Strength of Materials

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Lecture - 32

Deflection of Beams - III

Welcome to the third lesson of the seventh module which is the deflection of beams part III. In fact in the last two lessons on the deflection of beams we have discussed how the deflection of beams affects in general and why we need to evaluate the deflection in beams and the consequently the slopes in the beams. Now, in this particular lesson we are going to look into some more aspects of deflection. In fact in the last two lessons we have discussed the differential equation of the elastic curve and consequently we have derived the elastic curve I mean the equation for elastic curve.

Now here we look into another methodology which we call as the moment area method where we evaluate the slope and deflection in beams.

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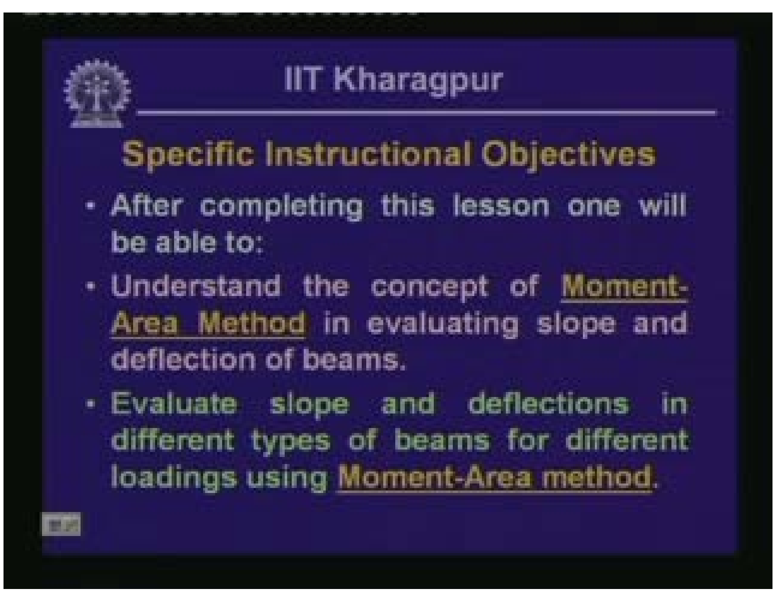

Hence it is expected that once you go through this particular lesson one should be able to understand the concept of moment area method in evaluating the slope and deflection of beams and one should be in a position to evaluate slope and deflection in different types of beams for different loading using moment area method. In fact, in the last two lessons we have solved quite a few examples wherein we have used the differential equation for elastic curve and we have derived or we have evaluated the equation for the elastic curve based on which we could evaluate or estimate the deflection in a beam at a particular location.

Also, we could compute the value of the slopes wherever needed along the length of the beam. Now in this particular lesson we are going to discuss that if we employ this particular method which is called the moment area method how do we compute the value of the slopes and deflection in beams.

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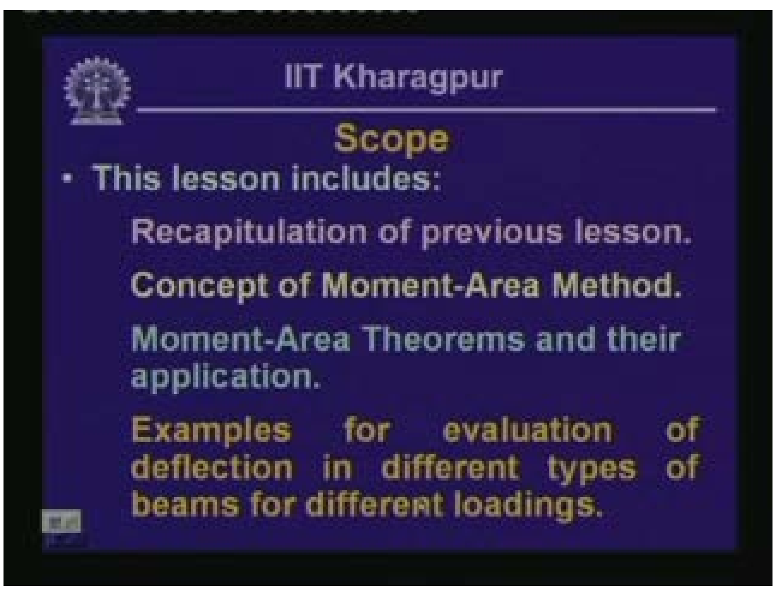

Hence the scope of this particular lesson includes....... we will recapitulate some aspects of the previous lesson. It includes the concept of moment area method; it includes moment area theorems and their applications. In fact based on this moment area method there are two theorems which you call as moment area theorems and we will state those theorems and consequently we will look into how they are applied in evaluating slopes and deflection in beams and then we will look into some examples for evaluation of deflection in different types of beams for different loadings.

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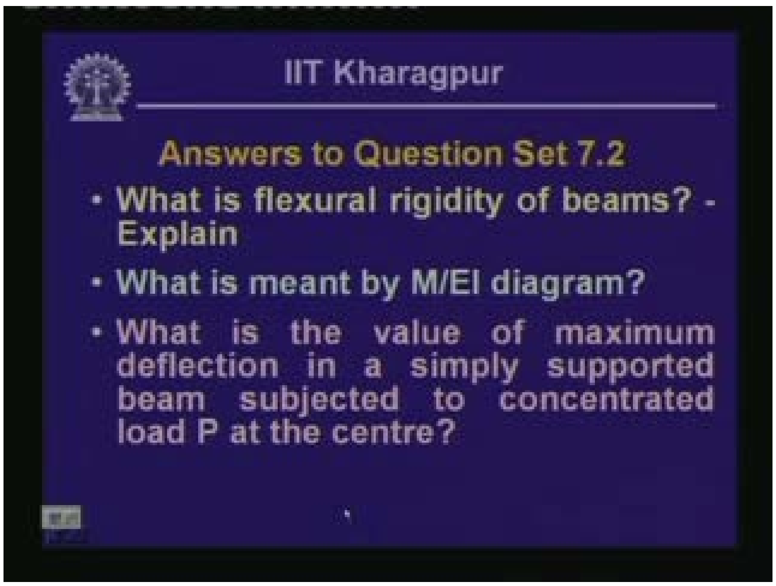

Now let us look into the questions which were posed last time. The first one is what is flexural rigidity of beams, explain.

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Now let us look into this aspect. We had indicated in the last lesson that EI this particular parameter EI is known as flexural rigidity where E is the modulus of elasticity of the material which we are using for the beam and L is called as the moment of inertia of the cross section taken about the neutral axis. This is the moment of inertia about neutral axis.

Now let me explain this with reference to a simple example. We have seen that for a simply supported beam which is subjected to uniformly distributed load of intensity q power unit length over a length of L the deflection which we get the maximum value of deflection is 5qL to the power of 4 by 384EI.

Now let us assume that we have two beams which are identical. That means they have the same length, they are subjected to similar intensity of the loading which is q and they are made out of the same material so that the value of E is same for both the beams. Now the only difference between these two beams is that in this particular case we choose a cross section; let us say that we choose a section which is a rectangular section, the width of which is b and let us call the depth as h 1 and for these particular beam we choose a cross section for which width is b but the depth is h 2 and let us assume that the depth h 1 is greater than h 2.

Now since all other parameters like qL E they remain constant then the parameter which is changing the depth is I because I for this rectangular is equal to b h cube by 12. So with the change in h there will be change in I, there will be change in the deflection delta.

Now physically if you look into that if you take a beam which is having a depth larger than the other one having other parameters constant then larger the depth the more the beam will have more strength and physically, given the ideal situation the larger depth of the beam will be in a position to carry more load.

Or in other words, if we keep the same load for both the beams the beam which is having larger depth will deform less. So we can call the beam which is having larger depth in comparison to the other so the other beam the larger depth beam is more rigid in comparison to the other beam. Hence the value of I if E is constant then EI this parameter governs the delta since other parameters remain constant.

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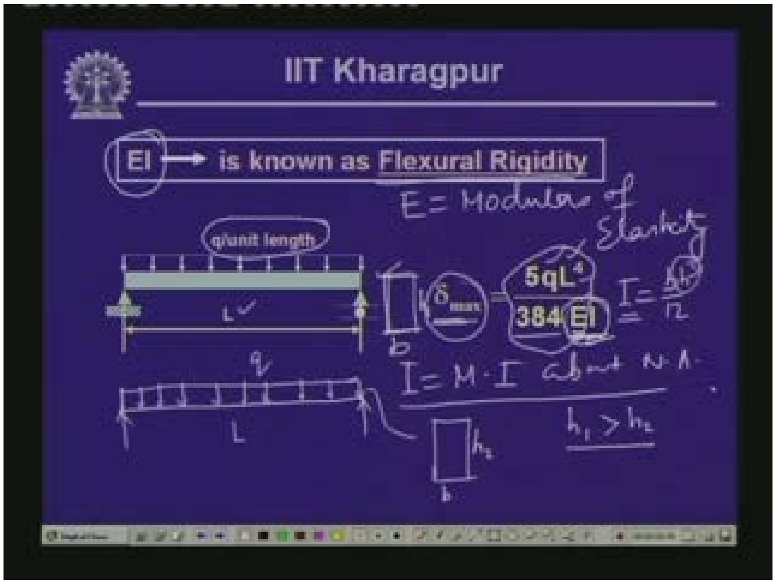

Now if the value of EI is higher, then the value of delta will be less and the vice versa; if EI becomes lower then delta will be more and physically that is justified. That means if EI is larger means for the material for the same E the I value is more that means the cross-sectional size is more and it is naturally going to carry more load or going to deform less and consequently you are going to get less deformation which is delta. Hence this particular term EI in fact physically signifies that how rigid a member is.

