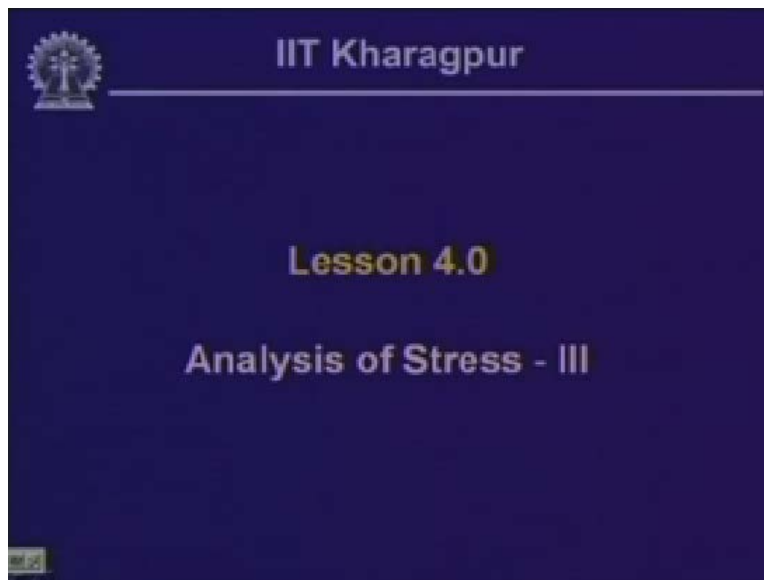


Strength of Materials
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Lecture - 4
Analysis of Stress - III


Welcome to the 4th lesson on the course Strength of Materials.

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Today we will continue our discussion on certain aspects of Analysis of Stress.

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Specific Instructional Objectives

- After completing this lesson one will be able to:
- Evaluate stresses on any plane through stress-transformation equations.
- Evaluate Principal stresses & locate principal planes for 2-D problems.
- Evaluate maximum shear stresses at a point on a body for 2-D problems.
- Construct Mohr's circle for stress.

In this particular lesson it is expected that once we complete it,

- One should be able to evaluate stresses on any plane through stress transformation equations
- Evaluate principal stresses and locate principal planes for two dimensional (2-D) problems. In fact in the previous lesson we discussed about the evaluation of principal stresses in three dimensional planes. Here we will be discussing evaluation of principal stresses and location of principal planes for 2-D problems.
- We are going to evaluate the maximum shear stresses at a point on a body for 2-D problems.
- We will also look into the concept of Mohr's circle for stress and we will demonstrate how to construct Mohr's circle for stress.

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Scope

- This lesson includes:
 - Transformation of Stresses for 2-D problems.
 - Evaluation of Principal stresses & maximum shear stresses
 - Construction of Mohr's circle
 - Examples for evaluation of stresses.

Hence the scope of the particular lesson is the derivation of transformation equation for evaluation of stresses for 2-D problems, evaluation of principal stresses and maximum shear stresses; construction of Mohr's circle. We will look into aspects of how we are going to draw Mohr's circle for the evaluation of the stresses at a particular point in the body. We will solve a few examples to demonstrate how the stresses can be evaluated at any particular point.

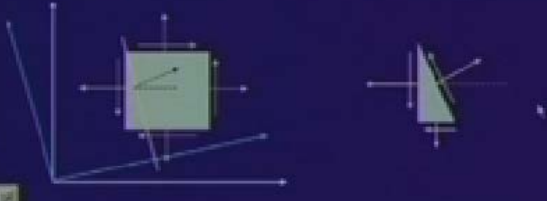
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Transformation Equations

- 2-D plane stress
- Consider a plane, outward normal to which makes an angle θ with x-axis.



Let us look into the derivations of the transformation of equations. We have discussed about the plane stresses in 2-D. If we consider the stress body at a particular point this is our reference x-axis and this is the reference y-axis. The stress which is acting in the x-plane the normal stress

is σ_x . The normal stress in the y-plane is σ_y and the shearing stresses are τ_{xy} . We are interested now to evaluate the stresses on a plane the normal to which is at an angle θ with respect to the x-axis. The plane is considered in such a way that the normal direction normal to the plane coincides with reference axis which we denote as x prime and y prime. Since the normal to this particular plane coincides or is parallel to the x' axis we call this plane as x' plane.

Now let us look into the state of stress on this particular plane, if we take out this particular wedge and if we designate this as A, B, and C the stresses which are acting on this particular part are σ_x normal stresses on this surface, σ_y the shearing stresses τ_{xy} . This being the x' plane the normal stress to this particular plane is $\sigma_{x'}$, and correspondingly the shear stress will be $\tau_{x'y'}$.

Considering the unit thickness normal to the plane of the board if we assume the area on line AC as dA which is length AC multiplied by the unit thickness then considering that this particular angle being θ , this particular angle is also θ the area on line AB can be designated in terms of the area dA which is $dA \cos \theta$ and area on line BC can be designated in terms of $dA \sin \theta$. Hence the forces which are acting on these planes are the stresses multiplied by the corresponding area will give us the force. If we wish to write down the equilibrium equations in the x' direction and y' direction then the equation looks like this: summation of forces in the x' direction that is $\sum F_{x'}$ is equal to 0.

The forces which are acting in the x' directions are $\sigma_{x'}$ into dA is acting in the x' direction minus σ_x acting on the area $dA \cos \theta$ and the component in the x' direction is $\cos \theta$; σ_y which is acting in the opposite direction of $\sigma_{x'}$ is the minus $\sigma_y dA \sin \theta$ the force and multiplied by the component $\sin \theta$.

The shearing stresses we have is minus τ_{xy} acting on BC which is $dA \sin \theta$ component along x direction is $\cos \theta$ minus τ_{xy} which is acting in the plane $x dA \cos \theta$, and component along $\sin \theta$ is equal to 0. This gives us the equation as $\sigma_{x'}$ is equal to $\sigma_x \cos^2 \theta$ plus $\sigma_y \sin^2 \theta$ plus $2 \tau_{xy} \sin \theta \cos \theta$. Writing $\sin^2 \theta$ and $\cos^2 \theta$ in terms of $\cos 2\theta$ we can write this as σ_x into $\frac{1}{2}(1 + \cos 2\theta)$ plus $\sigma_y \frac{1}{2}(1 - \cos 2\theta)$ plus $\tau_{xy} \sin 2\theta$.


