Strength of Materials

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Lecture - 33

Deflection of Beams – IV

Welcome to the fourth lesson of the seventh module which is on deflection of beam part IV. In fact in the last three lessons of this particular module we have discussed how to evaluate a slope and deflection in beams for different loading situations considering the differential equation of the elastic curve. And also we have looked into the method of superposition; how to employ method of superposition in evaluating slope and deflection of beams and subsequently we have looked in to the theorem of moment-area or moment-area method to evaluate the slope and deflection in beams.

Now in this particular lesson we will be concentrating on the evaluation of slope and deflection of beams employing moment-area method again. But in the previous examples we have looked into for the beams where the EI value or the flexural rigidity is uniform throughout the beam. Now here we will be considering the beams where EI may not be uniform throughout the beam, there could be variable moment of inertia or if there is a non-prismatic beam where it is varying continuously then what will be the values of slope and deflection in such beams.

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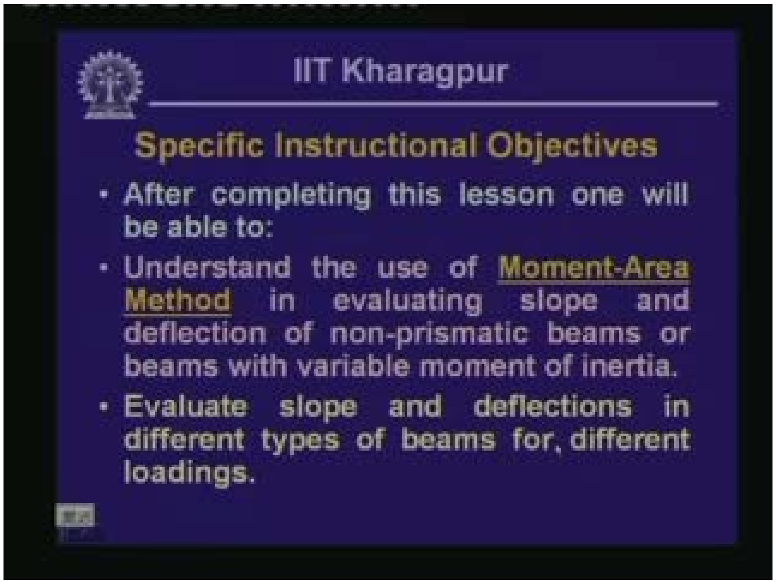

Hence it is expected that once this particular lesson is completed one should be in a position to understand the use of moment-area method in evaluating slope and deflection of non-prismatic beams or beams with variable moment of inertia. One should be in a position to evaluate slope and deflections in different types of beams for different loadings.

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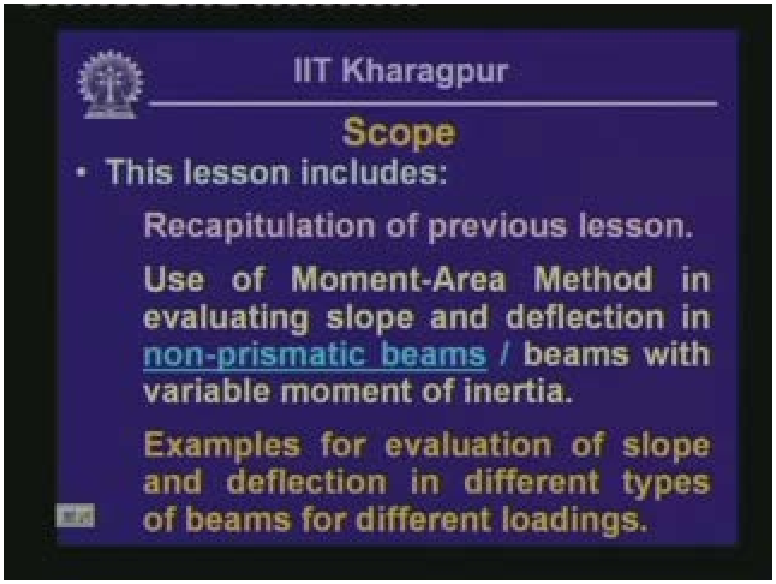

The scope of this particular lesson includes recapitulation of previous lesson and you will be doing that through the question answer sessions. We will be answering the questions which were posed last time and thereby recapitulate the previous lesson.

Use of moment-area method in evaluating slope and deflection in non-prismatic beams or beams with variable moment of inertia. Also, this lesson includes the examples for evaluation of slope and deflection in different types of beams for different loading. Well, next we will look in to the answers of the questions which were posed last time.

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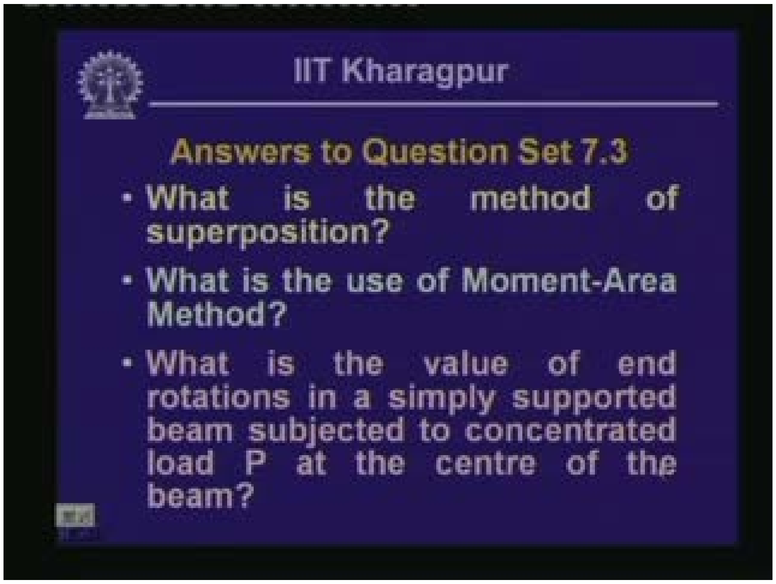

The first question is what is the method of superposition.

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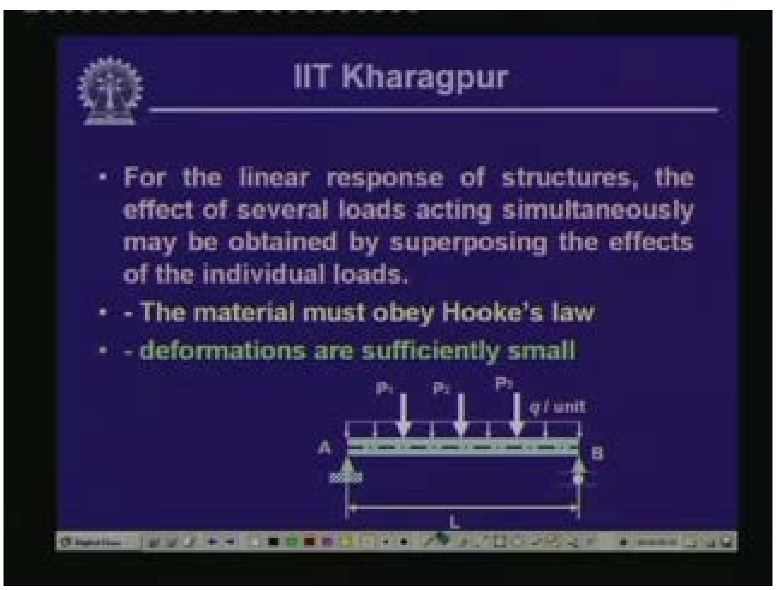

Now if you remember we said that in a particular beam if we are interested to evaluate the slope and deflection for multiple loadings like the loadings shown over here that this particular beam which is simply supported subjected to uniformly distributed load of intensity q per unit length and also it is subjected to concentrated loads P 1 P 2 and P 3. Now if we are interested to evaluate displacement or deflection of this particular beam at this point for these loadings then we can employ any of the methods which we have learnt; let us say the usage of differential equation of elastic curve, we can use differential equation of elastic curve which says that EI d 2 i dx 2 is the bending moment m and we can employ the to evaluate the deflection at these particular point for general loading as shown.

Also, we can evaluate the deflection of this particular point using this method of superposition wherein we use individual loading; instead of considering all the loadings acting on this particular beam we consider the loads one by one and evaluate the deflection at this particular point for each individual load and finally we sum them together to get the final deflection at this particular point. and it is expected that the results which we get in either case; that is in the first case when we use the differential equation for elastic curve for the whole lot of loading and compute the deflection and if we employ method of superposition by using differential equation or area moment theorem and if we evaluate the deflection at that particular point for individual loading and sum them up the result should be identical.

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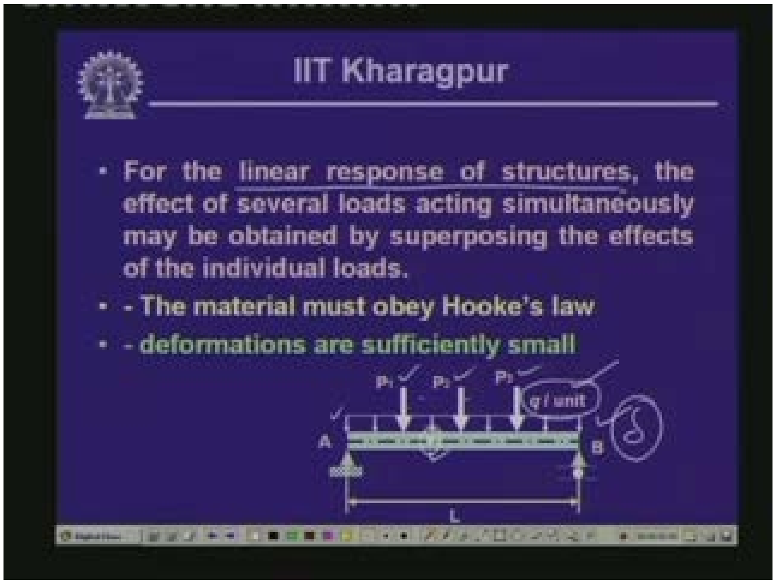

This is what the method of superposition says that for the linear response of the structure this is quite important that when the response of the structure is linear means that the deflection which we compute is a linear function of the load and then the effect of several loads acting simultaneously may be obtained by superposing the effects of the individual loads. For these you will have to satisfy these two criteria: one is that the material with which the beam is made of must obey the Hooke's law and the deformations are sufficiently small so that we can employ a superposition technique. This is what we call as superposition.

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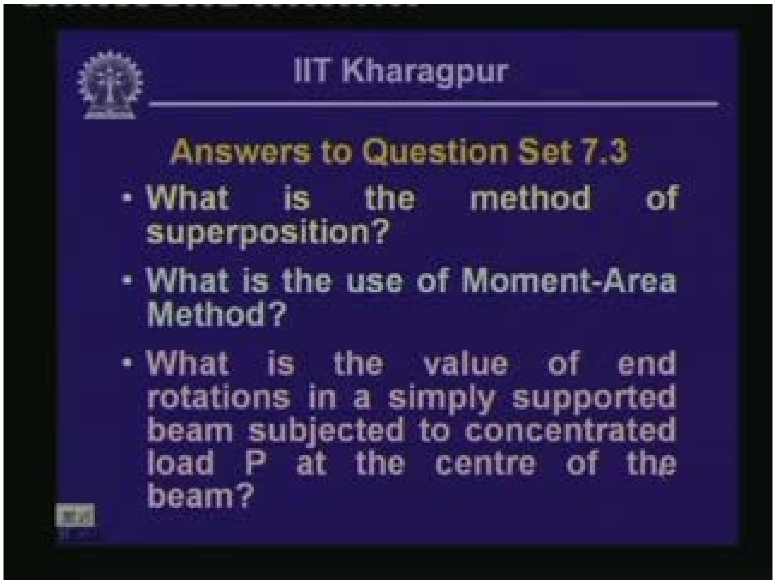

The second question was what is the use of moment-area method.

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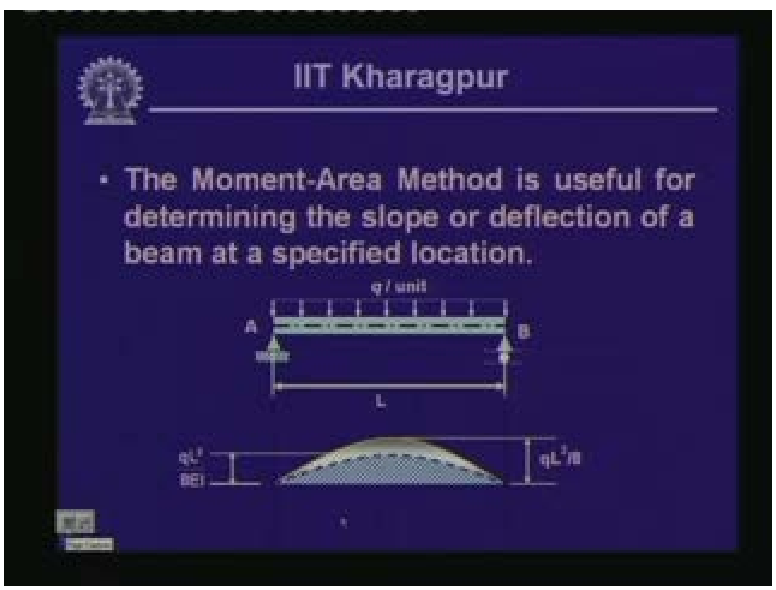

Now we have seen that in the last lesson we have discussed about the moment-area method and consequently we have discussed the two theorems moment-area theorems.

