Strength of materials

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Lecture 39

Springs-I

Welcome to the first lesson of the tenth module which is on springs.

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We have looked into the aspects of the strength, the stiffness and the stability on the members where in we have evaluated the stresses, we have evaluated the deflections and also we have looked into the stability aspects of the members.

Now we are going to discuss on a special topic on which we will make use of the formulae which we have derived and we will look into that on how to calculate the stresses in a member called spring. Hence it is expected that once this particular lesson is completed one should get an idea about the different types of springs that are commonly used.

That means we will look into what we really mean by spring and then what are the different types of springs that we come across and what are the types of springs that are commonly used.

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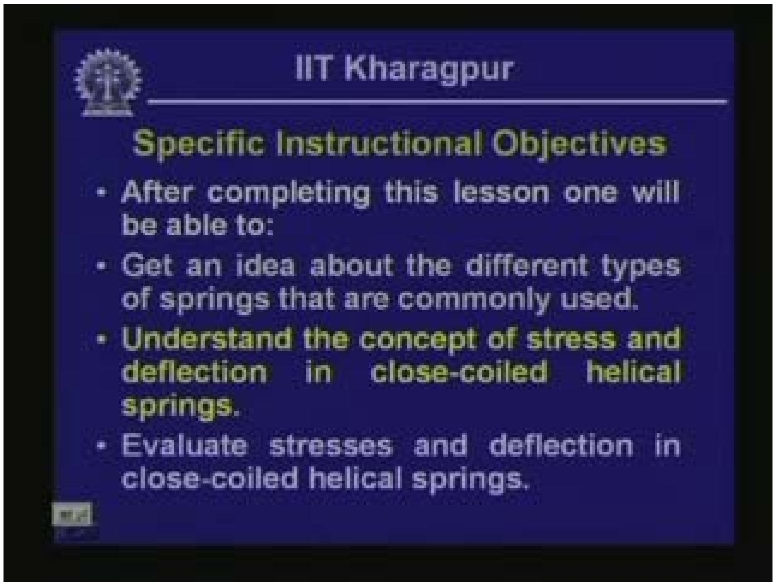

One should be able to understand the concept of stress and deflection in close coiled helical springs. We have taken a specific name called close coiled helical spring. We will look into the different types of springs and we are not going to discuss all the types in this particular course. We will be restricted ourselves specifically to the helical springs and we will look into those aspects of helical springs, where and how to evaluate the stresses in the springs.

One should be able to evaluate stresses and deflection in close coiled helical springs.

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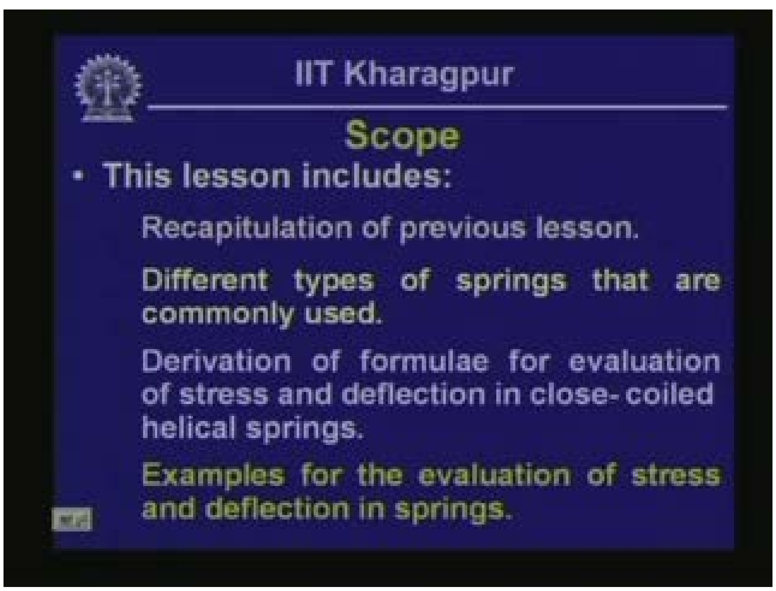

The scope of this particular lesson therefore includes the recapitulation of previous lesson.

In the previous module we have discussed about the stability of the columns and in the previous lesson on stability of columns, I have given you some questions.

We will be discussing the answers of those questions and to that extent we recapitulate the previous lesson.

Also we look into the different types of springs that are commonly used. This particular lesson includes the derivation of formulae for evaluation of stress and deflection in close coiled helical springs.

In fact we will make use of the already derived formulae for the members where we have seen that how to calculate stresses for different types of forces. The axial force or the shear force or the mini moment or the twisting moment etc, we will try to make use of that and see that a spring is subjected to what kind of stresses because of the application of the force.

We will try to combine the effect of these forces and avoid the stress in a spring. Also we will look into some examples for the evaluation of stress and deflection in springs.

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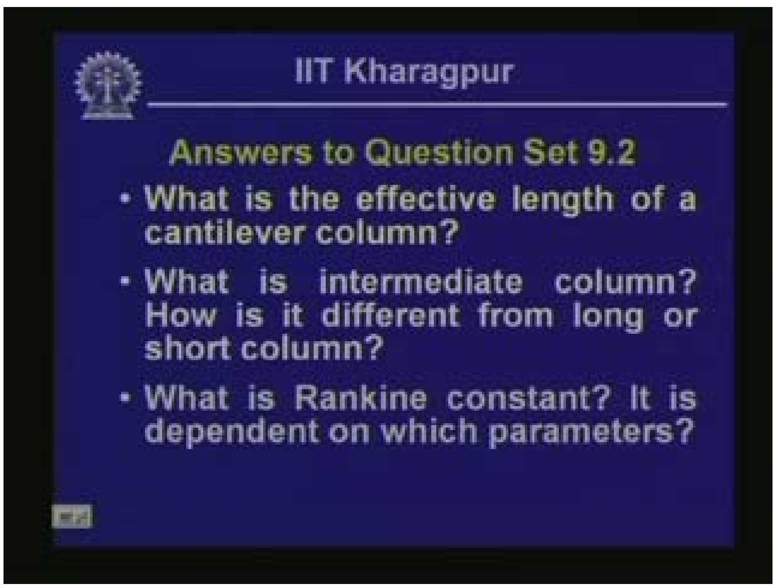

Before we go into the discussion for the springs, let us look into the answers of the questions which were posted last time.

The first question which was given was: What is the effective length of a cantilever column?

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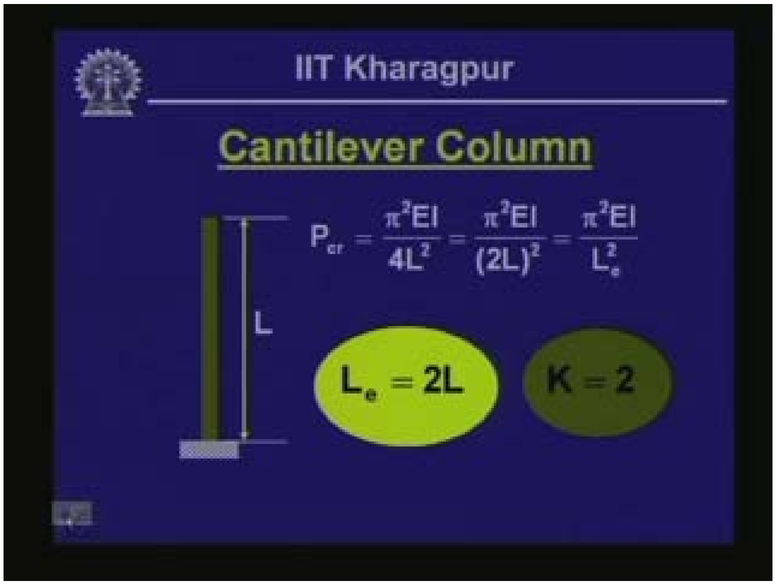

The previous lesson was on stability of columns.

We have seen that how to evaluate the critical load using Euler's formula which we have called as Euler's critical buckling load formula and was given by Leonhard Euler.

We have seen that if a column member is having different support conditions like if both ends are hinged or if you have one end fixed or other end hinged or if you have both ends fixed condition or if you have cantilever member were one end is fixed and other end is free then how do you evaluate the critical buckling load in such members.

Now the question is what is the effective length of a column member which is cantilever?

It means is fixed at one end and free at the other and the actual length of the member is L.

As we have seen, if use Euler's critical buckling load formula then P critical is equals to pi square EI over Le square.

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

In case of a member which is fixed at one end and free at the other, the critical load evaluation comes as pi square EI over 4L square.

