Strength of Materials

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Lecture - 34

Combined Stresses – I

Welcome to the first lesson of the eighth module which is on combined stresses part I. Or most precisely let us call it as the stresses which are developed due to the combined actions of loading in the members. In fact, so long we have discussed the action of different kinds of forces in the member which are acting in an individual form. Say for example when we have discussed about the axial force in a bar the force was acting normal to the section and we have computed the corresponding stresses which we have called as normal stress.

Subsequently, we have looked into that if a pressure vessel is subjected to pressure we have computed the stresses on the outer shell which is in the circumferential and the longitudinal direction. Subsequently, we have computed the stresses in a bar which is subjected to another kind of force which we have called twisting movement. And then lastly we have evaluated the stresses in the bar corresponding to another kind of force which we have called as the bending and the shear force. We have seen how to compute the bending movement and shear force in a beam which is subjected to transverse loading and consequently we have computed the values of stresses which we have termed as bending stress and shear stress.

Now, in all these cases you have noticed that we have computed these stresses for the individual loading thereby the stresses were for those individual loading access. Now we are going to consider the case where if these loads act simultaneously in a particular member what will be the consequence of the simultaneous actions of this loads and what will thereby be the combined stresses in the member.

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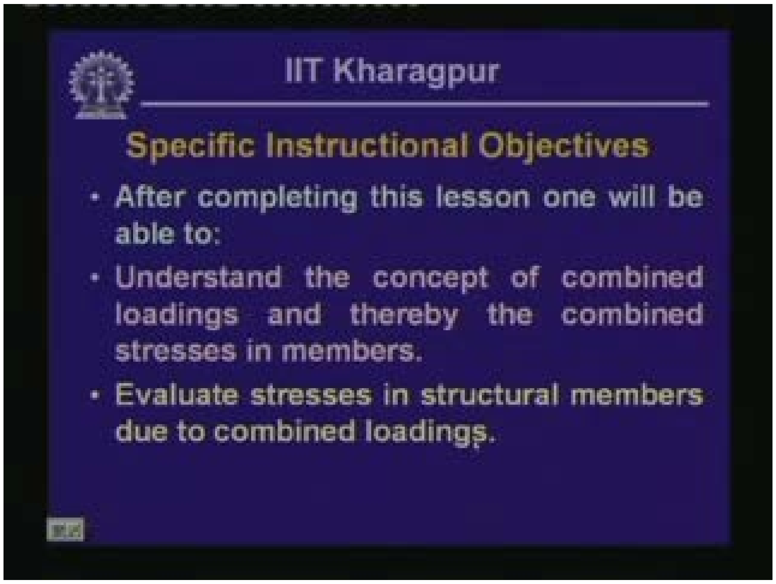

Hence, once this particular lesson is completed one should be in a position to understand the concept of combined loadings and thereby the combined stresses in members. Also, one should be in a position to evaluate stresses in structural members that are developed due to such combined loadings.

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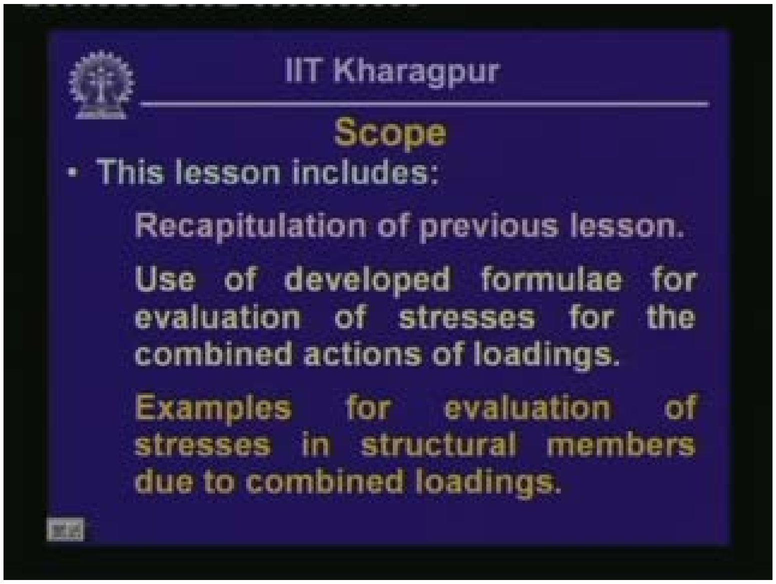

The scope of this particular lesson includes therefore, well, as we do the recapitulation of previous lesson we will be looking into the answers of the questions which I had posed last time and in the process we will recapitulate some of the aspects of the previous lesson. In this particular lesson we will make use of the developed formulae for evaluation of stresses for the combined actions of loadings. Now as we have noticed that in earlier cases for individual loading situations we have arrived at different stress formulae.

Say for example, when a member is subjected to a twisting movement we have seen that what is the relationship between the shearing states and the torsional movement. Consequently, we have looked into that if a member is subjected to bending then what is the relationship between the bending stress and the bending moment and also the relationship between the shearing stress with the shear force.

Now all these aspects we have looked into as I said in an individual form. Now in this particular lesson we look into, if there are actions of these loadings which are acting simultaneously or which are getting developed simultaneously because of some practical situations now how do we compute the values of the stresses because of such combined loading forms.

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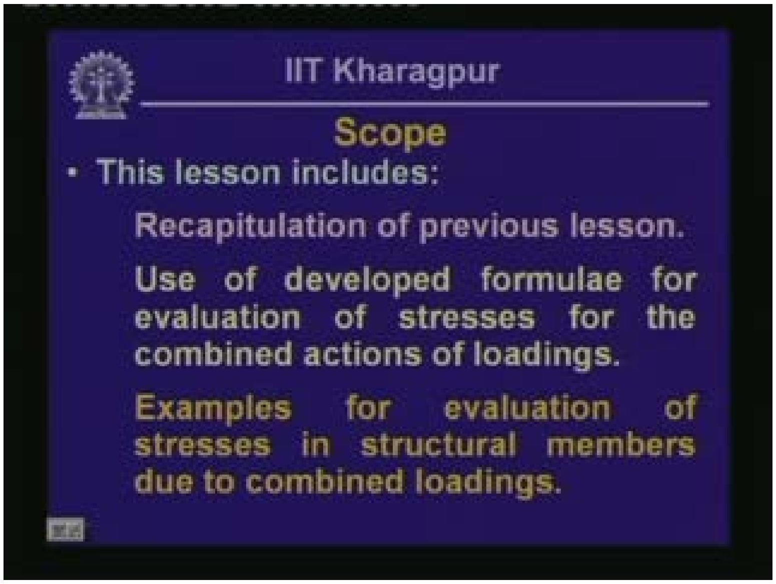

Also we will look into the examples for evaluation of stresses in structural members due to such combined loadings.

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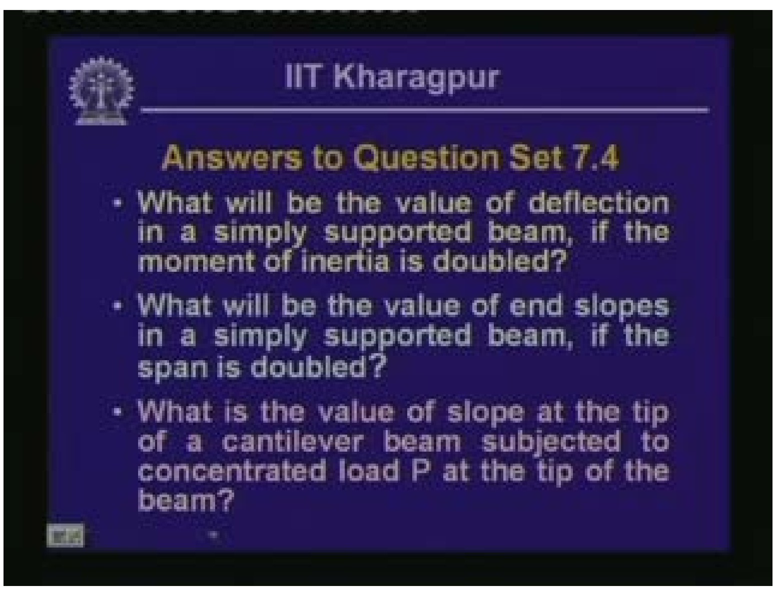

Well, let us then look into the answers to the questions which were posed last time. The first question was what will be the value of deflection in a simply supported beam which is simply a supported beam if the moment of inertia is double.

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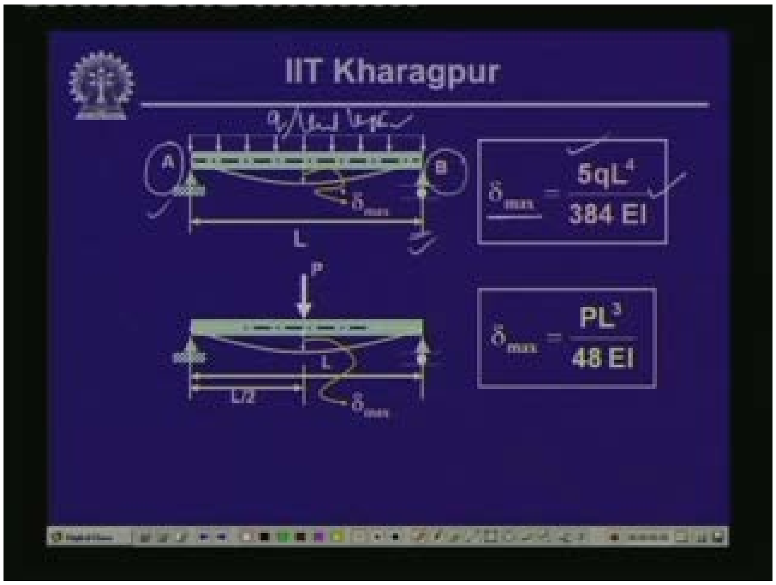

Now you must have noticed that while computing the value of the deflection we make use of the moment of inertia of the cross section and these are the expressions for the deflection values which we had evaluated in the previous lesson. Now if we have a simply supported beam which is hinged at n a and supported on roller thereby it is a simply supported beam which is subjected to uniformly distributed load. Let us say the intensity is Q per unit length.

Now, for this kind of loading on a simply supported beam the value of the maximum deflection delta is 5qL 4 by 384EI this we have seen, we can evaluate this either using the difference in equation for the beam deflection or we can make use of moment area theorem and we can compute the value of this deflection which we have seen in the previous module.

Also, if the same simply supported beam if it is subjected to a concentrated load at the center then the value of the maximum deflection is PL cube by 48EI this also we have seen in the previous module.

Now, if you look in to these two expressions here the moment of inertia value which is I comes at the denominator of this expression and the delta value is inversely proportional to the moment of inertia I. Therefore, irrespective of the loading what you have on the beam if this moment of inertia is doubled that instead of I it is twice the I then the value of the delta will become half than what it was earlier and so is the case in this particular situation.

So this particular question which was posed which is quite general in nature wherein we have not talked about the loading on the beam, we have not talked about the spanner such or the length of the beam but what we said is that if the moment of inertia is double then what is the impact of that on the deflection. Now, as we can see from this expression of the deflection, whether it is uniformly distributed load or a concentrated load, if the moment of inertia is doubled the deflection is going to be half of the previous value. This is what is the relationship between the deflection and moment of inertia.

