Strength of Materials

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Lecture – 38

Stability of Columns - II

Welcome to the second lesson of the ninth module which is on stability of columns part II. In fact, in the last lesson we have introduced the concept of the buckling in a member, a vertical member which is subjected to complexity force which we have termed as column and also we have looked into the stability aspects of different types of column members and thereby we have introduced the derivations which was proposed by Leonhard Euler which we normally call as Euler's buckling load formula.

Now in this particular lesson we are going to look into the aspects where the Euler's load can be applied or in other words, what are the limitations of Euler critical buckling load in applying in the column members and subsequently also we look into what are the other formulas that can be used for evaluating the critical load in a column member.

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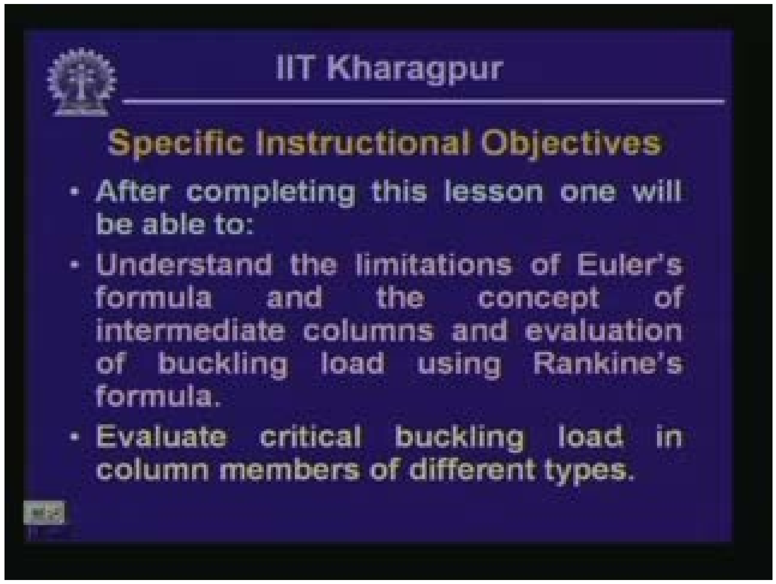

It is expected that once this particular lesson is completed, one should be in a position to understand the limitations of Euler's critical buckling load formula and also we look into the concept of intermediate columns and evaluation of buckling load using Rankine's formula. Also, one should be in a position to evaluate critical buckling load in different types of column members. When we talk of different types we mean that the column members are having different support conditions. As we have seen in the previous lesson the column members can be having the hinged ends or it can have fixed ends and also fixed and hinged or combinations of these and then how do you calculate the critical buckling load in such column members having these different types of supports.

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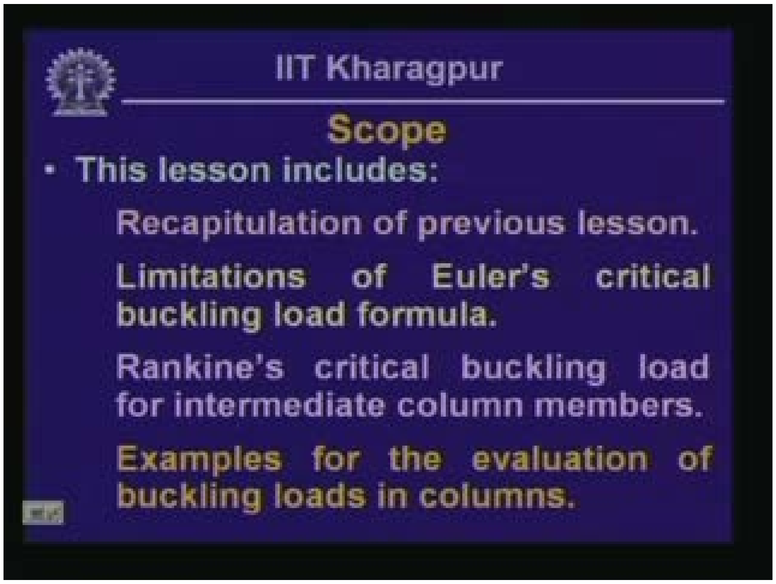

The scope of this particular lesson therefore includes the recapitulation of previous lesson. We will look into some aspects of the lesson which we have discussed in the previous class wherein we have given the concept of the buckling and the stability and we have discussed the Euler's critical buckling formula; we will look into some more aspects of that.

We will look into the limitations of Euler's critical buckling load formula and also this particular lesson includes this Rankine's critical buckling load for intermediate columns. In fact, we will look in to what we really mean by intermediate column and how do we evaluate the critical buckling load using this Rankine's formula. And also we will look into some example for the evaluation of buckling load in columns of different support conditions.

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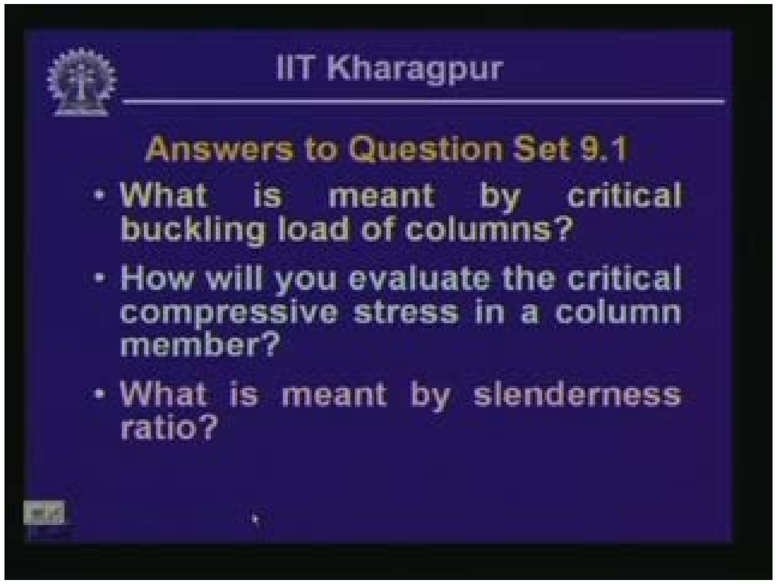

Well, before we proceed, let us look into the answers to the questions which we posed last time. The first question given was what is meant by critical buckling load of columns? Now let us discuss this with respect to the buckling model which I discussed last time in the previous lesson.

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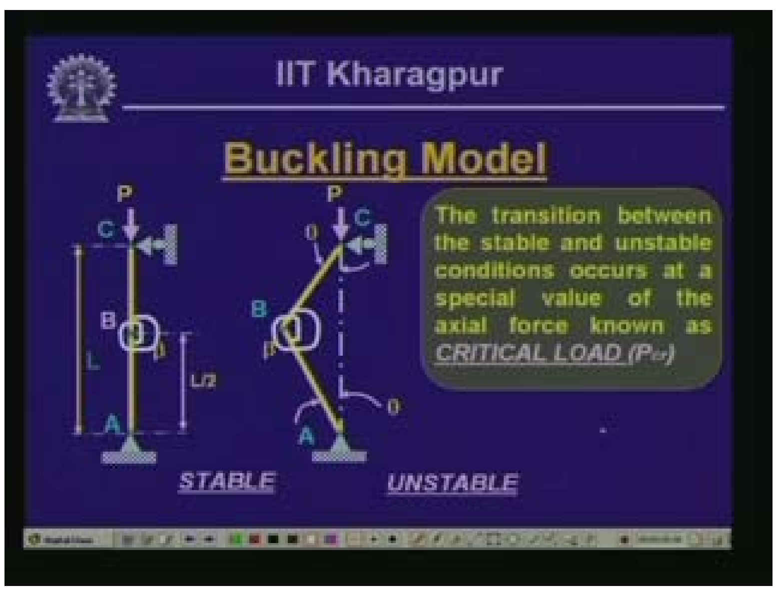

If u remember, the actual column supposing if we consider an idealized column which is hinged at both ends and which is perfectly straight subjected to axial load these we can model as having two digit bars AB and BC and connected with a rotational spring at point B, the spring stiffness of this rotational spring being beta; and this particular system where AB and BC are perfectly concentric the axial load P is acting in this member which is also concentric.

Now if we give lateral load to these or a little disturbance to this kind of a system then it is expected that the bars will move thereby an angle theta will be made by these bars and the rotational spring which is provided at B having the stiffness beta will produce a restoring movement the magnitude of which will be equals to the total rotational angle that these two bars will be undergoing which is twice 2 theta this spring is undergoing so beta times 2 theta is the restoring movement. If I call that as M R this is equals to beta time twice theta.

Now if we remove this disturbing load as we have given a disturbance and brought the columns bought this systems in this particular form, if we remove the disturbance it is expected that the bars will come back to its original position because this restoring movement given by the rotational spring will overpower the effect of this axial load and thereby this kind of a system we call as a stable system where the restoring movement is larger than the axial load P which is acting on this particular member.

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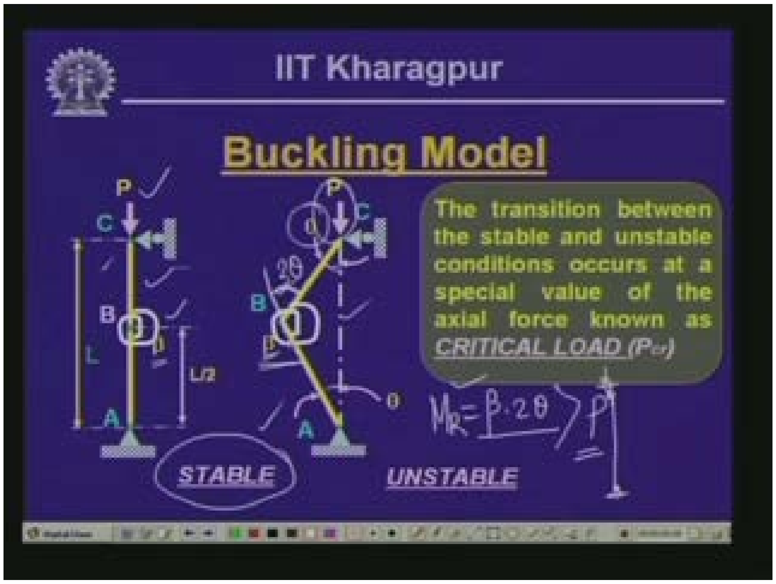

Now as you can see that when the rotational spring is giving a restoring movement the axial load P is giving opposing action that means it is trying increase the movement of the point B and thereby it tries to create destabilization in the system. now supposing if we keep on increasing this load to such a to such an extent that the axial load exceeds the restoring movement capacity that means the movement produced by this axial load through this movement if that exceeds the rotational the restoring movement capacity then the system will no longer be in the equilibrium position and it will fail and if we remove the external disturbance the restoring movement will not be or the spring will not be in a position to restore back the normal position and thereby the system becomes unstable.

Now between these two positions the stable and unstable position there is unique value of the load P which we call as the critical load. This is what is stated over here that the transition between the stable and unstable condition that occurs at a special value of the axial force which we term as critical load. In fact critical load is that load beyond which if we add a little load to the system the system will come unstable or it will fail by excessive deformation or unrestraint deformation which we call as buckling. So the member will no longer be in a stable state but it will become unstable and it will fail. That is the load; there is the limiting value of the load;

beyond which the member fails with little addition of the load we call that limiting load as the critical load.