Now they are useful in determining the slope or deflection of the beam at a specified location. Now, for using moment-area method as you have noticed that we need to employ the bending moment diagram of a particular beam for which we are interested to evaluate slope and deflection and from bending moment diagram we arrived at the M by EI diagram and that is what is our interest for employing moment-area method that we draw M by EI diagram from the bending moment diagram and using this M by EI diagram we compute the value of slope and deflection in a beam at a particular location where we are interested in.

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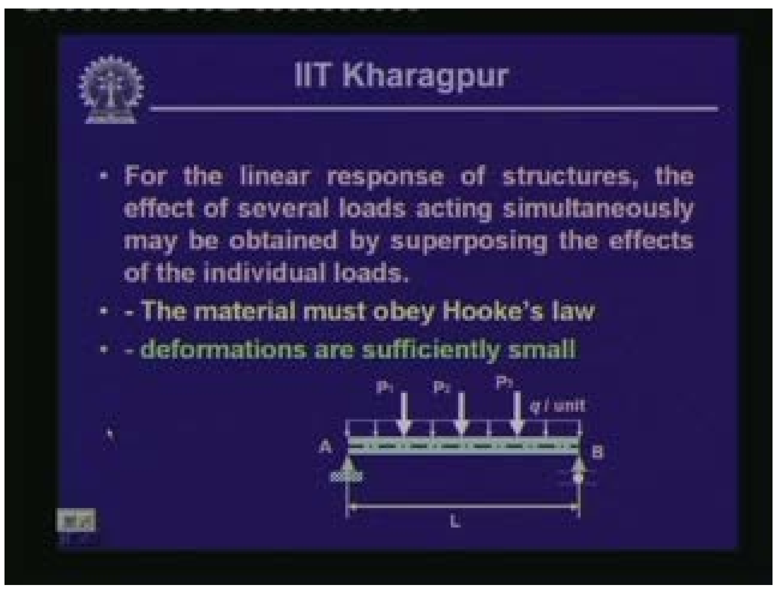

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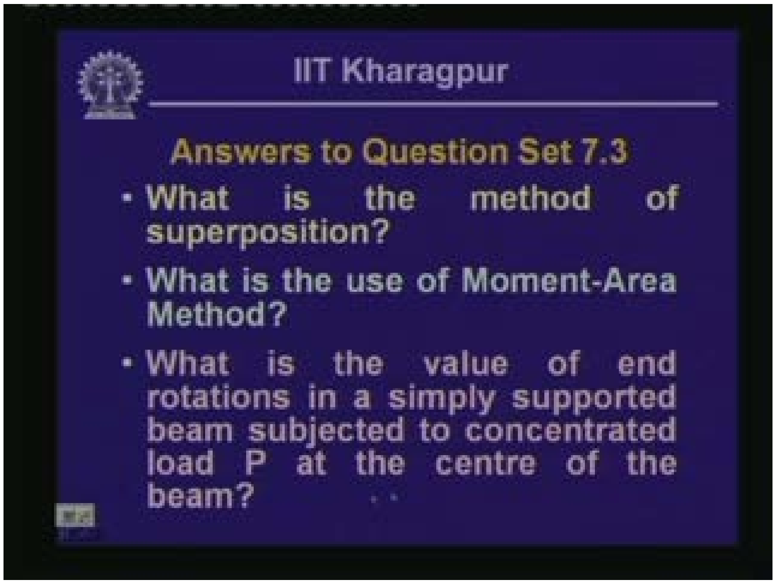

And the last question given was what is the value of the end rotations in a simply supported beam subjected to concentrated load P at the center of the beam.

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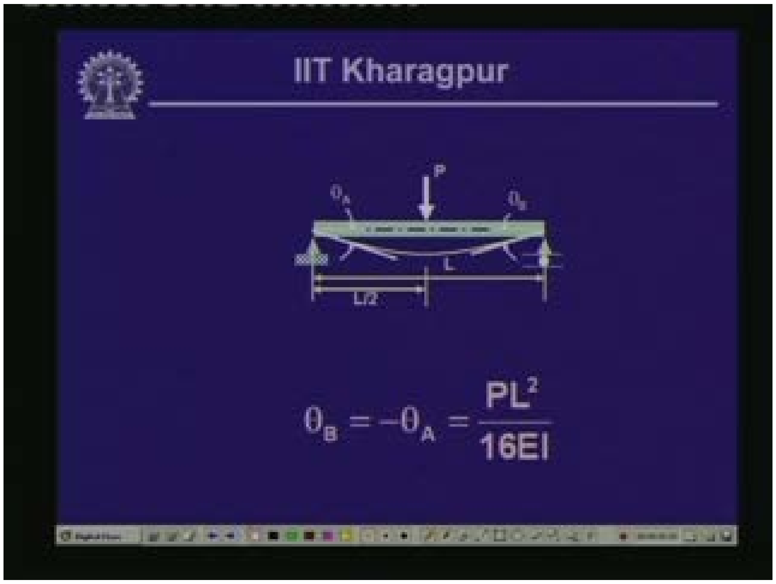

In fact we have discussed this example and we have solved it using differential equation of elastic curve as well as using moment-area method. and as we have seen, that when we compute the value of the rotation at the end the value of the rotation comes as PL square by 16EI so the value of the end slope is whether theta A or theta B because this particular beam is symmetrical and is symmetrically loaded plus these two, magnitude-wise are both theta n and theta B are in the same which is PL square by 16EI and from the sign convention as you know that from this axis it is rotating in a clockwise direction, (Refer Slide Time: 8:03) from this axis it is rotating in an anticlockwise direction so for this matter it is negative and according to our convention this is positive and this is what is indicated over here: theta B is equals to minus theta A and magnitude-wise it is PL square by 16EI. Those were the answers of the questions which we posed last time.

And you know, as I have told you that in the three lessons of this particular module, we have seen that how to compute the value of slope and deflection in a beam for particular support conditions and also for a particular type of loading how to compute the values using differential equation of elastic curve or moment-area methods or employing any of these using superposition method we can compute these values.

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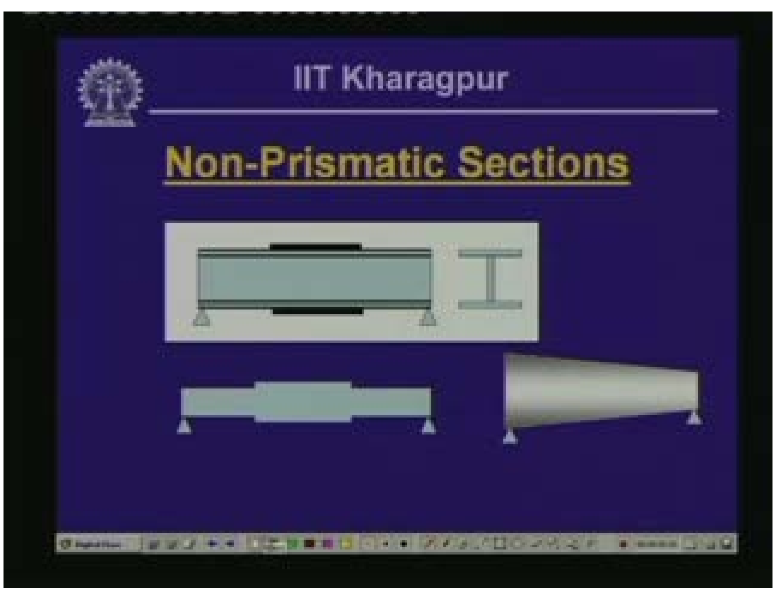

Now here in the previous examples whatever we have computed so far we must noticed that we have used the value of EI which we have defined as flexural rigidity we have kept constant throughout the span of the beam. That means the value of the moment of the inertia is constant. Or in other words, the cross-sectional parameters of the beam is not changed throughout the span of the beam.

Now the question is, if it is not constant, if EI is a very good parameter then what happens. So that is what we are going to look in to today. Now in this case let us see that many a times for economic reasons we use this i cross-section of the beam as we have seen that cross-sectional area which is in the form of I is useful because the areas are masked away from the neutral axis and as if this particular section contributes more to the moment of inertia I.

Now as we have seen that if the beam is a simply supported one subjected to different kinds of loading either concentrated load or distributed load, the maximum bending moment occurs at the center of the span, at the center of this particular beam. Now, to enhance the strength of the beam many a times in the central region we add some additional material so that the moment of inertia can be enhanced further. That means on the top of this we can add some plate on either side so that the moment of inertia of the cross section in this particular region can be enhanced.

Now as we have seen that the bending stress sigma is equals to M by L. now if we increase the value of I then the value of sigma comes down and thereby this will be more effective in carrying the stress if we can increase the moment of inertia in the central region.

Now if we do that then this particular beam becomes a section a different section it becomes the variable cross-sectional area. that means at this particular location this may be I, in this particular location this may be twice I or some fraction of values of I so thereby you have different values of moment of inertia in these regions and as a result if we like to employ the differential equation which is EI d 2 y dx 2 is equals to moment then we cannot employ the uniform expression for all the regions, we will have to write down this differential equation for different regions and we can compute it.

Now in this particular case it becomes rather simpler. But for a case where you have a beam of this particular form where the section of the beam is a non-prismatic one; that means any section you take the cross-sectional parameter is changing always and thereby the value of the moment of inertia I is changing and its function of x along the length of the span.

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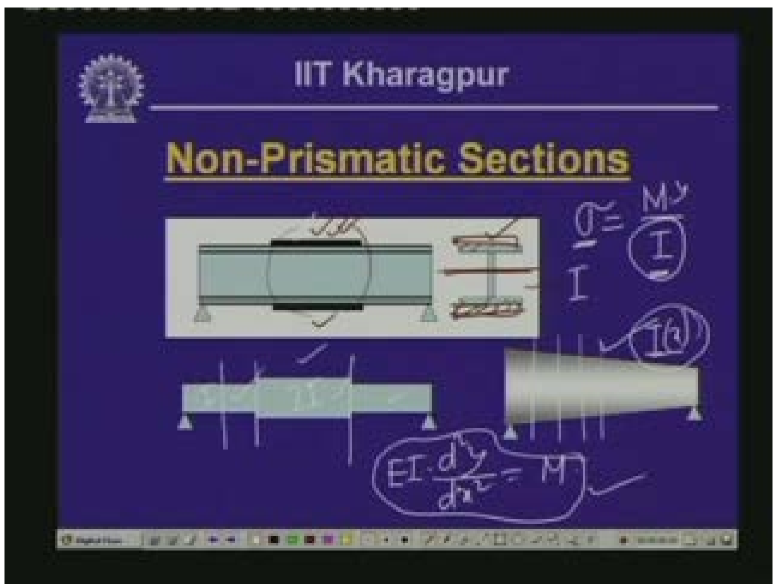

Now since this L is in the denominator as you have seen that when you compute that M by EI diagram now this becomes a little complex when we try to evaluate the slope and deflection. But still we can use the moment-area method in evaluating the deflection and the slope of such beams by making a little simplification. now instead of considering this non-prismatic one for a particular length of the beam we can make it a prismatic one and thereby we can think of that this particular section is composed of several prismatic parts and for this particular region we can write down the differential equation or we can evaluate M by EI diagram for this beam segment-wise and we can compute the value of slope and deflection.

Now, in the process the slope and deflection which we evaluate for the entire beam now though it is not going to be exactly the same if we consider the moment of inertia which is a function of x but nevertheless we can accept the solution with reasonable accuracy for using in the design practice. So we can employ this kind of technique for evaluating the slope and deflection in a beam. That is what we will be looking into while solving the examples and we will be demonstrating the effects of this using some examples and we will see that how do we compute the slope and deflection in beams which are non-uniform that means the moment of inertia is not uniform throughout the span of the beam or if the sections are non-prismatic.

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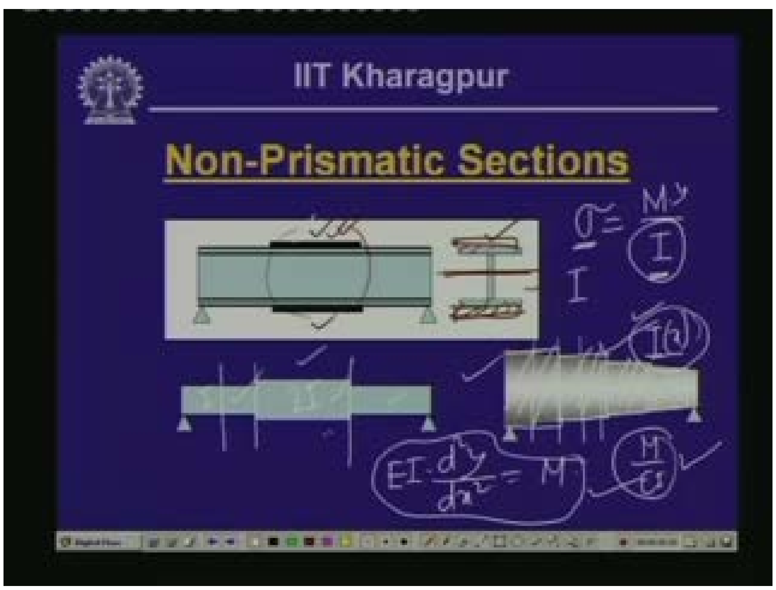

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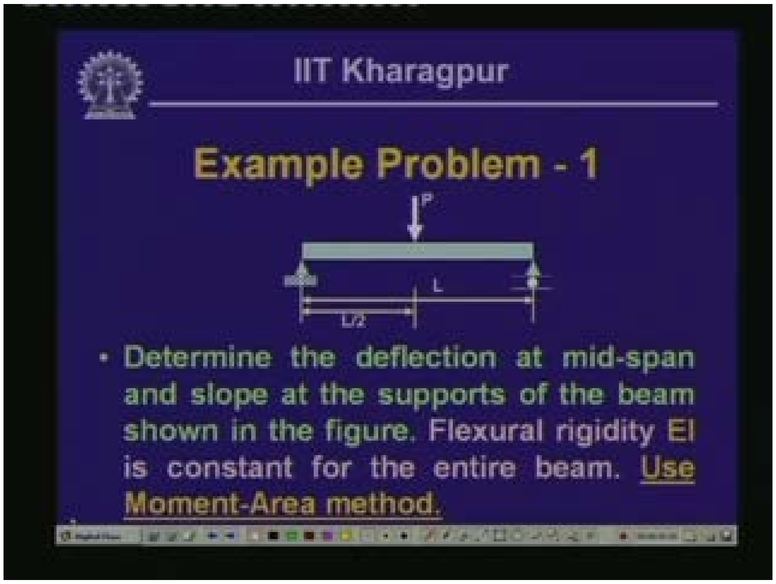

Well, let us look into the example problem which I had given to you last time for evaluating the slope and the deflection so the deflection at the mid-span and slope at the supports for loading indicated over here. And we will have to use moment-area method to evaluate the slope and deflection.