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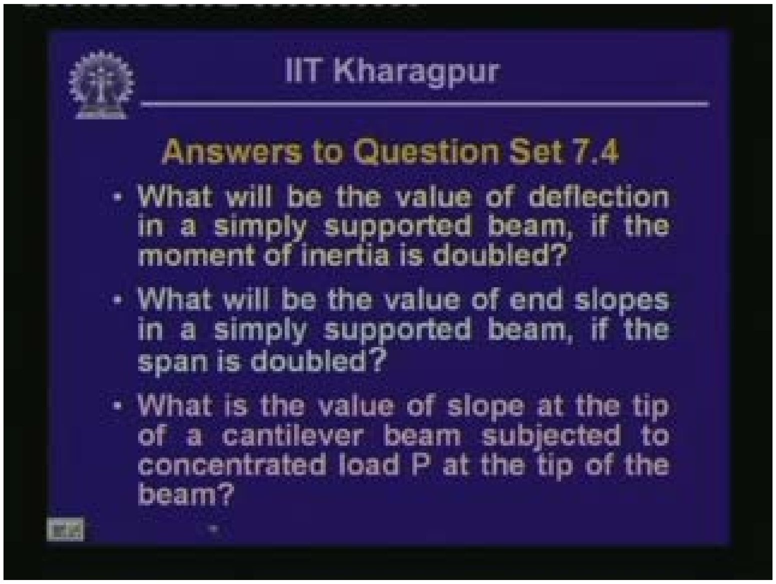

Consequently, the second question was what will be the value of end slopes in a simply supported beam if the span is doubled. This particular question also is quite general because we are talking about that that if the length of the beam or the span if it is doubled then what is the consequence on the end slopes that will be generated in the beam.

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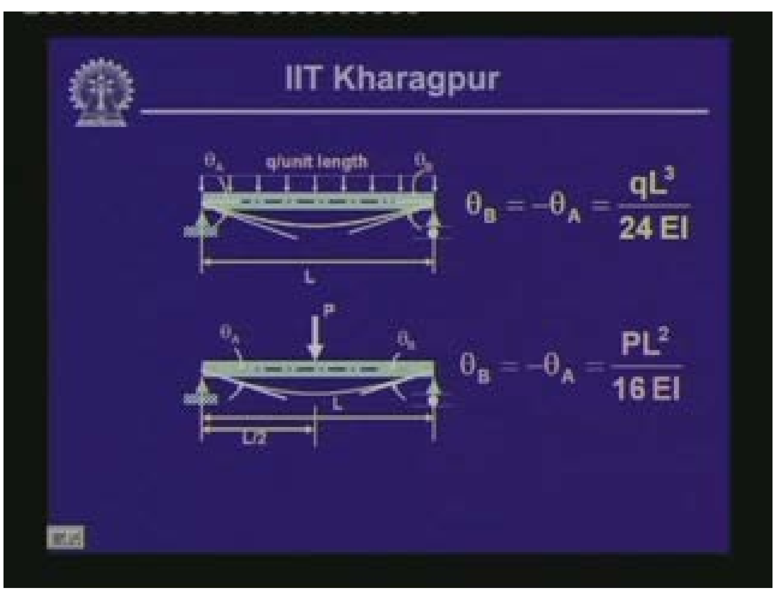

Here if you look in to, see the values of the end slopes as we have computed for these two different cases when the beam is subjected to uniformly distributed load of magnitude Q per unit length we have computed the value of theta B and theta A and we have seen the magnitude is qL cube by 24EI. Now if the same simply supported beam if it is subjected to a concentrated load at the center the value of the end slope comes as PL square by 16EI.

Now if you look into the expressions carefully you will find at here the span is having a power of 3 and here the span as a power of 2. So it is very difficult that way to say that you know in a general form that if the span is doubled then what will be the consequence of the slope. Now we can say that if the beam is subjected to uniformly distributed load then the end slope will be.......... since L becomes twice L so it will be eight times of the previous value whereas for a concentrated load if L becomes twice L then it will be four times of the previous load.

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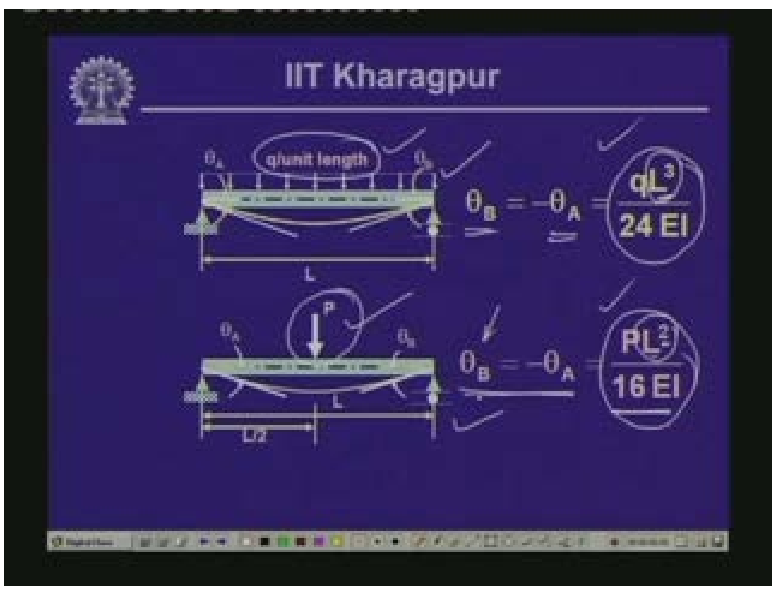

Therefore, what we can say is that if the span is doubled then the value of the end slopes are going to be increased or is going to be more than the previous value but it depends on that what kind of loading you have on the beam. If you have a uniformly distributed load as we can see over here it gets eight times of the previous value whereas when you have a concentrated load at the center it becomes four times of the previous values. Now, depending on the type of loading we have the values of the slope will change and accordingly if the span is doubled it will be multiplied by a factor but it is quite obvious that the slopes are going to be larger if the span is double.

The last question we had was; what is the value of slope at the tip of a cantilever beam subjected to concentrated load P at the tip of the beam.

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The cantilever beam as you know is fixed at one end and is free at the other and is subjected to a tip load P at the end of this beam and we had computed that the value of theta B the slope at end B is equal to PL square by 2EI as you can compute it very quickly using moment area theorem.

Now, as you know, the bending moment diagram if you draw, it is a triangular one having the value of this particular ordinate as PL by EI, this is the M by EI diagram, and as you know that the slope of this particular end (Refer Slide Time: 10:34) with reference to the tangent drawn at this particular point is nothing but area of the M by EI curve between A and B which is equal to half times PL by EI multiplied by the length and that gives you PL square by twice EI. Thus the slope and the negative sign indicates that it is moving in a clockwise form. According to our sign convention this clockwise rotation is negative. This is the magnitude of the end rotation when a cantilever beam is subjected to concentrated load at the tip of the beam.

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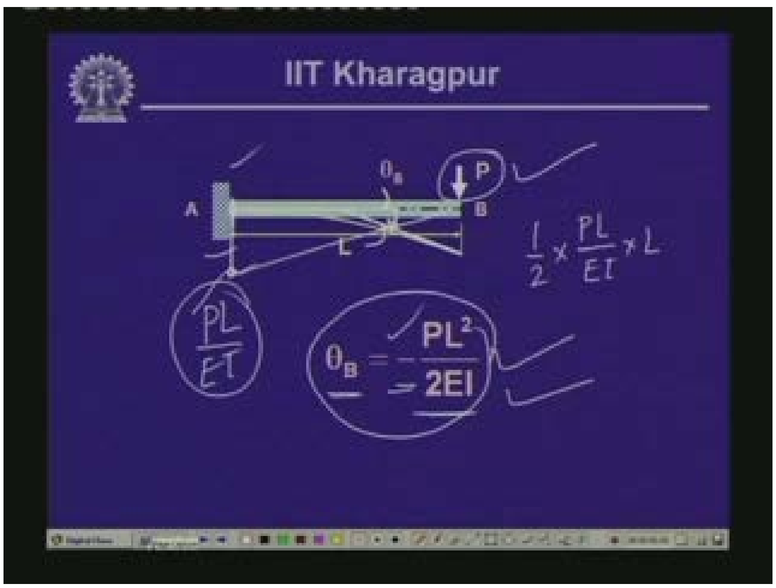

Well, these were the answers of the questions which were posed last time.

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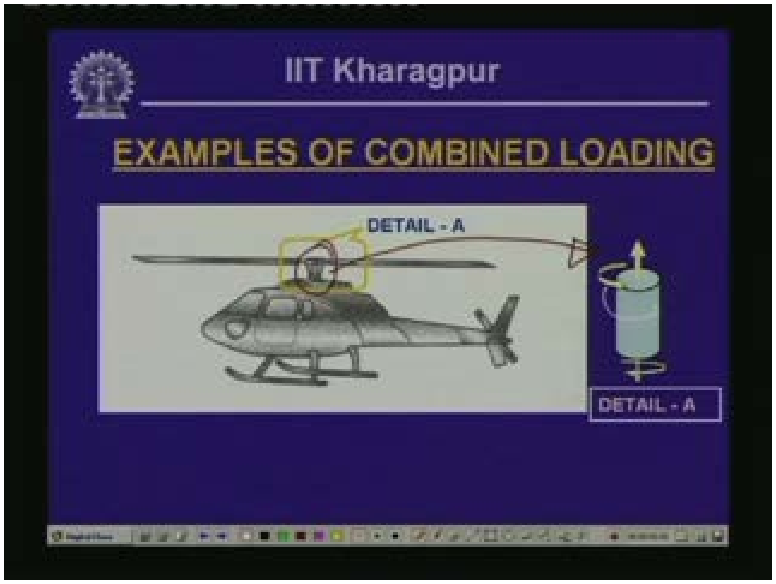

Now let us look into that if the members are subjected to............ or on what situations a member is subjected to the combined actions of loading. this is an example where this helicopter blades they are connected to this rotor, at this particular point there is a shaft which is like this and this particular shaft when this fan rotates gives a twisting movement to this shaft and also in the process when the fan moves then gradually it gets lifted up and the shaft experiences a pull in the vertical direction

So this particular member if we look into the shaft part of it, it is subjected to a load P which is an axial pull at the same time it is subjected to a twisting movement. So, at a particular point of time the shaft may be subjected to the combined actions of this axial pull and the twisting movement T.

Now, as you have seen from the previous cases when we have discussed individually these loading actions when a member is subjected to axial pull it is subjected to a normal stress which is P divided by the cross-sectional area.

Also, if a member is subjected to a twisting movement then there will be shearing stresses generated as we have already observed. Now when you have a combination of these two that means the member is subjected to an axial pull and at the same time it is subjected to a twisting movement that means there will be both normal stress as well as the shearing stresses. So we will have to compute these individual stresses and then combine them together suitably so that we can arrive at what could be the maximum possible stress that can generate in the member at any point. And as designers, we are concerned with that; what is the maximum possible value of this particular of any stress that can generate in that particular member; and we must safeguard the member against that maximum value of the stress.

Now let us look into another example where we come across the combined action of the loading. Here you see this beam member is hinged at this particular end, is pinned and at this particular end it is connected with a cable. The member is loaded in the transverse direction let us call by the load P 1 P 2 and so on. Now if we take the free body of this particular member, if we take a curve in this cable and if we take the free body of this particular part now the end reactions will be represented by the horizontal a vertical and the horizontal reactive forces then we will have this force which will be acting in the cable member and if we take the component of this cable, one will be in the vertical direction let us call this as F 1, another will be in the horizontal direction let us call that as F 2.

Now, this horizontal force the vertical force, now when we take the summation of horizontal force is equals to 0 we get the value of the horizontal reaction which is equivalent to F 2 so H is equal to F 2 and this force is acting along the center line of this member or the centroid of this member and thereby this is going to cause........... if I take any cross section then the stress at that particular section because of this load will be the axial load if we call that as H divided by the cross-sectional area and that is the normal stress that we will have.