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

We have seen this derivation in the previous lesson where in we have derived this particular expression. If we write the term 4L square in square form, then it will be (2L) square and there by 2L is equivalent to this term Le.

$$P_{cr} = \frac{\pi^2 EI}{(2L)^2}$$

We have called Le as effective length. Effective length Le is equals to twice the actual length. As we have seen that for the different support conditions the corresponding term, the coefficient of L is K where in, we have designated that as the effective length Le equals to K times L.

$$L_e = K \cdot L$$

and this parameter K changes with the support conditions and for cantilever case the value of K is equals to two.

$$K = 2$$

So the effective length for a cantilever column is equals to twice the actual length and the effective length coefficient K is equals to two.

$$L_e = 2L$$

This is the answer for the first question.

The second question is: what is the intermediate column and how is it different from long or short column?

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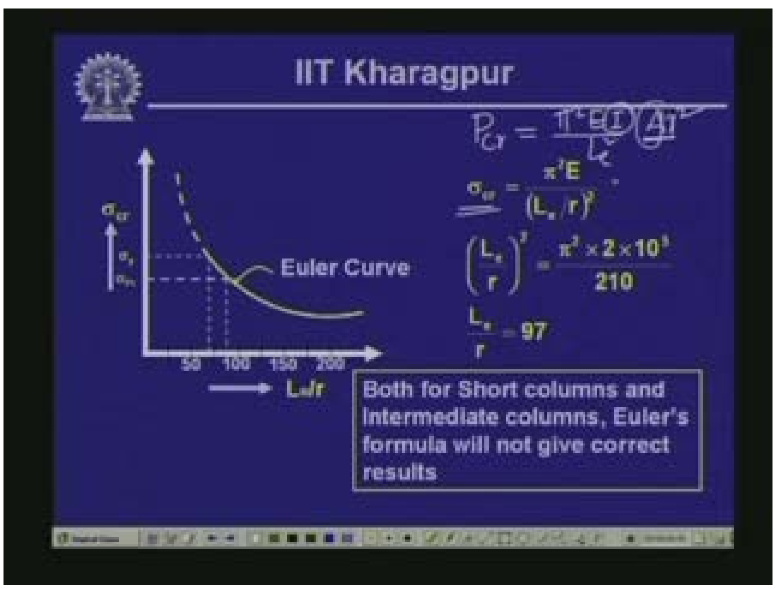

We have already discussed before about the aspects of short column, the aspects of long column and then we have defined what intermediate column is.

Now coming back to the discussion again with Euler's critical load, the critical stress as we have seen that the critical load is equals to pi square EI over Le square.

$$P_{cr} = \frac{\pi^2 EI}{L_e^{\ 2}}$$

And I we had represented in terms of the cross sectional area and the radius of direction as r square.

$$I = Ar^2$$

There by P critical divide by A gives a sigma critical.

Hence critical stress is equals to pi square E and I we take it as the denominator, so we have Le by r square. So pi square E by Le by r square is the critical stress.

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e / r)^2}$$

If Le by r becomes very small then the value of σ_{cr} will be large. The smaller value of Le by r indicates that the lower height of the column member and thereby that refers to a short column. If the stress becomes higher means that if it goes beyond the yield stress, then the material is going to yield. Hence it has no meaning when we say that the stress is much higher.

So we restrict ourselves to the limit of the yield stress.

We categorize those groups of columns as short column where L by r or Le by r is such that it produces stress which is above the critical stress or sigma yield stress.

In this particular case the critical stress is yield stress and the columns which come under that group we designate them as short column.

If Le by r becomes larger then the stress becomes lower.

Now the question is up to the stress limit of up to the proportionality, if we restrict ourselves to that limit then the Le by r which we get for a particular member corresponding to that we can find out the stress, and that stress will be somewhat below this stress limit at the proportionality limit state.

The columns which are governed by this particular expression given by Euler is valid when the stress is lower than the proportionality limit stress.

The short columns are the one which goes beyond the yield stress.

The long columns are the one where the stress is governed by these expressions and the yield or the critical stress is limited up to the stress at the proportionality limit.

In between these two stages that is, in between the state of short column and in the state of long column, we get set of columns which we designate normally as the intermediate column. The intermediate columns are in fact the ones which may fail by the combination of the yielding and the buckling.

As we have seen that in case short columns it fails by yielding or by crashing and in case of long columns it fails by buckling. The Intermediate column fails by the combination of these two yielding and the buckling.

They are in a particular range between these short columns and the long columns. Those columns are called as intermediate columns and for intermediate columns if we try find out the critical load using Euler's formula it will not give you the correct results.

The last question was: What is Rankine constant and it is dependent on which parameters?

As we have seen that for such intermediate columns where we cannot apply Euler's critical load formula for the evaluation of critical load, Rankine had proposed an empirical relationship for evaluating critical buckling load for such columns.

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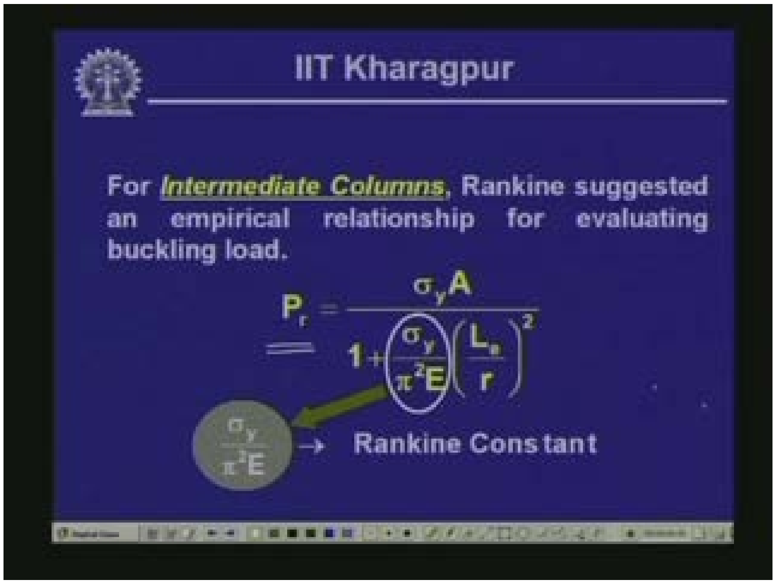

This is given by the formula,

$$P_r = \frac{\sigma_y A}{1 + \frac{\sigma_y}{\pi^2 E}\left[\frac{L_e}{r}\right]^2}$$

Where, σ_y is the yield stress of the material

A is the cross sectional area of the column member

E is modulus of the elasticity

Le is the effective length and r is the radius of the gyration

This particular parameter which is σ_y by pi square E, $\left[\frac{\sigma_y}{\pi^2 E}\right]$ is known as the Rankine's constant.

As you can see from this particular expression here we have a parameter σ_y which is the yield stress of the material and we have parameter E which is the modulus of the elasticity. So basically Rankine constant is dependent on the material properties, the modulus of elasticity of the material and the yield stress of the material with which the column member is a fabricated.

Let us look into the aspects which we are going to discuss today.

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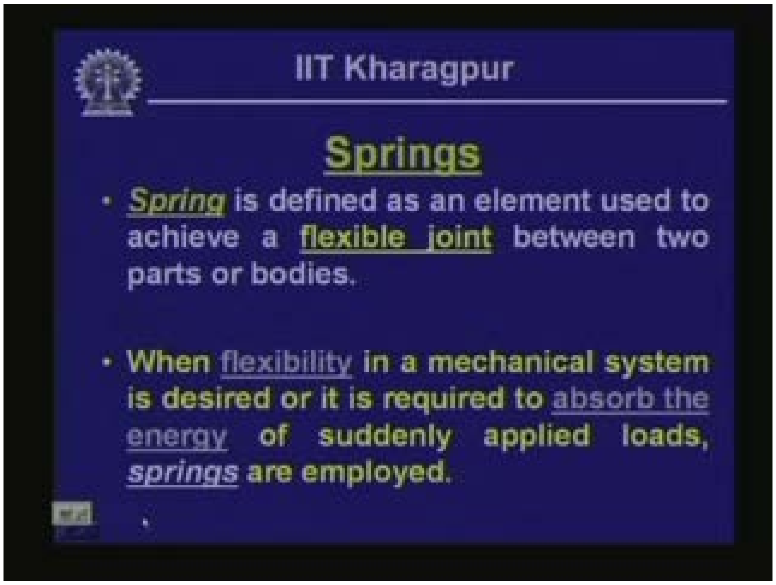

Spring is an element which is defined as that this is an element which is used to achieve a flexible joint between two parts or bodies.

As we have seen earlier that the deformation is not desirable in structural elements like beams or axial members or any other vessels kind of things, there are some situations where we need to have some amount of deformation to be introduced into the system. To achieve this kind deformation or flexibility, we introduce an element which we normally term as spring.