If you write $\sin \theta$ and $\cos \theta$ as $\sin 2\theta$, then plus $\tau_{xy} \sin 2\theta$; this we can write as $(\sigma_x + \sigma_y)$ by 2 plus $(\sigma_x - \sigma_y)$ by 2 $\cos 2\theta$ plus $\tau_{xy} \sin 2\theta$. This is the stress in the x-direction which is the normal stress $\sigma_{x'}$ which are written in terms of stresses σ_x , σ_y and τ_{xy} . Similarly, if we take equilibrium along $F_{y'}$; $\sum F_{y'}$ is equal to 0 we get $\tau_{x'y'}$ is equal to minus $\sigma_x \cos \theta \sin \theta$ plus $\sigma_y \sin \theta \cos \theta$ plus $\tau_{xy} \sin^2 \theta$, $\tau_{xy} \cos^2 \theta$ minus $\tau_{xy} \sin^2 \theta$. Hence we can write $\tau_{x'y'}$ is equal to $[(\sigma_y - \sigma_x) \sin 2\theta] + \tau_{xy} \cos 2\theta$. So these are the equations $\sigma_{x'}$ and $\tau_{x'y'}$ and they are the stresses on the plane which is at an angle of θ with respect to the x-axis.

Similarly, if we want to evaluate the stress in the y prime direction the normal stress σ_y prime, the stress σ_y prime is at an angle of θ plus 90 degrees, if we substitute in place of θ as θ plus 90 then $\sin(180 \text{ plus } 2\theta)$ is equal to $-\sin 2\theta$; $\cos(180 \text{ plus } 2\theta)$ is equal to $-\cos 2\theta$ and if we substitute these values in the expression for σ_y prime again we get σ_y prime is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ minus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ minus } \tau_{xy} \sin 2\theta$. Thereby if we add these two σ_x prime and σ_y prime this gives us the value as $\sigma_x \text{ plus } \sigma_y$.

The stresses $\sigma_x \text{ plus } \sigma_y \text{ plus } \sigma_z$ is equal to σ_x prime plus σ_y prime plus σ_z prime which indicates that irrespective of the reference axis system the summation of these normal stresses are constant which we called as stress invariants. So here this is to prove again that the normal stresses with reference axis is x primey prime σ_x prime plus σ_y prime is equal to $\sigma_x \text{ plus } \sigma_y$ is equal to constant and so are the other stress invariants. Hence we have obtained the stresses in the direction at an angle of θ as σ_x' is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$.

We have seen $\tau_{x' y'}$ is equal to $-\sin 2\theta (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \text{ plus } \tau_{xy} \cos 2\theta$. These are the transformation equations. That means we can evaluate stresses at any plane which is oriented at an angle θ in terms of the normal stresses σ_x, σ_y and τ_{xy} . Please keep in mind that the rotation of the angle θ we have taken as anti-clockwise and this is a positive according to our convention.

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Stresses on inclined plane

- $\sigma_{x'} = (\sigma_x + \sigma_y)/2 + \cos 2\theta (\sigma_x - \sigma_y)/2 + \tau_{xy} \sin 2\theta$
- $\tau_{x'y'} = -\sin 2\theta (\sigma_x - \sigma_y)/2 + \tau_{xy} \cos 2\theta$
- $\sigma_{y'} = (\sigma_x + \sigma_y)/2 - \cos 2\theta (\sigma_x - \sigma_y)/2 - \tau_{xy} \sin 2\theta$
- $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$

Hence these are the stresses which we have derived; σ_x prime is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \text{ into } \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$. τ_{xy} prime is equal to $(\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \text{ minus } \sin 2\theta \text{ plus } \tau_{xy} \cos 2\theta$. We have also seen σ_y prime like this and if we add σ_x prime plus σ_y prime we will get $\sigma_x \text{ plus } \sigma_y$.

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Maximum Normal Stresses

- For a maximum or minimum Normal stress,
- $d\sigma_x / d\theta = -2\sin 2\theta(\sigma_x - \sigma_y)/2 + 2\tau_{xy}\cos 2\theta = 0$
- This gives $\tan 2\theta = 2\tau_{xy} / (\sigma_x - \sigma_y)$
- Two values of the angle -2θ & $(180 + 2\theta)$

Now let us look into the position of the planes where the normal stresses are at maximum. We have obtained that the normal stresses on a plane σ_x prime which is at an angle θ is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$. If we take the derivative of the normal stress with respect to θ is $\partial \sigma_x \text{ prime} / \partial \theta$ is equal to $\text{minus } 2(\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \sin 2\theta \text{ plus } 2\tau_{xy} \cos 2\theta$. If we set this as equal to zero and take this on the other side then we get $\tan 2\theta$ is equal to $\tau_{xy} \text{ by } (\sigma_x \text{ minus } \sigma_y \text{ by } 2)$. Now this particular equation has two values of θ . One is θ_p with reference to the axis system we have the plane which is in the angle of θ_p . Also, we will get another angle which is at an angle of 180 as $\tan 180 \text{ plus } \theta$ is equal to $\tan \theta$. Hence we have one angle as $2\theta_p$ and another at angle of $180 \text{ plus } 2\theta_p$ which will us two values.