Now this particular load is acting at the center of the beam. Now, to employ the moment-area method as you know first you should know the bending moment diagram and the each ordinate of the bending moment diagram if it is divided by the flexural rigidity EI, the resulting diagram which we get that becomes M by EI diagram.

Now, to obtain the bending moment diagram for this beam what we need to do is, the first step is the evaluation of the reactive forces. So let us call this end as A and this end as B. So the reactive forces that will be acting at the end is R A and H A and at support B is R B as you have seen earlier and since there are no horizontal force H A would be equal to 0 so you are left with only R A and R B and if you compute the values, the values of R A and R B would be equal to the half this concentrated load which is acting which is P by 2.

And thereby, if we take any section over here then the moment will be equals to R A times x and at L by 2 it will be PL by 4 and again at this point and these two support points it will be 0.

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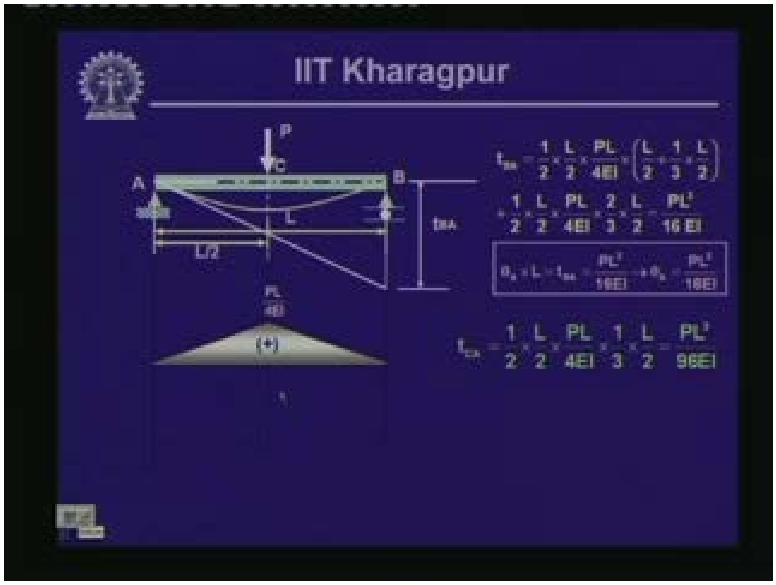

So if we plot the bending moment diagram the bending moment diagram looks like this that at the two ends the value of the bending moment is 0 and at the center we have the value of the bending moment as PL by 4 and when we divide this by EI this becomes PL by 4EI and truly speaking now this is M by EI diagram and not the bending moment diagram alone. So this is the bending moment diagram (Refer Slide Time: 16:16) divided by the flexural rigidity EI of each of the ordinate and thereby this is M by EI diagram.

Now we like to compute the value of the deflection at the center and beam symmetrical this is the maximum deflection that is what we have to obtain at this particular point and also we need to find out the slope of this beam. Let us call this as theta A. Now we employ the second moment-area theorem to evaluate the tangential division of point B with respect to point A.

Now as you know, the second moment-area theorem states that the tangential division of any point on the elastic curve, in this particular case let us say this is B and with reference to the tangent drawn on any other point on the elastic curve which is A here which is, if you draw the tangent from A and this is the tangential derivation which you call as t BA is equal to the moment of the M by EI diagram taken about the first point.

So if we take the moment of this M by EI diagram with respect to point B we get this tangential division of point B with reference to the tangent drawn at point A. So let us evaluate this value t BA. So t BA now here this whole triangle you can divide into two parts; one is this one; this is first one and this is second one. Now for first one if we take the moment of this triangle with respect to point B this is half and this distance is L by 2 so half L by 2 and to the ordinate is PL by 4EI and the c g of this particular part of the triangle is this distance which is one third of L by 2 and this is what has been taken over here; one third of L by 2 plus this distance L by 2 (Refer Slide Time: 18:23) so that is the moment of this particular part of the triangle.

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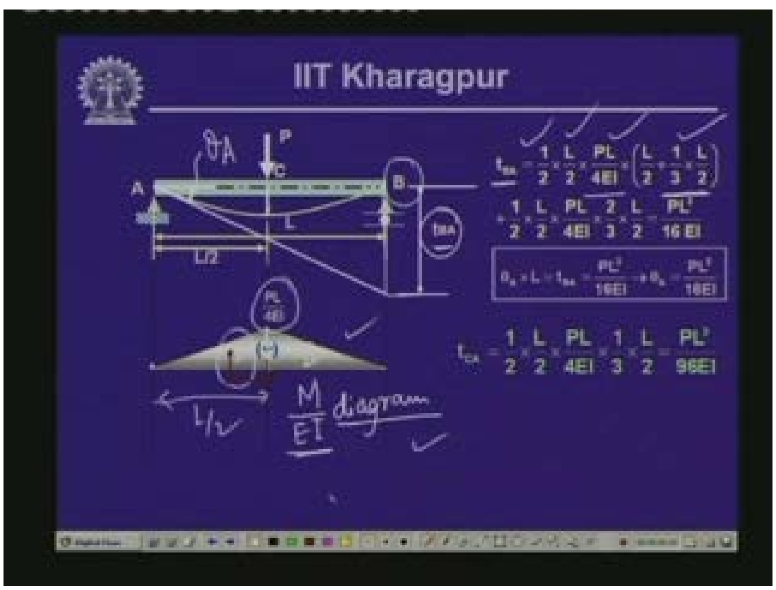

Then if we take the moment of this triangular part half L by 2 again its ordinate is PL by 4EI. Now this distance the c g distance from this end is two third of L by 2 and this is what is written over here two third of L by 2 and if you compute this this becomes PL cube by 16EI. So this particular length that means the deviation of this point B on the elastic curve with respect to the tangent drawn at point A this distance is t BA which is equals to PL cube by 16EI.

Also, if you look into this particular diagram AB and let us call this point as D, from this triangular part A BD we can write the distance BD is equals to this slope theta A multiplied by length. So BD is equals to the slope theta A multiplied by the length and BD which is t BA as we have seen as PL cube by 16EI this is equals to theta A multiplied by length L. So theta A is nothing but equal to PL square by 16EI and this is what is indicated over here.

Therefore, value of theta A is equals to PL square by 16EI and this we have seen earlier using differential equation of elastic curve.

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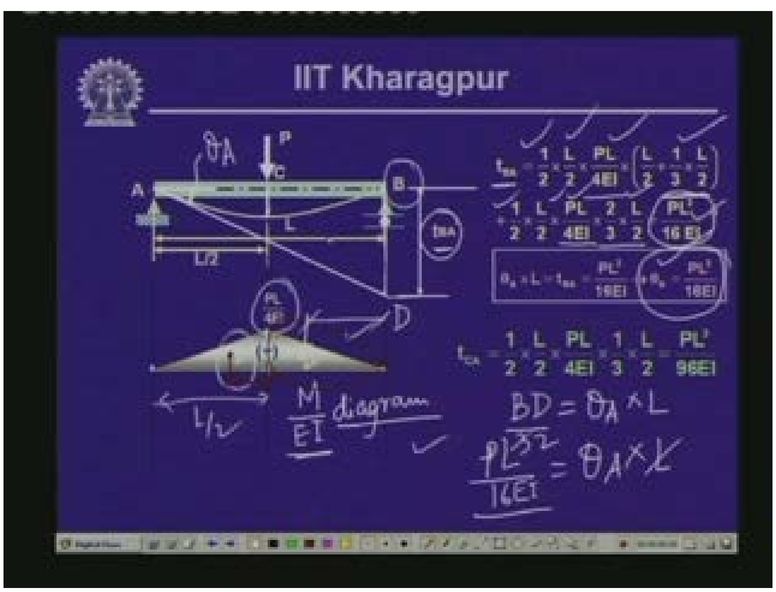

Now what we are interested in is to evaluate the deflection of point C which is going to be the maximum deflection in this particular case because this is a symmetrical beam and the load also is symmetrical with respect to the beam. Then if we like to compute the value of deflection which is from here to here (Refer Slide Time: 20:26) this is the value of deflection let us call that as delta; now if you like to compute delta now how do you do that?

Now at this particular point again if we try to compute the value of the tangential deviation of this particular point with reference to the tangent drawn at A we can evaluate this particular distance and this we call as t CA; the tangential deviation of point C with respect to the tangent drawn at A and the whole distance from this center to this particular point we can get this length as........... this length is equals to theta A multiplied by L by 2.

So, from the whole distance if we subtract this particular part we will get the value of delta. So let us compute then the value of the delta which is.................... now the t CA value the tangential deviation of point C with respect to the tangent drawn at A is equals to the area of the moment of the M by EI diagram between A and C taken about C. So the area of the M by EI diagram between A and C is half L by 2 times PL by 4EI and if we take the moment of this particular area with respect to point C then its distance is one third of L by 2 and this gives us a value of PL cube by 96EI. Hence, the value of delta as I said is equal to theta A multiplied by L by 2 minus t CA.

Now theta A as we have seen is equal to PL square by 16EI so this is PL cube by 32EI minus PL cube by 96EI so this gives us a value of..... this is PL cube by 32EI minus PL cube by 96EI so if you compute this this is going to be equal to PL cube by 48EI. And we have seen earlier that the maximum deflection which we get in a beam which is simply supported and subjected to a concentrated load at the center gives us a value of PL cube by 48EI.

Also please note over here that in this particular beam this being a symmetrical beam with symmetrical loading, now the elastic curve which we get also is symmetrical and as we have seen earlier that the values of the rotations which you get at the end they are the same.

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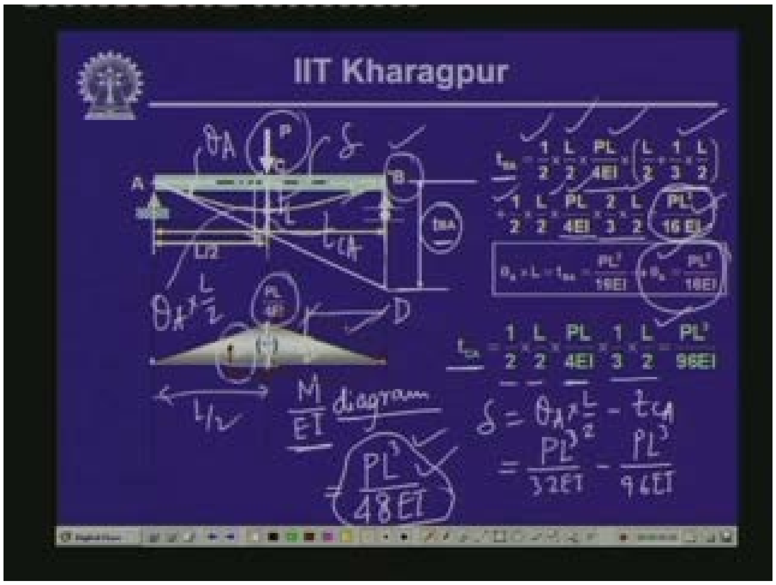

Now what will be the value of rotation over here; as we have seen that theta A equals to PL square by 16EI. Now if we draw a tangent at this particular point at the center point of this beam that becomes.............. now if we draw the tangent at this particular point of the elastic curve the tangent at this point is horizontal and as we exceed this gives us the maximum value of the displacement or the deflection where the tangent to the elastic curve is horizontal.

Now if we take the tangent at this particular point (Refer Slide Time: 23:46) at the central point and draw it up to this and if we consider these two points we can say that the tangential deviation of point A with respect to the tangent on at point C which is this equals to this distance which we can call as say t AC. That is tangential deviation of point A with reference to the tangent drawn at point C now this distance is nothing but equals to this particular distance which is the deflection. Hence we can compute the value of the maximum deflection from this particular part of the M by EI diagram as well.

Now what we can do is we can find out the moment of this particular part of the bending moment diagram which is between A and C and taking the moment of the EI diagram about A will give as the value of the deflection. Now if you do that the moment of this particular diagram is half, the area is L by 2 times PL by 4EI so this is the area multiplied by the distance of the c g from this end which is two third of L by 2; so two third of L by 2. Now if we do that then we have 16 48 so this gives us PL cube by 48EI. Hence we get the value of the delta by computing this part of the bending moment M by EI diagram; we will take this part of the M by EI diagram, take the moment of this with respect to A and we get the tangential deviation of point A with reference to the tangent drawn at C and that is nothing but equals to the deflection of this particular beam.

