# Strength of Materials

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**Lecturer - 35**

## Combined Stresses - II

Welcome to the second lesson of the eighth module which is on combined stresses part II. In fact, in the last lesson we have discussed about or we have introduced the concept of the combined stresses in a member wherein a particular member is subjected to a combination of loads; say for example, axial load and the torsion or the torsion and the bending or could be axial load and bending and various combination of these individual loads. Now we have also looked into that how do we compute the stresses when a particular member is subjected to the combinations of these different loads.

Now, in this particular lesson we are going to look into some more aspects of such combined loads and how do we analyze them and thereby we compute the values of the stresses.

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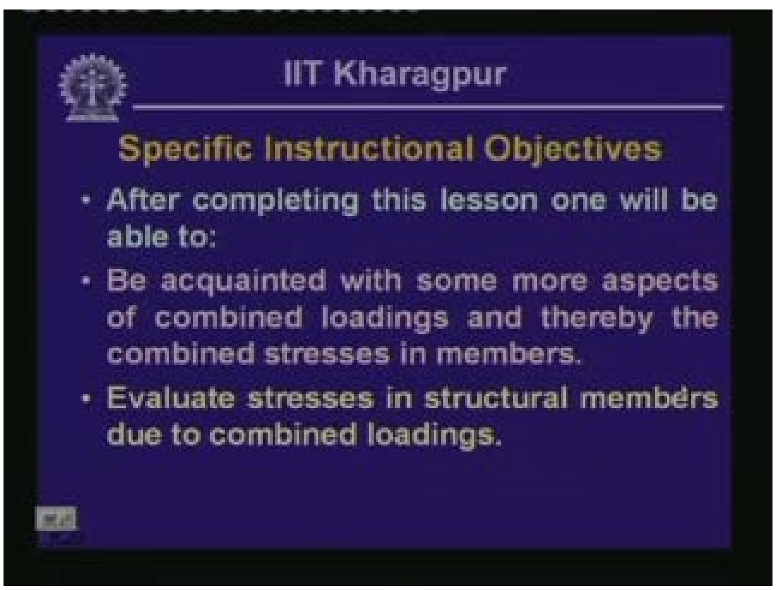

Hence, it is expected that, once this particular lesson is completed, one should be able to be acquainted with some more aspects of combined loadings in a particular member and thereby the evaluation of the combined stresses in members and one should be in a position to evaluate stresses in structural members due to such combination of loading or combined loading.

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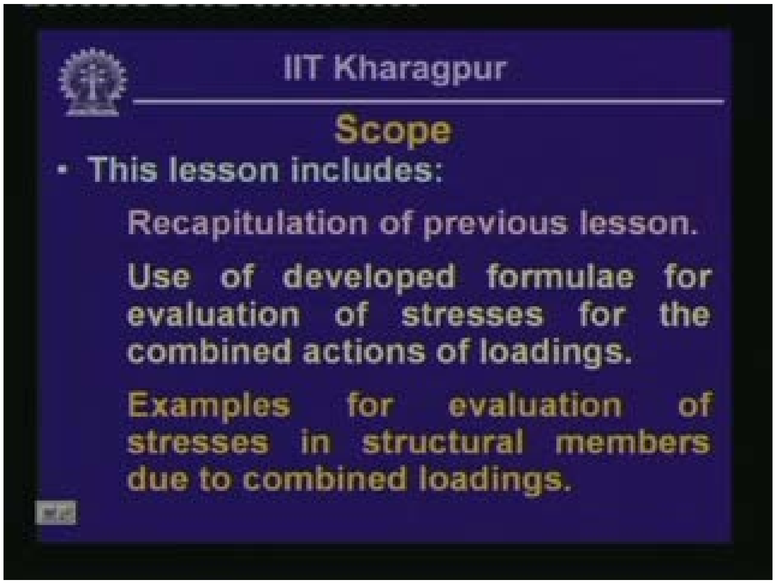


The scope of this particular lesson therefore includes the recapitulation of previous lesson. We will be looking into some aspects of the previous lesson which we have already discussed and that we will be doing through the question answer session.

Also, in this particular lesson we will be using the already developed formulae for evaluation of stresses or combined actions of loadings. Now, as I have told you, in the last lesson, that in the previous modules we have developed some formulae for the individual load cases. Like for example; we have looked into that how to evaluate stress in a member when that particular member is subjected to axial load. Or if a particular member is subjected to a twisting moment or torsion then how do we calculate the stresses because of the torsion in a member; or if the member is subjected to transverse loading for which there will be bending in the member and thereby there will be bending stresses or shear stresses and we have seen how to evaluate those stresses because of such bending moment and shear force.

Here we are going to make use of those formulae which we have derived earlier in evaluating the stresses because of the combined loading.

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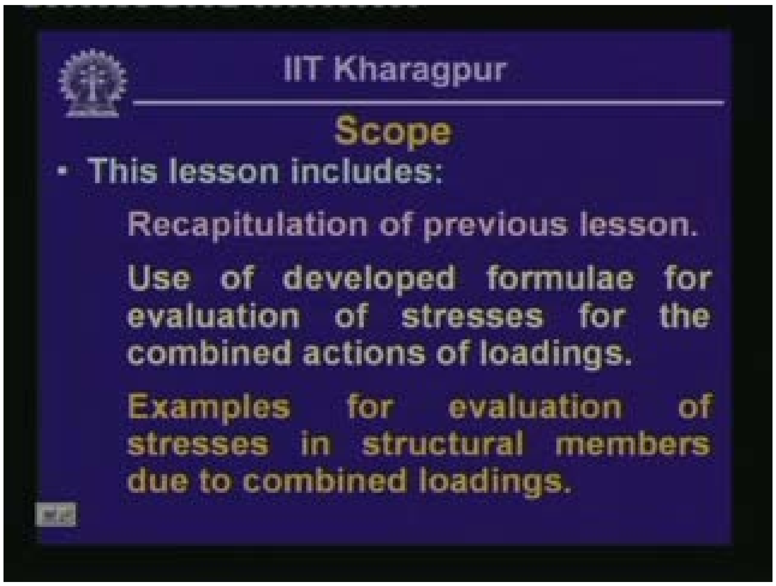


Also, we will be looking into some examples for evaluation of stresses in structural members which will be arising due to these combine loadings.

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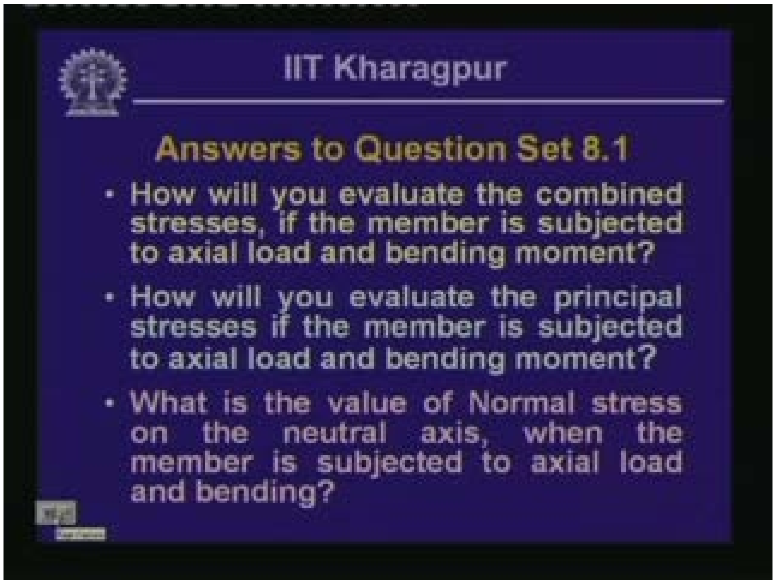


Now let us look into the answers of the questions which we had posed last time. The first question which we posed was how will you evaluate the combined stresses if the member is subjected to an axial load and bending moment. That means now that a particular member is subjected to not only the axial load but also subjected to a bending moment. Now, in such a situation how do you calculate the combined stresses?

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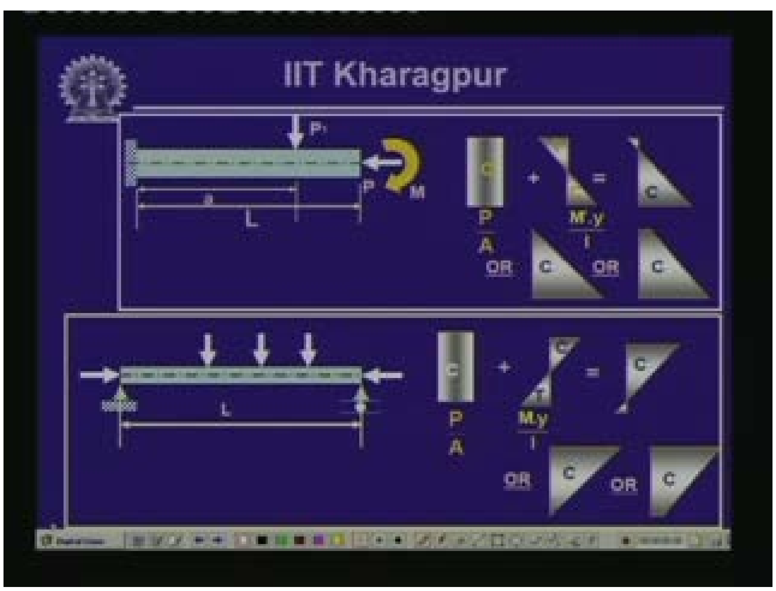

Now we had looked into in the last lesson that there could be cases with reference to either a cantilever beam or simply supported beam or for that matter any of such members where it is supported at suitable supports and is subjected to axial load as well as the transverse load.

When this particular member is subjected to as in this case or in this case (Refer Slide Time: 4:36) where you have the axial load and the transverse load and here you have the moment as well. Now if we like to calculate stress at any cross section in the member then the stress will be because of this axial load and this lateral load is going to cause a bending moment M and shear force V at this particular cross section and there will be stresses because of this bending moment M and because of the shear force V and finally we will have to calculate the resulting stress because of all three actions.

Now as you know we have seen that when a particular member is subjected to axial load that is subjected to a stress which we call as normal stress which is P by A and since the load is compressive so the stress also is compressive and which is uniform throughout the cross section.Now, because of the bending there will be bending stress which we evaluate from M y by I as we have seen earlier that sigma is equals to M y by I where M is the bending moment, y is the distance at which we are computing the bending stress from the neutral axis and I is the moment of inertia of the cross section with respect to the neutral axis.

As we have seen that this bending stress with respect to the neutral axis where the stress is 0 causes compression and tension in this particular case and the top part above neutral axis is under compression and the bottom part below neutral axis is under tension. Now, bending also produces normal stress as we have seen in the cross section. So if we combine these two normal stresses together, the normal stress produced by the axial load and the normal stress produced by the bending then we get a combined stress form either in this form or in this form or in this form (Refer Slide Time: 6:37) and we have discussed this why we get these three different situations.

Now, apart from this bending stress and the normal stress the transverse loading will cause a shear force as well and because of shear force there will be shear stress in the member at that particular cross section. So we got to compute the value of the shear stress at the point where we are computing the bending stress and the normal stress and as you know, the shear stress tau is equals to the Vq by Ib. And in this particular formula we have seen that how to calculate the value of the shear stress when we know the shear force at that particular cross section. So we know the normal stress because of the axial load, we can compute the value of the normal stress because of the bending and we can compute the value of the shear stress because of the shearing force which is acting.

Now the normal stresses because of the axial load and bending can be combined together to give the resulting normal stress and normal stress and shear stress the two stresses which are acting in two different planes we compute the values using either the transformation equations or using Mohr's circle.