You know, given the cross-sectional parameter note you. Hence this particular term flexure as we have seen comes from the bending of the beam which you call as the flexer as well, it is the flexing of the beam and hence this particular term EI which defines the rigidity of the beam against these flexural action we call EI as the flexural rigidity and that is the significance of this flexural rigidity in computing the deflection or the slope which we call as the deformation of the beam against lateral loading or the transverse loading.

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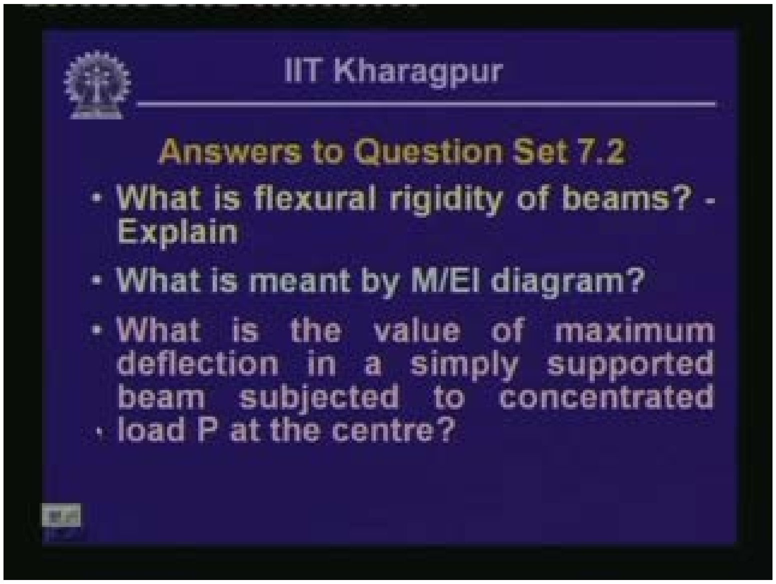

Now the second question was what is meant by M by EI diagram. Now this interesting and with reference to this present lesson we will be discussing in more detail.

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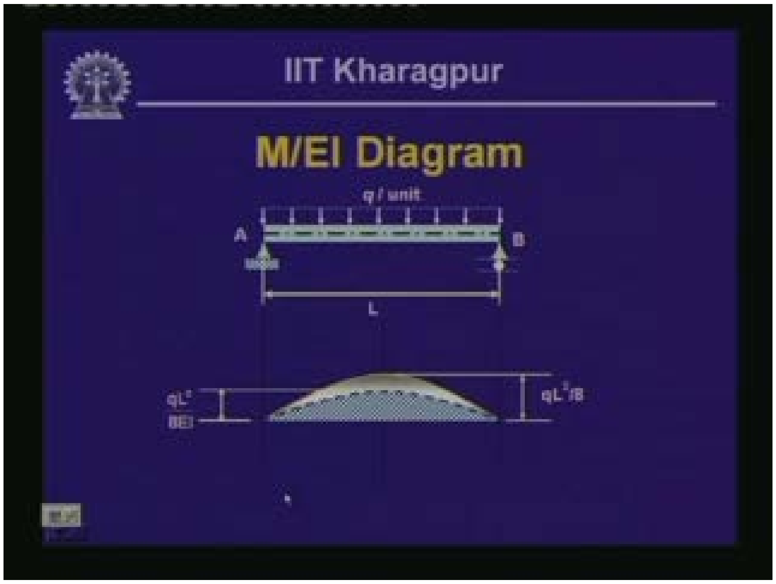

Now let us look into what is really meant by M by EI diagram. Now, as the terms you are acquainted with that particular value M or the particular designation M, we generally designate the bending moment in the beam as M. So basically when we were saying M by EI diagram that means it is related to the bending moment diagram.

Now again coming back to the example of a simply supported beam with a uniformly distributed load which is subjected to q per unit length, here as we have seen that the bending moment varies parabolically over the entire length and this particular diagram represents the bending moment diagram, the maximum magnitude of which is qL square by 8, if you compute this ordinate you will get qL square by 8.

Now, when we say by M by EI diagram what we mean is that each and every ordinate of this bending moment diagram if it is divided by the flexural rigidity of the beam member which is EI then we will get another diagram which is similar to the bending moment diagram in nature but will have the different value of the ordinates which is equal to M by EI the M value which we had for the previous diagram that is the bending moment diagram; the M by EI diagram would be having ordinate values which is lower because we are dividing each of the ordinates of the bending moment diagram by the term EI.

Hence the magnitude the maximum magnitude here for the M by EI diagram will be qL square by 8EI whereas the magnitude of the bending moment ordinate will be qL square by 8. So this particular diagram where we divide all the ordinates of the bending moment diagram by EI we call that as M by EI diagram.

Now it may so happen that in this particular case we have considered that EI value is constant throughout the beam. Now it may so happen that we have a beam wherein the central part is having larger value of EI thereby here as if we have the larger cross section over the rest of the beam. And let us say, here if you have a moment of inertia of 2I and other places we have the moment of inertia of i then the value of flexural rigidity in this zone is EI, in this zone also EI whereas in this particular zone it is 2EI. So when we But the bending moment diagram remains as it is like in this form.

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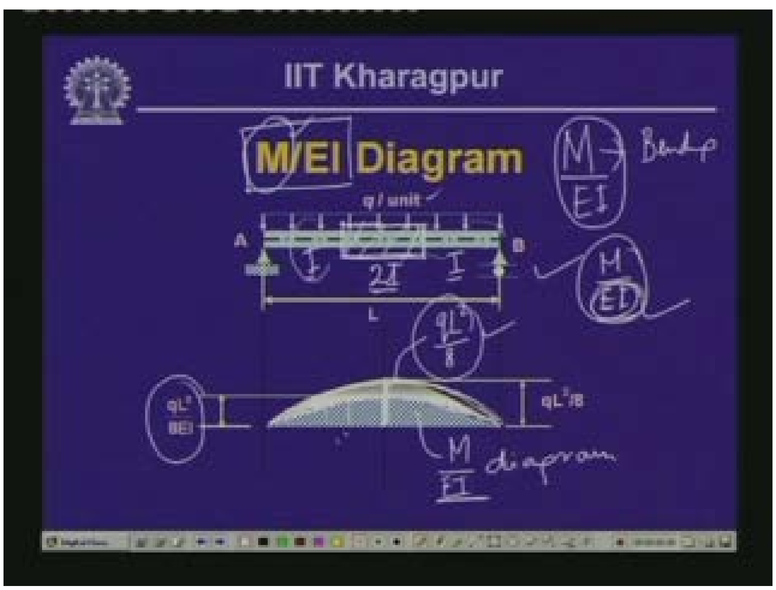

Now if the ordinate each of these ordinates are divided by the respective EIs; here it is EI and here it is 2EI then accordingly the M by EI diagram will change and will have a figure similar to this kind. That means we will have here it is divided by EI, at this point is divided by 2EI (Refer Slide Time: 11:25) hence the value will be less and then again it will go in this form exactly in the same orientation and again it will come in this form.

So you know, depending on the value of the moment of inertia and consequently the flexural rigidity of EI the M by EI diagram will change. So this particular diagram we called as M by EI diagram and then we will see we will make use of this M by EI diagram while discussing this moment area method for evaluating the slopes and deflections of beams as we will go along in few minutes.

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Now the last question which we had was what is the deflection the maximum value of the deflection of a simply supported beam which is subjected to a concentrated loop at the center. Now this particular problem or the example we have solved last time we have seen that if we employ the differential equation for evaluating the deflection curve or the elastic curve which is EI d 2 y by dx 2 equals to M then we get the expression called elastic curve y equal to function of x then there if we substitute x for L by 2 then we get the value of y which is maximum in this particular case because it is symmetrical with respect to the support and with reference to the loading and at this point if we draw the tangent to the elastic curve this will be horizontal hence this is the maximum deflection and delta max we get as pL 3 by 48EI where p is the concentrated load, L is the span of the beam and the load is acting at the midway between the beam which is at L by 2 and EI is the flexural rigidity which is constant throughout the length of the beam.