Thus, in a simply supported beam we can compute the value of the maximum deflection either the way we have done at first; that means we compute the tangential deviation of the support B with reference to the tangent drawn at A and thereby first you compute the value of slope and then at the center we compute the value of the deflection. Or else in a simpler form we can take half the area of the M by EI diagram, take the moment with respect to point A and compute the value of the deflection and you will get the identical result. So you can apply this technique as well for evaluating the deflection.

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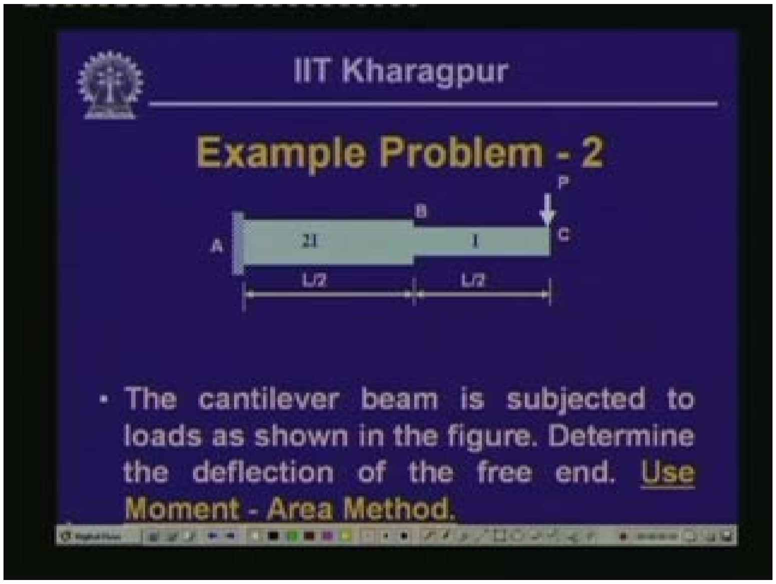

Well, now let us consider an example where the moment of inertia is not constant throughout the beam instead we have variable moment of inertia. Over the part AB we have the moment of inertia as 2I and over the part BC the moment of inertia is I.

Now, in this beam this is a cantilever it is fixed at one end and is free at the other, it is subjected to a concentrated load at the tip and we will have to compute the value of the deflection of the free end and we need to use the moment-area method.

Now if we do that the first step will be to evaluate the bending moment of this particular beam. If we consider first let us say uniform I, thereby we have the beam subjected to a concentrated load here P over the length L and if we compute the reactive forces that means removing this if we replace this with the reactive values we have vertical force, horizontal force and the moment and we can compute the value of the since we are interested in the bending moment diagram we compute the value of the moment so value of the moment is equal to if you call this as M, M is equal to minus P times L. So at this point it is PL and if you compute at any point it is P times x so it is varying linearly from 0 to the maximum value which is P times L.

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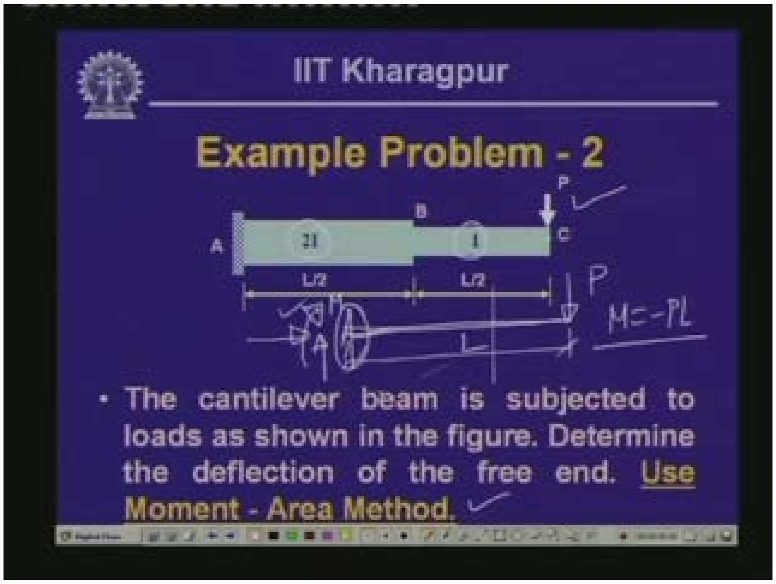

So let us look into the bending moment diagram of this and then we go for the M by EI diagram.

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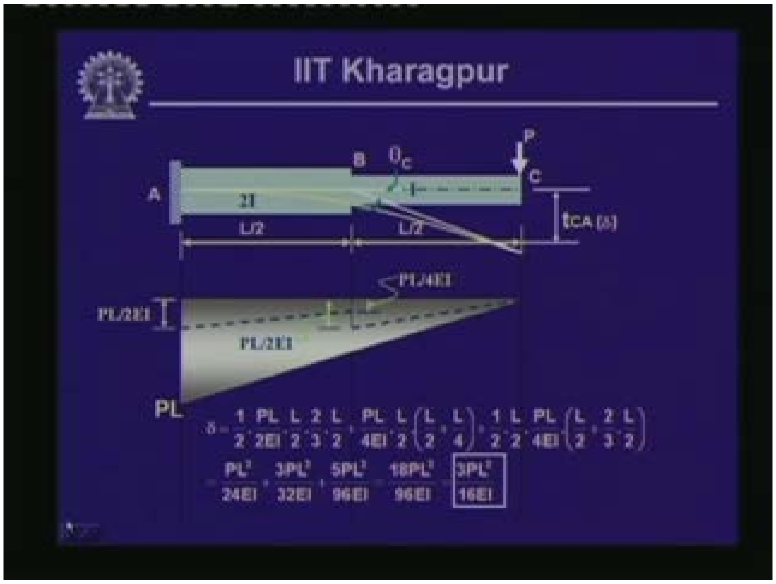

This is the bending moment diagram that means zero bending moment over here and at this point we have the maximum value of the bending moment which is equals to PL. So this external one shows the bending moment diagram which is varying from 0 to linearly from 0 to PL.

Now, when we like to employ the area moment theorem what we are interested in is to evaluate the M by EI diagram. And as we know that M by EI diagram is nothing but the bending moment diagram divided by the EI value at each ordinate and that gives us the M by EI diagram. Now if you look into this particular part B to C the value of the bending moment of inertia is I hence if we divide each ordinate of the bending moment diagram which is this particular part (Refer Slide Time: 28:42) so at this particular part the bending moment is equal to the value PL by 2.

Now if we divide this by the EI value then the ordinate of the M by EI diagram at this particular point becomes PL by twice EI. Now since at this particular point there is a transition from I to 2I so immediately next to this section the moment of inertia is 2I. So the change from PL by 2EI to it will be changing to PL by 4EI because bending moment is PL by 2 that divided by 2I so this is equals to PL by 4EI. This is the value over here.

From PL by 2EI it goes to PL by 4EI and then again at this particular point the bending moment is P times L that divided by twice I that means this is going to be equal to PL divided by 2EI. So at this point again the M by EI value is PL by 2EI. So from PL by 4EI it goes to PL by 2EI and varies linearly. So in effect this is going to be the M by EI diagram in this particular distance. So the M by EI diagram is something like this.

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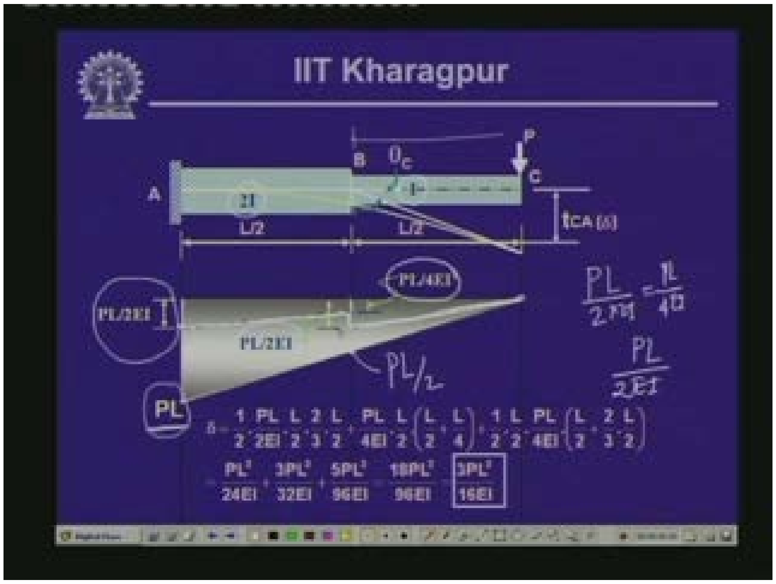

Now we will have to employ this M by EI diagram to compute the value of the slope and the deflection. Now for computing the deflection of the slope what we need is now we need to use the moment-area method and for using moment-area method we will have to employ the moment-area theorems and as you know that for employing moment-area theorems we will have to deal with the M by EI diagram.

And the second of the moment-area theorem states that the tangential deviation of any point on the elastic curve with reference to another point on the elastic curve or you draw the tangent at that particular point that gives you the deviation that is equivalent to the moment of the M by EI diagram between those two points taken about the former point. Hence, if we do that; that means on the elastic curve it is expected that the elastic curve is moving like this.

Now if we choose two points a point here and a point here (Refer Slide Time: 31:05) and if we draw the tangent at point A which is horizontal because here the slope is 0 dy dx is 0, now the tangential deviation of this particular point C with reference to A is this value which we have said as t CA. Thus, the tangential deviation of point C with reference to the tangent drawn at point A is equivalent to the moment of the M by EI diagram between A and C taken about C. Hence, if we take the moment of M by EI diagram between A and C about C then what we get?

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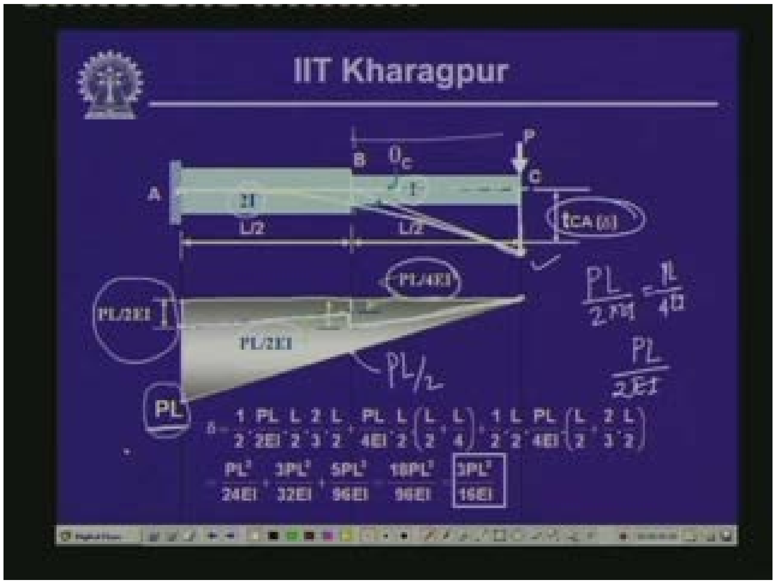

Now let us look into this expression for evaluating this t CA. So t CA is equals to........... now this M by EI diagram which is of this particular shape we divide into three parts: one is this particular triangle and then thereby we are left with this trapezoidal area and this trapezoidal area we can divide into two parts: one is this rectangular part with ordinate PL by 4EI and then this is the triangular part which is having PL by 4EI as the ordinate. So if we compute or if we take the moment of all these three areas with reference to point C then let us see what we get.

Now for this particular part, for this triangular area let us call this as area 1 which is half PL by 2EI is the ordinate, L by 2 is the length times two third L by 2 is the distance of the c g from here, this is two third of L by 2 so two third of L by 2 is taken over here so that is the moment of this particular area with reference to point C.

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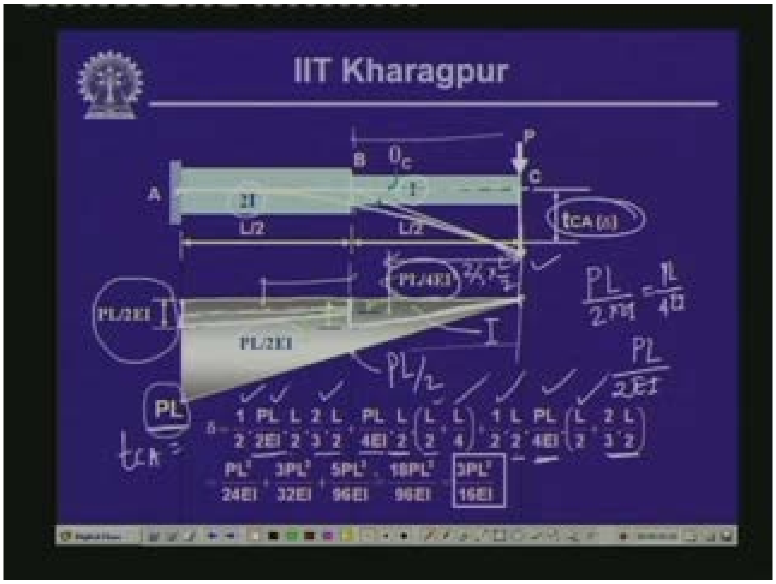

Now if we take this rectangular area, now the ordinate here is PL by 4EI over the length L by 2 so PL by 4EI multiplied by L by 2EI is the area and the c g distance here is from here to here is L by 4 and plus L by 2 so L by 2 plus L by 4 plus if we take this triangular area which is half it will be L by 2 times PL by 4EI. This is PL by 2EI and this is PL by 4EI (Refer Slide Time: 33:27) which we have taken constant so balance is PL by 4EI so half into L by 2 into PL by 4EI and the c g distance from this particular end is two third of L by 2 plus L by 2 this distance is L by 2. So it is L by 2 plus two third of L by 2.