Spring is an element where when it is loaded it undergoes movement such as either elongation or shortening. Once we remove the load, the element comes back to its original position.

The level of the working stress in the spring material is rather high in comparison to the other structural materials we use.

This what stated over as second point in the slide:

When flexibility in a mechanical system is desired, which means, when we like to introduce some amount of flexibility, then we introduce this element which is called as spring.

Also if you like to absorb some energy, for example, in many of places such as structural members or mechanical systems which are subjected to sudden loads and if these kind of sudden loads act on structural members, the structural member may undergo large deformation and thereby there could be stresses which goes beyond the yield value of that material.

To absorb this shock loading or the impact loading, we introduce these kinds of elements which are called as spring. These spring elements can absorb the energy which is being imported by the external load. Thereby it can absorb that external loading shock.

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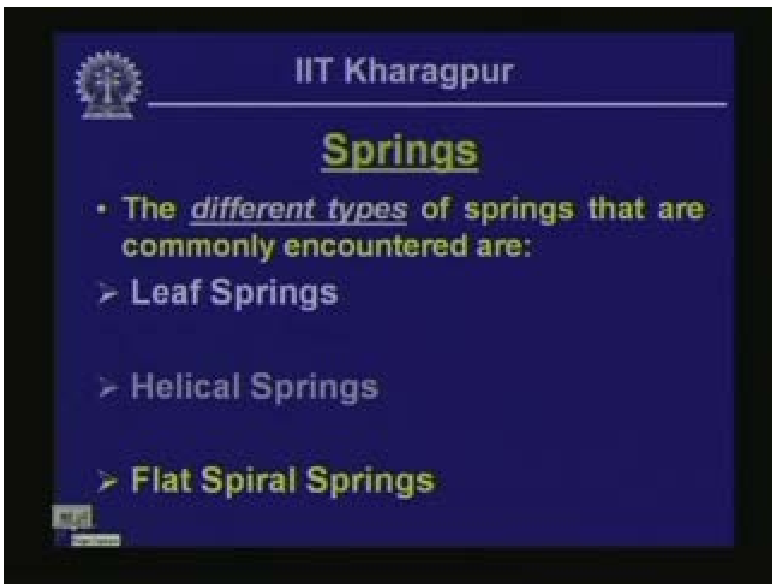

The different types of springs that are commonly encountered are leaf springs, the helical springs and flat spiral springs. There are other kinds of springs as well, but these are commonly used.

Let us look into these different types of the springs in detail.

Leaf spring is a spring which is also known as carriage springs which we can see in the transport vehicles.

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Basically we have number of plates which are joined together to form these kind of springs and these kind of springs can absorb the vibrated load that get transmitted from the vehicles.

In this particular type of springs the top plate is called as master leaf. These plates are called as leaves and that is why this spring is named as leaf spring. And as it is in semi elliptical shape these are also designated as semi elliptic spring.

There are clamps which hold all the plates positioned together and there are clips which are called as rebound clip to keep the spring in a position.

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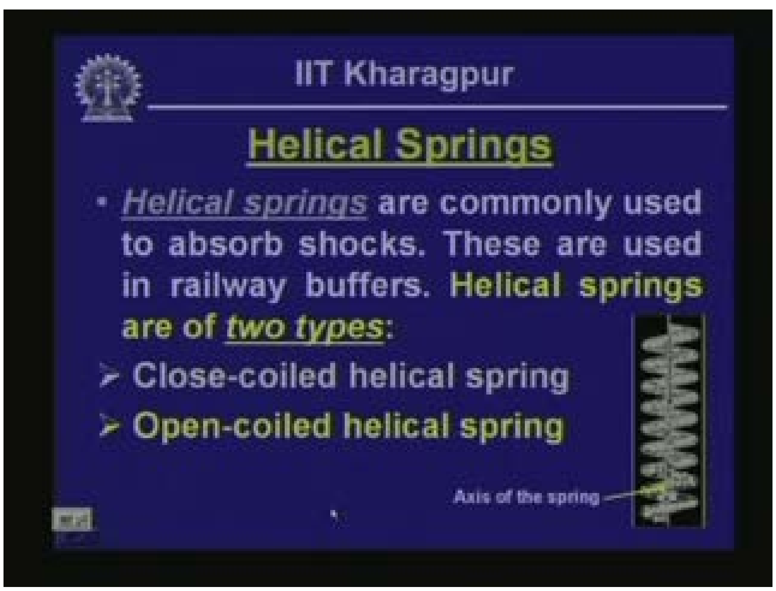

We have another kind of spring which are called helical spring. In fact in this particular module we will be looking in details about these helical springs. Helical springs are commonly used to absorb shocks. As I have told you that the springs are used to have desired flexibility in a mechanical system or springs are introduced to absorb shocks on the impact of the loads which comes suddenly on the member.

Helical springs are the springs which are generally used for absorbing such shocks. You must have seen that when a railway wagon comes into a platform. In a platform you have those buffers where the wagon comes and hits and then stops. The wagon comes in a speed and hits on the buffer and naturally it impacts some amount of energy on those buffers. The buffers are provided with the springs which can absorb the shock.

This is one of the examples where we use these helical springs. There are of course other areas were helical springs are used.

Helical springs are of two types. They are close coiled helical spring and other one is open coiled helical spring.

We will be looking into these two categories in greater detail. In this particular lesson we will be concentrating on the close coiled helical spring and subsequently we will look into the aspects of the open coiled helical spring.

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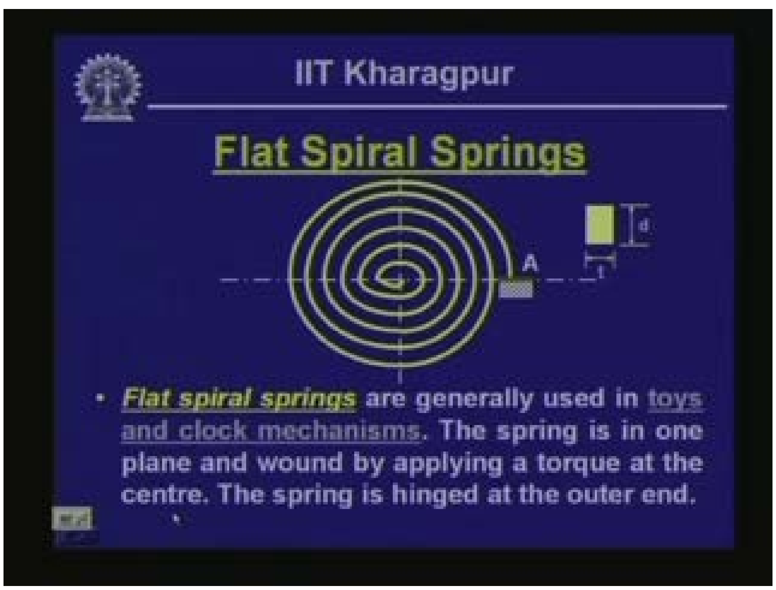

Apart from leaf spring and helical spring, another kind of spring which we commonly encounter is the flat spiral springs. In the toys are in clock mechanisms these kind of springs are used.

This particular spring is in one plane and they are wound in such way that we apply at a torque one end and the other end is hinged. So thereby the whole spring is in a dot position and the cross section of these particular spring material is something like a rectangular one having a thickness t and depth d.

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Apart from those springs which are very commonly used other kinds of springs which are used in some specialized cases. These springs are termed as Belleville spring or disk type of spring ring spring or volute spring.

These are the different kinds of the springs which are seen. But they are occasionally used in some special cases and we are not going to go into the details of these.

In this particular course, we will be restricting ourselves to the helical springs only.

As I said earlier, we will look into the two types of helical springs, the close coiled helical springs and then open coiled helical springs and evaluate the stresses in those kinds of springs if they are subjected to loads.

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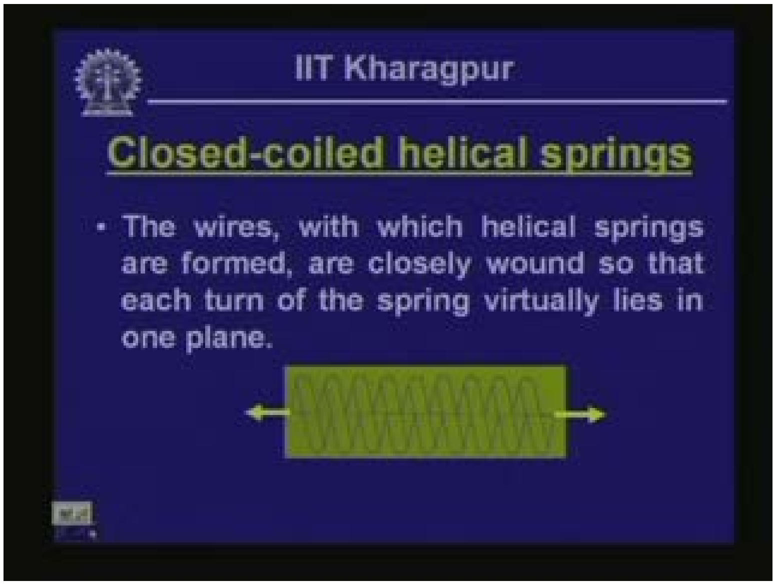

Let us look into the behavior or the characteristics of closed coiled helical springs.