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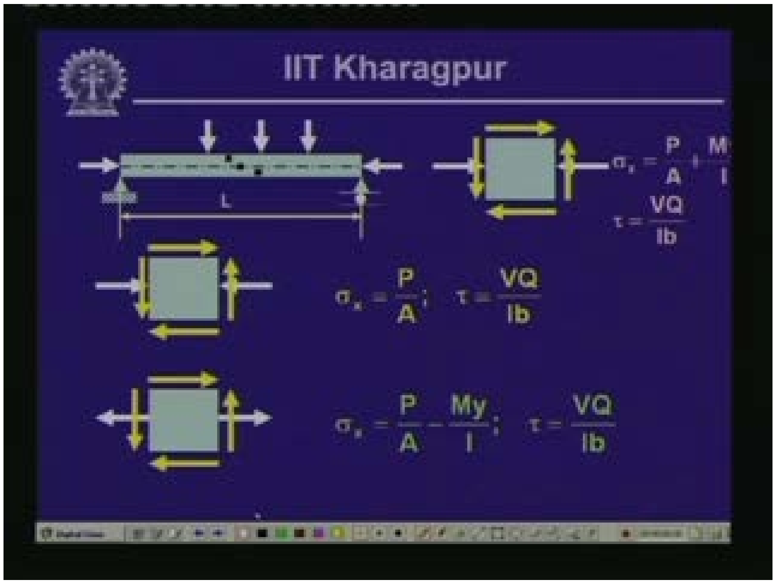

Now if you look into that in this particular figure at these three locations if we like to compute the value of the stresses; here we have this normal stress sigma which is the P by A plus My by I or plus and minus depending on the magnitudes of these values and tau is going to give you the shearing stress which is VQ by Ib. so we have the normal stress and we have the shearing stress and these resulting stresses because of the normal stress and the shearing stresses can be computed from the Mohr's circle as you know this is the sigma x and this is the tau positive direction (Refer Slide Time: 8:31).

Now here if you plot the values of the normal stress and the shearing stress as we are computing from this formulae and since you do not have any sigma y so sigma y and tau, and if you join these two points together you get the circle and thereby this gives you the maximum value of the normal stress which you call as the sigma 1 or the principal stress. In fact, this is the answer for the second question as well.

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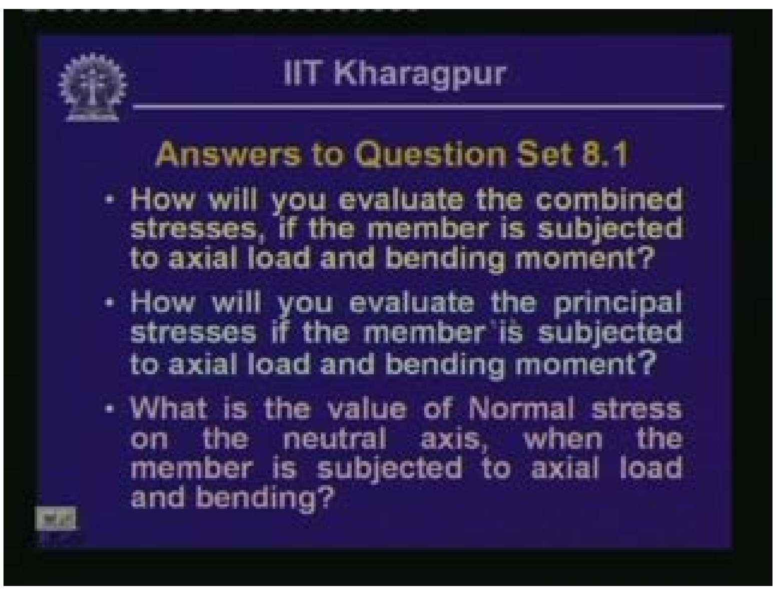


In fact, if you look into the question which we have for the second one that how will you evaluate the principal stresses if the member is subjected to axial load and bending moment; now as we have seen in the first one that when you have the axial load and the bending moment we can compute the values of the normal stress; axial will give you the direct normal stress and bending will give you the normal stress in terms of compression and tension with respect to the neutral axis and you can calculate the resulting normal stresses and individually you can calculate the shear stress from the shear formula which is VQ by Ib.

Now, using this if we like to calculate the resulting stress you call as the maximum value of the tensile or compressive stress which is the principal stress that is the maximum or minimum value; that we compute from the Mohr's circle.

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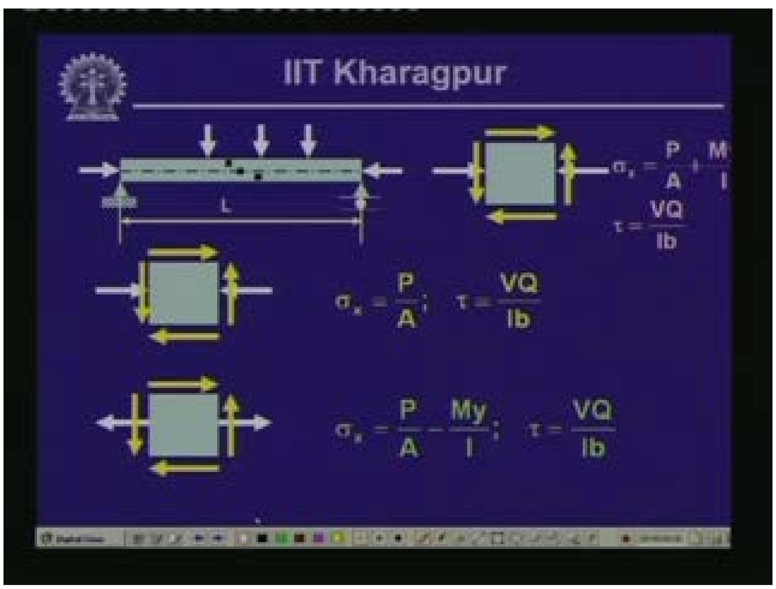


And this is what has been done over here that we can have this kind of a relationship that means this is for the normal stress and this is for the shearing stress (refer Slide Time: 10:01) and based on this normal stress and shear stress we can compute the value of the maximum principal stress sigma 1 or the minimum stress sigma 2 these are the principal stresses and also we can compute the value of the in-plane shear stresses that what is the maximum value of the shearing stress that will be occurring at any point.

Now, the third question which was posed was that what will be the value of the normal stress at the neutral axis. And probably I can answer this from this particular configuration itself or from this figure.

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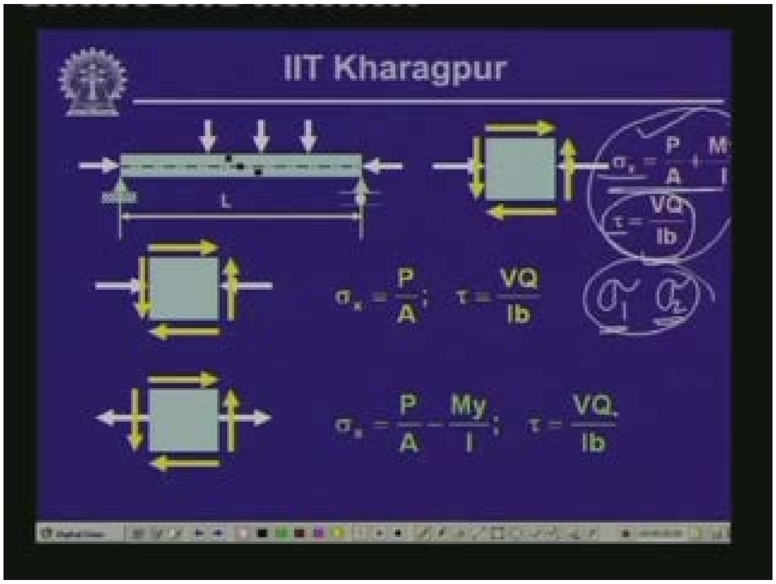


Now you see that we have an element chosen which is lying on the neutral axis. Now as you know that when you compute the bending stress, the bending stress gives you a distribution linear distribution with respect to the neutral axis where you get a compressive force or tensile force or tensile and compressing depending on the kind of loading you have. On the neutral axis the value of the normal stress which is arising from bending is equals to 0.

Now if you have the combination of axial and the bending then the normal stress which will be occurring at the neutral axis point or along the neutral axis is only for the axial stress which is the normal stress sigma x equals to P by A and there will not be any contribution from the bending along the neutral axis.

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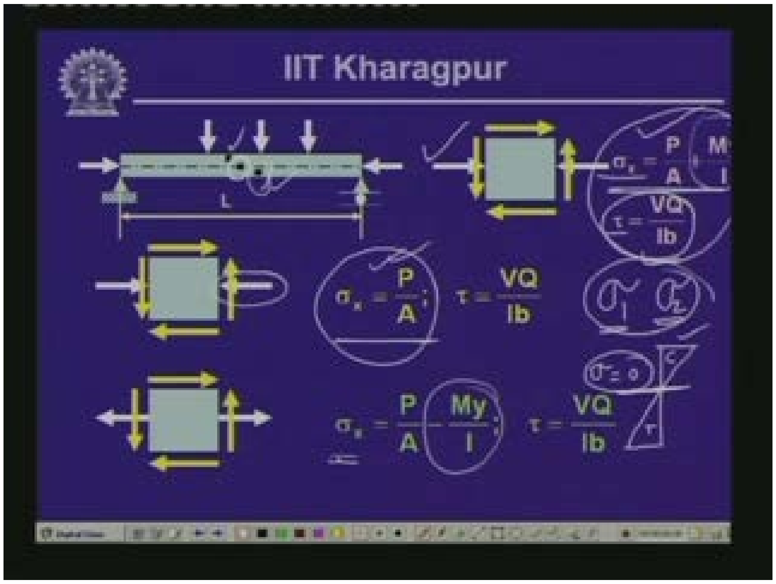


However, if you choose the point above the neutral axis or below the neutral axis, there you will have the contribution from the bending and thereby you will have the resulting normal stress. So as you can see that all three questions which we had related to the actions of the combined loading where a member is subjected to the axial load and bending moment. And when axial load and bending moment occurs in a particular member simultaneously, we compute the resulting normal stress, axial will give you the normal stress, bending also will give you the normal stress and we can sum them together to get the resulting normal stress and then because of the presence of the transverse load at that particular section there will be shearing force as well and shearing force will produce the shearing stress. So you can have the resulting stress from the calculated value of the normal stress and the shearing stress using the Mohr's circle.

Now you can visualize that when we had discussed in module 1 the transformation equation or the Mohr's circle of stress where because of the given stresses sigma and tau we could evaluate that what will be the value of the principal stresses. Now as you can see from the practical examples that a beam member is subjected to axial pole and the transverse load because of some loading situations where we compute the normal stress and the shearing stress individually and then combine them together using the Mohr's circle to find out the maximum or minimum normal stresses and the maximum value of the shearing stresses.

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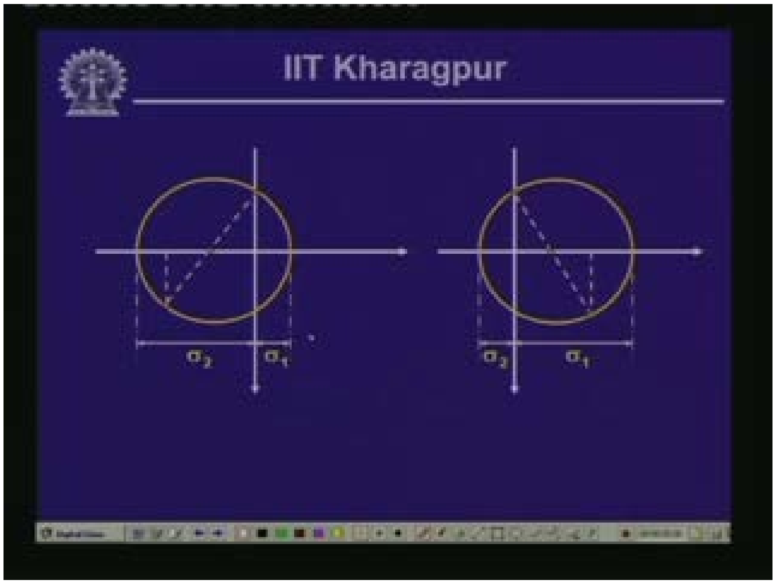


Well, this is what is shown over here that if you get the stresses normal on the shearing stresses then you can plot them in the Mohr's circle. Here the normal stress is negative and the shearing stress is positive and that is what you have getting corresponding to the first element; and when your normal stress is positive and the shearing stress also is positive you get the point over here; and since sigma y is 0 we get the other point over here, we join them together, this is the central line of the Mohr's circle and the maximum value of normal stress (Refer Slide time: 13:35) which is represented by this point in the circle gives you the maximum normal stress or the maximum tensile stress in the member and this is the value of the sigma 2 and minimum normal stress; and this is how we compute from the Mohr's circle.