Now if we add these three parts together the first part gives PL cube by 24EI, second one gives you PL cube by 32EI and the third part gives you 5PL cube by 96EI and if you sum them together, finally you get the value as 3PL cube by 16EI. So this is the value of........ t CI is nothing but equals to the deflection of this elastic curve over here that is what is the delta so delta is equal to 3PL cube by 16EI.

And also if we like to compute the value of the slope if we take the tangent at this particular point and here since the tangent is horizontal so this will give us the value of the slope which is equal to theta C and this theta C is nothing but the area between A and

C area of the M by EI diagram between A and C and that is as per the first moment-area theorem that the slope the differential slope between two points is equal to the area of the M by EI diagram between those two points. So here the differential slope between these two points is theta C and we will have to take the area of the M by EI diagram between these two points. And if we take that; that means the first part of the EI if we take all the areas and sum them together then we get the values and that will give you; (Refer Slide Time: 35:08) here if you take this as PL square by 8EI from the first part plus from here it is PL square by 8EI the area of the rectangular part and for the other triangular part it is PL square by 16EI. These are the three areas and if we sum them up we have this as 16EI and 5PL square by 16EI. So this is the value of theta at C.

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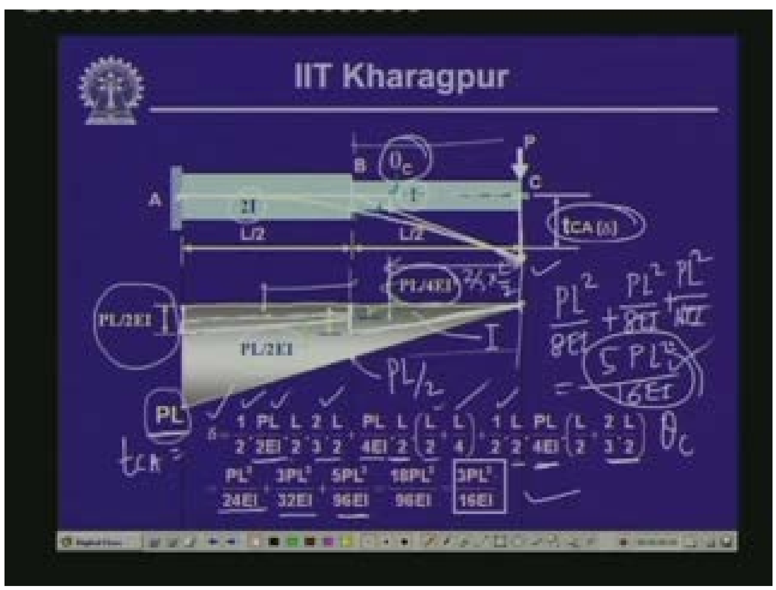

So theta C is equal to 5PL square by 16EI and the value of the delta is equal to 3PL cube by 16EI. Therefore, as you can see, we can compute the value of the deflection delta and the slope at the free end theta C even if the moment of inertia is variable using moment-area method.

Now that is what I was trying to explain to you that even if the beam is non-prismatic we can make it we can approximate in a segmental prismatic form and we can employ moment-area method to evaluate the slope and deflection in the beam and that is going to give you a close value to the exact one and we can employ that for a quick reference instead of going into a very rigorous analysis.

Now if we compare these values; in fact we have seen that if you have a beam which is having a constant flexural rigidity or the moment of inertia if it is constant right through we have seen what is the value of the deflection, what is the value of the slope for a cantilever beam etc; that means if we have a cantilever beam which is subjected to a concentrated load at the tip we have seen that the maximum value of the deflection comes as PL cube by 3EI and the slope as PL square by 2EI.

Now at this stage we have seen that what is the value of the slope and the deflection. Now let us make a comparison of these values to get an idea that what change we get physically really because of this non-uniform moment of inertia..

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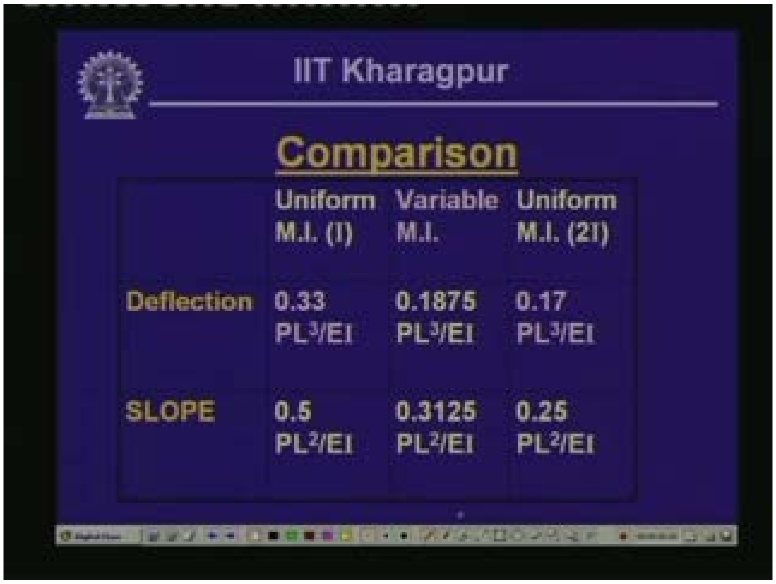

	Uniform M.I. (I)	Variable M.I.	Uniform M.I. (2I)
Deflection	0.33 PL^3/EI	0.1875 PL^3/EI	0.17 PL^3/EI
SLOPE	0.5 PL^2/EI	0.3125 PL^2/EI	0.25 PL^2/EI

Now you see that when you have uniform moment of inertia value I throughout the value of the deflection was PL cube by 3EI which is 0.33 times PL cube by EI. Now, at this stage we have seen, when we have variable moment of inertia where over half the length you have twice I and for the rest of the half you have moment of inertia as I that means when you have this variable moment of inertia over the section then the value of the deflection at the tip comes as 0.1875 PL cube by EI where you see that the value changes drastically from 0.33 to 0.1875. The change is of the order of around 43 to 44 percent.

Now, if instead of making 50 percent as twice the moment of inertia if you would have made that the whole beam is having moment of inertia of value 2I that means we provide a larger section throughout the length of the beam and then what happens is the value of the deflection which you get is 0.17 PL cube by EI which is half of this value when you have I.

Now if you notice over here the change from this where we have variable moment of inertia over the half of the length to where we have the full length as twice I the difference is not much. Now, also if you look into the value of the slope, in the uniform case we get 0.5 PL square by EI, when we have constant 2I we will get half of that 0.25, when we have 50 percent as 2I we get 0.3125 that is from 0.5 it gets reduced to 0.3125 0.3125 to 0.25 the reduction is not to that extent.

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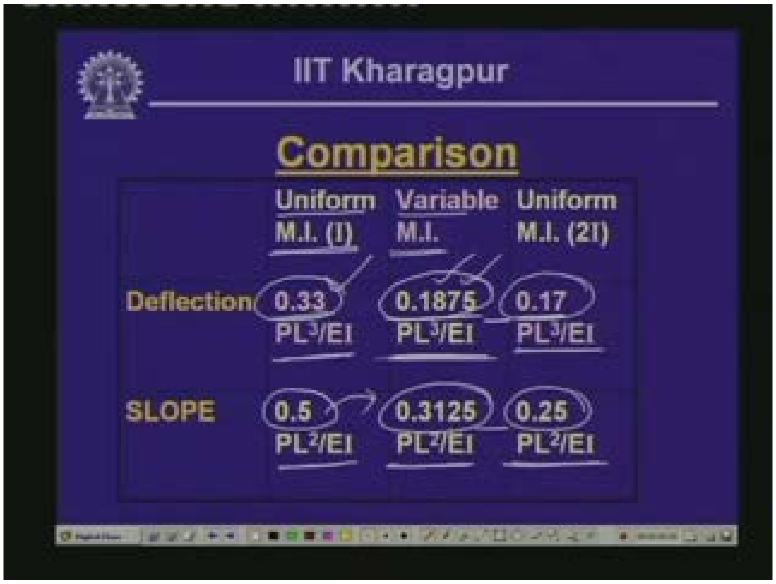

Now the idea is if you look in to physically that either you can have a beam with uniform cross section having moment of inertia as I or if you like to reduce the value of the deflection we increase the value of the moment of inertia to 2I throughout the entire span of the beam and thereby the deflection value will be half that when it was with I. now, instead of that if we provide the larger moment of inertia over the area where you have larger value of the moment; as you have seen in the cantilever beam the moment value is highest at the support point so towards the support point from the mid span towards the support point where we have increase the value of the moment of inertia twice than for the first half, we see that the value of the deflection it could reduce to a great extent and in the process in fact we have economized on the usage of the material as well.

Since we have used 2I only on the 50 percent of the length of the beam naturally we have not you know used the material over the entire length of the beam thereby we could make some economy on the selection on the section. So we could make a tradeoff between the reduction in the deflection and less usage of the material as well between the two extreme cases.

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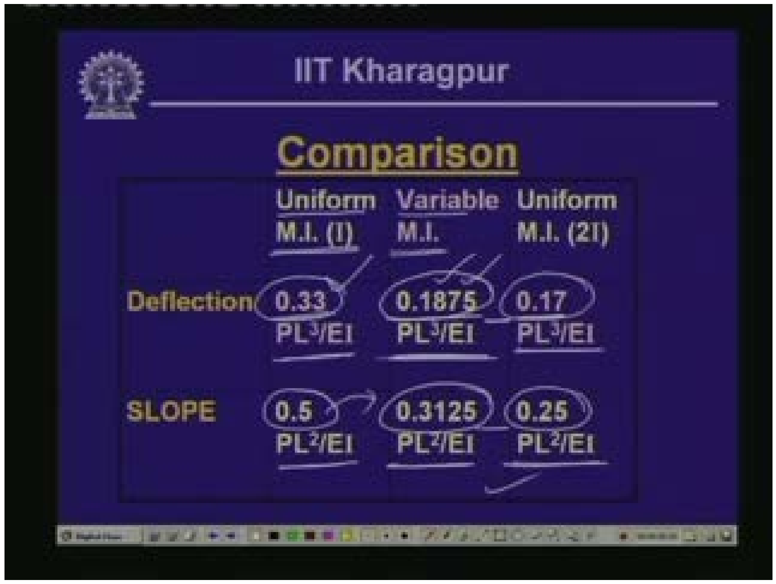

So, many a times we employ this particular situation in the design so that we can reduce the value of the deflection at the same time we can economize on the sections which we use.

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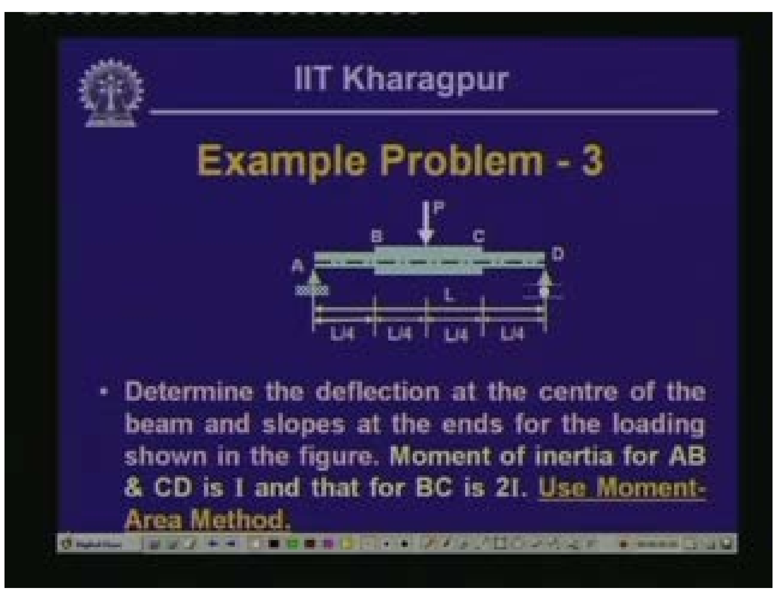

Well, let us look into another example of a variable moment of inertia when the beam is a simply supported one and is subjected to a concentrated load at the center.

Now as you have noticed that for this we get the bending moment diagram where the maximum value of the bending moment is at the center which is in this particular form and thereby in the central region if we have the moment of inertia of the section here this part is I (Refer Slide Time: 41:21), this part is I and this part is twice I as it is stated in this particular example. It says that: determine the deflection at the center of the beam and slopes at the ends for the bending for the loading shown over here and moment of inertia for AB and CD is I and for BC is 2I.

Now if we use moment-area method for evaluating the slopes and deflection here naturally first thing is that we will have to draw the bending moment diagram and as you know as the first step for drawing bending moment diagram you evaluate the reactive values R A and R B and for this case as we have seen R A and R B will be equal to P by 2. And if you compute the value of bending moment at any section over here we will have R A times x. So at different points you can compute the value of the bending moment which is linearly varying.

Now let us look into the bending moment diagram for this particular beam which is of this particular form which is shown over here.