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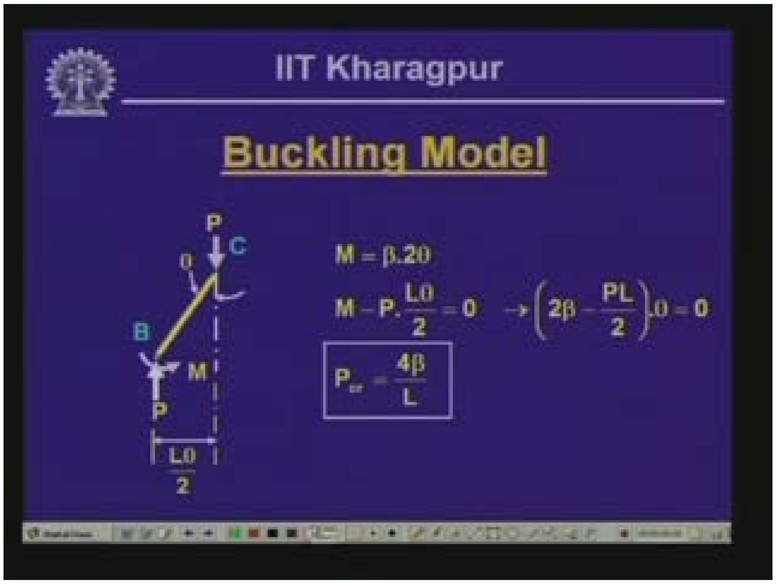

And as we have seen that last time we had derived for this particular buckling model that what will be the value of the critical load, now as we have seen that the restoring movement is equals to beta times twice theta the total rotation that it undergoes and if we take the equilibrium of the forces for this particular free body as we have seen that the horizontal force is equal to zero now if we take the movement of all the forces which is respect to B we get M minus P times L theta by 2 equal to 0 and thereby since theta equals to 0 will lead to the normal situation that means there is no movement in the bar and thereby question of instability does not come in, so if we put 2 beta minus PL by 2 is equal to 0 that gives us the value of P which is equal to 4 beta by L. Now this value of P becomes critical when this matches the restoring moment M. Now, as soon as when this particular state if we add additional load over here delta P then the system is going to collapse and that is the reason this load is called as the critical load P cr.

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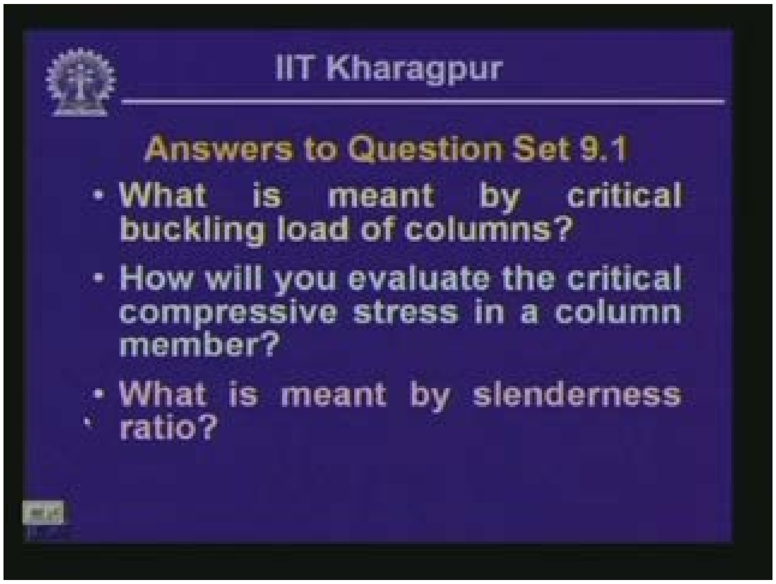

Now let us look into the other questions. The second question given was how will you evaluate the critical compressive stress in a column member? Now that we have discussed about the critical load that a column member when subjected to axial load when it reaches to criticality then what is the corresponding critical compressive load?

Now what we are interested in as we have seen earlier that if we like to evaluate the stress corresponding to that critical load then what is the...... which we term generally as critical compressive load how to compute that? And the third question what is given is what is meant by slenderness ratio? In fact I like to answer these two questions together, both the second and third questions simultaneously.

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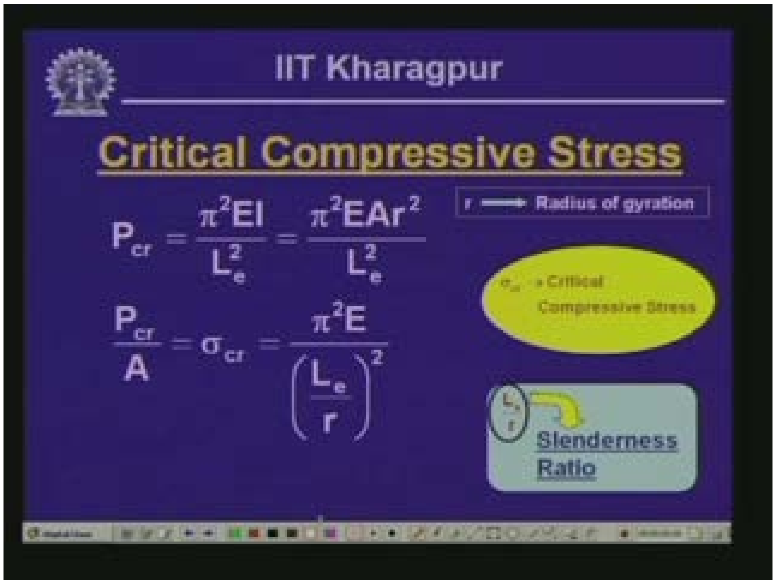

Let us look into the formula which we have derived or which was given by Leonard Euler that the P cr the P critical is equal to pi square EI over L suffix e square but by L suffix e we mean the effective length. As you have seen last time, we had discussed what we mean by effective length because for different support conditions we will have different values of this Le and this is what we call as the critical load.

Now if we replace this I the movement of inertia in terms of the cross-sectional area then I is equal to Ar square where r is known as the radius of gyration of the section. so if we replace I with Ar square and divide the whole of equation by cross-sectional area A then we get on the left hand side that P cr by A which we term as the critical stress sigma cr this is equal to pi square e divided by if we take r down it becomes L e by r square. So the expression for the critical compressive stress P cr as you have seen that P by A is the compressive stress the normal stress so here since we are computing the stress corresponding to the critical load P cr we call these as sigma cr the critical compressive stress and critical compressive stress sigma cr is equals to pi square e divided by L e by r square and this term L e by r we call as slenderness ratio.

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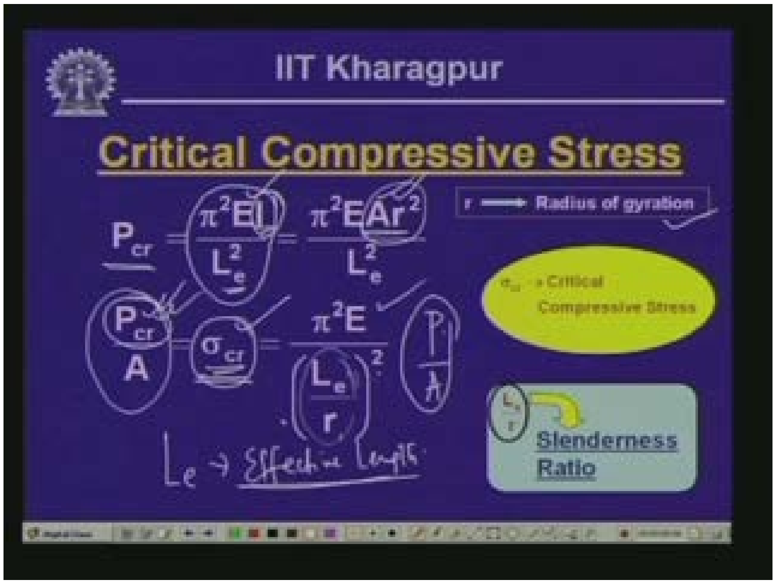

Here you see, from these two expressions we are getting two terms: one we call as the sigma cr the critical compressive stress and another term which is emerging out is the effective length L e by r ratio and this particular ratio we call it as slenderness ratio. It indicates that how slender or how long the member is with reference to its cross-external area. This is what is indicated over here that L e by r is the slenderness ratio and sigma cr is the critical compressive stress.

Now if you notice it carefully that when the value of L e by r will be larger, then the value of compressive stress sigma cr will be less and larger L e by r means the lower value of r. or in other words, what I can tell you is that if we have a cross section for which you have a smaller value of r thereby you will have larger value of L e by r thereby that will give you the minimum possible stress.

So if you have a section which is unsymmetrical; say for example, if we have a rectangular cross section and the moment of inertia about both the axes Ixx and yy are different then since r is nothing but equals to root of I by A the lower the value of the moment of inertia lower the lower will be the value of r so the moment of inertia about y axis in this particular section will be lower so r y is going to give us the lower value out of the two r values. Therefore, minimum of this r will give us the value of larger L e by r and thereby will have lower stress that is what is the critical stress.

So if you have a cross section wherein you have different values of the moment of inertia about two rectangular access system x and y thereby we must deal with the minimum value of the radius of direction so that you get critical value of the slenderness ratio which is L e by r which is larger. And as we are looking in to it here the larger the value of the slenderness ratio smaller will be the stress thereby, if you consider that particular stress with the cross-sectional area that will give you the load getting capacity of the member. Hence you will have to always look for that what is the minimum possible stress that will be required otherwise the member will fail by buckling if we go beyond that particular load. This is what is important when we talk about the stresses the slenderness ratio L e by r and the critical compressive stress sigma cr.

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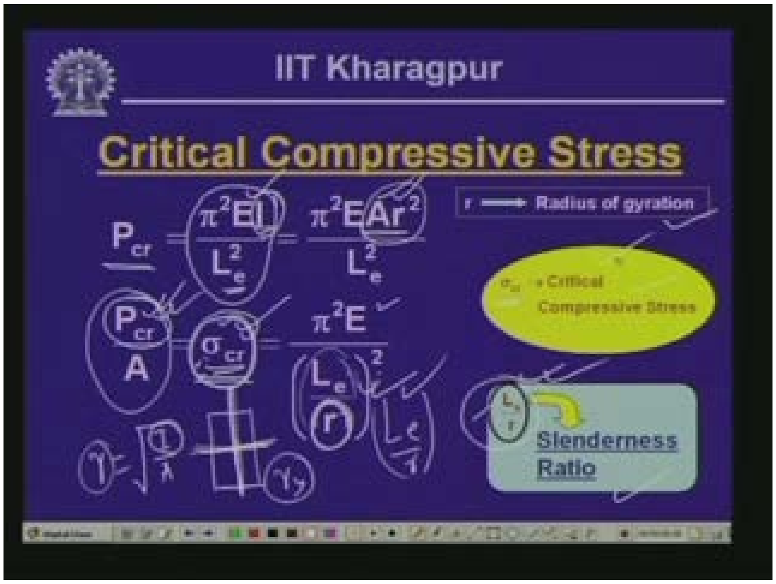

Well, having looked in to these questions let us once again look back to the values of the critical load that we had evaluated or which was derived by Leonard Euler for different column support conditions.

