Strength of Materials

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Lecture - 37

Stability of Columns - I

Welcome to the first lesson of the ninth module which is on stability of columns part I. In fact, in the previous modules we have discussed certain aspects of the stresses in members and consecutively we have looked into the effect of bending in a member where we have evaluated the deflection of the member. Now, in this particular lesson we are going to discuss different aspects which are the stability of a member which we designate as column.

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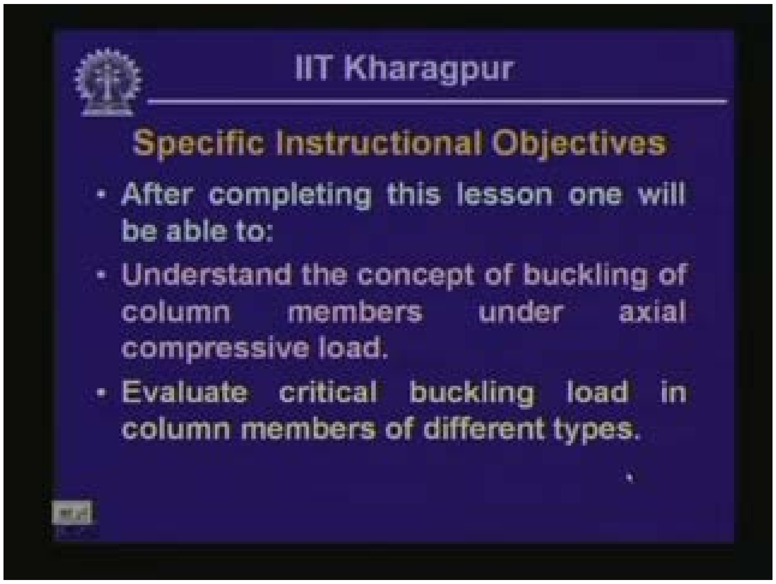

Now it is expected that once this particular lesson is completed, one should be able to understand the concept of buckling of column members. In fact, we will define what we mean by column member; which member we term as column and then we will look into certain characteristic features called as buckling; so buckling of column members under axial compressive load.

One should be in a position to evaluate critical buckling load in column members of different types. Now we will look into that what we really mean by critical buckling load that the member which is subjected to axial compressive load in which load it is going to buckle of deform.

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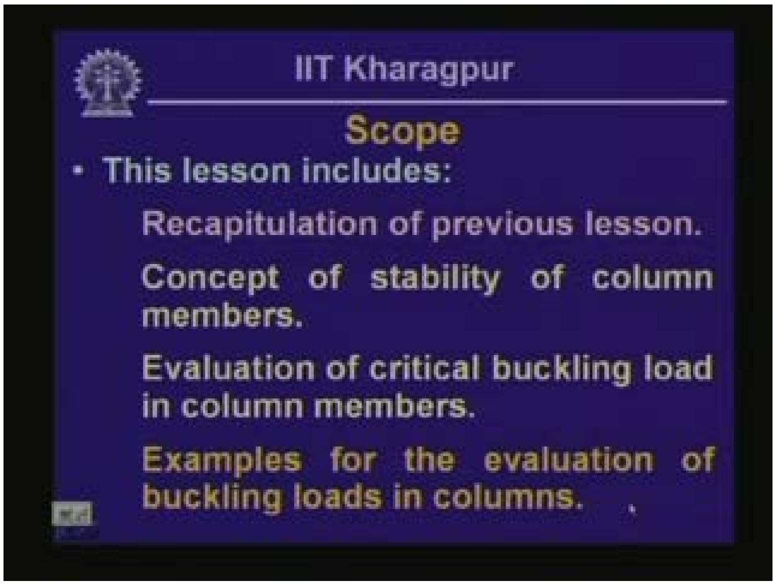

The scope of this particular lesson therefore includes the recapitulation of previous lesson. In fact, the last module we have looked into the aspects of combined stresses; we will look into certain aspects of the combined stresses while answering the questions related to that. And this particular lesson includes the concept of stability of column members, evaluation of critical buckling load in different types of column members. The different types of column members we mean, we look into that what are the different types of supports the column members have and what are the contributions of axial load in the buckling of those members.

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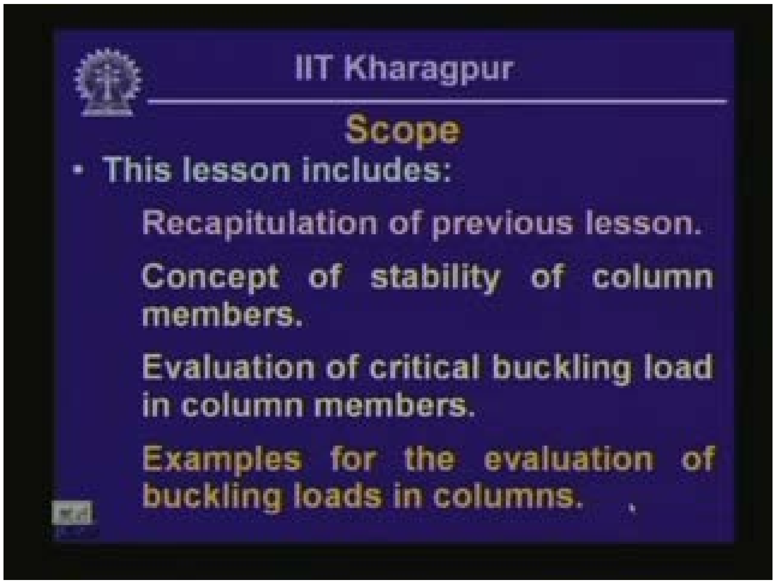

Then we will look into some examples for the evaluation of buckling loads in columns.

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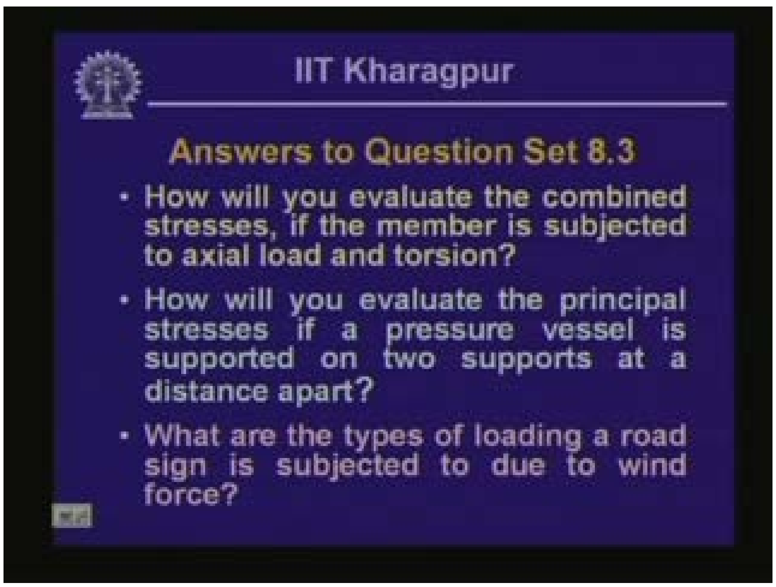

Well, now let us look into the questions which were given in the last lesson. The first question was that how will you evaluate the combined stresses if the member is subjected to axial load and torsion simultaneously.

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Now as I said, in the previous lesson we had discussed about the concept of the combined stresses and you have seen that if a particular member is subjected to an axial pull P along with the twisting moment T then what will be the effect of this combined loading in the member.

As you know that the axial load which is acting in the member which is concentric that means acting through the center of gravity of the member will contribute to the normal stress sigma which is equals to P divided by cross-sectional area. So everywhere wherever you take the element on the surface we will get or in the other point normal stress will be equals to P divided by A and the contribution of the twisting moment T will be In the form of shearing stress as we have seen while discussing the effect of the torsion and member. Thereby, if we take an element this will be subjected to the normal stress sigma and the shearing stress tau.

And as you know that if a particular element is subjected to the combined action of normal stress and the shearing stress we can plot these stresses in the Mohr's circle in terms of sigma and tau then we can compute the value of the maximum normal stress which we call as sigma 1 and the minimum normal stress which we called as sigma 2 we call these also as principal stresses and the maximum value of the shearing stress which we call as tau max. These stresses we can evaluate when they are subjected to the combined action of the axial load and the torsion.

Therefore, when a member is subjected to the axial pull or axial compressive force and a twisting moment, the resulting stresses are the normal stress and the shearing stress. Shearing stress comes from the twisting action and then when normal stress and the shearing stress act simultaneously we can compute the values of the maximum tensile or compressive stresses and the maximum value of the shearing stresses in the member.

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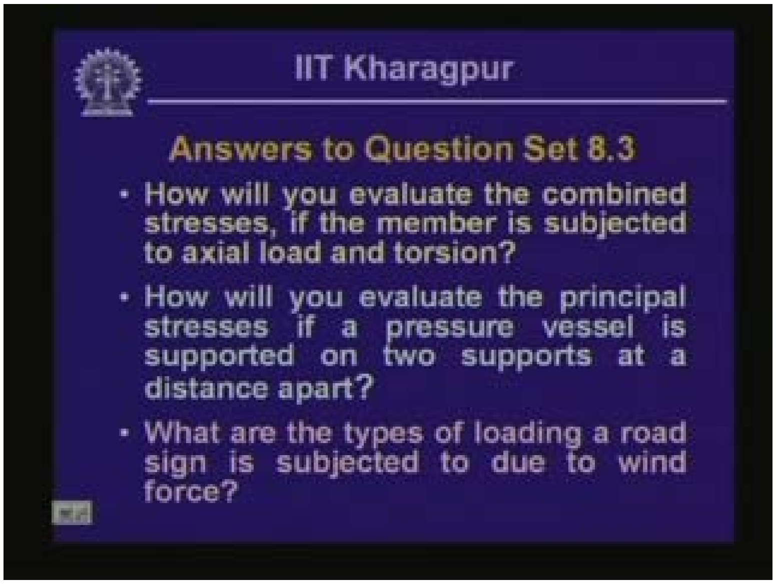

Now the second question posed was how will you evaluate the principal stresses if a pressure vessel is supported on two supports at a distance apart.

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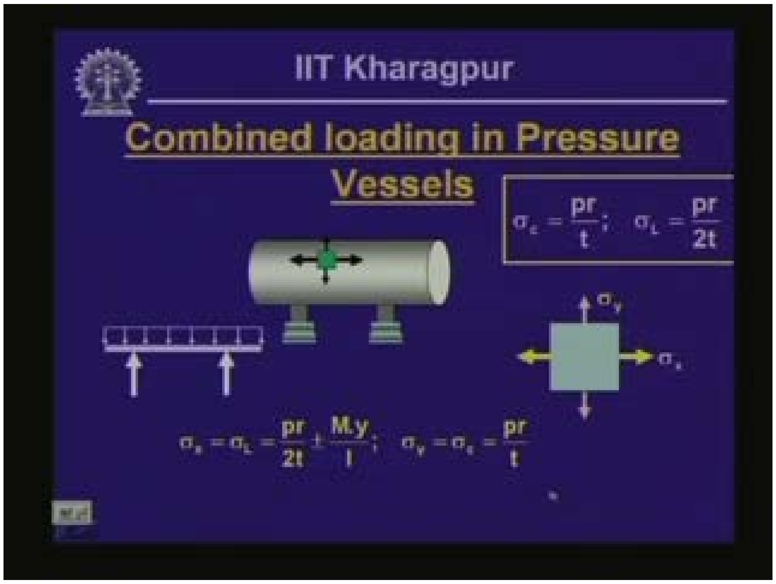

As you have seen that a pressure vessel when it is subjected to some liquid which is exerting pressure on the container we get two types of stresses which we call as sigma C the circumferential stress or the Hoop's stress which is given by sigma C equals to pr by t. Consequently, we get the longitudinal stress which we call as sigma L which is equal to pr by 2t where p is the internal pressure, r is the radius the outer radius of the cylindrical vessel and t is the thickness of the vessel. These are the stresses the sigma C and sigma L the circumferential stress and the longitudinal stress that we get on the surface.

When this particular vessel is supported on two supports thereby if we take the weight of this container along with the contained liquid, we can idealize that particular form in this particular form (Refer Slide Time: 6:33) where we have two supports with the load which is distributed over the entire length of the vessel. the entire length of the vessel we can consider as a beam member which is subjected to its own weight along with the contained liquid and supported at these two points which we can call as A and B.

Now as you know, this particular beam member when it is subjected to a load of q uniformly distributed will get a moment diagram which will look like this. now because of this bending there will be bending stresses and as you know the bending stress introduces the normal stress which is My by I and this depending on the position of the element it could be compressive or tensile when you are talking about with reference to the neutral axis so thereby that will contribute to the normal stress.