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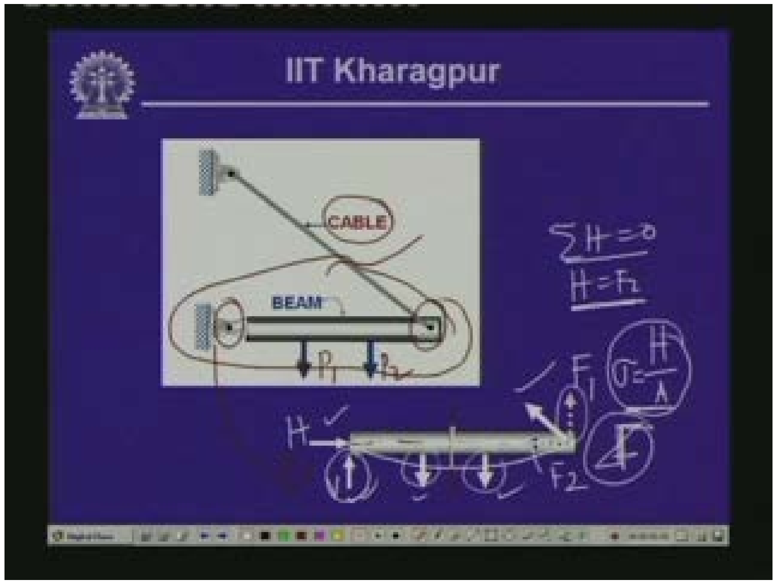

Now, apart from that, since this particular beam is acting with these supports; the vertical reaction here and the vertical component of this particular force also is going to hold this particular point so the whole member will undergo a deflection because of this load and there will be bending movement and shear force developed in this particular beam and because of the bending and the shear there will be bending stresses; and bending stress as you know, again is a normal stress which will be acting in the section in terms of the compression and tension so there will be compressive forces generated because of this axial force, there will be compressive and tensile stresses generated because of bending and there will be stresses generated because of the shear.

So at any section if we look into there will be normal stress, there will be the normal stress because of axial force, there will be normal stress because of the bending, there will be shearing stress and finally we will have to combine this normal stress and the shearing stress to find out what is the value of the maximum stress which you call as the principal stresses.

As we have seen in module 1, if you have the normal stress and the shear stress how do you compute the values of the principal stresses or the maximum normal stresses. As you can see here, now it is not only the axial force or the bending stresses alone, we have the actions of the axial force as well as we have the actions of the lateral force which is causing bending in the member and thereby such members are subjected to the combined actions of loading and thereby the combined stresses.

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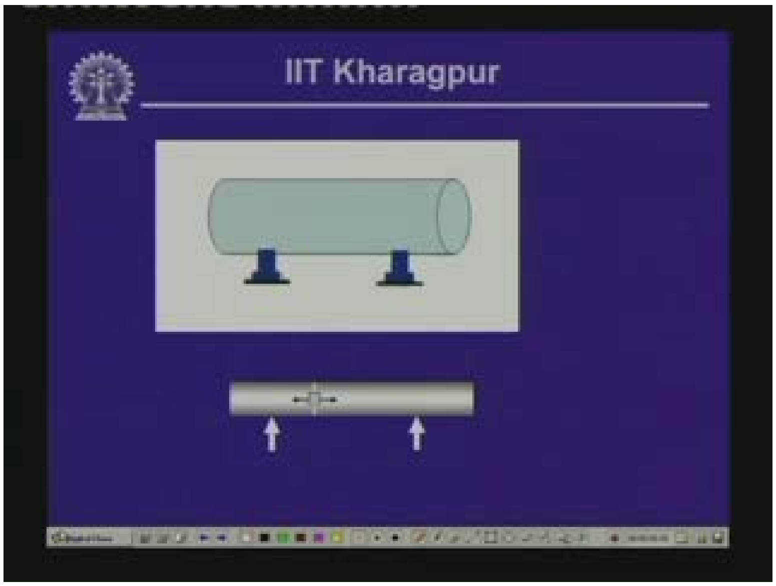

Now let us look into another example wherein we come across the combined loading actions. Now we have already seen earlier that if a pressure vessel is subjected to internal pressure P then how do we compute the longitudinal stress and the circumferential stress which is sigma which we have called as Hoop's stress circumferential stress sigma as pr by t and the longitudinal stress which is equal to pr by twice t. Now these stresses we have computed individually for the pressure which is acting inside.

Now if you look into that this particular pressure vessel is supported at this particular point and at this particular point; now naturally because of its own weight along with the liquid it will undergo bending over this particular part and because of this bending there will be stresses generated in this pressure vessel. so apart from the stresses which we get from the internal pressure we will have the stresses generated because of this supporting point; as a beam it is getting supported at these two points and thereby there will be deflection of the member and there will be stresses generated. So we will have to account for both the stresses so that finally the resultant stress which we get which we call as sigma they are the combinations of the internal pressure stress and the stresses which are getting generated because of the bending. So again this is an example for the combine loading situation.

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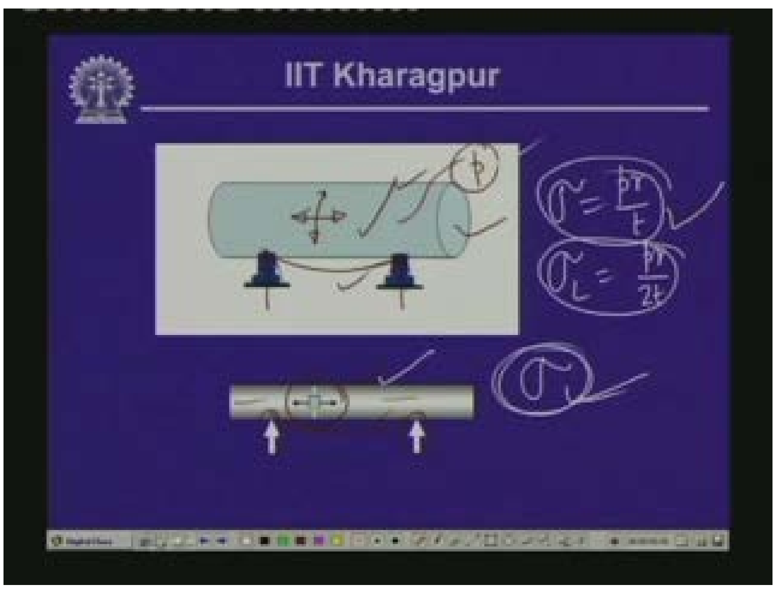

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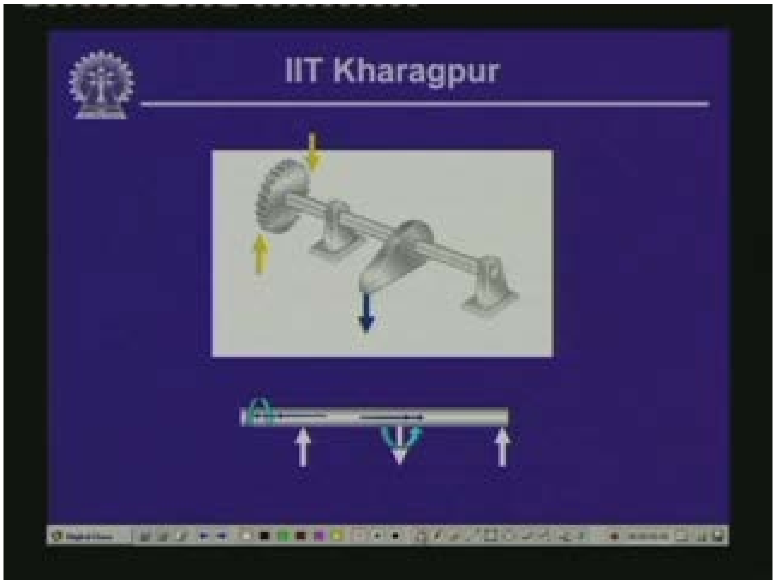

Now this is another case wherein we get the values of the loading in combined form. Let us say this is a shaft and this particular shaft is subjected to or is attached with a gear where we have a couple which is giving a twisting movement in the shaft. Also, if you look into this particular shaft it is having a member which is extensive with respect to the shaft center line and there is a concentrated load acting at the end of this particular member.

Now this particular load if we transfer to the shaft center line this will be along with the couple which is going to be a twisting movement in the shaft. So if we take the idealized form of this particular shaft this is supported at this particular end, this is supported at this point (Refer Slide Time: 19:02) through this particular support and at this particular point where this overhang is there the force is transferred to the shaft center line and thereby there is a concentrated load along with the twisting movement that is acting over here and also at this particular end we have the twisting movement that is developed because of this gear and if gear has some weight then there also there will be a concentrated load acting at this particular point.

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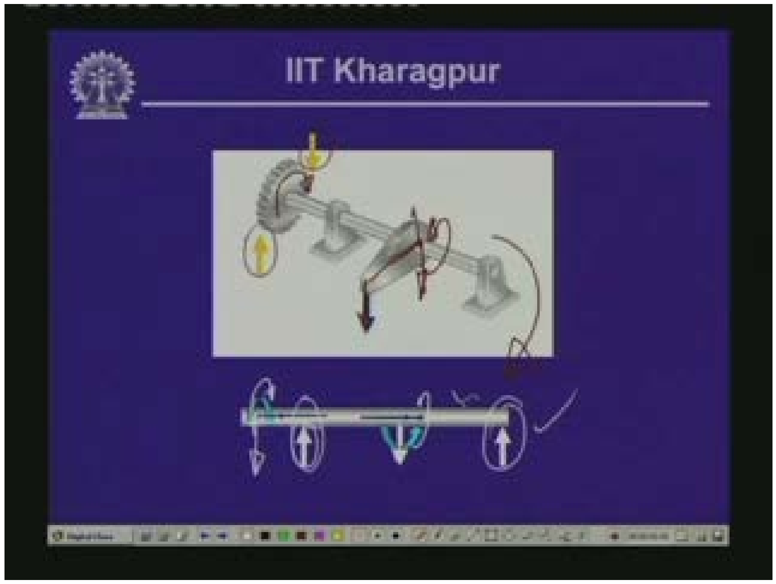

Therefore, if you look into this particular member over this particular length now this is supported here, supported here and on this part it is overhang (Refer Slide Time: 19:36) so it is a kind of an overhang team subjected to a concentrated load at this center, subjected to a concentrated load at this end and subjected to a twisting movement at this particular point and at this particular point. So this particular shaft is not only subjected to the transverse loading which will cause the bending in the member; thereby we will have the bending movement at the shear force; and consequently you will have the bending stresses and the shear stresses.

Also, because of the twisting movement there will be shear stresses in the shaft. So there will be combined stresses or combined shear stresses because of the shear force and the twisting movement and there will be normal stress because of the bending. And now, once we have this bending stress and shear stress then we can compute the values of the resultant stress as we have seen earlier in module 1 that using Mohr's circle we can find out the value of the maximum normal stress which we have called as principal stresses using the normal stress and the shear stress values.

So you see, in this example also this particular shaft is subjected to the combined actions of the loading.

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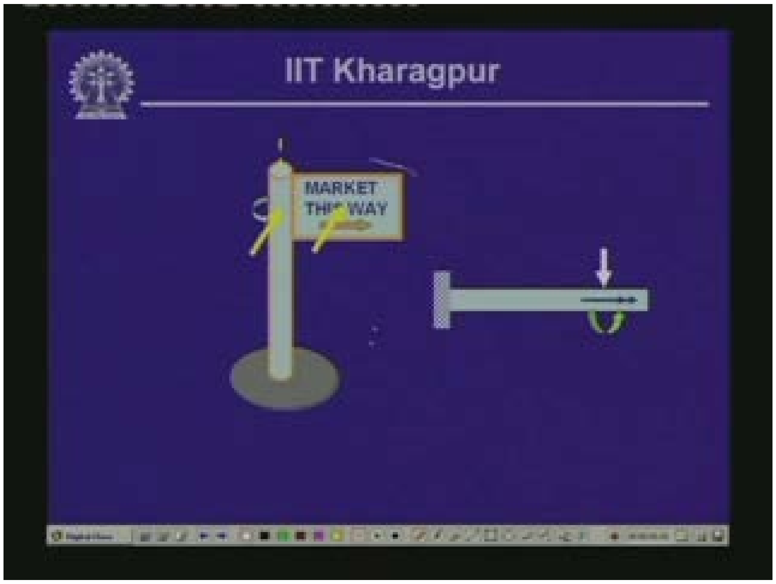

Let us look into another example which is quite common that in ctc you must have noticed that for giving directions for different places a board is used and this particular board is connected to a vertical shaft through this particular joint and this board is projecting from this particular vertical shaft. The vertical shaft is connected at the base by some rigid support. Now, this particular board which is projecting from the shaft is exposed to the forces like wind. Now, when the wind load acts perpendicular to this particular plate let us say the action of this resulting wind load which is acting at the center of this plate is P.

