Strength of Materials

Prof. S. K. Bhattacharyya

Department of Civil Engineering

Indian Institute of Technology, Kharagpur

Lecture – 36

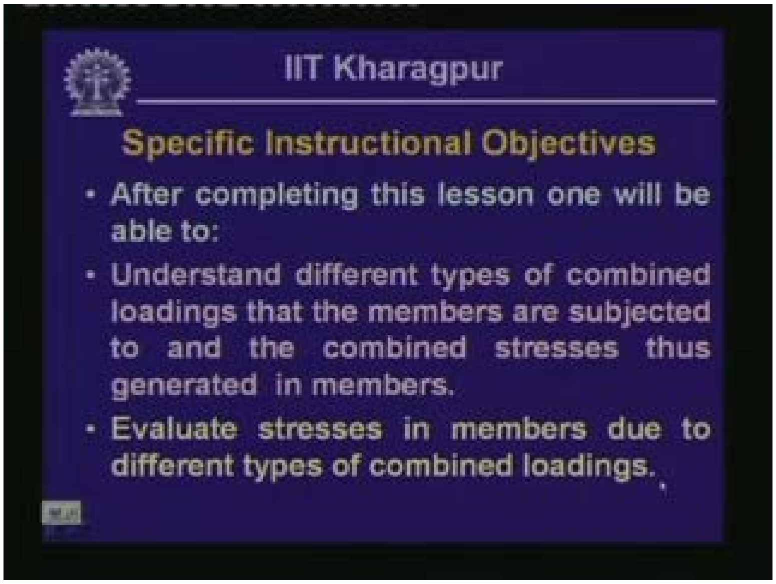
Combined Stresses - III

Welcome to the third lesson of the eighth module which is on combined stresses part III. Now the last two lessons of this particular module you have looked into that we have discussed several aspects of the combined loadings and thereby we have evaluated the combined stresses in members when they are subjected to different forms of combined loadings.

Now we have discussed that if a member is subjected to axial load and bending then what happens to the combined stresses; or if a member is subjected to twisting movement and normal axial force then what happens to the stresses; or if a member is subjected to the combined loading actions of the twisting moment and the bending moment or the shear force then what happens to the combined stresses. Those aspects we have looked into.

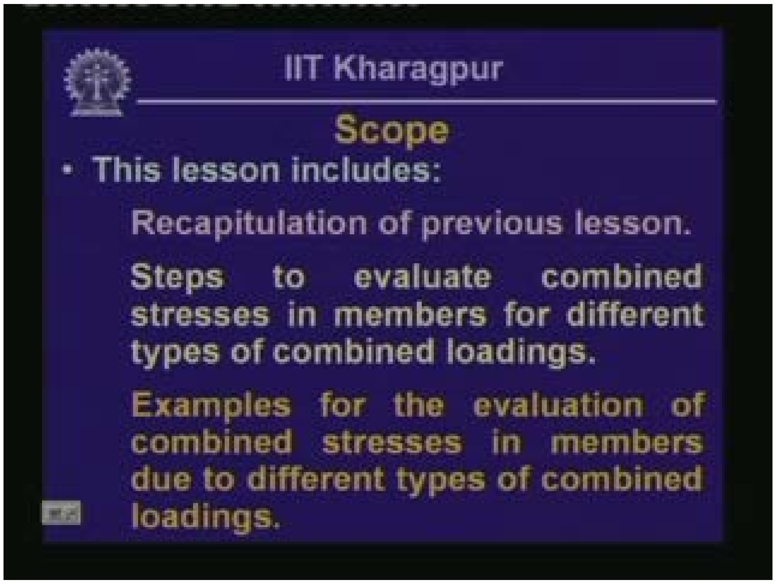
Now, in this particular lesson we are going to look into some more aspects of combined loadings were if a pressure vessel which we have earlier analyzed for the pressures only, now if they are subjected to the external forces like the axial pull or the compressive force or if they are subjected to twisting moment or if the whole vessel a cylindrical vessel is supported on to supports and thereby some bending is induced into the member, then in addition to the stresses that is being induced because of the pressure inside what happens to when they are subjected to these external loads as well. So we are going to look into those aspects in this particular lesson.

(Refer Slide Time: 02:27 – 03:03)



Hence it is expected that once someone goes through this particular lesson one should be in a position to understand different types of combined loadings that the members are subjected to and when we are talking about the different types of loadings different members are subjected to, it includes, of course the aspects whatever we discussed in the previous lessons as well as the aspects which will be looking into this particular lesson and thereby we should be in a position to evaluate the combined stresses generated in members. And also one should be in a position to evaluate stresses in members due to different types of combined loadings.

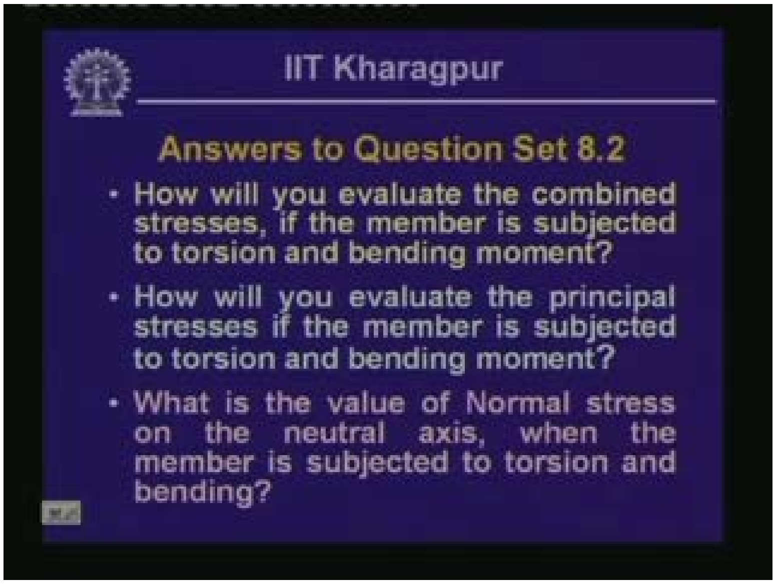
(Refer Slide Time: 03:03 – 04:09)



The scope of this particular lesson therefore includes......... we will be looking into some aspects of the previous lesson which we call as the recapitulation of the previous lesson part, then it includes the steps to evaluate combined stresses in members for different types of combined loadings. now we are in a position to more or less summarize that due to different kinds of combination of the loadings the individual loading we have seen in the previous modules, now, if a particular member is subjected to different kinds of load combinations then what are the stresses we have looked into in the last two lessons and in this particular lesson we will be looking into some more examples with particular reference to the pressure vessels and then we will be in a position to list out that what are the steps you will have to go through to evaluate the combined actions of combined loading actions in a member and thereby evaluate the combined stresses.

Also, we will be looking into some examples for the evaluation of the combined stresses in members due to different types of combined loadings.

(Refer Slide Time: 04:09 – 05:23)



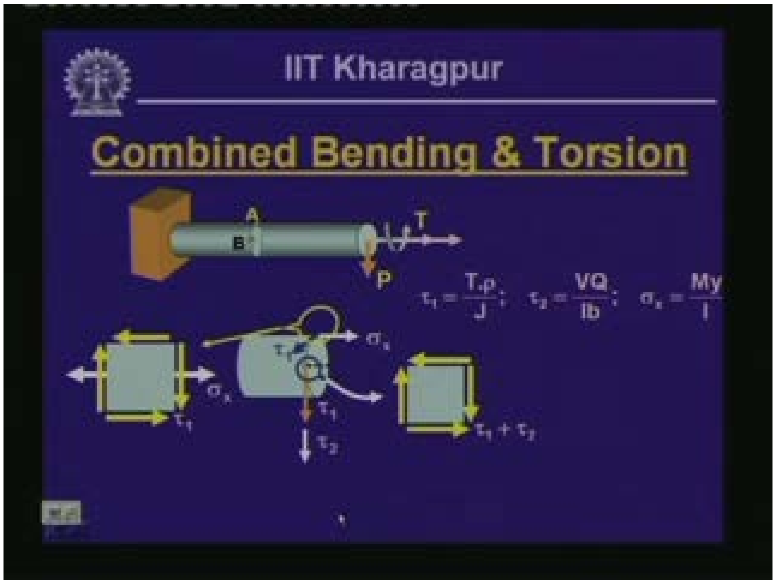
Well, before we go into the aspects of the pressure vessels let us look into the questions; the answers to the questions which I had posed last time; the first question was how will you evaluate the combined stresses if the member is subjected to torsion and bending moment together.

Now, in earlier cases we have seen that the member is subjected to the axial force and the bending. And in the last lesson we have discussed some aspects of the member subjected to twisting moment and the bending moment. Now the question is that how you are going to evaluate the combined stress in a member if the member is subjected to the combined actions of the twisting moment and the bending moment.

And in fact, let me discuss the other questions the second question also along with this. The second question reads as: how will you evaluate the principal stresses in the member if the member is subjected to torsion and bending. In both the cases it is subjected to torsion and bending. The first question is how you are going to evaluate the stresses the combined stresses and the second question is how you are going to evaluate the principal stresses.

Maybe I can answer these two questions together through this example.

(Refer Slide Time: 05:23 – 09:21)



Now if you look into this particular example which we have discussed last time that if a member of circular cross section is subjected to a twisting moment and this is a positive twisting moment whose vectorial direction is towards the positive x axis and it is subjected to a load, this is the cantilever beam (Refer Slide Time: 5:47) and this load generates moment at this particular section. So this particular member the cantilever beam is subjected to the action of the twisting moment T and a bending moment M at the section where this A and B is situated and also because of this particular loading situation this particular cross section will be subjected to a shear force B.

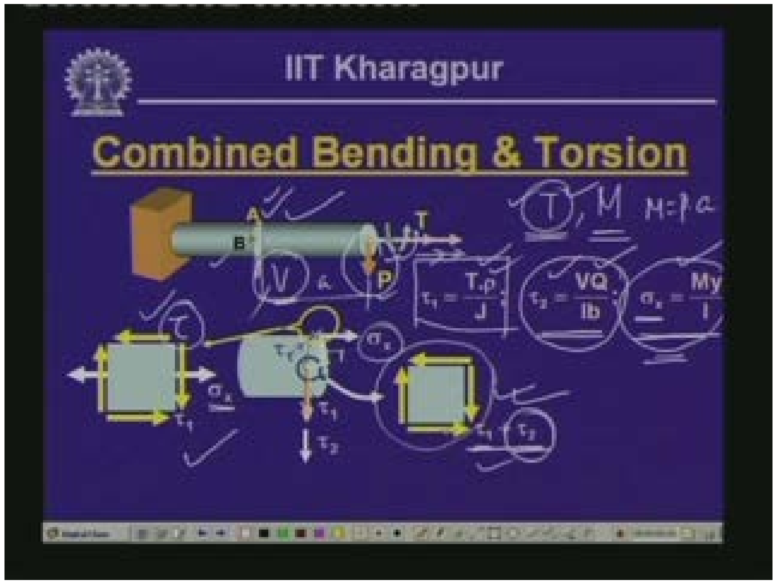
Now the stresses if we look into at this particular section at point A and B, because of the actions of these individual forces, the twisting moment and the bending moment and the shear forces; the stresses that are generated because of the twisting moment T is the shear stress which is given by T rho by J and the direction of the shear stress as it is shown over here because of this positive twisting moment we have the shear stress acting in this particular direction and at point B it is acting in the vertically downward direction so these are the directions of shearing stress at point A and B. This is the contribution of the twisting moment T in this particular member and the bending moment which is getting generated because of this load P, M is equals to P times A if we call this distance as A from the load point (Refer Slide Time: 7:03) then the stress is equals to My by I, now this bending stress is going to produce the normal stress and at the top point from the neutral axis if we take this radius as R then the normal stress there is sigma x.

Now point B in the neutral axis as you know the bending stress is not going to produce any stress at the neutral axis and therefore the normal stress at point B will be equals to 0. Now in this particular case since we have the load P which is contributing to the shear force V. Therefore we will have a component which is getting generated because of the shear force and the shearing stress as you know is equals to VQ by Ib and the top of this particular member will not be subjected to any shear stress because of the shear force but we will have the maximum shear stress at the neutral axis level which will be occurring at B.

Therefore, if we look into the resulting normal and the shearing stress that are occurring at A and B they will be like this that at point A we will have normal stress sigma x which is getting generated because of bending moment M, we will have the shearing stress tau which is getting generated because of the twisting moment but this point (Refer Slide Time: 8:25) will not experience any shear stress because of the vertical shear force V and the point B will have these kinds of stresses.