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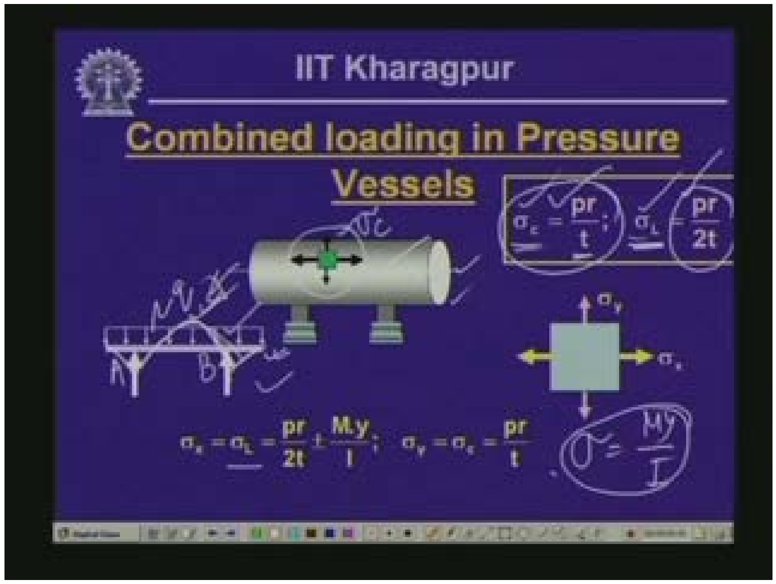

Now this stress the normal stress due to bending will get added or subtracted from the longitudinal or the normal stress we get from the pressure vessel and they are shown over here. Now pr by 2t is the contribution from the pressure vessel and My by I is the contribution from the bending. So this will give us the resulting normal stress in the x direction and this will give the normal stress in the y direction.

In this particular problem of course we have not considered the effect of the shearing stress. However, if you compute the value of the shear there will be an effect of the shearing stress as well. But if we do not consider the shearing effect then this element will be subjected to the normal stress sigma x and sigma y and from this again sigma x and sigma y since there are no shearing stresses on this element these are the principal stresses or the maximum value of the stresses the maximum tensile stresses that will be occurring in the x and y direction. Hence, at any direction we can compute the stress based on these stresses.

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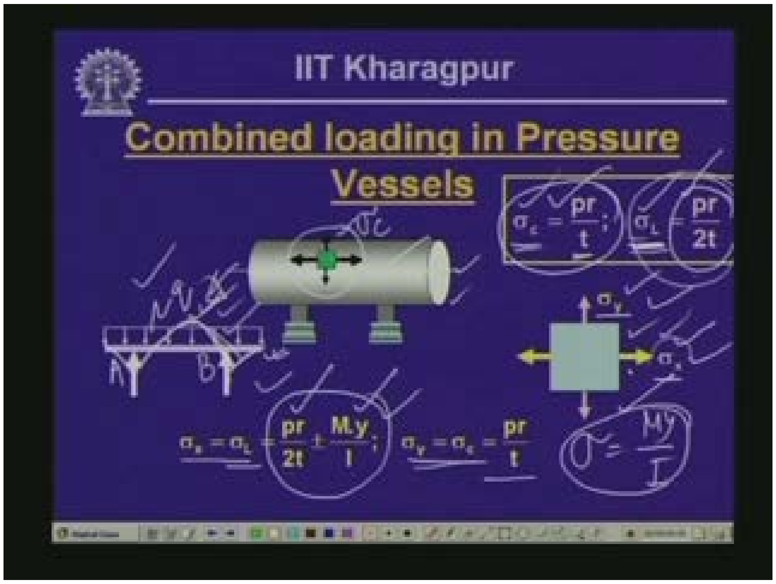

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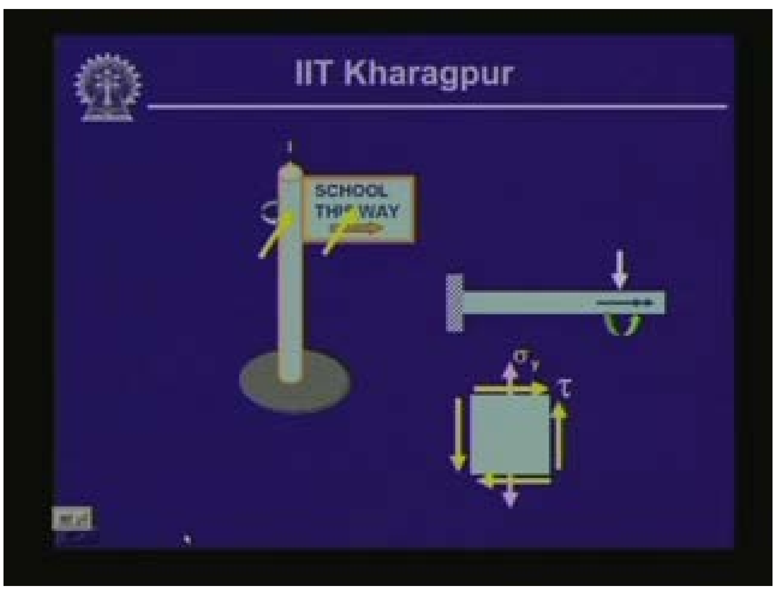

Well the third question was on a load sign which we normally use what kind of stresses this will induce the wind load will induce in the vertical member. Now as you have seen that the load sign which is projecting from a vertical shaft is subjected to the normal.............when the wind acts normal to this plate it will be subjected to a wind force which will be acting through the center of gravity of this particular plate; and if you call this load as P this load if we transfer to the vertical shaft member it will be transmitted with a twisting moment T. So, if we idealize this particular member in this form like you have a beam which is subjected to a lateral load P and a twisting moment T now this lateral load p will produce a maximum bending moment at the support which is equals to if you call this distance as a where P times a is the bending moment over here and this Pa the moment will introduce the normal stress sigma which is equals to My by I.

Also, at this section there will be shearing stress because of the shear force due to load P and that is equals to tau as equals to VQ by Ib and the twisting moment T will have the stress here will produce shearing stress which is tau which is equals to........ as you know that T by J equals to tau by rho so this is T rho by J.

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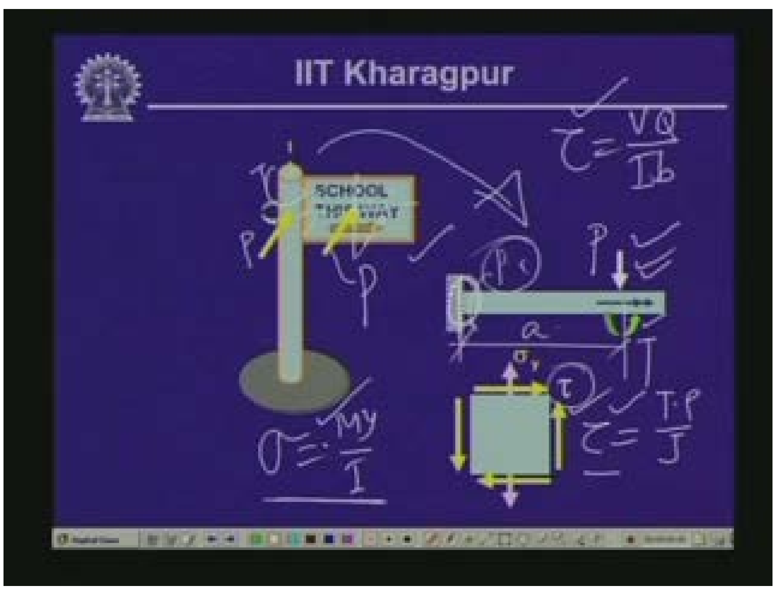

So the total effect of shearing stress you will have from the twisting moment as well as from the shearing force and there will be normal stress because of the bending. So the member the vertical member which is subjected to wind load the load sign which is subjected to the wind load will have the combined effect of the bending stress, the shearing stress and the shearing stress from the twisting moment. And once we know the normal stress and the shearing stress then we can compute the value of the.......... maximum value of the normal stresses or maximum principal stresses; if we plot them in the Mohr's circle we can compute the value of sigma 1 and sigma 2 which are the maximum normal stresses and thereby they are the principal stresses.

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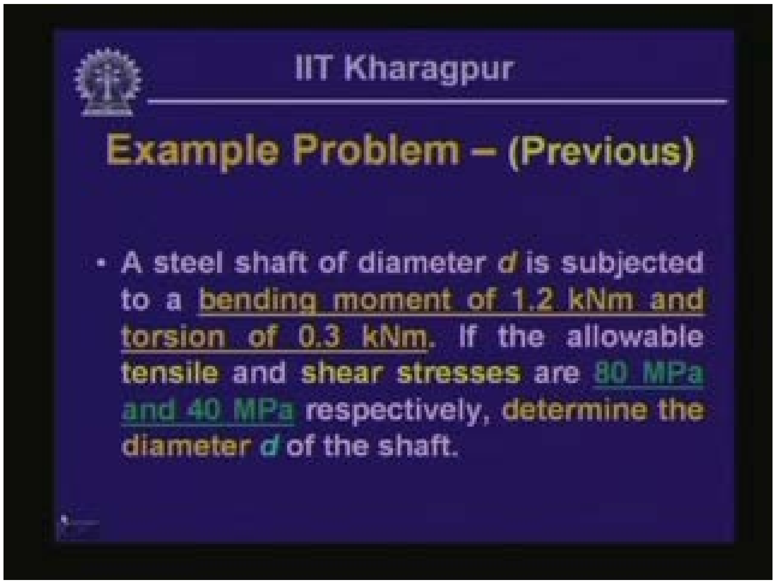

Well, before we go in to the stability aspects let me discuss the problem which I had shown to you last time and this is related to the combined effect of the bending and the twisting moment in a member.

A steel shaft of diameter d which is unknown is subjected to a bending moment of 1.2 kilo Newton meter and a torsion of 0.3 kilo Newton meter. Now if the allowable tensile and shear stresses are 80 MPa and 40 MPa respectively, we will have to determine the diameter d of the shaft. This is a situation where the shaft is subjected to combined action of bending moment and the twisting moment.

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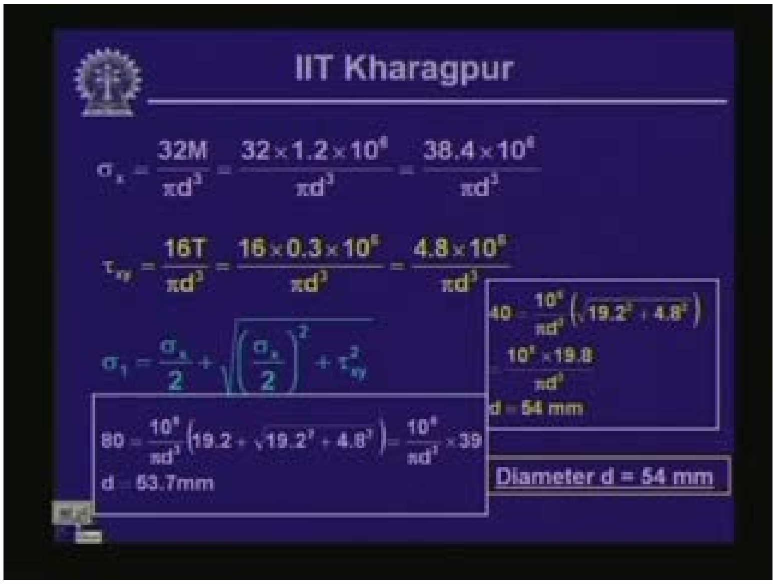

When the member is subjected to a combined action of bending moment and twisting moment then we get the stresses in this form. say for example; if we have the shaft of diameter d then as we know the sigma is equals to My by I and y here is the maximum distance from neutral axis which is d by 2 so this is equals to M times d by 2 and I is pid 4 by 64; this if you compute it, it comes as 32M by pid cube and this is what is indicated over here that sigma x is normal stress which is arising due to bending is equals to 32M by pid cube and here all the parameters are known because bending moment is given as 1.2 kilo newton meter which is 10 to the power 6 Newton millimeter and d is the parameter which we will have to define and if you compute this this comes as 38.4 into 10 to the power 6 by pid cube.

Now consequently, when we have the twisting moment that will introduce the shearing stress and again as you know that the value of the shearing stress will be equals to T rho by J and that if we can compute tau is equals to T rho by J; you know rho is the maximum distance from the center which is d by 2 again so T d by 2 and J is the polar moment of inertia which is pid cube by 32 pid 4 by 32 then it becomes as equals to 16T by pid cube. This is the shearing stress which is getting generated because of twisting moment T.

Now here also except d all other parameters are known and twisting moment is 0.3 newton kilo meter and thereby tau xy comes as 4.8 into 10 power 6 divided by pid cube.

Now, when you have this normal stress sigma x and tau xy we can compute the value of the maximum tensile stress sigma 1 either using Mohr's circle or using the transformation equation which is sigma 1 equals to sigma x plus sigma y by 2, now here sigma y means 0 so this is sigma x by 2 and since it is maximum tensile stress that is why you have taken the sign plus equals to the radius which is sigma x sigma y by 2 square plus tau xy square and sigma y being 0 again this is sigma x 2 square plus tau xy square.

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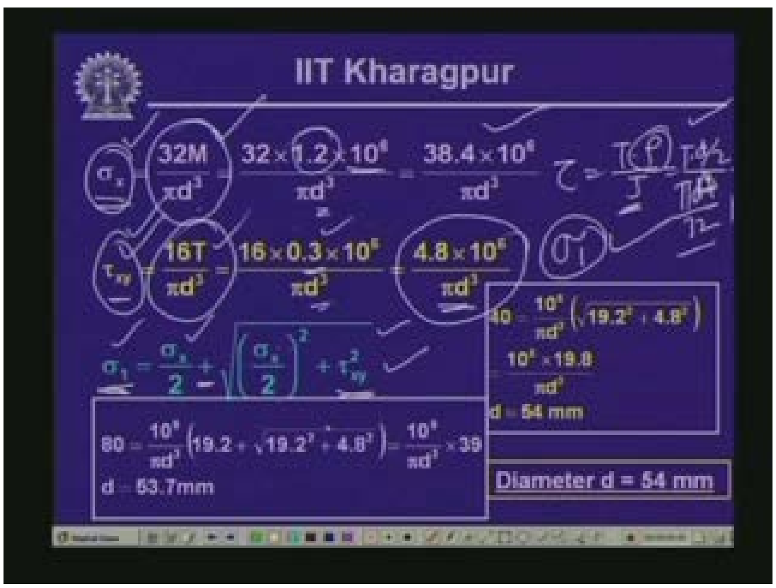

Now if you substitute the values of sigma x and tau xy in this expression; sigma x by 2 is 19.2, this is of course 10 to the power 6 divided by pid cube has been taken out and this is root of 19.2 square plus 4.8 square which is tau xy and this gives us a value of d if we compute from this expression we get 53.7 mm. This is the value of the diameter of the shaft which is corresponding to the maximum tensile stress of sigma 1 which is equals to 80 MPa.