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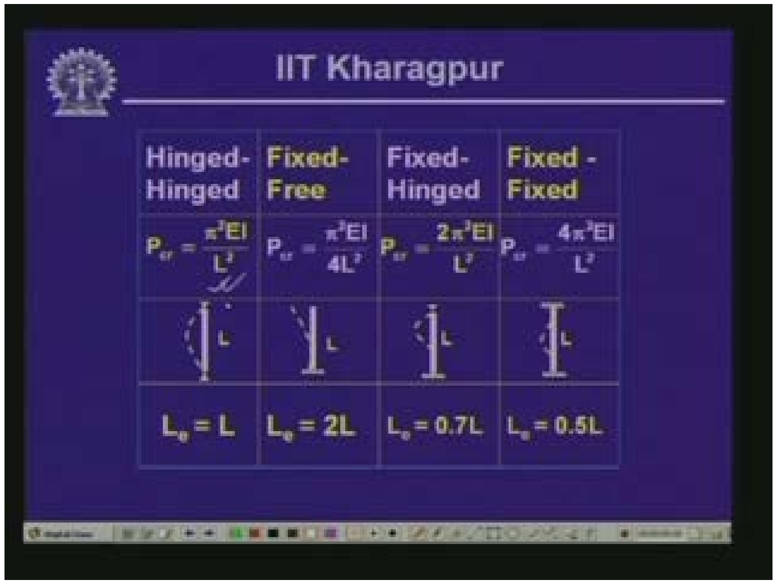

Now, first one which was an idealized column member we had considered wherein the hinge of the column members were hinged and that is what we have put as hinged and hinged and as we had seen the Euler's critical load corresponding to this kind of column having length L is equal to pi square EI over L square. When the support condition changes say the lower becomes fixed and the top becomes free which is that of a cantilever member this kind of member we call as cantilever member as we have seen in beams. now here the condition is a fixed free condition and the Euler's critical buckling load which we get corresponding to this is equals to P cr as pi square EI over 4L square.

Since here we have the coefficient of L as 1 (Refer Slide Time: 15:30) now if we try to write down everything as equivalent to this then we can write that as P cr as equals to pi square EI over 4L square this we can write as pi square EI over 2L square and this parameter 2L we call is equivalent to the single length L which is L e. So in the first case as you can see that P critical is equals to pi square EI by........... let me add a term K so K times L square where K times length is the effective length L e.

In the first case K is equals to 1; in the second case as we can see K is equals to 2. So the effective length L e is equals to twice the L and the value of K is equals to 2. Here the value of K is equals to 1, here the value of K is equals to 2 (Refer Slide Time: 16:30) where K we call as the effective length coefficient. That means we add this factor or the coefficient to the actual length to get the effective length of the member.

Likewise if we compute the critical load for the other two cases like you have fixed hinged and fixed fixed cases then for the fixed and hinged condition one end fixed and other end hinged we get the critical load P cr as equals to twice pi square EI over L square and this if we write in terms of pi square EI divided by.................... we can write this as L by root 2 square. So here the value of K is equal to 1 by root 2 which is equal to 0.707 and that is what has been written over here that L e is equal to 0.7L. Thus, the value of k here is 0.7.

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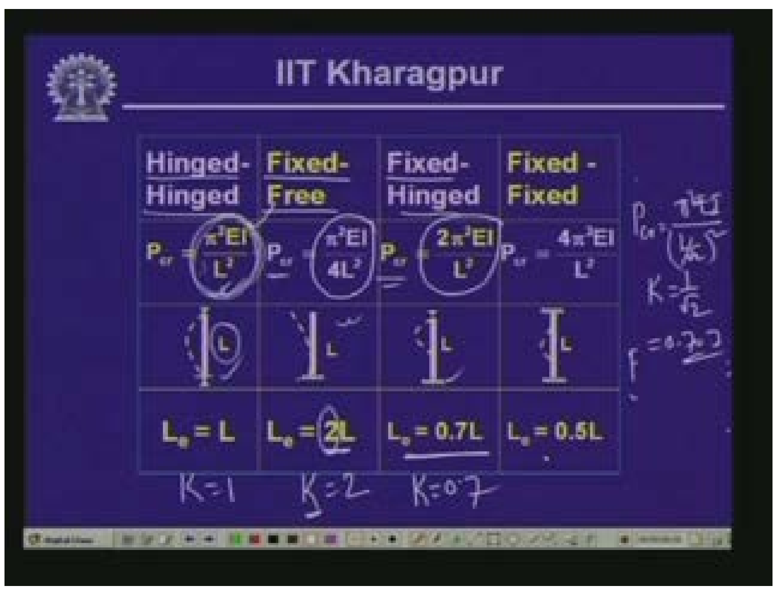

And if we consider a case where the supports are fixed at both ends, the column member is fixed at both ends then the critical load which we get is equals to 4 pi square EI over L square and thereby we can write P cr as equals to pi square EI by L by 2 square and thereby K is equals to half over here and that is what is indicated that effective length is 05 times L so k is equals to 0.5.

Thus we see that we get different values of the K or the coefficient of the effective length based on which we can compute the value of the critical compressive load for each of such column members. And as we have seen now that once we can evaluate the critical compressive load correspondingly we can evaluate the value of the critical compressive stress as well.

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Having looked into this let us look into the variation of this stresses and the assumptions with which this Euler’s formula were derived.

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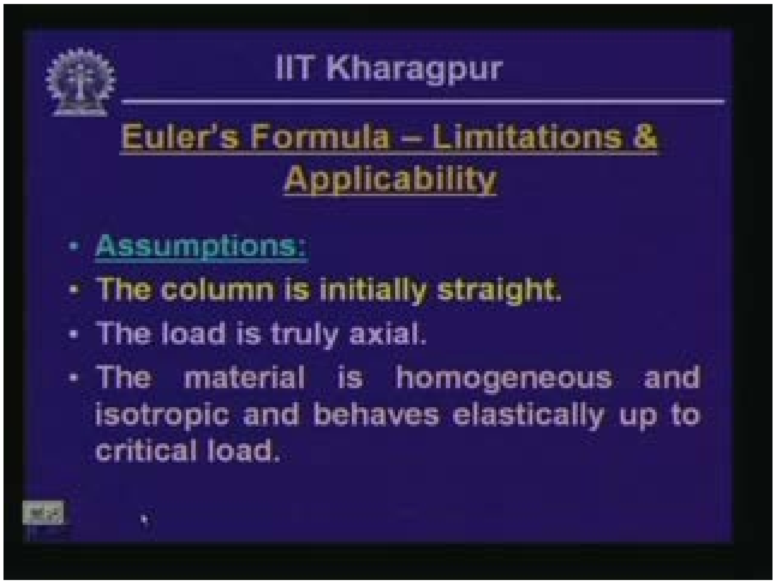

Now you see, when we have derived or when the formula was proposed by Leonard Euler it was assumed that the column member is perfectly straight that means we had considered an idealized situations that the column member is perfectly straight and subjected to a compressive load which is truly axial; that means it is passing through the centroid line of the cross external member of the column member. So, the column is initially straight, the load is truly axial and the material is homogenous and isotropic and behaves elastically up to the critical load. So up to the limit of the critical load we presume that the material behaves in an elastic manner and thereby the Hook's law is applicable. Therefore, beyond critical load there might be inelastic deformation or beyond buckling when the buckling occurs the failure subsequently could be in an elastic manner which is of course not in the scope of this particular lesson.

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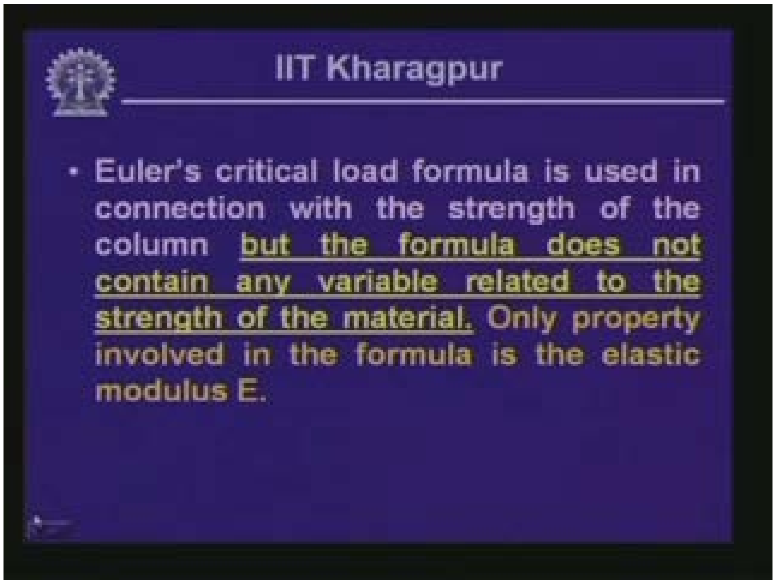

Now if you look in to the Euler's critical buckling load formula you will observe that we had the P critical as equals to pi square EI over L e square. Now we are talking about a critical load that means how much load a member can carry which must be related to the strength of the column. But unfortunately we do not have any parameter in this particular expression which signifies the strength of the member; instead what we have is the elastic modulus e only which is the material characteristics present in this particular expression. This is what is written over here; you see that Euler's critical load formula is used in connection with the strength of the column but the formula does not contain any variable related to the strength of the material and this is what is very important.

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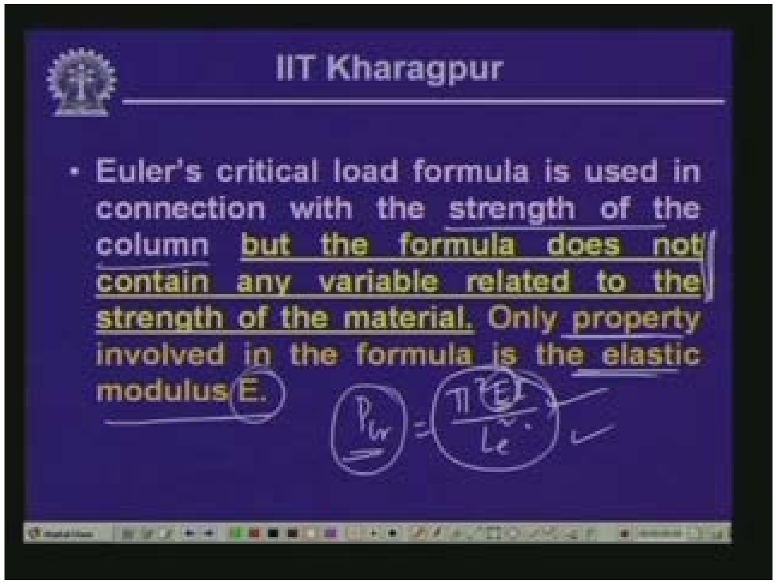

Hence, the only property that is invoked in this particular expression is e which is the elastic modulus of this material that we are using.

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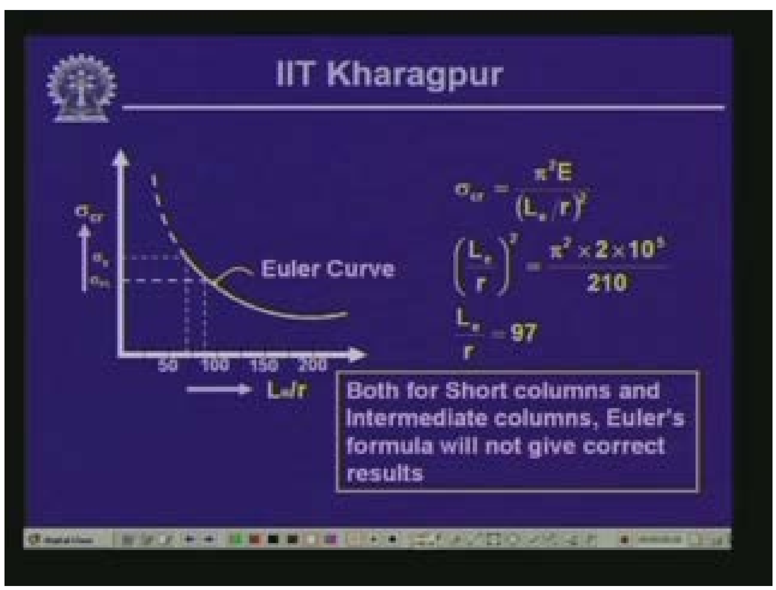

Well, now with this background if we look into the variation of the critical stress with the slenderness ratio L e by r we will find, as we have discussed in this particular section that sigma cr is equals to pi square e divided by L e by r square where sigma cr is the critical stress and L e by r we have designated as slenderness ratio.