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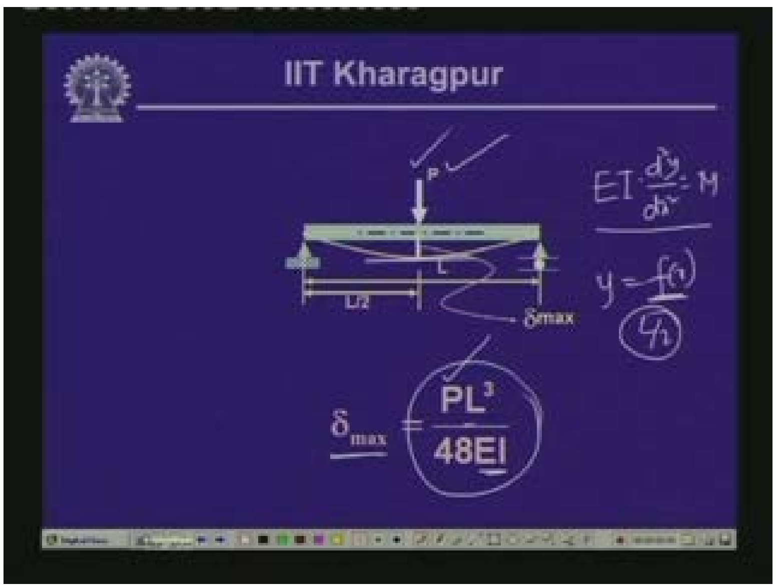

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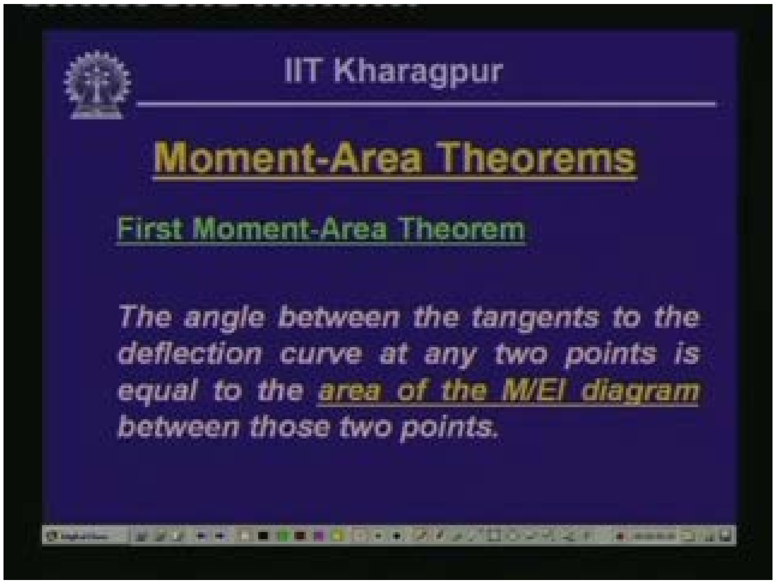

Well, then having looked into this let us look into some aspects of the moment area theorems. Now moment area method basically is a method for evaluating the slope and deflection for beams which are subjected to different kinds of loading.

Now, in the last two lessons we have looked into that if a particular type of beam that means it has some kind of support; neither a simple support nor a cantilever beam, when they are subjected to different forms of loading either uniformly distributed load or concentrated load, we have seen how to evaluate the slope and deflection by using the differential equation for the elastic curve. And consequently, by integrating the differential equation of the elastic curve, after writing the expression for the bending moment, we have computed the expression for elastic curve and based on that expression of the elastic curve we could compute the value of the deflection at different points along the length of the beam.

Now we are going to look into another method which is called as the moment area method and as the name signifies, we are going to make use of the bending moment diagram of a beam and in fact in this sense this is semi-graphical in nature that we will make use of the bending moment diagram in evaluating the slopes and deflection of the beam.

In deriving the equations or the differential equation for the elastic curve we assumed that the beam material is linearly elastic and the slope is small so that we could make some assumptions. Now, while deriving this moment area theorem here also we make similar assumptions like the material is linearly elastic and the slope is sufficiently small so that we can employ these moment area theorems in evaluating the slope and deflection.

In fact there are two theorems for this moment area method which is called the first theorem of moment area and the second theorem of moment area and based on these two theorems we can compute the values of deflection and the slope in a beam. Now the first theorem first moment area theorem goes like this that the angle between the tangents to the deflection curve at any two points on the elastic curve is equal to the area of the M by EI diagram between those two points.

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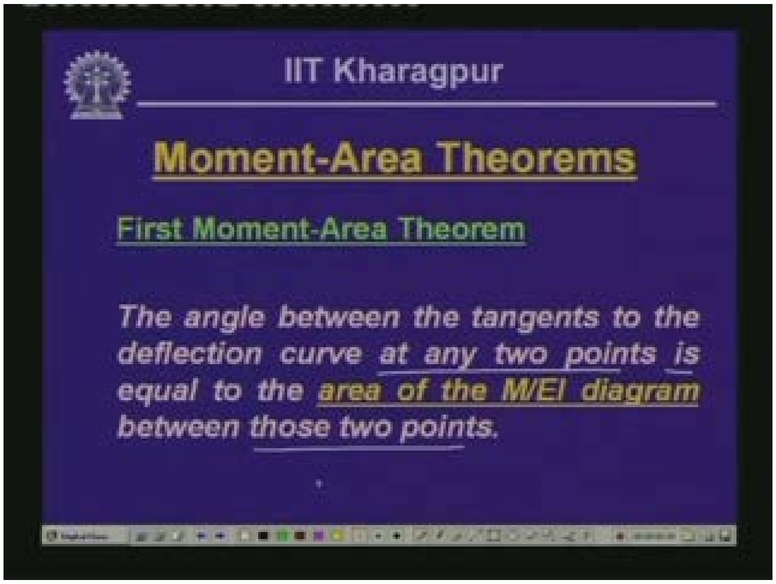

The first theorem says that the angle between the tangents to the deflection curve that is the elastic curve at any two points is equal to the area of the M by EI diagram. Now let us see that how really do we apply this theorem in evaluating or making use of this M by EI diagram.

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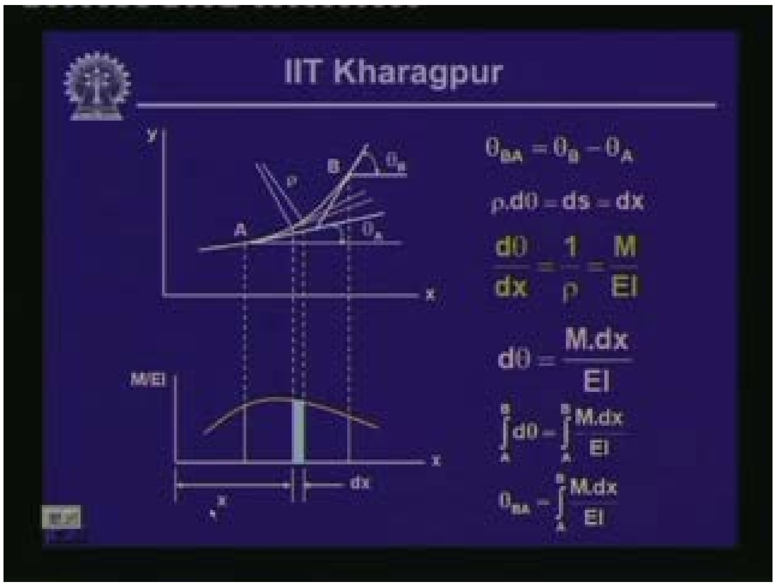

In this particular figure let us say that this is a part of the deflection curve and we choose two point A and B and at point A we draw a tangent so that this gives the slope of this particular point of the elastic curve which you call as theta A. Now if we draw a tangent at B of the elastic curve, now this particular tangent which has the slope of the elastic curve at point B which we call as theta B. So this particular angle is the difference of these two angles theta B and theta A and this we call as theta BA which is theta B minus theta A.

This is the part of the bending moment diagram and let us call this now as M by EI diagram that means the bending moment diagram has been divided by the term EI to obtain the M by EI diagram. Now on this elastic curve or around the axis of the beam we choose a small length dx and on the elastic curve we choose this length as ds. Now if we draw normal to the end of this ds this particular length is ds (Refer Slide Time: 17:46) this goes to the center of curvature and rho being the radius of the curvature. Hence let us assume that this particular segment ds forms an angle of d theta at the center. So this length ds can be written as rho times d theta. So ds is nothing but equals to rho theta.

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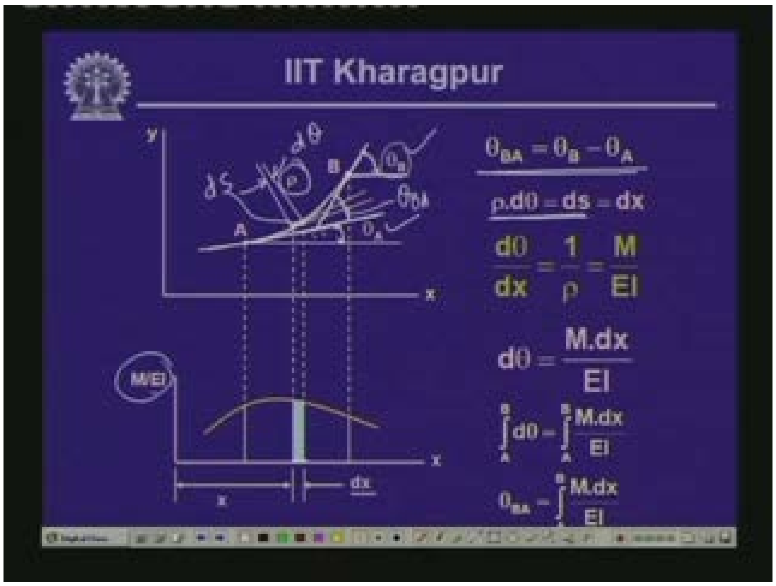

Now since this deformation is small and the slope is small we assume that dx is equal to ds. So in place of ds we write dx. So rho d theta is equal to dx or d theta by dx is equal to 1 by rho; taking this on this side and dx on this side d theta by dx gives us a value equal to 1 by rho. Now we already know from the bending equation that M by I is equal to sigma by y equals to E by rho. So, if we take the relationship between M by I and E by rho then 1 by rho is equal to M by EI and that is what which is used over here 1 by rho is equals to M by EI and therefore d theta by dx can be related to the M by EI d theta; dx is equal to M by EI. Or in other words, d theta equal to M dx by EI.