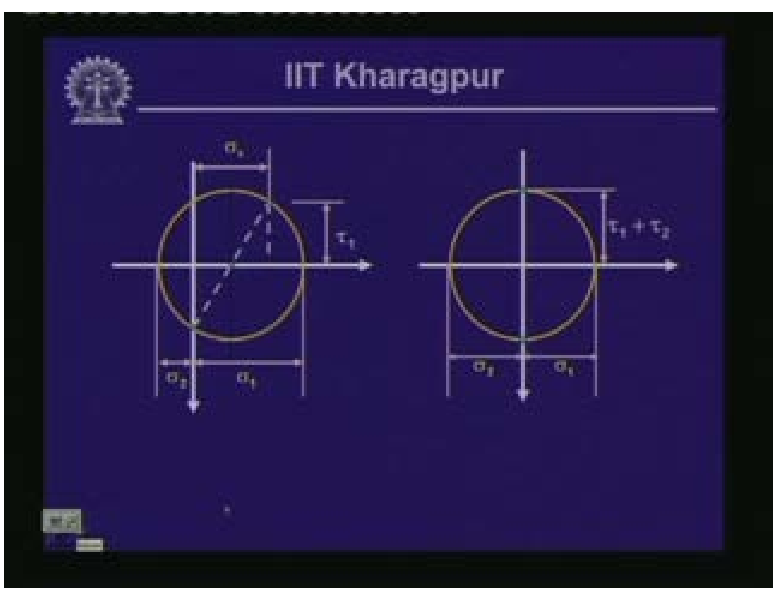
Here there are no normal stresses because it is lying on the neutral axis and we have only bending which is contributing to the normal stress therefore this particular point does not have any normal stress. But there will be shearing stress because of the twisting moment T and also you will have shearing stress which is getting generated because of the shear force B; thereby we will have a resulting shearing stress which is tau 1 and tau 2 at this particular point. So this particular point will be in a state of pure shear where we do not have any normal stress.

(Refer Slide Time: 9:07)



So these are the combined actions of loading because of the torsion and bending that is acting in the member where we get these kinds of stresses. Now the question is that if we like to find out the principal stresses; now as we have done in the past what we need to do is we need to plot these stresses in Mohr's circle.

(Refer Slide Time: 09:21 – 12:08)



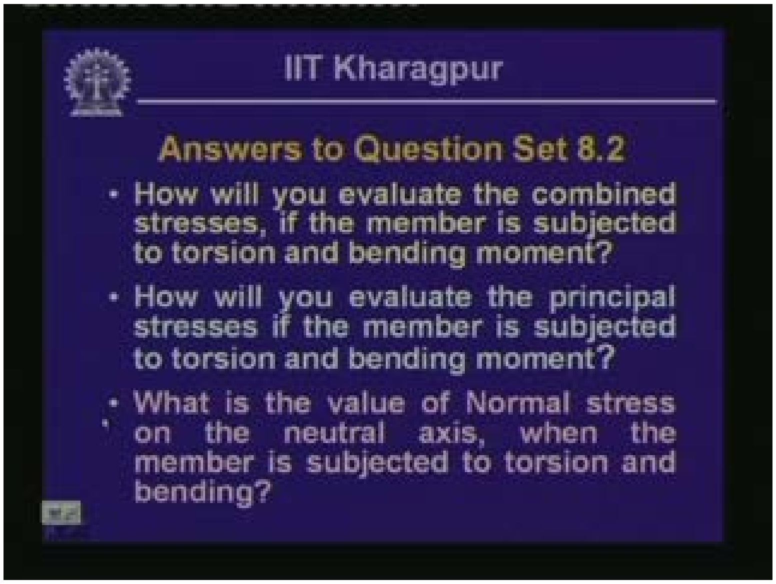
Now as you know that this is the positive sigma axis and this is the positive tau axis downwards. Now, for the first case if we plot the positive sigma x and negative tau this is the point which represents that plane and since we have sigma y as zero and tau as positive so this is the point (Refer Slide time: 9:47) which is representing the other plane.

Now if we join these two points together it cuts the sigma axis at this point which is the center of the Mohr's circle and with this O as the center and OA as the radius if we draw the circle, the maximum normal stress which we get is this particular point and this is the distance from the origin which is sigma 1 we call this as the maximum principal stress and the other point in the Mohr's circle in the diametrical direction on the sigma axis is the minimum normal stress which is equal to sigma 2.

Now this is the state of stress at point A of the previous figure and this is the state of stress at point B of the previous figure where we do not have any normal stress but we have the shearing stress. So, on the right hand side plane you do not have any normal stress the normal stress is 0 but we have the shearing stress. Now this shearing stress will have the value over here as zero normal stress and the shearing stress tau 1 plus tau 2; and on the other side again you have zero normal stress and positive tau 1 plus tau 2 in the perpendicular plane and once you join this it becomes the center of the Mohr's circle thereby the maximum normal which we get corresponding to that point B this is equals to sigma 1 and as you have seen earlier these value of sigma 1 is equals to tau 1 plus tau 2 the radius of the circle and the minimum normal stress of the minimum principal stress which you have is equals to sigma 2 and sigma 2 also is equals to tau 1 plus tau 2.

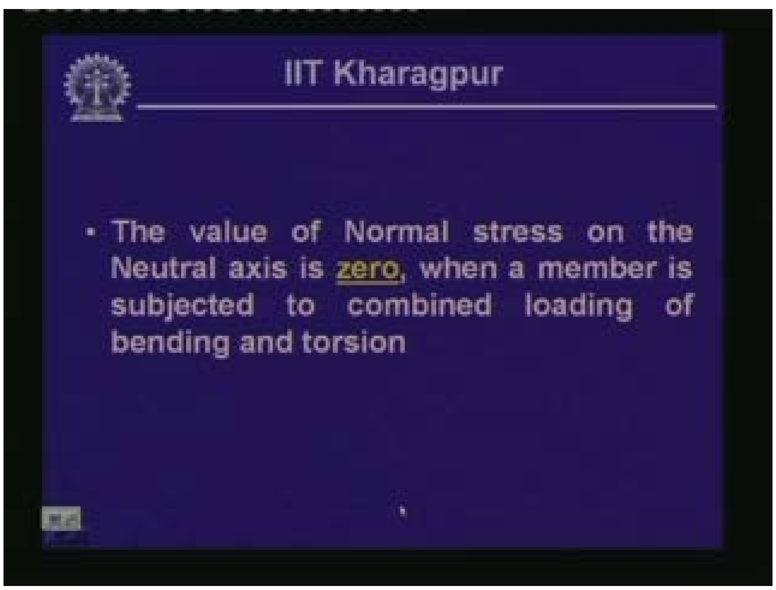
So you see you can absorb now that the member which is subjected to the combined actions of the bending and the twisting moment, I mean the twisting moment and the bending because of the load will be subjected to the combined stresses and we can compute the value of the principal stresses using the Mohr's circle which we had discussed in module 1 where we had that if you have the biaxial state of stress how to compute the principal stresses or stresses at any inclined plane with reference to the x axis at that particular point.

(Refer Slide Time: 12:09 – 12:17)



And the third question was that what will be the value of the normal stress on the neutral axis when the member is subjected to torsion and bending.

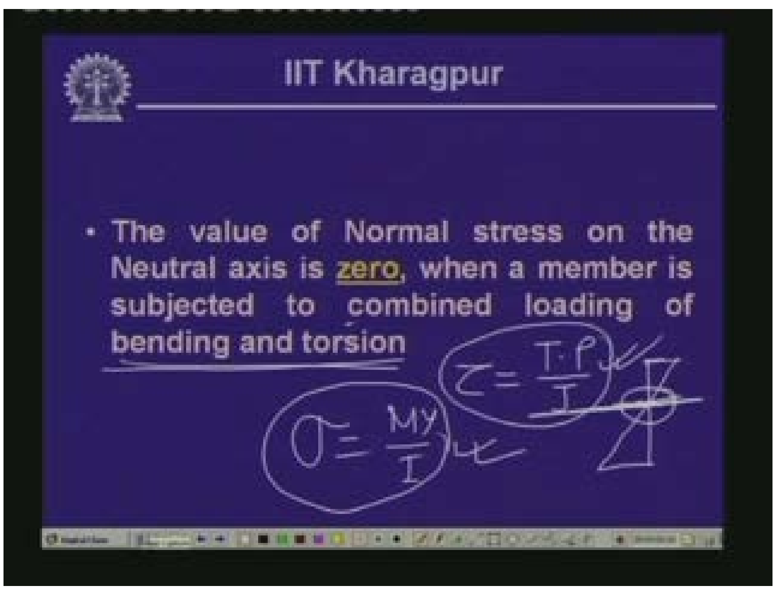
(Refer Slide Time: 12:17 – 13:22)



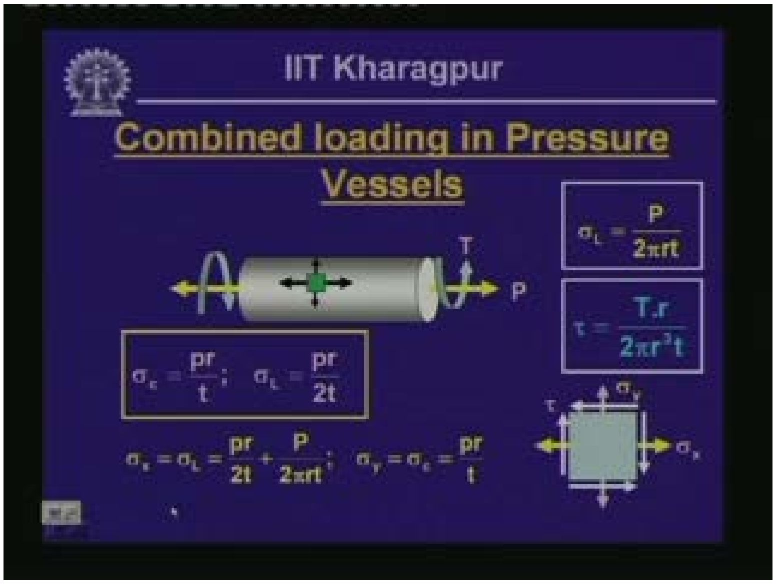
When it is subjected to torsion and bending what will be the value of the normal stress along the neutral axis. When the member is subjected to torsion and bending as you can see that torsion is producing shear stress which is T rho by J and bending is producing normal stress which is My by I. Now as you have noticed that bending stress in the cross section produces a linear distribution of the stress and on the neutral axis the stress is 0.

Now, since you do not have any other normal forces acting in the member thereby when a member will be subjected to combined action of bending and torsion the normal stress along the neutral axis will be equals to 0. So if we choose a point to evaluate the stress which is lying on the neutral axis then it will be subjected to shear stress only and that will be in a state of pure shear and it will not have any normal stress at that particular point when you are evaluating the combined effect of the stresses.

(Refer Slide Time: 13:20)



(Refer Slide Time: 13:22 – 18:46)



Well, then have looked into these answers. Let us look into the actions of the combined loading in a pressure vessel. In the previous module or in module 3 where we have discussed the effect of pressure in a cylindrical or spherical pressure vessels which is a thin wall pressure vessel how to compute the stresses exclusively because of the internal pressure. And as we have seen that because the wall is a thin wall and because of the pressure the stresses which we get is on the wall and they are in the circumferential direction and in the longitudinal direction; the stress in the circumferential direction we have called that as Hoop's stress and you have the longitudinal stress.

Now, if such pressure vessels which are subjected to internal pressure because of the content liquid if they are subjected to some kind external loading as well as it is indicated over here (Refer Slide Time: 14:24) the pressure vessel is subjected to a tensile pull P and also a twisting moment T externally then what will be the consequence of the stress say at this particular point A.

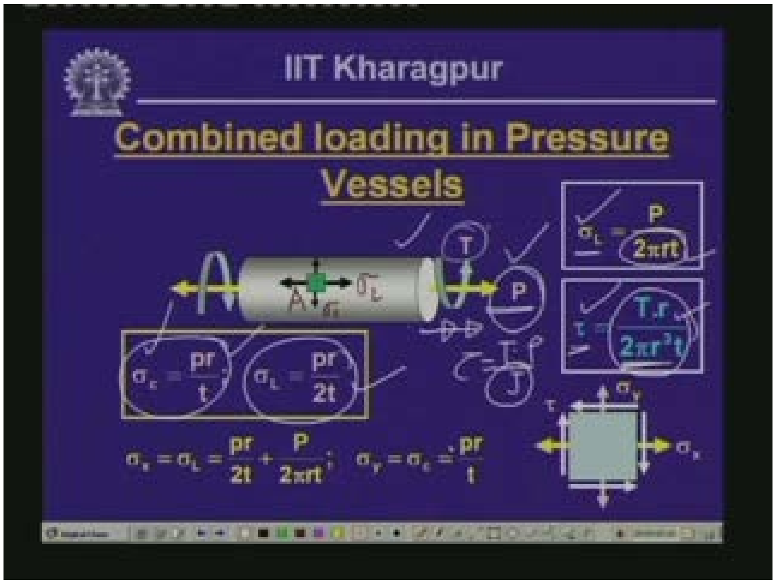
If we are interested to find out the stress at this particular point A, because of these combined actions of the loading what will be the state of stress? Now, as we have seen that when we have the internal pressure if we look into the effect of the loadings individually what is the contribution of this individual loading in the stresses then because of the internal pressure inside the vessel we will have the circumferential stress or the Hoop's stress as pr by T this we had seen earlier.