As I was telling you that if this value of slenderness ratio increases then the value of the sigma cr decreases. If the L e by r value becomes lower and lower that means the stress level will be higher and higher. Now what does that imply? That means if you have a very small L e by r or very small value of the slenderness ratio you will have very high value of the stress. But what does that physically mean?

Supposing if you have a stress which is much higher than the yield stress of the material then what is going to happen; the material is going to fail as soon as it crosses the yield stress. So the stress higher than yield stress makes no sense. So what happens is if we are talking about a column where the slenderness ratio is very low or as we have seen that in case of short column the critical stress thereby is the yield stress the yield stress is the critical value because once the member reaches to the yield stress the material is going to yield and as we have noticed earlier as we have discussed earlier that for a short column when it is subjected to axial load or even if the load is eccentric thereby it is going to give you the axial load and the bending and in terms of the combined stresses if you compute the normal and the bending stresses in the member, as soon as the stress level goes beyond the yield stress the member is going to fail by crossing that means the material will yield and the question of buckling will not arise in that particular situation.

Hence, this Euler's critical load formula has a limitation that beyond a certain value of L e by r we cannot use this Euler's critical buckling load formula. Now, if we consider the material as steel material which we know that the proportional limit stress the sigma PL is equal to 210 MPa and the yield stress say we consider as 250 MPa then if we consider that the proportional limiting stress which is equal to 210 MPa then we get a value of slenderness ratio L e by r as equals to 97. This indicates that if we use the slenderness value less than 97 then the stress level is going to go beyond the proportional limit.

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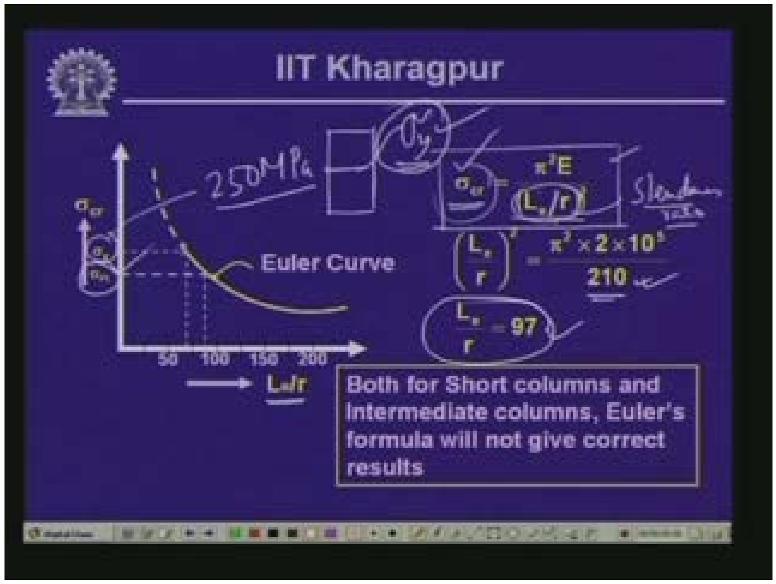

As we have noticed, if the stress level goes beyond the value of sigma y then the material is going to fail by crossing which is the criteria for a short column. Hence, there is a limiting value for the slenderness beyond which the Euler's critical load formula is applicable; otherwise it is not applicable for such type of members. So, in this particular curve as you can see where sigma cr is plotted against the slenderness ratio L e by r, there is a limiting value of the slenderness and this curve (Refer Slide Time: 24:41) or the Euler's curve is valid when L e by r is greater than this limiting value. When L e by r is higher....................let us call this L e by r as the limiting value. this is the L e by r which we have computed for steel and let us call this L e by r as the limiting value.

Now, when actual L e by r in the member is greater than the limiting value of the slenderness ratio then we can use the Euler's critical load formula. But if it is less than this value; if actual L e by r is less than the limiting L e by r value then we cannot use the Euler's column buckling formula. So what happens is you see that we are getting clearly two areas: one is that beyond the limiting L e by r or the slenderness ratio or higher the value of limiting value we can go for the Euler's critical buckling load formula based on which we can compute the critical load in the member and the other aspect is that as we can see that when the stress goes beyond the yield stress the material is going to fail by yielding.

So, for the short columns when sigma y is the critical stress we can evaluate what will be the load carrying capacity. So, between these two cases that you have a short column where the member is going to fail by yielding and a long column formula where beyond a limiting value of the slenderness ratio we are using Euler's column buckling formula now in between these two there could be some members which may failed in the combination of buckling and yielding and those members which are in between this short column and long column we call them as intermediate columns.

As we have noticed that intermediate columns will have L by r less than the limiting value of L by r or L e by r and thereby will not be in a position to apply Euler's critical buckling load formula for evaluating the critical compressive load for such members. For such intermediate members we use different formulas. In fact there is a formula which was proposed by Rankine we call that as Rankine's formula for evaluating the critical load in intermediate columns; so, for both short columns and intermediate columns in fact the Euler's column Euler's formula will not be applicable and it will not give you the appropriate results.

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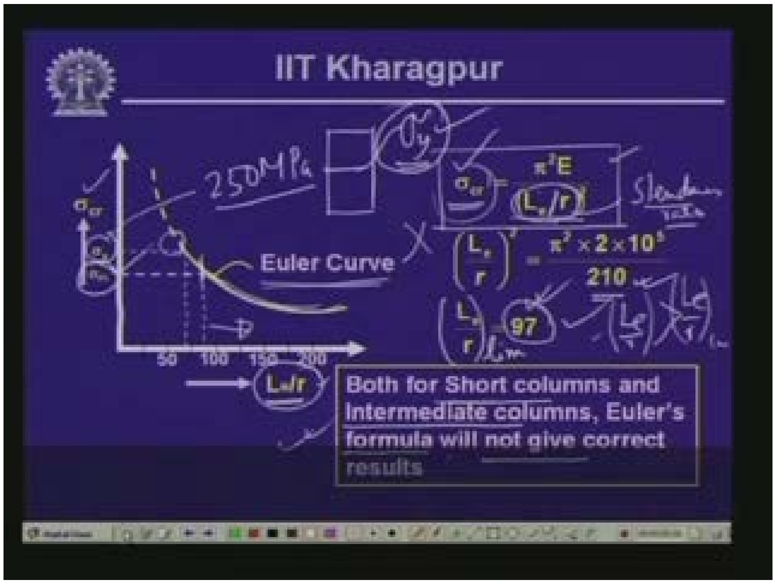

Now let us look into this Rankine's formula which was proposed by Rankine which we commonly call as Rankine Gordon formula.

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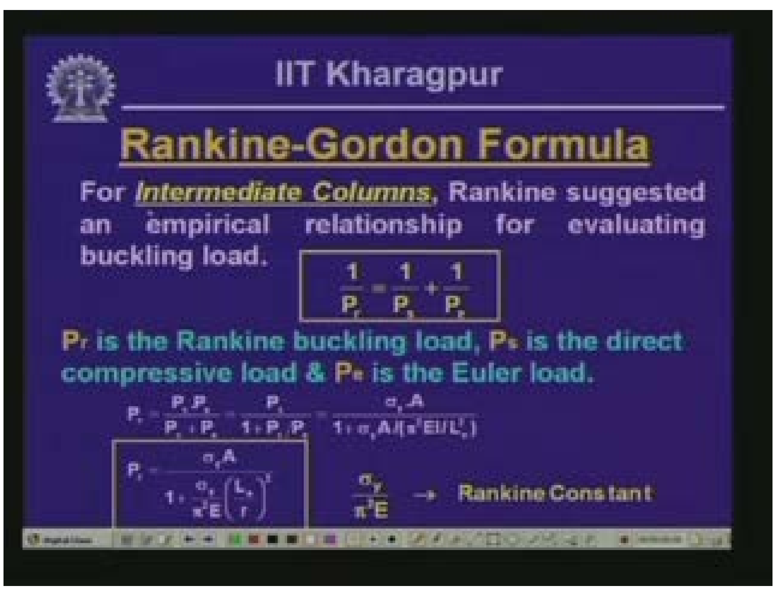

These are applicable for the intermediate columns and Rankine suggested that an empirical relationship for evaluating buckling load in this form which reads as 1 by P r is equal to 1 by P s plus 1 by P e where P r is termed as the Rankine's buckling load, P s is the direct compressive load which is equals to the yield stress multiplied by the area and P e is the Euler's critical buckling load formula or buckling load. So we have three terms P r, P s and P e where P r is the critical buckling load that is given by Rankine and that is what we are interested to evaluate and that is being evaluated in terms of P s and P e; P s is the load which is evaluated from the direct compressive stress and that is for the short column and as you know for the short column the critical compressive stress is nothing but the yield stress sigma y.

So this particular expression that 1 by P r is equals to 1 by P s plus 1 by P e if we evaluate this, this comes as P r is equals to P s multiplied by P e by P s plus P e and then if we divide the denominator and the numerator by P e we get this as P s by 1 plus P s by P e and as I said that P s is the direct compressive load which is equal to the yield stress sigma y times the cross-sectional area A so P s is equals to sigma y times A and P e as you know is the Euler's critical buckling load which is equal to pi square EI over L e square. That is what is substituted over here and replacing I replacing the I over here in the Euler's critical load formula I as Ar square we get these as P r is equals to sigma y times A by 1 plus sigma y pi square e times L e by r square.

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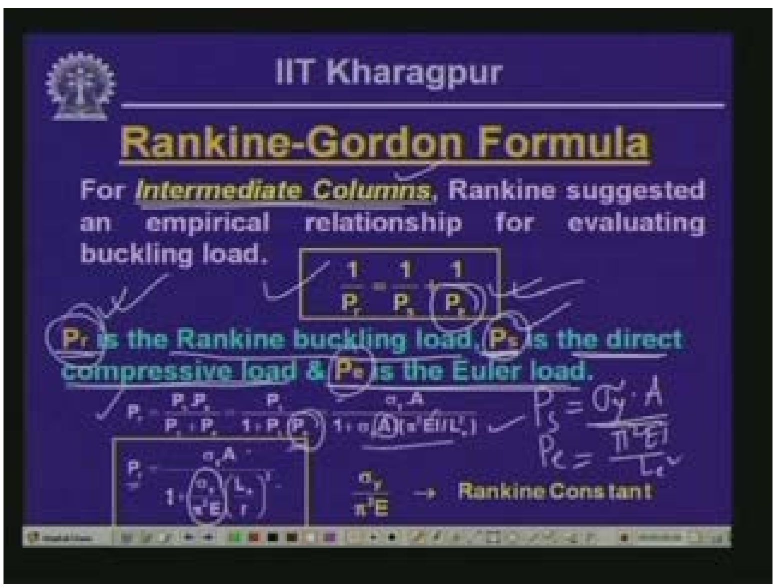

Now here the sigma y by pi square e which is dependent on that material the yield stress of the material and the modulus of elasticity of the material is commonly termed as Rankine's constant.