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Maximum Normal Stresses

- For a maximum or minimum Normal stress,
- $d\sigma_x/d\theta = -2\sin 2\theta(\sigma_x - \sigma_y)/2 + 2\tau_{xy}\cos 2\theta = 0$
- This gives $\tan 2\theta = 2\tau_{xy}/(\sigma_x - \sigma_y)$
- Two values of the angle -2θ & $(180 + 2\theta)$

Therefore this is the derivative of the normal stress and this is the derivatives of $\tan \theta$. We will get two values of this root 2θ and 180 plus 2θ . We have designated these as θ_P and 180 plus $2\theta_P$. In effect when we transform from this into the stress part we have angle θ_P and 90 plus θ_P and that indicates that we have two planes which is at an angle of θ_P and normal to this is the plane on which the normal stress is maximum, and then we have another plane which is at an angle of 90 degrees with reference to this particular plane because the other plane is at 90 plus θ_P . These are the two normal directions where one will be the maximum and the other will be the minimum. These are the two normal stresses.

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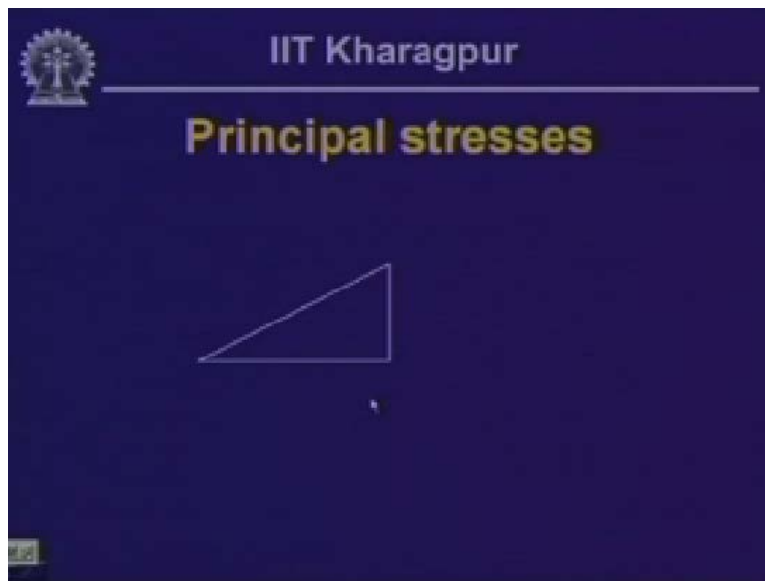
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Principal stresses

- $\tau_{x'y'} = -\sin 2\theta(\sigma_x - \sigma_y)/2 + \tau_{xy}\cos 2\theta$
- If $\tau_{x'y'} = 0$, then $\tan 2\theta = 2\tau_{xy}/(\sigma_x - \sigma_y)$
- Maximum & Minimum Normal stresses are the Principal stresses

Interestingly if we look into the expression τ_{xy}' is equal to $(\sigma_x - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta$. If we say τ_{xy}' is equal to 0 then we get $\tan 2\theta$ is equal to $2\tau_{xy} / (\sigma_x - \sigma_y)$ which is similar to the expression which we have obtained for $\tan 2\theta$ setting the derivative of the normal stress to 0. And since these two angles match this shows the planes where we have obtained the maximum and minimum principal stresses they coincide with the planes where the shearing stress is 0. And as we have defined before that the planes on which shearing stress is 0 the normal stress is designated as principal stress. Hence the maximum normal stresses which we have obtained are nothing but the principal stresses where the shear stresses are 0 and their angles are defined by θ_p where we have evaluated $\theta_p = \frac{1}{2} \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$.

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Let us evaluate the maximum values of the normal stresses and the principal stresses. We have seen that this angle $2\theta_p$ where $\tan 2\theta_p$ is equal to $2\tau_{xy} / (\sigma_x - \sigma_y)$. Hence the value of this hypotenuse R is equal to square root of $\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$. Hence value of $\cos 2\theta_p$ is equal to $\frac{(\sigma_x - \sigma_y)/2}{R}$ and $\sin 2\theta_p$ is equal to $\frac{\tau_{xy}}{R}$. If we substitute the values of $\cos 2\theta_p$ and $\sin 2\theta_p$ in the expression of the normal which we have evaluated σ_x' is equal to $\frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$.

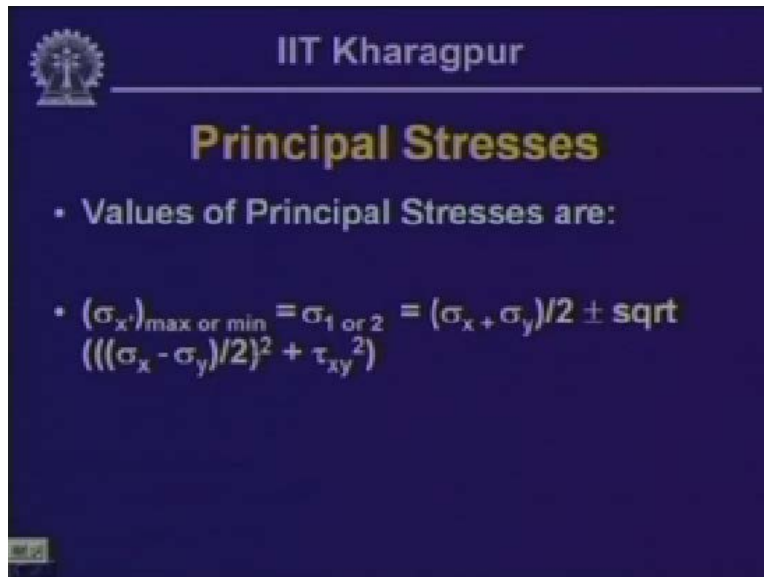
Now if we substitute for $\cos 2\theta$ and $\sin 2\theta$ in terms of θ_p this we get as the maximum stresses, so σ_x' maximum or minimum which are nothing but the principal stresses σ_1 and σ_2 is equal to $\frac{\sigma_x + \sigma_y}{2} \pm \frac{(\sigma_x - \sigma_y)}{2} \frac{(\sigma_x - \sigma_y)/2}{R} + \tau_{xy} \frac{\tau_{xy}}{R}$. This is equal to at this particular part, $\frac{(\sigma_x - \sigma_y)^2}{4R} + \frac{\tau_{xy}^2}{R}$ and as we have denoted the R as root of this, so the R^2 is the top part. So $\sigma_{1,2}$ can be

written as $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus R square by R}$, these get cancel so, this eventually gives us $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus square root of } ((\sigma_x \text{ minus } \sigma_y) \text{ by } 2) \text{ whole square plus } \tau_{xy} \text{ square}$. So this is the value of maximum stress or one of the stresses we get.