Also, the longitudinal stress is equals to pr by twice T which is indicated over here so this will be sigma longitudinal and this is sigma circumferential. These are the contributions from the internal pressure. Now let us look into that what will be the state of stress because of the axial pull P? When the vessel is being pulled by an external tensile pull then every cross section of this particular vessel will be subjected to a stress and this which have designated as normal stress and that normal stress is nothing but equals to this tensile pull divided by the cross-sectional area.

Now this being a thin-walled pressure vessel the cross section area as we had seen is equal to twice pi rt and the longitudinal stress contribution because of the axial pull is equal to P divided by twice pi rt. So this is the normal stress that is getting generated because of the axial pull P.

Now this particular member (Refer Slide Time: 16:18) also is subjected to a twisting moment. Now here whatever we have indicated is a positive twisting moment, the vectorial direction which is in the positive x direction. This particular element when they are subjected to this twisting moment on this surface it will be subjected to the action of shearing stress which is given by T rho by J; tau as you know is equals to T rho by J and J for this thin-walled cylindrical member is equal to twice pi r cube t and r being the extreme radius of the top point or the point where we are evaluating so tau is equal to tr by twice pi r cube t.

(Refer Slide Time: 16:57)

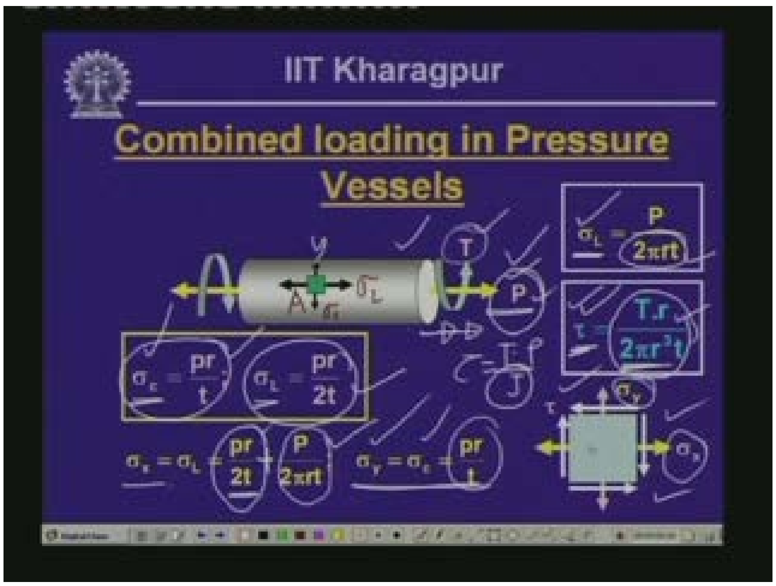


So as you can see now that individually these three loading conditions one is the pressure internally, another one is the axial pull and the third one is the twisting moment that is acting. individually they are generating........... sigma c is giving you the normal stress in the y direction, sigma L will give you the normal stress in the x direction; also, the axial pull is giving you the normal stress in the x direction and the twisting moment is generating the shearing stress.

So the total normal stress that you have in the x direction this is equals to this is the contribution of the pressure part from the internal pressure and this is the contribution of the this is the contribution of the this is the contribution of the external pull (Refer Slide Time: 17:43). Now here this should be; this is plus, this plus this will be the total stress in the x direction and in the y direction we have the circumferential stress which is equals to pr by T that is in the y direction so that is sigma y and in addition to that we have the shearing stress which is indicated over here. So this particular element now is subjected to sigma x, sigma y and tau.

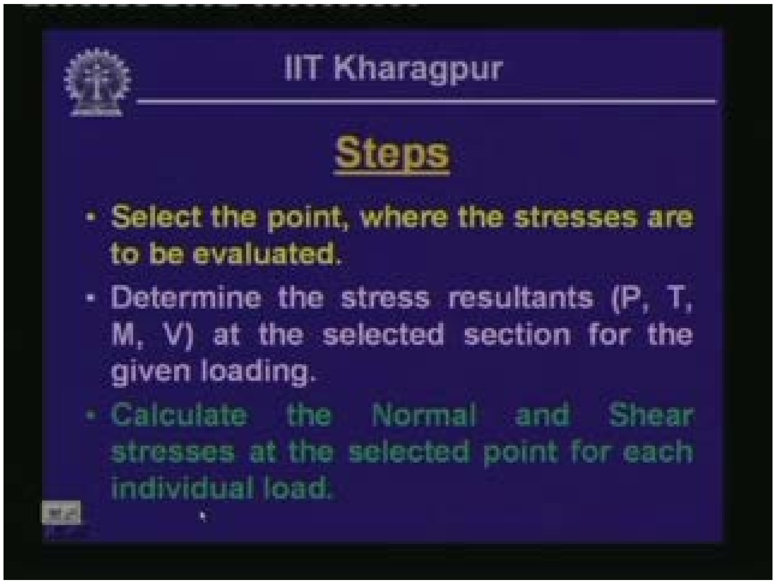
As we had seen earlier in module 1 that if a particular at a particular point in a body if we have the biaxial state of stress along with the shearing stress then how do you compute the resulting principal stresses. We can compute the maximum value of the normal stresses which are given by sigma 1 and sigma 2 maximum and minimum principal stresses and also the maximum value of the shearing stress we can compute from this combined state of stresses.

(Refer Slide Time: 18:43)



Now we can plot these values in the Mohr's circle and thereby we can get the values of the principal stresses.

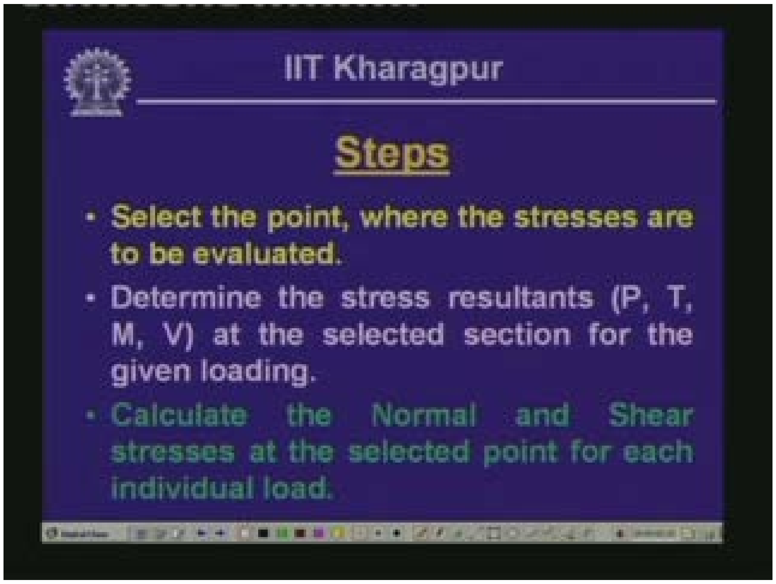
(Refer Slide Time: 18:46 – 22:25)



Now, since we have looked into different kinds of combined loading situation in members; in the previous two lessons we have seen that a member, when it is subjected to axial pull and bending then what is the combined state of stress. Subsequently, in the last lesson we had seen that if the member is subjected to a twisting moment and an axial pull then what is the effect or if it is subjected to the twisting moment and bending then what is the consequence in the combined stresses.

Now that we have seen that if a pressure vessel is subjected to internal pressure along with the external loading then how do you compute the stresses. Now if you look into all these cases more or less they follow a normal guideline or general guideline which can be listed out over here.

(Refer Slide Time: 19:36)



You see, what we are interested to do is that we are interest to evaluate stress at a particular point in a body which is subjected to the combined actions of the loading. So we select a point where we need to evaluate the stresses and this selection of the point generally is selected at a particular point where we expect that stress level for some of the loading actions could be maximum.

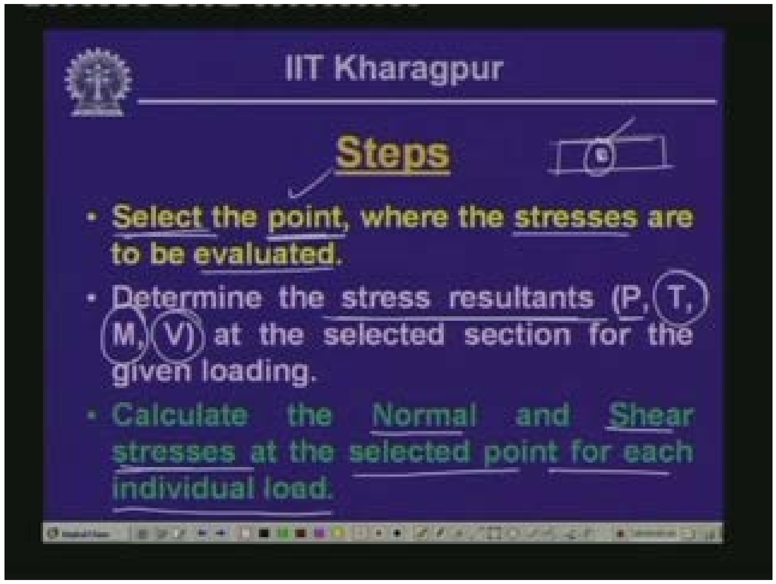
Say for example; if we draw the bending moment diagram of a member which is subjected to bending we know that where the maximum bending moment can occur. So we can choose that particular point where the maximum bending moment occurs and then compute the stresses corresponding to the bending and also we compute the stresses corresponding to the other actions of the loading and then we try to analyze that what will be the consequence of the combined loading action at that particular point.

Likewise we can select the point where we expect that the maximum shear stresses will be generated or where we have the maximum actions of the shear force and then at that particular point what is the consequence of other loading actions and thereby what will be the value of the combined stresses. So likewise we select some points if we know the positions of the maximum loading situations like bending and shear as I was describing we select those points or else we choose some points in the member and then we try to find out that which one gives you the maximum of all these maximum situations and that we treat as the worst situation for the load combinations or the combination of the combined stresses.

Secondly, what you need to do is that at that particular point the point which you select you need to determine the stress resultants. What are the stress resultants? As we have seen what are the axial ports that is acting at that particular section where that point lies or what is the value of the twisting moment at that particular section or what is the value of the bending moment at that particular section or what is the value of the shear force at that particular section or if it is a pressure vessel then what is the state of stress that is getting generated because of the internal pressure within that particular vessel.

So, at that particular point we try to find out first the stress resultant and then for each individual such stress resultant quantity what are the consequence on the stresses. So we calculate the normal and shear stress contribution of each individual load including the stresses that will be generated in the pressure vessel, the normal stresses.

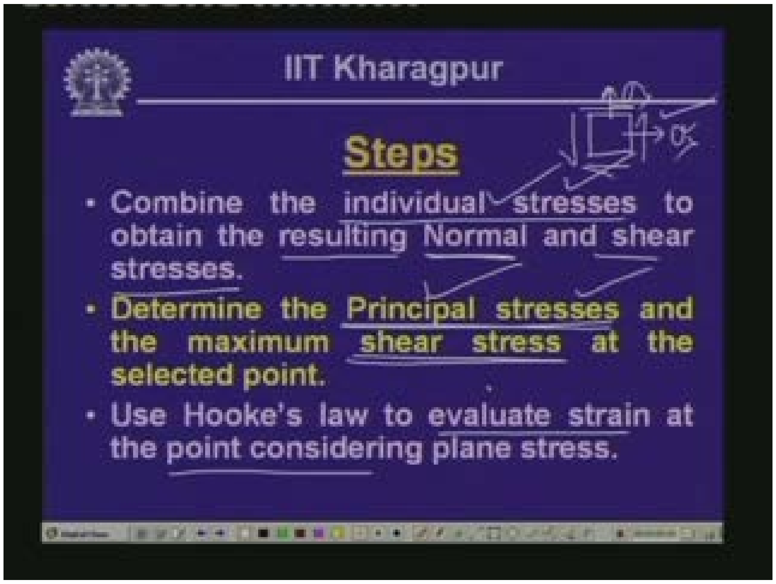
(Refer Slide Time: 22:20)



Once we get the normal and shear stresses for each individual load then what we do is that we try to combine suitably according to their Science; depending on compressive or tensile normal stress you have or the shearing stress you have we algebraically sum them up and arrive at what will be resulting value of the normal and shear stress at a particular point which we have selected to evaluate the stress at that point.