And again as you can see that the critical load for the member will dependent on this L e by r the slenderness ratio and the critical stress or the yield stress of the material. This is the expression which was proposed by Rankine for evaluating the critical buckling load for the intermediate column members.

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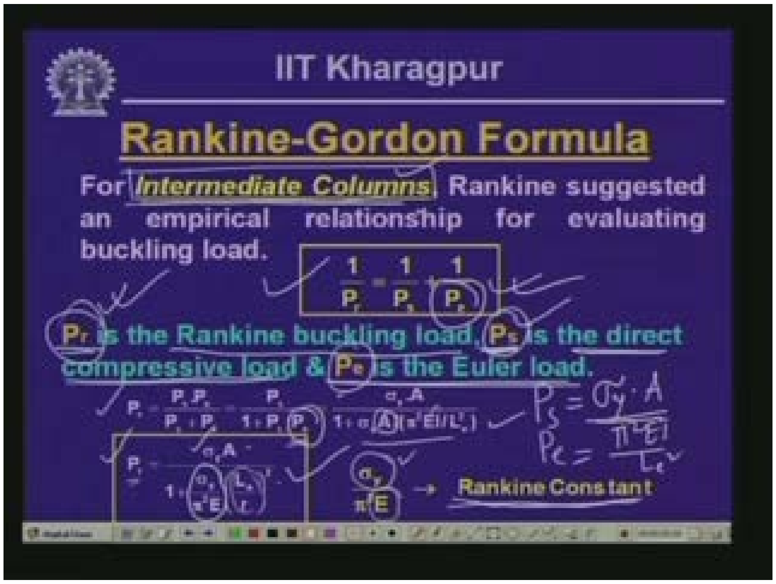

Now as we have seen clearly we have three distinct areas: one we have called as short column, another we have called as long column and now we have defined another column range which is between short column and the long column. For the other long column members we can use Euler's critical buckling load formula for evaluating the critical compressive load when the actual slenderness ratio L e by r ratio of the member exceeds the limiting slenderness value. as we have just now seen, for any material we can compute L e by r limiting for a particular material and when the column member is made up of that material, if we know the actual slenderness and when that actual slenderness exceeds the limiting value then we can use the Euler's column formula for evaluating the critical load; or if the actual L e by r is much less than the limiting L e by r value wherein the failure will be governed mainly by the yielding of the material; wherein we take the critical compressive stress as the yield stress of the material that multiplied by the cross-sectional area will give the critical load as that of a short column and in between these two where the members could fail in the combination of the crossing or yielding and the buckling those types of columns we call as intermediate column and we can evaluate the critical buckling load of those columns using Rankine's formula.

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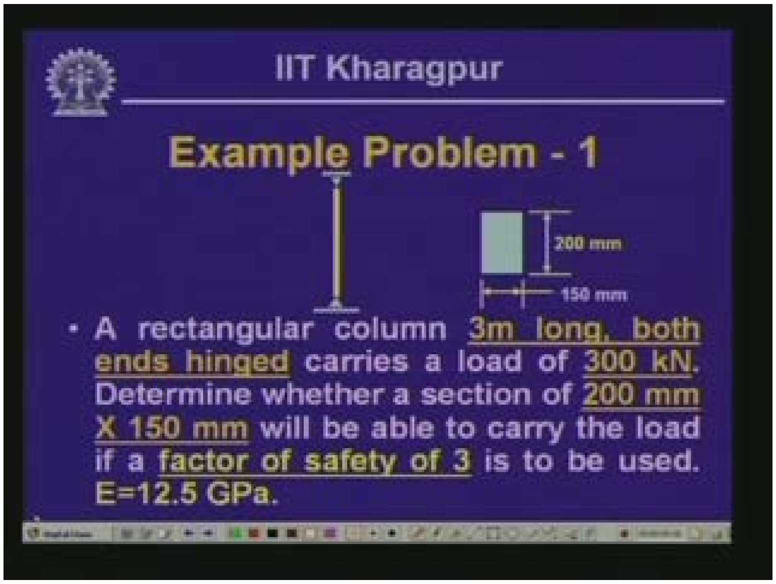

Now having looked into this with this background let us look into some of the examples. In fact, this particular example I had given to you last time and asked you to look into; let me give you the solution for this. This is the column which is hinged at both the ends and the length of the column member is 3m. The cross section of this column is a rectangular one having a size of 150 mm by 200 mm. Now it says that this particular member carries a load of 300 kilonewton. You will have to determine whether this particular section the cross section of 200 mm by 150 mm will be able to carry this load this 300 kilonewton load if a factor of safety of 3 is to be used for this purpose.

Now you see when we use a factor of safety of 3 it means that if a member is subjected to a load of P we should check the stress in such a way that it can withstand a load of three times P that is the meaning of that factor of safety of 3. So the section is to be chosen or the stress has to be evaluated in such a way that it can withstand a load of three times P and then only we can apply a load P and we say that the factor of safety applied to this member is 3. Thus, we will have to check whether the member can withstand a load of 300 times of 3 as 900 kilonewton.

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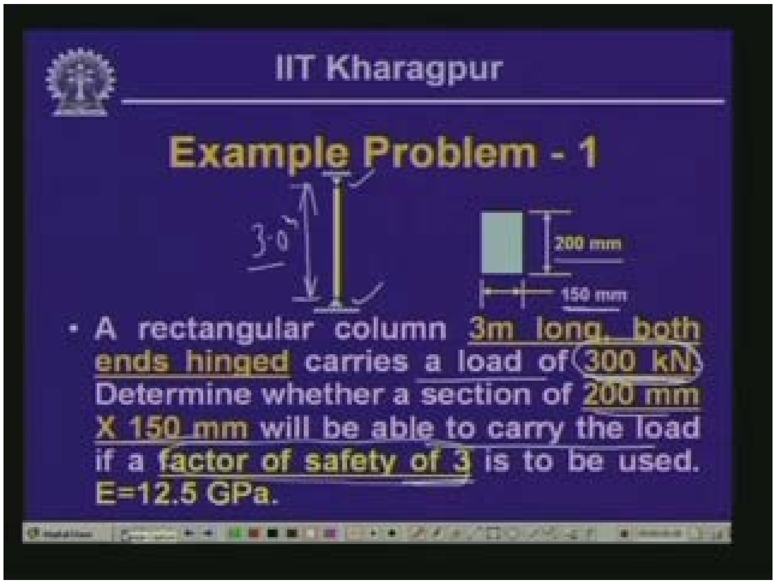

Now let us look into this that if we compute the value of the critical load using Euler's critical buckling load formula see that the value of the I y................ now as I was telling you the cross section is a rectangular one, 150 by 200 and the rectangular access system of this is xx and yy. Now, which can compute the moment of inertia of this section I x and I y?

As you know I x will be equals to 150 times 200 cube divided by 12 and I y will be equal to 200 times 150 cube by 12.

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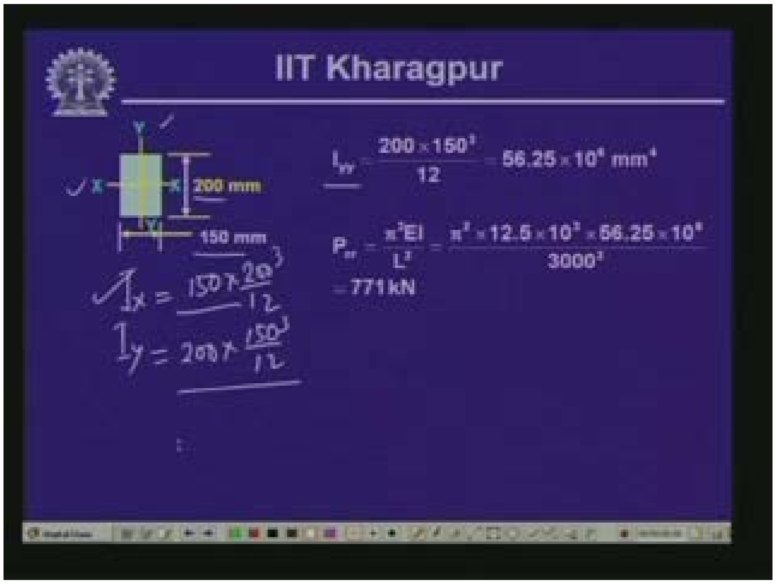

Clearly from this expression you can see that the value of I x should be higher than the I y and as you know that the value of r is equals to root of I by A; and for this particular section we will have two values of r which is r x and r y and r x will be equals to root of I x by the cross-sectional area and r y will be equals to root of I y by the cross-sectional area.

Since I y is less than I x so expectedly r y will be less than r x. So we compute I y which is going to give us the minimum possible value because beyond that if we apply load beyond that it is expected that it will buckle about the yy axis. And as I had shown you last time that if you take a member and apply a compressive load then if the member is a slender one a long one then it buckles about one of the axes and obviously it is going to buckle about the axes which is weaker if the two axes do not have the same strength like you do not have the same moment of intertia on both the axes.

Or in other words, the section is not a square one as we are dealing in this particular case. Since is a rectangular one, one of the moment of inertia is less in comparison to other one so it is weaker about yy axis in comparison to the xx axis and therefore is going to buckle about y axis.

As I had shown you last time or the derivation we have looked into we have considered the buckling of the member in one direction that means we have taken in the positive y direction. Now the question is whether the buckling can physically occur in this direction or it can occur in this direction. Now whichever direction it occurs our evaluation will also be the same. The expression for the critical load we will have will be unchanged. Now this we have (Refer Slide Time: 36:10) because of our positive axis direction.

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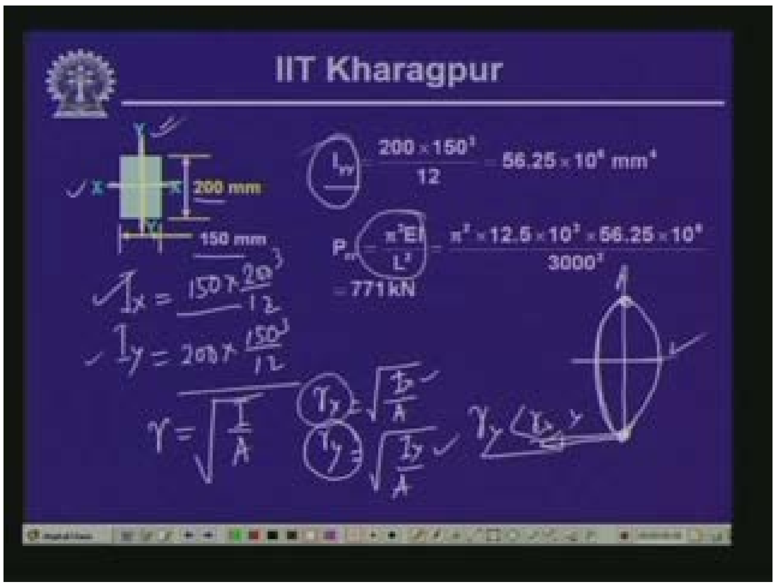

Thus if we apply the critical load formula given by Euler then we get a load value as equals to 771 kilonewton, e is given as.......... this is pi square and e as we have seen over here is 56.25 into 10 to the power of 6 is the I and e is given as 12.5 into 10 to the power of 3 and L is equals to 3000. Since this column member is hinged at both ends so the L e is equals to K times L and K in this particular case is equals to 1 and that is what is indicated over here and if we evaluate this we are going to get a value of P critical as equals to 771 kilonewton.