Now if we look into this particular expression; now for this particular segment ds two ends if we draw an tangent let us say these are the two tangents, since these are the normals at the end of ds and since this angle is d theta hence this particular angle is also going to be d theta.

Now it demonstrates that the change in angles between the ends of these two segments ds which is d theta this is nothing but equals to the area of the strip which is on the bending moment diagram, the area of this particular strip the ordinate of which is M by EI and the width is dx. So M by EI multiplied by dx is nothing but the area of the M by EI diagram over this small strip ds and that gives us the change in angle d theta between the two ends of this particular segment.

Now if we integrate this particular expression over the stretch A to B then integral d theta A to B is equal to integral M dx by EI between A to B and then if we integrate d theta from A to B we get this as theta B minus theta A which is nothing but equals to theta BA and theta BA then gives us integral M dx by EI between A to B. This means, now this the area of the M by EI diagram between the two points A and B which we have chosen on the elastic curve the area of the M by EI diagram between those two points.

So you see, now what we are getting is that the change in slope between two points on the elastic curve because the two points we have chosen is B and A and theta BA is the difference in slope between B and A which is theta BA which is nothing but equal to the area of the M by EI diagram between those two points.

So you see that this particular theorem demonstrates that if we know or we can physically plot the elastic curve of a beam and if we are interested to find out the difference in slope between two points on the elastic curve then simply if we can compute the area of the M by EI diagram between those two points that will give us a value of change in slope between those two points. So, by making use of the bending moment diagram; or in other words, M by EI diagram we can readily compute the values of the slope of the elastic curve at different points and that is one of the advantages of this particular method.

Now of course while using this particular method we have to be careful that the diagrams which we get for the bending moment diagram they are to be simple enough; like if you have a form like rectangular or square or a triangular form or a parabolic form or in which case we know the areas we can compute the area values directly for such diagrams and it becomes easier to implement it.

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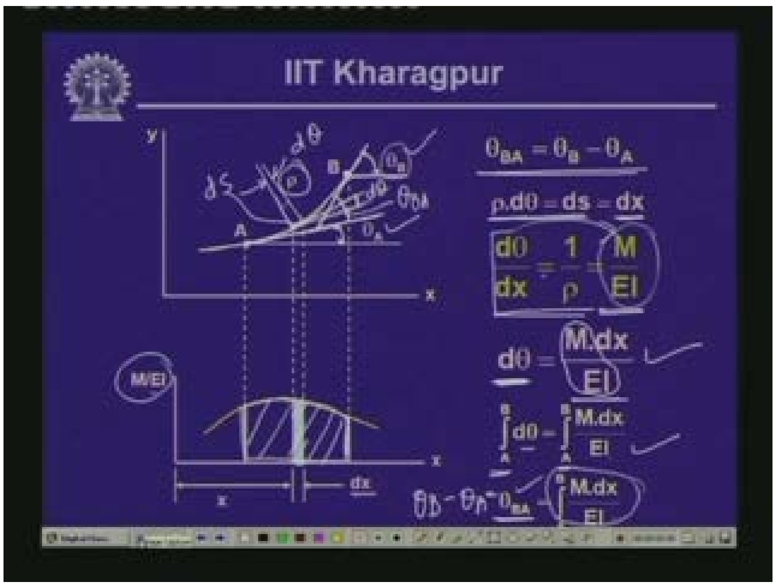

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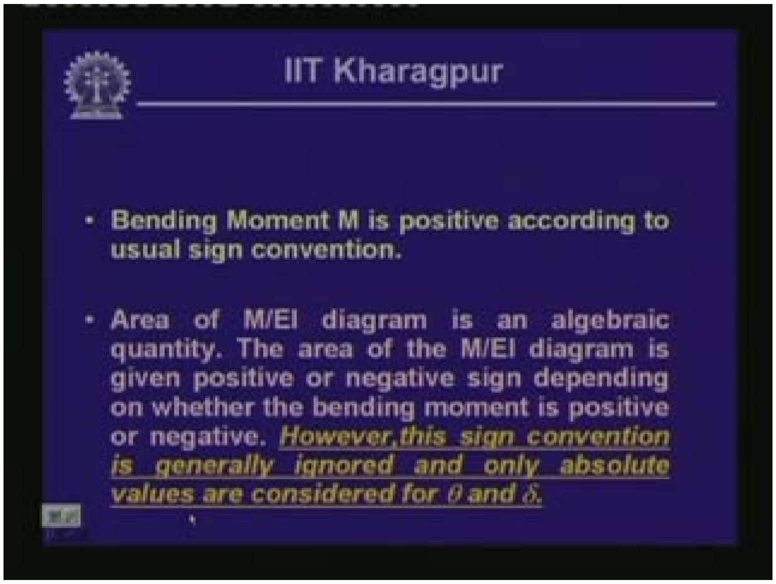

Now let us look into the some of the aspect of this particular theorem. Now here if you note that we had used the term M now that is for the bending moment and the value of the bending moment could be positive or negative or rather its sign would be as usual as we have assumed earlier, that means we have a counter clockwise moment counter clockwise moment will give you positive value of bending and the clockwise moment will give you the negative value of the bending. So we follow the users' sign convention as we have used so long.

Also, now area of M by EI diagram is an algebraic quantity. That means if we are choosing M by EI diagram between two points and if it so happens...... say for example we have the M by EI diagram which goes like this......... you know I am choosing arbitrarily; now if I am considering these two sections let us say this is positive and this is negative (Refer Slide Time: 23:30) then depending on the sign as we have between these two points we must consider these signs. So the area of the M by EI diagram is given a positive or negative sign depending on whether the bending moment is positive or negative.

But when we try to compute the values of the slope or the values of deflection we generally do not bother about the signs since we know physically, because of the loading how it is going to deflect we can we just compute the absolute value and then accordingly assign depending on whether it is having a clockwise moment or counter clockwise moment or how the deflection is going whether it is above the base line or below the base line. But only we are interested to evaluate the magnitude of these quantities that is the slope or deflection. However, this sign convention is generally ignored and only absolute values are considered for evaluating theta and delta. This is what is important while using this moment area theorem.

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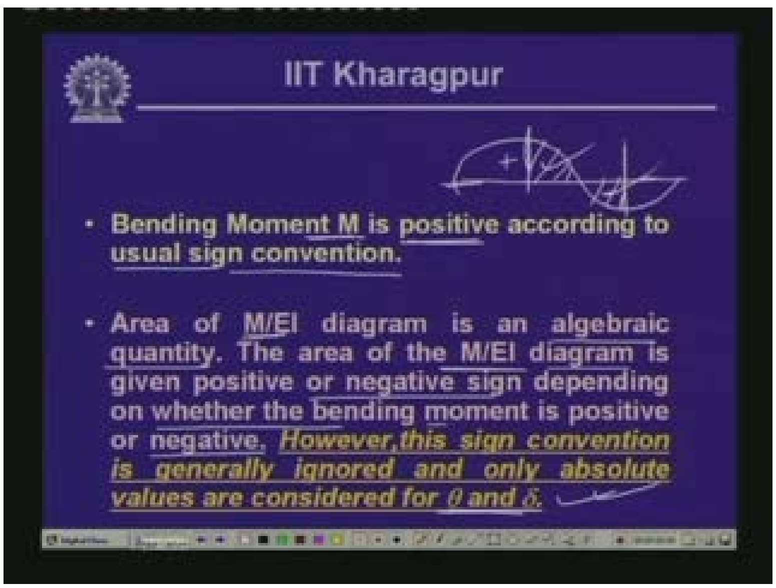

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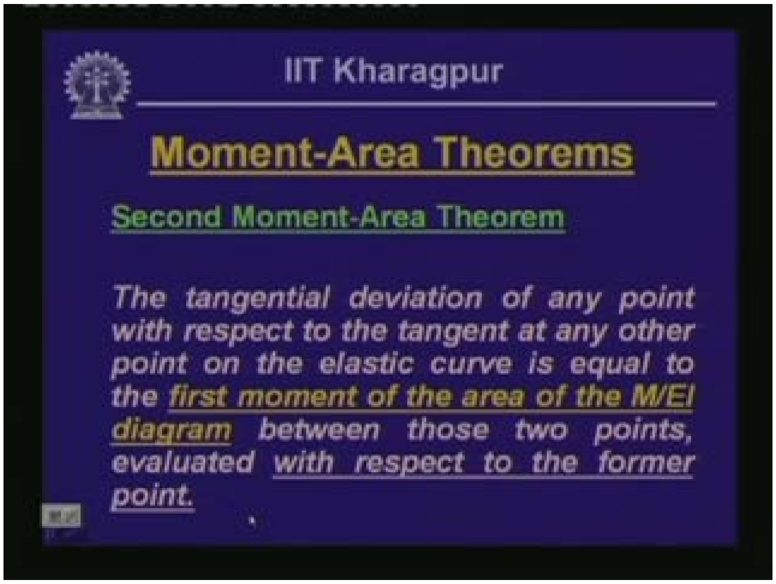

Now the second moment area states that the tangential deviation of any point with respect to the tangent at any other point on the elastic curve is equal curve is equal to the first moment of the area of the M by EI diagram between those two points evaluated with respect to the former point.

