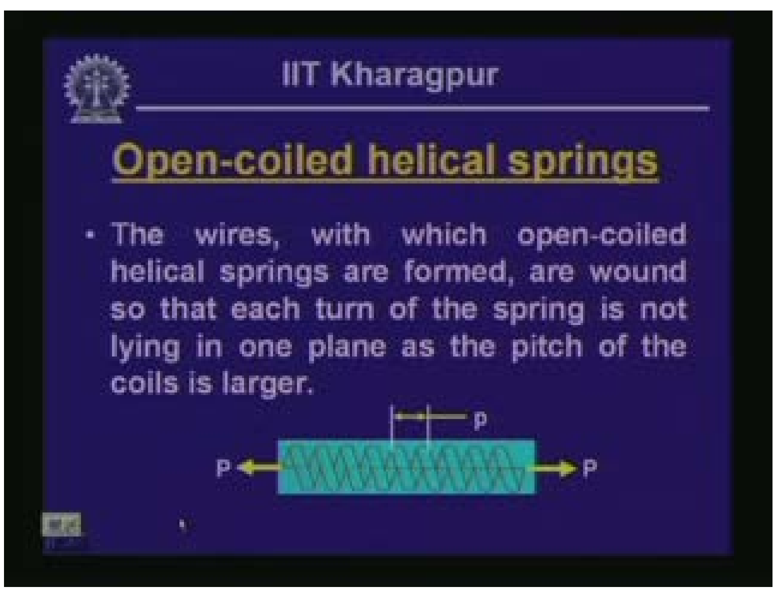

As you can see over here that the pitch is the distance between the two helix.

We have defined that pitch as ‘p’.

In case of close-coiled spring this p is very small so that virtually one of the wound lies in the same plane. Whereas in case of open-coiled, they do not lay in the same plane and the helix has an angle which is inclined with respect to the horizontal plane.

When we talk about the load transfer mechanism you will find the forces that an open-coiled spring will be subjected to is different from the closed one.

Let us look into that in detail.

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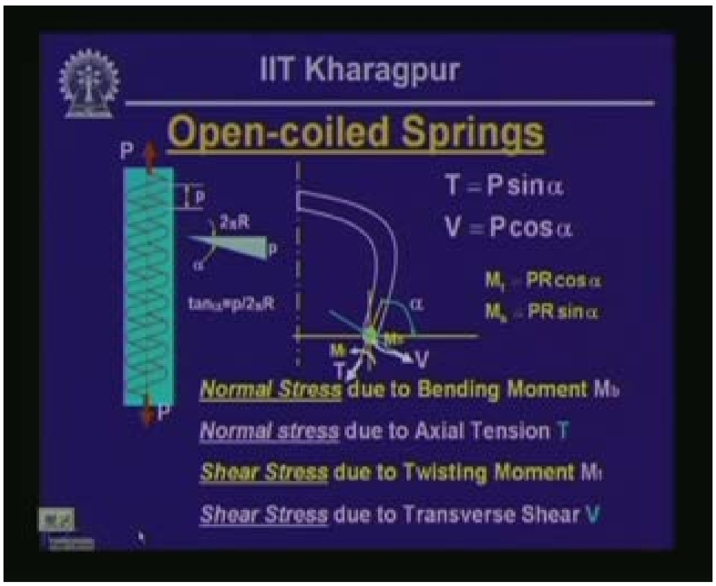

As we had seen in case of close-coiled springs that a spring is subjected to load p may be the top is held against some support.

If we consider one coil in the spring and thereby if R is the mean radius of this spring, then the length of this particular plane will be $2\pi R$ (a small helix of the spring has been marked and indicated in the below slide). If they are in the same plane the length of that wound will be $2\pi R$. But since there is an inclination, there is the helix which is inclined with respect to the horizontal plane because of the larger pitch, the length is going to be larger than the $2\pi R$.

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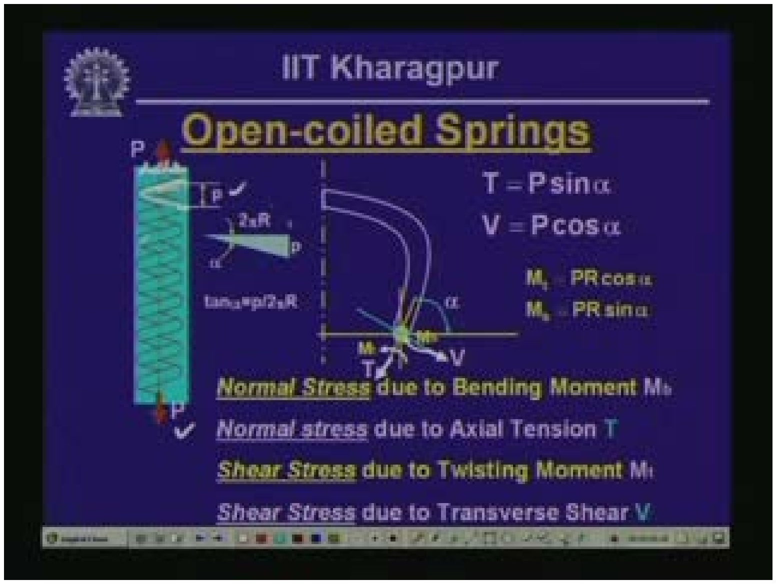


Let us look into the small triangle which is shown in the above slide.

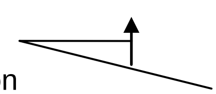
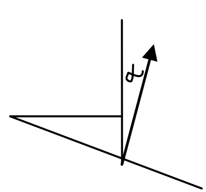
For this particular triangle, let us define the right side portion of the triangle as pitch P and the horizontal distance as $2\pi R$, if the wound lies in the same plane. Since it has an inclination, let us assume that the length gets extended and the helix angle between these two planes is equal to alpha.

From this particular triangle we can say tan alpha ($\tan_{\alpha}$) is equal to P divided by $2\pi R$ and that is how we have defined the helix angle alpha with which this particular wire is wound.

Let us look into the forces that this will be subjected to.

First if I transfer this axial force P to the straight coil which is in one plane will be subjected to a direct force P and a twisting moment P times R and the vectorial direction will be towards right side.

This is the position () of the open-coiled spring because of the inclined plane.

Basically, this is the direction of the axis of the spring wire  and the force which is acting is in this vertical direction , so perpendicular to this makes an angle alpha 

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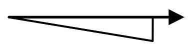
So we get the component as $P\cos\alpha$ and along the axis will be $P\sin\alpha$.

The component which is acting as $P\sin\alpha$ along the axis of the wire that will give us a tensile pull and the force which is normal to this cross section which is $P\cos\alpha$ will give as a shearing force V.

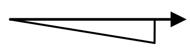
So the force P now contributes to two actions, one is the tensile pull T which is given by $P\sin\alpha$ and the shearing force which is acting perpendicular to it gives rise to V which is $P\cos\alpha$.

These are the two forces that the spring will be subjected to because of the load P.

P also has introduced a twisting moment which is P into R and the vectorial direction of which is  .

If we take the component of the moment along the position of the spring coil  and perpendicular to it which is $PR\cos\alpha$ will be introducing a twisting moment in the wire.

This is the action of the moment which we have defined as M_t which is equals to $PR\cos\alpha$.

If we take this vectorial direction  moment, the component perpendicular to this wire axis is equal to $PR\sin\alpha$ and this is going to cause a moment which is basically bending moment.

The first one we have is a twisting moment in the wire and the other one is going to cause a bending moment, because the vectorial direction is perpendicular to the wire axis and the moment which is acting is a bending moment in the wire.

The twisting moment that it was acting in that plane where the wound is perfectly in the plane has two components one is along the member axis which is causing twisting moment in the wire and other one, the vectorial direction which is perpendicular to the wire axis is a bending moment to the wire.