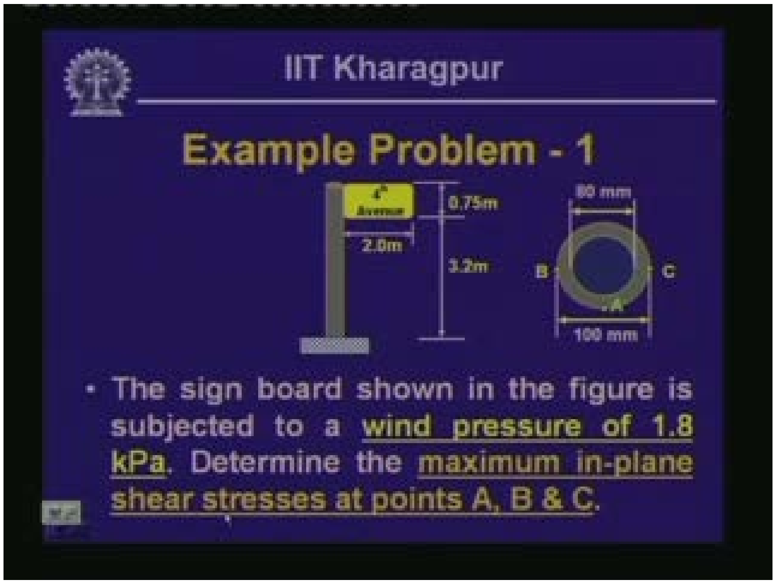
So combine the individual stresses to obtain the resulting normal and shear stresses. This is what is important. Thereby we now arrive at two stresses: one is the normal stress or rather three quantities: sigma x, sigma y and tau xy; sigma x and sigma y are the normal stresses and tau xy is the shear stress. So, at a particular point we arrive at these three quantities corresponding to of course the plain state of stress and then once we get that at a particular point once we know that what is the value of sigma x, what is the value of sigma y and consequently what is the value of the shearing stress then we can plot them in the Mohr's circle or we can use the transformation equation from which we can compute the values of the principal stresses and the maximum shear stress at that particular point and once we know the stresses then we can compute the value of the strain using Hook's law.

(Refer Slide Time: 23:48)



These are the steps more or less which are common for all these situations which you have analyzed so far or which we are going to analyze in this particular lesson. Wherever a member is subjected to such combined loading actions these are the steps we have to follow so that you can arrive at what will be the consequence of this loading at that particular point in terms of the stresses.

(Refer Slide Time: 24:14 – 27:56)



Well, then let us look into some of the examples on these combined loading situations. Now this is one of the examples which I had given to you last time wherein the actions will be in the form of bending and torsion.

Here you see this is a sign board which is connected to a vertical member which is a tubular member and it gives the direction of a particular strip. Here when this particular sign board is subjected to a wind pressure of 1.8 kilo Pascal; now what you need to do is you need to determine the maximum in-plane shear stress at point A, B and C these are the three points selected, this is the cross section of the vertical post (Refer Slide Time: 25:08) and this is let us say at the support point where we expect the maximum bending moment to occur because of the loading.

When the wind load occurs perpendicular to this board then the total load that will be exerted by the wind will be equals to the wind intensity multiplied by the whole area and that is expected to act as the resulting force at the center of gravity of this particular board. Now this cg of the board being eccentric with respect to the center of gravity of this particular vertical member the loading to this member is eccentric with respect to its vertical access. So if we like to shift this particular load to the central axis of this particular member this will be associated with a moment of magnitude; if we call this as load P this load P will be shifted to center P with a moment which is equals to P times e.

And incidentally this particular moment P times e is about an axis which is lying in this plane of the tube and this moment is nothing but a twisting moment of this tubular form. And since this particular vertical member is supported at the base and P at the top it is like a cantilever beam which is subjected to a concentrated load towards the free end and this load is going to cause a bending moment at the base which is equals to P times this vertical distance. And because of this P there will be bending at this level (Refer Slide Time: 26:48), also this cross section will be subjected to the shearing force.

Now, because of the bending there will be normal stress, because of the shearing there will be shearing stress and because of twisting there will be shearing stress. Let us compute now that what will be the result stresses because of these three actions at points A, B and C.

Now, before we go in to the calculation of stresses one point we must note that point A is the top surface of this tubular form and thereby when it is subjected to the shearing action the shear stress because of the vertical shear force will be 0 at point and it will be maximum at the diametrical point BC but the contribution of the shear stress in the twist from the twisting moment will have all the three points subjected to the shearing stress.

When we talk about the normal stress because of the bending, A will be experiencing normal stress but since B and C lies on the neutral axis B and C will not experience any normal stress because of the bending. With this discussion let us look into the calculation of the stresses at points A, B and C.

(Refer Slide Time: 27:56 – 32:15)

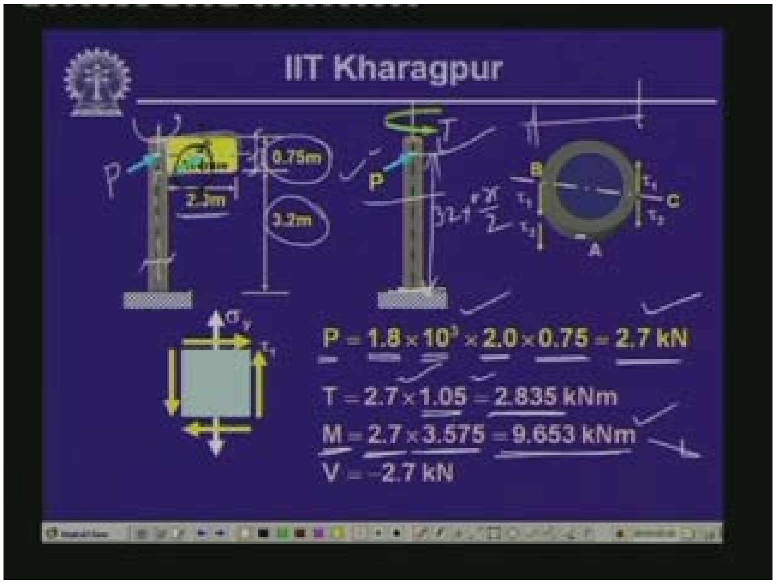
Now you see, the analysis of this particular loading action is indicated over here that you have the resultant load acting at this particular point at the cg of this particular board and this load is transferred to the vertical axis of the member and this is the load P along with a twisting moment that is what is indicated over here. Now this vertical post is subjected to a concentrated load P.

Now what are the distances; now this distance is 2m (Refer Slide Time: 28:35) and the diameter of the tube is 100 mm. So if we shift this particular load to the center of this vertical member it is 1m and this is 50 mm so 1.05 is the distance so the twisting moment T is going to be equal to this load multiplied by the distance 1.05. Now how much is this load P? P is equals to the wind pressure which is acting on the board area so 1.8 kilo Pascal multiplied by 10 to the power 3 so much of Pascal is the load acting on the which is distributed over the entire board and the board area is equals to 2m by 0.75 and that gives us a force of 2.7 kilonewton.

This 2.7 kilonewton force which is acting at the cg of this particular board if we shift it to the central axis of this tube then we have the load which is 2.7 kilonewton along with a twisting moment T which is equals to 2.7 into 1.05 which gives us a value of 2.835 kilonewton meter.

Now the concentrated load P which is acting at a distance of................ now the bottom of the board is at a distance of 3.2m from the support and then the center line of this board lies at half the distance between the two which is 0.75 by 2 so the distance of this particular load from the support is equals to 3.2 plus 0.75 by 2 which is equals to 3.575. So M is equals to then 2.7 into 3.575 which gives you 9.653 kilonewton meter as the moment at the base level.

(Refer Slide Time: 30:35)



And at this level if you take a free body diagram it is like a cantilever beam having a load here; if you take a section here (Refer Slide Time: 30:38) at this section you have a shear force B so B is equals to......... I mean, if you take the free body diagram like this you have the direction of the positive shear as this so V is equals to the minus P and this is what is indicated over here that V is equals to minus 2.7 kilonewton is the shear force you have in the member.

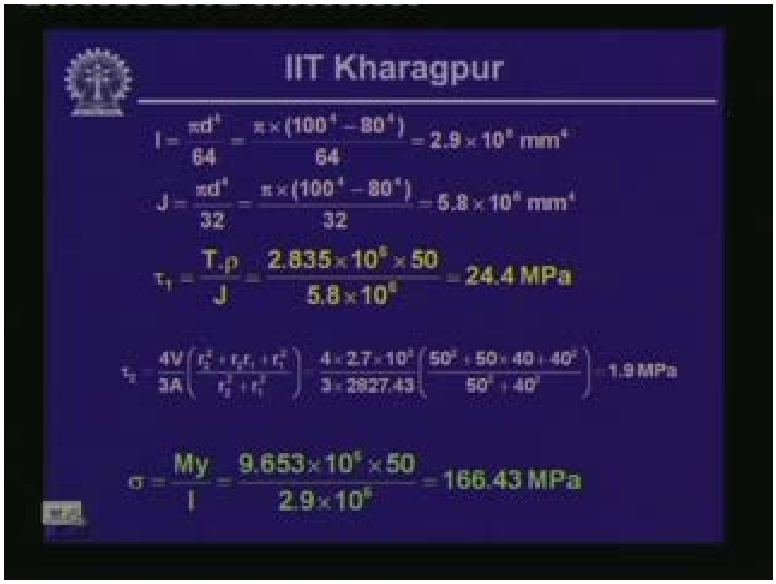
So, at this particular point, then, the three quantities which are acting is the twisting moment T, the bending moment M and the shear force V. And as I have told you that at point A we will have the normal stress because of the bending and we will have the shearing stress because of the twisting moment T which we have called as tau 1; and since this the vertical member the tubular member is a vertical member and the bending is going to cause a stress in the y direction and so we have the normal stress as sigma y but we do not any stress in the x direction; sigma x is 0.

So this particular element a will be subjected to a normal stress sigma y and a shearing stress tau 1 which is getting generated because of the twisting moment T. But point B and C which is lying on the neutral axis will not have any normal stress and we will have the shearing stress tau 1 which is because of the twisting moment T and we will have the shearing stress tau 2 which is because of the shearing force V.

(Refer Slide Time: 32:10)

Thus, if we compute the stresses now because of this moment twisting moment and the shear force then we get these values.

(Refer Slide Time: 32:15 – 34:59)

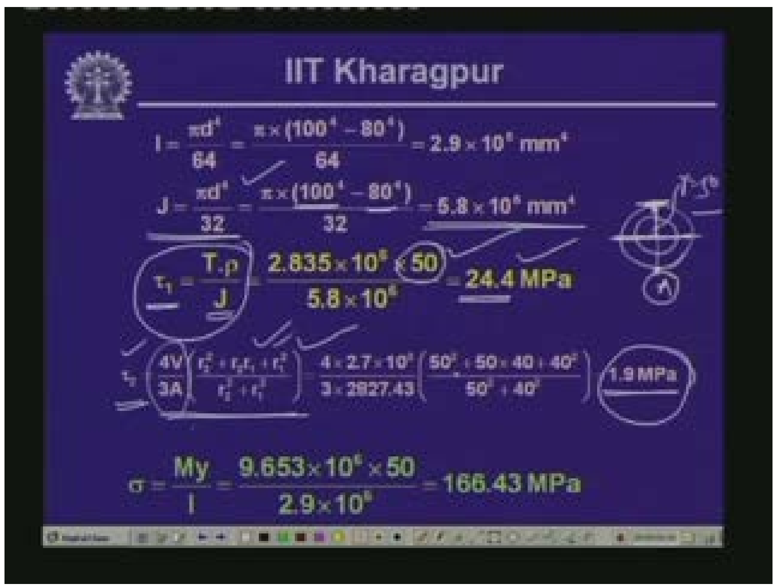


Now here you see that the shearing stress tau 1 is equals to T times rho by J and J is the polar moment of inertia which is given by this: J is equals to pid 4 by 32; for this tubular member this 100 is the outer diameter and 80 is the internal diameter so this gives us the value of 5.8 into 10 to the power 6 mm to the power of 4 and that you substitute here 50 is the radius because the external diameter is 100 so the maximum value where you get the shear stress maximum is r equals to 50 and this is what is used here so this gives you a value of 24.4 MPa; this is the value of the tau 1.

Now the shearing stress tau 2 which is because of the shearing force which will be acting at this level because at this point at point A the contribution of the vertical shear force is 0 because shear stress distribution as we have seen is 0 at the two outer surfaces and you have the maximum value of the shear stress at this neutral axis position and the value of which is given by this when it is a solid circular one we have only 4V by 3A but for a tubular member it gets modified by this expression which we have already seen while evaluating the shear stresses in a member.