Why we call these as closed coiled helical springs?

As you can see this wire which is forming a helix we call that as helical springs. These wires are wound in such a way that each turn of this helix is virtually in the same plane. That is what is indicated in the definition as closely wound and the important point here is that each turn of the spring virtually lies in one plane.

Since they are very closely wound we called these kinds of springs as closed coiled helical springs.

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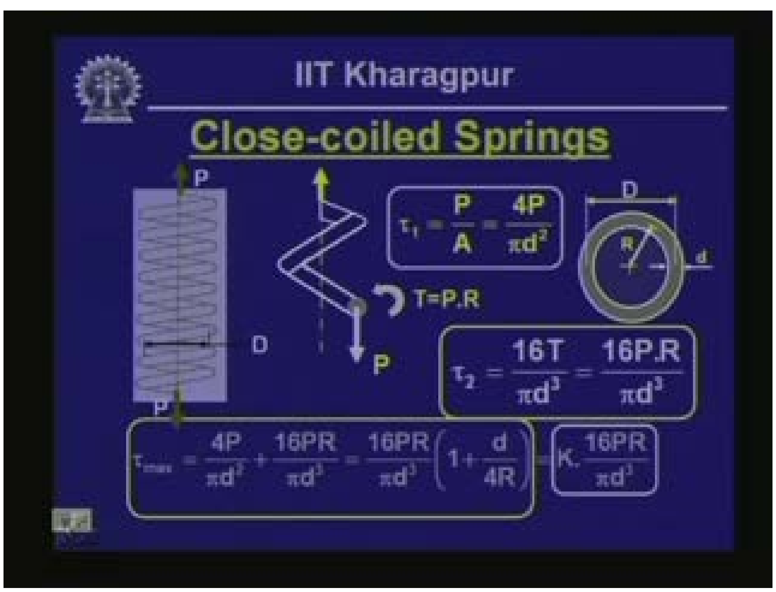


Let us look into how to calculate the stresses in such close coiled springs.

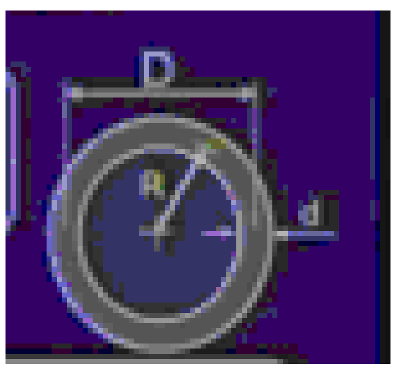
If a closed coil spring is subjected to a load p as indicated in the above slide.

The ends of these closed coil springs are provided with some forms, so that they are connected with some supports or some loads can be applied at these particular ends.
Several kinds of mechanisms are formed at the end, either in the form of a hook or a flat end which can be connected to a particular member where we can apply a load p in the system.

Suppose if we take a part of this spring, cut across and take a free body of this particular spring. These turns are normally called as coils.
These particular springs are having several coils along its length. If we look into one such coil where we have the load ‘P’ acting into the member, if we transfer that load in the section of the member this P gets transferred along with the movement and this particular movement here is a twisting moment T.

$$T = P \cdot R$$

In this particular coil, if we look into that we have one wire which is having diameter‘d’.
If you look into a plane view of one of such coils, say this is the circular form and this is the radius R up to the central line of the wire and this R is the mean radius of the spring.
R we called as mean radius of the spring and small d is the diameter of the wire with which the spring is form and the capital D is the diameter of the whole spring coil (the small cross section of the wire shown in the right side of the slide).



If we look into this action of these forces when it is transfer to this center of the wire cross section where this load P is acting as a shearing force and thereby it will produce a direct shear stress.
And the twisting moment T which is acting in this solid circular section will be producing a shearing stress as we have seen earlier.
We have seen that if a solid circular section is subjected to twisting moment T then you get the shearing stress.

So as you can see that if you consider the circular wire which is subjected to a shearing force P will produce a direct shear stress and because of the twisting moment you will have a shearing stress component which is the function of the twisting moment.

Generally in spring we consider the direct shearing stress in the form average stress which is equals to P by A.

$$\tau_1 = \frac{P}{A}$$

So the cross sectional area of this spring wire is,

$$A = \frac{\pi d^2}{4}$$

where small d is the diameter of the spring wire and P is the shearing force that is acting in the wire.

So the shearing stress tow 1 is equals to 4P by πd^2,

$$\tau_1 = \frac{4P}{\pi d^2}$$

because of the twisting moment T which is equals to P into R,

$$T = P \cdot R$$

we get another component of the shearing stress which is 16T by πd^3

$$\tau_2 = \frac{16T}{\pi d^3}$$

This we have seen earlier that T by J is equal to tow by row.

And tow as we calculate is equals to T row by J

$$\tau = \frac{T \cdot \rho}{J}$$

and row for a circular one is d by 2 and J is $\pi d^4$by 32 and thereby we get tow as 16T by $\pi d^3$and the value of T the twisting moment in this particular case is equals to P times R where P is the force which is acting through the axis of the spring and R is the mean radius of the spring coil. So P times R is the twisting moment that will be acting in the spring wire. So if we replace T as a P times R, this becomes 16PR by πd^3.

$$\tau_2 = \frac{16T}{\pi d^3} = \frac{16P \cdot R}{\pi d^3}$$

The total shearing stress that will be subjected to, is the combination of these two shearing stress which is tow1(τ_1) and the other is tow2(τ_2).

So we call this as maximum shearing stress tow max is equals to 4P by πd^2 plus 16PR by πd^3.

$$\tau_{\max} = \frac{4P}{\pi d^2} + \frac{16PR}{\pi d^3}$$

Again, the first one is from direct shear stress and the other one is originating from the twisting moment.

If we take 16PR by πd^3 out, we get a factor which is 1 + d by 4R

$$\tau_{\max} = \frac{16PR}{\pi d^3}\left[1 + \frac{d}{4R}\right]$$

and the bracketed term shown above is called as K, which is stress concentration factor.

$$\tau_{\max} = K \cdot \frac{16PR}{\pi d^3}$$

Basically if you look into the 16PR by πd^3 is the shearing stress which is getting generated because of the twisting moment of magnitude P into R. So the shearing stress which is getting generated because of the twisting moment is multiplied with the factor K which we call as stress concentration factor.

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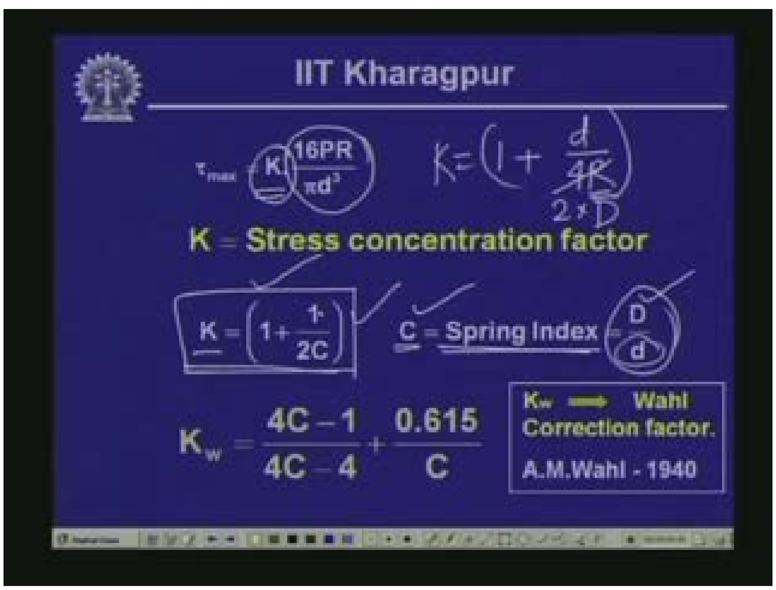

Now K as we have seen is equals to 1 + d divided by 4R.