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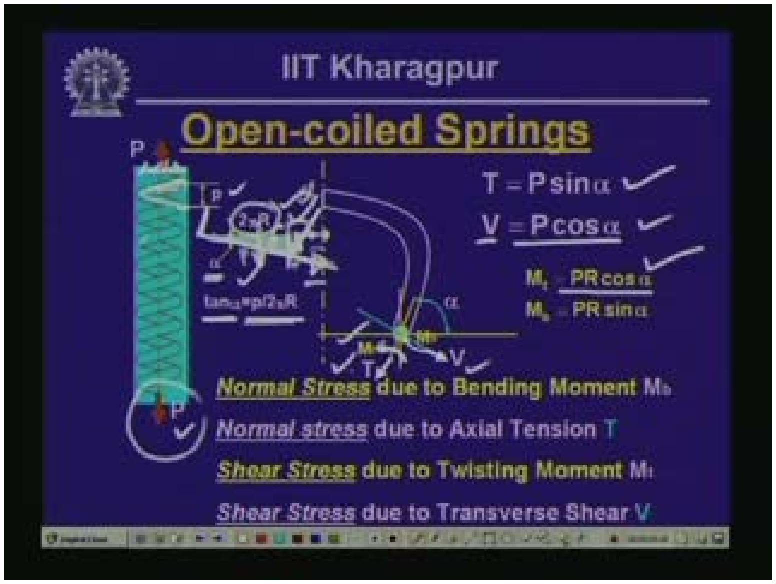

Hence as you can see that the force P has two components, one is T another one is V. T is the direct tension that is causing in the wire and V is the shear that is being caused in the wire because of P which are $P\sin\alpha$ and $P\cos\alpha$.

The twisting moment P into R will have two components one is causing twisting moment which is equals to $PR\cos\alpha$ and another one is causing a bending moment which is equals to $PR\sin\alpha$.

Thus the axial force P introduces four force components in the open-coiled springs and they are the tensile pull in the wire T, a shearing force V, the twisting moment M_t and a bending moment M_b.

The stresses corresponding to each of these forces will be, because of the bending moment M_b there will be normal stress and which is $\sigma = \frac{MY}{I}$. This is the bending normal stress.

Then there will be normal stress because of the axial tension T, T divided by cross sectional area will give the normal stress.

Then because of the direct shear V, we will have the shear stress which we call as V divided by the cross sectional area.

And the shear stress because of the twisting moment M_t will be, as we know that $\frac{T}{J}=\frac{\tau}{\rho}$, so $\tau=\frac{T\rho}{J}$ will introduce a shearing stress.

So as you can see that the axial force which is acting in the a open-coiled helical spring will be introducing four components of the forces which are the tensile pull along the wire, the shearing force, the twisting moment in the wire and a bending moment in the wire.

All these four force components will introduce stresses in the member which are the normal stresses and the shear stresses.

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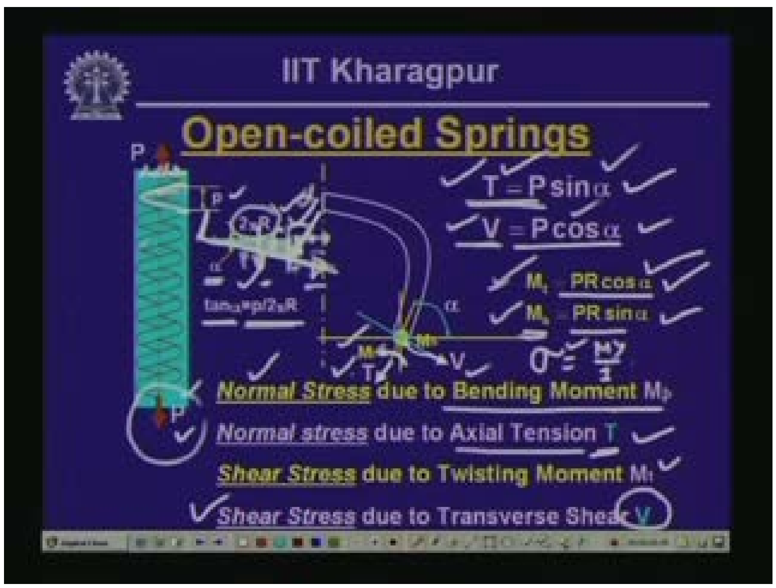

These two normal stresses which are resulting from the bending moment and the direct tensile pull, we can have a combined normal stress,

The shearing stresses which are resulting from the twisting moment and the transverse shear, we can have a resulting shear stress.

From the normal and the shearing stress we can compute the resulting value of the maximum normal stress and the maximum shear stress using more transformation equations.

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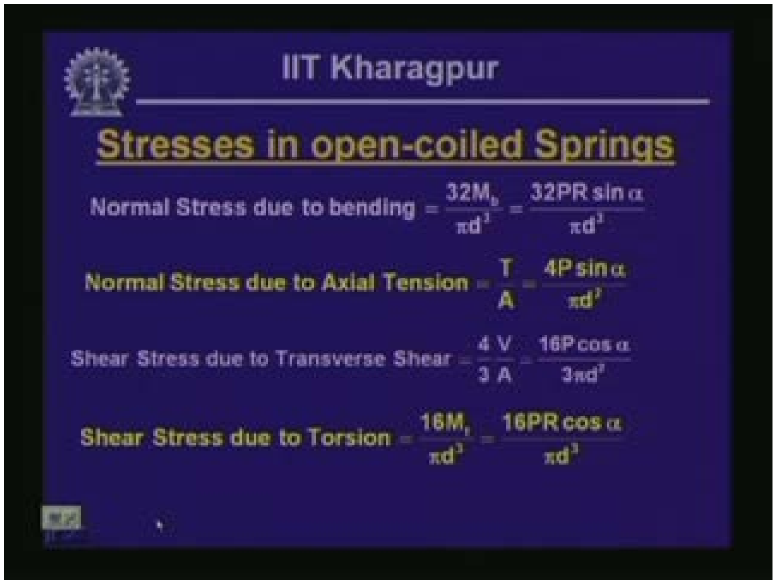

The resulting stresses are shown in the above slide.

To explain these in detail:

First one, we have a normal stress due to bending which is equal to $\frac{32M_b}{\pi d^3}$, because it is a circular in cross section and we know that $\sigma = \frac{MY}{I}$.

As you know for a circular cross section I is equals to $\frac{\pi d^4}{64}$ and $Y = \frac{d}{2}$, so sigma equal to, $\frac{M \cdot d/2}{\pi d^4/64}$ and this gives us $\frac{32M_b}{\pi d^3}$ and M_b as you have seen is equals to $PR\sin\alpha$.

If you substitute, it gives $\frac{32PR\sin\alpha}{\pi d^3}$ as normal stress due to bending.

The second one, we have normal stress due to axial tension is equals to the tensile pull divided by the cross sectional area. Tensile pull as we have seen is equals to $P\sin\alpha$ and cross sectional area is $\frac{\pi d^2}{4}$. So the normal stress due to axial tension is $\frac{4P\sin\alpha}{\pi d^2}$.

The third one, shear stress is due to transverse shear. As we have seen that the transverse shear equals to V which is $P\cos\alpha$ and the maximum shear which you get at the diameter of the circular section is equal to $\frac{4V}{3A}$, as $A = \frac{\pi d^2}{4}$, the shear stress due to transverse shear V is $\frac{16P\cos\alpha}{3\pi d^2}$.

The last one, shearing stress due to the torsion which is $\frac{T}{J} = \frac{\tau}{\rho}$, so $\tau = \frac{T\rho}{J}$ and $\rho = \frac{d}{2}$ and $J = \frac{\pi d^4}{32}$. This gives raise to $\frac{16M_t}{\pi d^3}$ and twisting moment we had seen as $\pi PR\cos\alpha$. So shearing stress due to torsion is $\frac{16PR\cos\alpha}{\pi d^3}$.

If we combine the two stresses such as the normal stress with the normal stress, we will have the resulting normal stress which we call $\sigma_{\max}$. And from the two shear stress we will have the $\tau_{\max}$.

We can make use of this $\sigma_{\max}$ and $\tau_{\max}$ to find out what will be the maximum and the minimum principal stresses and the maximum shear stress because of these two actions.