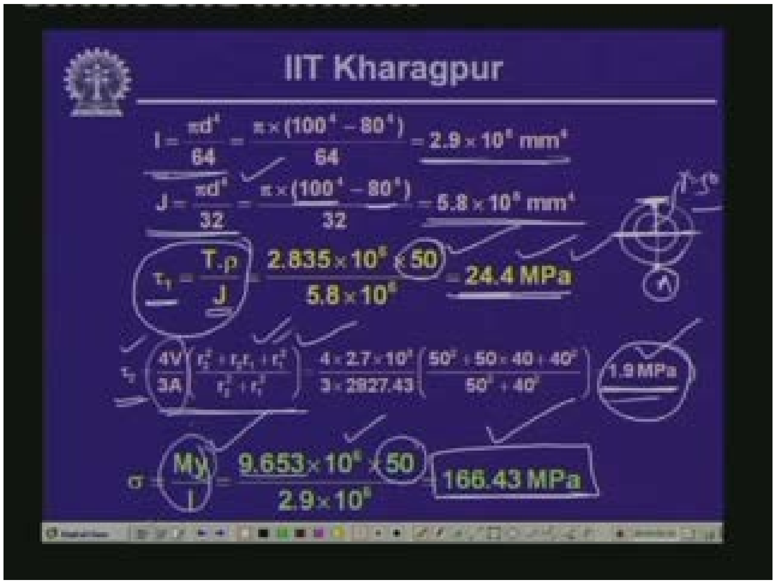
You know, if you remember that when we evaluated the value of the shear stress in a solid circular section or a tubular section we were able to calculate only the diametrical position because at other positions we cannot calculate because the section is not parallel to the y axis. And the value of the maximum shear stress which we had obtained at that particular cross section it was fourth third of shear force divided by the cross-sectional area; and when it is a tubular section it gets modified in terms of those radii and this is the expression for evaluating the shear stress.

(Refer Slide Time: 34:13)

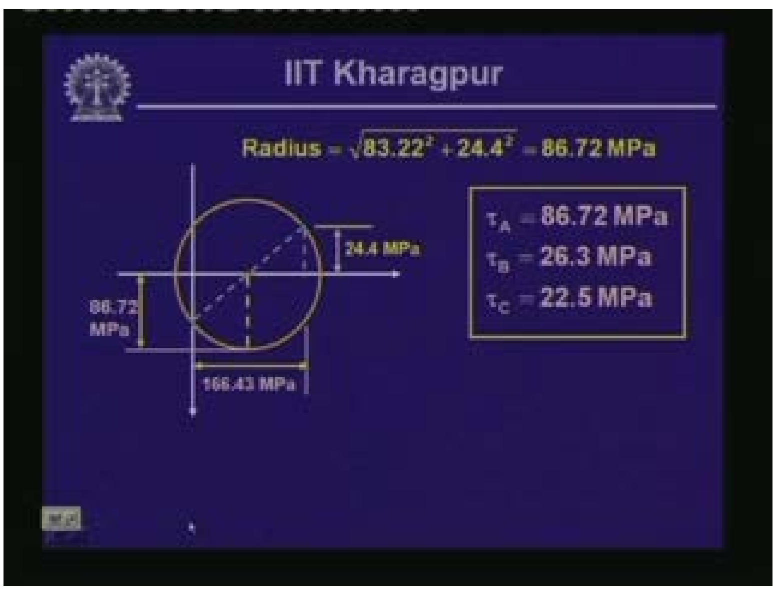


So the shear stress which we get from the shear force is equals to 1.9 MPa and the normal stress which we get because of the bending which is My by I; bending moment as we have seen is 9.653 into 10 to the power of 6 so much of Newton millimeter, 50 is the extreme distance where we are computing the stress and I is the moment of inertia which is pid 4 by 64 and this is 2.9 into 10 to the power 6 mm to the power 4 so that gives us the stress of 166.43 MPa. This is the value of the normal stress, this is the value of the shearing stress which we are getting for the twisting moment and (Refer Slide Time: 34:54) this is the value of the shearing stress which we are getting from the shear force.

(Refer Slide Time: 34:56)



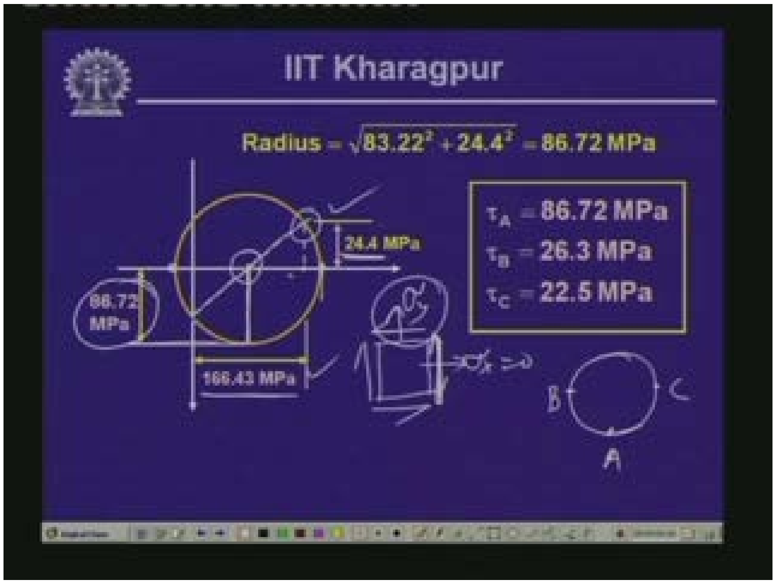
(Refer Slide Time 34:59 – 38:53)



With these stresses now if we try to find out the resulting stress in the member............ because we are interested to find out what will be the values of shearing stress at point A, at point B and at point C; and the resulting shearing stress because of these individual loading actions; what is the individual loading that is acting at that particular cross-sectional support; they are the twisting moment, the bending moment because of the eccentric loading and the shear force because of the loading that is acting on that member.

As you have seen, we have obtained the value of the normal stress at 166.43 and the element on which we have the sigma x is 0; sigma x is 0, sigma y we have as 166.43 and we have the twisting moment I mean the shearing stress which is negative on this, so at this particular point we have a positive shear stress as is plotted over here that we have zero value of the stress and we have sigma y and the negative of the shear stress over here so this 24.4 is the shearing stress that is getting generated because of the twisting moment. So if I join these two points we get this (Refer Slide time: 36:22) as the center of the Mohr's circle and if we draw the circle then these are the two points which gives us the value of maximum normal stresses, the maximum principal stresses as we call it and the radius of the circle gives us the value of the maximum shearing stress.

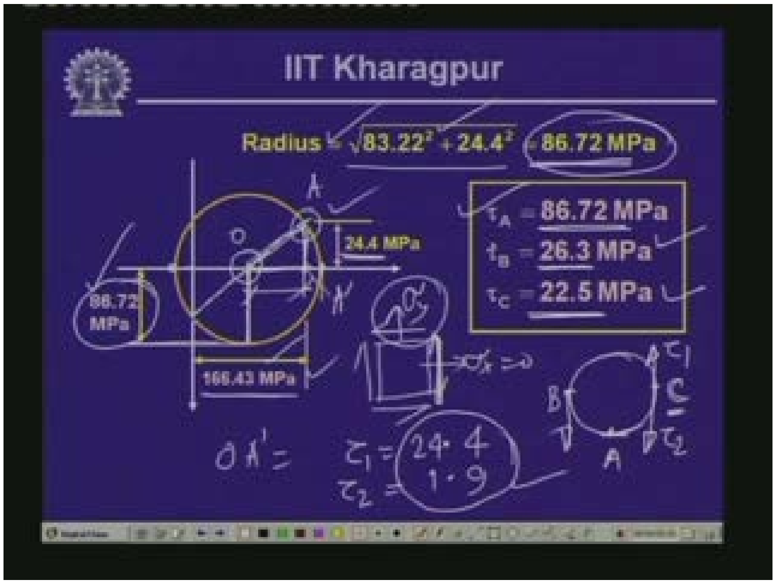
(Refer Slide Time: 36:40)



So, if we compute; now as you know that this distance is 166.43 from here so this particular distance if we call this as O and this as A and this as A dashed then the magnitude of OA dashed will be equals to............ as you know that sigma x plus sigma x minus sigma y by 2 and that is what is the half of the sigma x being 0 so we get half that stress as the normal stress and this is the shearing stress and radius of this will be equals to d square plus d square and this is what is indicated over here the radius equals to 83.22 square plus 24.4square which is equals to 86.72 MPa and that is the radius of this Mohr's circle and that gives us the value of the maximum shearing stress at that particular point.

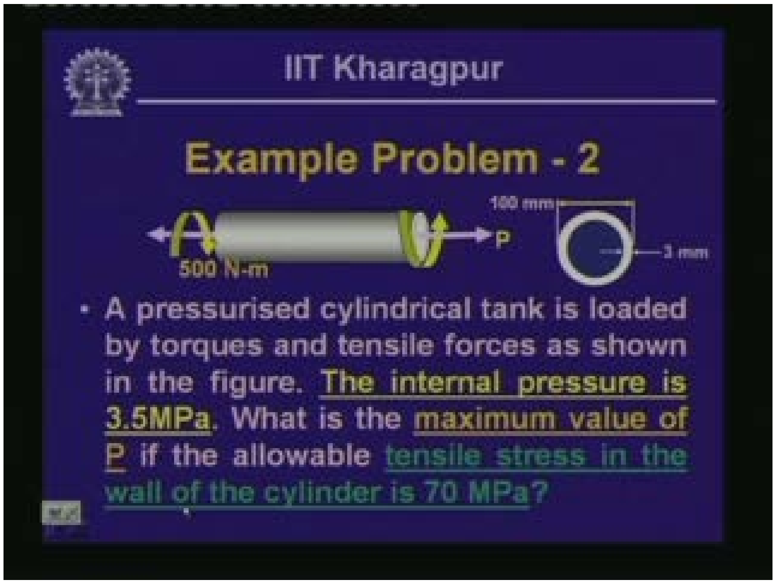
Hence, at point A the resulting shear stress which we had from the loading actions is equals to 86.72 MPa and at point P and point C as you have seen that we have shearing stress from twisting moment as equals to 24.4 and the shearing stress from the shear force we have as 1.9. So if we combine this together tau 1 plus tau 2 is the shearing stress that will be acting at B which is equals to 26.3 MPa and at point C we have the difference of tau 1 and tau 2; tau 1 is acting in this direction and tau 2 is acting in this direction so the resulting shearing stress at C is equals to 24.4 minus 1.9 which is equals to 22.5 MPa.

(Refer Slide Time: 38:22)



Therefore, you see, then these are the values of the shearing stresses tau A, tau B, tau C at three points A, B and C which are getting resulted from the wind load which is acting on the sign board. So you see that when we are using such signs which are eccentric with respect to the vertical post the vertical post is subjected to the actions of the shearing stress and also of course it is subjected to the normal the maximum principle stresses as well as you can from this particular Mohr's circle.

(Refer Slide Time: 38:53 – 41:10)



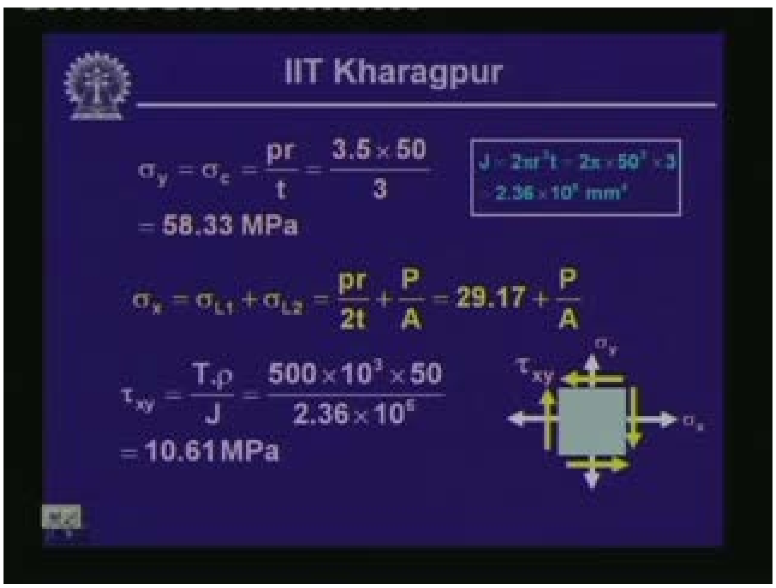
Well, let us look into this particular example which is best on the discussion which we had in this particular lesson where we have discussed the effect of the external loading on a pressurized vessel. Here we have a pressurized cylindrical tank that means it is subjected to the actions of internal pressure, also it is acted on by loads which is a tensile pull and there is a twisting moment.