R is the main radius. We can write 4R as 2R multiplied by 2R. And 2R is nothing but is the diameter of the coil which is D.

This particular term capital D by small d is defined as 'Spring index' and designated as the parameter C which is the ratio of the coil diameter D to the wire diameter d.

If we substitute for D by d as C, we get K equals to 1+ 1 by 2C.

$$K = 1 + \frac{1}{2C}$$

This is termed as the stress concentration factor.

As we have seen here while evaluating this stress concentration factor K or while designating that bracket term 1+ d by 4R as stress concentration factor there we have accounted for the shearing stress because of the direct shear which is in an average sense and also we have not considered the curvature of the spring coil.

This particular aspect the maximum value of the shearing stress as it happen in the circular cross section or if we take the curvature of the spring coil into account, this particular factor gets modified and in 1940 A.M. Wahl had proposed the modification of this particular factor which we call as Wahl's correction factor K_w and K_w is given by this particular expression.

When we need to evaluate the stresses precisely, we use this Wahl's correction factor K_w instead of K which is four C minus 1 by 4C minus 4, plus .615 by C.

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Now we can compute the value of the stress if we know the amount of load a spring is subjected to. As we have seen now, the load which will be acting through the axis of the spring will produce a direct shearing stress and a twisting moment into the spring coil. The twisting moment eventually will lead to the shearing stress.

So you will have two components or two shearing stresses, one from the direct shear force and another one from the twisting moment. It is a combined action of these two shearing stresses, axial force or a shear force P and the twisting moment T which is equal to P times R will give the resulting shearing stress in the member.

When a spring element is subjected to axial load, it undergoes deflection or deformation. We need to find out that how much elongation or compression which we call in general as the deformation in the spring it undergoes because of the application of the axial load P.

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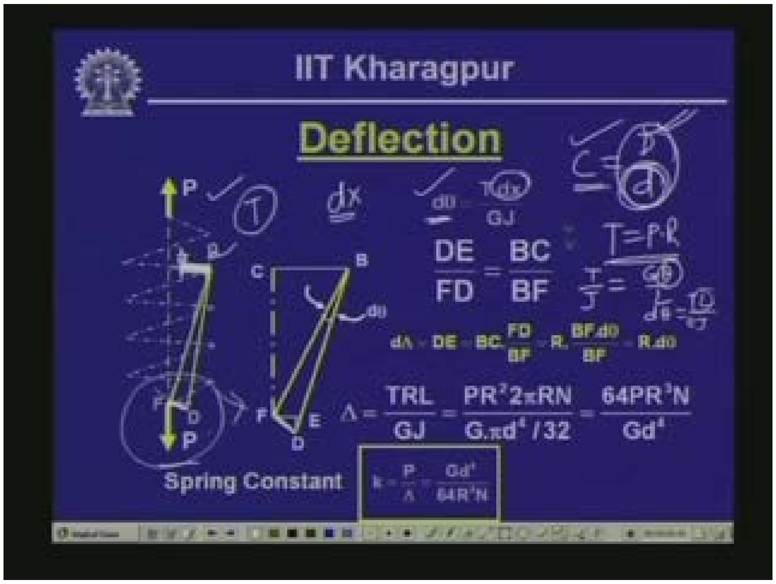

Let us look into this deflection in detail.

If a spring is subjected to a load P then as we have seen that the wire with which the spring is wound is subjected to a load P and a twisting moment T.

We consider a spring were the spring index is rather high which is, C equals to D by d

$$c = \frac{D}{d}$$

where in the diameter of the coil is larger or the wire diameter is very small. For such larger spring index or for springs with having larger spring index, if we consider a small element in the spring let us call these as distance AB.

Virtually this becomes like a straight length that means you have a straight rod which is subjected to a twisting moment. In this particular case we ignored the effect of the axial or the deflection that will be produced by the direct shear. We will be evaluating the deflection that will be caused by the twisting moment T.

If we look into this particular segment AB say having length dx, because of the twisting moment T which is equal to P into R this is subjected to, will produce the rotation.

The relative rotation between the two sections A and B, if we call that as d theta ($d\theta$), then we can compute this value of the relative rotation $d\theta$ from T.dx by GJ.

$$d\theta = \frac{T.dx}{GJ}$$

As you know that T by J is equals to tow(τ) by row(ρ) is equals to G theta by L.

$$\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\theta}{L}$$

So θ for a smaller element will be $d\theta$ which equals to TL by GJ and L here is the length dx for this particular element AB.

$$d\theta = \frac{TL}{GJ}$$

If we think of that is A fixed and B is moving which means d theta($d\theta$) will give the rotation of the segment AB. And because of the rotation of this segment AB, the point F of this load point, will undergo a movement and let us call this movement as FD.

Basically this movement will be in an arc length, but since AB is a small segment we consider that FD also will be small, so that FD is perpendicular to this length BF.

This particular piece from figure1 is exaggerated in the figure2.

The distance FD can be written as BF the distance, times $d\theta$.

$$FD = BF.d\theta$$

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If we take the component of this FD, one will have horizontal component another with a vertical component.

In this particular spring where we have considered, one segment is producing one horizontal component FE, since this particular spring will have number of coils and this horizontal component will be produced by each of these segments, diametrically opposite points in the spring, those horizontal components eventually will cancel out. So the components which will be remaining are the vertical ones and the vertical component is the DE, the distance DE.

If we look into this particular triangle which is FDE, and the triangle FBC, then the perpendicular DE divided by the hypotenuse FD is equal to BC by BF.

$$\frac{DE}{FD} = \frac{BC}{BF}$$

As we have seen that FD equals to BF times $d\theta$, if we write that then DE is equals to BC times FD by BF and FD we write as BF $d\theta$,

$$DE = BC.\frac{BF.d\theta}{BF}$$

The distance BC is nothing but equals to the mean radius R of this spring coil. So this particular expression becomes R times $d\theta$

$$DE = R.d\theta$$

Remember that this is the vertical component which is DE and this is generated because of this segment which AB undergoing a twisting moment T.

We call this as small deformation $d\Delta$. If we sum up all these deformation in the entire spring coil then we integrate this over the entire length of the spring.

So we integrate this $R.d\theta$ as $\int R.d\theta$, thereby we get the total deformation which we have designated as delta (Δ).

This is equal to in place of $d\theta$. If we place T dx by GJ, TR by GJ will come out and interprets dx over the length will you give length L.

So the delta or the total deflection that we get in the vertical direction is equals to TRL by GJ

$$\Delta = \frac{TRL}{GJ}$$

and as you know the twisting moment T is equals to P times R so this becomes PR square by GJ and the length L is the total length of the spring coil.

$$\Delta = \frac{PR^2L}{GJ}$$

A spring which is having a number of coils and one such coil if we take which is having circular form having mean radius R, the length is twice πR. If we have N such turns then N times twice

πR will give us the total length of the spring and this what is indicated in the below formula. L is equals to twice πR times N, where N is the number of turns we have in the spring.

$$\Delta = \frac{PR^2 2\pi RN}{GJ}$$

As you know J is the polar moment of inertia of the wire of the spring and the diameter of the wire is small d so this is $\pi d^4 / 32$.

If we substitute these values we get the value of delta (Δ) as equals to 64PR cube N by Gd to the power 4.

$$\Delta = \frac{64PR^3N}{Gd^4}$$

This is the value of the deflection that we get in the spring and mind that we have computed the value of this deflection considering the effect of the twisting moment only and we assume that the effect of this direct shear in the deflection is rather insignificant.

Thereby from this expression we arrive at a parameter which is quiet important, spring constant which is normally designated as small k. Spring constant k is equal to P divided by delta (Δ), the axial load divided by the deflection that it undergoes, which is equal to GD to the power 4 by 64 R cube times N.

$$k = \frac{P}{\Delta} = \frac{Gd^4}{64R^3N}$$

As you can see all the parameters GDRN they are basically the parameters of the spring coil.

D is the diameter of the wire with which the spring is made.

R is the mean radius of the spring coil.

N is the number of turn.

This particular parameter spring constant is of importance, because that gives as the behavior of a spring.

If we know the spring constant from which if we like to permit certain deformation in the member, we know how much load that can be applied on the spring.

So spring constant parameter is one of the important parameter for a spring.

Having looked into the aspects of the stresses that are generated in a spring and how to evaluate the deflection of a spring because of the load, let us look into the examples through which we can evaluate these parameters.