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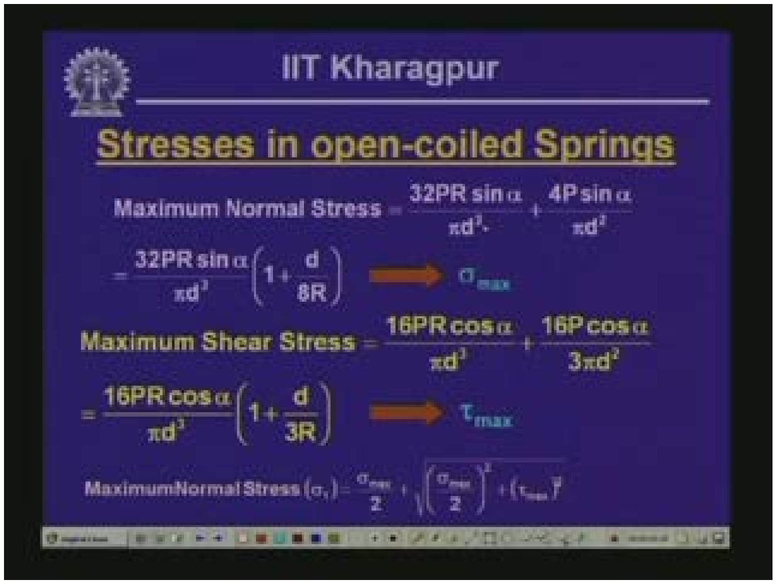

Here in the above slide, it is indicated that the maximum normal stress is equal to $\frac{32PR\sin\alpha}{\pi d^3}$. This is because of the bending and $\frac{4P\sin\alpha}{\pi d^2}$ is because of the axial tensile pull.

If we combine these two we get this as $\frac{32PR\sin\alpha}{\pi d^3}$ which is because of the bending part times $\left(1+\frac{d}{8R}\right)$.

If you look into this particular $\frac{d}{8R}$, 8R means it is four times 2R, twice R is D the diameter of the spring. Hence, $\frac{d}{8R} = \frac{d}{4D}$ and this particular ratio is very small in comparison to 1.

This indicates that the normal stress because of the bending is much higher then the normal stress because of the axial tensile pull. So the contribution of the axial tensile pull in the normal stress is very insignificant in comparison to the normal stress that is being produced by the bending.

Likewise if we combine the shear stresses, $\frac{16PR\cos\alpha}{\pi d^3}$ is the shear stress because of the twisting moment and $\frac{16P\cos\alpha}{3\pi d^2}$ is the shear stress which we get because of the transverse shear V.

If we combine these together then $\frac{16PR\cos\alpha}{\pi d^3}$ is the shearing stress because of the twisting moment and $\left(1+\frac{d}{3R}\right)$, again here if you look into $\frac{d}{3R}$ ratio is very small in comparison to 1 and thereby the contribution in the shear stress because of the twisting moment is much higher than the contribution of the direct shear stress because of the transverse shear V.

In fact subsequently we look into for evaluation of the deflection of the spring because of the axial load. We have computed the value of the deflection primarily from the actions of the twisting moment and the bending moment, because the actions of the axial tensile pull and the transverse shear force are very small compared to twisting and bending moments.

Once we have the maximum value of the sigma which we have called this as $\sigma_{\max}$ which is the sum of the two normal stresses and sum of the two shear stresses we have called them as $\tau_{\max}$, we can compute the maximum normal stress σ_1 which is equals to $\frac{\sigma_{\max}}{2}+\sqrt{\left(\frac{\sigma_{\max}}{2}\right)^2+\left(\tau_{\max}\right)^2}$

The value under the root is the maximum shear stress that will be generated in the spring wire.

This expression as we had seen earlier that we can obtain from the transformation equations or we can evaluate from the Mohr's circle.

In the Mohr's circle if we plot the sigma and tau because sigma y is zero. If you join them together then we get this as sigma by two and this as the radius is sigma by two square plus tau square. This will give as the value of σ_1.

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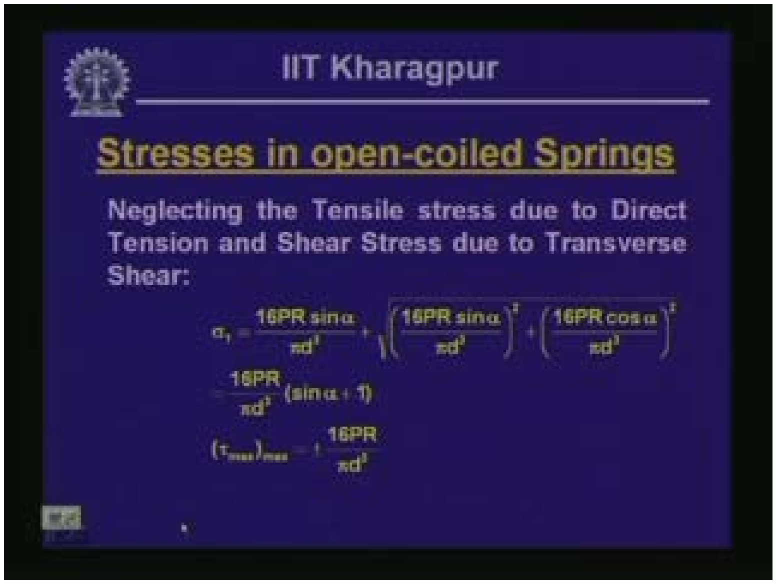

Now, if we neglect the axial tensile pull due to direct tension and the shear stress due to the transverse shear then the value of the maximum normal stress σ_1 we can write as follows:

The normal stress which is getting generated because of the bending which is $(\frac{32PR\sin\alpha}{\pi d^3})/2$, becomes $\frac{16PR\sin\alpha}{\pi d^3}$, and this tau is getting generated from the twisting moment which is $PR\cos\alpha$ gives you,

$$\frac{16PR\sin\alpha}{\pi d^3} + \sqrt{\left(\frac{16PR\sin\alpha}{\pi d^3}\right)^2 + \left(\frac{16PR\cos\alpha}{\pi d^3}\right)^2}$$

And if we take out this $\frac{16PR}{\pi d^3}$ from the root, we are left with sin square alpha plus cos square alpha which is equal to 1.

Hence this gives us $\frac{16PR}{\pi d^3}(\sin\alpha + 1)$, which is the value of the maximum normal stress.

As you know that this particular part which is under the root gives us the value of the maximum shearing stress. So the maximum shearing stress is equals to $\frac{16PR}{\pi d^3}$.

These are the two values of maximum normal stress and the maximum shear stress when we disregard the tensile stress because of the direct tension and the shearing stress due to the transverse shear.

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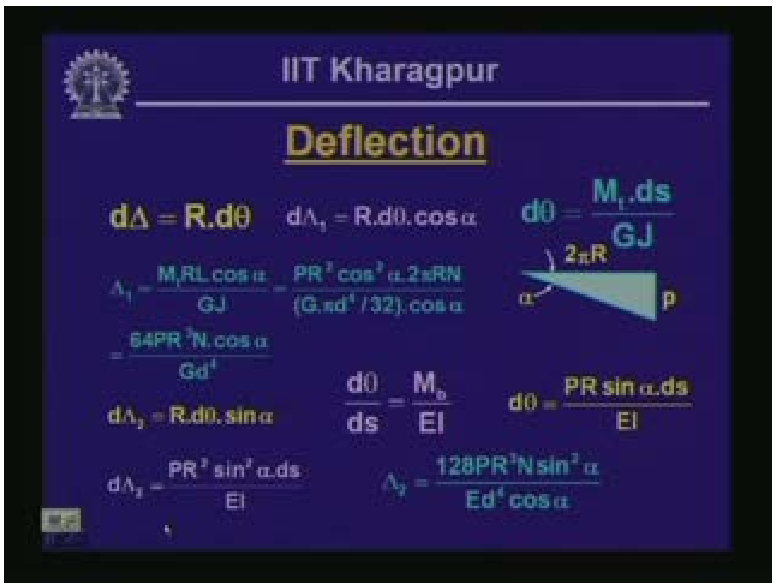

Having looked at the stresses, let us look into the aspects of the deflection that it will be occurring in an open-coiled helical spring.

As we had seen that incase of close-coiled helical spring that d delta is equals to R times d theta ($d\Delta = R \cdot d\theta$).

If you remember that the close-coiled helical springs, when it was subjected to the axial pull, we had evaluated the vertical component of the deflection that it is undergoing because of the twisting moment. There of course we had disregarded the effect of the transverse shear, we had considered the deflection because of the twisting moment.