Now the internal pressure given is 3.5 MPa. What you need to do is that you have to find out the value of maximum value of this load P axial pull P so that the tensile stress in the wall of cylinder does not exceed 70 MPa. So if you have to limit the tensile stress in the wall to 70 MPa then what will be the maximum value of this P and the value of the twisting moment given is 500 Newton meter, the diameter of the tank is 100 mm and the thickness of the wall is 3 mm and the internal pressure that is acting is 3.5 MPa.

As you can see, in this particular vessel we have three loading actions. One is that we have internal pressure and that will give rise to the two stresses as we have seen. They are the circumferential stress and the longitudinal stress because of the internal pressure. Also it is acted on by the axial pull and because of the axial pull at every cross section the member will be subjected to the normal stress and the member is subjected to a twisting moment and because of the twisting moment the wall of the pressure vessel will be subjected to the shearing stress.

So you see there will be resulting normal stress and the normal stresses will be generated from the internal pressure of the liquid and because of the axial pull of the vessel and the shearing stresses will be generated because of the twisting moment. So once we have the resulting normal stress and the shearing stress then we can compute the maximum tensile stress that will be generated in the wall of the vessel.

(Refer Slide Time: 41:10 – 44:35)

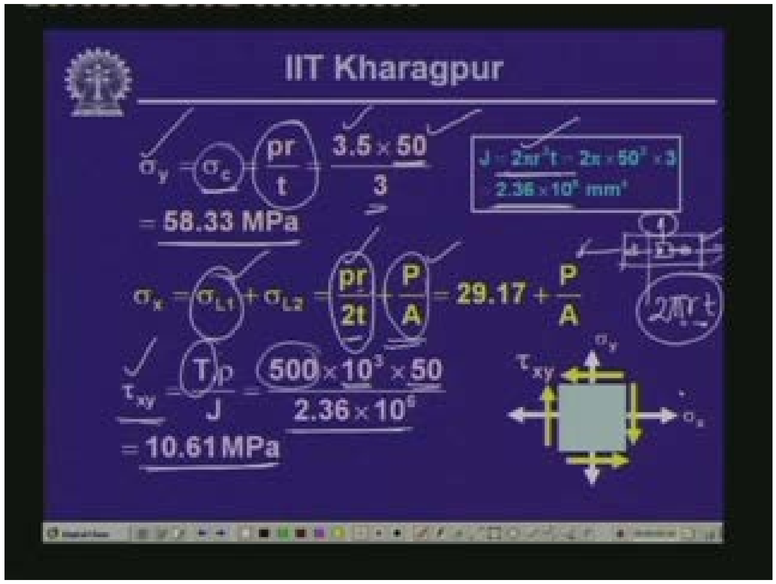


Let us look into the computations of these individual quantities so that we can find out what will be the maximum value of tensile stress. As you know that the circumferential stress or the Hoop stress because of the pressure internal pressure is equals to pr by T in the thin cylindrical wall cylindrical vessel. Now P is given as 3.5 MPa which is Newton per millimeter square, r is the radius where you are calculating the stress which is equals to 50 mm the external radius and T is the thickness of the wall so that gives us a value of 58.33 MPa and as you know the stress in the circumferential direction is the y direction and that is why we call that as sigma y the normal stress in the y direction.

Now, in the x direction because of internal pressure there will be a stress which we have called as normal stress sigma L1 and that is equals to pr by twice T and that is the internal pressure p, r is the radius and two times the thickness will give us the longitudinal pressure or the normal pressure in the x direction. Also, because we have the axial pull in the pressure vessel so it will be experiencing; this point will be experiencing (Refer Slide Time: 42:31) a normal stress and the magnitude of this particular normal stress will be equals to P divided by A.

P is unknown to us and A of course equals to twice pi r T for the thin-walled hollow section here where T is the thickness and r is the radius the external radius. And tau xy the twisting moment T will give rise to the shearing stress on the wall which is equals to T rho by J and J is equals to twice pi r cube T as you know for this hollow section and that is equals to 2.36 into 10 to the power 6 mm power 4 so if we substitute the value of T, T is 500 Newton meter so multiplied by 10 to the power 3 to make it Newton millimeter, rho is 50 mm, J is equals to 2.36 into 10 to the power 6 so, that gives us a value of 10.61 MPa that is the value of tau xy.

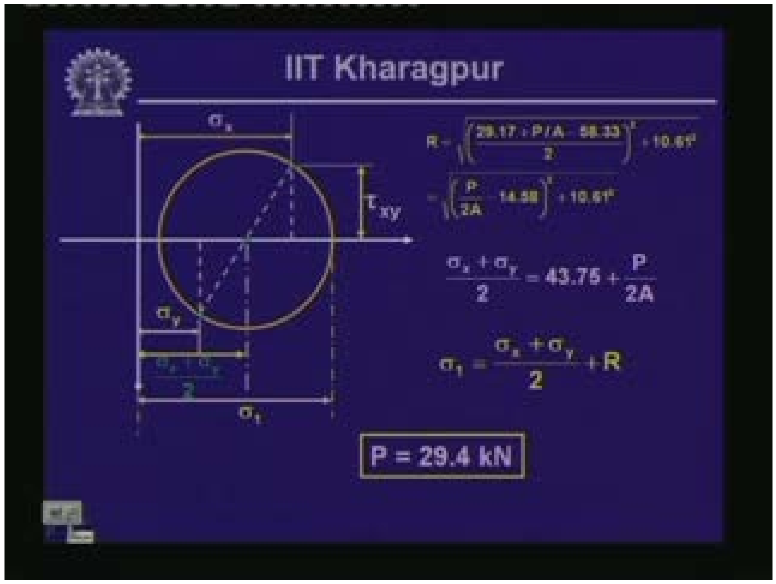
(Refer Slide Time: 43:33)



Thus, you have normal stress sigma x which is the sum of the effect of the internal pressure and the external loading; you have the normal stress sigma y which is getting generated because of the internal pressure and you have the shearing stress tau xy which is getting generated because of the twisting moment T.

Now if you have this stresses the resulting combined stresses in terms of sigma x, sigma y and tau xy now because of the internal pressure, the twisting moment and the axial pull so, for all three loading actions we have individually first computed the normal stress and the shearing stress now we have combined these normal stresses and shearing stresses to find out that at that particular point because of these combined loading actions we have the resulting normal stresses sigma x, sigma y and the shearing stress tau xy.

(Refer Slide Time: 44:35 – 51:16)



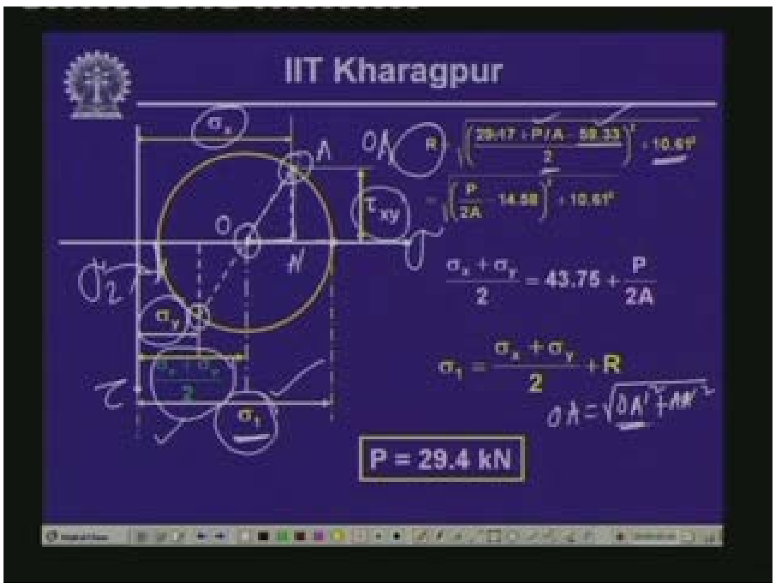
Now if we make use of this to find out what will be the maximum normal stress at that particular point then we plot them in the Mohr's circle; we have the value of sigma x which is computed and sigma x is equals to....... we have seen as 29.17 plus P by A and probably as we can see from the previous one that value of the sigma x is equals to 29.17 plus P by A and the value of sigma y is 58.33 and tau xy is 10.61 then this is sigma x and this is sigma y; sigma x and tau xy is indicated by this particular point and sigma y and tau xy is indicated by this particular point, now if we join them together this is the center of the Mohr's circle where it cuts the normal stress axis sigma axis and this is the tau axis.

Now, considering O as the center and let us call OA as the radius we plot the Mohr's circle and thereby this particular point gives us the value of the maximum stress or maximum normal stress which we call as sigma 1 and this is the value which gives us the minimum normal stress which we call as sigma 2 (Refer Slide Time: 45:56) so you have the maximum value sigma 1 is the maximum tensile stress because it is a positive sigma.

From this particular circle as you know the center distance from the origin is equals to sigma x plus sigma y by 2 as we had seen earlier. So the value of sigma 1 is equals to sigma x plus sigma y by 2 plus the radius of the circle. now the radius of the circle OA is equals to the OA square is equals to this square plus this distance square if we call this as OA dash, so OA is equals to root of OA dash square plus AA dash square.

Now OA dash square is equal to sigma x minus sigma y by 2 and AA dash is equals to tau xy. This is what is indicated over here (Refer Slide Time: 46:58) you see 29.17 plus P by A is sigma x minus 58.33 is sigma y so sigma x minus sigma y by 2 square plus tau xy square this gives us the value of the radius which is OA.

(Refer Slide Time: 47:01)



Now if you calculate this it will come in this form that P by 2A minus 14.58 square plus 10.61 square and sigma x plus sigma y by 2 then is equals to 29.17 plus p by a plus 58.33 divided by 2 and that gives you a value of 43.75 plus P by twice A.

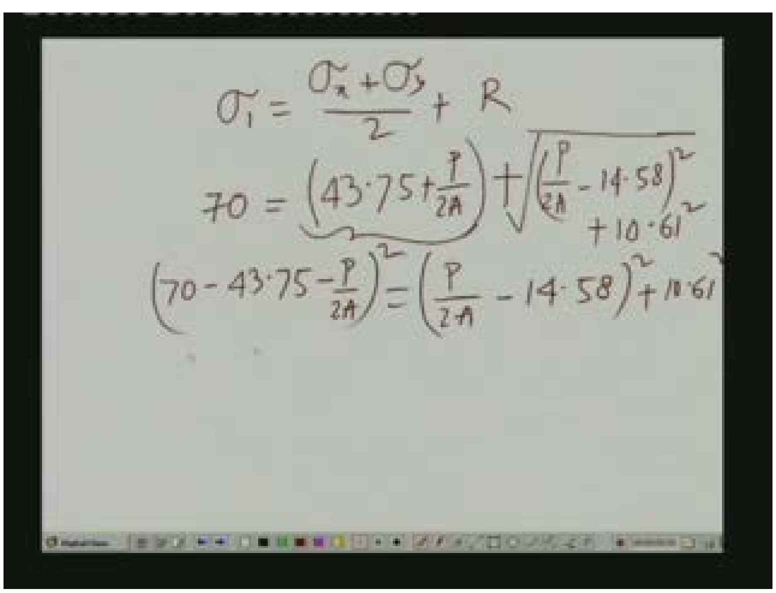
Now as you have seen that we have sigma 1 is equals to sigma x plus sigma y by 2 plus the radius r and as it is indicated that these values of sigma 1 the maximum tensile stress should not be more that 70 MPa. So if you take this as the limiting value and if you substitute these values of sigma x sigma y then we can get the value of P.

Now let us look into that if you substitute the value of sigma x and sigma y then what we get. Now here let us say we have sigma 1 as equals to sigma x plus sigma y by 2 plus the radius r (Refer Slide Time: 48:12).

Now sigma x plus sigma y by 2 if you substitute this is going to be equals to 43.75 plus P divided by twice A. this is the value of sigma x plus sigma y by 2. sigma x as you have seen is 29.17 plus P by A and sigma y is equals to 58.33 so that divided by 2 will be give this; plus R as you have seen is equals to P by twice A minus 14.58 the square of this plus tau xy square which is 10.61 square.