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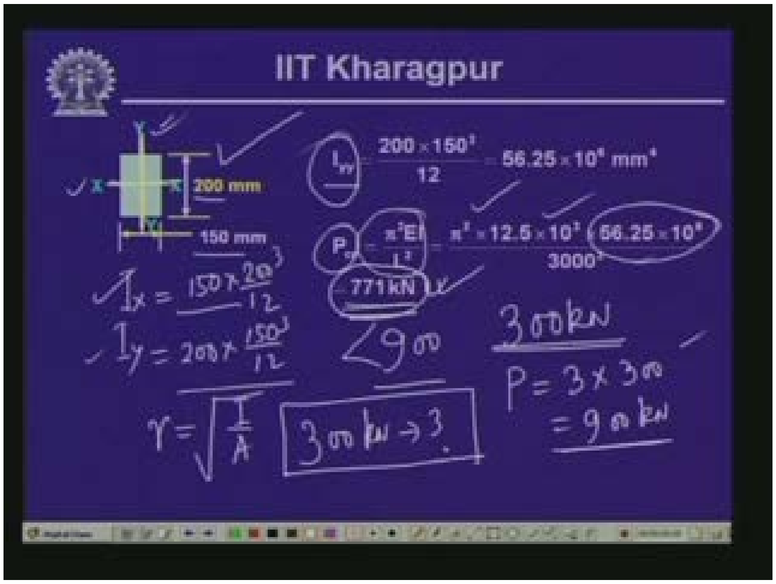

As I was telling you that we will have to apply a factor of safety of 3 to this particular member and thereby to have a 300 kilonewton load n this column member we will have to check the section for a load P as equals to 3 times 300 which is equals to 900 kilonewton. Since we find that using Euler's critical load formula the critical load is 771 kilonewton which is less than 900 kilonewton then this particular section will not be appropriate to apply a load of 300 kilonewton with a factor of safety of 3. So, to fulfill these two aspects; that means we will have to apply a load of 300 kilonewton with a factor of safety 3 will not be appropriate for this section or this section will not be able to carry that load.

Now if you have to satisfy that that means you have to have 300 kilonewton load on the member with a factor of safety of 3 naturally then you will have to change the cross-sectional area, you will have to go for higher cross-sectional area so that you can satisfy this particular criteria.

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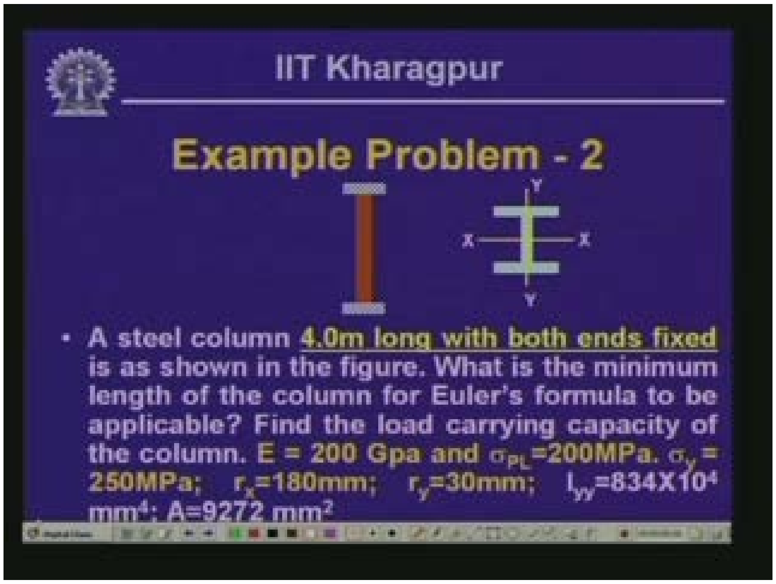

Well, let us look into another example and this particular example is a steel column of length 4m and the ends of this particular column is fixed; both the ends are fixed. Now what is the minimum length of the column for Euler's formula to be applicable?

First of all we have to find out, though it is given that the length of the column member is 4m we will have to find out the length for which we can apply the Euler's critical load formula for such situation and the member property is given as r, e of this is 200 Gpa Giga Pascal, the stress at the proportionality limit is equals to 200 MPa, the yield stress of the material is 250 MPa and the values of the radius of the direction about x and y axis r x is 180 and r y is 30 mm as it is expected that the moment of inertia about y axis is less than the moment of inertia about x axis and thereby the value of the radius direction about y axis is less than the radius of direction about x axis and the value of the moment of inertia about y axis is given, the cross-sectional area of this particular member is given.

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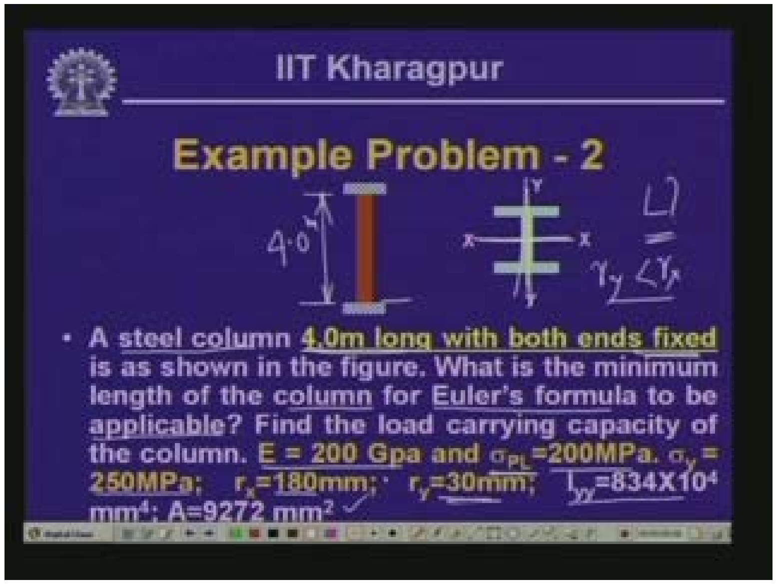

Now the question is let us first find out that what is the value of the limiting length up to which the Euler's critical buckling load formula can be applied.

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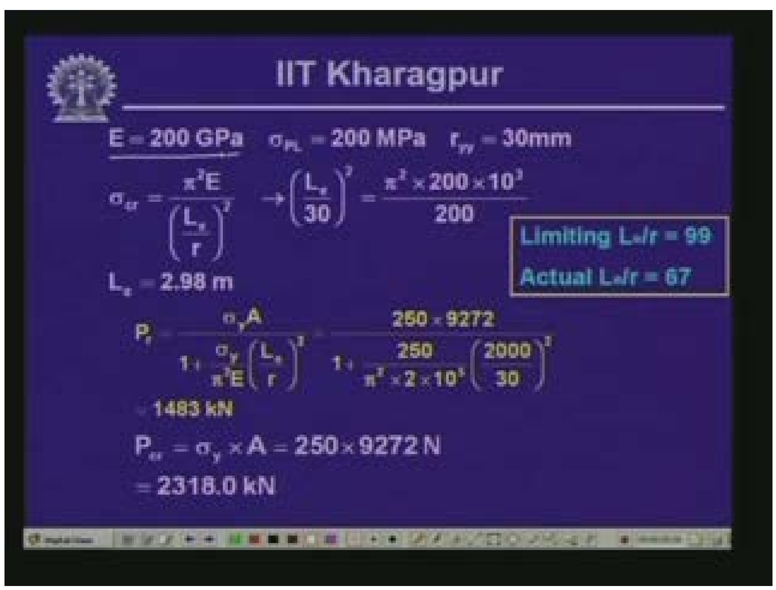

Now the value of e is given as 200 GPa, the stress at the proportional limit is given as 200 MPa; now, from the critical stress expression that sigma cr is equals to pi square e by L e by r square as we have seen right now from these we can write that L e by r square; r of course we have taken as r minimum which is 30 this is equals to pi square, e is given as 200 Giga Pascal times 10 to the power 3 so much of MPa divided by 200 MPa so this gives us a value of the length L e as equals to 2.98m. That means this is the minimum length that is needed for the member so that we can apply the Euler's critical buckling load formula. And mind that this is the effective length L e.

Now here (Refer Slide Time: 41:07) the member which we have considered is a fixed ended member and thereby as we have seen that the value of K for fixed ended member is equals to 2 because for a fixed ended member the critical load is equals to 4 pi square EI by L square and thereby the L by becomes L by 2 so the K value becomes half so with K as 0.5 and for 4m the length here is going to be equals to 2m. So effective length then in this particular case is equals to 2m and this (Refer Slide Time: 41:43) being less than the length the minimum length we need thereby we cannot apply the Euler's critical buckling load formula.

Therefore, as you can see that limiting L e by r that means this length divided by the minimum r where r is 30 if we use, the L e by r we get as 99 and the actual L e by r that means 2000 divided by 30 is the actual value of the L e by r; L e here is 2000 which is 0.5 times 4000 and this equals to 67.

Now this value is less than the limiting L e by r value and therefore Euler's critical load formula will not give us the value of the critical load. So the options what we have is to look into that if this member fails by the yielding that means it reaches to the yield stress value then the value the of critical load will be the yield stress multiplied by the cross-sectional area which gives us a load of 2318 kilonewton.

Now, if we consider this in the intermediate range that means it may fail in combination of the buckling and the yielding, neither in the buckling range because we cannot apply the Euler's load formula and on the short column range where it goes to the yielding that is sigma y times A the other limiting value, now if we consider that it fails in the combination of the buckling and yielding then it comes in the category of intermediate column unless we look into what is the critical load we get if we use Rankine's formula for evaluating the critical load.

Now the critical load which we get corresponding to the rankines formula which is given as sigma y times a divided by 1 plus sigma y by pi square 3 times L e by r square, now sigma y of the yield stress is given as 250 MPa, 9272 is the area of the cross section then we have 1 plus 250, 250 is again the sigma y, pi square e is 2 into 0 to the power of 5 and L e by r value is equals to 2000 by 30 square. Now this gives us a value of 1483 kilonewton.

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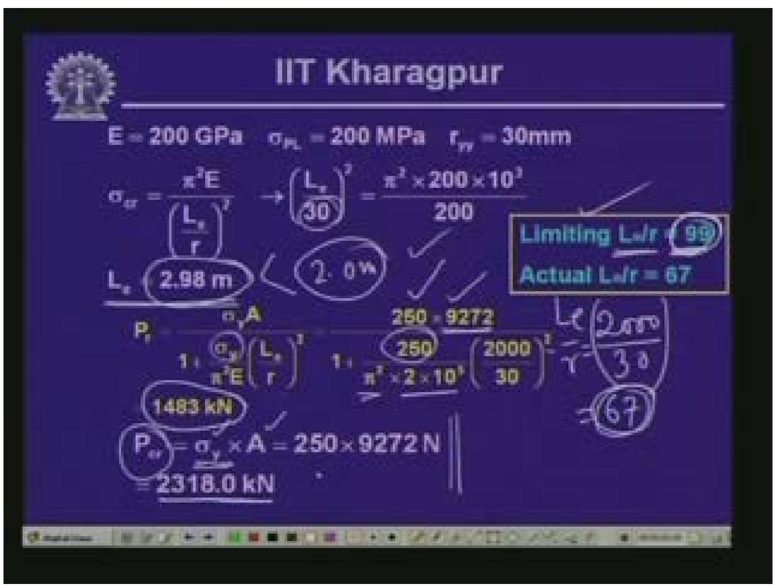

Now as you can see that if we go up to the yield stress level or up to the crossing level then the load which we can apply is this; and if we consider that the column might fail since the L e by r value which we have got the actual L e by r we have obtained less than the limiting value there is a possibility that the member is going to fail in the combination of the buckling and the yielding and thereby we need to limit ourselves to a load of 1483 so the maximum load that we can apply is equals to 1483 so that the member does not fail either by buckling or by yielding or in combination of the two.