And we had shown you that because of the twisting moment, it undergoes a vertical deflection which is equals to $R \cdot d\theta$, where d theta is the rotation that is being generated because of the twisting moment T, which is P times R.

Let us take the above as our basis to evaluate the deflection in open-coiled spring. We have seen that the $d\Delta$ as $R\cdot d\theta$ for a close coil. The difference between the close coil and open coil is the helix angle. In the open-coiled spring, the angle which is very large, and the length $2\pi R$ virtually becomes the length of hypotenuse side of the triangle (shown in the slide).

It has an inclination alpha. The deflection which we had is equals to $R\cdot d\theta$. But in this particular open-coil, the deflection is equal to $R\cdot d\theta\cos\alpha$ which is because of the twisting moment T and because of the bending moment it will have $R\cdot d\theta\sin\alpha$.

As we have seen that $d\theta$ is the rotation of the spring, because of the twisting moment M_t and $d\theta$ is equals to $\frac{M_t ds}{GJ}$, where ds is the length along the spring wire.

M_t the twisting moment which we have seen as $PR\cos\alpha$.

If we substitute in the equation $d\Delta_1 = R\cdot d\theta\cos\alpha$, for $d\theta$ as $\frac{M_t ds}{GJ}$, then we have $\frac{M_t R}{GJ}\cdot\cos\alpha$.

Here, the rotation $d\theta$ is the small increment in the vertical deflection $d\Delta_1$ which is caused by the small element ***ds***.

If we like to get the whole deflection over the entire spring which is integral of $d\Delta_1$ over zero to L, ($\int_0^L d\Delta_1$) then we will have ds $\int_0^L ds = L$ which is the length.

When the wire is wound in one plane the length of the wire is equals to $2\pi R$. Because of this inclination it has taken this particular length (hypotenuse side). Let us call this length as L'. So $L'\cos\alpha$ is equals to $2\pi R$.

L' therefore is equal to $\frac{2\pi R}{\cos\alpha}$.

If there are N number of turns, then the total length of the spring will be $2\pi R$ into N and that divided by the $\cos\alpha$ will give us the final length of the spring.

Here we have substituted then for integral ds as l as you can see in this expression which is $\frac{M_t RL\cos\alpha}{GJ}$ and for M_t, we have substituted as $PR\cos\alpha$ which gives as $PR^2\cos^2\alpha$ and for L we have substituted this as $\frac{2\pi RN}{\cos\alpha}$.

Once we substitute the J as $\frac{\pi d^4}{32}$, we get $\Delta_1 = \frac{PR^2\cos^2\alpha}{(G.\pi d^4/32)}\frac{2\pi RN}{\cos\alpha}$. If we simplify this particular expression, we get $\frac{64PR^3N\cos\alpha}{Gd^4}$.

This is the value of Δ_1 the deflection that the spring undergoes because of the twisting moment M_t which is equals to $PR\cos\alpha$.

Now, because of the bending moment which is equal to $PR\sin\alpha$, will cause the deflection in the vertical direction which is $d\Delta_2 = R.d\theta.\sin\alpha$.

In this particular case, as we know that $\frac{M}{I}$ is equals to $\frac{\sigma}{Y} = \frac{E}{R}$.

And $\frac{1}{R}$ is nothing but curvature which is equals to $\frac{d\theta}{ds}$.

So $\frac{d\theta}{ds} = \frac{M_b}{EI}$, here M_b is the bending moment and therefore $d\theta = \frac{M_b.ds}{EI}$.

In $R.d\theta.\sin\alpha$, there, if we replace $d\theta$ as $\frac{M_b.ds}{EI}$ and hence $d\Delta_2 = \frac{R.M_b.ds.\sin\alpha}{EI}$, if we integrate then $d\Delta_2$ becomes Δ_2 and integral ds will give us the length and length will be again, $\frac{2\pi RN}{\cos\alpha}$.

If we substitute M, ds and I, we get the value of $\Delta_2 = \frac{128PR^3N\sin^2\alpha}{Ed^4\cos\alpha}$ and as you know that the value of the bending moment M is equals to $PR\sin\alpha$ and here we had $\sin\alpha$, that gave $\sin^2\alpha$ and I is $\frac{\pi d^4}{64}$.

If we combine the value of Δ_2 with Δ_1 that we have from the twisting moment, we get the total deflection that will be caused in the spring because of the twisting moment and the bending moment.

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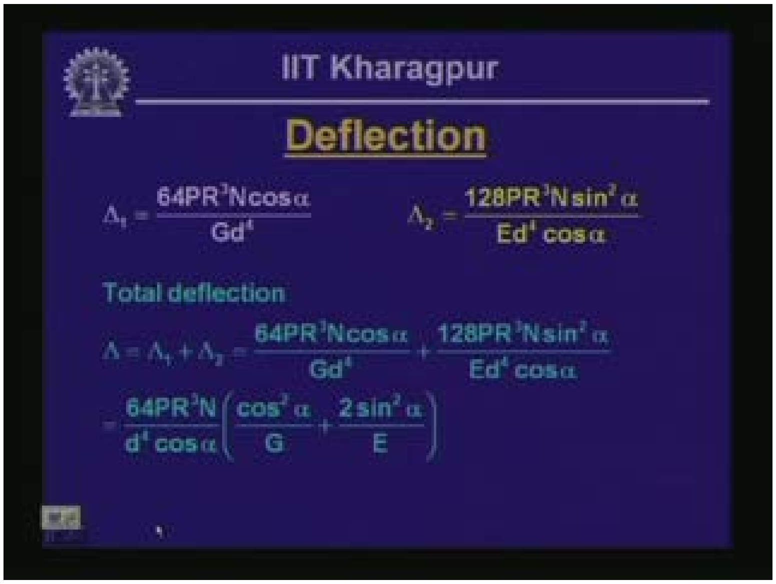

Here are the two expressions that we have obtained for Δ_1 because of the twisting moment M_t and Δ_2 because of the bending moment M_b.

Note that we have disregarded the deflection in the spring because of the axial tensile pull and the transverse shear.

As we have seen that the effect of the tensile pull and the transverse shear is very small in comparison to the twisting moment and the bending moment and accordingly we have disregarded those components.

As you can see that the Δ_1, the deflection because of the twisting moment is $\Delta_1 = \dfrac{64PR^3N\cos\alpha}{Gd^4}$ and the deflection because of the bending moment which is $\Delta_2 = \dfrac{128PR^3N\sin^2\alpha}{Ed^4\cos\alpha}$.

The total deflection will be sum of these two which is $\Delta_1 + \Delta_2$ equals to $\dfrac{64PR^3N}{d^4\cos\alpha}\left(\dfrac{\cos^2\alpha}{G} + \dfrac{2\sin^2\alpha}{E}\right)$ and as you know, P is the axial pull, R is the mean radius of the spring coil, N is the number of turns and d is the diameter of the wire with which the spring is formed, G is the shear modulus and E is the modulus of elasticity and alpha is the helix angle which is tan inverse P by $2\pi R$.

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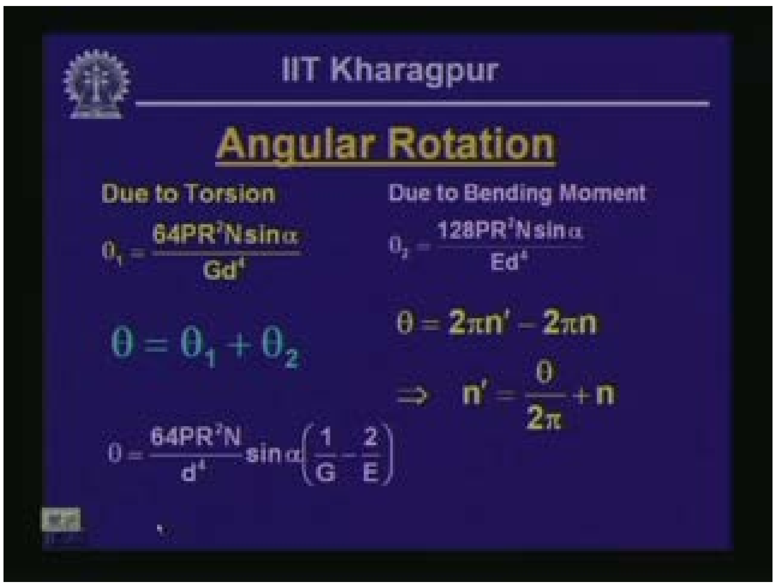

Now, because of this load, not only the spring will undergo a deflection, it will under angular rotation as well.