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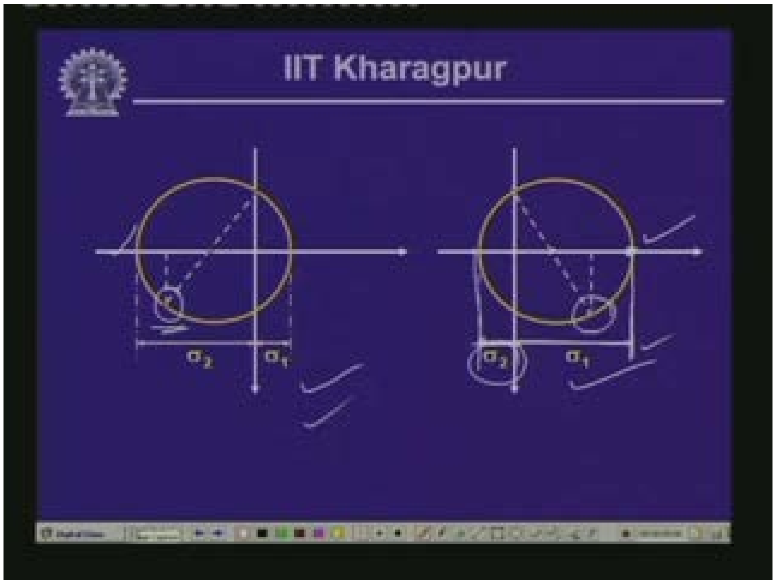


Now let us look into that if a member is subjected to an axial pole along with the twisting moment

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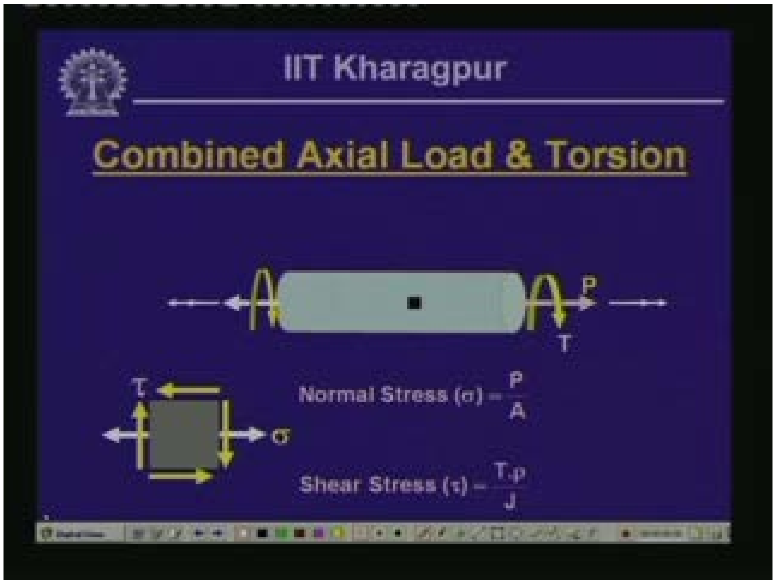

In the last lesson we had looked into some numerical example where a member was subjected to the axial load and the transverse load which was causing bending in the member, thereby the member was under the action of axial load and the bending. Now in this particular case we are going to consider that if a member is subjected to the axial load and a twisting moment then what happens to the combined action of this loading.

Now if we are interested to find out the stress at this particular point let us say this point is A (Refer Slide Time: 14:34) where we like to compute stresses because of the loading actions like you have the axial pull P and there is a twisting moment which is positive in the sense that as we have defined earlier that when the vectorial direction points towards the positive x axis we call that as a positive twisting moment and at this end a positive twisting moment is acting in this member.

As we have seen earlier that because of this twisting moment we get the shear stress at different points which is given by t rho by J. Now, from the center if we go along the radius this is rho, so maximum shear stress will be acting on the surface where R is maximum.

Now, given the value of the twisting moment T we can compute the value of the shear stress if we know the radius of this cross section and J is the polar moment of inertia which is Ix plus Iy or in this particular case it will be Iy plus Iz as we have called x as the a longitudinal axis. Thus, we have the actions for the axial load as a normal stresses which is P by A and twisting moment will give us the shear stress tau.

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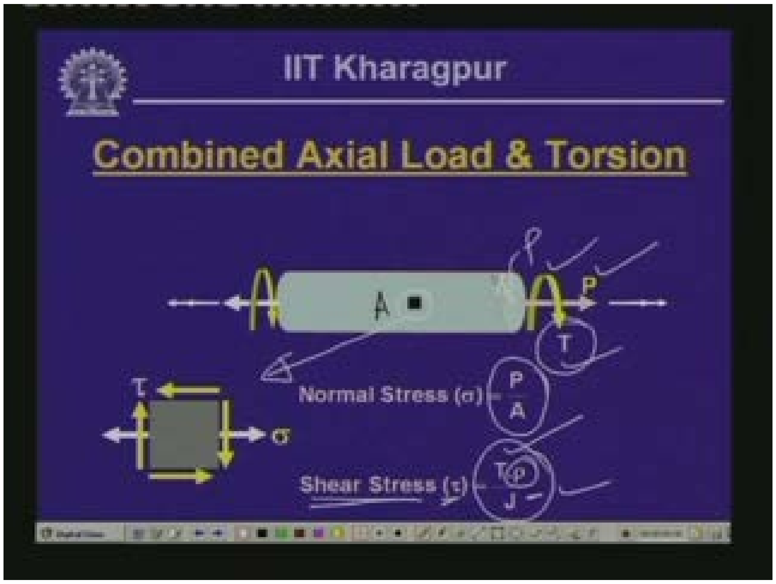


Therefore, if we look into this particular element this element will be subjected to the shearing action because there is a twisting moment which is a positive twisting moment, on this element (Refer Slide Time: 16:06) we will have the shear which will be acting on this form and that is what is represented over here which is resulted from this twisting moment T and these are the complementary shear because of this shear and sigma is the normal stress that is acting.

Now if this particular element is subjected to a normal stress sigma and shearing stress tau then the maximum normal stress we can compute again from the Mohr's circle of stress, this is the sigma axis, this is the tau axis and we can represent this particular plane where we have normal stress sigma and shear stress here is negative so this is sigma and shear is negative (Refer Slide Time: 16:49) so, on this side the point will be taken and for the other side the normal stress is 0 and we have a shearing stress which is positive so the point will be here.

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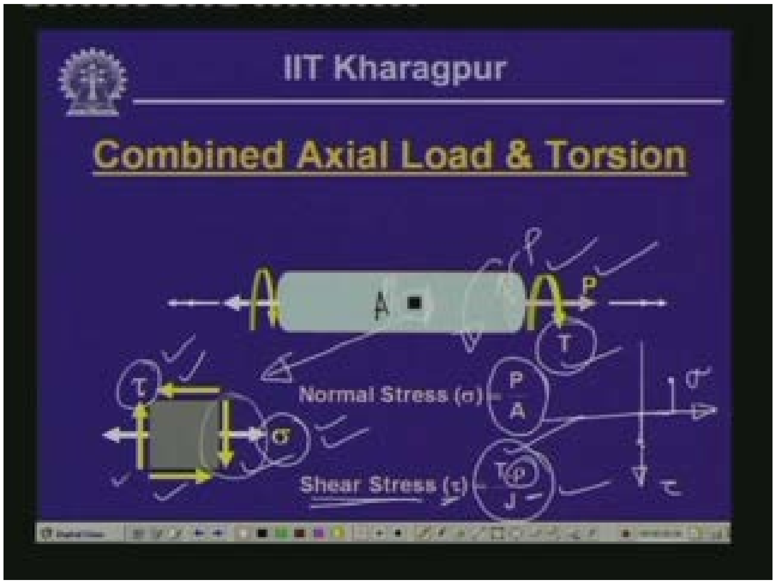


Now if you join these two points, the line which will cross the sigma axis at the point which is the center of the circle, so if we draw a circle with this radius then we get the resulting Mohr's circle and this is the maximum value of the normal stress which we get from this particular circle and this is the minimum value of this normal stress this will call as sigma 2. So this is sigma 1 and sigma 2, we can compute the value of the normal stresses; also, this is the maximum and the minimum value of the shearing stresses; this is the positive tau, this is the negative tau and this is the maximum shear stress that will be occurring at that particular point because of the action of axial pull and twisting moment.

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We will look into some examples on how to compute these using numerical values.

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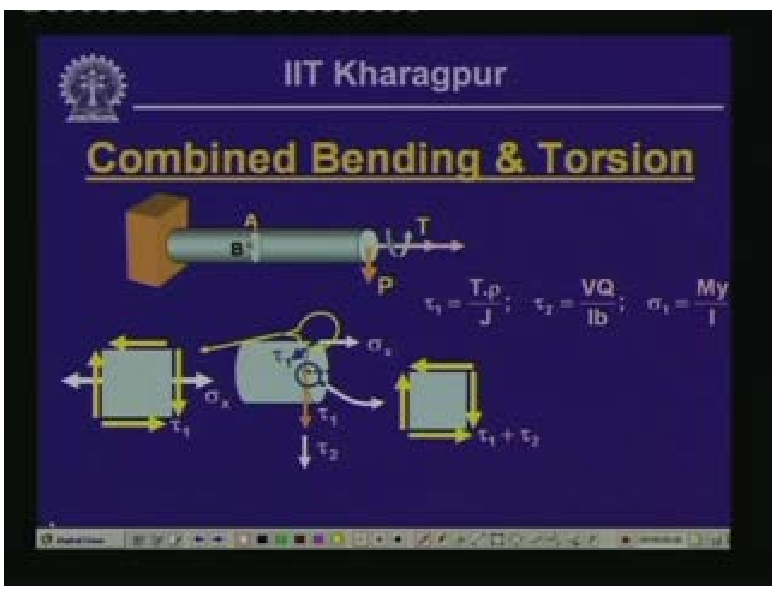


Now let us look into that, if a particular member is subjected to, again a combined loading action but here the individual loadings are bending and twisting moment which are acting simultaneously in a member. We had seen the action of a twisting moment in a bar that how do we calculate the shearing stress because of the twisting moment. Also, we have looked into that if a beam is subjected to transverse loading then how do we calculate the bending moment and the shear force and consequently how do we compute the bending stress and the shearing stress.

Now if the member is subjected to the twisting moment as well as the bending then what we will be the consequence or how do we calculate the stresses in such members. If you look into this that here we have a twisting moment which is acting (Refer Slide Time: 18:39); again it is a positive twisting moment and a load P is acting at the end of this cantilever beam, this is fixed at this particular end.

Now this load which is acting at the free end of the cantilever beam will produce a moment at this section where we are interested to find out the stresses. We are interested to find out the stresses at point A and B on this particular cross section let us say which is at a distance of A from the free end. Now at this particular cross section the load P is going to cause a bending moment M which is equals to P into A and also there would be a shear force which is equals to P.

In fact, if you take a free body diagram, there we have P here, on this the moment will be in this direction and the shear force will be in this particular direction. Therefore both moment and shear will be negative. Now physically if you look into this particular member when this is loaded at the tip, the deflected form will be in this and thereby the top surface will be on the tension and the bottom will be under compression and that we can verify from the numerical values as well or from the expressions as well.

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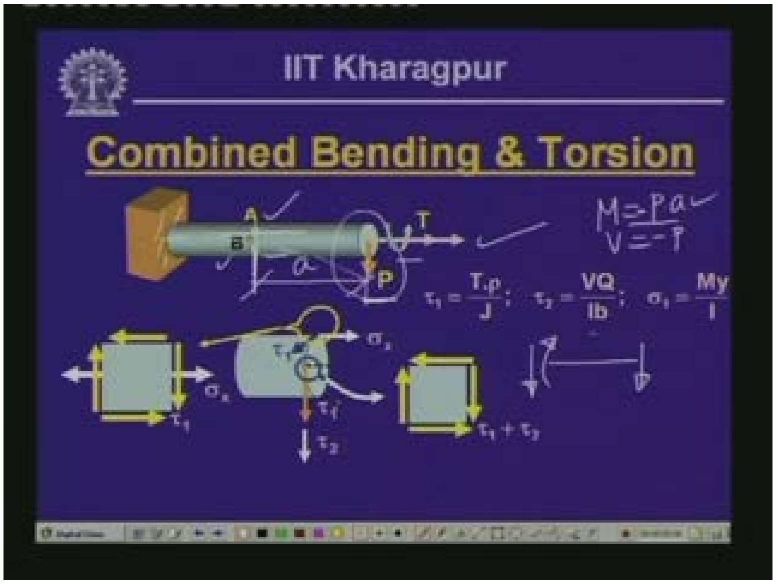


Now you see, M is minus P into a and as you know that sigma is equals to minus My by I, so for M if we put minus Pa to a so this is plus Pa y by I so thereby the stress is becoming positive and positive indicates our tensile stress. From the expression also you get the same value as you are looking here physically.