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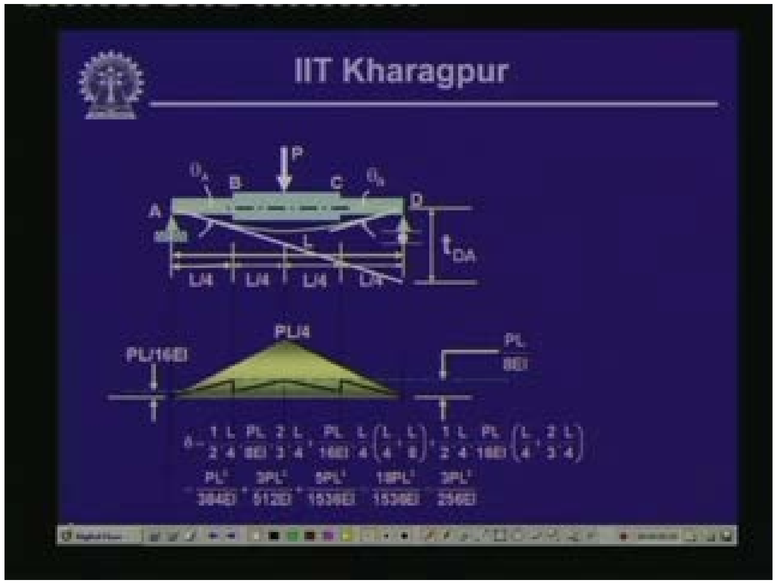

Now here as you can see that the maximum value of the bending moment is PL by 4 at the center which is P by 2 times L by 2 and at this point the value of the bending moment is P by 2 times L by 4 which is PL by 8 and so is at this point as PL by 8.

Now what we need to do is that we need to draw the M by EI diagram and for drawing M by EI diagram we divide the ordinates of the bending moment diagram by respective EI values. And as you can see over here that for these two parts you have one EI value, for this part BC you have another EI value, so over here for this the flexural rigidity is EI, for this flexural rigidity is EI and for this flexural rigidity is 2EI.

So when we divide by these magnitudes EI and 2EI at this point we get PL by 8EI as this ordinate. now immediately at this particular point particular section we have a change from I to 2I and thereby there is a change in the M by EI value over here which is going to be equal to PL by 16EI and at this point again it is PL by 4 is the bending moment divided by twice I so this is PL by 8EI and here also it is PL by 8EI (Refer Slide Time: 43:46). So these three ordinates they are called identical which is PL by 8EI and at these two points where there is a change over from I to 2I we have the values of the ordinate CI as PL by 16EI.

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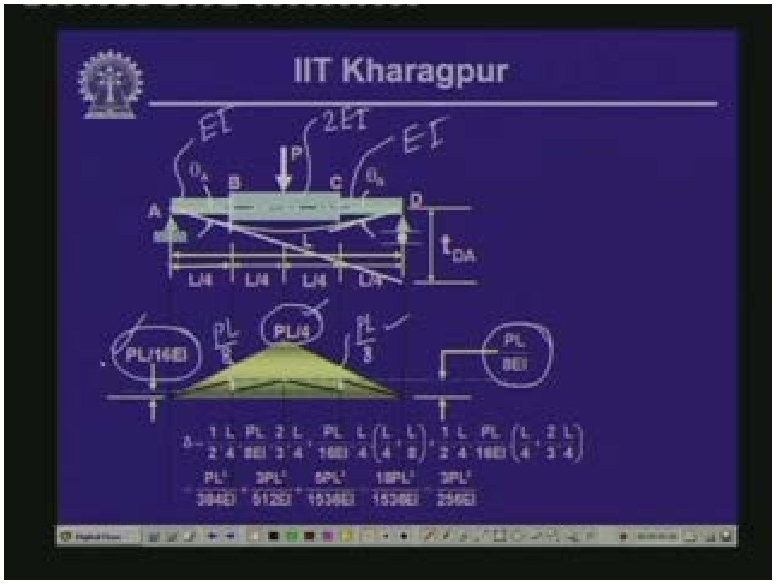

Therefore, the M by EI diagram as we can see here is like this, it comes over here, changes over here then goes like this (Refer Slide Time: 44:10) goes over here, changes here and then comes like this. This is the M by EI diagram for the beam between A and D.

Now what we need to do is that we need to compute the value of the bending moment delta at the center and we need to compute the value of the slope. now if we compute the value of the delta; now as we have done earlier, as I was stating you that at this point if we draw the tangent to the deflection curve and since this is symmetrical it is expected the deflection will be maximum here and the tangent to the elastic curve is going to be horizontal and thereby the tangential deviation of point A with reference to the point if we call this as say E with reference to E so this is the tangential deviation which is t AE and that is nothing but equals to the moment of the M by EI diagram between A and E taken about point A.

Now this M by EI diagram which you have drawn we can divide into three segments: one is this triangular part, then we have this trapezoidal part and this trapezoidal part we can divide into two areas: one is the rectangular part and the other one is the triangular part and if we take the moment of this particular half with reference to point A then what we get.

We have for the first part half into L by 4 is the length, now PL by 8EI is the ordinate and the distance two third of L by 4 is the moment of this area with reference to point A. then for this rectangular part (Refer Slide Time: 46:01) we have PL by 16EI as the ordinate, we have L by 4 as the length that is the area and then the moment of this particular area with reference to point A which is equals to l by four plus half of L by 4 which is L by 8. So this is the moment of this rectangular area with reference to point A.

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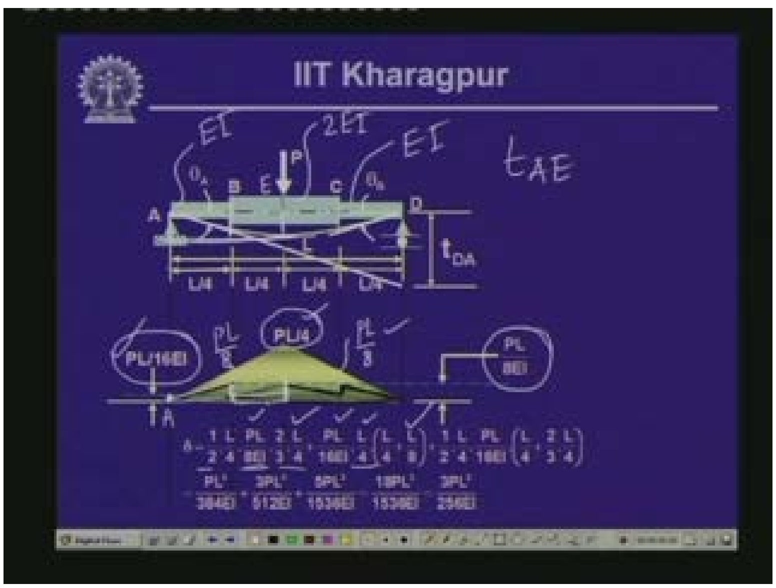

And thirdly, we have this triangular area which is half times L by 4 is the length, PL by 16EI this is PL by 8EI is the total ordinate minus PL by 16EI so balance is PL by 16EI and the moment of this c g from moment of this particular area with reference to this distance is L by 4 plus two third of L by 4.

Now if we evaluate these individual moments of individual areas we get this as PL cube by 384EI; 3PL cube by 512EI; 5PL cube by 1636EI and if you combine them it comes as 3PL cube by 256EI. This is the value of the deflection when centrally you have the moment of inertia as twice that of the end part of it. So in the two end zones we had the moment of inertia as I and in the central zone we had the moment of inertia as twice I. So we have added additional material at the central part of the beam to increase the moment of inertia value and thereby we are reducing the value of the bending stress because sigma I equals to M by EI; by enhancing I we are reducing the bending stress at that region and consequently we are reducing the value of the deflection as well.

Now that is what we see over here. The value of delta comes as 3PL cube by 256EI.

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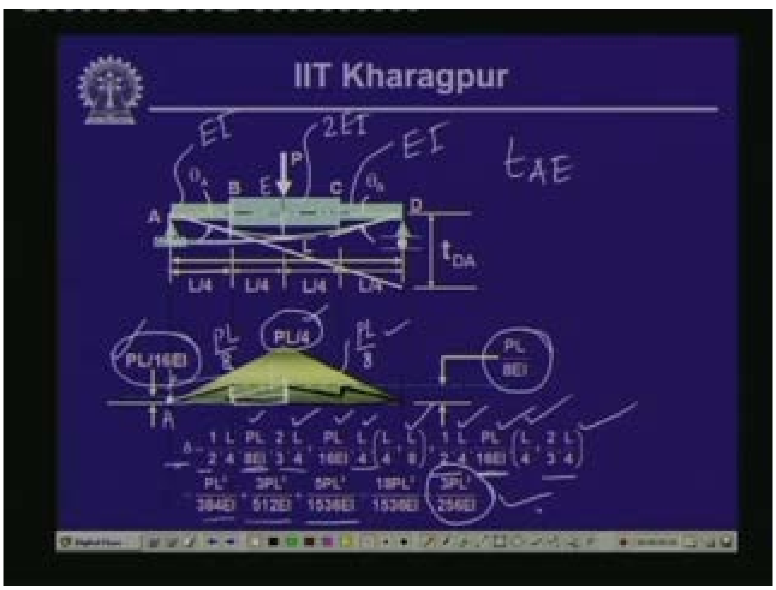

Also, if we compute the value of the slope at this particular end; now when we draw the tangent to this elastic curve at this point and when we draw the tangent to the elastic curve at this point (Refer Slide Time: 48:06) since this is 0 so this equals to the slope theta A. Hence we can take the area between these two points to evaluate the value of theta A and consequently we get the value of theta A as............. if we add up this area, now area is equal to half L by 4 times PL by 8EI which is PL square by 64EI plus we have for this part of the area which is PL square by 64EI again so PL square by 64EI plus we have for this particular part which is PL square by 128EI and if we add them together we have 128EI and this is 224 so 5PL square by 128. So this is the value of the slope which we have and this being symmetrical the value of theta A and theta B magnitude-wise it will be this value: 5PL square by 128EI.

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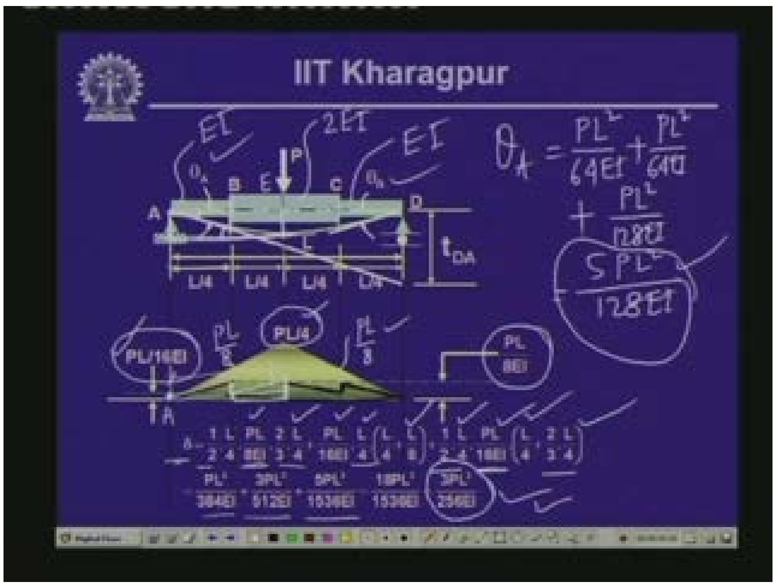

Now if we again make a comparison of the values which we have seen earlier, now in a simply supported beam where the value of the moment of inertia is uniform right through where the flexural rigidity is uniform then we have seen the value of the maximum deflection is PL cube by 48EI and the values of the slope as PL square by 16EI.

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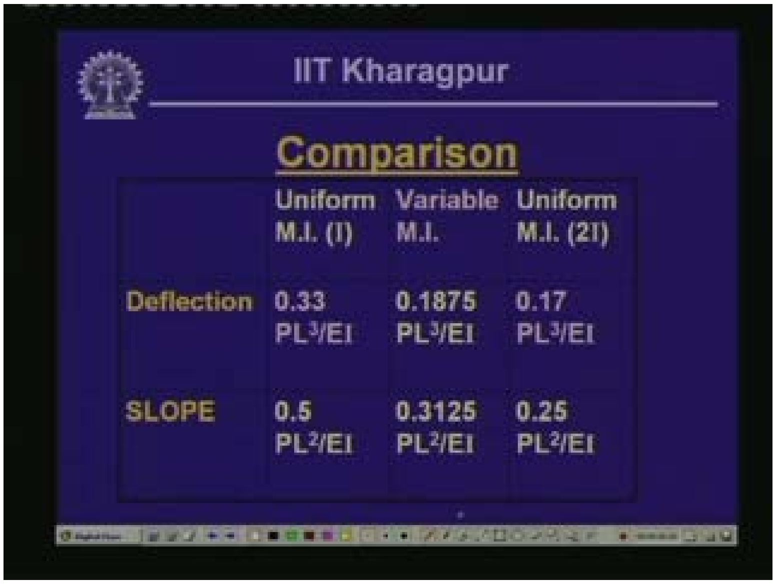

Consequently, if we make the value of I as 2I right through that means if we have uniform flexural rigidity but instead of EI now it is twice EI that means we are adding more material to it then naturally the values of the deflection will be half of the previous one and so will be the slopes.

Now, if we make the combination of these two which we have seen right now and also we have seen the maximum value of the deflection which is 3PL cube by 256EI and consequently we have looked into the value of the slope.

Now if you make a comparison you see that the value of the deflection which you get for constant I is 0.0 to 0.08PL cube by EI and in contrast when you have this variable moment of inertia you have a value of 0117 a drastic reduction from this to this and if you have uniform twice I then your value is 0.0104. In fact, from this to this the reduction is not to that extent and so is in case of slope. You see, here will get 0.0625, for the variable moment of inertia we get 0.0391 and when it is becoming twice EI we are getting 0.0313; hence the reduction again from here to here is not that much to the extent as we achieved over here.