Now if you have to satisfy the criteria of the maximum shearing stress which is equals to 40 MPa; and as you know, the maximum shearing stress is nothing but equals to the radius of the Mohr’s circle which is this (Refer Slide Time: 14:51) that is sigma x by 2 square plus tau xy square and if we substitute that this is 10 to the power 6 pid cube taken out, we have root of 19.2 square plus 4.8 square and this gives us a value of 19.8 into 0 to the power of 6 divided by pid cube.

And as you know, the limiting shearing stress is 40; from this we get a value of d as 54 mm. Now as you can see we have computed the value of diameter from the maximum tensile stress, also we have computed from the criteria of maximum shearing stress. And since both the criteria have to be satisfied because you cannot provide a diameter which will satisfy one and will not satisfy the other, then naturally it will not stand it will fail so you will have to provide the maximum value of the diameter corresponding to the two; satisfying both the stress criteria and thereby the diameter of the shaft comes as 54 mm. So the shaft diameter has to be 54 mm so that both the normal stress and the shearing stress criteria are satisfied.

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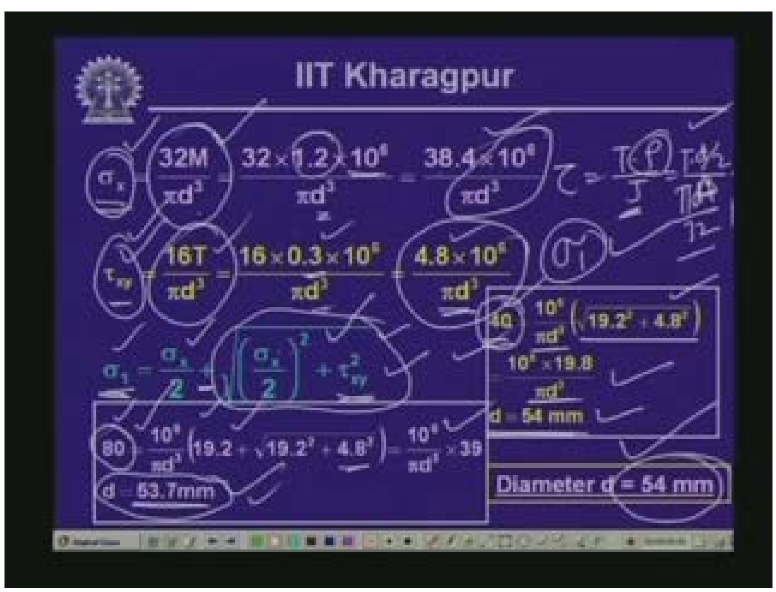

This was the solution for the example which we had given last time.

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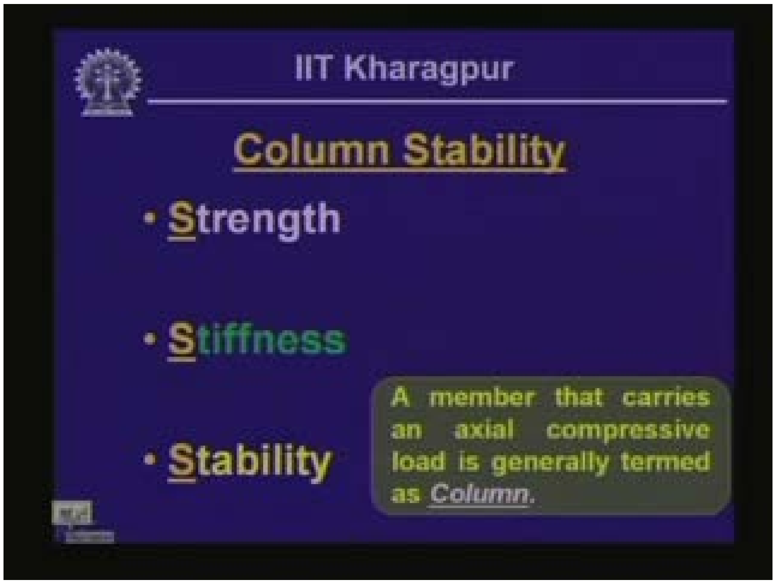

Having discussed about the effect of these combined stresses now let us look into the stability aspects of the column. Before you really go into the stability part, let us look into these two aspects. As you remember that in the first module we have discussed that we are concerned about the three Ss in the strength of material: one we call as the strength and in the strength as you have seen we have computed the value of the stresses.

Say for example; if you have an axial load; axial load divided by the cross-sectional area gives you the normal stress. Also, we have computed the shearing stress from the twisting moment or we have computed the normal stress from the bending moment. Now these stresses as we have seen must withstand the stresses that can be of any material that we are providing for constructing any of these elements.

Naturally that says that the strength of that particular material is such that it can withstand these stresses what we have designated as the stress criteria for a particular member. Consequently, we have looked into that if you have a member which is subjected to load it will be undergoing deformation. Let us take for example that when you have a member subjected to the axial load P we had corresponding deformation delta which was equals to PLl by AE.

Now if we like to relate the delta with P we can write these as P as equals to AE by L times delta. Now here if we define like this if we like to have unit deformation that load which we need is corresponding to this particular term AE by L and this is what we call as stiffness of the element. That means this is the axial stiffness of the member which will need this much of load P to produce unit displacement delta or unit deformation delta.

Consequently, say for example; in a particular beam simply supported beam subjected to uniformly distributed load we have seen the delta is equals to PL cube by 48. Now here also if I like to write to P in terms of delta we can write P as equals to 48EI over L cube times delta. Now if you like to have unit deformation or deflection the corresponding load which we need is equals to this amount which we term as stiffness of the member. Or if we are talking about say the twisting moment as we know that tau is equals to T rho by J this will again be equals to G theta I mean T by J is equals to t tau by rho is equals to G theta by L so T is equals to GJ by L times theta.

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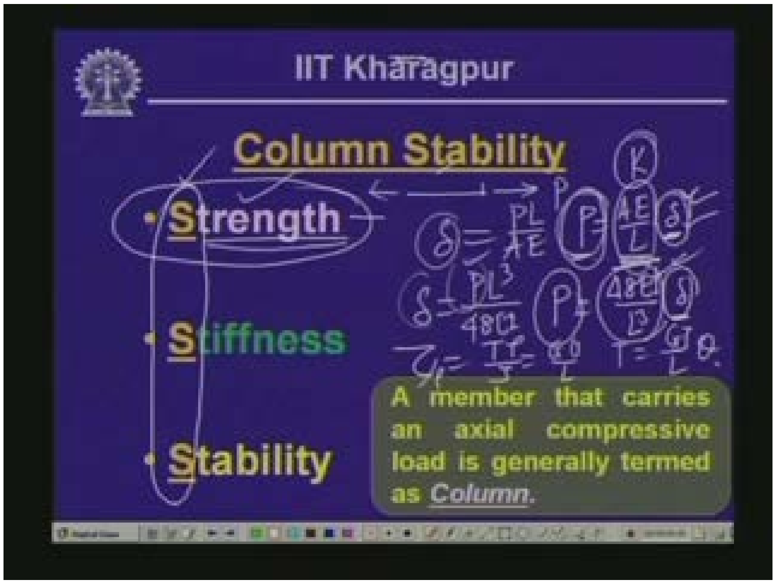

Here also if we like to have unit rotation the corresponding twisting moment that we need is equals to GJ by L and that is what is twisting stiffness. Now you see, in each of these cases as we are looking into that we need stiffness of a member whether in terms of A by L or EI by L cube or GJ by L they are the characteristics of the member concerned where the cross-sectional area or the moment of inertia or the polar moment of inertia introduced or connected with. These are the parameters which we call as the stiffness. So these are the two aspects the strength and stiffness parameter we have already looked into.

Thirdly, the aspect which we are going to look into is the stability of a member. Now in this particular lesson, of course we will be discussing about the stability with reference to the column member. So strictly speaking, stability does not mean the stability column alone; it can be stability of different members as well. But in this particular lesson we will be looking into or in this particular course rather will be looking into the stability of the column members only. A column member is a member that generally carries axial compressive load and we call those kinds of members as column members.

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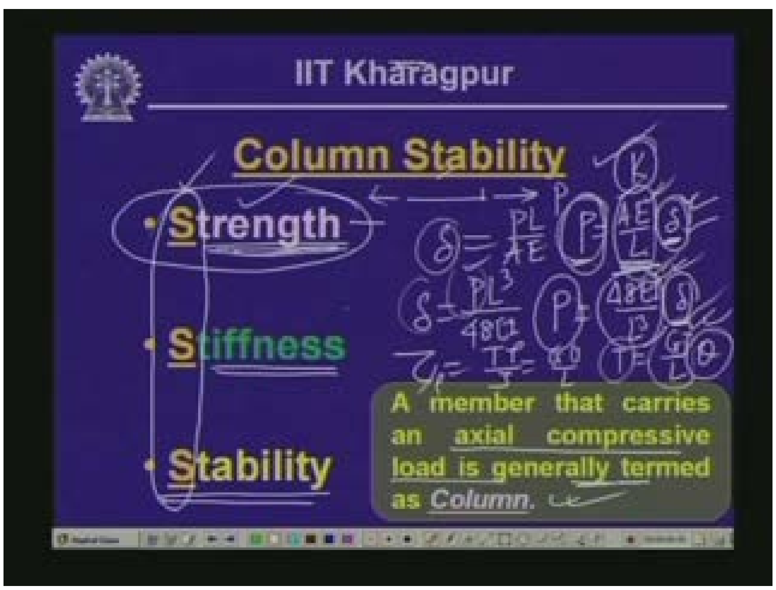

Now let us really look into what we mean by the stability of columns.

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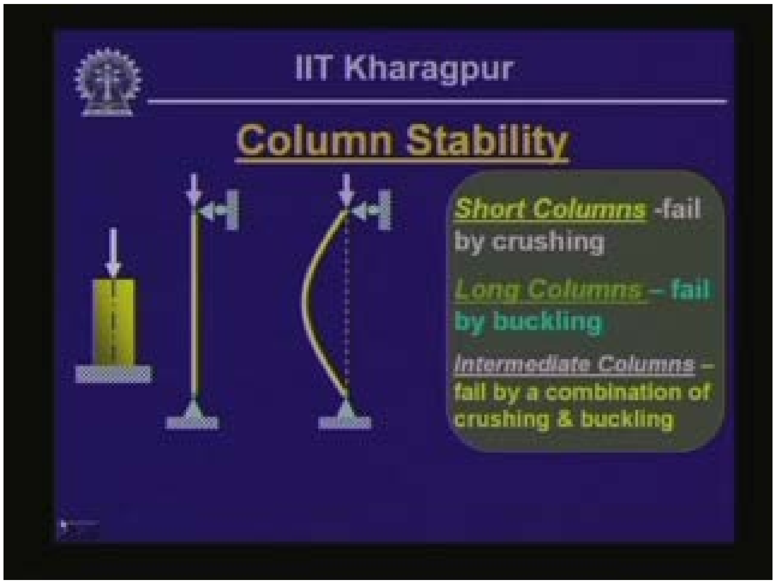

Columns, we can categorize into three groups: one we call as short columns where the axial load is applied to a member in such a way that everywhere we get the stress. As you have seen, sigma will be equals to P divided by A the normal stress and on this member if you keep on applying this load P will fail by crushing or yielding of the material and the stress will go beyond the permissible value of the material.

Another category we call as a long column where the member fails by buckling. Now let us look into this aspect from these two very small experiments. Now here, I am considering two members the cross section of which is rectangular one and they are on the same cross section. If you look into these two members I have one member which is of shorter length and I have another member which is of little longer length and the cross section of these two members are same and they are rectangular in nature. If you look into the cross section they are the identical.

When I am talking about this shorter length member, if I am applying the axial load you will find that the stress at any level will be P divided by........... the load applied divided by the cross-sectional area. Now if I take the longer member and if I try to apply the load you will find that this particular member is going to bend in a form and this is what we call as buckling. Now this particular member, as you can see that we are applying the axial load and it has taken a bent shape thereby it has undergone some amount of bending and this is what we call as buckling.

In case of the shorter member the stress which we had obtained they are by the crushing of the member or the stress level at any point goes beyond the permissible value of the material stress; in case of the long columns or in case of the member which is a cylindrical one, when we apply the load P if we increase this load gradually, as you have observed that it takes this particular bent shape and this is what we call as buckling.

Now this particular shape (Refer Slide Time: 22:59) is little unstable; if we try to apply a lateral load there will be deformation and this deformation will be in an unrested manner and that is what we call as buckling; the deformation of the member in an unrested manner causes failure of the member and we get intermediate columns which can fail by the combination of the crashing and buckling. Now for this it is very difficult to arrive at the theoretical basics so we resort to some experimental or empirical formulae for evaluating the critical load for the intermediate columns.