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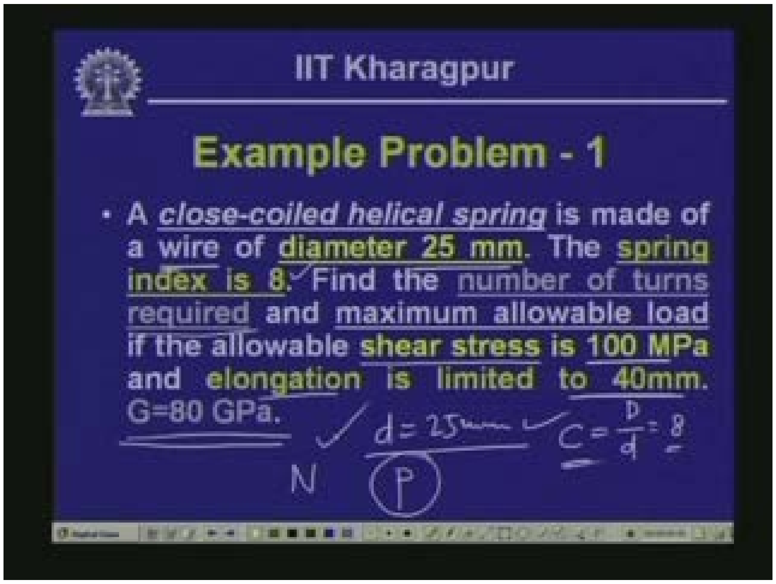

The example problem here is a close a coiled helical spring is made of a wire of diameter 25 millimeter. That means the small d is equal to 25 mm. The spring index value is given as 8 and as you know spring index C is equals to D by d. We will have to find out the number of turns those are required and maximum allowable load if the allowable shear stress is 100 MPa and elongation of the spring is limited to 40mm. The value of the shear module G is given as 80 Giga Pascal.

You will have to find out the number of turns that means the value of N and the maximum allowable load the P that can be allowed on this particular spring.

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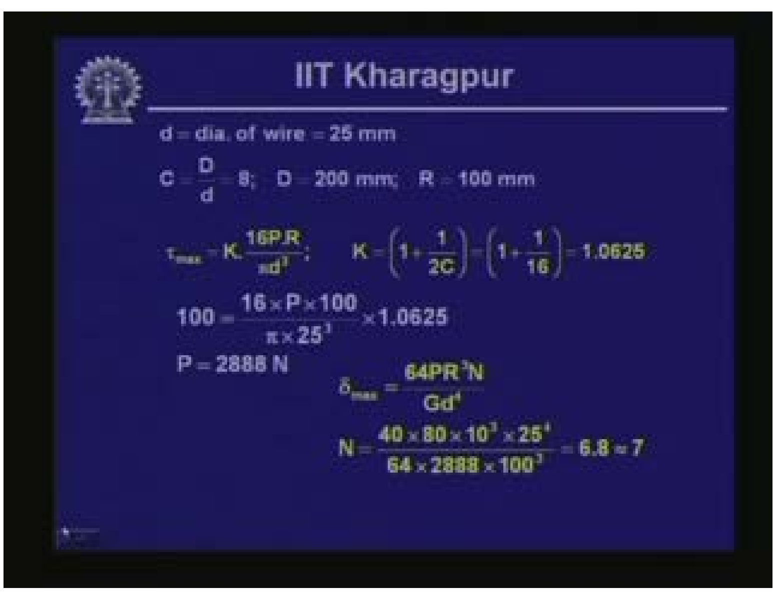

As I said, the values which have been given are; the diameter of the wire which is equal to 25mm and the spring index C is equal to D by d which is equal to 8.

From these we can evaluate the diameter of the spring coil which is equal to 200mm and the mean radius is equal to 100mm. So from the expression of the shearing stresses as we have seen the maximum shearing stress is equal to K times 16 PR by πd^3

$$\tau_{\max} = K \cdot \frac{16PR}{\pi d^3}$$

where K is the stress concentration factor and it is given by the below expression as 1 plus 1 by twice C.

$$K = 1 + \frac{1}{2C}$$

C is given as 8 and hence 1 plus 1 by 16, which gives a value of 1.0625

$$K = 1 + \frac{1}{16} = 1.0625$$

If we substitute all this values in the expression for the stress because stress is to be limited to 100 Mega Pascal, say

$$100 = \frac{16 \times P \times 100}{\pi \times 25^3} \times 1.0625$$

then the value of P from this expression comes as 2888 newton

$$\mathrm{P = 2888N}$$

also it is stated that the maximum value of the deflection of the spring is restricted to 40mm. so if you restrict the deflection of the spring to 40mm and from the expression of the deflection which we have seen is equal to $64PR^3$ times N by Gd to the power 4 where in N is the number of turn. From these we get N is equals to 40 for the delta max ($\delta_{\max}$),

$$N = \frac{40 \times 80 \times 10^3 \times 25^4}{64 \times 2888 \times 100^3} = 6.8 = 7$$

G is shear modular which is 80 Giga Pascal. So 80 times 10 to the power 3 MPa,

times‘d’ is 25. So 25 to the power 4,

divided by 64,

P as we have calculated is equal to 2888 and R is equal to 100, so100 cube.

This equation gives as value of 6.8. If you round it of, the number of turns you will get 7. so the spring with the which I mean a wire of diameter 25mm has been used to form a can carry a load of 2888newton and if we like to limit the displacement or the deflection of the spring up to 40mm then the number of turns those are required are 7.

Let us look into another example.

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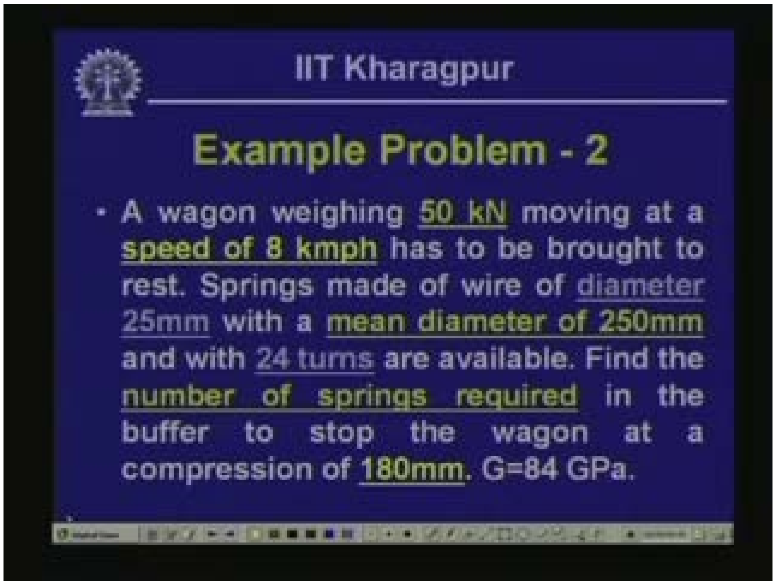

This is an interesting problem.

A wagon weighing 50 kilo Newton (kN) moving at a speed of 8 kilo meter per hour (kmph) has to be brought to rest. Springs are made of wire of diameter 25mm with a mean diameter of 250 mm of the coil and with 24 turns are available.

We will have to find out that how many numbers of such springs are required for the wagon to be brought to rest with a compression of a 180mm that means the maximum deformation that you can allow on the spring is equals to 180mm.

The value of the shear modulus is given as G equal to 84 Giga Pascal.

As you can note that the wagon which is moving at a velocity has to be brought to the rest. When it goes and hits into the buffer the kinetic energy of the wagon is transferred into the energy that can be absorbed by the spring.

We need to find out that if a force is applied on the spring and if we can allow a maximum compression of as it is given 180mm, then what is the work done by those springs to absorb the thrust or the load which is imported on those springs by the wagon and then a the wagon can be brought to the rest.

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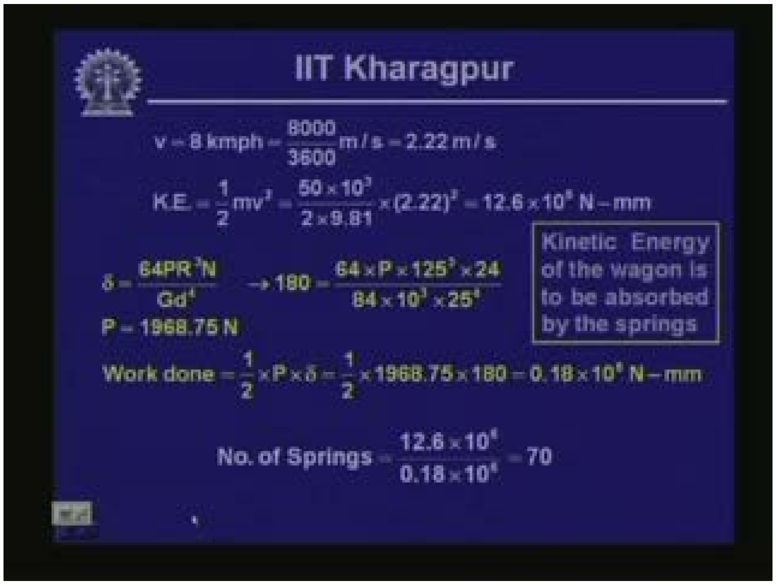

As you are aware that the kinetic energy of the wagon which is equal to half mv square ($\frac{1}{2}mv^2$) where m is the mass of the wagon and v is the velocity with which it is moving.