Now the question is that when this moment and the shear force is acting on this particular cross section then what are the kinds of stresses we are going to get at point A and point B. Now as you know that at point A because of the twisting moment there will be shearing stress tau 1 which is equals to T into rho by J where T is the twisting moment, rho is the value of the radius of outer point and J is the polar moment of inertia of the cross section. Also, there will be shearing stresses because of the shear force V. But the point A which is on the top surface, there as we have seen the shear stress because of the shear force is equals to 0. So, the only contribution of the shear will be from the twisting moment at point A. And at point A because of this axial because of the bending because the transverse load P is causing bending at A which is equals to minus p into a will have a normal stress which is M y by I and this is sigma x (Refer Slide Time: 21:40). So sigma x is equals to My by I as it is indicated over here.

At the top point we will have a normal stress sigma x and there will be shearing stress tau 1 which is arising from the twisting moment only and this is what is the representation of the stress pattern at that particular point A. Now at point B which is lying in the neutral axis, now as you have seen that at the neutral axis level the stress because of bending is equals to 0. So there will not be any contribution from the bending thereby the value of the normal stress for that particular point will be 0.

So all we will have at that particular point is the shearing stress is which is arising from the twisting moment and the shearing stress which is arising from the shear force value. so tau 1 is arising from twisting moment and tau 2 is equals to VQ by Ib; depending on the amount of the shear force you have at that particular section we can compute the value of tau 2. So, if we combine these two together tau 1 and tau 2 depending on the signs obviously that tau 1 and tau two I have shown in general as tau 1 plus tau 2 but depending on the signs it will have this value so you will get a resulting shearing stress at this particular section and you will not have any normal stress.

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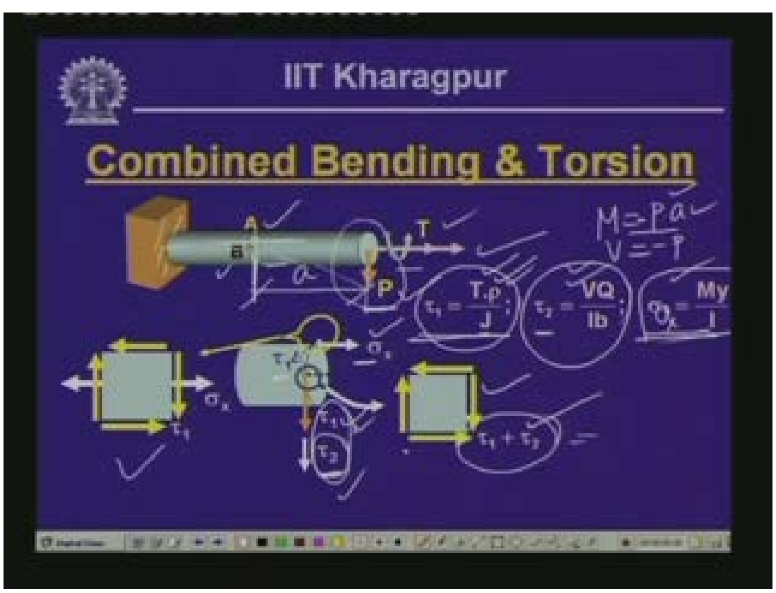

Hence, these are the two situations that we get corresponding to A and B and if we plot these stresses in the Mohr's circle then you get the situations in this form.

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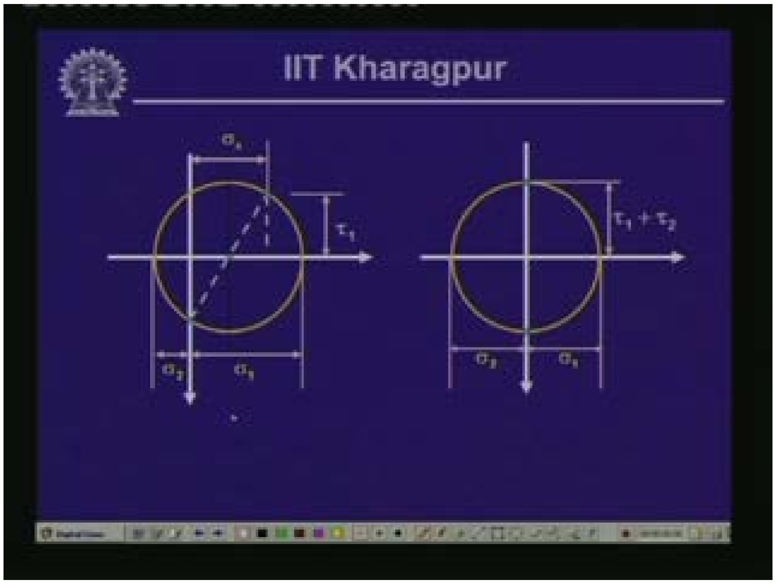


In the first case where we have the normal stress sigma x and tau 1, this is the point which is representing the stress (Refer Slide Time: 23:24) sigma y is 0 and you have the shearing stress only on that particular plane. So, if you join these two points this particular point gives you the center of the Mohr's circle; so as the point and this as the center and this as the radius if you plot the circle you get the corresponding Mohr's circle of stress and the maximum value of the sigma on this particular circle is this which we call as sigma 1 the maximum normal stress and this particular point is going to give as the minimum normal stress and the maximum value of the shearing stress at that particular point will be equal to the radius which is this; this is tau max.

And corresponding to the other one where we do not have normal stress but we have the resulting stress only tau 1 plus tau 2 so you see we do not have any normal stress but on this the normal stress is 0, you have only the values of the tau so this is the center of the circle and if we plot the Mohr's circle we get the normal stress as this which is equals to the value of tau 1 plus tau 2 and the minimum normal stress also is equal to tau 1 plus tau 2 which we have seen is the case of the pure shear.

That means at that particular point the state of pure shear is prevailing and corresponding to that, as we have seen in the past that how to compute the value of the normal stresses we get the normal stress as equals to the shearing stress.

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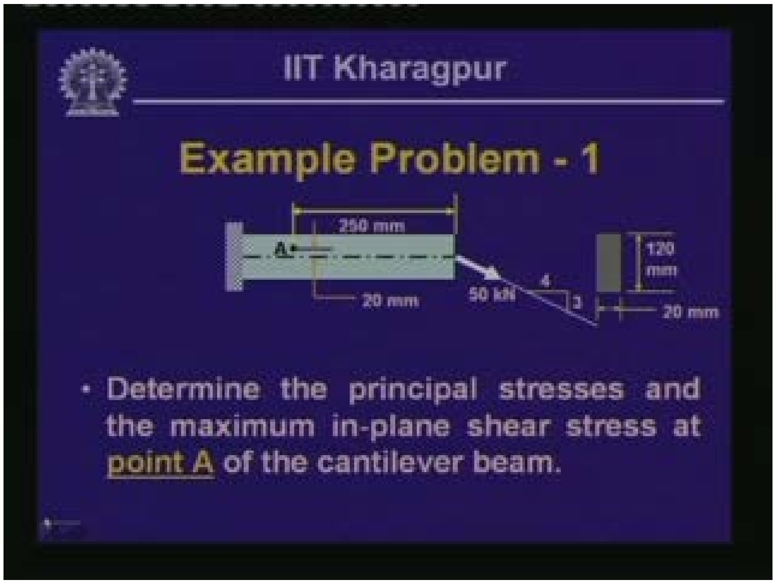


Well, let us look into the examples. this is the example which I had given to you last time wherein this is the cantilever beam which is subjected to axial pull or the axial load which is inclined at an angle of let us say theta, the magnitude of this load is 50 kilonewton and this load is acting at an inclination of theta with respect to this axis of the beam.

The cross section of the beam is a rectangular one having width of 20 mm and the depth of 120 mm. Now we will have to compute the value the value of the maximum in-plane shear stress and of course the principal stresses. So you have to compute the principal stresses and the maximum in-plane shear stresses at point A of the beam.

Now, point A as you can see is 20 mm away from the neutral axis of the beam and the point is at a distance of 250 mm from the edge cantilever beam. So we will have to compute the value of the principal stresses here and the value of the in-plane shear stress.

Now this particular load which is acting at an inclination with reference to the beam axis which is at an angle of theta can be decomposed into two directions: one is in the axial direction and another one is in the perpendicular direction and thereby this axial direction force will cause an axial pull or thereby there will be normal stress because of that and the transverse loading which is component in the vertical direction will cause bending and a shear at this particular cross section.

So first we will have to find out that what are the magnitudes of the bending moment and what are the magnitudes of the shearing force that is acting at this particular section so that we can compute the value of the principal stress and the in-plane shear stress.

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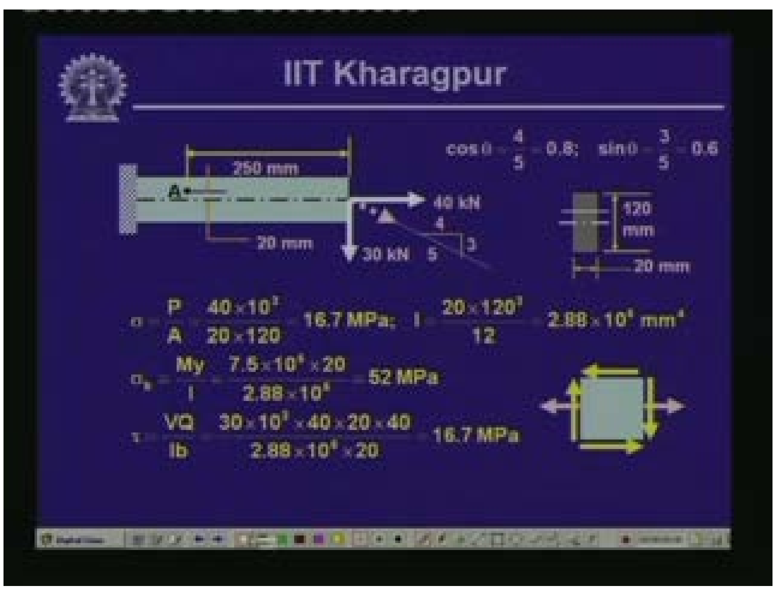


Now this is what is indicated over here. First thing is that we have decomposed this inclined load which is at an angle of theta in two directions: one in the axial direction, one in perpendicular to this axis direction. Now, since this is given as 4 and 3 so the hypotenuse is 5 thereby cos theta is equals to 4 by 5 which is 0.8 and sine theta is equals to 3 by 5 is equals to 0.6. So this value is 50 cos theta cos theta being 0.8 so this is 40 kilonewton and the vertical component is 50 sine theta which is 0.6 is equals to 30 kilonewton. So we have an axial force which is of magnitude 40 kilonewton and we have a transverse load which is 30 kilonewton which is acting at the tip of the beam.

Now what we are interested in is to evaluate the stress at point A which is located as 20 mm from the axis of the beam and 250 mm from the edge of the beam from the free end of the beam. As we have seen, the axial load 40 kilonewton will cause a normal stress at this particular cross section which is equals to P by A. So P is 40 kilonewton converted into Newton divided by the cross sectional area which is twenty times 120 mm square and thereby we get a stress of a 16.7 MPa.

Now the point A will be subjected to bending moment and the magnitude of the bending moment will be equals to the load times this distance 250. Now if we take the free body at this particular section we have a load here which is 30 kilonewton, we have the axial load which is 40 kilonewton and at this end you have the bending moment M and the shear force V. So if we take the moment of the forces with respect to this then we have moment plus thirty times 0.25 is equals to the is equals to 0. So bending moment as we get as 7.5 kilonewton per meter and this is of course negative since they are acting in the same direction; and this will give, if we call this as V (Refer Slide Time: 29:34) so V plus 30 is equals to 0 from of the summation of vertical forces is equals to 0 so that gives V as equals to minus 30 kilonewton.