Now we have obtained that, σ maximum or minimum, let us call it as σ_1 is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus square root of } ((\sigma_x \text{ minus } \sigma_y) \text{ by } 2) \text{ whole square plus } \tau_{xy} \text{ square}$. We have seen that the normal stresses are the constants summation of $\sigma_x \text{ plus } \sigma_y$ is equal to $\sigma_x \text{ plus } \sigma_y$ is equal to $\sigma_1 \text{ plus } \sigma_2$; because these are the two normal stresses at perpendicular plane.

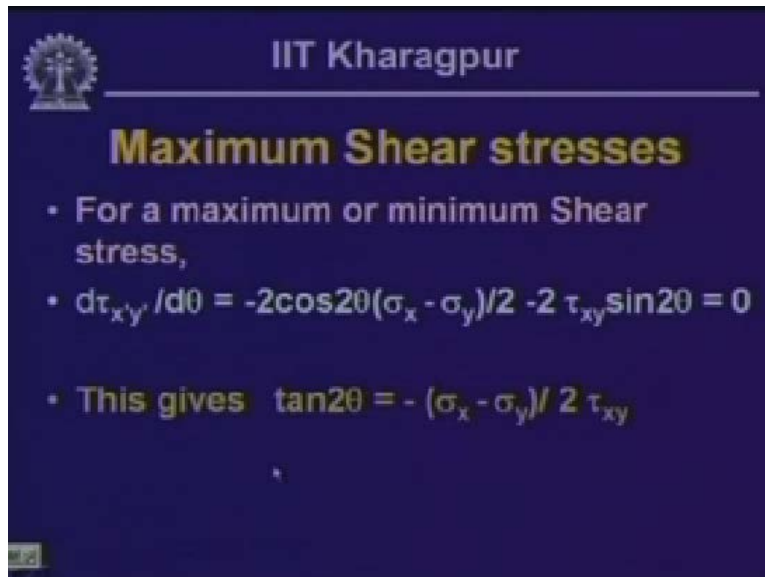
We can write $\sigma_1 \text{ plus } \sigma_2$ which are two normal stresses at perpendicular plane as equals to $\sigma_x \text{ plus } \sigma_y$, σ_2 from here is $\sigma_x \text{ plus } \sigma_y \text{ minus } \sigma_1$; σ_1 is given by this, which can be write this is equals to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ minus square root of } ((\sigma_x \text{ minus } \sigma_y) \text{ by } 2) \text{ whole square plus } \tau_{xy} \text{ square}$. Hence the stresses σ_1 or σ_2 is given as $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus or minus square root of } ((\sigma_x \text{ minus } \sigma_y) \text{ by } 2) \text{ whole square plus } \tau_{xy} \text{ square}$. Hence these are the values of principal stresses, maximum and minimum principal stresses.

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Now these are the values of principal stresses maximum and minimum values are $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus square root of } ((\sigma_x \text{ minus } \sigma_y) \text{ by } 2) \text{ whole square plus } \tau_{xy} \text{ square}^0$.

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Maximum Shear stresses

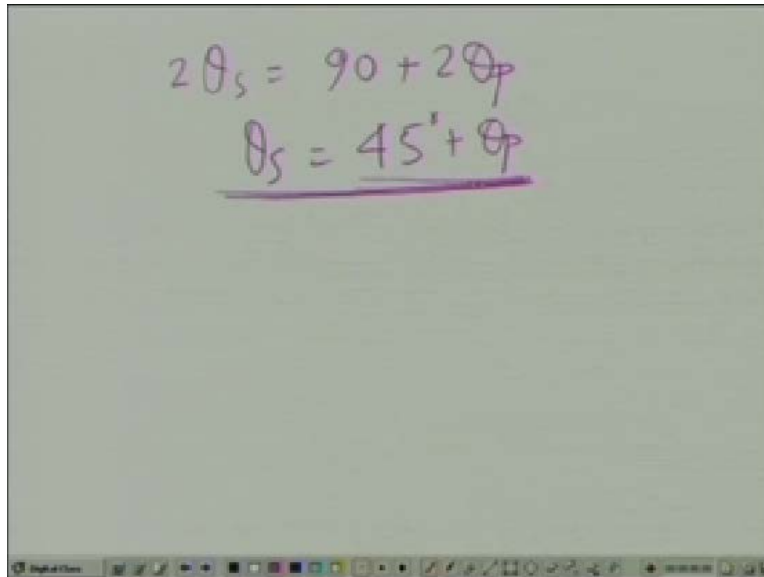
- For a maximum or minimum Shear stress,
- $d\tau_{x'y'}/d\theta = -2\cos 2\theta(\sigma_x - \sigma_y)/2 - 2\tau_{xy}\sin 2\theta = 0$
- This gives $\tan 2\theta = -(\sigma_x - \sigma_y)/2\tau_{xy}$

Maximum shear stress:

We have seen shear stress on any plane, $\tau_{x'y'}$ is equal to minus $(\sigma_x \text{ minus } \sigma_y)$ by $2 \sin 2\theta$ plus $\tau_{xy} \cos 2\theta$. If we take derivative of this with respect to the θ , $\partial \tau_{x'y'}/\partial \theta$ is equal to minus $2(\sigma_x \text{ minus } \sigma_y)$ by $2 \cos 2\theta$ minus $\tau_{xy} 2 \sin 2\theta$. So $\tau_{xy} \sin 2$ is equal to minus $(\sigma_x \text{ minus } \sigma_y)$ by $2 \cos 2\theta$; or $\tan 2\theta$ is equal to $(\sigma_x \text{ minus } \sigma_y)$ by 2 by τ_{xy} . Here also as we have noticed earlier two values of θ defining two perpendicular planes on which the shear stress will be maximum and those angles being, $2\theta_s$ and 180 degrees plus $2\theta_s$. So in the stress body it will be θ_s plus 90 , or θ_s and θ_s plus 90 perpendicular plane on which shear stress will be maximum.