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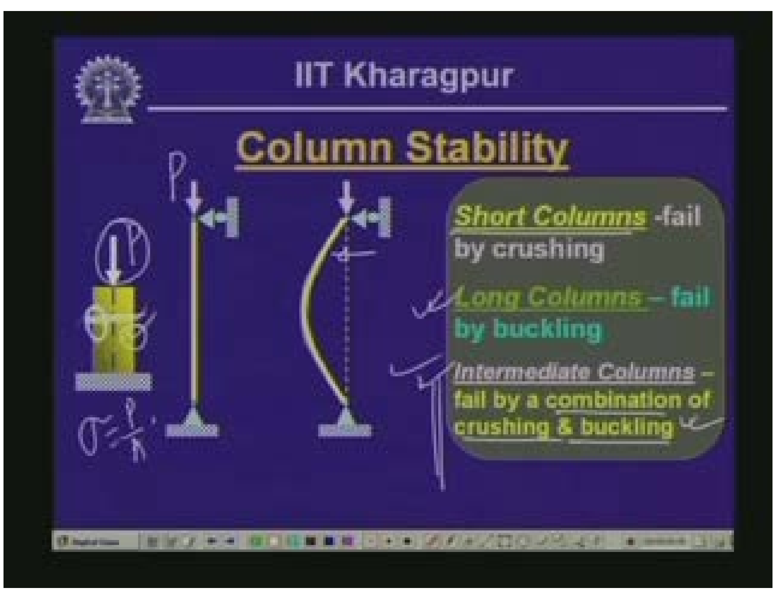

Now if we try to look into the animation part of it that if you have the member when they are subjected to load as you can see it is in a straight form; and when the first load is added as you can observe it is in straight form you see, now it is still straight, now we have added some load and now it has deformed slightly, and when you add load further it goes in an unrested manner.

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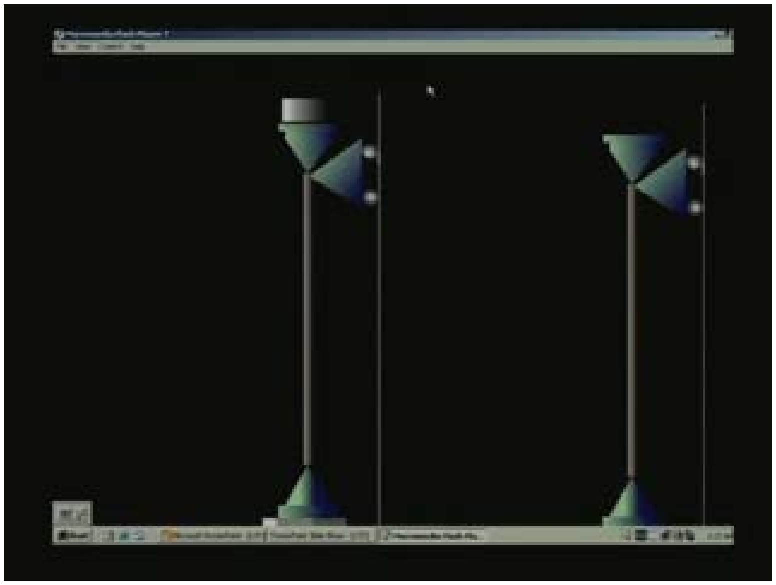

Now let us discuss this stability aspect of column through a model which you call as a buckling model.

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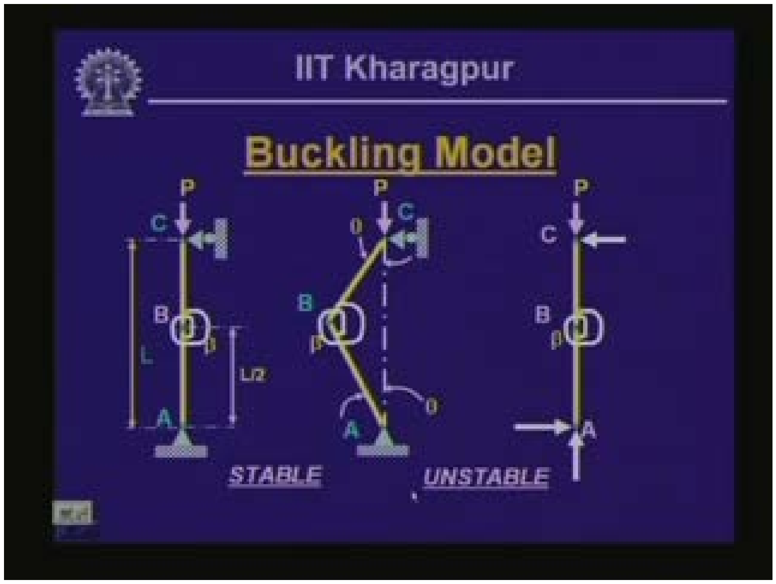

In this buckling model we consider two rigid bars AB and BC and they are connected by a pin at point B. And at point B we add a rotational spring having a stiffness beta. Now when this particular member AB and BC they are concentric they are subjected to a load P it will be in a stable form. Now if we add a little disturbance to the member then the point B will move so that the deformation which we get, the member will undergo a rotation called theta which we presume as a small deformation. When this particular member point B moves then the rotational spring which we have at point B will have some amount of moment generated which will try to bring this member back once we remove the disturbance. That means this particular moment which will be equivalent to the stiffness times the total rotation these two members undergo which is equals to 2 theta so the restoring moment for the spring M is equals to beta times twice theta.

This particular restoring moment will try to bring these two members in their straight form. Now, on the other hand what happens is this particular load P will try to increase this deformation at point B. that means it will try to increase this rotation theta. As you can see that the effect of the axial load which is trying to deform the member or buckle the member is against or opposite to the restoring moment that has been extracted by the rotational spring at point B.

Now this particular module you can compare with the previous example as I was showing that you have a long member subjected to axial load. Now there the bending or buckling is over the entire length of the member; now here that elasticity is introduced through the rotational spring only.

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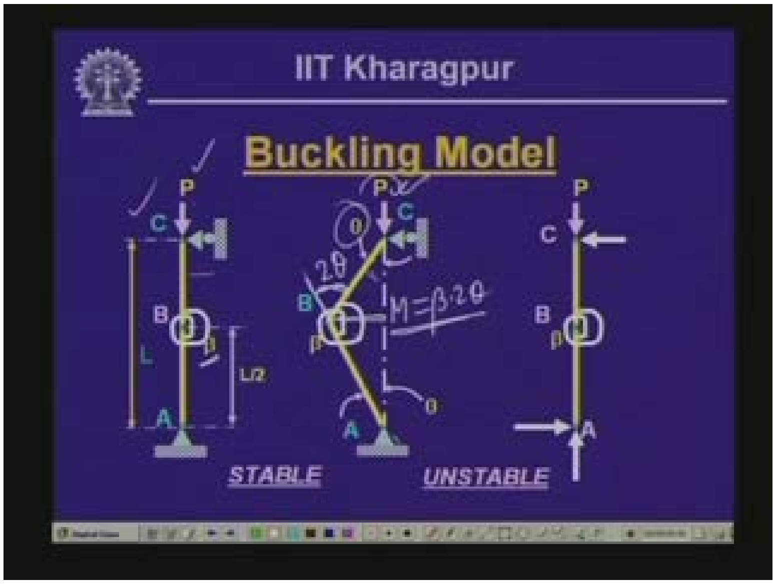

Now in this particular state if we remove the disturbance it will come back to its original position. If we go to the particular load when................ so that particular configuration when after removal of the disturbance it comes back to its original position we call that particular state as a stable state. In that anti-shaft state again if we keep on increasing this load P a situation will come when the deformation will be large and thereby the member is going to collapse and now the restoring moment will not be able to withstand the effect of the compressive load or thereby the load P which will be applied will be greater than the restoring moment that is being offered by the rotational spring and that particular state we call as unstable state.

Hence, you see that one state we get as stable state and another state we get as unstable state corresponding to two values of the axial compressive force P. Now in between these two states; between the stable state and unstable state we get an exclusive value of P which is the boundary between the stable and unstable state and that load we call as the critical load. Now at that particular load if the member is subjected to in a stable equilibrium position little disturbance in that member can cause a failure of the member and that is why call that particular load as a critical buckling load.

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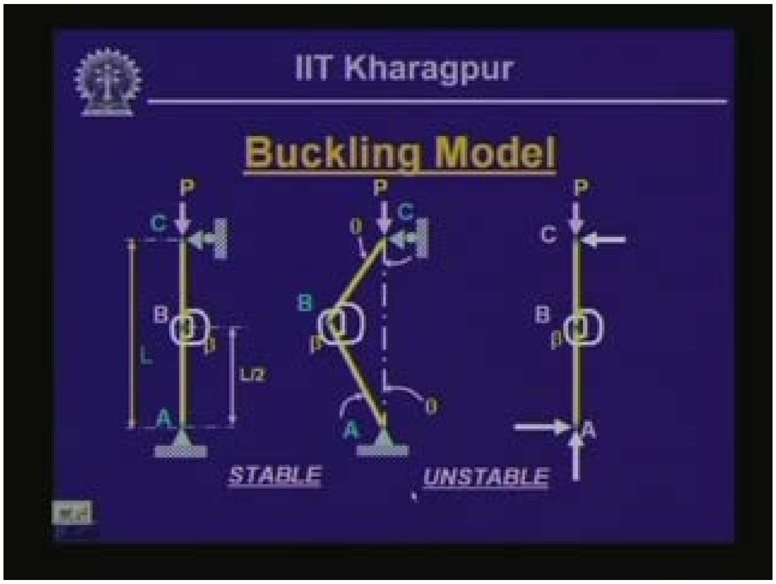

Now if you like to analyze that what will be the critical buckling load for this particular model we take the free body diagram of this whole thing the whole member and if you see that this horizontal force if you call it as H and if you take the moment of all the forces with respect to A, we see that H is going to be equals to 0.

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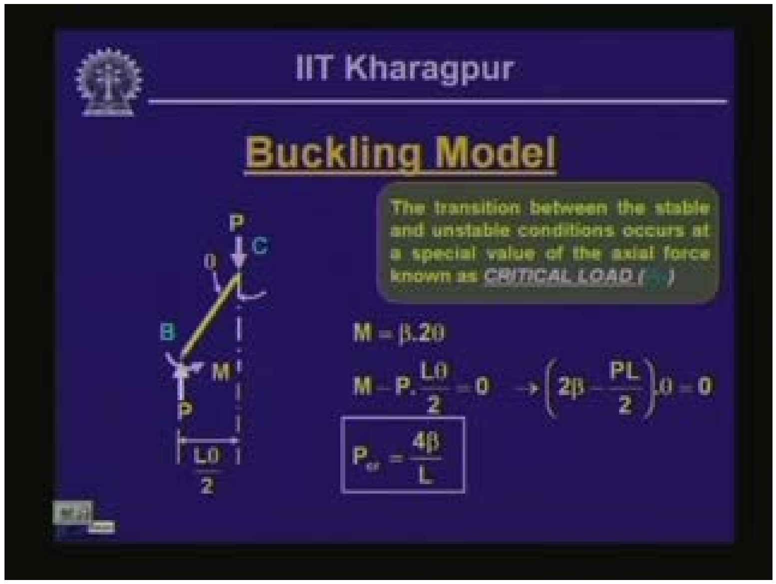

Now if we take the free body of the top part of the member the part PC which is like this where you have the small deformation theta here, the axial force P is acting and the restoring moment M and the resistive force P the reactive force P is acting at point B; now as we have seen that M is equals to beta times twice theta is the restoring moment as we have called it; well, it is trying to restore its original position, if P is small if restoring moment is larger than the P it will come back to its original position and hereby the column is a stable member if the disturbance is removed.

But if this restoring moment becomes smaller than the P or P becomes larger than the restoring moment the member is going to collapse. Therefore, at the point when we are calling that P is in the boundary between this stable and the unstable state that means at that particular point of time the load is just equal to the restoring moment. So when axial compressive load P is equal to the restoring moment in this particular model then it will be in a just state of equilibrium and any additional load will cause instability of the member or the ember is going to collapse due to the unstable situation.

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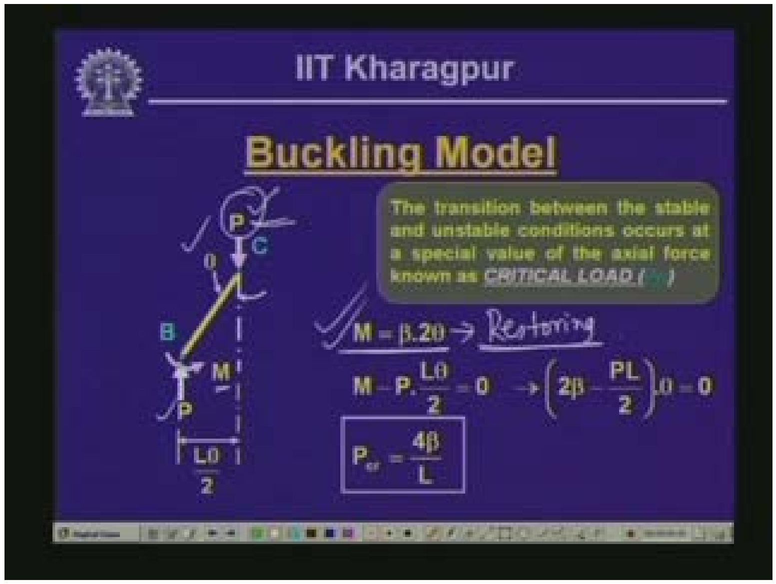

Thus, if we equate that or if we take the moment of all these with respect to B and M minus P times now this is theta (Refer Slide Time: 30:35) and this length is L by 2 so this distance is L by 2 times theta so P times L theta by 2 minus or M minus PL theta by 2 equals to 0; now this gives you M as equals to 2 beta theta so 2 beta minus PL by 2 theta is equals to 0.