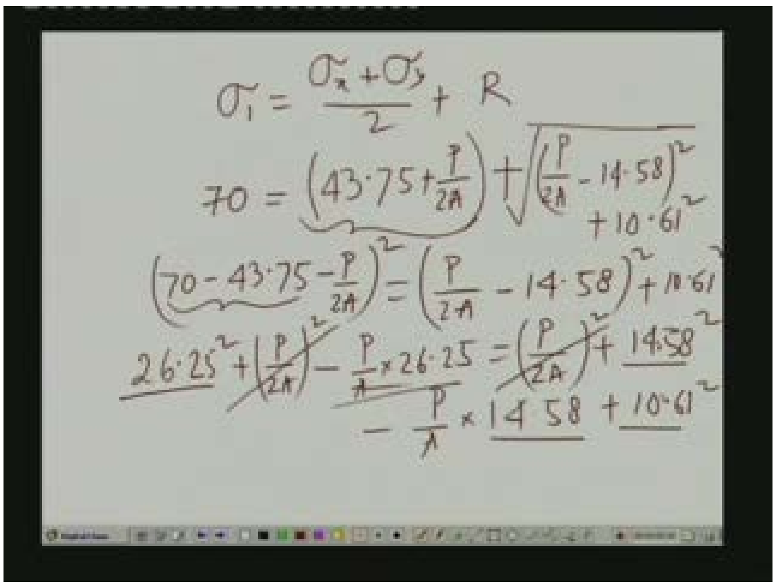
Now sigma as we know that it is limited to 70 so if we take this part on the left hand side then it becomes 70 minus 43.75 minus P by twice A square of that this is equals to root of......................... this we are squaring so the root goes off and we have P by twice A minus 14.58 square of that plus 10.61 square.

(Refer Slide Time: 49:26)



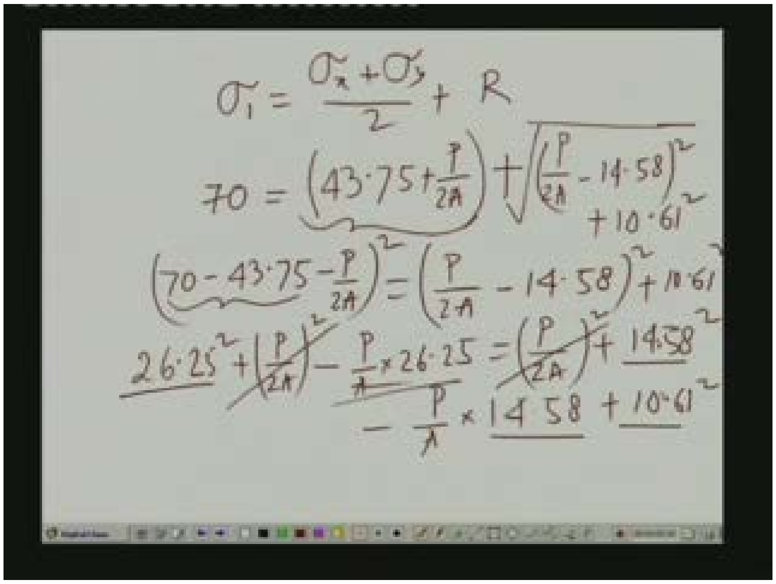
This part if you compute you will have......... this is 26.25 minus P by 2A square. So if I expand this we will have 26.25 square plus P by twice A square minus 2 of these two terms which is P by A into 26.25 and this is equals to P by 2A square plus 14.58 14.58 square minus P by A into 14.58 plus 10.61 square. As you can see from here that P by 2A square term gets canceled and these if you take on the other side you have P by A into 25.25 minus 14.58 and the other terms we can combine them together.

(Refer Slide Time: 50:21)

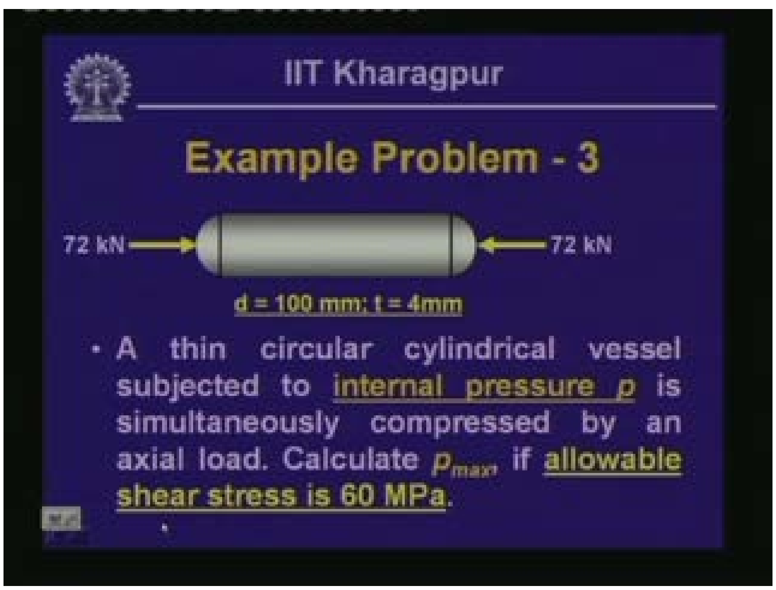


Now if you do that; if you take this (Refer Slide Time: 50:21) minus this minus this that becomes equals to 364 and this 364 is equals to P by A into 26.25 minus 14.58 and if you compute from this you will get A as you know is equals to twice pi rt and if you substitute the value of r as 50 and t as 3 we will get the value of P as 29.4 kilonewton and that is the load that the maximum amount of load that you can apply this is what is written over here that the maximum value of the load is equals to 29.4 kilonewton that can be applied so that the stress does not go beyond 70 MPa, the maximum tensile stress in the vessel at that particular point should not go beyond 70 MPa and the limiting value of P is equals to 29.4 kilonewton.

(Refer Slide Time: 50:52)



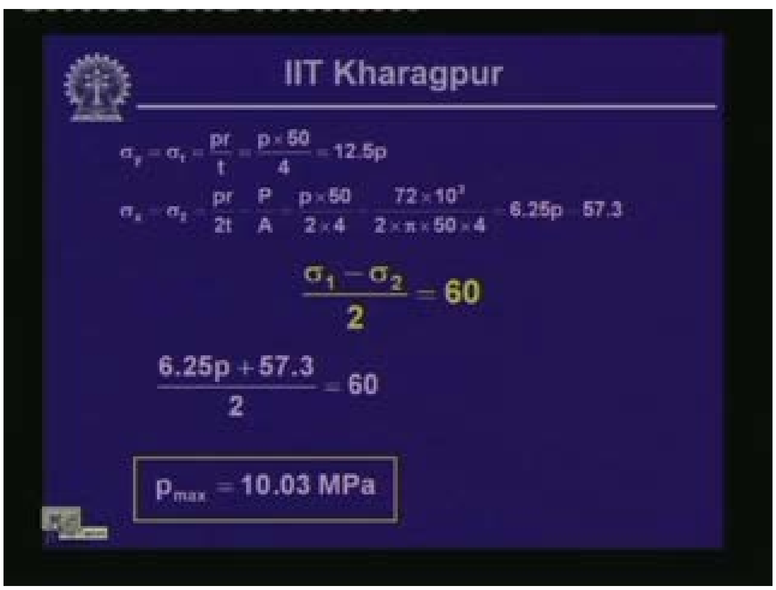
(Refer Slide Time: 51:16 – 52:38)



Well, let us look into another example which is again related to this pressure vessel were a thin circular cylindrical vessel is subjected to an internal pressure P which is not known to us and it is simultaneously compressed by an axial load of magnitude 72 kilonewton. So, if the allowable shear stress is 60 MPa then what will be the value of the maximum internal vessel. We will have to compute the value of the P max if the shearing stress is limited to 60 MPa.

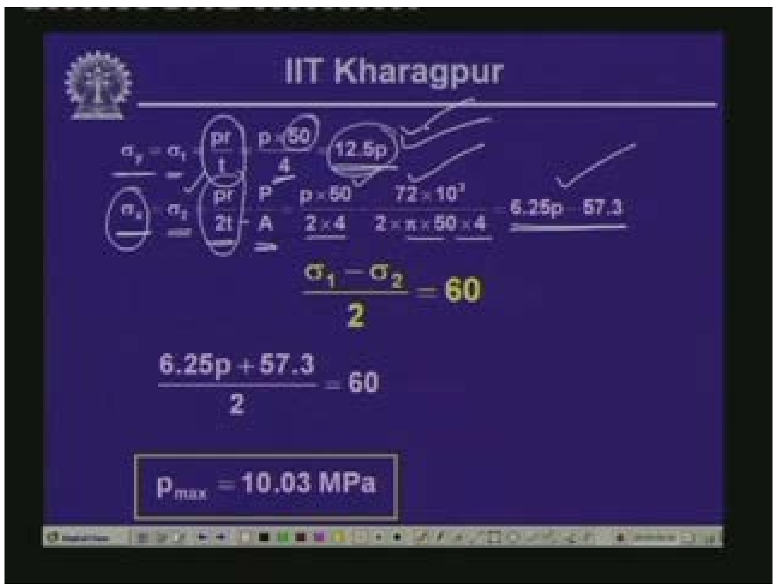
Now if we choose a point in the vessel wall we can find out the value of the normal stresses: the longitudinal and the circumference stresses because of the pressure which we had said sigma L and sigma C and this external force of axial 72 kilonewton will give a compressive stress compressive normal stress in the vessel wall. Therefore, if we take the resulting of this then we can find out that what will be the values of longitudinal stress and the value of the circumferential stress. And since here there are no shearing stresses acting so these will be the principle stresses and thereby we can compute the shearing stresses directly from them.

(Refer Slide Time: 52:38 – 55:22)



Here you see that we have computed these values and the value of the normal stress in the y direction which we call as the circumferential stress is equals to pr by T and r being 50 and T being 4, we have sigma y as 12.5 p and p is unknown to us; and the normal stress in the x direction or in the longitudinal direction sigma x we have called that as sigma 2; because of the internal pressure we will have the longitudinal stresses pr by twice T; also, because you have the axial compressive load p so at every cross section the normal stress will be equals to P divided by A and since again it is a thin-walled pressure vessel the area will be twice pi rt so since that is compressive pr by twice T is tensile and P by A is compressive so that is subtracted so p times 50 is the radius times twice T minus capital P is 72 kilonewton and twice pi rt if we substitute we get a value of 6.25p minus 57.3. This is the resulting normal stress in the x direction and this is the resulting normal stress in the y direction.

(Refer Slide Time: 53:54)

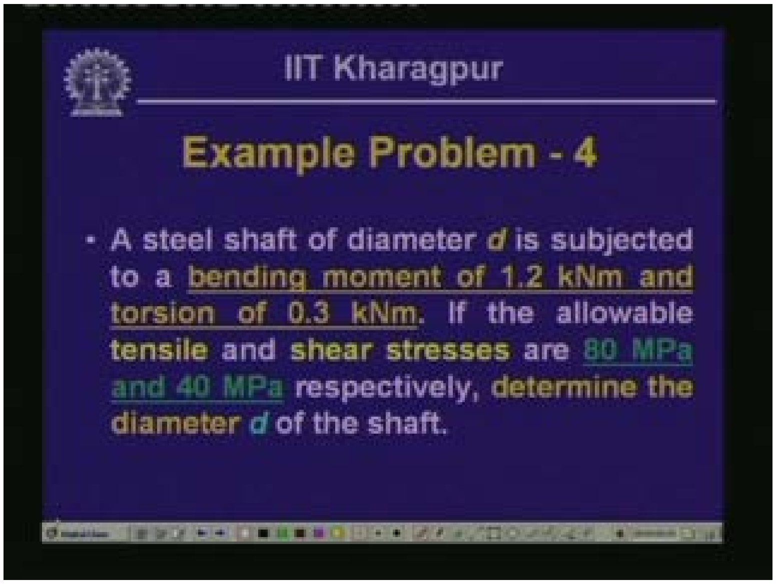


Here we do not have any external loading on the member which can contribute to the shearing stress. Since there are no shearing stresses at that particular point so those normal stresses as we know from the Mohr’s circle the places where we do not have the shear stress there we have the principal stresses. Here, these are the values of the principal stresses let us call them as sigma 1 and sigma 2.

Now as we know the maximum value of the shearing stress tau max is equals to sigma 1 minus sigma 2 by 2 and as it is indicated in this particular problem that the maximum value of the shearing stress that can go up to 60 MPa. So if we have to limit the value of the shearing stress to 60 MPa then what will be the value of the P.