So it is again the tangential deviation of a point with respect to another point on the elastic curve so this equals to the first moment of the area of the M by EI diagram between the points under consideration and the moment is to be taken with respect to the fresh point where we are evaluating the tangential deviation with respect to the other so with respect to the former point. This is the second moment of second moment area theorem. Now let us look into how do we apply this theorem for evaluating the information as we need for evaluating the slope and the deflection now again.

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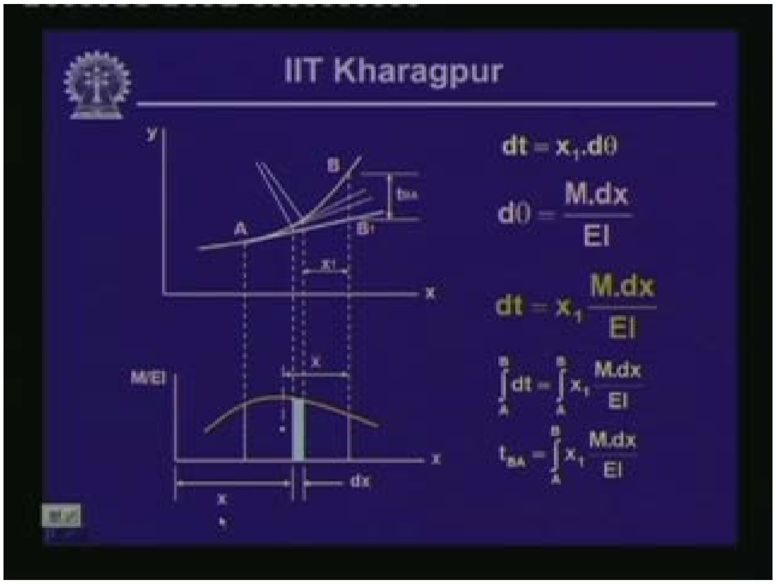

Again we consider a segment of the elastic curve an on this segment we choose two points A and B and we draw a tangent at point A and from B we drop a vertical line and let us say this tangent cuts this vertical line at B 1. In fact from B B 1 is the tangential deviation of B with respect to the tangent drawn at A. So the second moment area theorem states that the tangential deviation of a point with respect to the tangent drawn from any other point is equal to the moment of the M by EI diagram between those points and taken above the former points.

So the moment the moment of this M by EI diagram between two points A and B taken about B will be the will be the case as per this second moment area theorem.

Now let us look into this. Now if we go exactly the same way as we have done earlier that means from the first moment area theorem, now here we choose a small segment say of length ds and as usual this makes an angle of d theta at the center and considering this as a small deformation problem or the slope is small, this ds length we approximate as dx and at the end of the ds if we draw the tangent we assume that this cuts this vertical line and we call this length as dt (Refer Slide Time: 27:22). This length B B 1 where this tangent A from cuts the vertical line from B, B B 1 we call as t B A and this small segmental length we call it as dt on line B B 1. And since this deflection is small you know the slope is small so the axis of the beam the distance x 1 as we have chosen the distance of the strip dx from B which is x 1 we assume that the length dt can be computed in terms of this x 1.

Now as we have seen that if this angle is d theta, this angle is also d theta between the two tangents so this is d theta, hence if we call this length as x 1 x 1 times d theta will be dt. This is what written over here that dt is equals to x 1 times d theta. Now here we assume that slope is small and thereby the length x 1 which is along the axis of the beam is equivalent to the tangent length which is cutting the vertical line B B 1 giving the distance dt so dt is equal to x 1 times d theta.

Now we have seen in the previous theorem that d theta equals to M dx by EI so in place of d theta if we substitute M dx by EI dt equals to x 1 times M dx by EI so it shows that the distant dt between the two ends of the segments of this ds is........ M by EI again is the ordinate of this particular strip. Now M by EI M by EI times dx is the area of this particular strip of the length dx and if we take the moment of this area with respect to the point B is x 1 times this area that is what is the dt.

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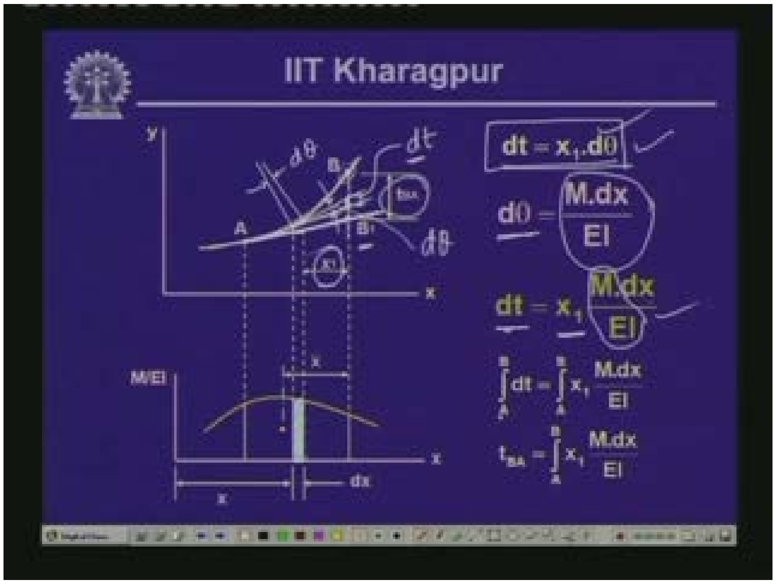

Now if we integrate that dt from A to B which will eventually give us B B 1 so t at P minus t at B 1 is equals to t BA this is equal to x 1 times integral x 1 times integral by EI between A to B; this indicates that this is the moment of the M by EI diagram between points A and B taken about the point B and integral x 1 dx basically is the.................. M by EI is the ordinate, dx integral over the length and the distance of that, now this point is the c g of this particular part of the M by EI diagram and let us call that distance as x bar. So this area times the x bar, now x bar mind that is with reference to the point B for which we are evaluating the tangential deviation is the moment of this particular area. So the tangential deviation of B with respect to the tangent drawn at A which is t BA is equals to the moment of the M by EI diagram between B and A and the moment is taken about the point B that is the statement of the moment area theorem.

Therefore, now as we have seen that we have two theorems of moment area method, the first theorem says that if we are interested to evaluate the difference in slope between the two points of the elastic curve then that is equal to the area of the M by EI diagram between those two points. And the second moment area theorem states that if we are interested to find out the tangential deviation of a point on the elastic curve with respect to the tangent drawn on the other point with reference to the other point then that is equal to the moment of the M by EI diagram between those two points A and B and the moment is taken about the form of point which is B; that will give us the tangential deviation of the elastic curve of the point with reference to the other point where we have drawn the tangent.

Now we make use of these two theorems to evaluate the values of the slope and the deflection at any point in the beam as we have done in case of using the differential equation for elastic curve that we have written down the expression for the bending moment, we have employed the differential equation, we have integrated it and we have computed the value of y at different points.

Now we will make use of this semi-graphical method by employing the bending moment diagram to compute the value of slope and the deflection in the beam for different kinds of beam with different kinds of loading. But please keep in mind that moment area method becomes useful or suitable when the bending moment diagram becomes relatively simpler; like if you have a regular diagram like a square one or rectangular one or triangular one or a parabolic configuration for which you can really evaluate the value of the area and consequently you can take the moment of those areas with reference to the point then there you are evaluating the tangential deviation. So, moment area method can be employed to evaluate the values of the rotation of the slopes and the deflection at any point in the beam.

Now let us look into some of the examples so that this becomes clear.