Here if you see that either theta could be equals to 0 or this bracketed equals to 0. If theta is equals to 0 then naturally this is in a straight form so there is no buckling. Hence, this bracketed term is equals to 0 so that gives us a value of P equals to 4 beta by L and this is what we are calling as critical load and this critical load is independent of theta that indicates that whatever may be the value of the theta the critical load P is equals to the 4 by L times that stiffness the stiffness of the rotational spring which is analogous to the stiffness of the module which we are looking into the long member which we were looking into.

Hence, the critical load is equals to 4 beta by L. Now what does we mean by this particular critical load. This is the boundary between the stable and the unstable situation. If we have load less than P critical that means the member is in a stable form that means once we remove this disturbance it will come to a perfect state a perfect vertical state and if in such a state if we add disturbance in the critical load if we add any disturbance to it then it is going to cause the collapse or the member will become unstable.

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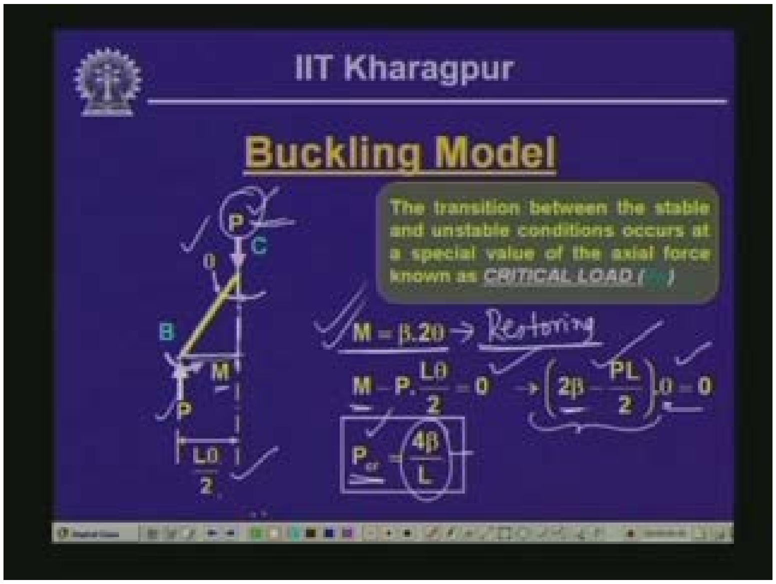

So what we need to look into is that the difference state which we can compare with this particular situation that if a ball which is placed in a member which is upwardly concave, here wherever it is the position of this particular ball it will come back to its lowest position and thereby this is a stable form and that you can plot with respect to load verses theta if you say this as theta that means in a stable form once you deform it if you add some disturbance to the member and bring it back it will always bring it will come back to the perfectly vertical position and this is a stable state.

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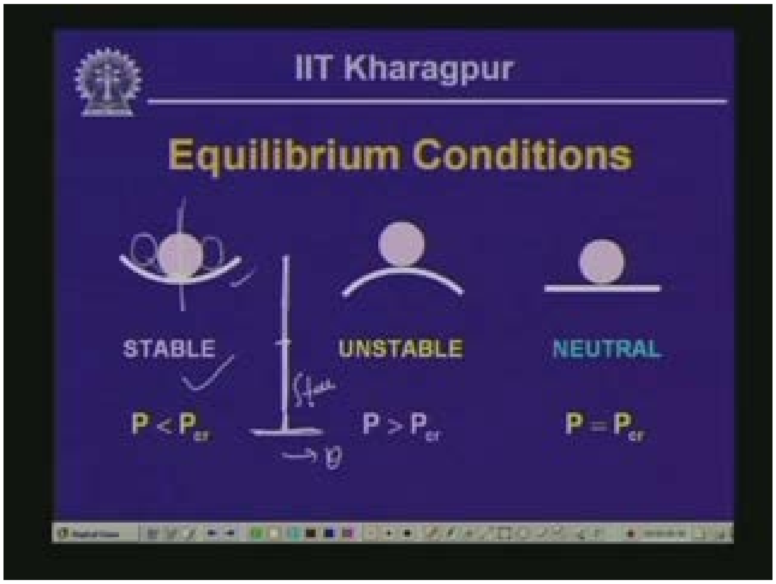

Now, this is a state which is unstable. That means at this particular point at this particular point it is stable but a little disturbance to this can cause failure of this particular form this will move. That means at this particular state at a particular state when load has gone to the criticality the member can be just vertical that means the theta could be equals to zero so it is just in a critical state; if we give a little disturbance this way or that way that can cause the collapse of the whole member and that is what we call as the unstable state; this is what is being represented through this particular diagram.

Here (Refer Slide Time: 33:46) P is greater than the P critical and at this particular point......... so this is the unstable state and this is the stable state and this particular one is the neutral state. Now here wherever you place this position it will always remain in that particular position and that is what we call as the just equals to the critical load; this we call as the neutral position.

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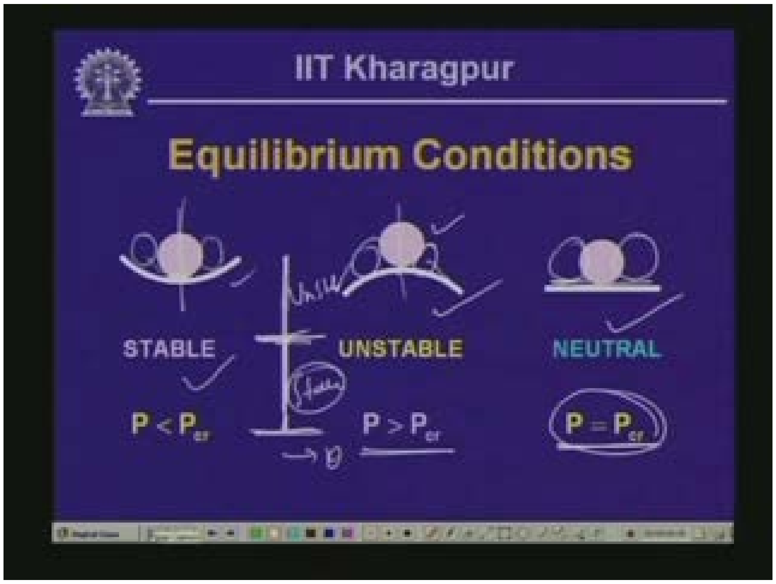

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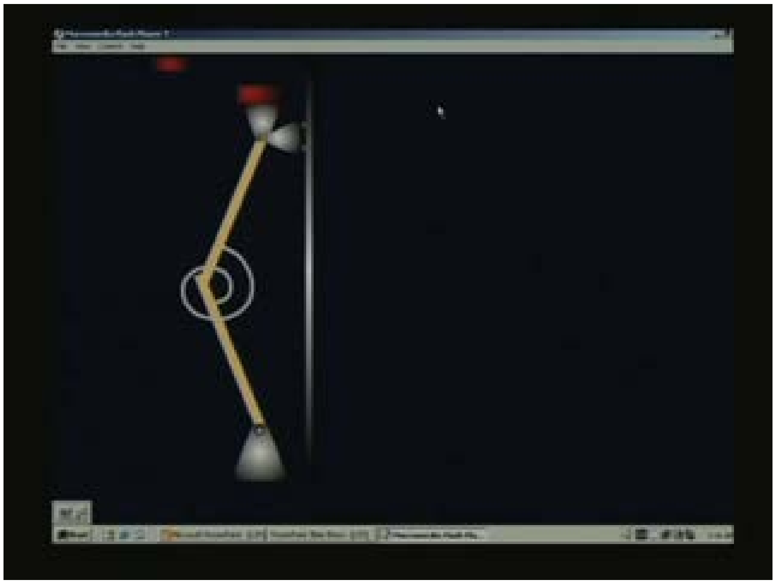

Now we can look into this particular form where you see that once you remove this it comes back to its original position because the restoring moment is more than the axial force and that is why the restoring moment is putting it back in its original position. But if the load becomes the larger you see that it collapses that means if the axial force become higher, then the member collapses.

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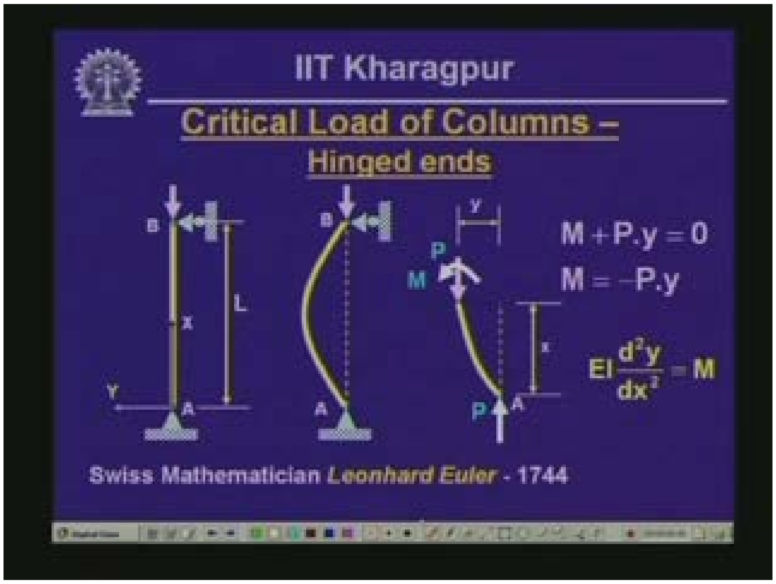

Well, then we come back to the original discussion again after looking into the aspects of the model where we have looked into the buckling model. Now we are interested to evaluate that what will be the critical buckling load or a member and for that we start with a situation that we have a column member which is pinned at both ends and this is an idealized form of the column where the axial load P is acting perfectly concentric with respect to this member.

Now here we choose the axis system in such a way that x goes along the member length and y is perpendicular to this. That means if we rotate by 90 degree we get the situation that x is in the positive x direction and y is in the positive y direction as we have assumed for a beam member. The length of the member is L. As you have seen, at the criticality when we reach to the P critical the other slightly bent form of this member we call this as theta the member can be in the equilibrium state beyond which to this particular state if we add any disturbance then it is going to cause a failure of the member.

We are interested to evaluate this critical load (Refer Slide Time: 36:07) corresponding to which it will be in a state of equilibrium. Now if we take a section at a distance of x from end A from (0, 0) point then the free body diagram of this particular part of the deformed state of the member is this where this distance is equals to y at a distance of x and at this point you have the reactive force P and the moment M.

Now if we take the moment of all the forces with respect to say this point we call this as d (Refer Slide Time: 36:46) then moment M plus P times y is equals to 0 or M is equals to minus P into y. Now as we have seen that the flexural equation or the deferential equation for evaluating the deflection of a beam member was EI d 2 y by dx 2 which was equals to M and M the bending moment is here equals to minus p y. Based on these particular aspects we like to derive the critical load for this particular member and this derivation was presented by the great Swiss Mathematician Leonhard Euler in the year 1744 wherein he had given this particular derivation based on which we compute the value of the critical load P critical and thereby we generally designate this critical load as the Euler's critical buckling load of a column member and or it is a Euler critical buckling load formula. The expression which we get for evaluating the critical load we call that as the Euler critical buckling load formula.

Now let us see how we get this value of the critical load corresponding to this.

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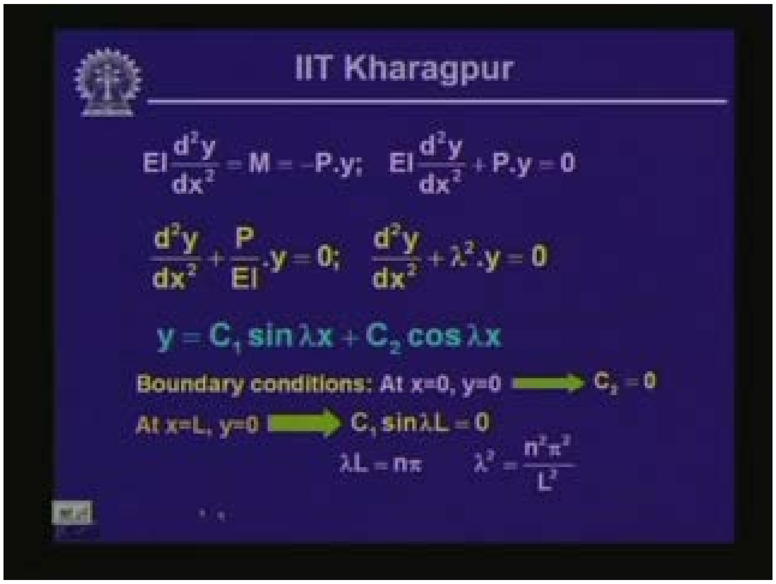

Now as we have seen; EI d 2 y dx 2 is equals to a moment which is equals to minus P times y. Now if we take this on the other side we have EI d 2 y dx 2 plus P into y is equals to 0. Now if we divide the whole equation by EI we have d 2 y dx 2 plus P by EI times y is equals to 0. Now we designate P by EI term as equals to lambda square. So P by EI is equals to lambda square so this is d 2 y by dx 2 plus lambda square into y is equals to 0.

Now if we choose a solution of this y as equals to e to the power mx then we get an expression that M square plus lambda square into e to the power mx this is equals to 0. Since e to the power mx equals to 0 will give y equals to 0 which is meaningless so M square plus lambda square is equals to thereby M is equals to plus minus i lambda.