Now basically since this board is connected to the vertical shaft or the vertical rod now this load will be transferred to this because finally the load has to be transferred to the base below. So this force if we transfer to the vertical member then it will be associated with a moment and this particular moment will be acting about the axis which is normal to the vertical shaft and this moment we call generally as the twisting moment.

Therefore, if we look into the idealized form of this particular configuration; if we look into the idealized form of this vertical shaft this looks like this that one end of this member is connected which is simulating this particular support, the other end is free (Refer Slide Time: 22:20) and at this particular point the load concentrated load is getting transferred to the shaft along with the twisting movement; the vectorial direction of this is this and the twisting movement is acting in the member. Now, because of this transverse load the beam will be subjected to a bending moment and there will be shear force as well. So this beam will have the actions of bending moment M, will have the actions of shear force; also, we have the twisting moment T which will give rise to the shearing stress tau, bending will give rise to the normal stress sigma, V will give rise to the shearing stress tau.

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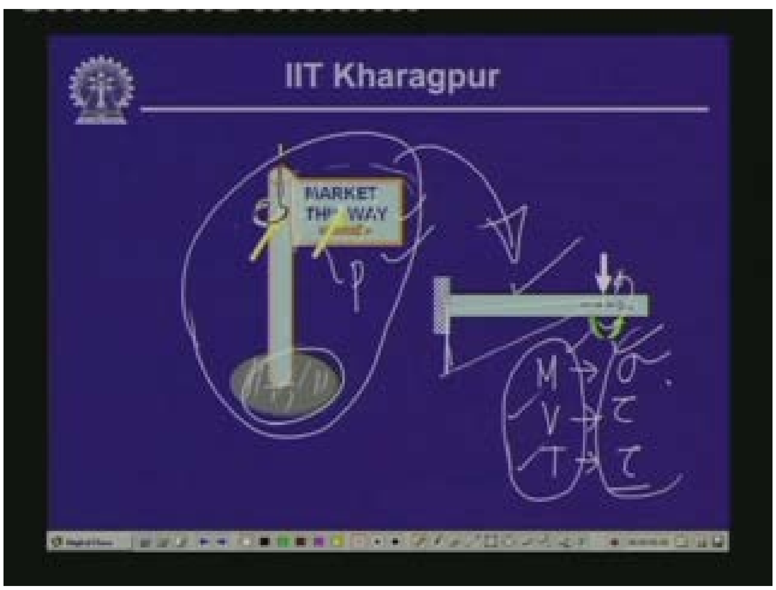

Thus, you see that this particular member is subjected to the action of bending moment, shear force and the twisting movement and thereby we will have the resulting normal stress and the shearing stresses or each individual load we can compute the values of the normal stress and the shearing stresses and once we know the values of the normal stress and the shearing stress then we can compute the resultant stress in terms of the normal stress and the shearing stress.

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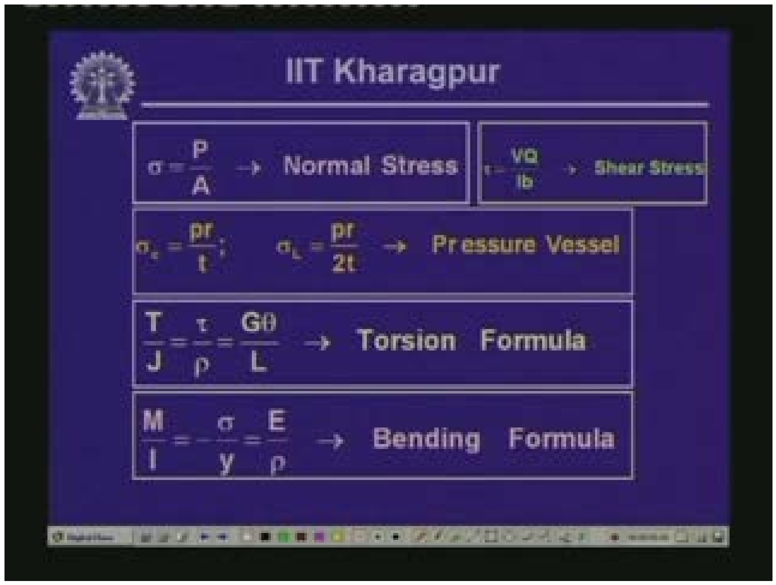

Well, let us look into these again. If we go back to the expressions which we have already derived, now in the first module we had looked into that if you have a axial load in the member then the normal stress we compute as sigma equals to P by A. Subsequently, we have seen that if the pressure vessel is subjected to internal pressure then it will be subjected to stresses which we have called as Hoop's stress or the circumferential stress which is equal to pr by t. Also, we have computed the longitudinal stress which is pr by twice t.

Then subsequently we have looked into that if a shaft is subjected to a twisting moment or torsion then the torsion formula is T by J is equals to tau by rho is equals to G theta by L and this indicates that from these first two equality we can compute the value of the shearing stress tau which is equals to t rho by j; so if you know twisting moment we can compute the value of the shearing stress and subsequently we had looked into the bending formula where the moment is related to the bending stress sigma and we know that sigma is equals to minus M y by I so sigma the bending stress is related to the bending moment M.

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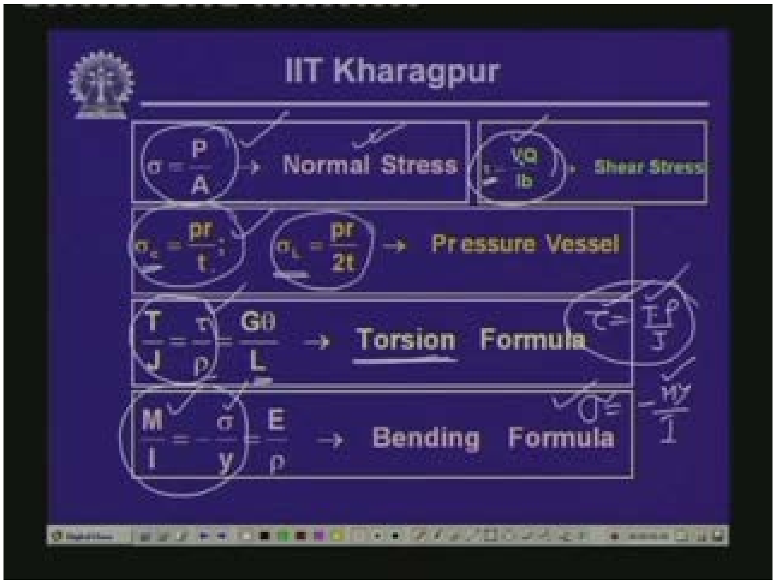

Also we had looked into that if we have shear force V then the shearing stress tau is equals to VQ by Ib. These are the individual cases as we have evaluated earlier. Now we will make use of.......... in this particular lesson as such we are not going to derive any formulae but we will make use of this formulae which we had derived earlier for individual load cases.

Now, depending on the situations wherever we will have combinations of these stresses we will make use of those individual formula and combine them suitably to arrive at the resultant stress in the member. Because finally we are concerned with that, what is the maximum possible stress that can be generated in a member because of such combined loading actions.

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Well, now let us look into a case where we get a combined loading form wherein we have a vertical member which is fixed at the base and at the top we have a rigid bar on which a load P is acting which is eccentric with respect to the center line of this member or the cg of this vertical member.

Now if we transfer this load which is eccentric with respect to the cg of the vertical member, if we try to transfer to the center of gravity of this vertical member then what happens? This load we transfer over here (Refer Slide Time: 26:24) and since here we did not have any load so we will have an opposite load and this opposite load along with this will form a couple. So the transfer of this force to the cg of this particular vertical member is a load and a moment and this is what is shown over here.

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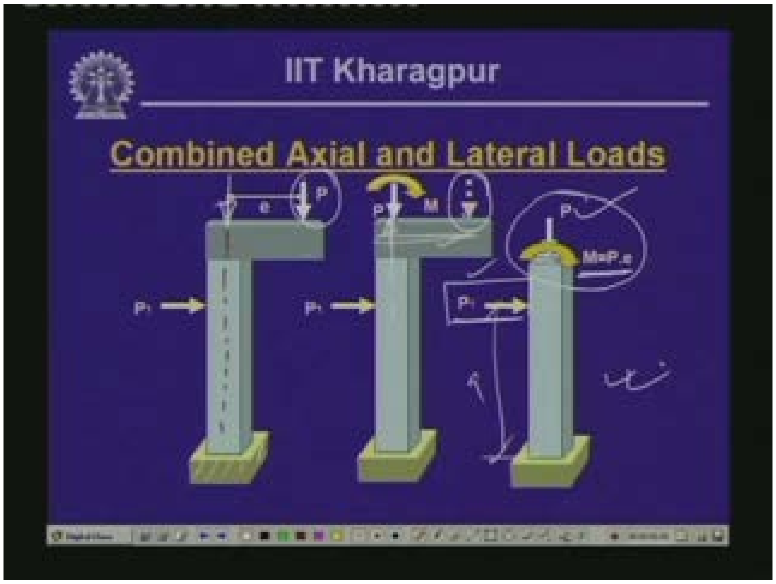

Now if you remove this vertical bar I mean this horizontal member then the vertical bar we have the effect of this eccentric load as a load P which is concentric now which is acting through the cg of the member and we have a moment which is equal to P times e where e is the eccentricity or the distance of the cg from this loading point. Also, this particular member is subjected to concentrated load P 1 which is acting at a distance let us say A from the support.

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Now if we idealize the particular situation then probably we can describe the same thing in this particular form that we have a cantilever member which is fixed at one end and free at the other and at this end we have a concentrated load P which is acting through the center of this particular member along with moment M bending moment M and we have a transverse load P which is acting at a distance of a from the support.

Now, the actions of these loads will be that the concentrated load P which is acting through the center of gravity of the member will cause a normal stress. It is like a member is subjected to a compressive load. So at any cross section if we compute the stress will be P divided by the cross-sectional area which we have called as a normal stress.

This is what is indicated over here that the stress distribution is uniform which is P by A. Now since this is compressive so we have called this as C which means this is a compressive load. Now, because of this bending moment M at any section if you compute; now on this side of the load it will be just the M (Refer Slide Time: 28:26) but if we take a section over here and draw the free body then there will be moment due to this lateral load or the transverse load and the external moment M. So at any section we can find out the bending moment because of this transverse load P and moment M. And if we call that moment as M then the corresponding stress in this member will be M y by I as we have seen in case of......... if we call this as M dash M dash is the combination of M and P effect of P and finally we get the bending stress which is M dash y by I.

Now this is going to cause the tensile and compressive stress. Physically if you look into this particular member when it is subjected to the action of bending in this particular form which is in a clockwise direction and this particular load is also going to cause a moment in a clockwise direction, so on this phase on the top phase we will get a tensile stress and at the bottom phase we will get a compressive stress. This is what is indicated to about here that we have a stress distribution which is tensile at the top and compressive at the bottom which is varying linearly across the depth.

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Now if we combine these two stresses; since we are writing in general terms P and M now there could be three situations. now if I draw a section through the, or if draw a line through the neutral axis, below neutral axis the member is always under compression;

under the action of axial load and under the action of the bending both are subjected to compressive stresses and thereby below neutral axis we will find that you have always the stresses as compressive stresses. But at the top there could be different situations; situations like, if P by A the direct normal stress which we get is compressive in nature and bending stress is causing a tensile stress as the nature; if tensile stress is larger than this compressive stress that means if M dashed y by I if it is greater than P by A then we will get a tensile stress at the top.