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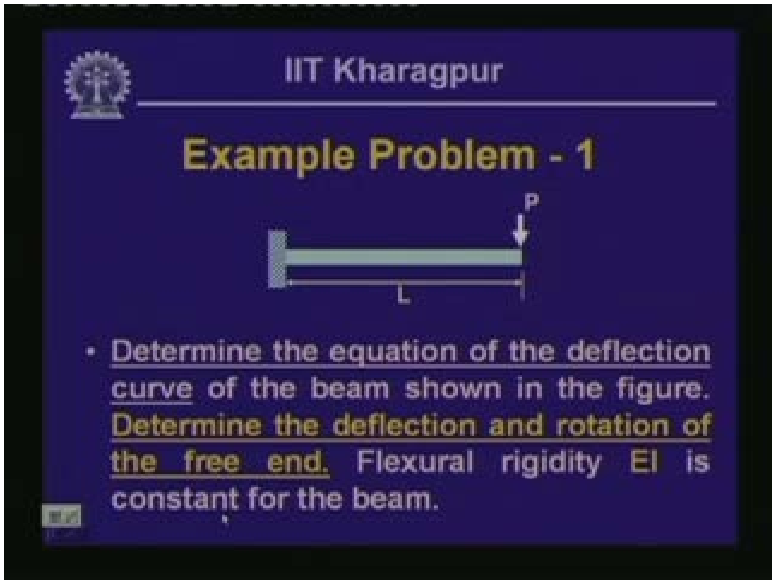

Now, before we go to the examples for evaluating the or for using the moment area theorem let us look into the example which we did in the last lesson. In the last lesson we did this particular problem that we have a cantilever beam and we have the load at the tip of the cantilever beam which is a concentrated load, now what we had to do was to arrive at the equation of the elastic curve and deflection and rotation of the free end.

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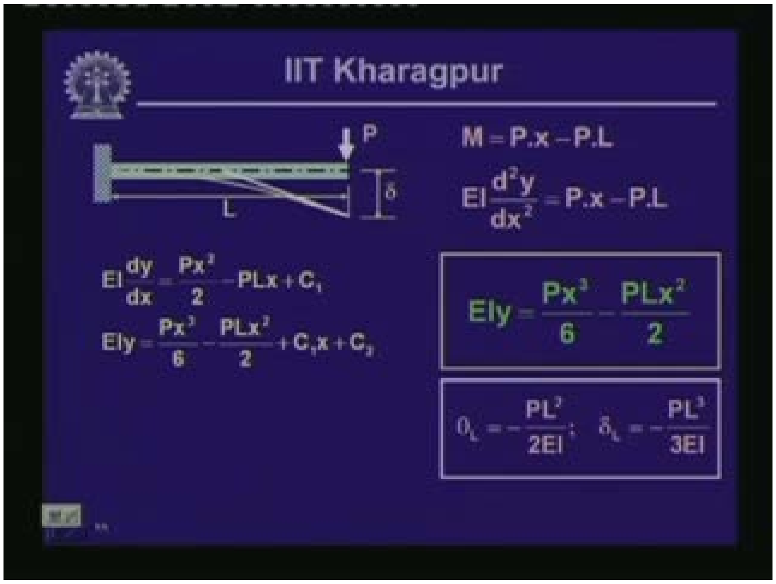

Now as we have seen that we could arrive at the elastic curve based on the equation of the differential equation of the elastic curve which is EI d 2 y by dx 2 equals to moment and moment here we first compute the reactive values the vertical reactive value, the horizontal one and the moment and based on these reactive values the moment at any segment, (Refer Slide Time: 33:57) if I take a cut over here we get the value of moment as P into x minus P into L and this if we employ then we get the equation as this: EI d 2 y by dx 2 equals to Px minus PL.

Now if we integrate this then we get EI dy dx equals to Px square by 2 minus PLx plus C 1. And subsequently if we integrate this once again we get EIy is equals to Px cube by 6 minus PLx square by 2 plus C 1 x plus C 2.

Now for evaluating these two constants C 1 and C 2 we need to employ the boundary conditions. And as you know, at the fixed end at x equals to 0 at the fixed end both the slope and deflection are zero and the fixed end being at x equals to 0 so at x equals to 0 y equals to 0 and at x equals to 0 dy dx also equals to 0.

Now if you substitute these boundary conditions at x equals to 0 and dy by dx equals to 0 this gives us that C 1 equals to 0 and at x equals to 0 y equals to 0 if we substitute we get C 2 equals to 0. This gives us that the value of the deflection curve or the elastic curve EIy is equal to Px cube by 6 minus PLx square by 2.

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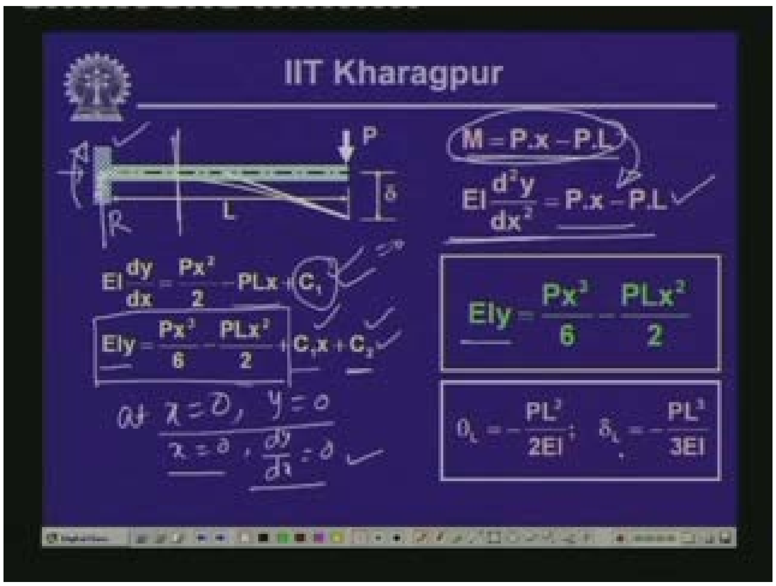

Now if we like to compute the value of the deflection at the tip which is at x equals to L, we substitute the value of x equals to L over here then we get Px cube by 6 minus PL cube by 2 and eventually we get PL cube by 3 EI delta equals to minus PL cube by 3EI and we get minus sign because it is in the opposite direction of positive y direction so this is the value of the delta which we get.

Now eventually if we compute the delta the maximum value of delta we get is at the free end of the beam and that is the value of delta L which we call as the maximum delta or delta max. Now, when we like to compute the value of theta at L that means that if we take a tangent to this elastic curve at this particular point then here the tangent to the elastic curve is along the axis because it is parallel or along the line of this particular axis hence this gives us the value theta B because theta B minus theta A theta A is 0 here so this is theta B and we call this as A and this as B.

Now the value of theta B or theta L at x equals to L if we substitute here at dy dx since C 1 is equals to 0 so it will be PL square by 2 minus PL square so that will give us PL square by 2 if we divide by EI theta is equals to PL square by 2EI. This is the value of the slope of the elastic curve at the free end and this is the value of the deflection of the beam (Refer Slide Time: 37:14) which is subjected to a concentrated load.

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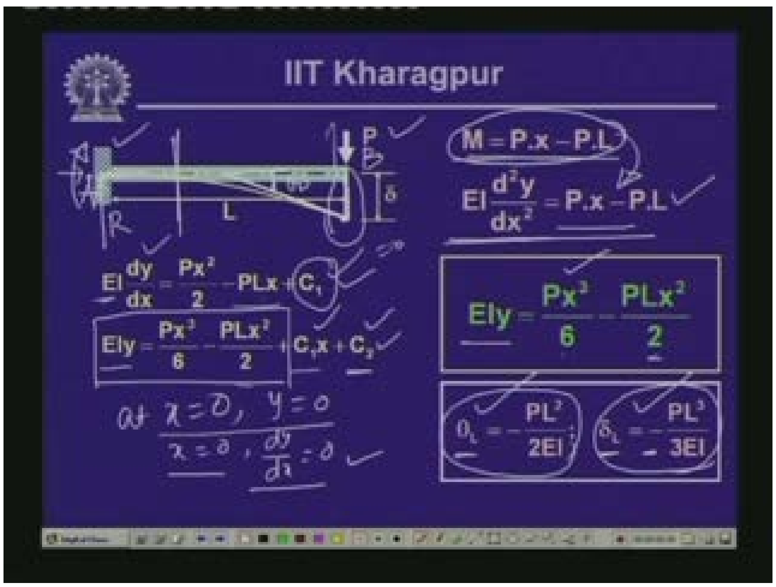

Now, as I have told you that from the governing equation which is EI d 2 y by dx equals to bending moment M, we have derived different forms of the differential equation. That means if we take the derivative of that differential equation once again we get EI d 3 y dx 3 which is equal to sheer and as we know that V is equals to minus dm dx so EI d 2 y dx 2 is moment M, EI d 3 y dx 3 is dm dx which is plus 2 minus V.

Now we can employ this particular form of the differential equation as well and we can get the solution in the same form. So, if we do that then how do you go about it? Now let us look into that.

Now here (Refer Slide Time: 38:10) we have computed the values of the reactive forces R and the moment M. R is equal to P. Now if I take a cut over here if we take a free body diagram here you have R, here you have moment and at this cut we have of course V and moment M. Now this R is equal to P. So if we take the vertical equilibrium of the forces then V plus P is equal to 0 or V is equal to minus P and that is the shear which we have and that is where from here to here you get the same value.