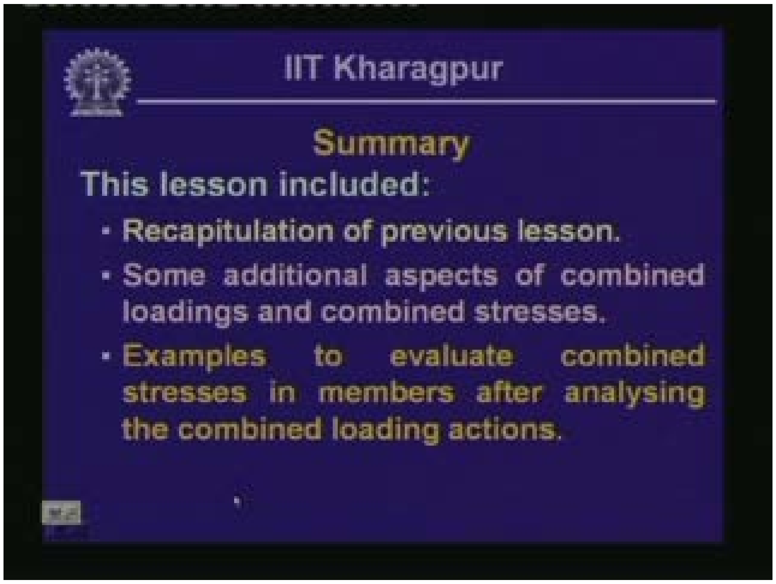
Now we can compute the value of shearing stress in terms of sigma 1 and sigma 2 or in terms of P. Now sigma 1 is equals to 12.5p minus we have 6.25p plus 57.3 so this is the value of sigma 1 minus sigma 2 so that gives us 6.25p plus 57.3 and that divided by 2 gives you the shearing stress that is equals to 60 and from this if we compute we get the maximum pressure that can be applied which is equals to 10.03 MPa. So because of this combined action this is the maximum value of the pressure that can be applied inside the cylindrical vessel.

(Refer Slide Time: 55:22 – 55:51)



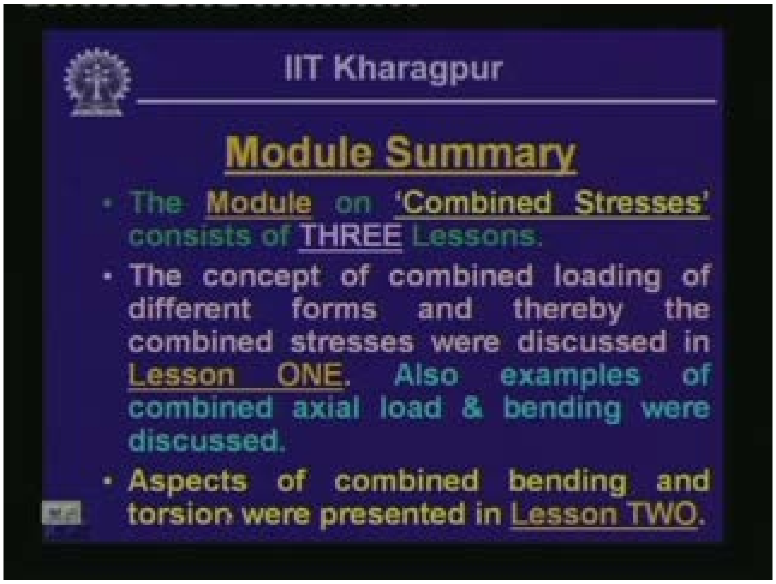
Now we have the another problem where a steel shaft of diameter d is subjected to a bending moment of 1.2 kilonewton meter and a torsion of 0.3 kilonewton meter. If the allowable tensile and shear stresses are 80 MPa and 40 MPa respectively what will be the diameter of the shaft. Now this problem is given to you. Look into this particular problem. We will be discussing about this particular problem in the next lesson.

(Refer Slide Time: 55:52 – 56:14)



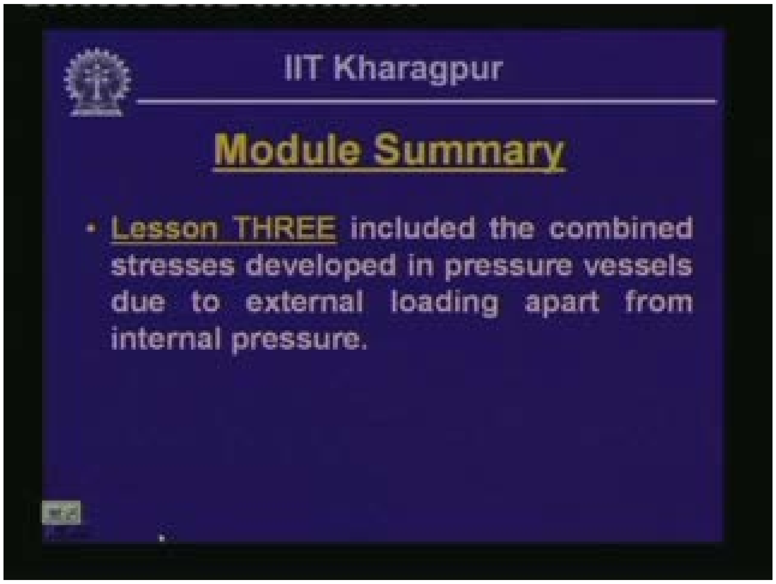
Well, then to summarize what we have done in this particular lesson is that we have looked into some aspects of the previous lesson; we have recapitulate the previous lesson. Now we have looked into some additional aspects of combined loading and that is in the pressure vessel and also we have looked into some examples to evaluate combined stresses in members after analyzing the actions of the combined loading.

(Refer Slide Time: 56:14 – 56:42)



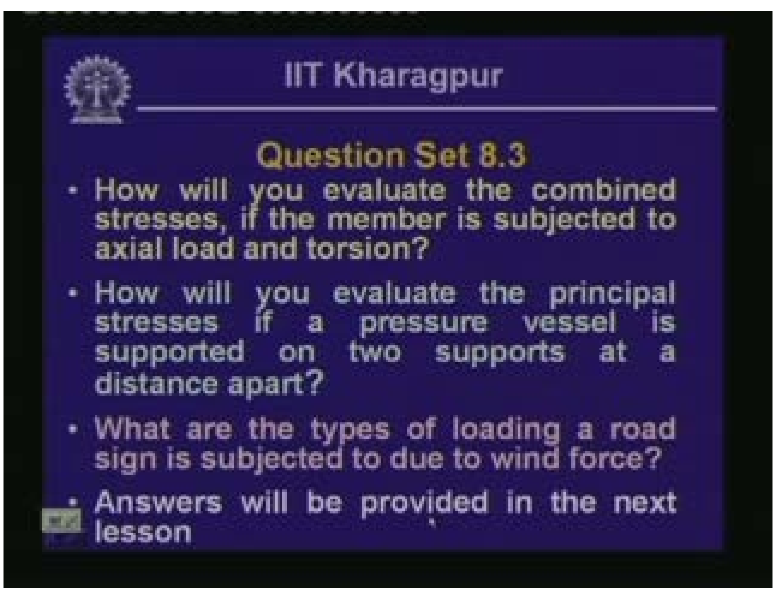
Now this is the last lesson of this particular module and this module on combined stresses we had three lessons and as you have seen in the lesson 1 we have introduced the concept of combined loading and also we have seen various forms of combined loading in members and we had solved some examples which are related to the combined axial force and the bending. In the subsequent lesson in lesson 2 we had looked into some aspects of bending and torsion.

(Refer Slide Time: 56:42 – 56:52)



And in the third lesson or the lesson which we have discussed today there we have discussed some aspects of the combined loading in the pressure vessels. So that concludes the lesson on the module on the combined stresses.

(Refer Slide Time: 56:52 – 57:14)

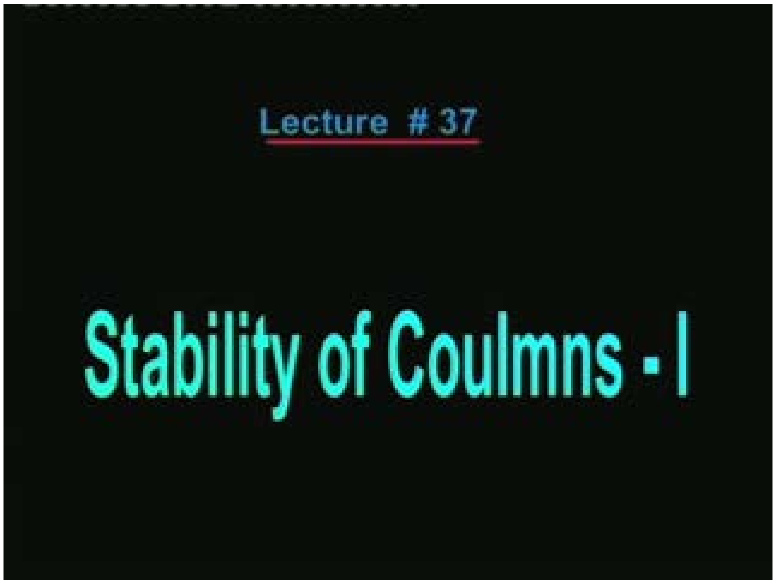


And these are the questions set for you. How will you evaluate the combined stresses if the member is subjected to axial load and torsion? How will you evaluate the principal stresses if a pressure vessel is supported on two supports at a distance apart? And what are the types of loading a road sign is subjected to due to wind force?

We will look into the answers of these questions in the next lesson, thank you.

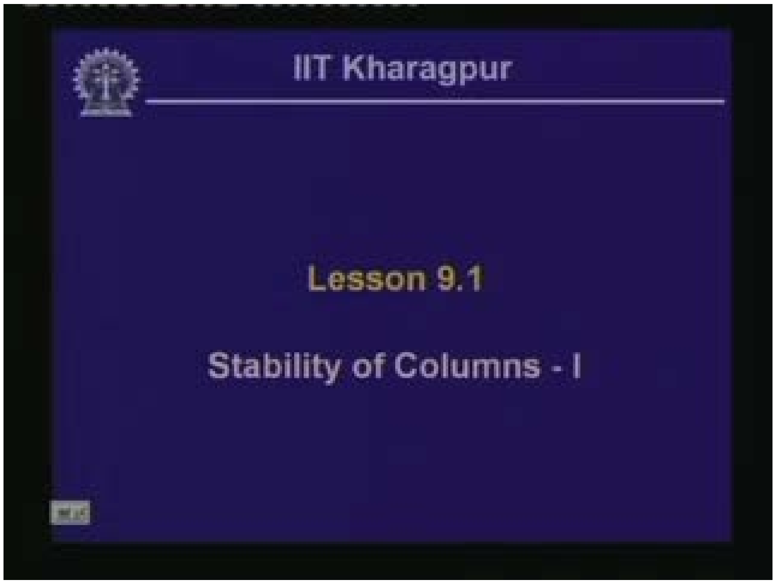
(Refer Slide Time: 57:22 – 57:28)

(Refer Slide Time: 57:28 – 57:30)



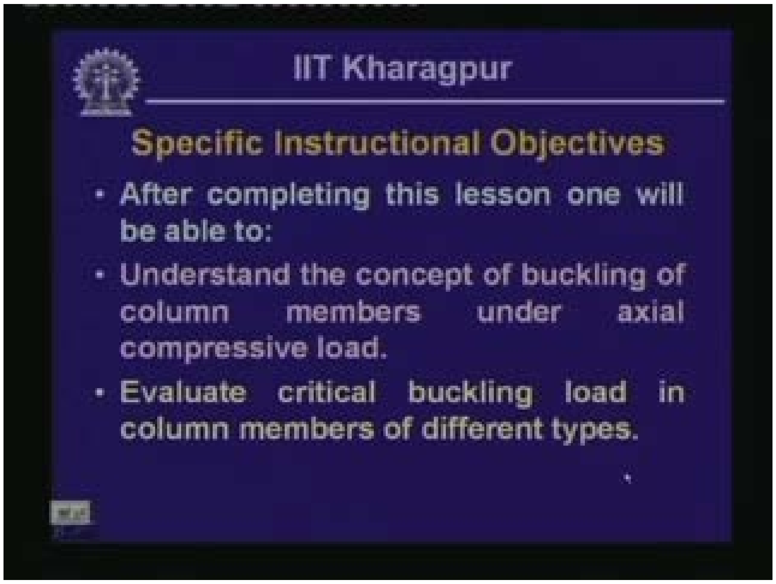
Welcome to the first lesson of the ninth module which is on stability of columns part I.

(Refer Slide Time: 57:41 – 57:50)



In fact, in the previous modules we have discussed certain aspects of the stresses in members and consequently we have looked into the effect of bending in a member where we have evaluated of the deflection of the member. Now, in this particular lesson we are going to discuss different aspects which are the stability of a member which we designate as column.

(Refer Slide Time: 58:08 – 58:40)



Now it is expected that once this particular lesson is completed one should able to understand the concept of buckling of column members. In fact, we define what we mean by column members; which member we call as column and then we will look into certain characteristics features which we call as buckling so buckling of column members under axial compressive load; and one should be in a position to evaluate critical buckling load in column members of different types.

Now we will look into that what we really mean by critical buckling load that the member which is subjected to axial compressive load; in which load it is going to buckle or deform.

The scope of this particular lesson therefore includes the recapitulation of previous lessons. In fact, in the last module we have looked into the aspects of combined stresses.

We will look into the certain aspects of the combined stresses while answering the questions related to that and this particular lesson includes the concept...................