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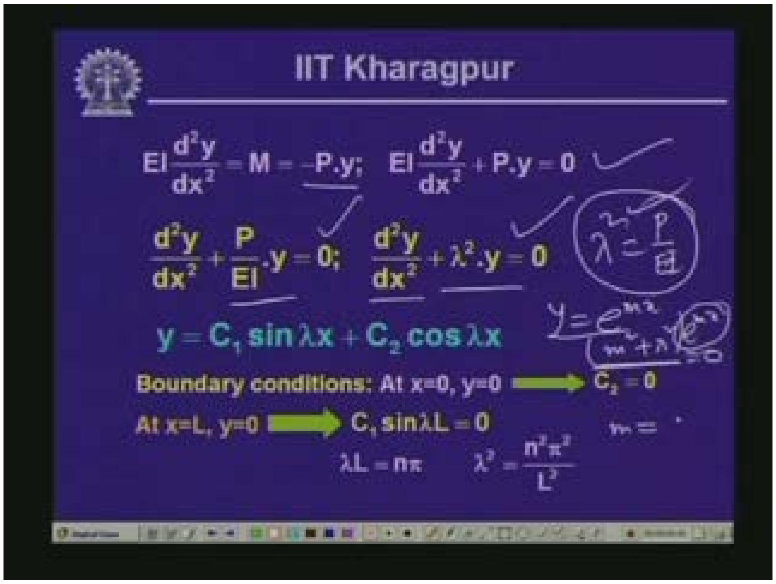

Corresponding to this solution we have the solution y as equals to C 1 sin lambda x plus C 2 cos lambda x where C 1 and C 2 are the unknown constants. Now the boundary conditions which we have for this particular member; this member is pinned at both ends, it is hinged at both ends so at x equals to 0 the deflection y is equals to 0 of this particular point and that gives us the value of C 2 as equals to 0. And also at x equals to L the deflection is 0 the deformation is 0 over here so at x equals to L if we substitute y equals to 0 we get C 1 sin lambda L is equals to 0.

From this particular expression as you can see that either C 1 can be 0. If C 1 equals to 0 that means y equals to 0 that means there is no deformation in the member. But since C 1 cannot be 0 will lead us to a solution which does not mean buckling or deformation of the member then sin lambda L has to equals to 0. Now if sin lambda L equals to 0 it gives that lambda L in general forms equals to n times pi that means for different values of pi 2, twice pi, thrice pi the value of sin lambda L will be equals to 0.

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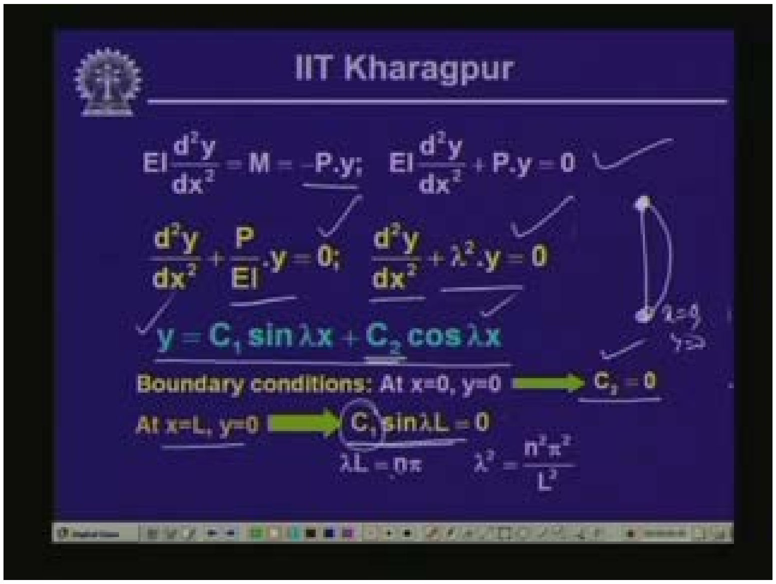

Now if we square this up we get lambda square is equals to n square pi square by L square and lambda square as you know is equals to P by EI so P by EI is equals to n square pi square by L square and therefore P is equals to n square pi square EI over L square. So we get then P is equals to n square pi square EI over L square.

Here the minimum value of n; n can have values 1 2 3; now minimum value of n will lead to the critical load for this particular member and if we substitute n or we take n as equals to 1 then the critical load for this becomes y square EI by L square. So, if we have load less than pi square EI by L square in a column which is hinged at both ends then it will be in a stable form and if we go beyond that load then that particular member is going to collapse or fell in buckling; may be the stress level could be lower than the yield stress; the P by A if you compute that could be less than the yield stress but the member will fell in excessive deformation which we call as buckling.

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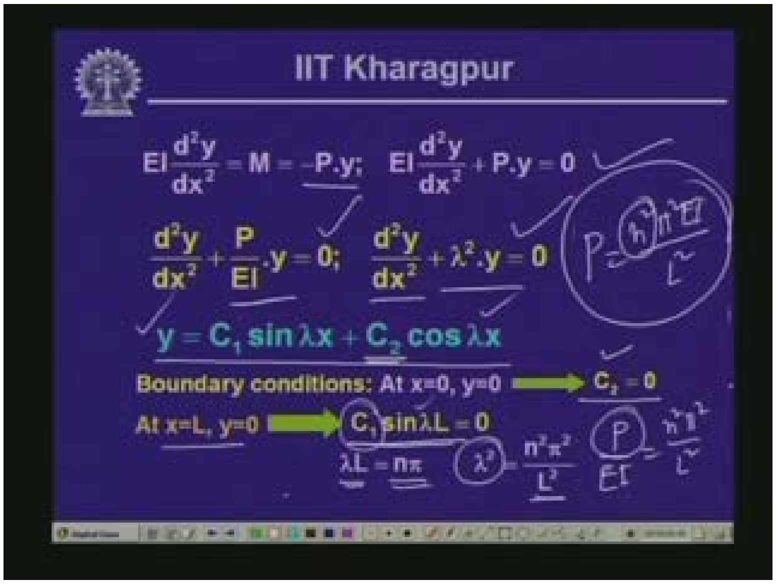

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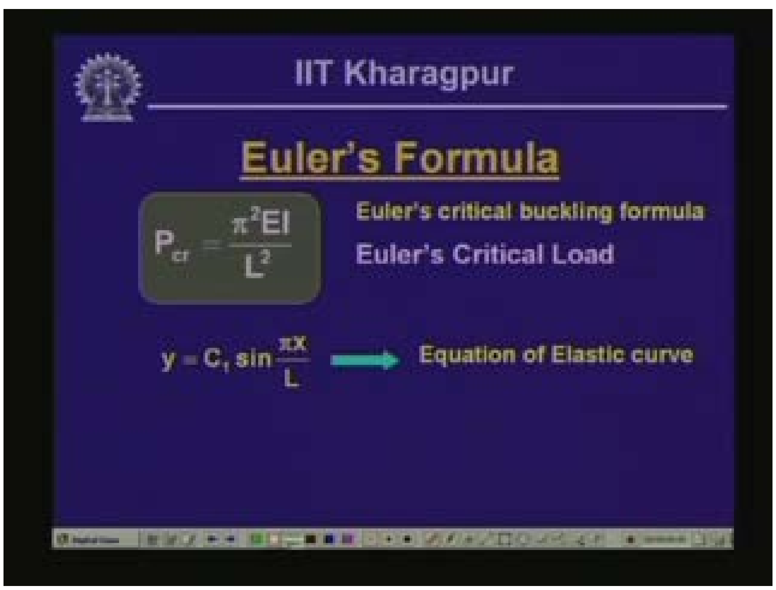

Now this is what is indicated over here. The critical load P cr is equals to pi square EI over L square and this is what we call as Euler's critical buckling load formula or Euler's critical load; P critical is known as Euler's critical load. And consequently, as we have seen y is equals to C 1 sin lambda x for lambda if we substitute pi L or in a general formats in pi L this will give you n pi L x the equation of the elastic curve. Now since C 1 is not determined uniquely, so naturally the value of the deformation which we get is not uniquely defined so we can have an arbitrary value or arbitrary shape of the particular deformation.

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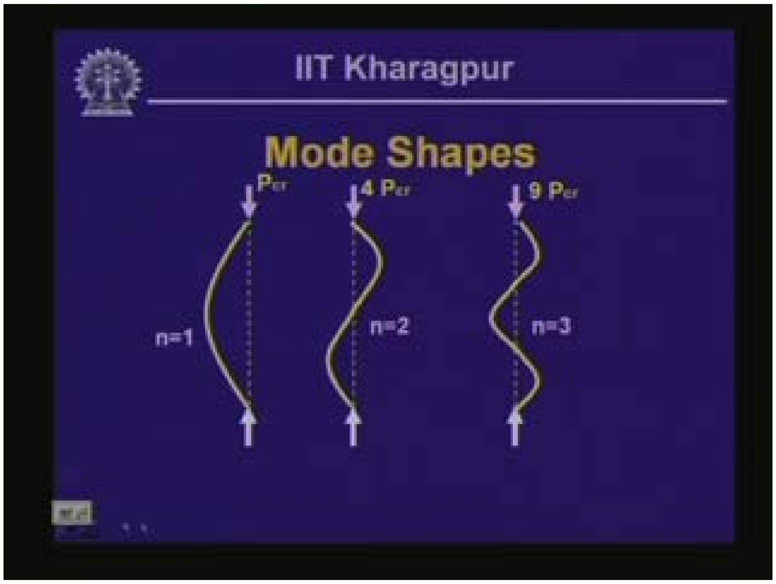

And this is what the deformation shown over here. You see, when n is equals to 1 this is the way the member is going to deform and you have the corresponding load as the critical load. Now when n is equals to 2 we are going to have the critical load as four times P critical and since it is larger than this critical load naturally at this point you will have to some kind of a support so that you get deformation in this particular pattern. When n is equals to 3 then the load can go up to nine times P critical corresponding to this one and thereby at the third point you need to have a some kind of support and thereby the column can take a shape of this particular form. Now these are the shapes which we generally call as the mode shapes.

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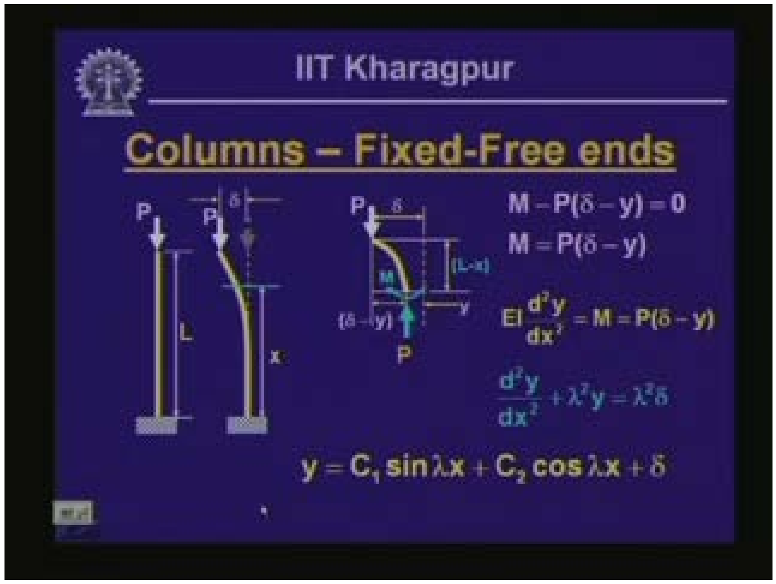

Now let us look into the critical load corresponding to a column member where the supports are of different types. Here one end is fixed and the other end is free. In the previous case we had a member where both ends were pinned or hinged. Now here one end is fixed and the other end is free. Now if that happens, at the critical state it may deform in this particular configuration and beyond which if we add disturbance to this particular member this is going to cause the collapse of the member and thereby corresponding to this state we call the load as critical load.

Again now if we take free body diagram of this particular part which is at a distance of x from the origin which is let us call as A then this is the free body diagram. Now at this particular point (Refer Slide Time: 44:18) this distance from here is equals to y and thereby at the top we have taken the maximum deformation of the member with respect to its original position as delta thereby this particular length is going to be equals to delta minus y. So if we take the moment of all the forces with respect to this particular point then we have moment M minus P times delta minus y equals to 0 and this is equals to the value of the moment.

Now, if you substitute the value of the moment in this deferential equation for the elastic curves which is EI d 2 y dx 2 equals to P times delta minus y this will give us d 2 y dx 2 plus P y equals to P delta; will have EI d 2 y dx 2 plus P y equals to P delta. Now if you divide this by EI you will have d 2 y dx 2 plus lambda square y because P by EI we have called as lambda square equals to lambda square delta. Now it will have two solutions: one is the complementary solution considering this as 0 and then corresponding to this we will have a particular solution. When this is equals to zero as you have seen the solution is equals to C 1 sin lambda x and C 2 cos lambda x. when you have the right hand side part we can evaluate the particular solution if we assume say solution y is equals to ax square plus bx plus c in this form then we will have d 2 y dx 2 as equals to twice A and thereby if we substitute in that way we have twice A plus lambda square y which is ax square plus bx plus c this is equals to lambda square delta.