If this M y by I is equals to P by A then at the top we will get a stress which is equals to zero and if M dashed y by I is less than P by A then we will get a stress which is compressive at the top. So you can get either this configuration or this configuration or this configuration depending on the magnitude of the axial load P and the bending moment M. Thus, at the bottom since the axial compressive force is giving a compressive stress everywhere and the bending is giving the compressive stress at the lower part that is below the neutral axis then at the bottom always you have the compressive stress whereas in the upper zone you can have either tensile stress or compressive stress or zero stress depending on the magnitude of the bending and the axial force you have. This is what is represented over here.

Now we can take another example of this kind of combined stress situation where the beam is a simply supported one I mean a support hinge support on one end and regular support of the other and is subjected to a compressive force P and you have transverse load let us say P 1 P 2 P 3. Now this compressive force which is acting through the center of gravity of the member at any cross section if you take the stress will be P divided by A which is compressive in nature which is indicated over here. But this transverse loading which is acting in this beam is going to cause the bending of the beam, this going to deflect (Refer Slide Time: 32:16) and thereby there will be bending of the beam.

And physically if you look into, the lower part will be under tension because of this loading and the top part above neutral axis will be under compression. This is what is indicated over here that if you compute the bending stress which is M y by I it will generate the compressive stress above the neutral axis and we will have tensile stress below the zone below neutral axis.

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Now if we combine these two together then we will have the compressive stress and the neutral axis position of course will shift and we will have the compressive stress above the neutral axis and the tensile stress at the bottom if M y by I is greater than P by A or this part will be 0 (Refer Slide Time: 33:05) if M y by I is equals to P by A and this will be positive stress or the compressive stress if M y by I is less than P by A.

Again we get a similar situation at the top but the only thing is that the position changes over here; we get the compressive stress always at the top. But the bottom part of it either it could be tensile or could be zero or could be compressive; depending on the magnitude of the axial load you have, depending on the magnitude of the transverse load you have and consequently how much bending moment it is generating and how much bending stress is generated because of this bending.

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Once we combine these two loading conditions together then we can get the effect of the combined stresses in the member and we can see that whether the material can withstand the maximum value of the stress that is getting generated because of such combined actions in the member and that is what is our objective in this particular situation.

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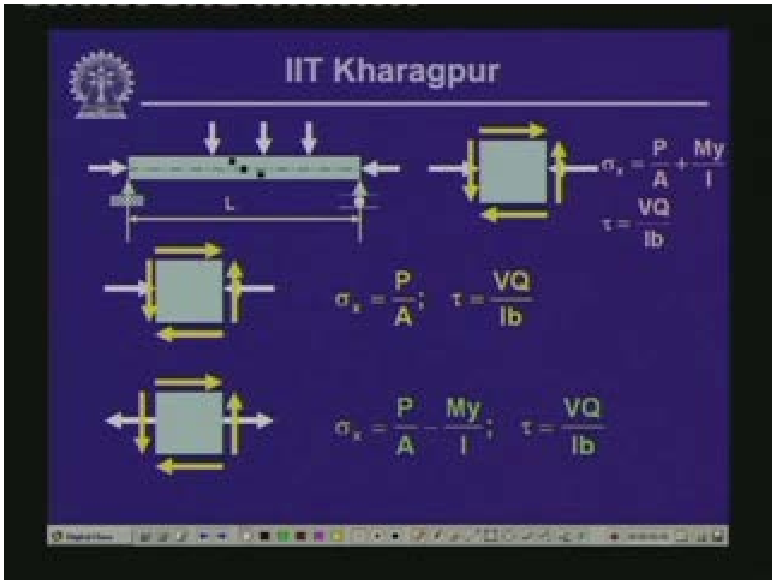

Now if we look into the effect of such combined loading in the member or at different points about the stresses as you know that it is going to produce the normal stress sigma as the axial force, the lateral force is going to cause the normal stress sigma and the shearing stress tau. So we are going to have total normal stress sigma and we are going to have the shearing stress tau. And finally we need to compute the value of the resultant stress because of this normal stress and the shearing stress.

Now if we consider a small element in this particular zone which is above the neutral axis but below the top surface of the beam then the stress we can represent in this form. If we enlarge that particular part we will have a compressive stress which is the normal stress and this normal stress is because of two actions: one is P by A which is because of the axial force acting in the member and another one is because of the bending which is equals to M y by I and this is going to give us the compressive stress again at the top of the beam or above the neutral axis that is why it is summed up together. But basically this is negative stress because it is acting in the opposite direction of positive x and this of course, if we take the sign conventions properly or if we take say axial pull is I mean the compressive force is negative the bending also is causing compression at the top so that is negative so if I sum them up is going to give me the negative stress which is acting in the opposite to the positive stress direction and that is what is indicated over here.

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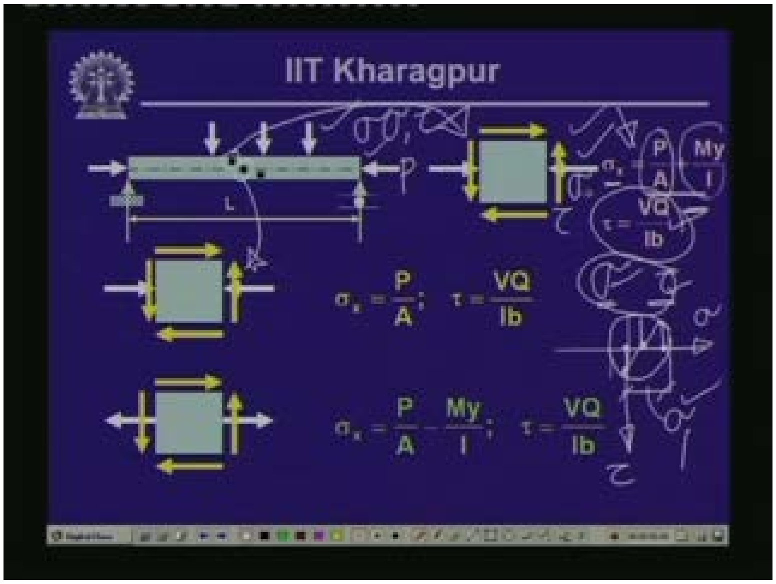

And we have that value of the shearing stress tau which we have seen is computed as VQ by Ib so this is the shearing stress tau and this is the normal stress sigma and once you have the sigma and tau as you know that you can plot them in the Mohr's circle then we can get the value of the maximum value of the normal stress. So you have sigma and tau and here the sigma is 0 and then the tau and if you join them together this is the center, if you draw the circle then this is going to give us the maximum value of the normal stress which is sigma 1. So at that particular point we can compute the value of maximum normal stress and the minimum normal stress and also what is the value of the maximum shearing stress.

Now if we consider a point which is on the neutral axis then the stress distribution can be indicated by this. Now as you know that because of the bending stress which is M y by I at the neutral axis L the value of the bending stress is 0, you have the compression and the tension depending on the nature of the moment but along the neutral axis the stress is 0. So when we are considering a point on the neutral axis there we will have the normal stress because of the axial compressive force only; there will not be any normal stress because of the bending and that is what is indicated over here.

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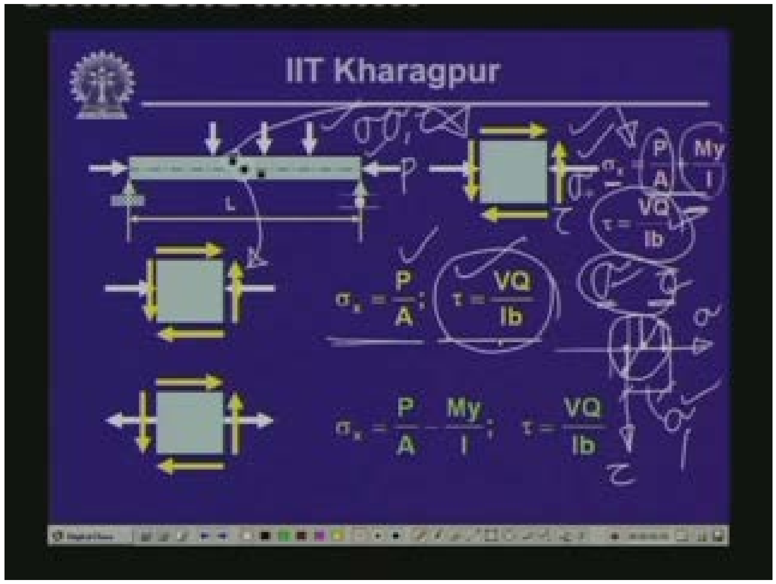

Here you can see that the sigma is equals to P by A on this particular element and consequently we have tau equals to VQ by Ib and incidentally the shearing stress at this particular location will be maximum as we have seen. Again we will have the normal stress sigma and the shearing stress tau and based on these values we can compute the value of maximum principle stresses.

And in the third point which is below neutral axis if we plot the stresses now here assuming that the compressive stress or the tensile stress which is getting generated because of the bending which we are computing from M y by I is larger than P by A and thereby there is resulting tensile stress below neutral axis. And if that happens then the normal stress which we have over here is shown as a tensile pull and along with that we have the shearing stress tau. So again based on this sigma and tau we can plot in the Mohr's circle as what will be the value of the principal stress sigma 1 and sigma 2.

But the only aspect you should keep in mind now is that here the sigma is in positive in earlier case sigma was negative. In fact earlier we have to draw sigma in the opposite direction which is in the negative direction and tau in the positive direction. This is what we get. Consequently the values of the normal stress and the shear stresses will be like this. In this particular zone sigma x is equals to P by A minus M y by I and if we presume that M y by I is higher than P by A and thereby we get a tensile pull and the shearing stress is equals to VQ by Ib.

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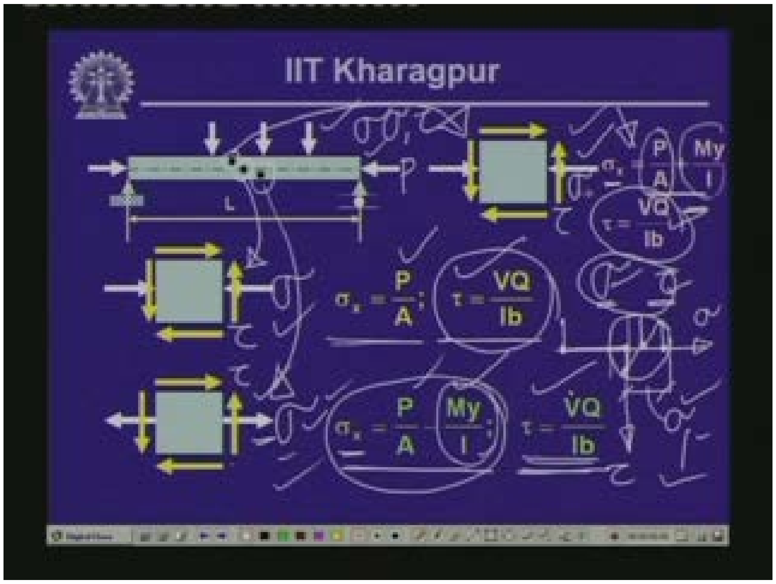

Well, so long whatever we have considered that basically we are combining the individual actions of the load. That means whether you have the axial load or the transverse load or a twisting moment what we are trying to do is we are trying to compute the stresses for the individual actions and combining them suitably either when you have total normal stress or the shearing stress and then we are combining them to Mohr's circle.

Now the question is that these are basically nothing but the superposition of the stresses of the individual actions. Now this is possible; if when we are talking about these axial loads or lateral load or the transverse load which is causing bending, now this is possible if the slope of the deformation is small. Now if we get a very large deformation of the member then this kind of superposition is not allowed and you cannot calculate the combined stresses in this form from the way we are calculating the moment.