Now if we say that EI d 3 y dx 3 this is equal to minus V this is equal to......... since V is equal to minus P this is equal to P; or if we integrate this we have EI d 2 y dx 2 this is equal to Px plus C 1. Now at this stage if we like to find out the value of C 1 now we see that at x equals to L that means at the free end the moment is equal to 0. Now if we substitute that particular condition we get that C 1 is equal to minus PL. Now if we substitute the value of C 1 over here we get EI d 2 y dx 2 is equal to Px minus PL and that is the point where we had started that M is equal to Px minus PL and then we have integrated after substituting in this differential form and we have obtained these values.

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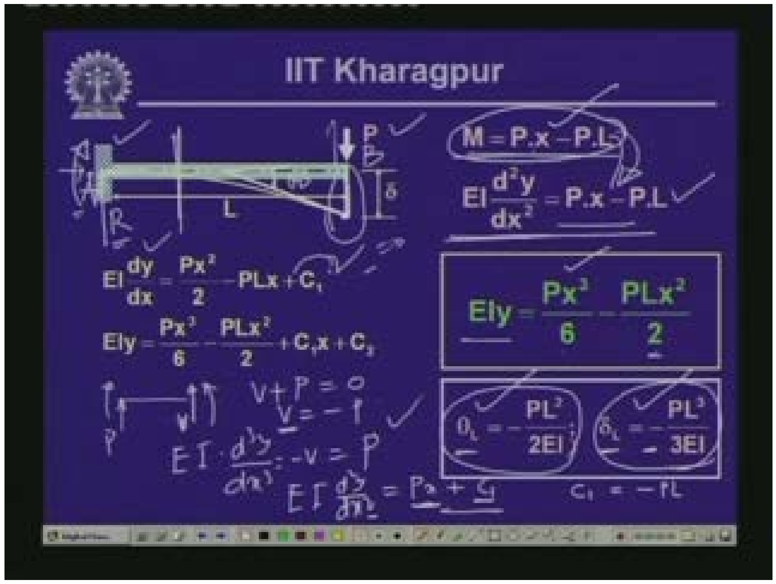

So even if we start from this particular expression that EI d 3 y dx 3 is equal to minus V we arrive at the same value. So you can employ any of these forms of these governing differential equations. That means EI d 2 y is equal to the bending moment M. Now you can employ the other form of the differential equation as well which is EI d 3 y by dx 3 which is shear which is convenient in this particular case or if you have a distributed loading over the entire span of the beam you could use EI d 4 by dx 4 is equal to minus q as well which we had seen in the case of a simply supported beam which is subjected to uniformly distributed load.

So any of these forms of the differential equation according to the suitability you can use it and the final form of the results which you will get the value of the maximum deflection or the rotations at different points we will have the identical results.

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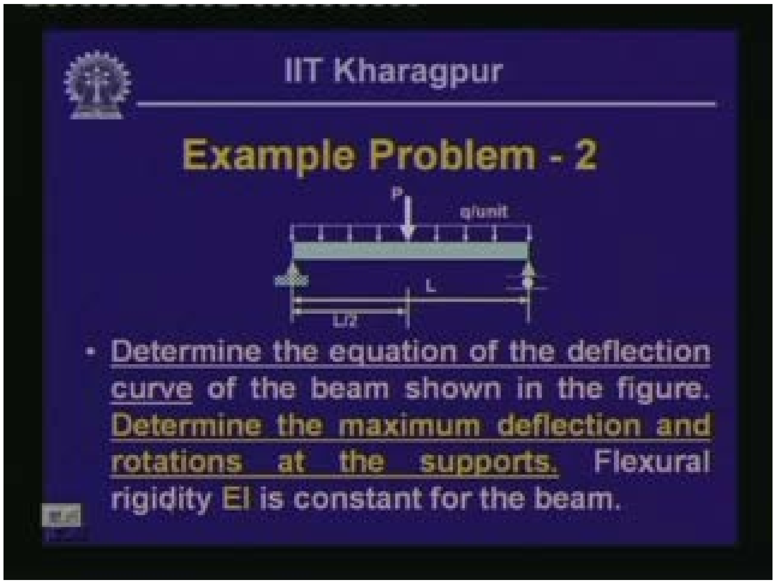

Now let us look into another example problem. In fact this example problem, in fact this example was given to you last time and in the last lesson we had discussed that if a beam is subjected to several loads of different forms; like if you have uniformly distributed load you have several concentrated load on a beam and if you are interested to find out the displacement at any point or the deflection of the beam at any point the length of the beam then the method which we had discussed so far that means we compute the bending moment at different segments and substitute in the differential equation and we solve the differential equation in those different segments, apply appropriate boundary conditions in those segments and arrive at the final form of the elastic curve.

We can employ the same technique for evaluating the deflection for multiple loads. Or else what we can do is that we can apply the method of superposition. This means that we assume that instead of all the loads acting simultaneously we split it. We assume that the beam first as it is subjected to uniformly distributed load, secondly it is subjected to a concentrated load alone and several such concentrated loads individually are acting and for each of these cases we compute the deflection at the point where we are trying to find out the deflection individually and if we sum them up then we get the identical value of the deflection as we were getting earlier.

The reason behind this is that the deflection which we are computing is a linear function of the load which we are applying and hence this superposition is valid and you can get the deflection by using this superposition technique.

Now let us look into this particular problem that first if we compute it the way we are computing now and subsequently if we impose the method of superposition then what happens.

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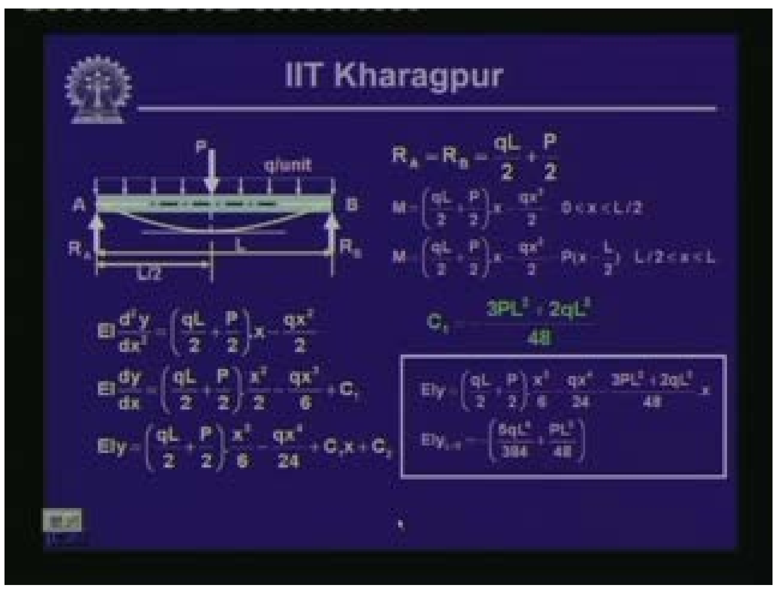

Now you see here, in this particular beam we have the concentrated load and the uniformly distributed load which is q per unit length and as usual we compute the values of the reactive forces; since we do not have any horizontal load in the beam so value of h a the value of horizontal reaction at a is equal to zero hence we have the values of R A and R B which is equal to.......... and this being symmetrical you can make out clearly from here that q times L is the total load that divided by 2 will be distributed between R

A and R B then the concentrated load P half will go by here and half will come over here (Refer Slide Time: 43:29) and that is why it is qL by 2 plus P by 2 is the value of the reactive forces R A and R B.

Now here we have two segments this is segment 1 and this is segment 2. Now, for the segment 1 if we take a cut over here and draw the free body of the left hand (Refer Slide Time: 43:42) then the expression about the bending moment comes as R A times x which is qL by 2 plus P by 2 times x minus the uniformly distributed load over a length x which is q x square by 2 and this is valid between the range 0 to L by 2; x is between 0 to L by 2.

Now, subsequently if we take the other part segment 2 if we cut over here and draw the free body of this part of the segment which is at a distance of x then the value of the moment is equals to again R A times x which is qL by 2 plus P by 2 times x minus qx square by 2 because of the uniformly distributed load minus the effect of this concentrated load which is at a distance of L by 2 at the 0 0 point so this is x minus L by 2 and this is valid between L by 2 to L. So L by 2 is less than equal to less than L. So these are the two expressions for the bending moment at two different segments.

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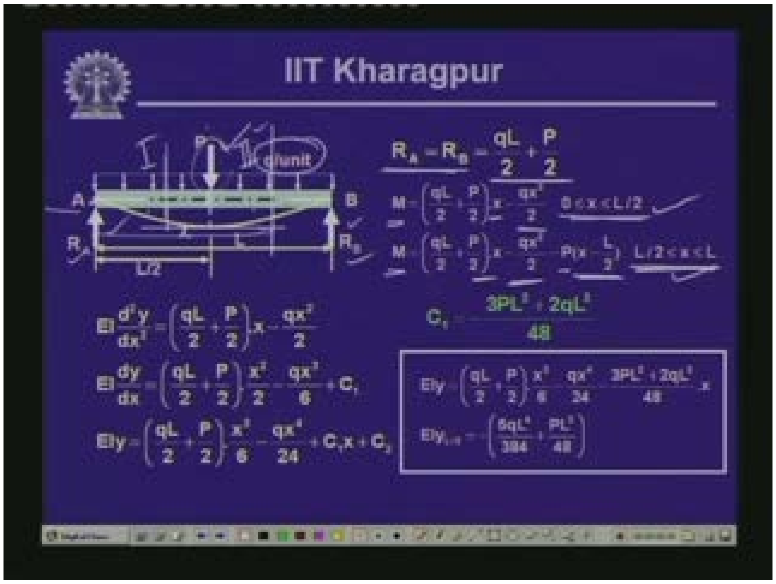

Now incidentally since this is a symmetrical beam or a beam which is having symmetrical loading, hence if we can compute for this segment the other segment will also follow the identical result but with a mirror image.