As it is indicated, the velocity of the wagon is 8kmph. If you reduce it to or in terms of meter per second, 8km is 8000meter divided by hour. Hour is converted in second as 3600.

$$v = 8kmph = \frac{8000}{3600} m/s = 2.22 m/s$$

This gives as 2.2 meter per second.

The weight of this wagon has been given as 50kN.

$$m = \frac{w}{g}$$

and g equals to 9.81 meter per second square.

This is what has been given w divide by g

$$K.E = \frac{1}{2}mv^2 = \frac{50 \times 10^3}{2 \times 9.81} \times (2.22)^2 = 12.6 \times 10^3 N - mm$$

and 50kN of 10 to the power 3 is multiplied with to get convert it newton and 2.22 square is the v square.

This gives as value of 12.6 into the 10 power 6 newton millimeters.

Now let us look into that if we have to limit the deformation of the spring to180mm then what is the load that can be taken by the spring.

From the expression of the deformation or the deflection that delta is equal to 64PR cube N by Gd to the power 4,

$$\delta = \frac{64PR^3N}{Gd^4}$$

we can evaluate the value of P because the value of R, the mean diameter of the spring is given as 250mm. Thereby the radius is 125mm and the number of turns given for the spring is 24 and the diameter of the wire with which the spring is formed d is equal to 25mm.

With these values if we substitute in the above formula, we have P is unknown

$$180 = \frac{64 \times P \times 125^3 \times 24}{84 \times 10^3 \times 25^4}$$

$$P = \frac{84 \times 10^3 \times 25^4}{64 \times 125^3 \times 24} = 1968.75N$$

the shear modulus G is given as 84 Giga Pascal, so 84 times 10 to the power 3 of MPa.

This gives as a value of the P which is equal to1968.75 newton. This is the value of the load that can be applied to the spring, so that the deflection criteria are satisfied.

As I said that the kinetic energy of the wagon is to be absorbed by the springs by applying that load and allowing the deformation which we call as work done by the spring. That means to achieve or to have a maximum deformation of the spring up to 180mm the load as you can see that you can apply P which is 1968.75 newton. This load will do some amount of work in pushing the spring by 180mm.

So work done by the spring is equal to half P times delta.

$$Workdone = \frac{1}{2} \times P \times \delta = \frac{1}{2} \times 1968.75 \times 180 = 0.18 \times 10^6 N - mm$$

P is 1968.75 newton and the delta or the deformation is equal to 180mm.

This gives us a value of the work done by each of the spring as 0.18 into 10 to the power 6 newton millimeter.

As we have seen that the kinetic energy of the wagon is equal to the 12.6 into 10 to the power 6 newton millimeter. This is to be brought to the rest.

So the number of springs that we need is equal to this kinetic energy divided by the work done by each of these springs.

$$No.\,of\;\;Springs = \frac{12.6 \times 10^3}{0.18 \times 10^6} = 70$$

Hence 70 springs should to be used in order to bring the wagon to the rest, by absorbing the energy which is being imported by the wagon.

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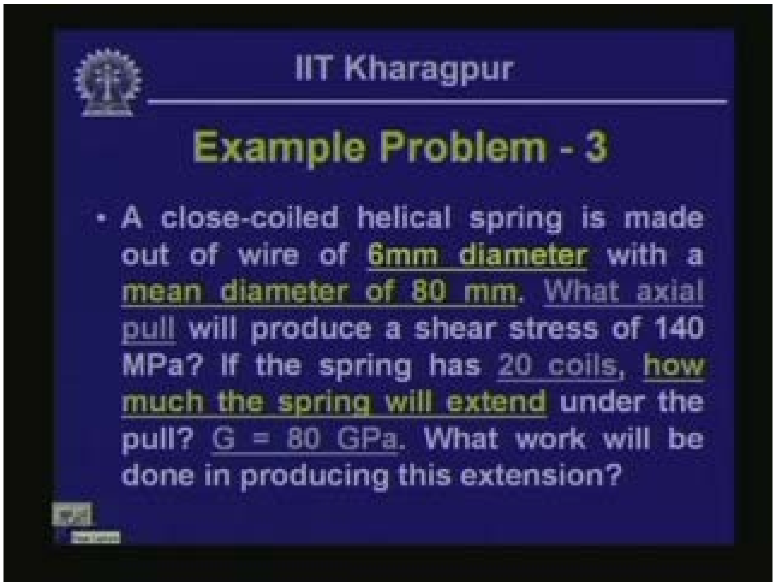

We have another example where it says that a close coiled helical spring is made out of wire of 6 millimeter diameter with the mean diameter of 80mm. What axial pull will produce a shear stress of 140MPa?

You will have to find out what axial pull that you can apply. That means the value of the P can be applied if the spring has 20 coils or 20 turns, then how much the spring will extend under this pull. That means you will have to find out the deformation delta and what work will be done in producing this extension. The value of shear modulas G is given as 80GPa.

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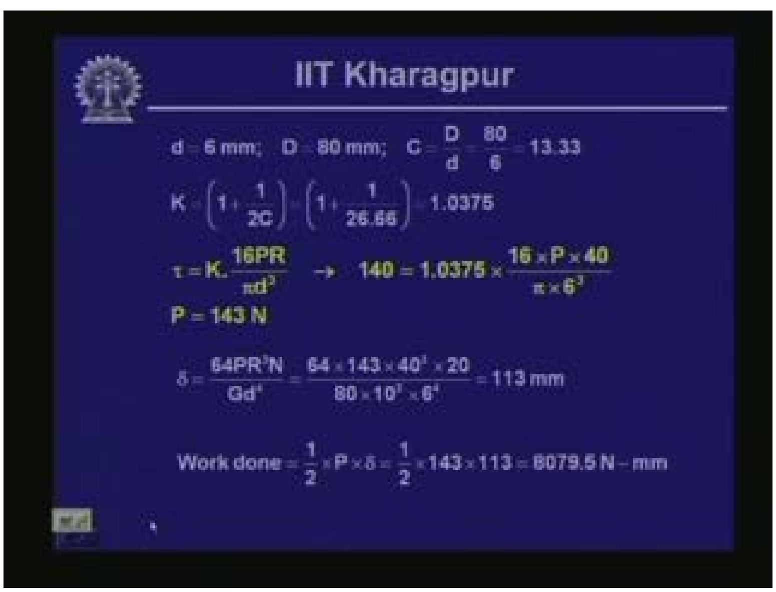

The values given are the diameter of the wire with which the spring is made is 6mm, the mean diameter of the spring coil is 80mm. Thereby the spring index C is equal to D by d which is equal to 80 by 6 gives a value of 13.33.

If we use this value of the spring index, then stress concentration factor K as we have seen is equal to 1 plus 1 by twice C.

As C is 13.33 and this value of K comes as 1.0375.

It is stated that the maximum shearing stresses that can be imported or that is allowed is equal to 140MPa. What will be the load corresponding to that?

If we substitute in the expression for the stress,

$$\tau = K \cdot \frac{16PR}{\pi d^3} \rightarrow 140 = 1.0375 \times \frac{16 \times P \times 40}{\pi \times 6^3}$$

Stress is equal to the stress concentration factor k multiplied by $\frac{16PR}{\pi d^3}$.

Since the diameter of the coil is 80mm, the R is 40mm.

The diameter of the wire is 6mm, so this is pi times 6 cube.

This gives a value of P which is equal to 143 newton.

The value of the deformation of the deflection delta as we have seen the expression for the delta is equal to $\frac{64PR^3N}{Gd^4}$ and we have evaluated the value of P which is equals to 143 newton.

The mean radius of the coil which is 40mm, so 40 cube and the number of the turns in the coil is equal to 20.

The shear module value G is 80GPa converted to MPa, so 80 times 10 to the power 3.

d is the diameter of the wire which is 6 to the power 4.

This gives a value of delta as 113mm.

$$\delta = \frac{64 \times 143 \times 40^3 \times 20}{80 \times 10^3 \times 6^4} = 113mm$$

$$Work\ done = \frac{1}{2} \times P \times \delta = \frac{1}{2} \times 143 \times 113 = 8079.5N - mm$$

The load which can be imported on the spring coil is equal to P which is 143 newton and the deformation that occurs because of this load P is equal to 113mm.