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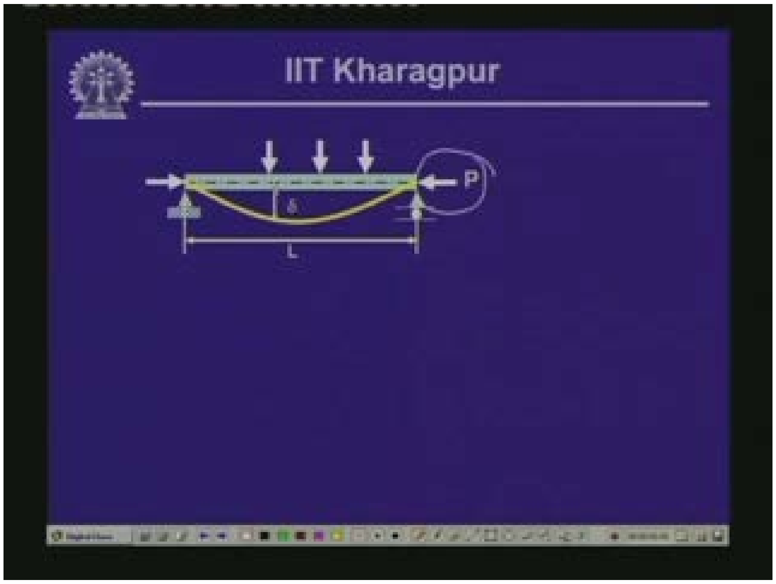

It means that if you have the axial pull and if your deformation is large now at any point where the deformation is delta then you will have the value of M because of this P is equals to P times delta. Now this moment if we add to the bending moment let us say M dashed then if this M I mean its suitable is sign plus or minus, now with M dash and M if they are comparable because of this large deformation then this superposition of stresses will not be allowed.

But if the value of M is insignificant in comparison to M dash where delta is small then we can ignore this and we can use the superposition and we can compute the values of the stresses at different points because of these combined actions. And this is one of the important aspects that should be kept in mind.

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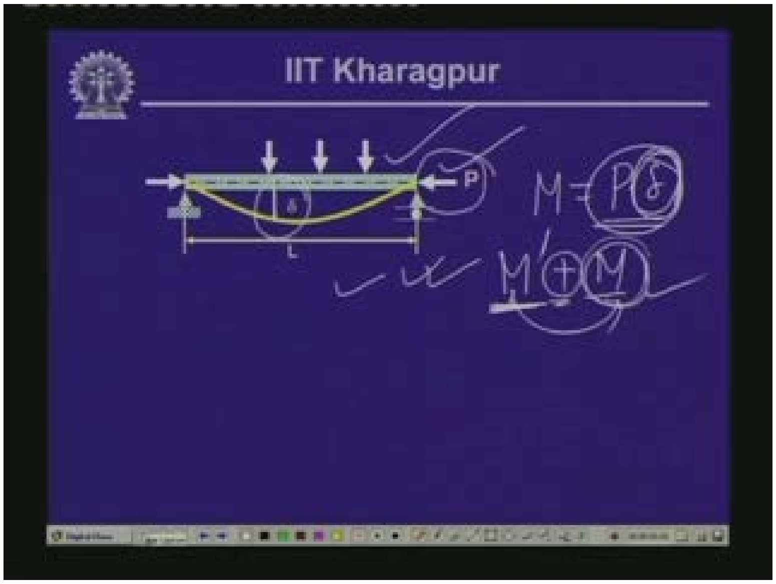

Well, then having discussed the actions of different loads or the different load combinations let us look into some example where a member a beam member which is fixed at one end and free at the other which you call as the cantilever is having a bracket at this particular end and is subjected to a pull of 100 kilonewton at this bracket. So a 100 kilonewton force is applied to the bracket as shown in the figure. Now what you need to do is that compute the normal stresses developed at point A. Now this point A and point B is on the outer surface of the beam at the top and the bottom surface. So you got to compute the normal stresses developed at points A and B. Now given that the cross section of the member is T and the cross-sectional area is 8000 mm square and the value of the moment of inertia is equal to 50 into 10 to the power 6 mm to the power 4.

Now here the position of the neutral axis for the section is given and the neutral axis is at a distance of 100 mm from the top surface and on the bottom surface it is 200 mm. Now this particular member which is the force which is inclined this is inclined at an angle of 17.5 degree with respect to the horizontal and the position of the load which is acting in the bracket is at a distance of 100 mm from the top surface of the beam. And the position of this bracket from this end is 1m.

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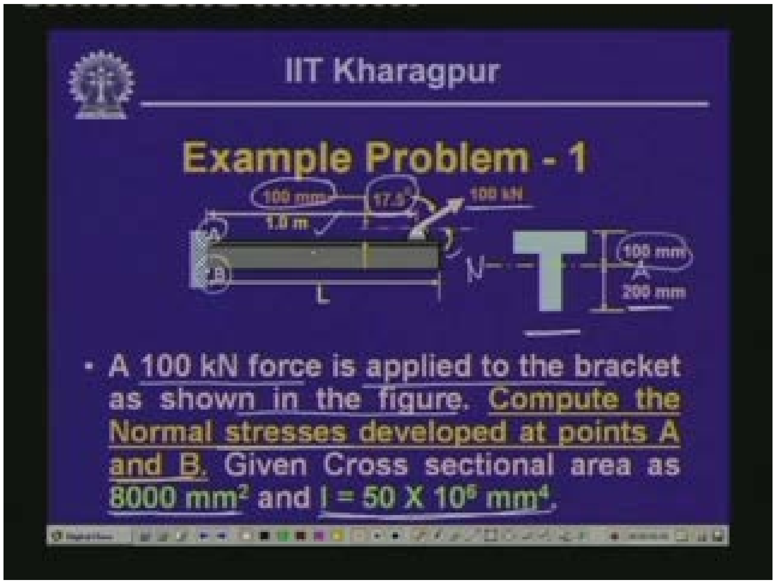

Now what is the action of this particular load?

This particular load which is acting at this bracket point we can decompose in two parts: we can decompose in the horizontal direction, we can decompose this in the vertical direction since we know this angle theta. Now this particular horizontal load which is acting away from the top of the beam if we transfer that beam to the load to the c g of the member then thereby it will be an axial pull to the member. But if you do that it will be associated with a moment. So this load can be transferred to the c g of the member with a load and a moment and the vertical component of the load is going to cause a bending in the member. So let us analyze that first on how we do that.

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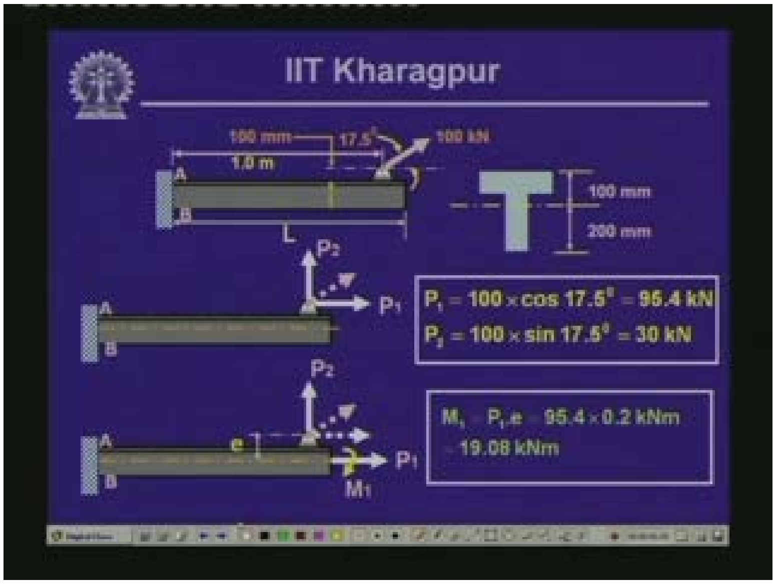

Now here this 100 kilonewton load is acting at an angle of let us say theta which is equals to 17.5 degree here. Now the first step is we decompose this load into two directions: one is P 1, another one is P 2 and P is 100 kilonewton so P 1 is 100 cos 17.5 degree so this is equal to 95.4 kilonewton and the vertical component of this P 2 is equal to 100 sine 17.5 degree which is equal to 30 kilonewton so we have two force components of this 100 kilonewton axial pull which is p 1 and p 2 now this P 1 is acting away from the c g of the cross section of this member.

Now if I transfer this horizontal load at the c g of the member then it will be associated with the moment as it is indicated over here. Now this load P 1 is transferred to the c g of the member and it is associated with moment M which is equal to load P 1 times this eccentricity e. So the value of the moment M is equals to P 1 into e which is 95.4 the axial pull and 0.2 the distance e and that gives you 19.08 kilonewton meter as the moment.

Therefore, we have the value of the moment and the axial pull. Now this axial pull is through the center of gravity of the member and thereby it gives a normal stress at any cross section. Now we have the bending moment M 1 acting at the end and we have the transverse load P 2 which is acting at a distance of 1m from the support, this is acting 1m distance from the support.

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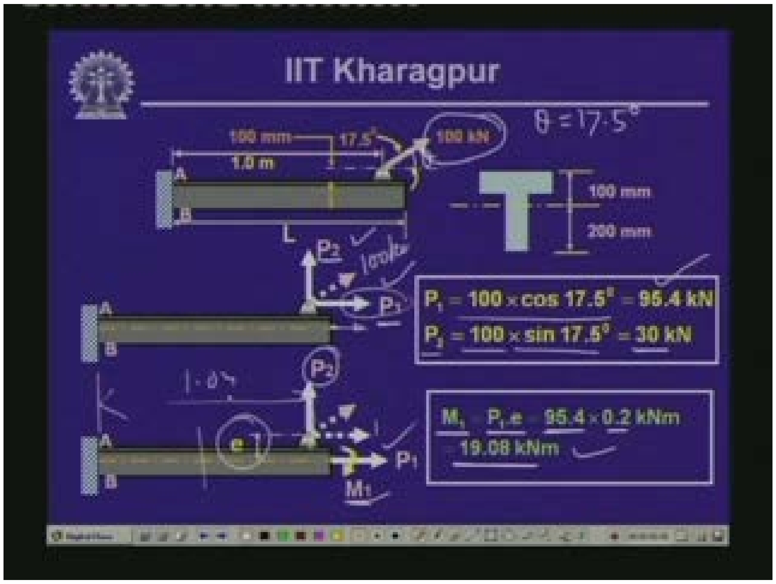

Now let us look into what is the action of this load individually.

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Now if you have this that axial load P 1 which is acting at the c g of the member now this going to give us a normal stress. Now, at any cross section if you compute the value of the stress sigma which is equals to P 1 by the cross-sectional area A we have called that as positive stress because it is tensile and according to the sign convention our tensile is positive so tensile stress axial tensile stress we have called as positive. So sigma is equal to P by A which is 95.4 into 10 to the power 3 so much of newton divided by 8000 mm square so this gives us 11.925 Mega Pascal.

Now let us look into the action of this bending moment M 1. Now this moment M 1 which is acting at this end, for this at any cross section if we take the free body we will get the M as negative of M 1 and therefore this is the bending moment diagram where the bending moment is negative everywhere.

Now because of this as you can see even physically that at the top we get a tensile stress, at the bottom we get a compressive stress. And as we know for tensile our sign convention is positive and compression is negative. Now if you compute the value of sigma which is M y by I sigma equal to minus M y by I and M is negative, y at the top is positive which is 100 and I for this member is 50 into 10 to the power 6 so we get a value of plus 38.16 MP a. This is the value of the stress which is acting at the top of the beam.

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Now consequently if you compute the stress sigma B at the bottom of the beam this equals to again minus minus of M now; why? Since it is downwards so it is minus so thereby we get a minus sign minus 76.32 MPa which is compressive at the bottom.

Now let us look into the action of this particular load which is the vertical load at the distance of 1m from the support. For that if we draw the bending moment diagram the bending moment diagram is a triangle and the value of this is equals to P 2 into 1m is equals to P 2 kilonewton meter at P 2 is 30 so this one is 30 kilonewton meter which is the moment over here.