Therefore, we have a shear force of minus 30 kilonewton and bending moment of minus 7.5 kilonewton per meter. Now because of this negative this will become a positive stress because sigma is equals to minus My by I and that will give us and since y is positive, since it is about the neutral axis so we will have a stress normal stress which is tensile which is arising because of the bending. And also, we have the axial pull which is a tensile force in nature so we have a tensile stress because of the axial pull.

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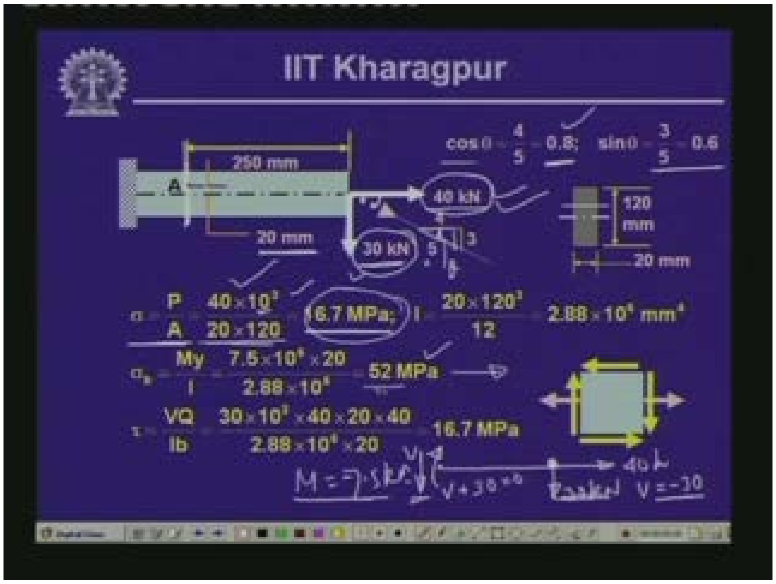


So these two stresses since they are of the same nature because they are tensile we can add them together. The normal stress because of the bending is equals My by I and as we have seen here M is equals to 7.5 kilonewton per meter into 10 to the power 6 will give you Newton millimeter times y, y is 20 because point A is located at 20 mm away from the neutral axis and I is equals to 2.88 into 10 to the power 6 which is b h cube by 12.

Therefore, once you substitute these values and the resulting value if you calculate it comes to 52 MPa. So 52 MPa is the normal stress because of the bending and 16.7 MPa is the tensile stress because of the axial pull. So the total stress that we have because of the bending and the normal stress is equals to 68.7 MPa. This is the value of the normal stress sigma. Let us call this as sigma x that is being in the x direction.

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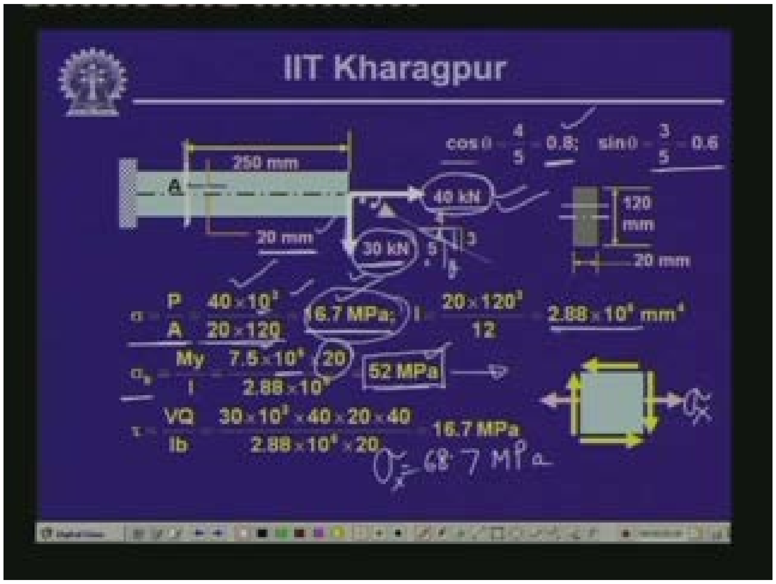


Now, in the y directions we do not have any normal stress so that is 0. The shear force which we have seen as V equals to minus 30 kilonewton will produce the shearing stress at this particular section A which is equals to VQ by Ib and V is 30 into 10 to the power 3; now Q is the first moment of area of the cross section above the section where we are computing the stress.

We are computing the stress at this particular section (Refer Slide Time: 32:01) which is 20 mm away from the neutral axis so this section being 120 the half of it is 60 so this particular part is 40 so the area of this part is 40 times 20 and the distance of the c g of this particular section with respect to neutral axis is 40. So we have 40 times 20 as the area and 40 is the distance of the c g with respect to the neutral axis of the section about which we are computing the stress. So this is Q and this is I and this is B (Refer Slide time: 32:33) B is 20 mm.

Thus, if we compute this we get the shearing stress as 16.7 Mpa. And since this is negative it is acting in the negative direction as we have taken our sign convention.

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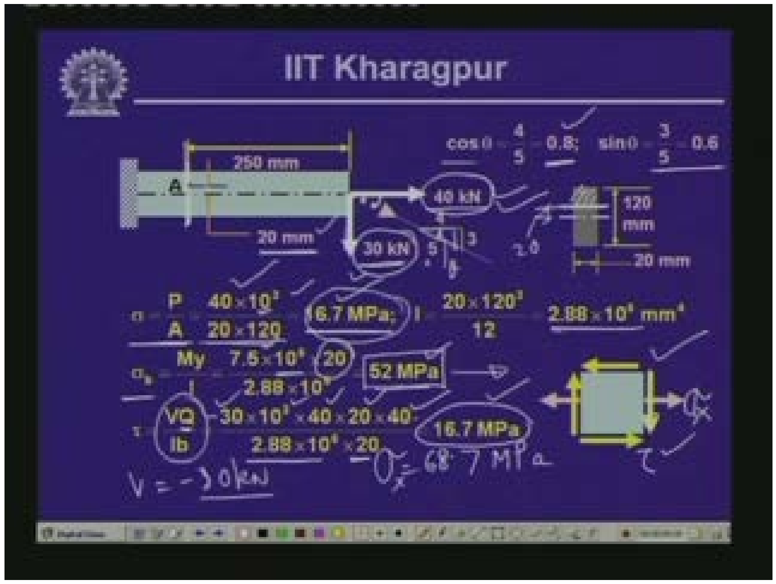


This is the state of stress that we have at that particular point A wherein we have the normal stress sigma x and the shearing stress tau. Now as you know that at a particular point when we have the combination of the normal stress and the shearing stress we can calculate the value of the maximum normal stresses and the maximum shearing stresses using the transformation equation or using the Mohr's circle.

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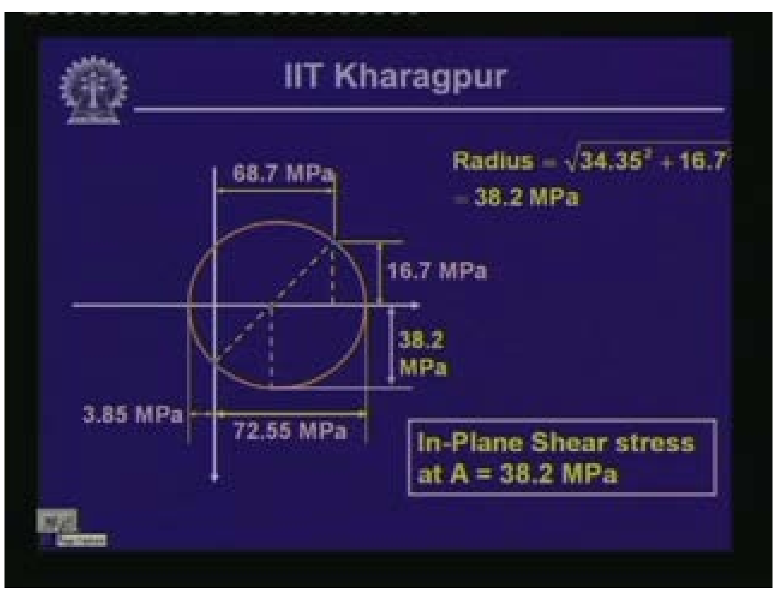

Now if we plot these stresses in the Mohr's circle then we get the stress values in this form that at this particular point where we have the negative shear stress and the positive sigma gives us this particular point; positive sigma is of magnitude 68.7 that is the total normal stress and this is the shearing stress 16.7 Mpa and on the other plane we have the shearing stress only the normal stress is 0. So if we join these two points wherever it cuts this sigma axis this gives us the center of the Mohr's circle. So with this as center (Refer Slide Time: 33:51) and if we take this as radius we get the Mohr's circle and this is the point which represents the maximum value of the stress. Now the value of the radius will be equals to the root of this distance let us say this as O, this point as A and this as A dashed. So the radius OA dashed will be equals to root of OA square plus AA dash square. Now OA is the distance which is ................ now from here to here we know the stress is 68.7 and the center divides this into two equal halves so 68.7 divided by 2 will give this particular distance because this is sigma x minus sigma y by 2 this particular is sigma x minus sigma y by 2 and since sigma y is 0 this is sigma x by 2 which is 34.35 square; and this is 16.7 square so we get the radius value as 38.2 Mpa.

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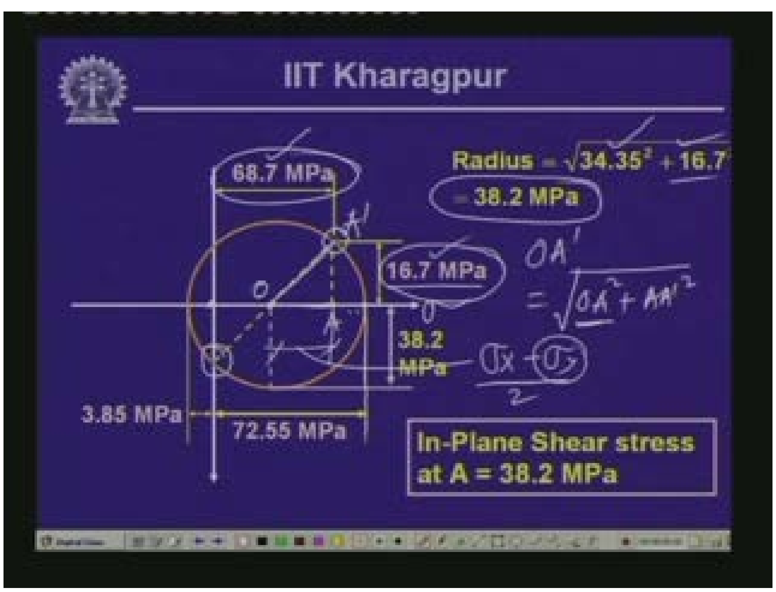

Now, when we like to compute the value of this particular stress sigma 1; sigma one will be equals to........ if we call this as O dash, sigma 1 will be equals to O dashed O plus the radius R and O dashed O as you know is s68.7 divided by 2 which is 34.35 so 34.35 plus 38.2 will give us the stress which is 72.55; this is the value of the maximum normal stress sigma 1. And consequently, from this radius if we subtract this, we get the minimum value of the normal stress which is 3.85 which is 38.2 minus 34.35. And the maximum value of the normal I mean the shearing stress is the radius of this particular Mohr's circle and this radius is equals to 38.2 Mpa. This is the value of the in-plane shear stress; so the maximum in-plane shear stress that is obtained is 38.2 Mpa.

Therefore, as you can see, that this particular stress normal stress being positive that is tensile in nature so the maximum value of the tensile stress that you have is 72.55 Mpa which is the maximum principal stress; you have the maximum value of the compressive stress which is 3.85 Mpa which is the minimum value of the principal stress and then you have the resulting in-plane stress the maximum value of the in-plane stress the shearing stress which is 38.2 Mpa which is the radius of the Mohr's circle.