This is the limiting load in this particular case. For this particular member as it has been said what is the minimum length of the column for Euler's formula to be applicable is as we have seen is 2.98m and mind that its effective length is 2.98m so in this particular case hence it is fixed onto its column and as we seen for fixed ended column member the value of the effective length coefficient is half that means we will have to have a column length of 2.98 times 2 that means around 6m length you need for a fixed ended column member where we can apply the Euler's column buckling formula. Or else we will have to go for either the short column formula which is the yield stress multiplied by the area or by the Rankine's formula which is for the intermediate column.

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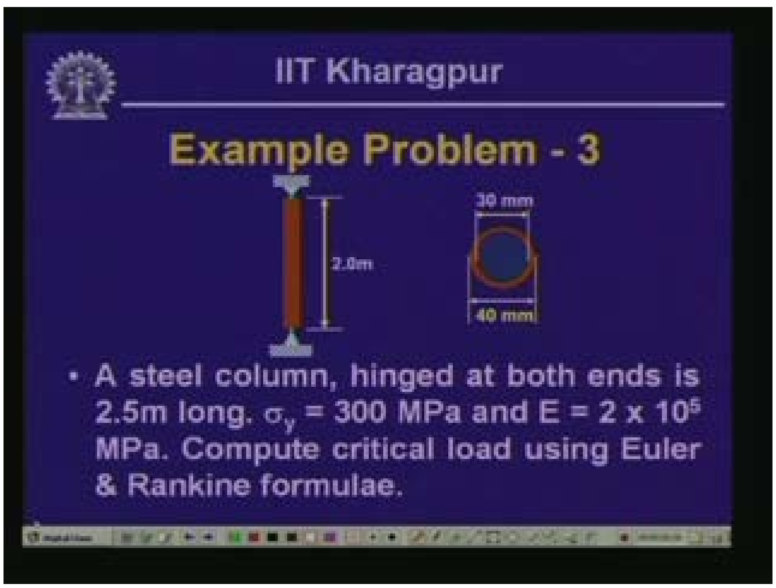

Now let us look in to another example problem wherein this particular member is having a cross section of that of a tube. So it is a tubular member which is subjected to a compressive load and this particular member is hinged at both the ends. That means this is hinged hinged column for which we have seen that P critical is equals to pi square EI over L square and thereby the coefficient K is equals to 1; the effective length coefficient is equal to 1.

Here you will have to compute the critical load using both the Euler's and Rankine's formula. We will have to find out the value of the critical load that this member can carry using both the

Euler and Rankine formula and the value of the yield stress is given as 300 MPa and the value of e is equals to 2 into 10 to the power of 5 MPa.

So let us look into that what will be the critical L e by r. This being a tubular member as you know that if we compute the value of the moment of inertia and the cross-sectional area, the cross-sectional area will be pi by 4 and d outer square minus d inner square and the moment of inertia will be pi by 64 times d outer square minus d inner square to the power 4.

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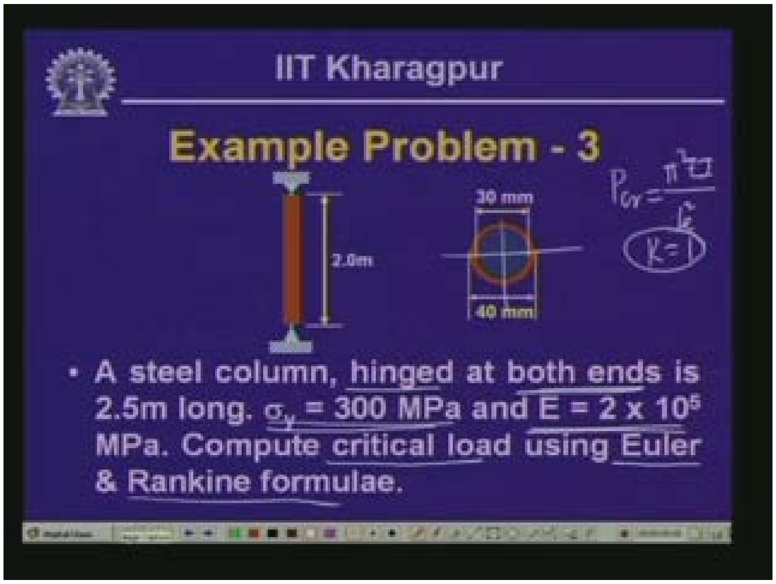

Hence, let us look into the values.

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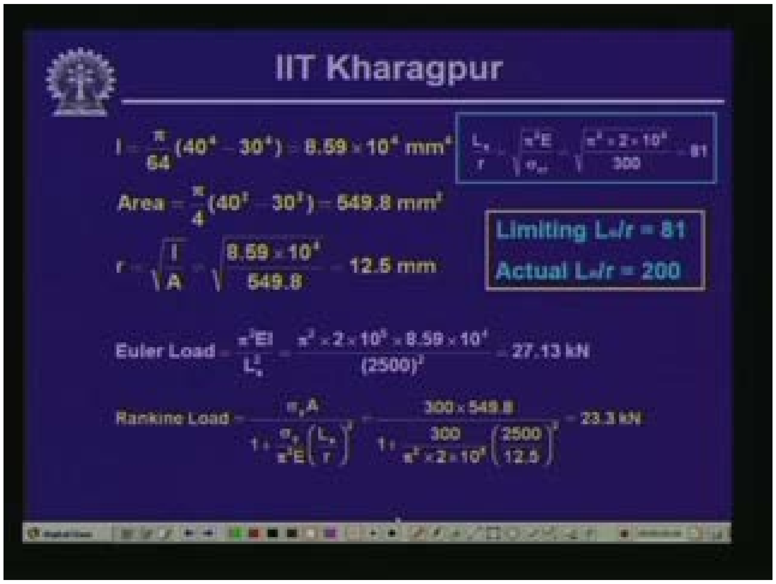

You see, this is the value of the moment of inertia which is pi by 64 times d outer time to the power of 4 minus d inner time to the power of 4; this gives us a value e8.59 into the power of 4mm to the power of 4 and the area is equals to pi d square by 4 where d outer square minus d inner square and this gives us an area of 549.8 mm square.

Now if we compute the value of the radius of direction r which is equal to root of I by A this gives us a value of 12.5 mm. Now if we compute the limiting value of the L e by r which is equals to root of pi square e by sigma cr which is the critical stress and critical stress here is given as 300 MPa and value of e is 200 GPa or 2 into 10 to the power 5 MPa this gives us a value of L e by r as 81. So this is the limiting value of the slenderness ratio L e by r.

And what is the actual L e by r in this particular case?

The length of the member which is given over here is 2m sorry this is 2.5m (Refer Slide Time: 48:47) as it is written over here it is 2.5m and this is hinged at both the ends so L is equals to L or K is equals to 1 in this particular case so we have used a value of 2500. So if we compute now the value; actual L e by r if we divide these by 12.5 we get the value of actual L e by r as 200.

Now since this actual L e by r is higher than the limiting L e by r so we can apply Euler's critical buckling load formula for evaluating the critical load in the member. We have used Euler loads which is pi square EI over L e square which gives us pi square e is 2 into 10 to the power of 5, I is 8.59 into 10 to the power of 4 and L is 2500 because K is equals to 1 so this gives us a value of 27.13 kilonewton if we use Euler's critical buckling load formula.

Now if we use Rankine's critical buckling load formula then we get sigma y which is again given here as 300, length is 2500, cross-sectional area as we have computed is equal to 549.8 mm square and e is 2 into 10 to the power 5, we get a value of 23.3 kilonewton. Now you see that we have now two values of the critical load: one is corresponding to the Euler's critical buckling load and another one is corresponding to Rankine's critical buckling load.

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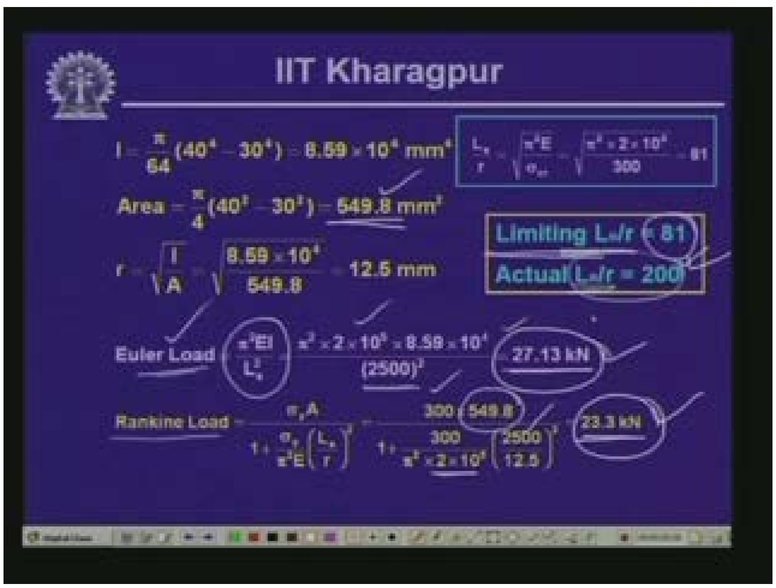

Since we have tasted that in this particular case the value of the actual slenderness ratio L e by r is higher than much higher than the limiting value hence the stress level will be much lower; as we have seen in the critical stress versus the slenderness ratio called that if you have larger value of the slenderness ratio then the corresponding stress is much lower than the yield stress and thereby the failure which will be dominated in such columns will be more in terms of buckling rather than going for the yielding. Hence here since the L e by r value or L e by r ratio which is much higher than the actual or the limiting L e by r we can use the Euler's formula or Euler's critical load we can take as the guiding critical load for the member. So the critical load for this member will be 27.13 kilonewton.

Now here, in this particular example (refer Slide Time: 51:37) as it is shown over here the length of the member is 2.5 and this is a mistake this is not 2m but this is 2.5m.

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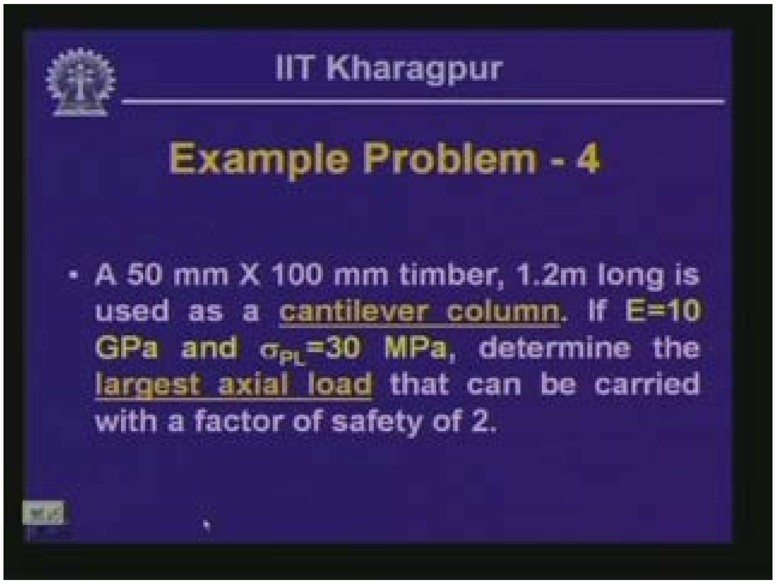

Well, let us look into another example wherein we have a member which is made out of timber, the cross section of which is a 50 mm by 100 mm rectangular one and length of the member is 1.2m. Now this particular member is used as a cantilever column; the value of e is given as 10 GPa 10 Giga Pascal and the stress at the proportionality limit is 30 MPa; we will have to determine the largest axial load that this member can carry with a factor of safety of 2.