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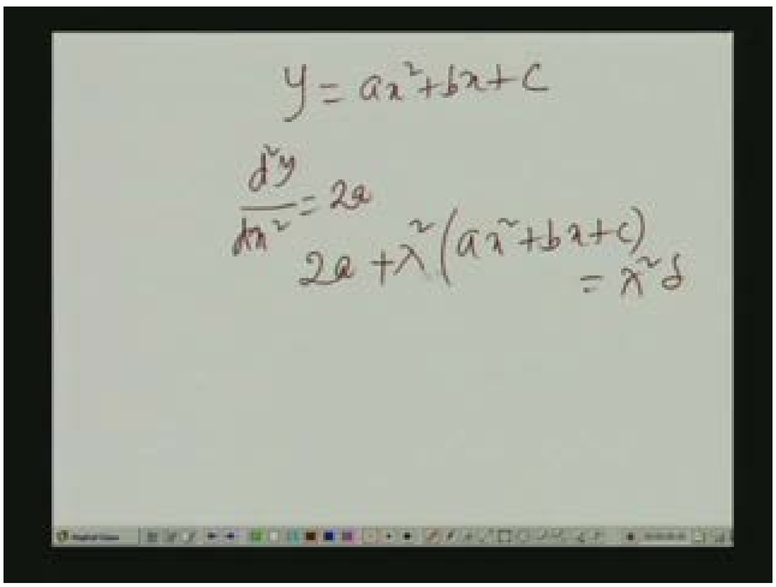

Now if you take the coefficients of x square and x on either side; since on the right hand side we do not have any term on x square and x thereby we will have a as equals to 0 and b also as equals to 0. Now consequently we will have that lambda square c is equals to lambda square delta and thereby c is equals to delta is the solution. So we will have y as equals to delta is the particular solution for this particular case and thereby we will have the solution in this form that y is equals to C 1 sin lambda x and C 2 cos lambda x plus delta. Now we will have to solve this C 1 and C 2 from the boundary condition and as you can see; at x equals to 0 y is zero here and x equals to 0 y............ dy dx is also 0 and the slope is zero here and also you will have to satisfy x equals to L and the y is equals to delta.

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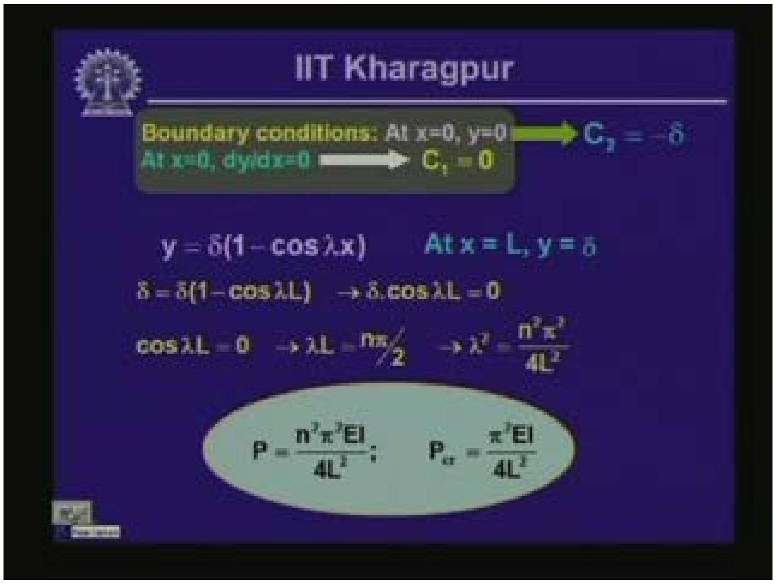

Now if we satisfy this now what we get is this that x equals to 0 and y equals to 0 will lead you to C 2 equals to minus lambda and at x equals to 0 dy dx is equals to 0 will lead you to C 1 equals to 0 and thereby we get y as equals to delta into 1 minus cos lambda x and we will have to satisfy that at x equals to L y is equals to delta thereby we get the delta is equals to delta into 1 minus cos lambda L and thereby delta cos lambda L equals to 0 and this gives us that either delta equals to 0 or cos lambda L is equals to 0.

If delta is 0 then there is no deformation of the member and there is no buckling and if cos lambda L is equals to 0 that gives you then what will be the value of the lambda L and lambda L in general will be equals to n pi by 2 from which if we substitute the minimum value of n we get the critical load as equals to pi square EI by 4L square.

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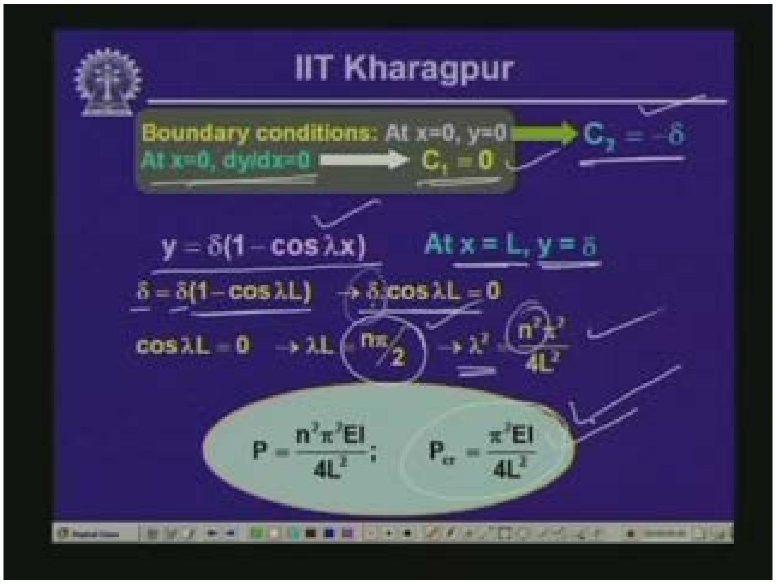

This is the critical load for a column member which is fixed at one end and free at the other.

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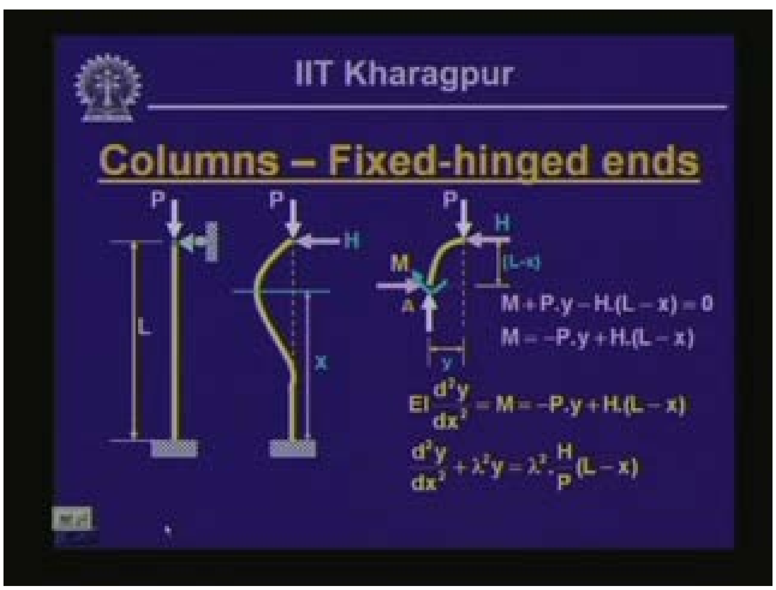

Now let us look in to the case where the column is fixed at one end and hinged at the other. Now it is expected that the deformed state will be in this particular form and if we try to find out the critical load corresponding to this state this is P critical and since this is the hinged end you have horizontal force h; so if you take the free body diagram of this particular part these are the moment, horizontal force and the vertical force, the reactive force is here; if we take the moment with respect to A we will have M plus P times y where y is this distance and since we are taking the section of x from the origin, let us call this as b (Refer Slide Time: 48:59) this is L minus x so minus H times L minus x equals to 0 so this is the value of the bending moment M. And these if we substitute here we get EI d 2 y dx 2 is equals to M which is equals to minus P y plus H into L minus x and this if we take on the other side and divide by EI then we have d 2 y dx 2 plus lambda square y is equals to lambda square H by P times L minus x. Lambda square you keep in mind as equals to P divided by EI. Now to write here lambda square we have multiplied with P and divided by P.

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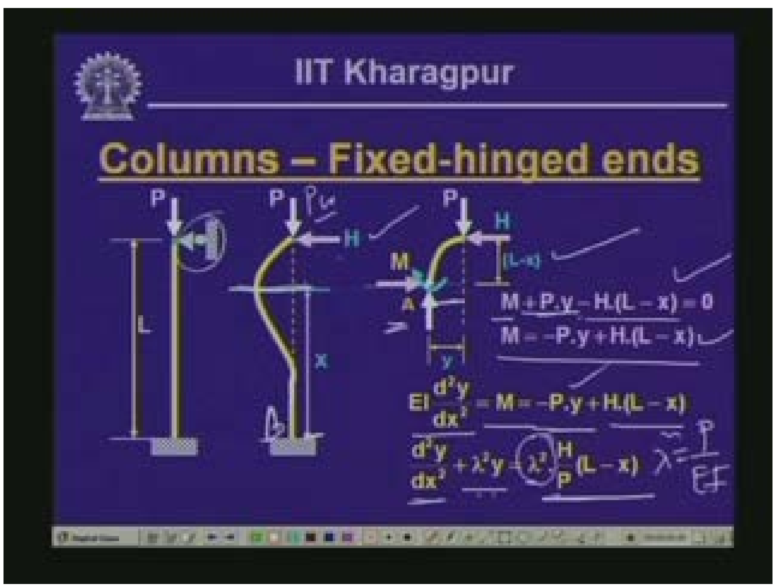

Now if we write down the solution for this as you have seen in the previous two cases the solution of this will be equals to the complementary solution and the particular solution. The complementary solution is C 1 sin lambda x plus C 2 cos lambda x and a particular solution is H by P into L minus x.

The boundary conditions are that at x equals to 0 and y equals to 0 will lead to C 2 equals to minus H L by p; at x equals to 0 dy dx equals to 0 because it is a fixed end and that gives us the value of C 1 equals to H by P lambda and these if we substitute the value of C 1 and C 2 we get the expression for y as this.

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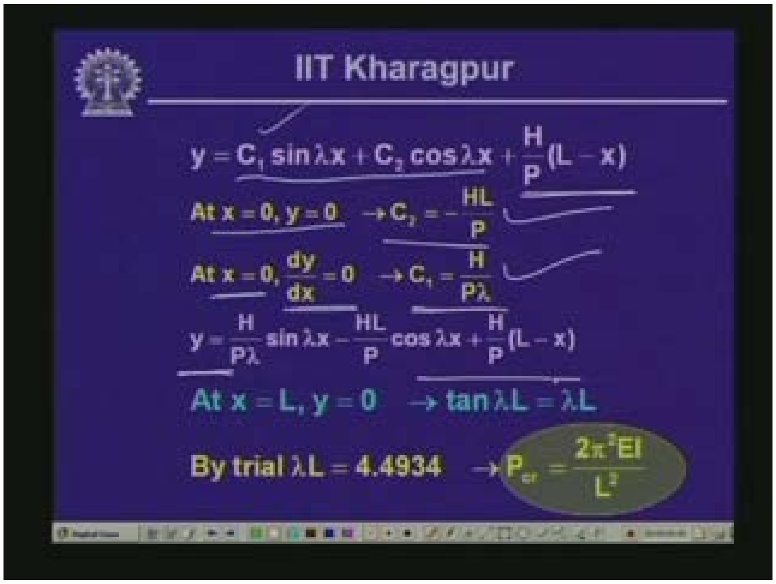

Also, if you look in this at x equal to L (Refer Slide Time: 50:24) this is the fixed end and this is the hinged end so you have a deformation in this or at x equals to L you will have to satisfy that y is equals to zero so if we do that if we substitute that at x equals to l and y equals to then from this expression we get this form: tan lambda L is equals to lambda L x equals to L means this term goes off, you have cos lambda L, this is sin lambda L, this is equals to 0 so that gives us a value of tan lambda L equals to lambda L.

Now this particular equation (Refer Slide Time: 50:51) is solved by trial and we get a value of lambda L equals to 4.4934 and thereby when you square this up lambda square L square is equals to this square which is equals to twice pi square so the minimum value of load P will be equals to 2 pi square EI over L square. Thus, when a member a column member is fixed at one end hinged at the other the critical load is 2 pi square EI over L square.

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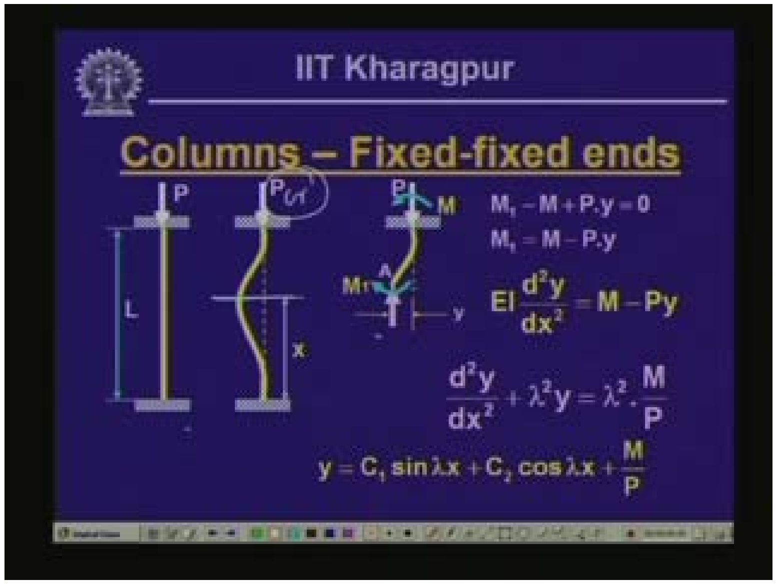

Now let us look into the case where we have a member which is fixed at both ends and what will be the critical load corresponding to that. Again if you take a free body of this particular member at a distance of x then the forces are like this: you have the bending moment, you have the resistive force reactive force and at this support you have the moment M. Now if you take the moment of all the forces with respect to A then we have M 1 minus M plus P times y equals to 0. So M 1 therefore equals to M minus Py where M is the support moment of this particular column.