Now if we look in to the values of $\tan 2\theta_s$ and compare with the values of previously calculated values of $\tan 2\theta_p$ we find that $\tan 2\theta_s$ is equal to minus 1 by $\tan 2\theta_p$ and $\tan 2\theta_p$ we have already evaluated earlier as τ_{xy} by $(\sigma_x \text{ minus } \sigma_y)$ by 2 . So this is equals to minus $\cot 2\theta_p$ which we can write as, $\tan 90$ plus $2\theta_p$. This indicates that, $2\theta_s$ is equal to 90 plus $2\theta_p$. Or θ_s is equal to 45 degrees plus θ_p . This indicates that maximum shear stress occurs in the plane which is at angle 45 degrees with maximum or minimum principal shear stresses.

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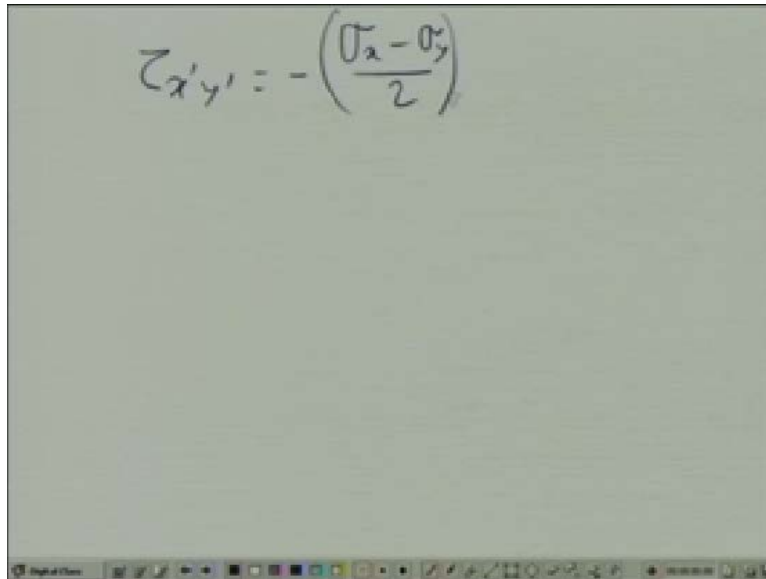

$$2\theta_s = 90 + 2\theta_p$$
$$\underline{\theta_s = 45 + \theta_p}$$

This is the value of $\tan 2\theta$ evaluated, hence we find that two mutually perpendicular planes on which maximum shear stresses exists; and of maximum and minimum shear stresses form an angle of 45 degrees with the principal planes is just seen. Now let us look in to the value of principal stress where shear stress is at maximum.

Now we have calculated that $\tan 2\theta$ is equal to minus $(\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \text{ by } \tau_{xy}$. If we place this in geometrical form, this we have to take a $2\theta_s$, this is τ_{xy} , this is minus $(\sigma_x \text{ minus } \sigma_y) \text{ by } 2$. Hence the value of R again, square root of $((\sigma_x \text{ minus } \sigma_y) \text{ by } 2)^2 \text{ plus } \tau_{xy}^2$ square. Likewise then the $\cos \theta$, rather $\cos 2\theta_s$ is equal to $\tau_{xy} \text{ by } R$ and $\sin 2\theta_s$ is equal to minus $(\sigma_x \text{ minus } \sigma_y) \text{ by } 2R$ if we substitute the values of \cos and \sin in the values of the normal which is σ_x' is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$.

Now in this in place of $\sin 2\theta$ and $\cos 2\theta$ if we substitute $\cos 2\theta_s$ and $\sin 2\theta_s$ we will get the values of normal stress. Also we will get the values of shear stresses explained. Now if you substitute these values we get this is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$, these two terms get cancelled once you substitute the values.

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$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta$$

Also we have seen the values of shear stress as, $\tau_{x'y'}$ is equal to $\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$. If we substitute the values of $\sin 2\theta$ and $\cos 2\theta$, we have obtained as $\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$, so this is $\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2 \cos^2 2\theta$ by R. Then R is equal to square root of $\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2 \cos^2 2\theta$. So this is the tau max. In fact the minimum stress is the negative of this. So tau max or min is equal to plus or minus square root of $\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2 \cos^2 2\theta$. These are the values of maximum stresses and we observed that the value of the normal stress on the plane where shear stress is maximum is equal to $\frac{\sigma_x + \sigma_y}{2}$.

Now if the normal stresses are the principal stresses then we get that maximum shear stress is equal to $\frac{\sigma_1 - \sigma_2}{2}$ from this expression, if the σ_x is σ_1 , and σ_y is σ_2 , and this is being the principal stress τ_{xy} is equal to 0, so tau max gives us the value in the terms of principal stresses as $\frac{\sigma_1 - \sigma_2}{2}$. This gives the maximum shear stress in the terms of principal stresses.

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Planes for maximum shear stresses

- Two mutually perpendicular planes
- Planes of maximum or minimum shear stress form an angle of 45° with the principal planes.
- Planes of maximum shear stresses are NOT free from normal stresses.


Hence, we say that planes of maximum shear stresses are not free from normal stresses. As we have seen in case of principal stresses, the principal stresses acts on the shear stresses are 0, whereas the planes on which the shear stresses are maximum there we do have normal stresses and the value of normal stress is $(\sigma_x \text{ plus } \sigma_y)$ by 2.