Now this is the positive moment. Now if you compute the stress at the top; now because of this the bending which is going to happen physically this member is going to be lifted up in this form so we will have tensile stress at the bottom and compressive stress at the top. So as per our sign convention this is negative at the top and positive at the bottom

(Refer Slide Time: 48:42) that is what is represented here. Now, stress sigma is equals to minus M y by I where M is positive so this is minus 60 MPa so giving the negative or the compressive stress at the top; sigma B the stress at the bottom is equals to M y I, y is minus 200 so this is plus 120 MPa. So these are the values of the stresses that we get at point A and B because of three individual loading actions. One is the axial pull, another one is the moment and another one is for the vertical load.

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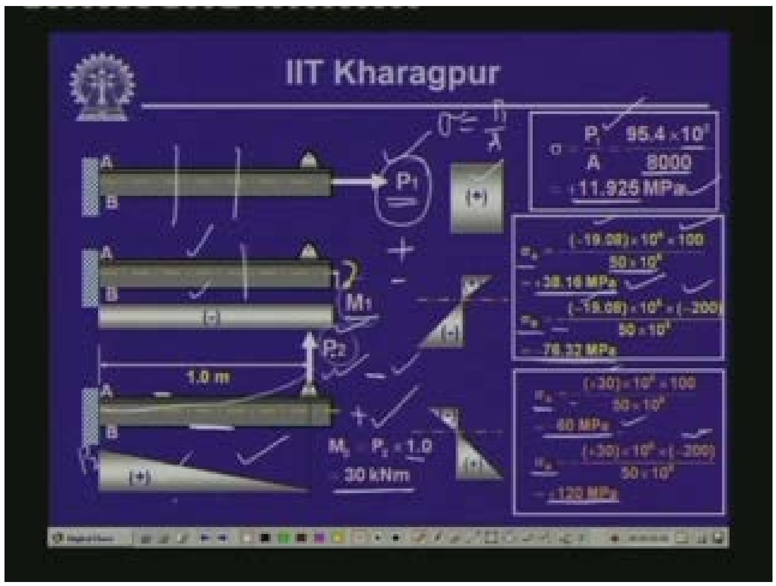

Now you see that the inclined load which we had that first we have decomposed in horizontal and vertical directions and the horizontal load which was away from the center of gravity of the beam we had first pulled that load in to the c g line and when we do so since it is eccentric loading it gets associated with a couple. So the cantilever beam now subjected to this axial pull which is concentric then we have a moment a couple acting at the tip of the beam and we have a vertical load which is acting at a distance of 1m from the support which is causing a bending in the beam.

Thus, we have seen the three individual actions.

Now this is a case of combined stress so we combine them together. the individual stresses we join them together to get the final effect at point A and B. Now if you do that so at A we will have plus of this, (Refer Slide Time: 50:11) plus of this and minus of this and at B we will have plus of this, minus of this and plus of this. So let us look into them finally what we get of the stresses. These are the values of the stresses that we get that at A we have plus P by A, this is plus because of the tip moment M 1 and this is because of the vertical load which is acting in the beam.

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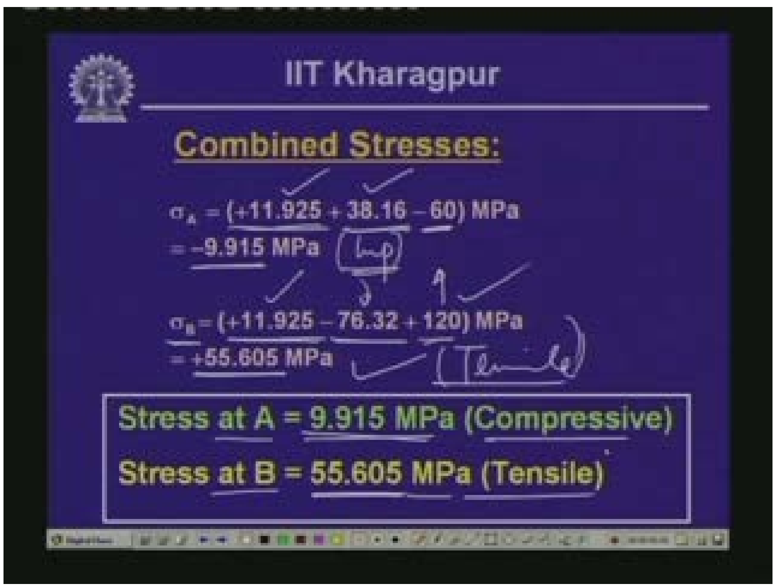

So, resulting of this gives us minus 9.915 that means this is a compressive stress that is acting at the top of the beam. and at the bottom we get sigma B which is equals to 11.925 which is P by A minus 76.32 this is because of the couple and this is because of the vertical load which is causing 120 MP a. So finally if I join these three together we get a trace of 55.605 MPa which is again tensile in nature. So we have stress at A which is 9.915 compressive and we have stress at B which is 55.605 MPa which is tensile.

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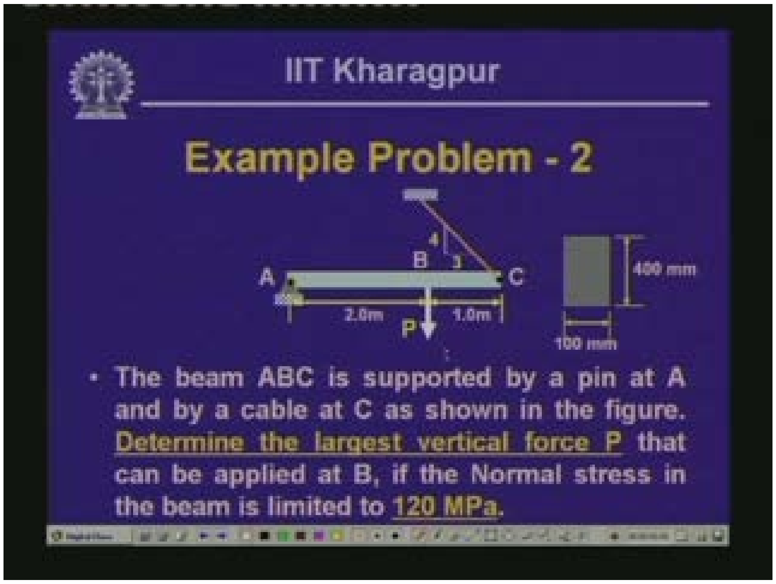

Now let us look into another example where we have a beam which is simply supported the beam A B C is supported by a beam at A and by a cable at C at this point (Refer Slide Time: 51:32). Now what you will need to do is you will have to determine the largest value of P so that the normal stress in the beam is limited to 120 MP a. If the normal stress value is 120 MPa then what will be the value of P? The cross section of the beam is 100 by 400.

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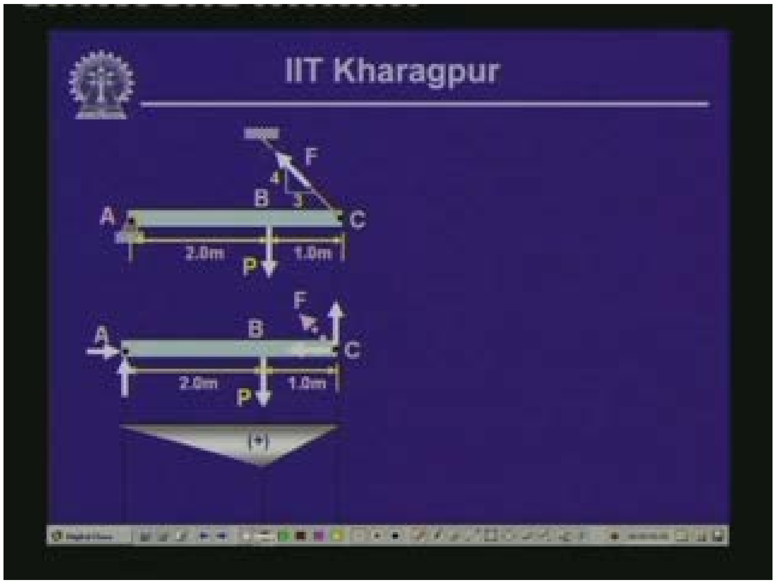

Now if you look into the free body of this particular beam it will be like this that if you take the cut in the cable then the cable force let us call that as F and again as we have done in the previous example we take the components of this force in the vertical and the horizontal direction then thereby if this is theta now you see that these distances are; this is 4, this is 3 (Refer Slide Time: 52:15) so this 5 and this is theta so cos theta is equals to 3 by 5 which is equals to 0.6 and sine theta is equals to 4 by 5 which is equals to 0.8.

So if cos theta will give you the horizontal one if you call this as say F 1 and if you call this as say F 2; so F 1 is equal to F cos theta which is 0.6, F 2 is equals to F sine theta which is 0.8. Now if you take the summation of horizontal force as 0, if you call this as H and this as V, H is equals to F 1 which is equals to F into 0.6.

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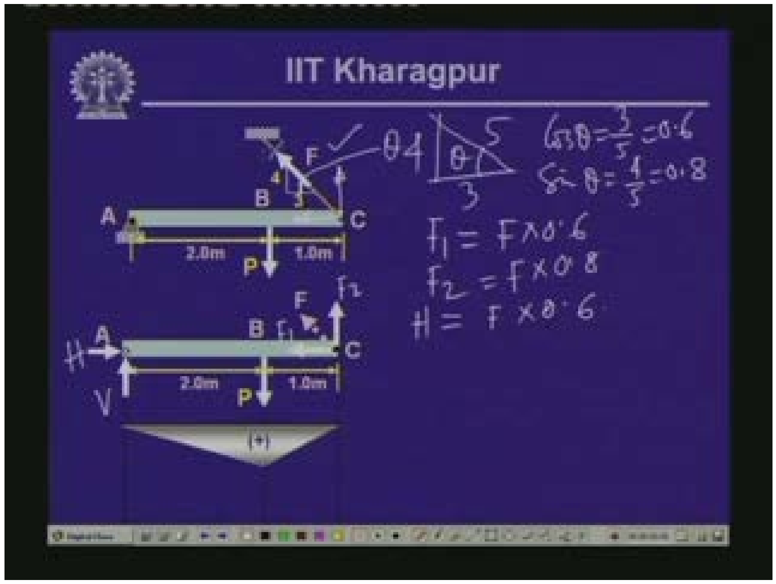

Now if you take the moment of the forces with respect to A, we have F 2 into three is equals to P into 2 and thereby F 2 is equals to 2 P by 3. Now F 2 we have seen is equals to F into 0.8 so F into 0.8 is equals to 2 P by 3 or F is equals to 2 P by 2.4. If this is F then the vertical force F 2 we have got as 2 P by 3 and F 2 plus V will give you P so V is equals to P by 3.

Now if you compute the bending moment because of this; now you can consider this as a simply supported beam subjected to load P over here and thereby you will get a bending moment of this form where the value of this is equals to 2 P by 3 kilonewton meter; 2 P is in kilonewton so this particular member will be subjected to the action of this axial force which is F 1 which is equals to F into 2.6 where F being 2 P by 2.4 this is equals to P by 2. So you have the axial compressive force of P by 2 and you have the bending because of the lateral load P which is of magnitude P by 3. So you have the action of axial stress which is normal stress and because of bending you are going to get the normal stress.

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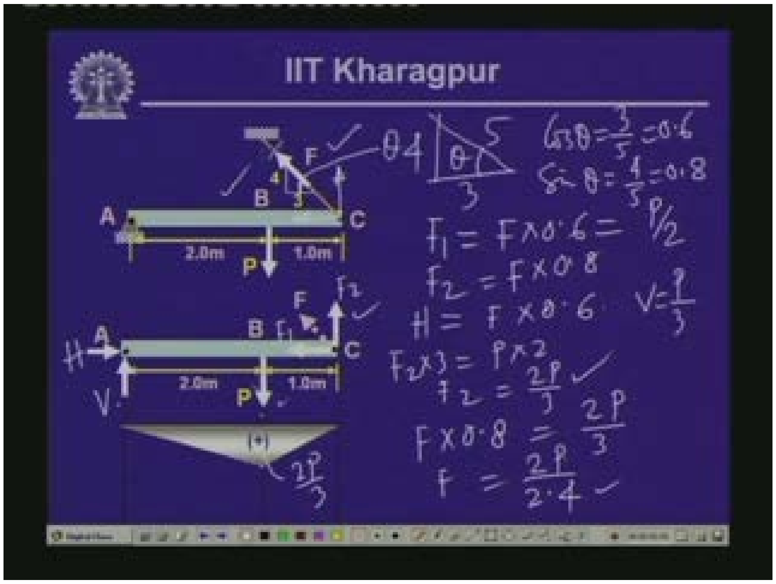

Now physically if you look into, because of this load P at the top you will get compressive stress and at the bottom you will get tensile stress. Now if we combine the compressive stresses because of the axial compression and the bending you will get the maximum compressive stress and if that will limit to 120 then we will get the value of the maximum stress.