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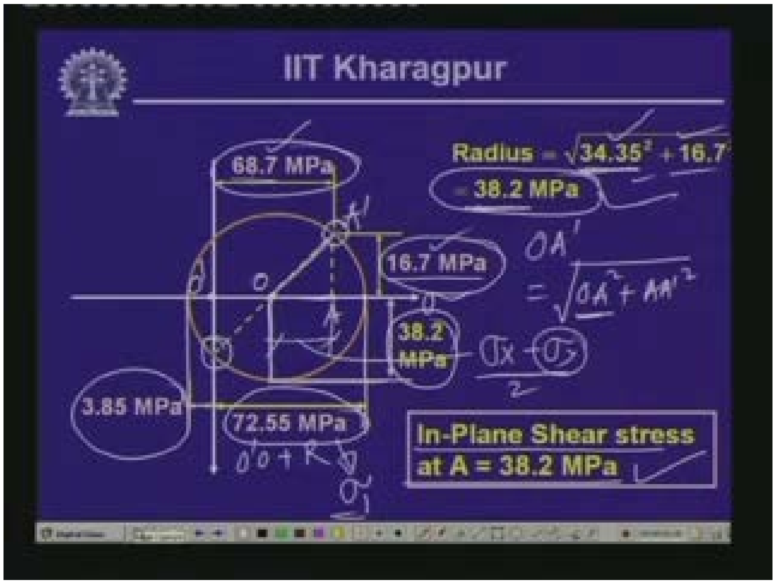

Therefore, once we know the individual stresses we can compute the resulting stress using this Mohr's circle.

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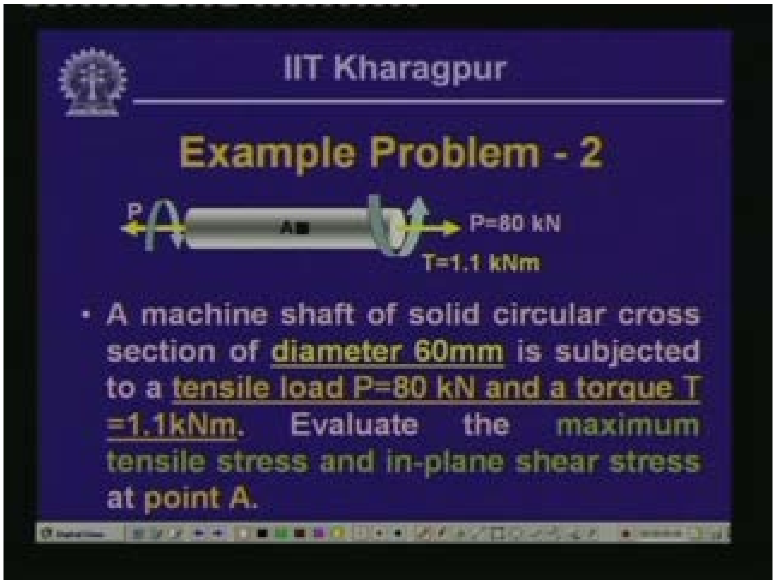


Now let us look into the second example which is the combination of the axial pull and the twisting moment. This particular bar is subjected to an axial pole of 80 kilonewton and also it is subjected to a twisting moment T which is 1.1 kilonewton meter. This particular shaft machine shaft is of solid circular cross section of diameter 60 mm. This is the circular cross section (Refer Slide Time: 37:04) it is a solid circular cross section and the diameter of this particular shaft is 60 mm. This is subjected to a tensile pull of 80 kilonewton and a twisting moment T equals to 1.1 kilonewton meter.

Now we will have to evaluate a maximum tensile stress and in-plane shear stress at point A. Here this is the point A where you have the where you will have to evaluate what will be the maximum value of the tensile stress and what will be the value of the in-plane shear stress.

As we were discussing today about the stresses that will be developed because of this axial pull and the twisting moment; as we have seen that the axial pull is going to give us the normal stress that P divided by the cross-sectional area the axial pull divided by the cross-sectional area will give us the normal stress and the twisting moment is going to produce the shearing stress. So at the point A on the surface you are going to have the normal stress and the shearing stress and if you the normal stress and shearing stress you can compute the value of the maximum normal stress and the maximum shear stress from the Mohr's circle. Now let us look into that.

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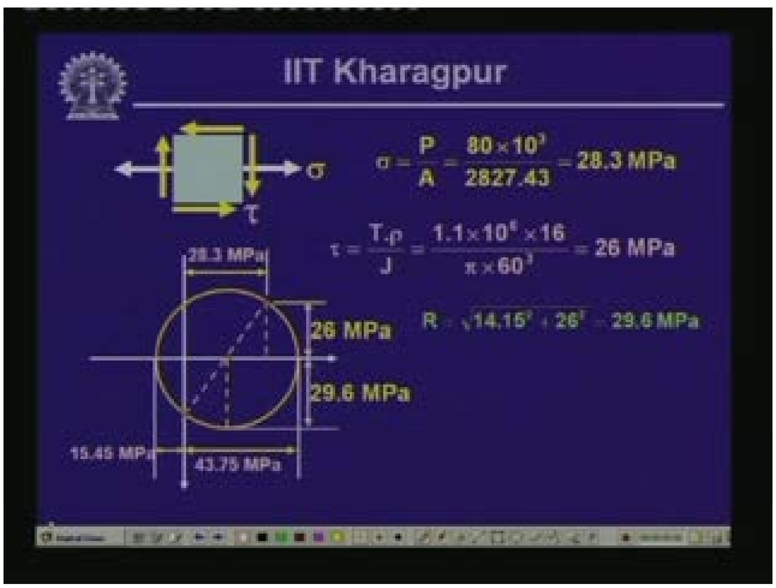


Now here you see that the normal stress which we are going to get because of the axial pull is equals to sigma that is equals to P by A and P is equals to 80 kilonewton, so 80 times 10 to the power 3 so much of Newton divided by the area; now here, since the diameter of the cross section is 60 mm and it is a solid circular cross section so area is equals to pi by 4 times 60 square and that gives as the value of 2827.43. So the value of normal stress sigma is equals to 28.3 MPa and the value of the shearing stress tau as it gets developed because of the twisting moment T is equals to T times rho by J and in terms of the diameter if we write tau as equals to T; rho as d by 2 and J as pi d 4 divided by 32.

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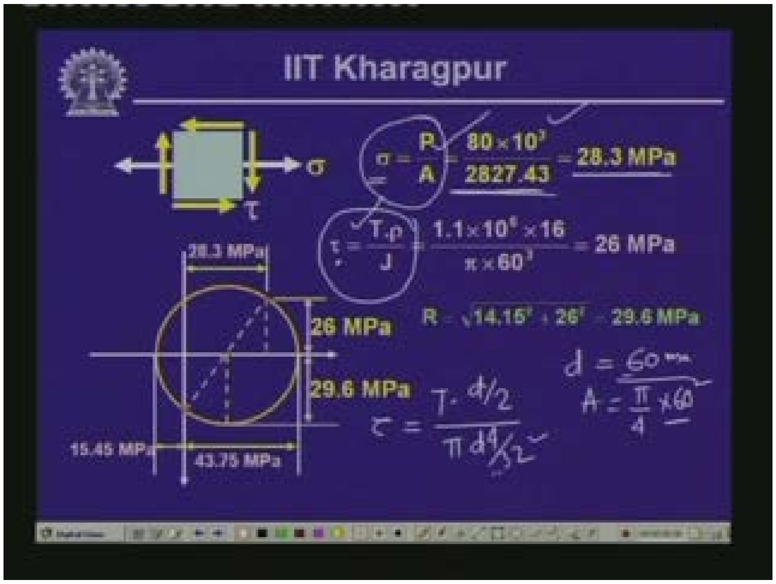


As you know that polar moment of inertia for the circular section is pi d 4 by 32 and rho is the maximum distance from the center which is d by 2 over here so we are going to get the maximum shearing stress which is d by 2. So, eventually this gives us 16 T by pi d cube.

Now this is what has been used over here. So 16 times T where T is 1.1 into 10 to the power 6 divided by pi times d cube and d is 60 that gives us the value of 26 MPa. This is the value of the shearing stress and this is the value of the normal stress (Refer Slide Time: 40:04) that is developed at point A because of the action of the axial tensile pull and because of the twisting moment. Therefore, because of the actions of these two individual loading which are acting simultaneously, we are getting the stresses the normal stresses and the shearing stress of this much of magnitude.

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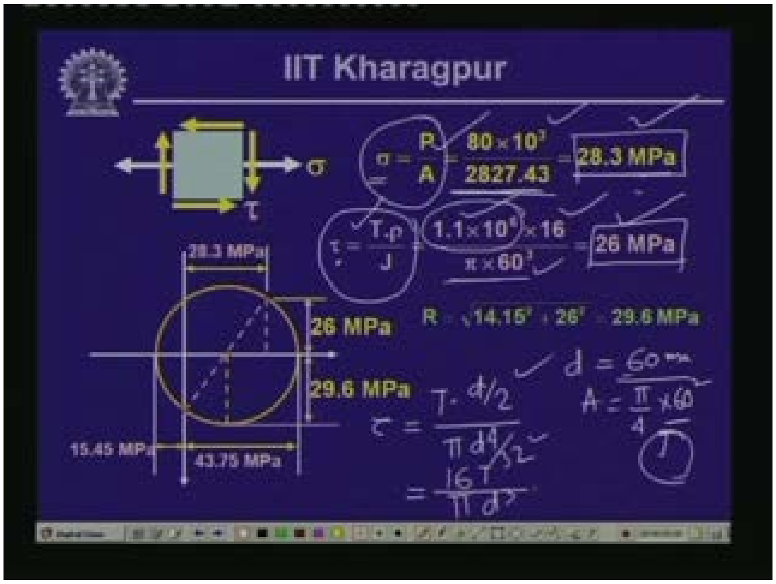


We get a normal stress of 28.3 Mpa and shearing stress of 26 Mpa. Now if we plot these stresses in the Mohr's circle and the values or the directions are shown over here; sigma being a tensile stress is positive and tau here is giving shearing stress which is in the negative direction.

Now if we plot this particular stress distribution sigma and tau, that represents this particular point where sigma is equals to positive 28.3 and tau is negative 26 Mpa and on the other plane we do not have the normal stress but the shearing stress is present which is also equals to 26 Mpa and if we join these two points together we get the center of the Mohr's circle. So taking this point as the center let us call that as O and OB as the radius we draw the Mohr's circle and this particular point represents the value of the maximum normal stress which is represented by this particular distance which is equals to.............. as you know this particular distance which we call as OO dash and OO dash is equals to 28.3 by 2 because sigma y is 0 and this distance being sigma x minus sigma y by 2 so we have 28.3 by 2 which is 14.15 and this is also 14.15 and the shearing stress here is 26.

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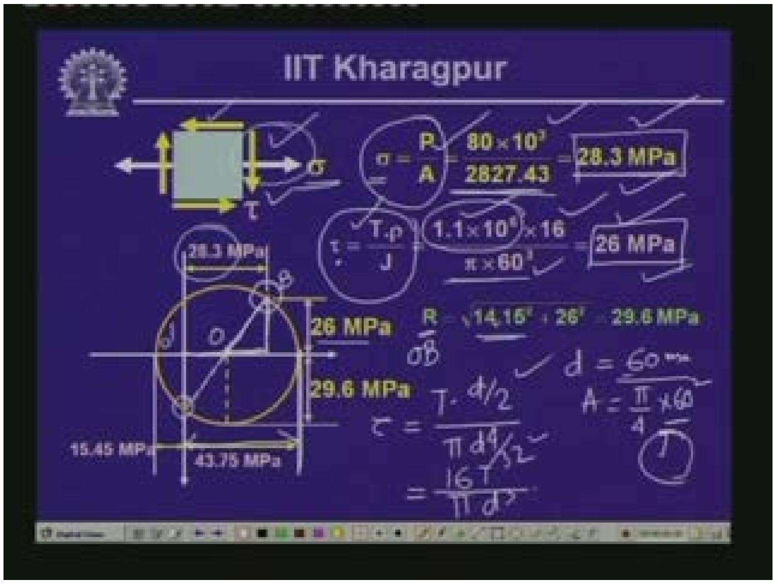


The radius OB, this is OB, this is equal to 14.15 square plus 26 square and we get a value of 29.6 MPa. So the value of this distance is equal to distance OO dashed plus the radius and OO dashed is equals to 14.15 and 14.15 plus we have the radius which is equals to 29.6. So if we add that then we get the value which is 43.75 so much of MPa; this we get for the maximum principal stress and this being positive this is the tensile stress that will be occurring in the member at that particular point and this is the minimum principal stress which is equals to the radius 29.6 minus 14.15 and that gives us a value of 15.45 Mpa.