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Mohr's Circle of Stress

- Otto Mohr of Germany suggested use of stress circle in 1895

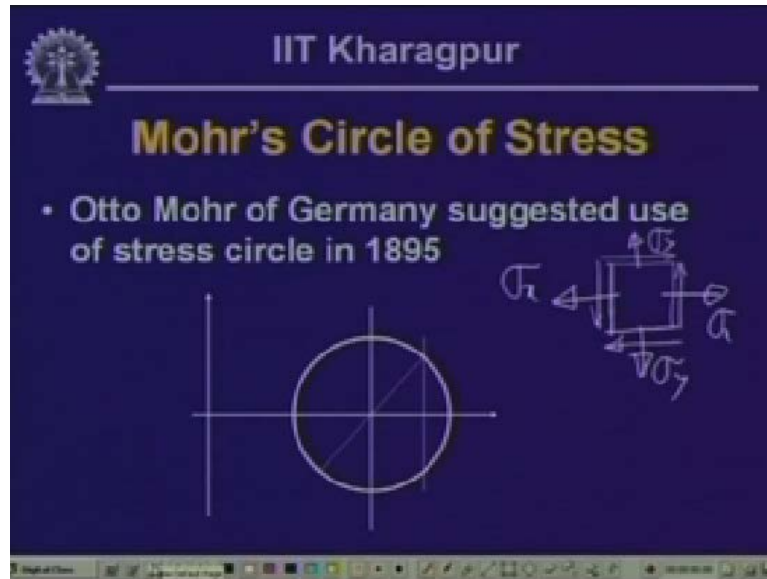


The diagram shows a Cartesian coordinate system with a horizontal axis representing normal stress and a vertical axis representing shear stress. A circle is drawn in the first quadrant, centered on the horizontal axis. The circle intersects the horizontal axis at two points, representing the principal stresses. A diagonal line is drawn from the origin through the center of the circle, representing the direction of the principal stress. The circle's radius represents the maximum shear stress, and its diameter represents the difference between the principal stresses.

Now let us look into another representation or the evaluation of the stress from a concept which is given by Otto Mohr of Germany in 1895 which we popularly designate as Mohr's Circle of

Stress. As we have seen we have a stress body in which the normal stresses are σ_x and σ_y and we have the corresponding shear stresses, now we can represent this stress system in terms of this circle.

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Now let us look in to the expression for normal stress and shearing stress. σ_x prime is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2 \text{ plus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$. Now τ_{xy} prime is equal to $\text{minus } (\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \sin 2\theta \text{ plus } \tau_{xy} \cos 2\theta$. From the first of this equation you can write this as σ_x ' minus $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ is equal to $(\sigma_x \text{ minus } \sigma_y) \text{ by } 2 \cos 2\theta \text{ plus } \tau_{xy} \sin 2\theta$. Now, if we square this equation and the second equation and add them up we get $(\sigma_x$ ' minus $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2)$ whole square plus τ_{xy} prime square is equal to $((\sigma_x \text{ minus } \sigma_y) \text{ by } 2)$ whole square and $(\sin^2 2\theta \text{ and } \cos^2 2\theta \text{ is } 1) \text{ plus } \tau_{xy}$ square; the other terms get canceled.

This particular equation can be represented as $(x \text{ minus } a)$ whole square plus y square is equal to b square. This particular equation is a well known equation which is that of a circle where the centre of the circle lies at the coordinates $(\text{plus } a, 0)$ the radius of which is equals to b and x represents σ_x prime, and y represents τ_{xy} prime. If we draw a circle whose centre is at $(a, 0)$, where a is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ on the σ_x prime axis which is representing x , with radius of b is equal to square root of $((\sigma_x \text{ minus } \sigma_y) \text{ by } 2)$ whole square plus τ_{xy} square then we get the circle, and that is what is represented as here in terms of Mohr's circle.

The centre of this particular circle is at a distance of from the origin, consider this as σ_x axis or σ axis and τ axis then, this at the distance of $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$ which is the average stress. This


particular point represents the stress which we have at the particular body which is σ_x, σ_y , and τ_{xy} . This particular point, point on this particular circle represents the value of σ and τ at a particular orientation which is representing a plane.

Here this particular point we are representing as σ_x and τ_{xy} . The σ_x and τ_{xy} on this circle is representing this particular plane. Hence this being σ_x and this being $(\sigma_x + \sigma_y) / 2$, the distance here, this particular distance, σ_x minus $(\sigma_x + \sigma_y) / 2$ is equal to $(\sigma_x - \sigma_y) / 2$. This particular distance is τ_{xy} . So eventually this particular distance is the square root of $((\sigma_x - \sigma_y) / 2)^2 + \tau_{xy}^2$ which is that of radius which is r .

This particular point represents the maximum normal stress on which this normal stress is acting; this is the minimum normal stress and from this plane we rotate $2\theta_p$ angle, one of the maximum normal stress plane, and if rotate by another 180 degrees, another plane representing the maximum normal stress. This maximum normal stress we call as maximum principal stress which we represented as σ_1 , and this we represent as minimum principal stress as σ_2 .

This particular point and this point in the circle represents the maximum value of the shearing stress τ_{xy} is equal to radius is equal to square root of $((\sigma_x - \sigma_y) / 2)^2 + \tau_{xy}^2$ square. So plus tau and minus tau are the maximum and minimum shear stresses. If we look into the plane this particular plane is representing principal stress and this particular plane representing maximum shear stress and the angle between these two is 90 degrees which is twice of that in the body. As we have seen that the angle between the maximum principal plane and the plane on which maximum shear stress acts is at an angle of 45 degrees which is being represented here as $2\theta_p$ is equal to 90; θ is equal to 45 degrees.