Now if we compute the value of the maximum normal stress this is equals to, that P by A now axial force is P by 2 that divided by A plus M y by I where is M is 2 P by 3 into y is 200 and I the moment of inertia which is equals to 100 into 400Q by 12.

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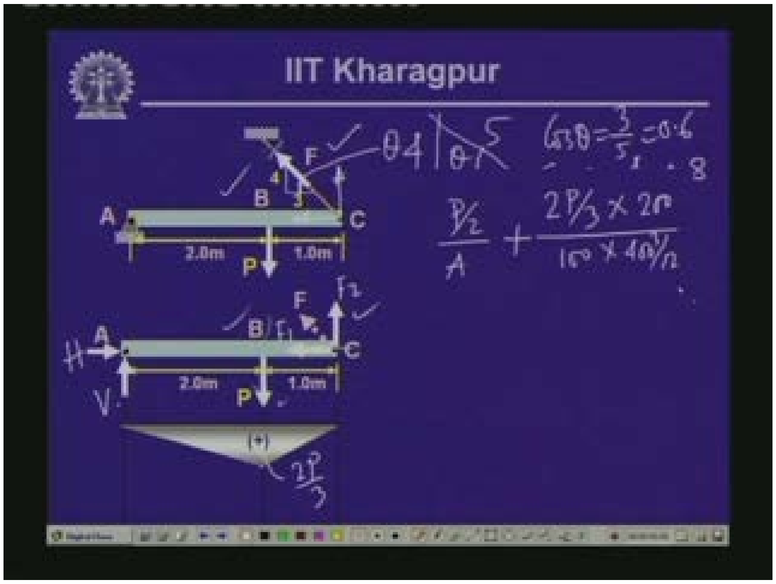

So if you compute this we will get the values as........... and this is equals to the stress sigma and sigma is equals to 120 so this is equals to........ if you compute or if you substitute the value of A as 8000 you will get this as 0.0125P plus you have 0.25P because of this. Now if you combine these two you get 0.2625P and this is equals to 120 and thereby you have the value of P as equals to 457 kilonewton.

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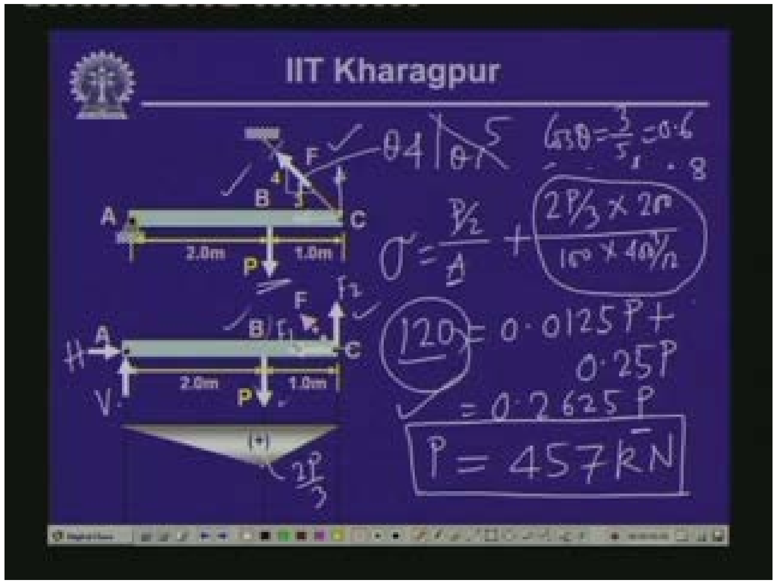

So you see that we can compute the maximum value of load P which can be applied here so that the stress does not go beyond 120 and here we get the combined actions of this axial stress as well as the bending stress.

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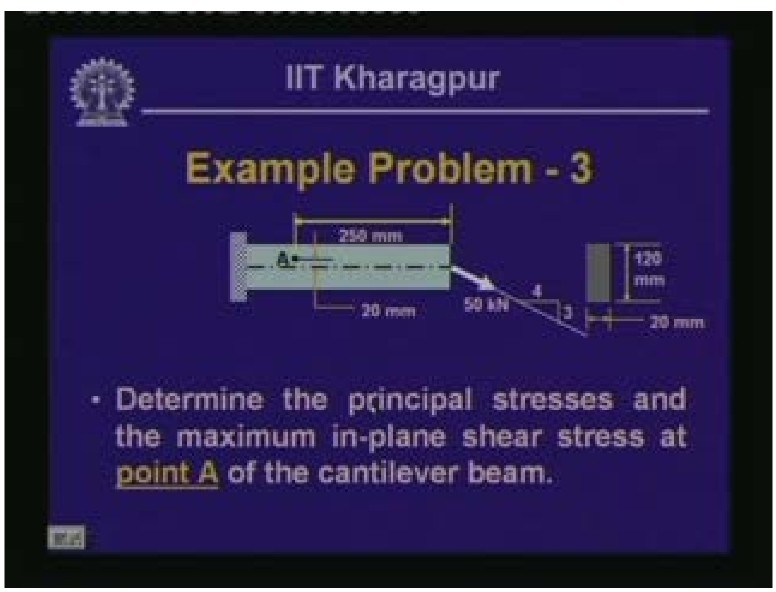

Well, we have another example problem that you need to determine the principle stresses and the maximum in-plane shear stress at point A which is at a distance of 20 mm from the centroidal axis and it is subjected to load of 50 kilonewton acting at the center of the beam in an inclined form and you look into this problem and we will discuss about this in the next lesson.

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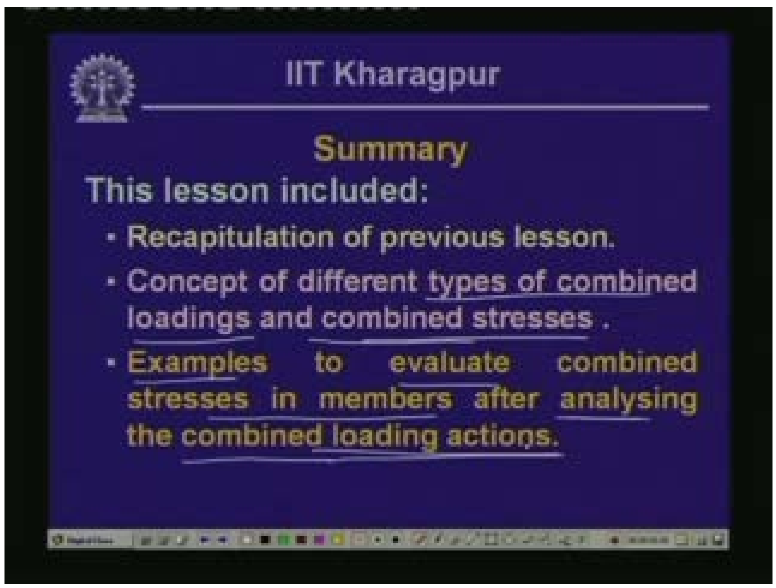

Well, then to summarize, here in this particular lesson we have looked into the aspects of the previous lesson, we have recapitulated some aspects of the lesson we have already done in the last lesson, then we have look into concept of different types of combined loading and thereby the combined stresses that is generating and we have looked into some examples to evaluate the combined stresses in members and after analyzing them properly for the individual loading cases and finally what are the actions of the combined loading situations.

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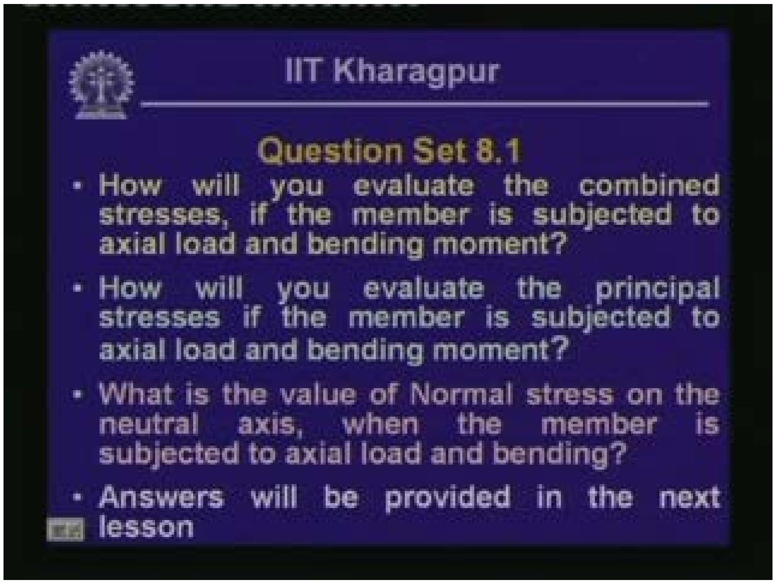

These are the questions given for you; how will you evaluate the combined stresses if the member is subjected to axial load and bending moment; how will you evaluate the principal stresses if the member is subjected to axial load and bending moment and what is the value of normal stress on the neutral axis when the member is subjected to axial load and bending.

Well, look into these questions, try to go through the lesson; we will give you the answers of the questions in the next lesson.

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Strength of Materials

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Lecture – 35

Combined Stresses – II

Welcome to the second lesson of the eighth module which is on combined stresses part II.

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In fact in the last lesson we have discussed about or we have introduced the concept of the combined stresses in the member wherein a particular member is subjected to combination of loads; say for example, axial load and the torsion or the torsion and the bending or could be axial load and the bending and various combinations of these individual loads. Now we have also looked into how we compute the stresses when a particular member is subjected to the combinations of these different loads.

Now in this particular lesson we are going to look into some more aspects of such combined loads and how do we analyze them and thereby compute the values of the stresses.

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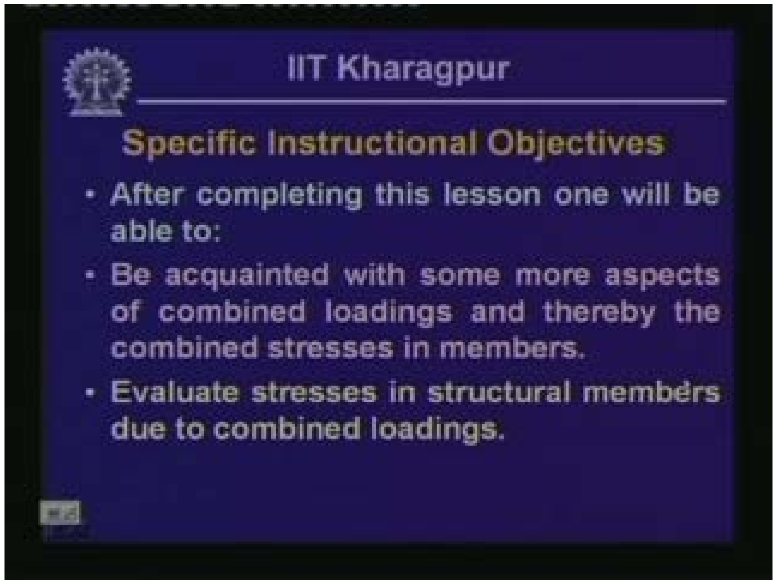

Hence it is expected that once this particular lesson is completed one should be able to be acquainted with some more aspects of combined loading in a particular member and thereby the evaluation of the combined stresses in members.