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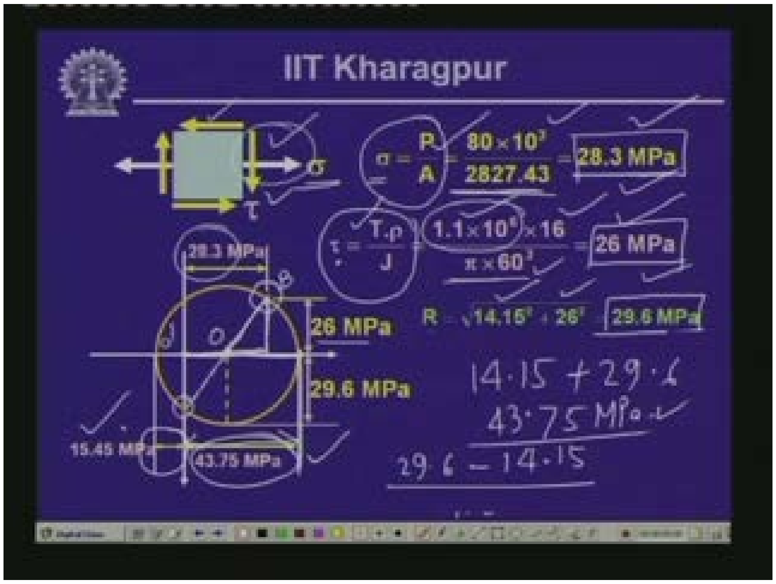


Hence, the maximum tensile stress that is occurring at that particular point is 43.75 Mpa, the maximum compressive stress that is occurring is 15.45 Mpa and the maximum value of the shearing stress, the in-plane shearing stress that is occurring which is the radius of the Mohr's circle is equals to 29.6 MPa; these are the values which we wanted to have.

As you can see, that because of the action of the axial pull of 80 kilonewton and a twisting moment of 1.1 kilonewton meter, the point A on the surface of this particular part is subjected to a tensile stress which is equals to 43.75 MPa and in-plane shear stress which is equals to 29.6 MPa. These are the stresses that is acting on this particular member.

And if you recollect from the previous lesson, we had shown you that this particular type of force distribution or the load combination comes in the case of a shaft which is used in the helicopter fan which moves; you know, it is subjected to a twisting moment and because of this the lift is subjected to an axial pull as well.

Therefore, if we like to compute the value of the stresses then we get the stress in this form in such shafts.

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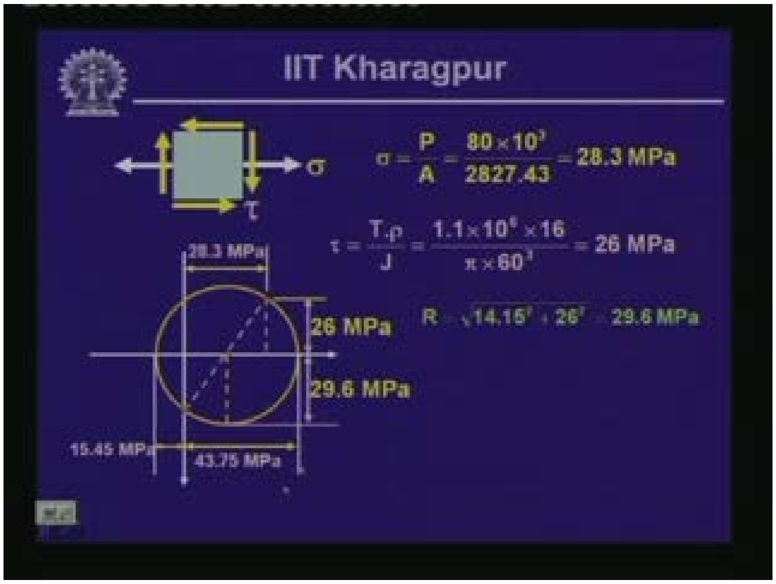


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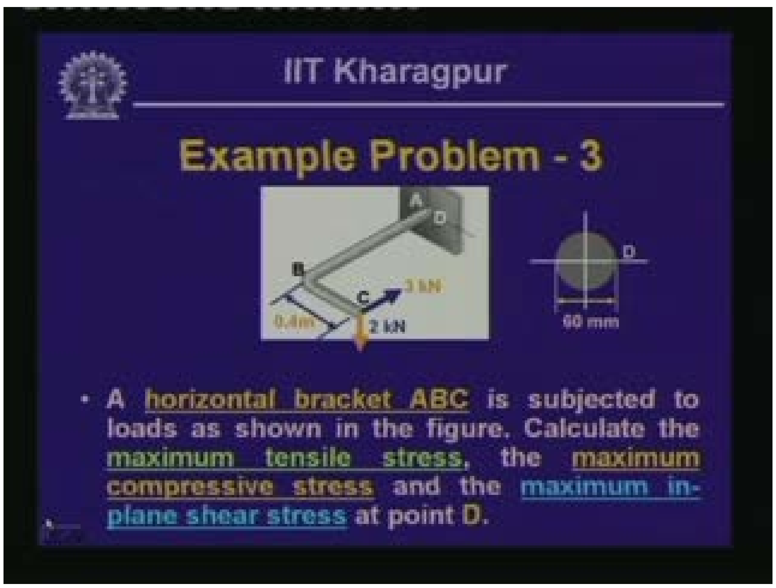

Well, let us look into another interesting problem where this is a horizontal bracket A b and C, it is in the horizontal plane; now this is subjected to a load at the tip; one is a vertical one which is of magnitude 2 kilonewton and another one we have a load which is 3 kilonewton which is acting parallel to the arm AB. Now what we need to do is that we need to compute the value of the stresses at a point D which is at the support which is at A.

Now, if we look into the cross section, the cross section again is a solid circular one with a diameter of 60 mm and we are interested to find out the stress at this particular point D. Now what we will have to do is that we will have to compute the maximum tensile stress maximum compressive stress and maximum in-plane shear stress; you will have to calculate these three quantities at point D for this particular member.

Now as you can visualize that this particular forces which are acting at the tip, if we transfer these forces at point B they will be associated with some moments.

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Now let us analyze the forces first that how we transfer these forces to this tip of this particular beam. This bracket A B C we can reduce it to a cantilever beam AB which is fixed at A and free at B; and in the process what we can do is we can shift this end loading to the point B which is the tip of this cantilever part.
Now the vertical force which is of magnitude 2 kilonewton if we shift at point B then this is associated with a moment which is going to be a twisting moment. so this vertical force 2 kilonewton is transferred at B with a vertical force 2 kilonewton and a moment T which is going to be a twisting moment for the bar AB.

The horizontal force which is of magnitude 3 kilonewton if we shift to this point B this gives us an axial thrust to member AB and is associated with the moment which is a bending moment at point B with magnitude 3 kilonewton times this distance which is equals to 0.4m. Then this particular member AB............. now if we disregard the member BC the part BC of the bracket, the member AB is subjected to a vertical load of magnitude 2 kilonewton, a horizontal load of magnitude or axial load of magnitude 3 kilonewton and a twisting moment T at the tip and a bending moment M at B.

Now the vertical load which is acting at the tip at point B of magnitude 2 kilonewton is also going to produce a bending moment at A. Now if we look into the cross section you will find that load P will cause a bending moment about the Z axis. Now already we have a moment M which is acting about y axis. We have a moment which is acting about y axis; load P is going to cause a moment about z axis and we have T which is a twisting moment which is acting about the axis which is perpendicular to the board. Along with that we have the axial thrust which is of magnitude 3 kilonewton. We have an axial compressive force which is equals to 3 kilonewton. Now the interesting part is that we are interested to compute the value of the stress at point D.

Now if you look into this particular moment which is being produced by load P of 2 kilonewton, now this is being the neutral axis, the stress on this because of this moment will be 0. So at point D the moment which is being produced by 2 kilonewton does not have any effect. So this particular point will be subjected to a compressive stress because of the axial compressive force, will be subjected to a compressive stress because of the bending which is acting about y axis and will be subjected to shearing action because of the twisting moment T that is acting at the end B. so let us compute And also at the end A because of this vertical load 2 kilonewton there will be a shearing force component which will be producing the shearing stress. So there will be shearing stress produced because of the twisting moment T, there will be shearing stress produced because of the shear force V, there will be bending stress because of M which is acting about Z about y axis and there will be normal stress because of the axial compressive force.

The normal stress sigma because of the axial compressive force is equals to the axial compressive force divided by the area which is equals to 1.061 MPa. Now the bending stress is equals to My by I. Now the bending moment as we have seen is 3 kilonewton horizontal load times 0.4 mm 0.4m is the leverom so 3 times 0.4 gives you 1.2m kilonewton meter as the bending moment, so 1.2 into 10 to the power 6 into y is 30 because we are computing from the neutral axis at a distance which is equals to 30 so My and I is the moment of inertia which is equals to pi d 4 by 64 which gives us this value.

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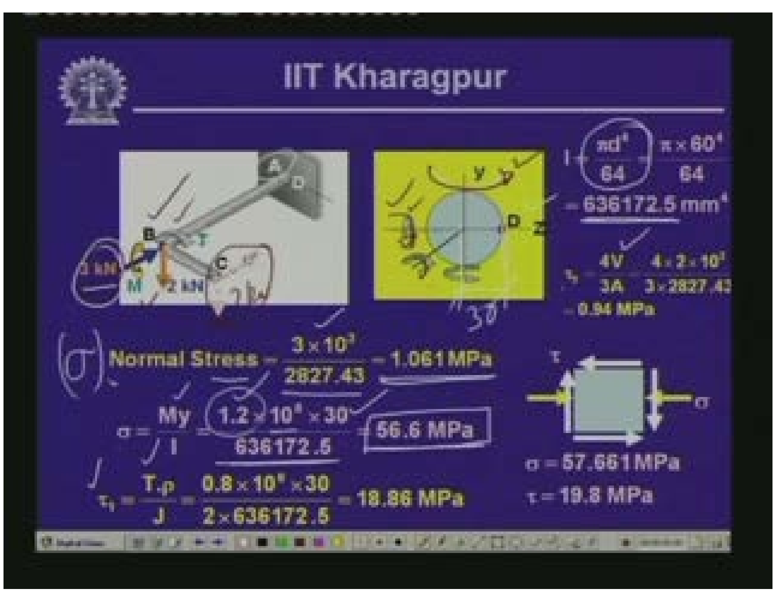

So once we substitute that we get the bending stress as 56.6 MPa which is compressive at this particular point. So we have a normal stress which is compressive, we got a normal stress from the bending which is compressive, so if we add these out we get the total normal stress sigma which is equals to 57.661 MPa.

Now we have the shearing stress tau which is arising from the twisting moment T and twisting moment T is equals to 2 into 0.4m which is equals to 0.8 kilonewton per meter and rho is again the distance which is 30 and J is twice the I so we get the stress as 18.86 MPa. That is the shearing stress which is arising from the moment.

Also, the shearing force at this end is equals to this 2 kilonewton. So, for the 2 kilonewton as we know, that the shear stress distribution across the diameter of a circular cross section as we have seen in on the module of the shear stress that the values equals to four third V by A. So if we compute the value tau 2 equals to 4V by 3A we get the magnitude of the shear stress as 0.94 MPa.

Hence, if we add these two tau 1 and tau 2 we get the resulting shearing stress that is acting at D is equals to 19.8 MPa. So you see this is the element which is at D which is subjected to a normal compressive stress of this magnitude and a shearing stress of this magnitude.

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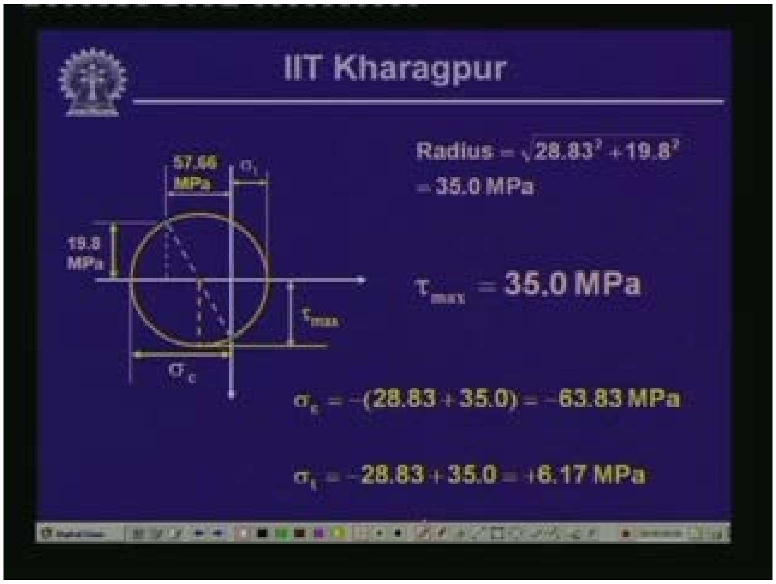


Now if we plot these stresses in the Mohr's circle then we get the stresses like this that we have.................. at this point sigma which is negative and tau also is negative so this is the point which represents the plane where you have the normal stress and the shearing stress and this is the plane perpendicular plane where we do not have any normal stress but the shearing stress again is 19.8 and if we join these two points together we get this as the center where it cuts the sigma axis and considering this as the center and taking this OA as the radius if we plot the circle we get the Mohr's circle of stress and this is the stress which is the maximum compressive stress and this is the stress which is going to give us the maximum tensile stress.