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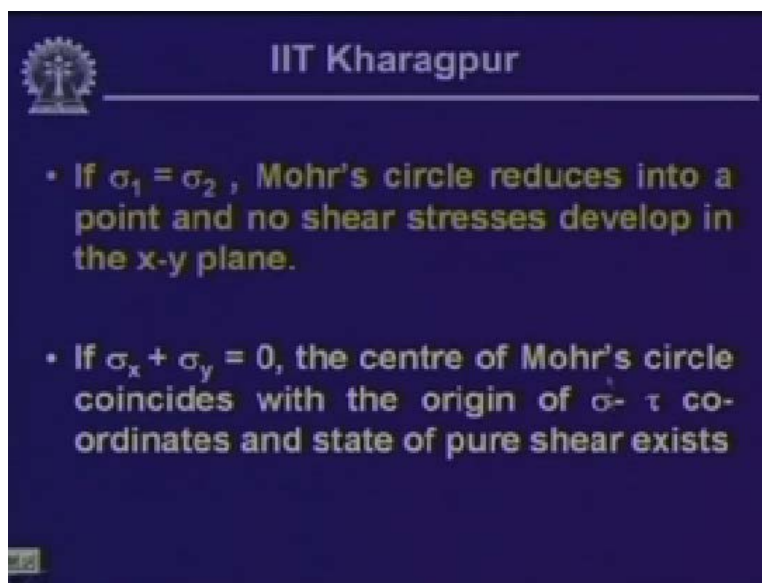
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Observations from Mohr's circle

- State of stress at a point based on Mohr's circle:
- Maximum Normal stress is σ_1 and the minimum is σ_2 . No shear stresses exist with these, hence Principal stresses.
- Maximum shear stress is equal to the radius of the circle. $= (\sigma_1 - \sigma_2) / 2$

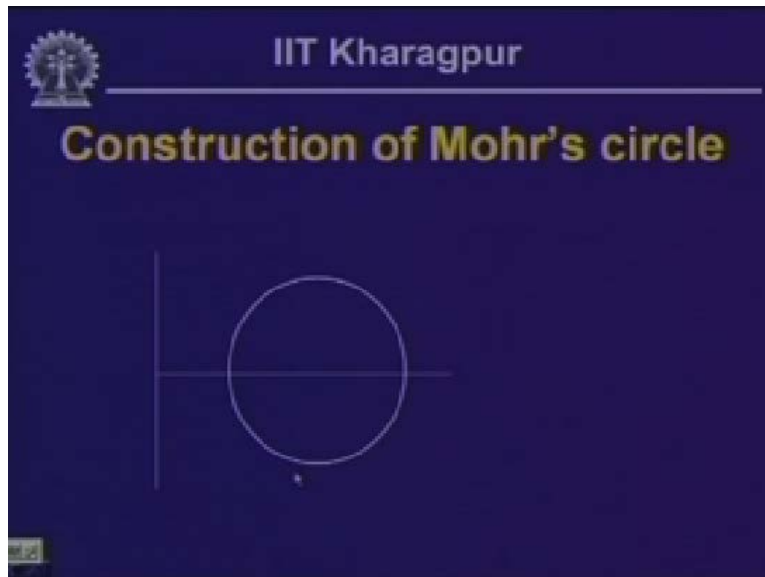
Hence from the Mohr's circle we can observe that the maximum normal stress is σ_1 which we have designated as maximum principal stress. The minimum shear stress is σ_2 , which is minimum principal shear stress and at those two planes we have seen that no shear stress exists. Because that being on the σ axis, the value of shear stresses is 0 and hence they are the principal stresses. Also, the maximum shear stresses is equal to the radius of the circle which is square root of $((\sigma_x \text{ minus } \sigma_y) \text{ by } 2)$ whole square plus τ_{xy} square and the radius is nothing but equals to in terms of σ_1 and σ_2 has $(\sigma_1 \text{ minus } \sigma_2) \text{ by } 2$ which is we have seen through our transformation as well.

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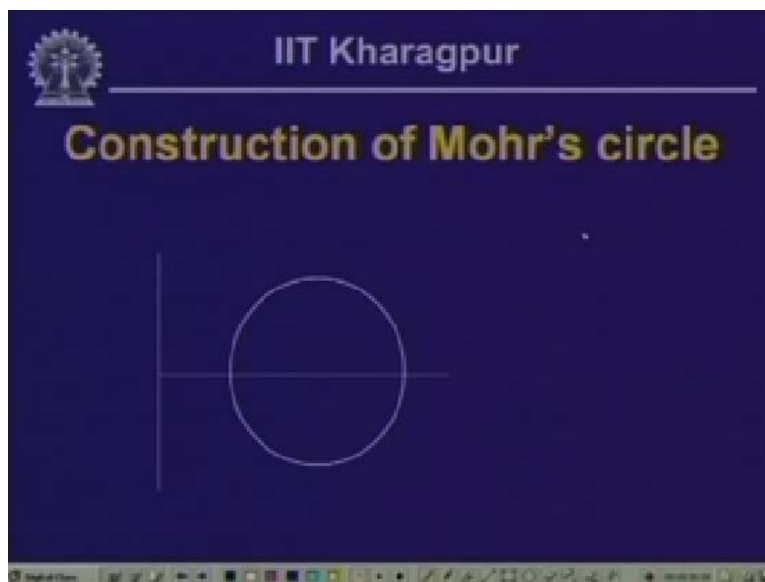
If σ_1 and σ_2 are equal then, Mohr's circle reduces to a point and there are no shear stresses will be developed in the x, y-plane. And if σ_x plus σ_y is equal to 0, then, as we have seen, centre of the circle is located at a distance of (plus a, 0) which is on the axis and plus a is equal to $(\sigma_x \text{ plus } \sigma_y) \text{ by } 2$, the average stress. If $\sigma_x \text{ plus } \sigma_y$ is equal to 0, the centre coincides with origin as zero point at the σ tau as reference axis. Hence, at any point on any plane which is on the circumference of the circle representing any plane at the particular orientation, the values we will get are the maximum principal stress as the tau and also maximum shear stress as tau. This we call as the state of pure shear. Maximum and minimum principal stresses are also equal to the maximum shear stress. These are the important observations from the Mohr's circle.

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Now let us look into how we construct a Mohr's circle. Basically we need to do that, if we know the stresses: σ_x , σ_y and τ_{xy} at a particular point, then we should be able to evaluate the stresses at any plane, which are at σ_x prime, τ_x prime and τ_y prime from the orientation of the plane with reference to the x-plane. Now those stresses can be evaluated, either from the transformation **of the** equations as we have just seen or we can evaluate the stresses from the Mohr's circle as well. Now let us look at how to construct a Mohr's circle based on the given state of stress.