(Refer Slide Time: 34:42 - 35:07)

If we hold the spring as shown in the above slide and the load is applied at the bottom of this spring, then the spring will be subjected to deflection and also it will undergo angular rotation.

We need to find out the angular rotation and because of this angular rotation, it will try to increase this wound and bending will try to reduce this wound.

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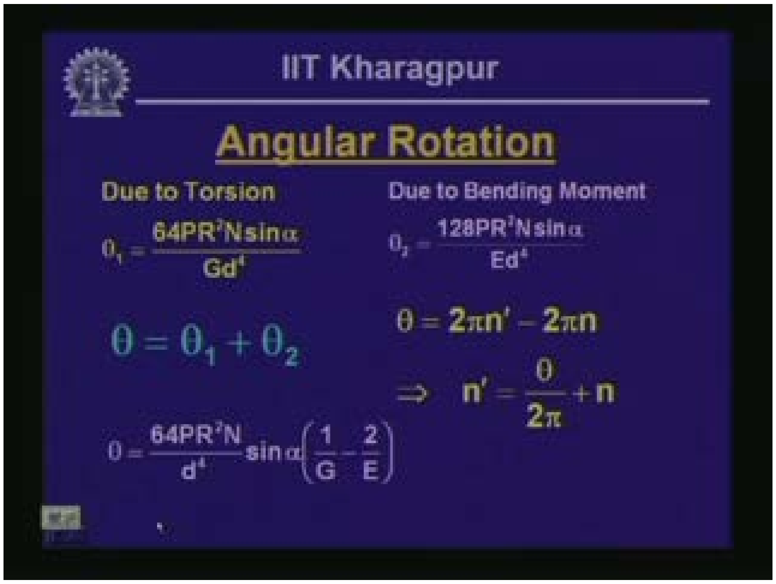

We need to find out the value of the θ angular rotation.

Due to twisting moment, we will have the rotation $\theta_1 = \frac{64PR^2N\sin\alpha}{Gd^4}$.

And due to bending moment will have the rotation which is $\theta_2 = \dfrac{128PR^2N\sin\alpha}{Ed^4}$.

Algebraically, we can add them up as $\theta_1 + \theta_2$.

But as we have noticed that because of the twisting moment the angular rotation will have a clockwise movement which will try to increase the number of wounds and in case of bending it will have an anti clockwise movement and that will try to unwind the spring.

So the final angular rotation with respect to the top will be the difference between the two which is $\theta_1 - \theta_2$ and if we try to find out the final θ, which is $\theta = \dfrac{64PR^2N}{d^4}\sin\alpha\left(\dfrac{1}{G} - \dfrac{2}{E}\right)$.

Now, because of this angular rotation, it is expected that there is going to be increasing the number of wounds. If we call this as n', then $2\pi n'$ is the rotation that it undergoes in a deformed state and earlier it was $2\pi N$.

That increase is $2\pi n' - 2\pi N$ from which we can compute that increase in the number of wounds which is n'. Where n' is equal to $\dfrac{\theta}{2\pi} + n$.

'n' is the number of wounds that the spring had originally and because of the application of the load P, this particular spring has undergone an angular rotation θ. This will give us the number of increase in the wound which is n' because of the axial load P applied on the spring.

Hence these are the stresses and the deflection on the rotation that an open-coiled spring is subjected to.

We have now seen the difference between the close-coiled helical spring and the open-coiled helical spring. We have seen the load transfer mechanism into these two types of system.

In case of close-coiled helical spring the wound is such that virtually the helix lies in one plane and thereby it is subjected to a direct shear and a twisting moment which is P times R.

Where as in case of open-coiled helical spring because the helix is under angle with respect to the horizontal plane and thereby the axial force which is acting through the center of the helix introduces four components of the forces. They are the axial tensile pull in the wire, the transverse shear in the wire, the twisting moment and a bending moment and these four forces caused four stresses which we have seen as individual normal and the shearing stresses.

From those individual normal and the shearing stresses we computed the maximum value of the normal stress and the shearing stress that the wire will be subjected to. Also consequently because of the application of the load we have seen that how the spring undergoes deflection and the angular rotation, which can be computed, which can increase the number of wounds or decrease the number of wounds depending on the application of the load.

Let us look into some examples where in we can make use of this formula for evaluating the stresses in open-coiled helical spring.
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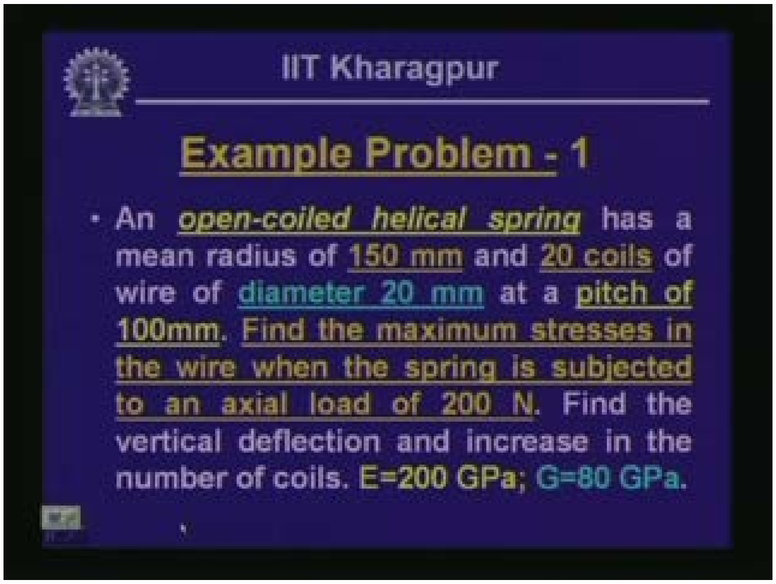

The first example is that “an open-coiled helical spring has a mean radius of 150mm, which is the value of R is 150mm. And it has 20 coils, which is the number of turns N is 20 and the diameter of the wire is 20mm at a pitch of 100mm which is the p.

We will have to find out the maximum stresses in the wire when the spring is subjected to an axial load of 200 newton. Also you will have to find the vertical deflection and increase in the number of coils.

The value of E is given as 200GPa and value of G the shear modulus as 80GPa.”

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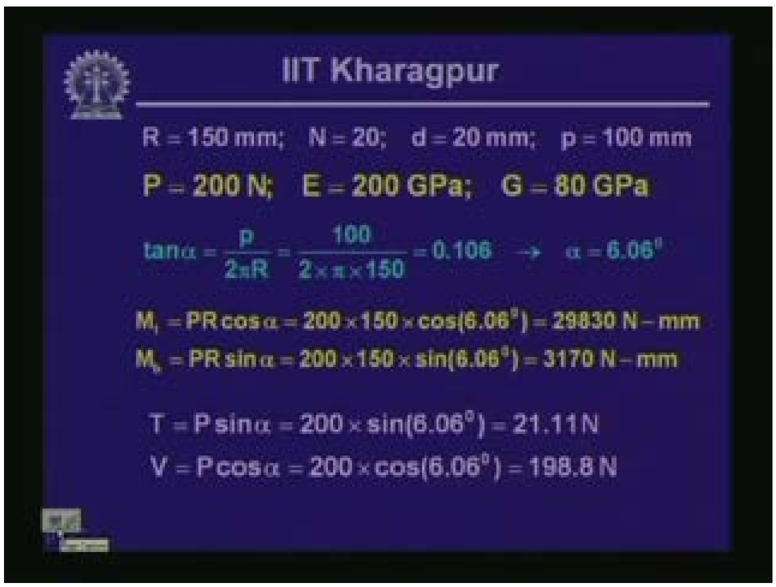

Let us look into how to compute the rest of the stresses.

The values given are

R is 150mm,

N is 20,

d the diameter of the wire is 20 mm,

Pitch of the helix is 100 mm

P the load which is acting is 200N.

Value of E =200GPa and G is 80GPa.

First, we calculate the value of alpha the helix angle.

$$\tan\alpha \text{ is equals to } \frac{P}{2\pi R}.$$

Substituting the values for P as 200 and R as150, we get α as 6.06°.