Now the value of the radius OA is equals to as we have seen is half of this 57.66 and 19.8 and if we take these two values 57.66 divided by 2 will give you 28.83 that square and vertical distance is 19.8 square so this is going to give us the value of the radius which is equals to 35 MPa. This is the value of the radius of this Mohr's circle.

Now what we need to do is that we need to compute the value of the maximum compressive stress and the maximum tensile stress. The maximum compressive stress as you can see from this particular diagram this is equals to let us call this as point B so OB is equals to the half the stress of 57.66 plus the radius. Now let us call this point as O dash so OO dash is equals to 28.83 which is half of 57.66 and OB is equals to the radius which is equals to 35.

So if we add these two 28.83 plus 35 you get the stress which is the normal stress and that is on the other side of the positive sigma axis so this is negative and this we call as the compressive stress. this is the magnitude 63.83 MPa is the compressive stress that is occurring at that particular point and then the other side the positive value of the Mohr's circle which we get the normal stress which we get on this side which is of positive magnitude so 28.83 is this particular distance (Refer Slide Time: 54:26) and the radius is equals to 35 and if we add that we get 6.17 MPa; this is the value of the maximum tensile stress.

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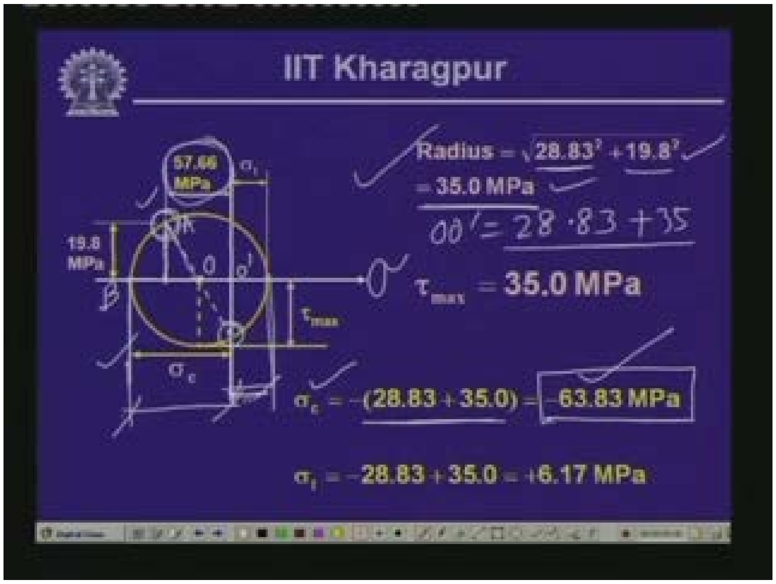


Hence, at the member, at that particular point here the magnitude of the maximum compressive stress is 63.83 MPa and 6.17 is the 6.17 MPa is the maximum tensile stress. Now the value of the maximum shearing stress in-plane shearing stress if you look into will be given by the radius of this particular circle which is given by this particular distance (Refer Slide Time: 55:03) and this is what is equals to the maximum value of the tau. That is on the tau axis we get this as the point, this is the maximum value of the shearing stress and tau max is equals to the radius which is equals to the 35 MPa.

Therefore, we have these three quantities now: the maximum compressive stress as 63.83 MPa, maximum tensile stress as 6.17 MPa and the maximum shearing stress as 35 MPa. These are the values at that particular point when the bracket is subjected to tip loads: one in the vertical direction, another in the horizontal direction which is parallel to AB, now these two forces compute the stresses at the support which gives you the maximum tensile, maximum compressive and the shearing stresses of the magnitudes as we have calculated over here.

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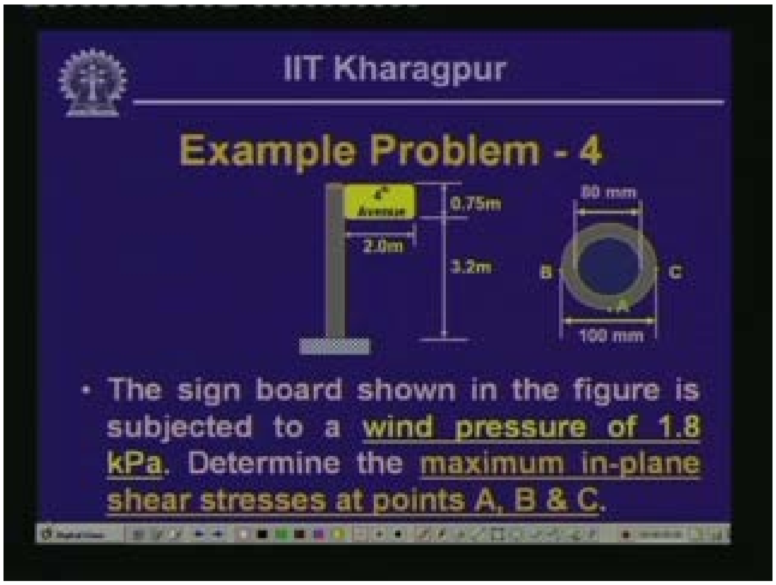


Well, we have another example problem over here. Now this is the sign board as we have discussed last time that many a times we use these signs for giving the directions and when the wind forces act on such signs it produces combined force actions on the vertical members and this particular board is subjected to a wind load of 1.8 kilo Pascal; you will have to find out the maximum in-plane shear stresses at three points A, B and C. So you solve this problem; I am going to discuss this in the next lesson.

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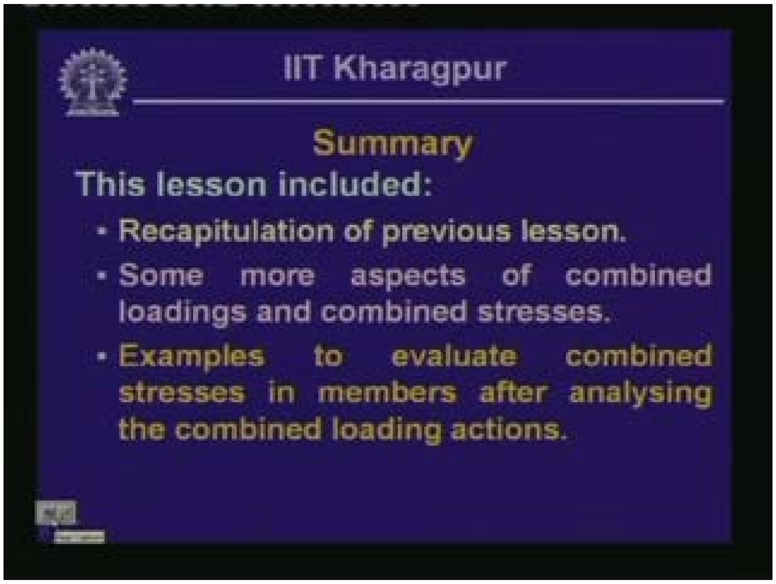


Well, then to summarize, in this particular lesson we have looked into the aspects of the previous lesson. Now we have looked into some more aspects of the combined loading actions. In the last lesson we had discussed that, what are the forms of different combined loading that a member is subjected to. Now in this particular case we have looked into some more aspects of such combined loading and therefore and thereby we have looked into some examples to evaluate combined stresses in member after analyzing the members for the proper loading.

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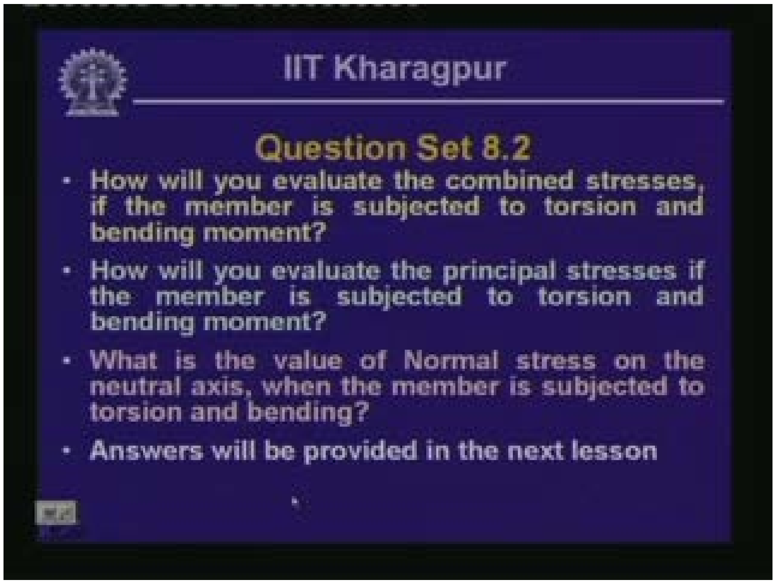


Now these are the questions given for you. How will you evaluate the combined stresses in the member if the member is subjected to torsion and bending moment? How will you evaluate the principal stresses in the member if the member is subjected to torsion and bending moment and what is the value of normal stress on the neutral axis when the member is subjected to torsion and bending? Well, we will discuss this in the next lesson. Meanwhile you can go through the lesson and look into these questions, thank you.

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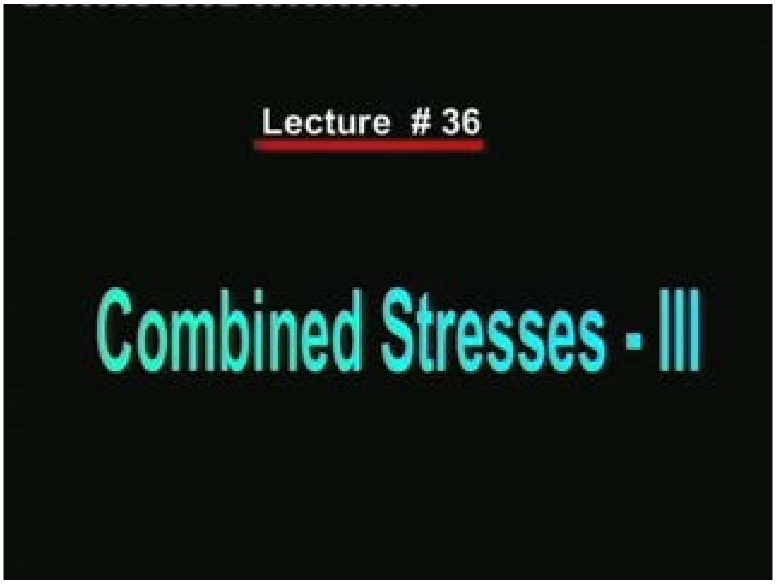


Welcome to the third lesson of the eighth module which is on combined stresses part III. Now, in the last two lessons of this particular module we have looked into that we have discussed every aspects of the combined loading and thereby we have evaluated the combined stresses in members when they are subjected to different forms of combined loading.

Now we have discussed that if a member is subjected to axial load and bending then what happens to the combined stresses or if a member is subjected to a twisting moment and a normal axial force then what happens to the stresses or if a member is subjected to the combined loading actions of the twisting moment and the bending moment or the shear force then what happens to the combined stresses. Those aspects we have looked into.

Now in this particular lesson we are going to look into some more aspects of combined loading where if a pressure vessel which we have earlier analyzed for the pressures only; now if they are subjected to the external forces like the axial pull or the compressive force or if they are subjected to twisting moment or if the whole vessel a cylindrical vessel is supported on two supports and thereby some bending is induced into the member then in addition to the stresses that is being induced because of the pressure inside then what happens to when they are subjected.....................