So the work done by this particular load is equals to half P times delta and this is equal to 8079.5 newton millimeter.

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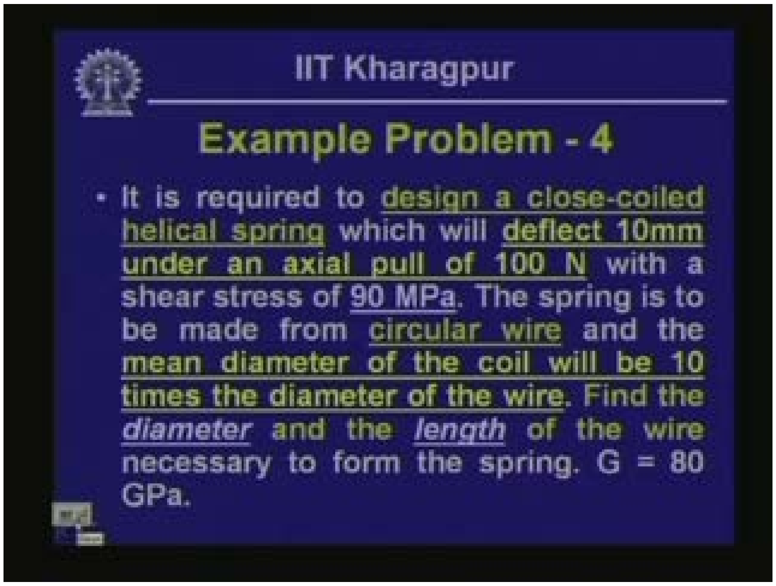

We have another example where it is required to design a close coiled helical spring which will deflect 10mm under an axial pull of 100 newton with an allowable shear stress of 90MPa and the spring is to made from circular wire and the mean diameter of the coil will be 10 times the diameter of the wire. So it is stated that the D is equal to 10 times the d or spring index is given as 10. We will have to find out the diameter and the length of the wire to form the spring. G is given as 80GPa.

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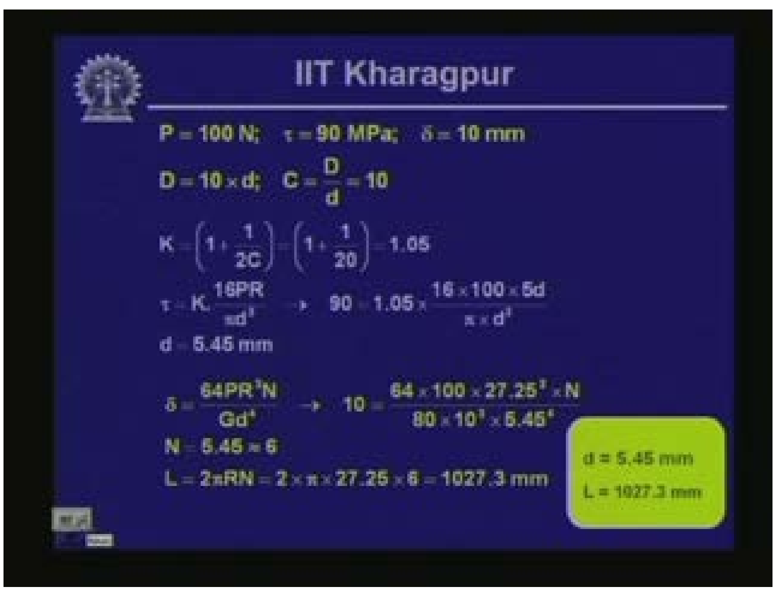

What is given is the load P as 100 newton and the shearing stress allowable is equal to 90MPa and also the maximum deflection that is allowed is equal to 10mm.

The spring index C which is the ratio of the D to the d is equal to 10 or the mean diameter of the coil is equals to 10 times the diameter of the wire. From the spring index C, we can compute the value of the K which is 1 plus 1 by twice C and this gives as value of 1.05.

From the expression of shearing stress tow is equal to K times $\frac{16PR}{\pi d^3}$. Limiting the shearing stress to 90,

$$\tau = K.\frac{16PR}{\pi d^3} \rightarrow \ 90 = 1.05 \times \frac{16 \times 100 \times 5d}{\pi \times d^3}$$

R is half D which is five times d and d is the parameter which we will have to evaluate. This value of d comes out as 5.45mm. The diameter of the wire that is to be used for forming the spring is 5.45mm.

The expression of the deflection the delta is equal to $\frac{64PR^3N}{Gd^4}$. If we limit the deflection to 10mm and since all other parameters are known expect the number of turns N, we can compute the value of N and it comes out as 5.45. We use a round figure of this number of turns which is 6. As we have seen the length of the wire that will be required for one turn is $2.\pi.R.$ and hence for N turns it is, $2 \times \pi.R.N$ where R is the mean radius which is 27.25mm and N is the number of turns which is 6.

Hence the length of the wire which we need is equal to 1027.3mm and the diameter of the wire which we need is 5.45mm for forming this spring.

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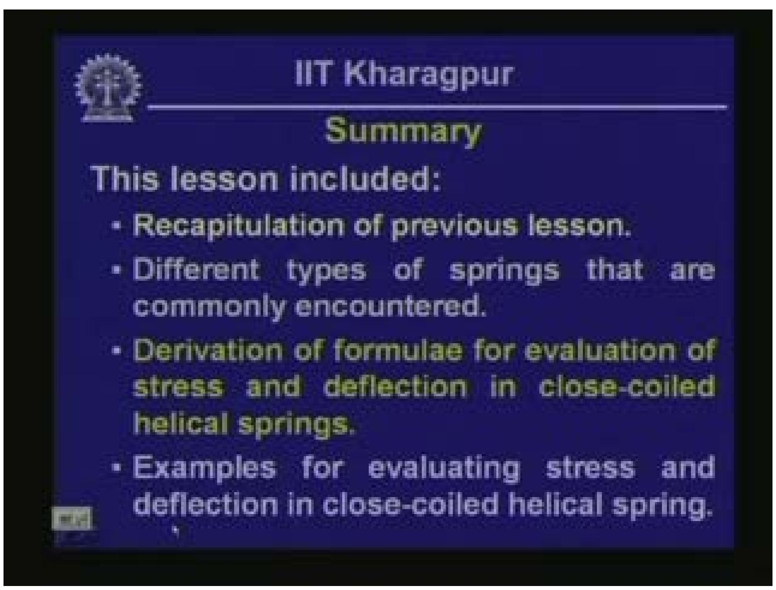

To summarize, in this particular lesson we have looked into the aspects of the previous lesson through our question/answers also we looked into the different types of springs that are commonly encountered.

We have looked into the derivation of formulae for evaluation of stress and deflection in close coiled helical springs and also we have looked into some examples for evaluating stress and deflection in close coiled helical springs.

These are the questions given for you.

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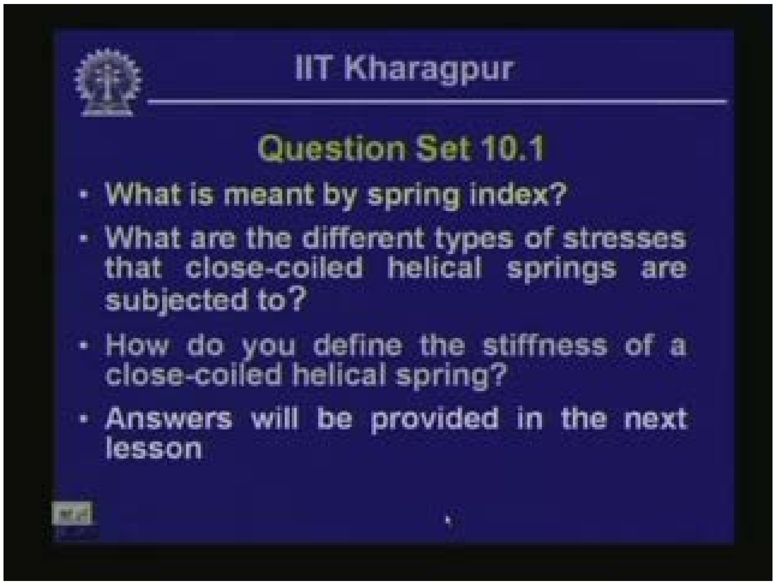

What is meant by spring index?

What are the different types of stresses that close coiled helical springs are subjected to?

How do you define the stiffness of a close coiled helical spring?

Answers to these questions will be given to you in the next session.

Once you go through this particular lesson you should be in a position to answer all these questions.

We will be looking into the aspects of the open coil helical springs in the next lesson.

Thank you.