This is the helix angle with which the wire is inclined with respect to the horizontal plane.

Once we know the value of α, we can compute the four force components that will be acting in the spring which is twisting moment M_t is equals to $PR\cos\alpha$, the bending moment M_b is equals to $PR\sin\alpha$, the axial tensile force that will be acting in the spring wire is equals to $P\sin\alpha$ and the transverse shear V is equals to $P\cos\alpha$.

The twisting moment which is calculated as 29830N-mm by substituting the values of P as 200, R as 150 and $\cos(6.06^\circ)$.

The bending moment is calculated as 3170N-mm by substituting the values of P as 200, R as 150 and $\sin(6.06^\circ)$.

The value of the axial tensile pull T is equals to $P\sin\alpha$ which is equals to $200 \times \sin(6.06^\circ)$ which is equals to 21.11N.

The transverse shear V is equal to $200 \times \cos(6.06^\circ)$ which is equals to 198.8N.

These are the force components the spring wire is subjected to.

Let us compute the value of the stresses corresponding to each of these force quantities.

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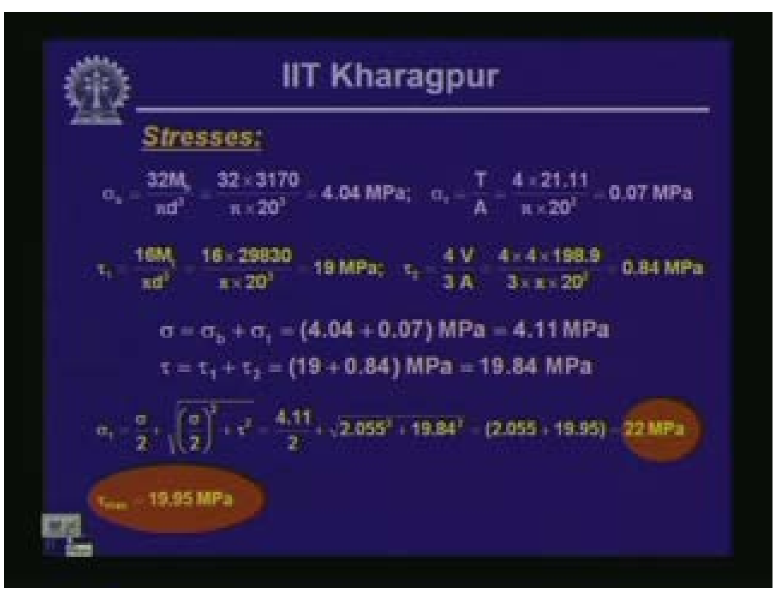

The values of the bending stress sigma as you know is equals to $\sigma_b = \frac{32M_b}{\pi d^3}$.

Substituting the value of M_b as 3170N-mm and d as 20mm, gives us a stress of 4.04MPa.

And also the normal tensile stress which is getting generated because of the tensile pull is equals to $\frac{T}{A}$ and the tensile pull which we have seen is equals to 21.11N divided by $\pi d^2 / 4$. Substituting the value of d as 20 gives us the value as 0.07MPa.

Here you can look into these numerical values that the normal stress which is getting generated because of the axial pull is very small in comparison to the normal stress that got generated because of the bending moment M_b. This is insignificant in comparison to this bending stress.

Let us look into the value of shearing stress that is getting generated because of the twisting moment.

If we call that as $\tau_1 = \frac{16M_t}{\pi d^3}$, where M_t is the twisting moment and we have seen that as 29830 and the value of d is 20, which gives us a value of 19MPa as the shearing stress because of the twisting moment.

The shearing stress because of the transverse shear is equals to $\frac{4V}{3A}$. This is the maximum stress as we have observed that when we have computed the shear stress in a circular cross section. We had observed that the maximum stress occurs at the center and the value of the maximum shearing stress is equals to $\frac{4V}{3A}$ and that is what has been taken here.

V as we have seen as equals to 198.9 and area A is $\frac{\pi d^2}{4}$, this gives as a value of 0.84MPa.

If you look into this particular stress that is getting generated because of the transverse shear V is very small in comparison to the shear stress that is getting generated because of the twisting moment M_t. This τ_2 is almost insignificant in comparison to the shear stress τ_1.

If you look into the other values, the stresses which have generated because of the bending and the direct axial, they are also is much less in comparison to the shearing stress that is getting generated because of the twisting moment.
Of course this depends on the helix angle as well. If the alpha angle varies then there will be change in this stresses.

The total normal stress which we have is the normal stress because of the bending and the normal stress because of the tensile pull.
This comes as equals to 4.11MPa. The total shearing stress that we have is $\tau = \tau_1 + \tau_2$ which equals to 19.84MPa. This is the value of the shearing stress that is getting generated because of the twisting moment and the transverse shear.

Once we have this value of sigma and tau , we can substitute these values in the expression or we can plot them in the More's circle to get the value of the maximum normal stress.

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{4.11}{2} + \sqrt{(2.055)^2 + 19.84^2}$$

This is the expression for evaluating the normal stress and if we compute the value of the maximum normal stress, it comes out as 22MPa.

The maximum shearing stress which is given by the expression: $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}\ which\ is \sqrt{(2.055)^2 + 19.84^2}$ and this gives as a value of 19.95MPa as maximum shear stress.

Let us look into the value of the deflection that the spring will be subjected to because of the action of this load which is acting in the spring.

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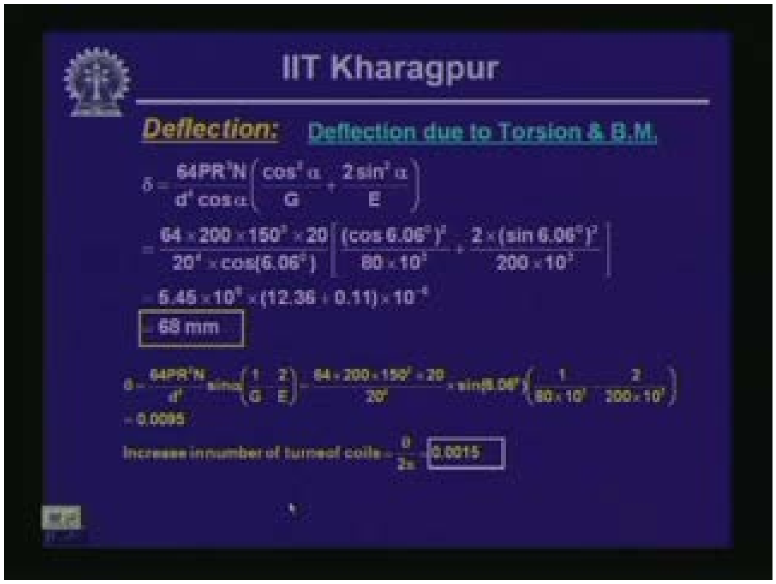

We have seen that the deflection expression for deflection,

$$\frac{64PR^3N}{d^4\cos\alpha}\left(\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right).$$

The value of alpha is computed as 6.06°, the number of turn is given as N=20, R is 150, the load P is equals to 200N, the shear modulus G is equal to 80GPa, it is converted into mega pascal as 80×10^3 and this E is 200GPa which is 200×10^3 mega Pascal. So everything is in terms of newton and millimeter and therefore the value of δ is in millimeter.

If we substitute these values for P, R, N, d, α and G, then the value of the δ the deflection which we get is equals to 68mm. So because of the application of load P which is acting in this open-coiled spring, P is causing a deflection of 68mm.

We have observed that not only the spring will undergo the deflection but it will undergo angular rotation as well.

The value of the angular rotation is $\theta = \frac{64PR^2N}{d^4}\sin\alpha\left(\frac{1}{G} - \frac{2}{E}\right)$.

If we substitute the values of P, R, N, d, G, E and α ,

$$\theta = \frac{64 \times 200 \times 150^2 \times 20}{20^4} \sin(6.06^\circ)\left(\frac{1}{80 \times 10^3} - \frac{2}{200 \times 10^3}\right).$$

Once you simplify, you will find that the value of $\theta = 0.0095$ and as we had seen that n', the increase in the total number of coil is equals to $\frac{\theta}{2\pi} + n$, where n is the number of turn that we had initially and n' is the number of turns that have been introduced after this angular rotation. So the increase in the number of turns is n' minus n which is equals to $n' - n = \frac{\theta}{2\pi}$ and this is what has been computed as $\frac{0.0095}{2\pi}$ gives as a value of 0.0015.