Now if you look into that, that the cross section of this member is a rectangular one having a size of 50 mm by 100 mm, the length of the member is 1.2m and this is a cantilever column; now the meaning of a cantilever column is that it is fixed at one end and is free at the other and this is subjected to a compressive load P. So we will have to find out how much of load P we can apply to this particular member having this particular section so that we can have a factor of safety of 2. You keep this aspect in mind that will have to impose a factor of safety of 2 and we will have to decide about what P load we can apply on this member.

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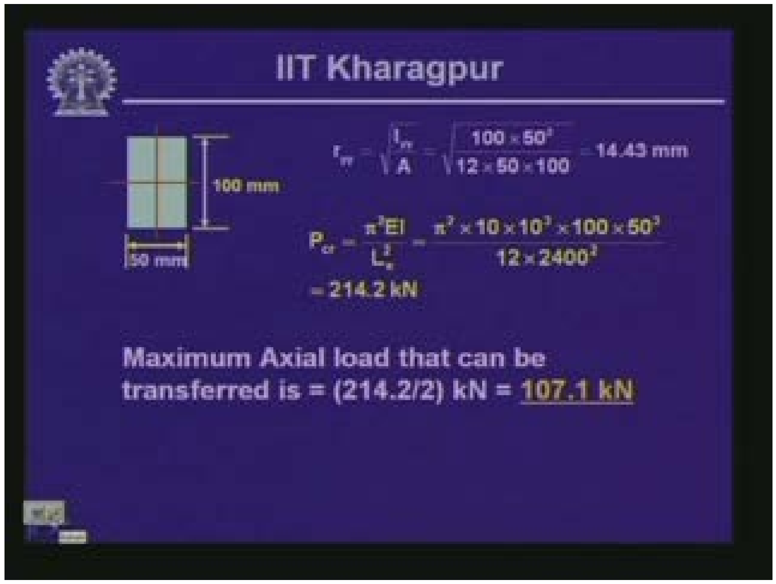

Now, if we look into the cross section of this; as we have seen earlier for this rectangular cases you have the xx and the yy at the two rectangular axis system and the width of the member let us consider as 50 and depth as 100 mm so we can compute the value of I x and I y and as you know I x will be equals to 50 times 100 cube by 12 and I y will be 100 times 50 cube by 12 and since I y will be less than I x thereby r y will be less than r x for this particular section.

Now if we compute the value of the r y which is equal to root of I y A I y by A and I y as I said is equals to 100 times 50 cube by 12 and area is equals to 50 times 100 so the value of ry comes as 14.43 mm. For this if we compute the value of.............. the length of the member is equals to 1.2m and since this is a cantilever member we have seen that for a cantilever member pi critical load is equals to pi square EI over 4L square thereby this is equal to pi square EI divide by 2L square and 2 is the value of K which is the effective length coefficient so this twice L as we called as L e and K becomes 2; so L is given as 1.2m thereby the effective length is 2400 and that is what is indicated over here that P critical which we use is equal to pi square EI by L e square. Now e here is 10 Giga Pascal so 10 into 10 to the power 3 MPa; I is 100 into 50 cube by 12 and 2400 square so this gives us a value of 214.2 kilonewton.

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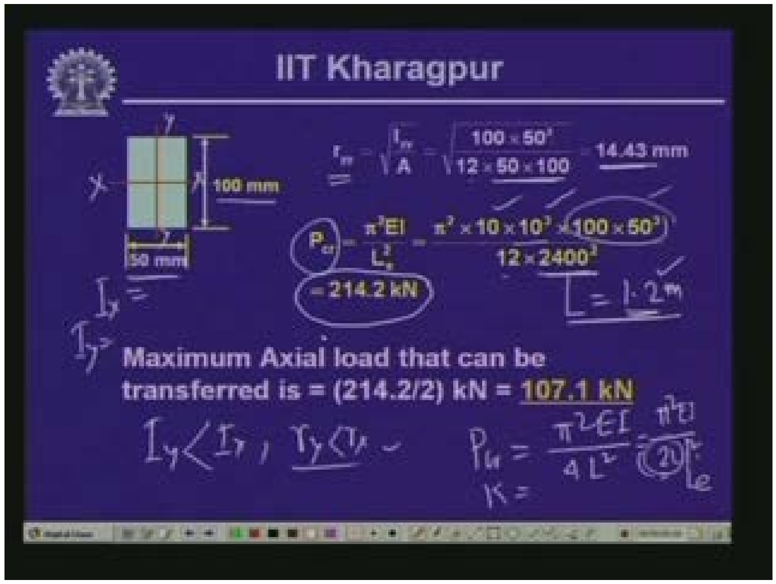

Now as it has been indicated that we will have to find the load in terms of a factor of safety of 2. That means you will have to apply a load P in such a way that you can achieve a factor of safety of 2 that means if we apply a load of twice P the member should be in a position to withstand that stress. So, the maximum load that should be applied or it should be limited to is half the actual load because this critical load which we compute from Euler's critical load formula is not with any factor of safety so we will have to impose the factor of safety to this. So, if we divided this load by 2 the factor of safety value the load comes as 107.1 kilonewton. So the maximum load that you can apply on this particular member is equal to 107.1 kilonewton.

Now if we apply a load higher than this 107.1 the wen will find that the member may fail by buckling. But the question is that we have applied a factor of safety of 2 so even in this particular case even if we exceed by this it may not fail immediately unless we have some other effect on this member which can cause the failure on the member.

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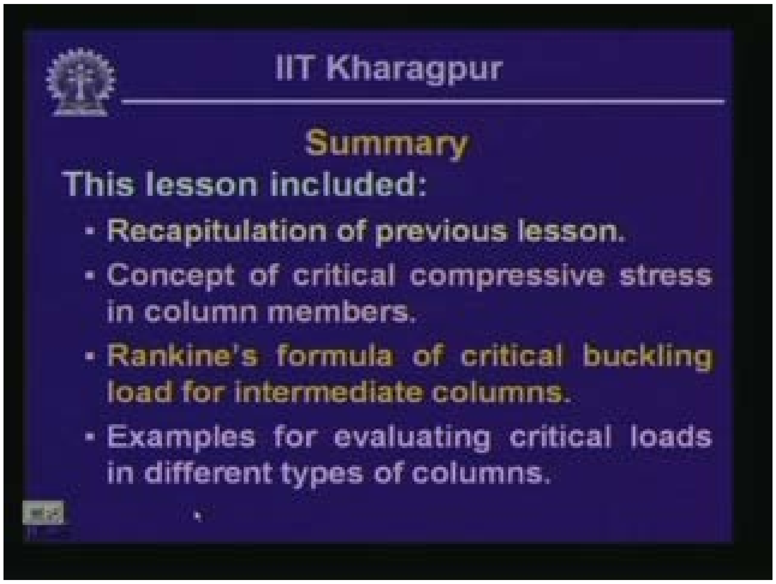

Well, then to summarize; in this particular lesson we have looked into some aspects of the previous lesson. As we have seen, in previous lesson we have discussed about the Euler's critical buckling load formula and in this particular lesson we have seen what are the limitations of Euler's critical buckling load formula and also what are the different values of K which we have termed as the coefficient of the effective length for different support conditions of the member which are either 1 or 2 or 0.7 or 0.5 depending upon different conditions we have.

Also, we have looked in at the concept of critical compressive stress in column members. We have discussed about the Rankine's formula of critical buckling load which are applicable for intermediate columns and we have looked into some examples for evaluating critical loads in different types of columns.

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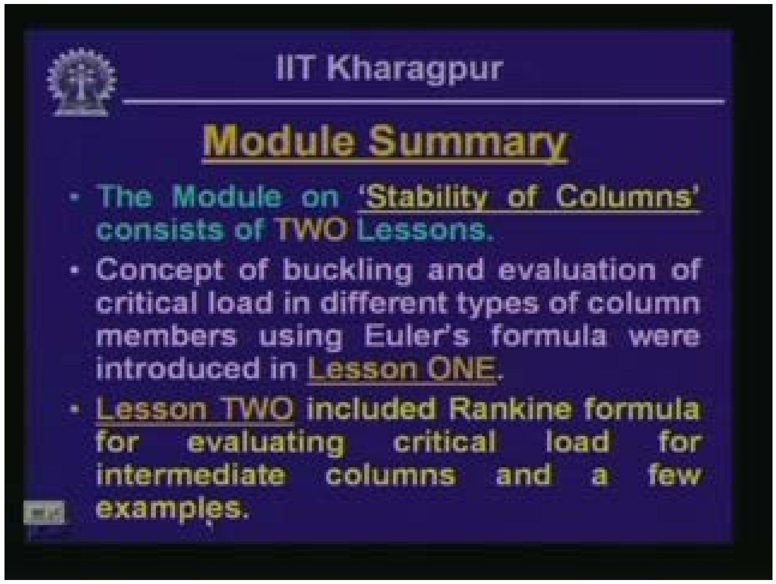

Now with this lesson we will come to the concluding part of this particular module which is on stability of columns. Stability of columns basically we had two lessons. In the previous lesson we had introduced the concept of stability the buckling and thereby we discussed about the Euler's critical load formula which are applicable for columns. In this particular lesson we looked into what are the different phases of column; the short column, the long column and the intermediate columns and then the critical load corresponding to the intermediate column and then we have looked into the formula which is applicable for evaluating the critical load in intermediate column given by Rankine.

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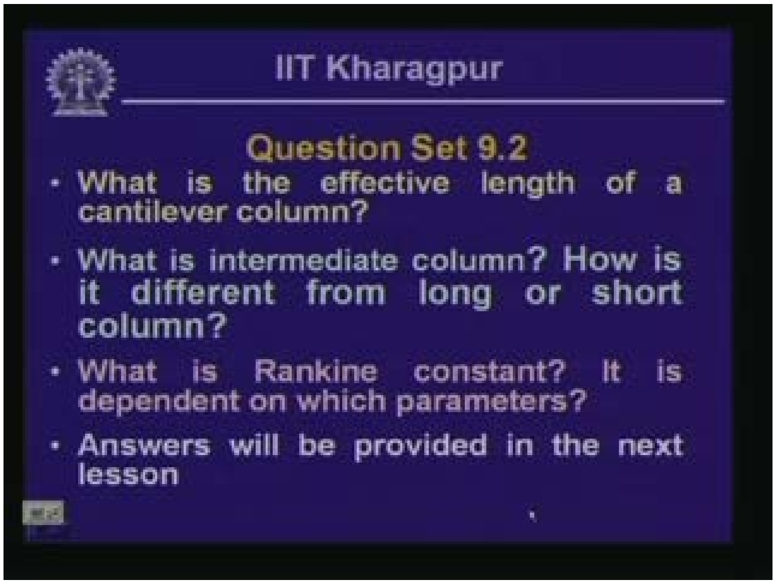

Now these are the two lessons which we had and consequently we had looked into some examples which can be evaluated using this formula. Now the questions set for you are this.

What is the effective length of a cantilever column?

What is intermediate column and how is it different from long or short column?

And what is Rankine constant? It is dependent on which parameters?

We will look into this; you will get the answers in these two lessons itself. The answers for this will be given in the next lesson, thank you.