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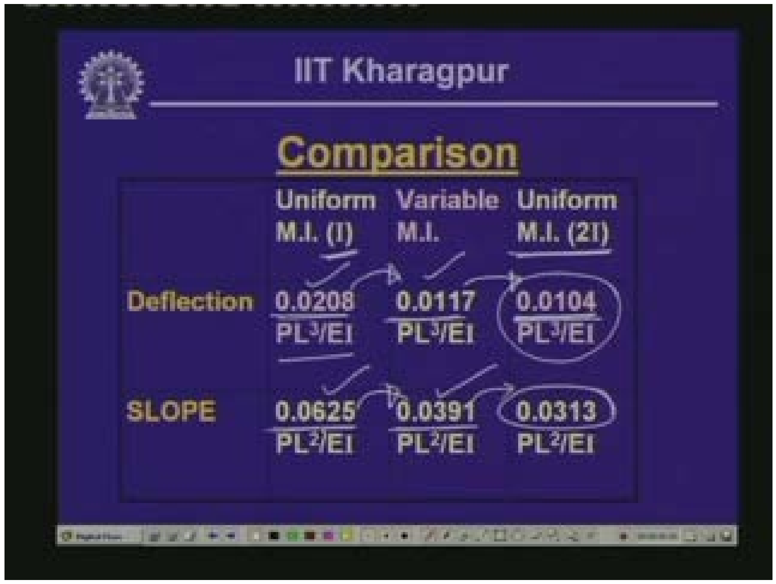

Therefore, by making variable moment of inertia in fact we can achieve economy both in terms of the deflection or the stresses and as well as the economization on the uses of the material and thereby where we have the larger value of the bending moment if we go for a larger cross-sectional area thereby you can have less stress, you can have less deflection, you can have less values of the rotation at the end.

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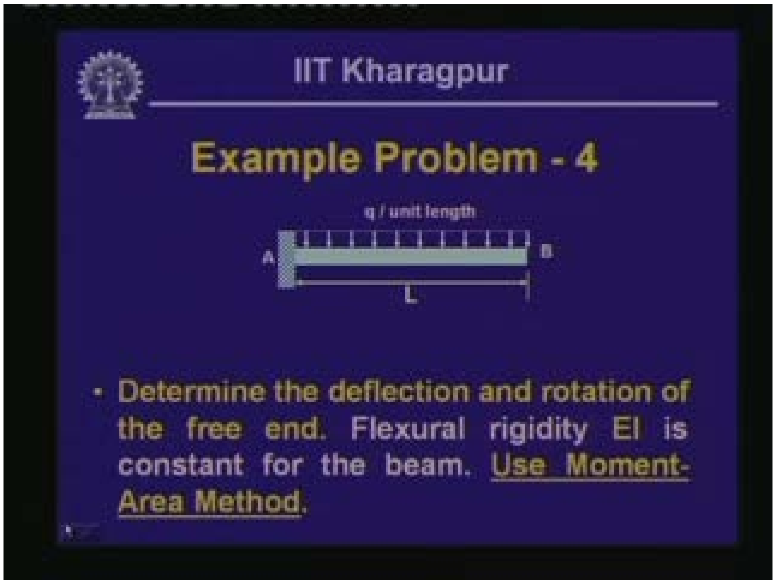

Well, we have another example where it is a cantilever beam subjected to uniformly distributed load and the flexural rigidity EI is constant for the beam; you will have to use moment-area method.

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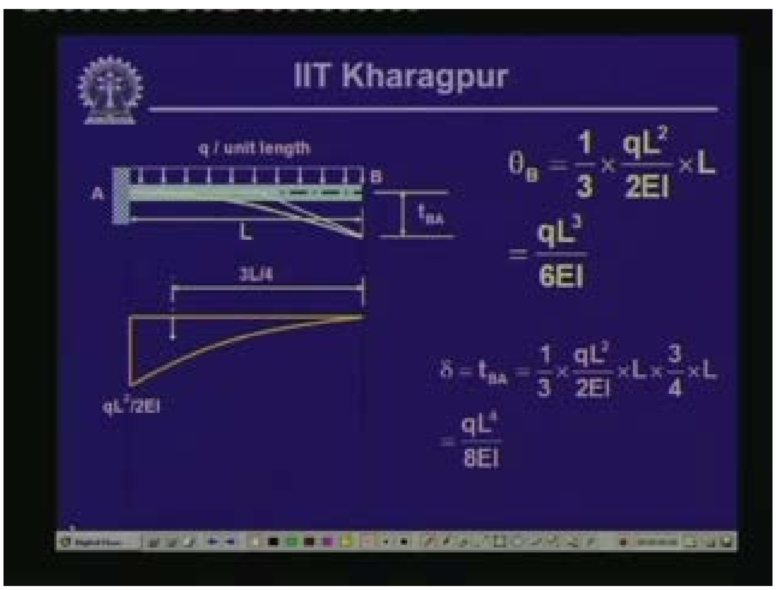

Now here of course you will get a diagram which is not a linear one and so long we have seen the diagrams which are linear one and thereby it was easier to adopt the moment-area method. Now for this, as you know that when you compute the value of the slope theta B it is the area between M by EI diagram which is in fact one third for this parabolic variation; the one third of the rectangular area which is one third and as you know the magnitude of the maximum bending moment is qL square by 2 divided by EI will be qL square by twice EI so it is one third.

We took qL square by 2EI over the length L so this is equal to qL cube by 6EI. and the value of the delta which is the tangential deviation of point B with reference to the tangent drawn at A is nothing but equals to the value of the delta which is the deflection of this particular beam; that should be equal to the moment of the M by EI diagram between these two points so this is the M by EI diagram and if we take the moment of this diagram with reference to point B then we get the value of the delta which is area is one third 2qL square by 2EI times L is the area and moment the c g distance from here is 3L by 4. So if we multiply that we get qL 4 by 8EI which we have seen earlier as well.

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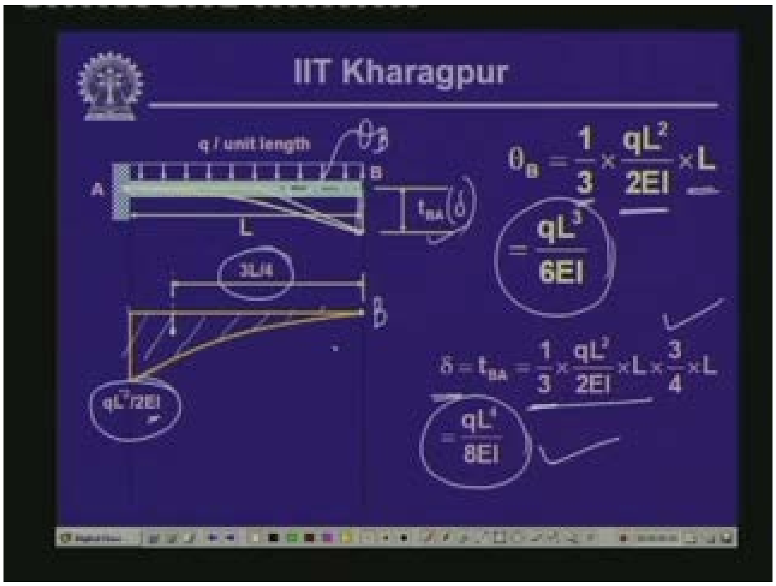

Therefore, by employing moment-area method to this we can compute the value of the slope and the deflection of this particular beam.

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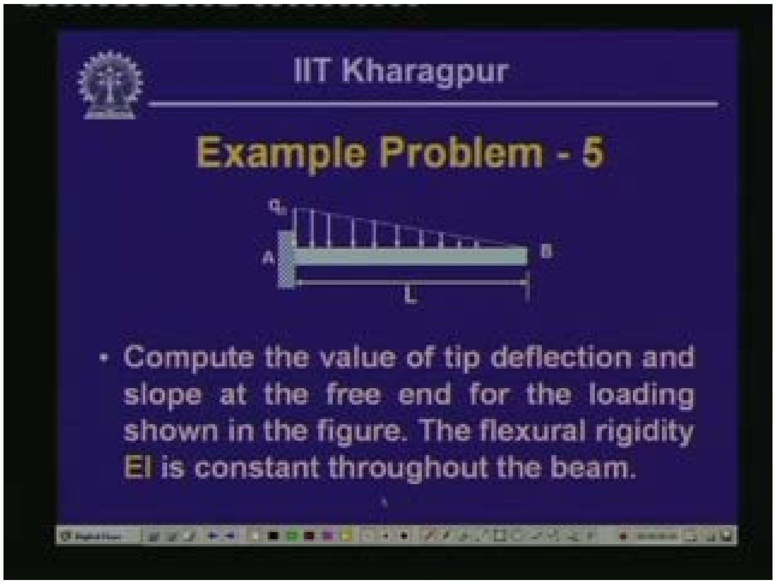

Well, we have another example over here. We have to compute the value of the tip deflection and slope of the free end for the loading shown over here. Here the loading is a triangularly varying load with zero intensity over here and maximum intensity q 0 at this particular end. The flexural rigidity EI is constant throughout the beam. You have to compute the value of the tip deflection and slope at the free end.

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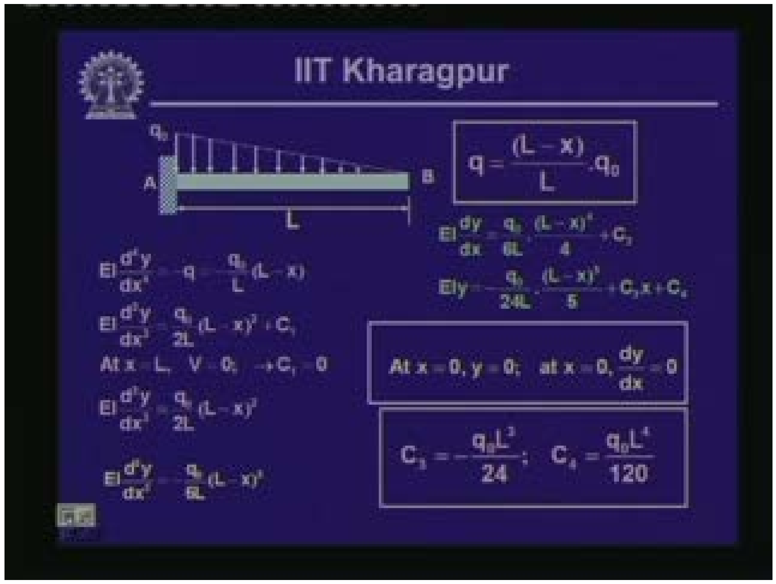

Now here we can employ the differential equation of the elastic curve. At any section from this support at a distance of x if we say that the value of loading is q since it is linearly varying is equals to....... in terms of this maximum value q 0 will be L minus x by L times q 0; and as we know that EI d 4 y dx 4 is equal to minus q which is equal to minus q 0 by L into L minus x over here and if we integrate this we have EI d 3 y dx 3 is equal to q 0 by twice L L minus x square plus C 1. Now at x equal to L the shear force is equal to 0 so d 3 y dx 3 equal to 0 at x equals to L and that gives us C 1 equals to 0; hence EI d 3 y dx 3 is equal to this q 0 by twice L L minus x square.

Now if we integrate this again we have EI d 2 y dx 2 as equal to minus q 0 by twice L L minus x cube by 3 which is minus q 0 by 6L L minus x cube plus of course you have the constant C 2. Now at x equals to L again the bending moment is 0, so that gives us C 2 which is also equal to 0 so this is the expression for the bending moment.

Now if we integrate this further we have EI d y dx as equal to q 0 by 6L into L minus x to the power 4 by 4 plus C 3 and consequently EI is equal to this where we have unknown constant C 3 and C 4.

Now at x equals to 0 y is 0 these are the boundary conditions and at x equal to 0 d y dx also is 0 and if we substitute that we get the value of C 3 and C 4 as this.

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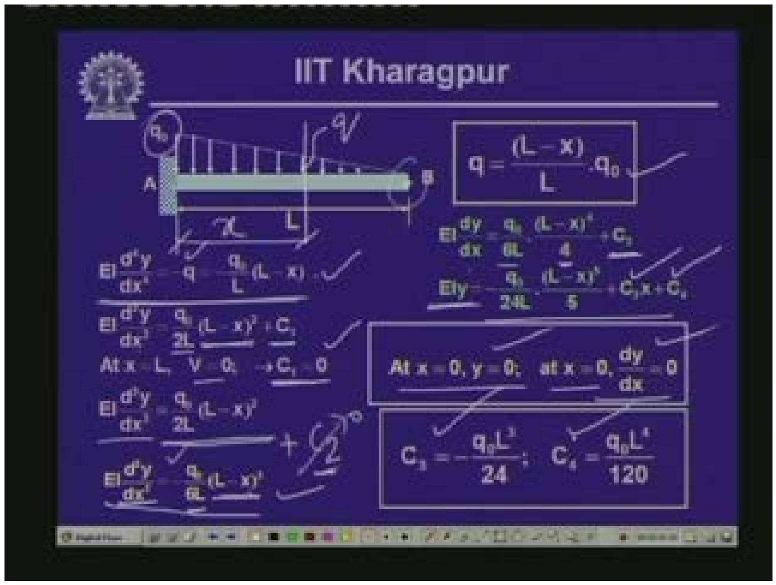

Now if we substitute the value of C 3 and C 4 we get the expression for y as this that q 0 by 120EIL into L minus x to the power 5 plus 5L to the power 4 x minus L to the power 5 and dy dx as this.