This is the number of turns that has been increased because of the axial pull P that is acting on the spring.

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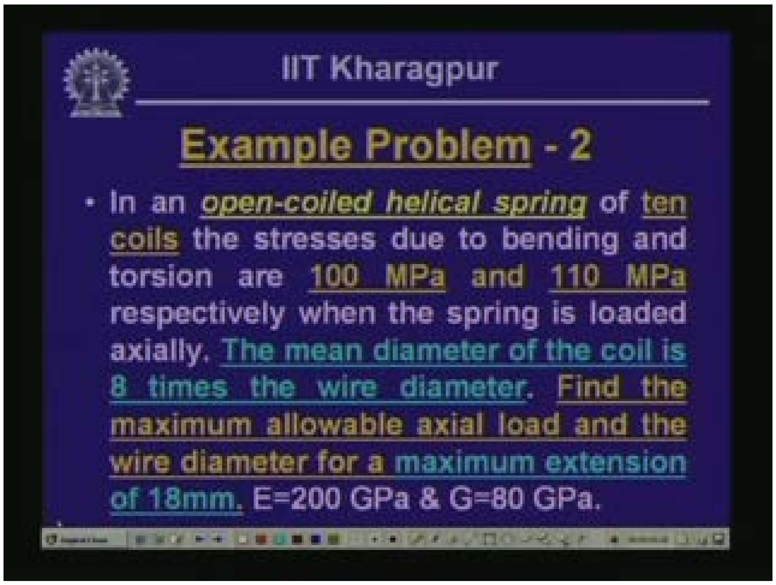

Let us look into another example:

"In an open-coiled helical spring of 10 coils the stresses due to a bending and torsion are 100MPa and 110MPa respectively when the spring is loaded axially. The mean diameter of the coil is 8 times the wire diameter, that is D is 8 times d. Find the maximum allowable axial load P and the wire diameter d for a maximum extension of 18mm. The value of E and G are given as 200GPa and 80GPa."

The value of delta is given. We will have to find out ‘P’ and ‘d’.

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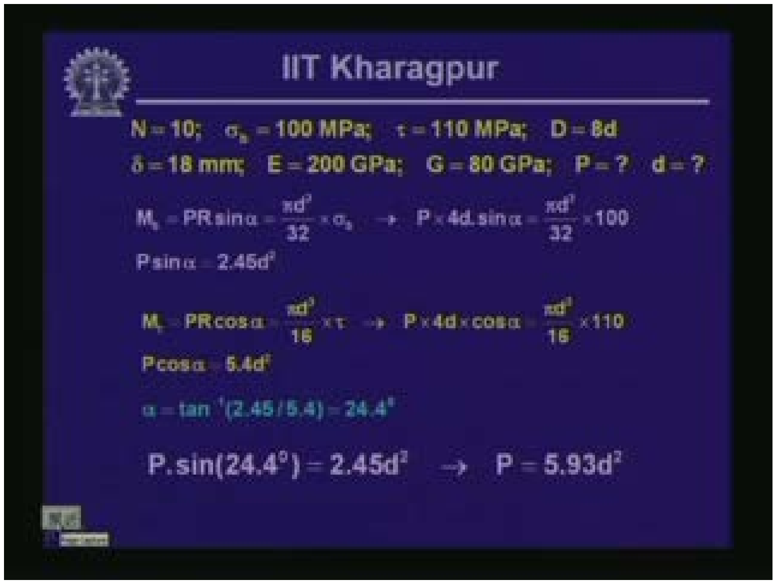

If we look into these values given;

The number of coils is 10.

The permissible bending stress is equal to 100 which mean the normal stress that is getting generated because of the bending.

The permissible shear stress because of the twisting moment is 110MPa.

The nominal diameter D of the spring is equals to 8 times d.

The delta the deflection is equals to 18mm

E is given as 200GPa

G is given as 80GPa

We have to compute the value of P, D and d.

As we have seen that the bending moment M_b is equals to $PR\sin\alpha$.

If we write the stress sigma as $\sigma = \frac{MY}{I}$ where $Y = \frac{d}{2}$ and $i = \frac{\pi d^4}{64}$, therefore this $\frac{MY}{I}$ equals to $\sigma = \frac{32M}{\pi d^3}$ and thereby $M_b = \frac{\pi d^3}{32}\times\sigma_b$.

M_b which is $PR\sin\alpha$ *is equal to* $\frac{\pi d^3}{32}\times\sigma_b$.

Since D is 8d. So R is 4d.

Hence $P\times 4d\sin\alpha = \frac{\pi d^3}{32}\times\sigma_b$.

The bending the stress σ_b is given as 100MPa. Substituting in the above equation gives as a value of $P\sin\alpha = 2.45d^2$.

The twisting moment M_t is equals to $PR\cos\alpha$ and again as we know that $\tau = \frac{T\rho}{J}$ and $\rho = \frac{d}{2}$, $J = \frac{\pi d^4}{32}$.

Hence $\tau = \frac{16T}{\pi d^3}$ and therefore the twisting moment is equals to $\frac{\pi d^3}{16\tau} \times \tau$ and that is what is indicated as

$$M_t = PR\cos\alpha = \frac{\pi d^3}{16\tau} \times \tau$$

and tau is given as 110MPa which is a permissible tau.

R we have seen is equal to 4d.

Hence if we simplify the above equation, we get

$$M_t = P \times 4d \times \cos\alpha = \frac{\pi d^3}{16\tau} \times 110$$

$$P\cos\alpha = 5.4d^2$$

If we take the ratio of M_b and M_t , $\frac{P\sin\alpha}{P\cos\alpha} = \frac{2.45}{5.4}$.

That gives us a value of α which is tan inverse of $\frac{2.45}{5.4}$.

Hence the helix angle $\alpha = \tan'\left(\frac{2.45}{5.4}\right) = 24.4^\circ$.

If we substitute the value of alpha in the equation $P\sin\alpha$ we get

$$P\sin(24.4^\circ) = 2.45d^2$$

$$Hence,$$

$$P = 5.93d^2$$

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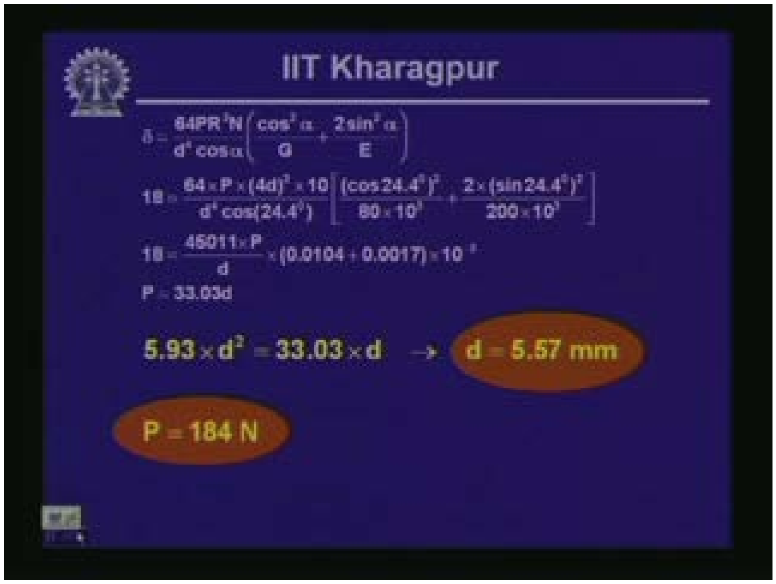

As we know the limiting deflection which is equal to 18mm and the value of the delta is given by

$$\delta = \frac{64PR^3N}{d^4\cos\alpha}\left(\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right)$$

If we substitute the values of G, E, alpha, and N is given as 10 and R as we know is equals to 4d, we get $P = 33.03d$.

We have seen that $P = 5.93d^2$ and now we have $P = 33.03d$ from the limiting deflection delta.