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Now let us say that at a stress body, we know the stresses: σ_x , σ_y , and τ_{xy} . As we have seen the centre of the circle is located at a distance of $(\sigma_x \text{ plus } \sigma_y)$ by 2. Now in this there are two ways of constructing a circle, one is either we can represent σ in this direction and tau in the positive direction as we have noted earlier in that case, the angle will be in a clock wise direction, θ angle in the Mohr plane will be in clockwise direction which will be opposite to the convention which we have assumed while deriving this transformation equation, where we take θ in anticlockwise direction.

Another way to construct is we take σ in the positive x-direction and tau in the opposite direction, in that case the representation of angle in the Mohr's plane, is in anticlockwise direction which matches with our transformation of the equation. So let us represent the σ in the positive x-direction and the tau in the lower direction as positive. Thereby the angle will be represented in the anticlockwise direction which is considered as positive and matches with our physical stress system.

Now if we represent this stress σ and tau on this particular plane, thereby this particular point on the circle represents this plane, plane-A, where the normal stress is σ and shearing stress is tau. Likewise in the perpendicular plane we have stress as σ_y and shearing stress as tau which is at an angle of 90 degrees, with respect to this A-plane. In Mohr's circle if we go 180 degrees with respect to this, we get another point, which is plane-B where we have σ_y and τ_{xy} .

Now this if we take the centre, as this O, and radius as OA and OB, and plot a circle, this gives us the Mohr's circle where in this point represents the maximum normal stress σ_1 . This represents the minimum principal stress which is σ_2 . Now if we are interested to find out stress on any plane, whose normal to this particular plane is oriented at angle of θ , now this particular plane which is represented by this point A, from here it moves at angle of 2θ , and joins from the centre of this the point which we get on the circle represents the plane normal to which is at angle θ from the x-plane. Thereby the stresses that act over here together is the normal stress and the shearing stress on this particular inclined plane and that is how we compute the stresses in the Mohr's circle.

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Example Problem - 1

- A mass M is hung by two steel wires as shown in the figure. The cross sectional area of wire AB is 200 mm^2 and that of BC is 400 mm^2 . If the allowable tensile stress of the wire material is 100 MPa , what mass M can be safely supported by the wires?

Dimensions are in mm

Now having known about the principal stresses from the transformation equations and through the Mohr's circle let us look into some of the problems. Let us understand how to evaluate the mass which is safely supported by the wires. Now if we take the free body diagram of this particular part, then here there is the force F_1 to balance it, the reactive forces. Here there is force F_2 to balance it.

If we draw the free body diagram of this whole part then we have F_1 in this particular direction and F_2 in this particular direction and the mass which is hanging from this particular point. Let us call it this force as Mg . Now let us call this angle as θ_1 , and this as θ_2 . From the given data, we can compute $\cos \theta_1$ is equal to 0.6 and $\sin \theta_1$ is equal to 0.8 ; $\cos \theta_2$ is equal to 0.923 , and $\sin \theta_2$ is equal to 0.385 . Now if we take the summation of horizontal forces as 0 , that gives us the $F_1 \cos \theta_1$ is equal to $F_2 \cos \theta_2$.

Now it has been indicated that the maximum tensile stress that the wires can withstand as 100 MPa and cross sectional area where the two wires are given, thereby if we compute, F_1 will give area times stress which is equals to 40 Kilo Newton . F_2 thereby is given 20 Kilo Newton . From this expression if we substitute the values for $\cos \theta_1$ and $\cos \theta_2$, we find that if we substitute for F_2 for F_1 is equal to 40 Kilo Newton , and F_2 is equal to 40 into 0.6 by 0.923 is equal to 26 Kilo Newton which is greater than 20 Kilo Newton . So instead of using F_2 , let us use, F_1 so that if we represent F_1 in terms of F_2 this is 20 into 0.923 by 0.6 which is less than 40 .

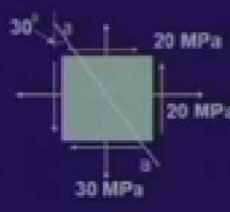
Hence if we compute the values, if we take the summation in the vertical direction, summation of forces we get, that Mg is equal to $F_1 \sin \theta_1$ plus $F_2 \sin \theta_2$ and thereby the values from M this you will get as 3293.4 Kg .

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Example Problem - 2

- The state of stress at a point in a body is as shown in the figure. Find the Normal and shear stresses acting on plane a-a using transformation equations.




You can look into this particular problem, you can compute the state of stress in transformation equations, also you can use the same problem to solve through Mohr's circle of stress.

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Example Problem - 3

- Solve the Example Problem-2 using Mohr's circle of stress.



These questions which we have posed in the last time they quite straight forward, the first one is the maximum normal stress and shear stress in an axially loaded bar, which we know as P by A is the normal stress and P by $2A$ is the shear stress and thereby τ the shear stress is equal to $\frac{\sigma}{2}$, half the normal stress. We have discussed about stress variants and we know that the value of shear stress on the principal plane is zero.

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Summary

This lesson included:

- Development of Transformation equations for evaluation of Stresses for 2-D problems.
- Evaluation of Principal stresses & maximum shear stresses
- Construction of Mohr's circle
- Examples to demonstrate the evaluation of stresses at a point.

To summarize: what we have done today in this particular lesson is developed the transformation equation for evaluation of stresses for two dimensional problems; evaluation of principal stresses and shear stresses, we have shown how to construct Mohr's circle for the evaluation of stress and some examples to demonstrate the evaluation of stresses at a point.

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Question Set 4.0

- What is meant by state of pure shear?
- What is the co-ordinate of the centre of Mohr's circle and what is its radius?
- What happens if $(\sigma_x + \sigma_y) = 0$ and $\sigma_1 = \sigma_2$?

These are the questions:

- What is meant X the state of pure shear?
- What is the co-ordinate of the centre of the Mohr's circle and what is its radius?

- What happens when $(\sigma_x \text{ plus } \sigma_y)$ is equal to 0 and σ_1 is equal to σ_2 ?