Now EI d 2 y dx 2 equals to moment which is equals to M minus Py is substituted here and thereby you get again taking Py on the other side we have d 2 y dx 2 plus lambda square y is equals to lambda square times M by P. And again it will have the solution the complementary solution and the particular solution and thereby the y is equals to C 1 sin lambda x plus C 2 cos lambda x and this is the complementary solution and M by P is the particular solution as you have seen in the previous example. Therefore, this is the expression for y where C 1 and C 2 are unknown constants that are to be evaluated from the boundary conditions.

Now what are the boundary conditions you have here?

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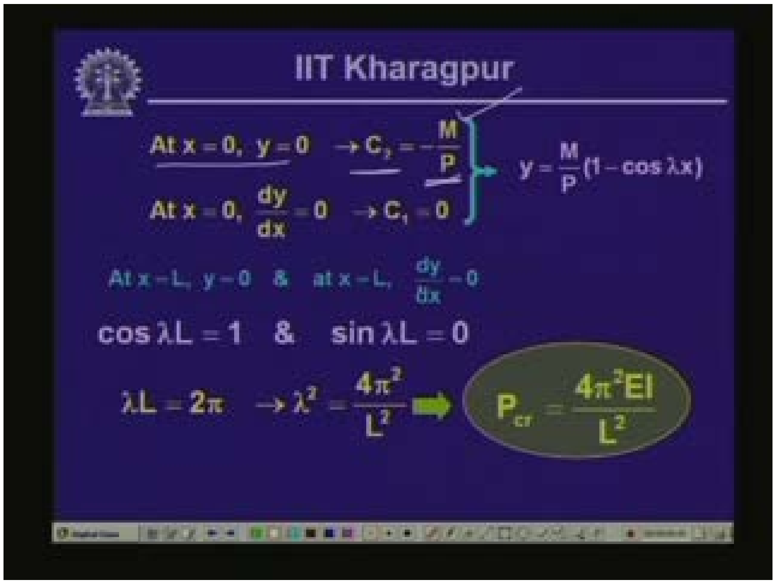


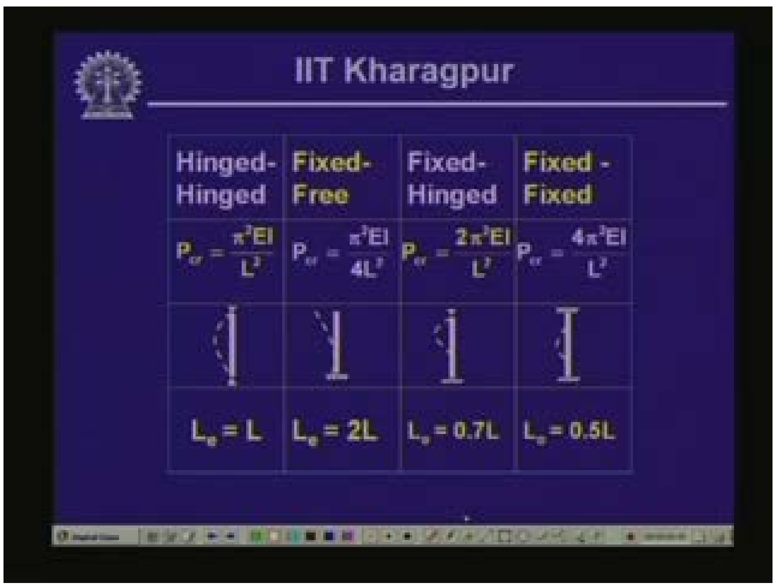
The boundary conditions are that at x equals to 0 y is 0 and that gives us C 2 is equals to minus M by P and at x equals to 0 since it is fixed it is dy dx equals to 0 that gives us a value of C 1 as equals to 0 and thereby we have y is equals to M by P into 1 minus cos lambda x; this is the expression for the elastic curve which is y equals to M by P into 1 minus cos lambda x.

Also, you will have to satisfy that since the member is fixed at both ends so at the top end where x is equals to L, there both y is equals to 0 and dy dx is equals to 0. Now at x equals to L y equals to 0 it gives us cos lambda L is equals to 1 and at x equals to L if we take dy dx equals to 0 that gives us sin lambda L is equals to 0. So we have to satisfy both the criteria that cos lambda L should be equals to 1 and sin lambda L should be equals to 0. Now this can be satisfied only if lambda L is equals to 2pi or multiples of 2pi.

Therefore, the minimum value corresponding to this will be 2pi and if we square it up we will have lambda square equals to 4 pi square by L square and since lambda square is equals to P by EI so P is equals to 4 pi square EI square over L square thereby critical load corresponding to the minimum value which is 2pi is equals to 4 pi square EI over L square. So you see that we could evaluate the critical load for different members or for the column members with different support conditions.

First we have evaluated corresponding to this where both the ends were hinged; subsequently we have seen that one end is fixed and the other end is free, subsequently we have seen that one end is fixed and the other end is hinged and now that we have seen that both the ends are fixed so in such cases what are the values of the critical load. And as you have seen that in all the cases the values of the critical load as we have obtained, they are the functions of the member properties like the I and L and you have the material property which is e. The L is having some coefficient factor which we can evaluate now.

(Refer Slide Time: 55:04 - 56:24)



Now if you look into in general for these cases you see for the hinged hinged we had P critical is equals to pi square EI over L square but the deformed state was in this particular form. Now if we try to write down the critical load for all the cases in terms of this pi square EI over L square corresponding to this case where your support conditions are fixed and free you have P critical is equals to pi square EI by 4L square.

Now let me call that P critical is equals to pi square EI over L e square. Let me call this L e as length equivalent. So when it is L square L e equals to L and when it is 4L square L e is basically twice L. So that is what is indicated over here (Refer Slide Time: 55:56).

In this particular case it pi square EI over L by root 2 square and 1 by root 2 is 0.7 thereby equivalent length here is 0.7L and in this particular case as you can see that this is pi square EI by L square by 2 square so L e is equals to 0.5L. These are the four cases as we have determined and correspondingly the effective lengths are given by this particular row.

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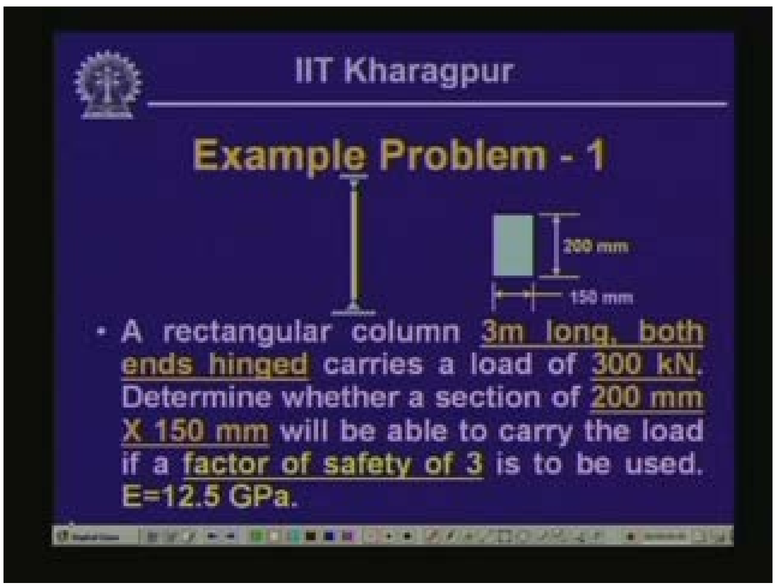
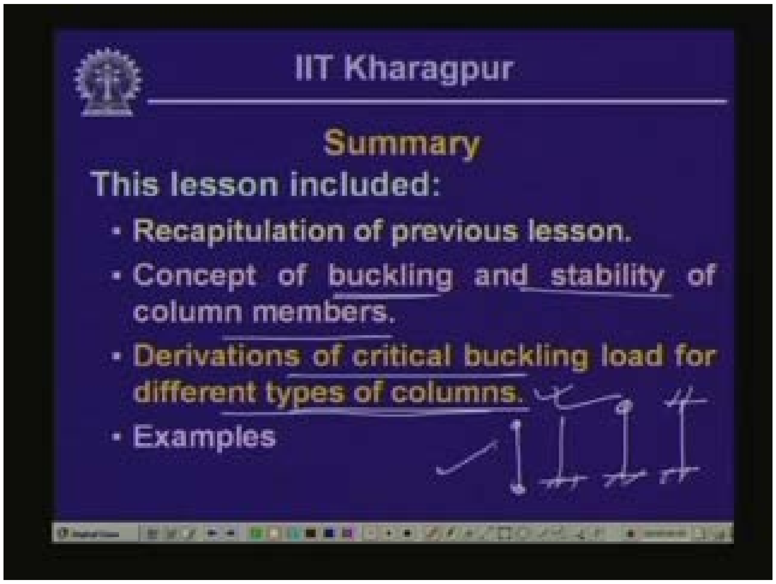


Well, we have an example for you which is a rectangular column 3m long which is hinged at both ends, carries a load of 300 kilonewton. You will have to determine whether a section of 200 mm into 150 mm which is rectangular one will be able carry this load if we introduce a factor of safety of 3 and the value of E is given as 12.5 GPa. Now this problem is assigned to you. Look into this. We will discuss in the next lesson.

(Refer to slide time: 56:59 - 57:45)



Hence to summarize in this particular lesson we have looked into some aspects of the previous lesson. Also, we have introduced the concept of the buckling and stability of different types of column members. We have derived the critical buckling load for different types of columns; different types of columns in the sense we have taken the columns with the different n conditions either they are hinged ends or you have fixed and free or you have fixed and hinged or you have fixed at both ends. These are the possible cases that we can have and corresponding to this we have derived the critical buckling load. Well, we have given one example and some more examples we will be discussing in the subsequent lesson.

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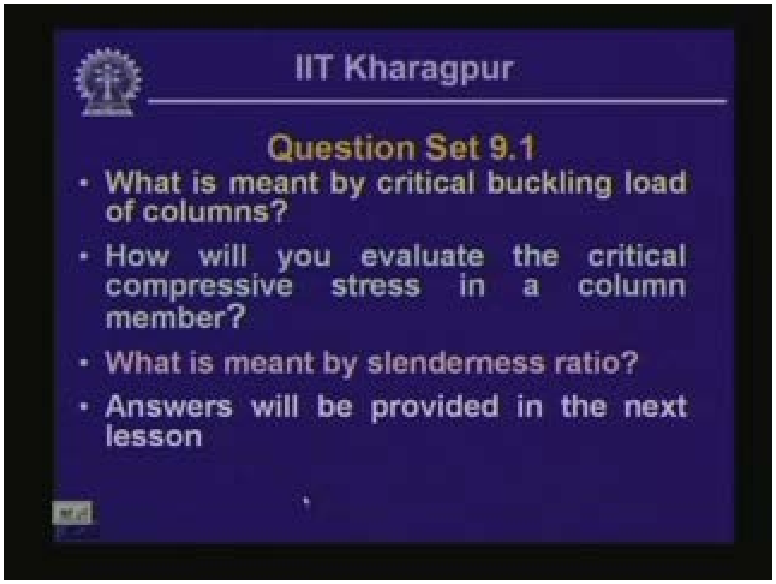


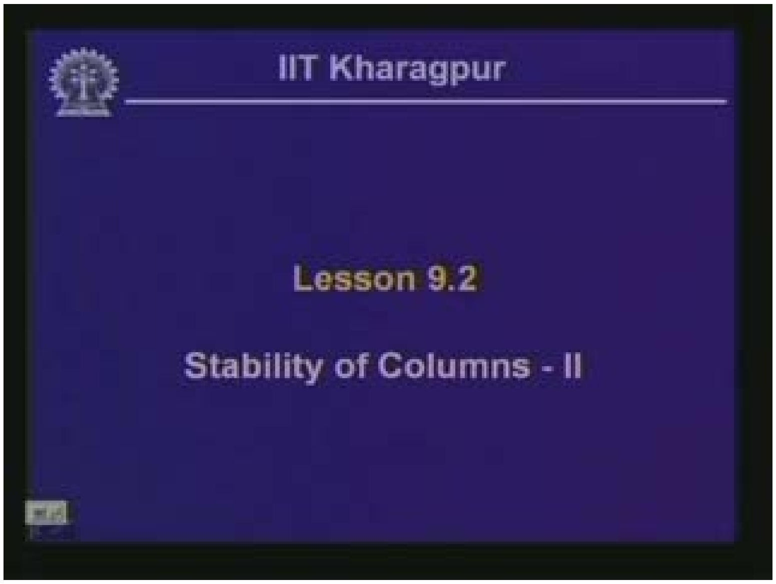
And these are the questions set for you that what is meant by critical buckling load of columns? How will you evaluate the critical compressive stress in a column member? And what is meant by slenderness ratio?

Look into these questions. Some of the questions you will be in a position to answer from the lesson we have discussed and we are going to give you the answer for this in the next lecture, thank you.

Next lecture preview

Stability of Columns – II

(Refer Slide Time: 58:29)



Welcome to the second lesson of the ninth module which is on stability of columns part II in fact in the last lesson we have introduced the concept of the buckling in a member vertical member which is subjected to a compressive force which we have termed as columns and also we have looked into the stability aspects of different types of column members. we will be introduced to............ which was proposed by Euler which we normally call as Euler's buckling load formula. now this particular lesson we are going to look into the aspects that where Euler load can be applied or in other words what are the limitations of Euler's critical buckling load.................................