If we equate these two, we get the value of d the diameter of the wire as $d = 5.57mm$ and if we substitute this value of d in the expression $P = 33.03d$, we can get the value of P which is equals to 184N.

Hence these are the values of the allowable P and the diameter of the wire that is to be used for forming the spring.

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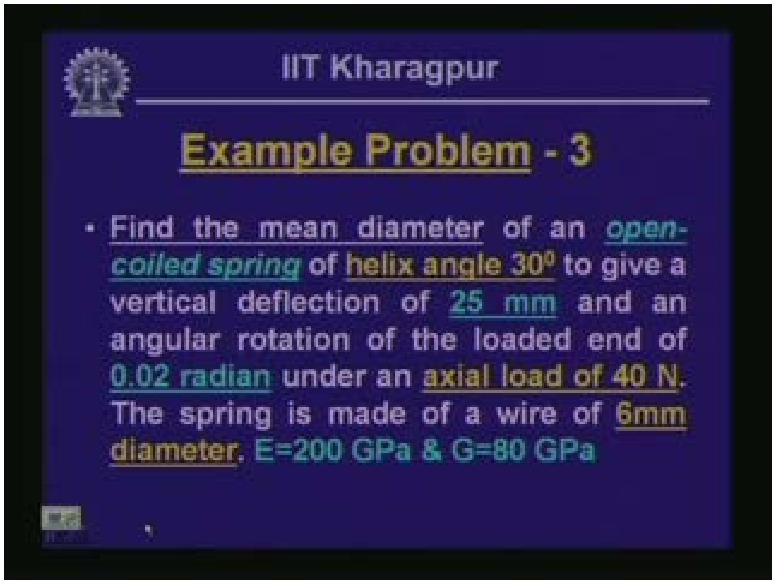

Let us see another example;

"Find the mean diameter of an open-coiled spring of helix angle 30^0 to give a vertical deflection of 25mm and an angular rotation of the loaded end of 0.02radian under an axial load of 40N. The spring is made of a wire of 6mm and the value of E and G are given as 200GPa and 80GPa respectively."

If we look into this example, the value of alpha is given as 30^0, the deflection delta is given as 25, theta is given as 0.02radian and P is equals to 40N, the value of d is 6mm.

Now we will have to find out the mean diameter D or the mean radius R.

(Refer Slide Time: 55:18 - 56:19)

Delta is given as $\delta = \dfrac{64PR^3N}{d^4\cos\alpha}\left(\dfrac{\cos^2\alpha}{G} + \dfrac{2\sin^2\alpha}{E}\right)$ and since the limiting delta is 25mm, if we substitute the values except this R and N, then we get the value of R^3N as equals to 923.4×10^3.

We know the limiting angular rotation is 0.02radian. If we substitute the values in the formula, $\theta = \dfrac{64PR^2N}{d^4}\sin\alpha\left(\dfrac{1}{G} - \dfrac{2}{E}\right)$ for except R and N, we get the value of R^2N is equals to 8.1×10^3.

If we divide R^3N by R^2N, we get the value of R which is equals to 114mm. Hence the diameter is equal to 228mm for this particular spring.

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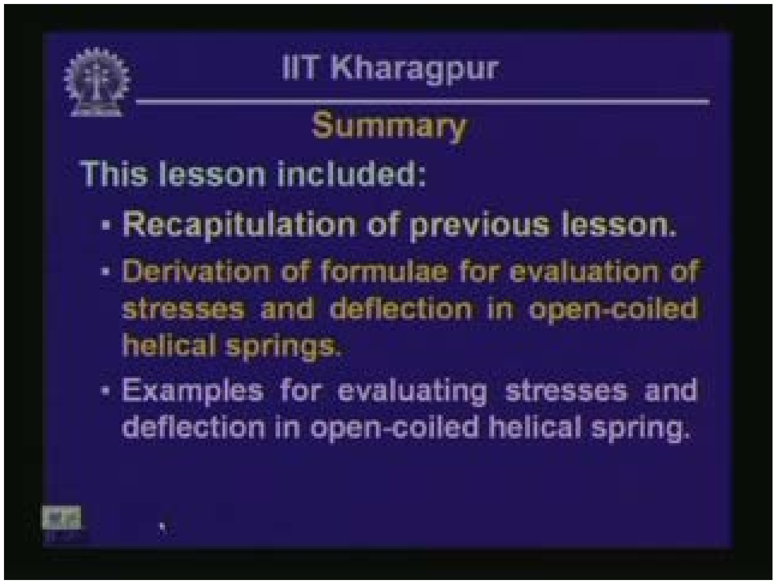

To summarize, in this particular lesson we have looked into some aspects of the previous lessons and also we have derived the formulae for evaluating the stresses and deflection in open-coiled helical springs.

We have also looked into the examples for evaluating stresses and deflection in open-coiled helical spring.

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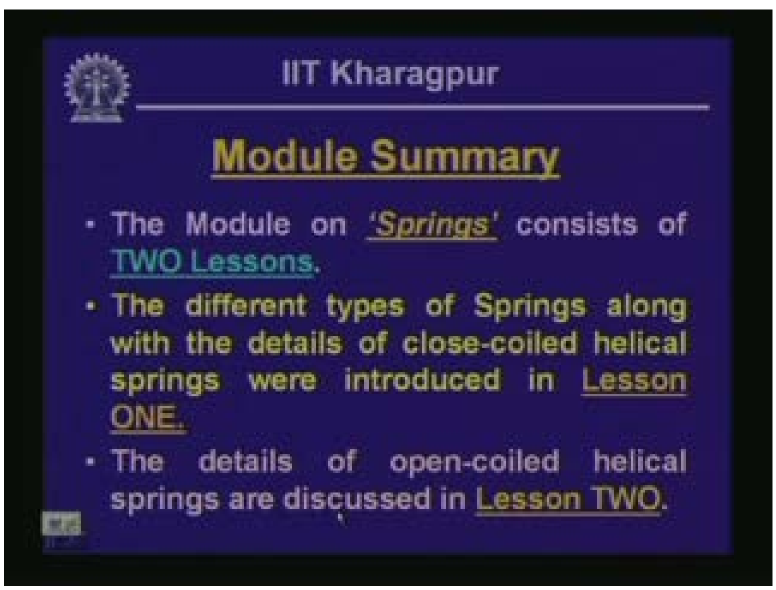

To summarize the module, this particular module consists of two lessons.

In the first lesson, we were introduced to the concept of the spring, we had looked into the different types of springs and we had discussed several aspects of the stresses and deflection in close-coiled springs and in the second lesson of this particular module we have looked into the stresses and deflection in open-coiled helical springs.

Here are the questions for you to go through.

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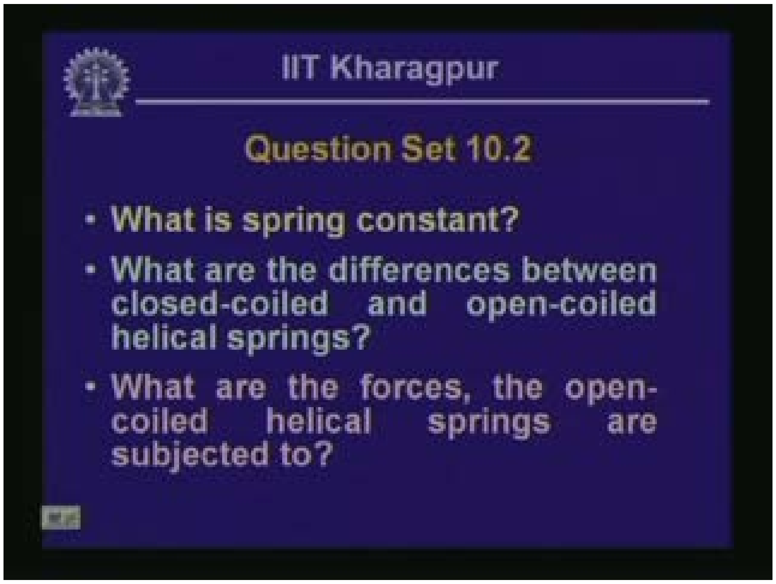

What is spring constant and what are the differences between closed coiled and open-coiled helical springs?

What are the forces, the open-coiled helical springs are subjected to?

Look into these questions and if you go through both the lessons of this module, you will get the answers for these questions.

Thank you very much.