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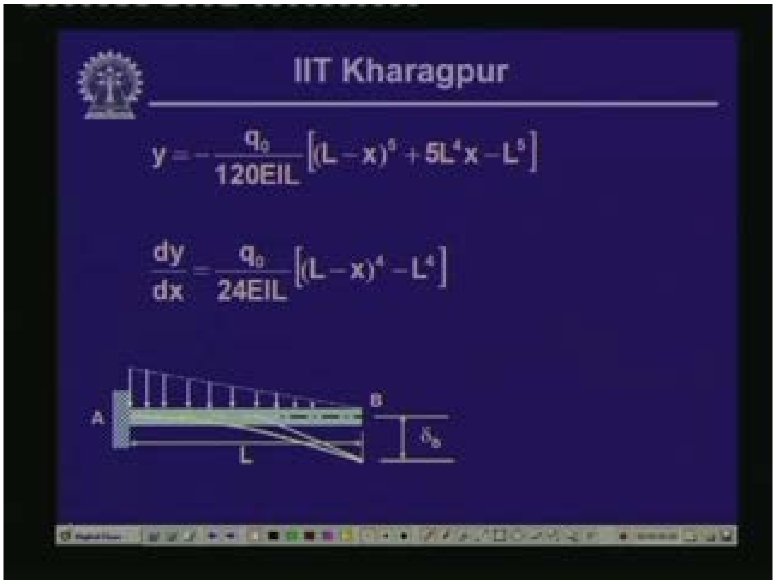

And if you substitute x as equals to L you get the value of y at the tip and dy dx at the tip. Now x equals to L means this goes off so you have 5L to the power 4, L to the power 5 minus L to the power 4 so this becomes q0 by 30 q0 L4 by 30EI that is the value of the deflection which is again negative so it is downwards and at x equals to L if you compute this you get minus q0 L cube by 24EI as the slope of this particular free end.

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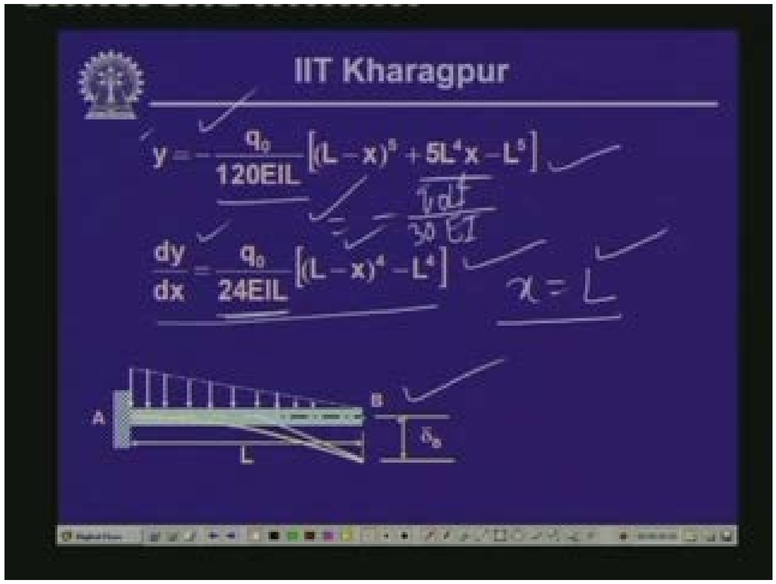

Well, then to summarize in this particular lesson we have seen the evaluation of slope and deflection of non-prismatic beams using moment-area method and also we have looked into the examples to evaluate slope and deflection of different types of beams for different loadings.

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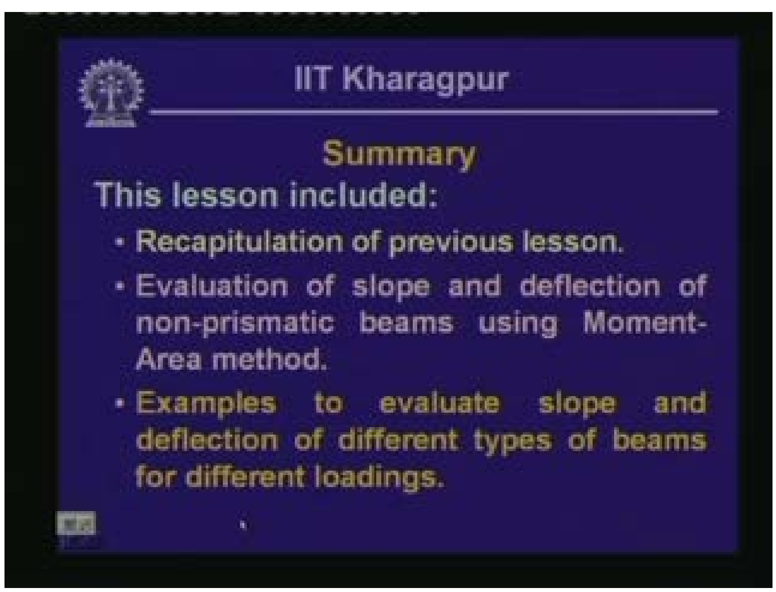

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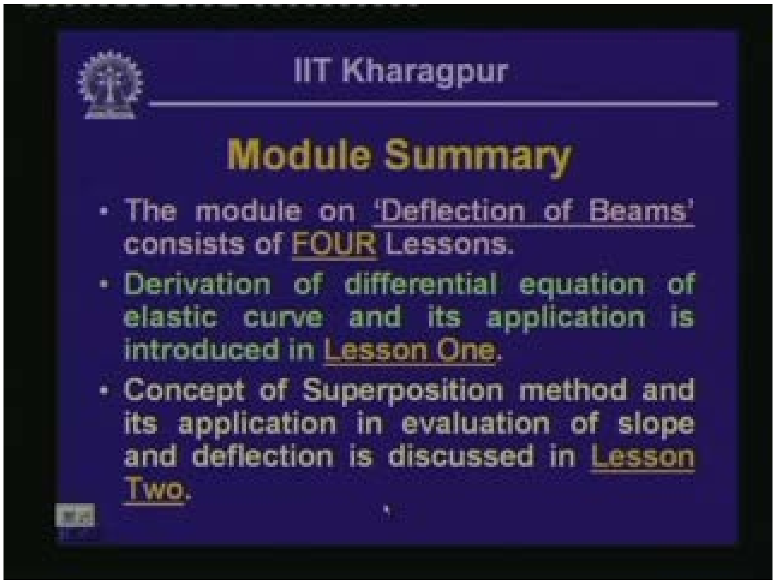

Now this is the last lesson of this particular module and in this module as we have seen it includes four lessons. In the first lesson we have discussed about the derivation of differential equation of elastic curve and its applications; then subsequently we have introduced the concept of superposition method in the lesson two.

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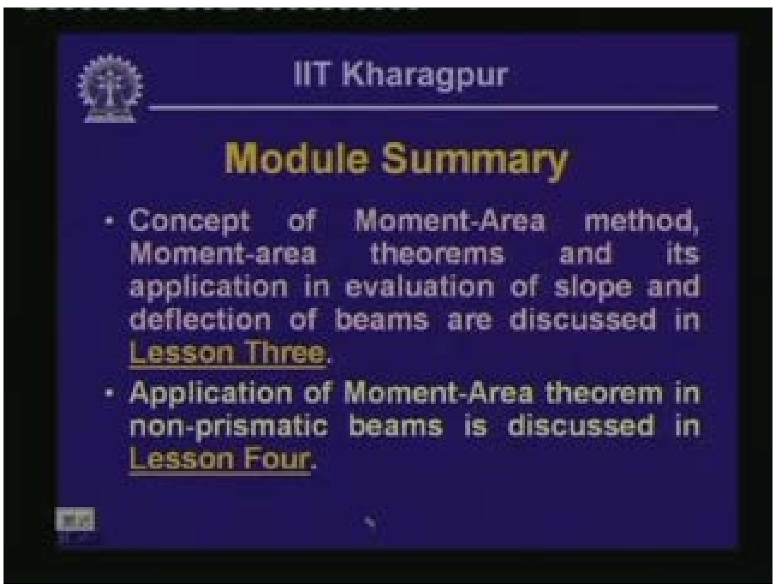

In the third lesson we have discussed about the concept of the moment-area method the moment-area theorems and its application in evaluation of slope and deflection. And in this particular lesson today we have seen the application of moment-area theorem in non-prismatic beams or beams with the variable moment of inertia.

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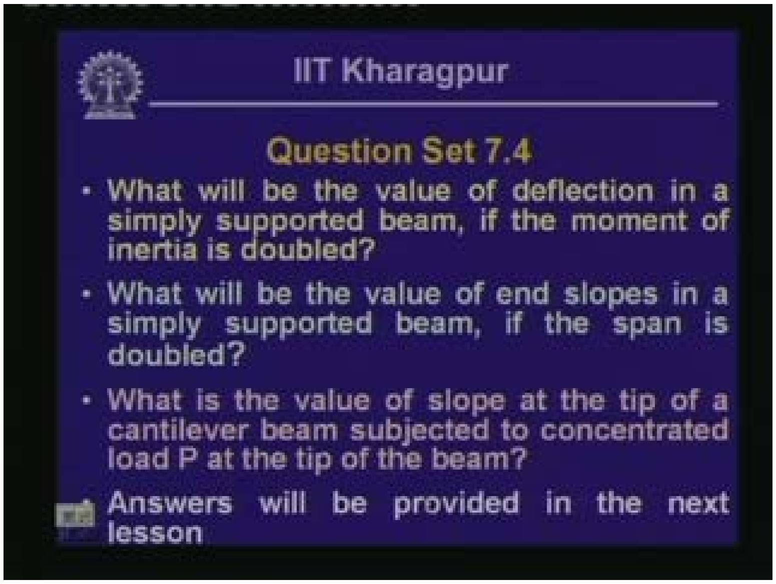

Now these are the questions set for you.
What will be the value of deflection in a simply supported beam if the moment of inertia is doubled?
What will be the value of end slopes in a simply supported beam if the span is doubled?
And what is the value of the slope at the tip of a cantilever beam subjected to the concentrated load P at the tip of the beam?

Now, answers for this will be given to you in the next lesson, thank you.

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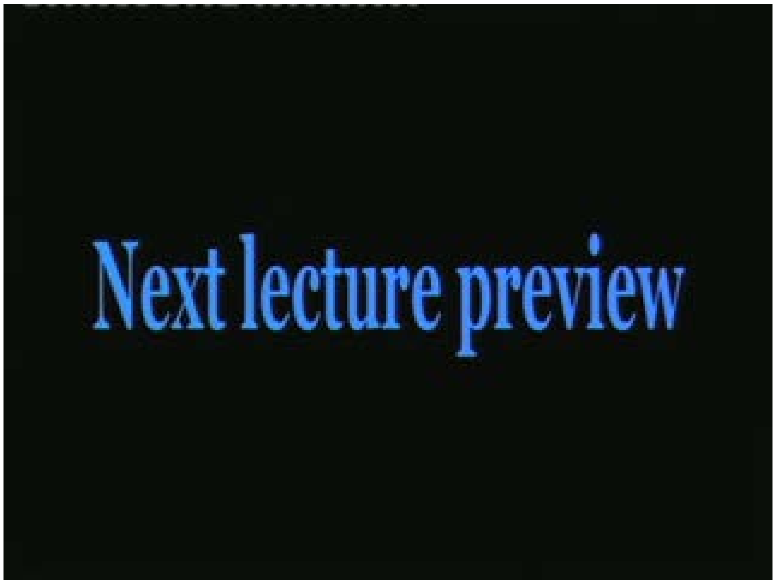

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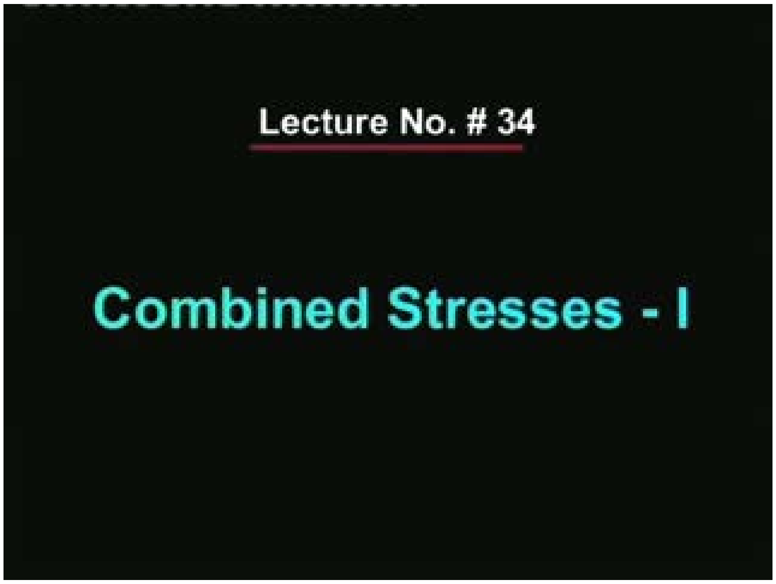

Welcome to the first lesson of the eighth module which is on combined stresses part I. Or more precisely let us call it as stresses which are developed due to the combine actions of loading in the member.

In fact so long we have discussed the action of different kinds of forces in the member which are acting in individual forms. Say for example when we have discussed about the axial force in a bar the force was acting normal to the stresses and we have computed the corresponding stresses which were called as normal stress.

Subsequently we have looked into that if preservation is subjected to pressure we have computed the stresses on the outer shell which is in the circumferential and the longitudinal direction. Subsequently we have computed the stresses in a bar which is subjected to another kind of force which we have called as twisting moment and then lastly we have evaluated the stresses in the bar corresponding to another kind of force which we have called as the bending and the shear force. We have seen how to compute the bending moment and shear force in a beam which is subjected to transverse loading and consequently we have computed the values of stresses.