Now if we take the first segment and apply this in the differential equation which is EI d 2 y dx 2 which is equal to moment M so here qL by 2 plus P by 2 into x minus qx square by 2 that means we are taking the first part of it, the moment expression which is valid into the first segment and if we integrate this we have EI dy dx equals to qL by 2 plus P by 2 and x will give x square by 2 minus this is x cube by 3 so this is qx cube by 6 plus C 1 and on further integration of this we have EIy equals to qL by 2 plus P by 2 into x cube by 6 minus qx 4 by 24 plus C 1 x plus C 2. So this is the expression for the elastic curve where we have the unknown boundary C 1 unknown constant C 1 and C 2 which are to be evaluated applying the boundary conditions.

Now since we are using only the first segment we should impose the boundary conditions which are valid within that first segment. Now if you look into that at x equals to 0 this particular support then that deflection is 0. Also, if we draw the elastic curve as we are expecting this being a symmetrical beam that means the beam subjected to a symmetrical loading, now if we draw the tangent to the elastic curve at this particular point at the half way point then that becomes horizontal. That means the value of the dy dx at x equals to L by 2 should be equal to 0.

Now if we substitute that x equal to 0 y equals to 0 we get that C 2 equals to 0. And if we substitute that at x equals to L by 2 dy dx is 0 then from this expression we can get the value of C 1 and eventually C 1 comes as at x equals to L by 2 3PL square plus 2qL cube by 48 with a minus sign. So the value of or the expression for the elastic curve which is EIy is equal to qL by 2 plus P by 2 x cube by 6 from here minus qx 4 by 24 from here and C 1 times x where C 1 is minus 3PL square plus 2qL cube by 48 times x. This is the expression for the elastic curve of the beam.

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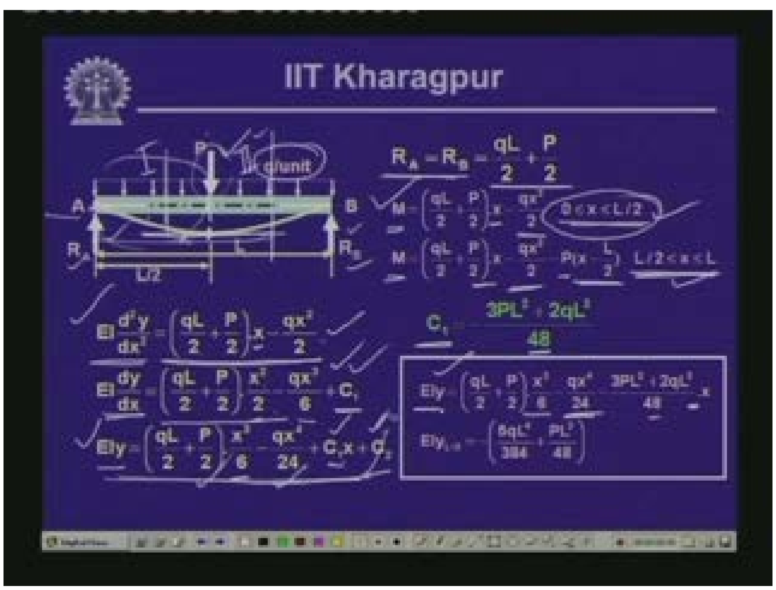

Now, since we are interested to find out the deflection at the center which is eventually going to be the maximum deflection because this beam is subjected to the symmetrical loading both in terms of the uniformly distributed load as well as for the concentrated load, it is symmetrical, hence we expect that the maximum deflection will occur at the center of the beam. Now, if we compute the value of y at L by 2 this gives us the value of minus qL 4 by 384 plus PL cube by 48 and in fact if we take EI on the other side we get y at that point divided by 1 by EI.

Now if we try to apply...... so this is the normal way the way we are computing the value of deflection in a beam using the differential equation for the bending. Now if we like to employ the method of superposition let us see what we get applying the method of superposition for this particular case.

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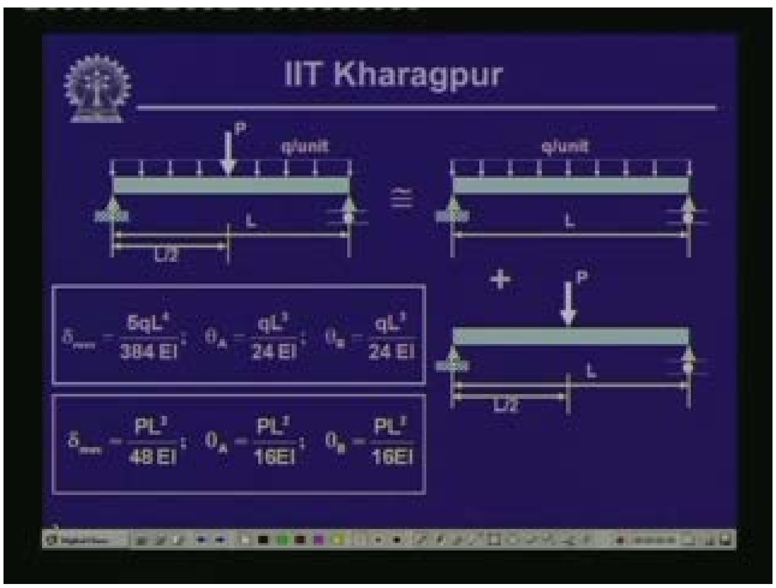

Now here, this particular beam this particular loading on this beam can be divided into two parts. The first part we take the beam subjected to the load which is q power unit length and this particular part we consider the beam which is subjected to the concentrated load P. Now as we have seen that when a simply supported beam subjected to an uniformly distributed load q per unit length then for that the values for the maximum deflection at the center is equal to 5qL 4 by 384EI and the values of the slope at A and at B which is theta A and theta B which is equal to qL cube by 24EI.

Now here of course we have not given the sign of the slope. As we have noticed earlier that if you take the deflection curve now the slope at this point (Refer Slide Time: 49:41) is a clockwise one which is negative and the slope at this end is anticlockwise which is positive but the magnitude-wise they are qL cube by 24EI.

Now if we consider this particular beam where we have the beam with a concentrated load P at the center then correspondingly we get the value of delta as PL cube by 48EI and correspondingly the values of the slope at A and B at A and B if we compute it this is theta A and this is theta B they are PL square by 16EI. Again we have not used the sign, only we have used the magnitudes of the slopes.

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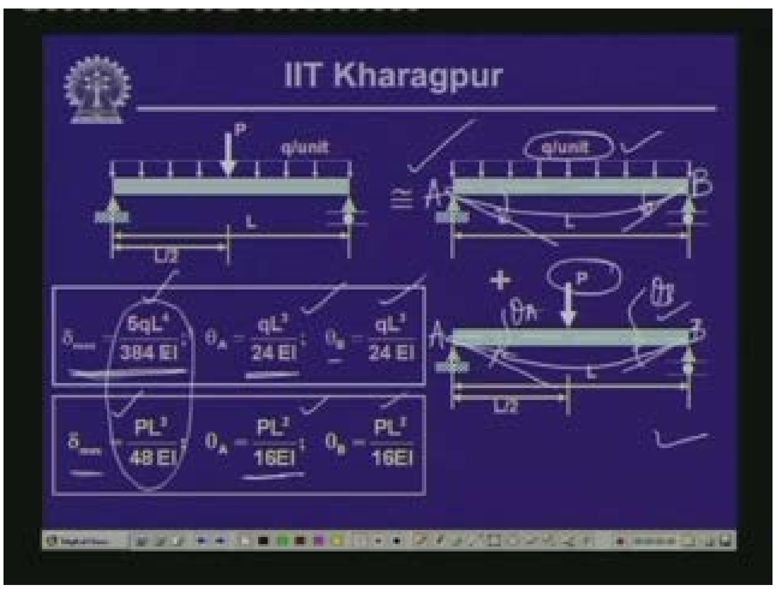

Now as you can see that if we combine these two deflections together; that means since this particular beam we are analyzing and this beam is consisting of these two systems of loading, so if we compute the deflection which we have computed here and here, if we sum them up then it will give the result for this particular case. So the total delta for this beam will be this plus this which eventually we have seen in the previous case. And also the value of the slope will be the sum of these two which is that of this plus this (Refer Slide Time: 51:09).

Thus, by employing this superposition technique we can split the loading system of the beam where it is subjected to several loads either in the form of uniformly distributed load or in the form of concentrated load; we can split the loading system in the beam in different forms or in a simpler form and then we can compute the value of deflection and the slope at a point where we desire and then we can sum them up to get the results for the loads which are acting simultaneously.

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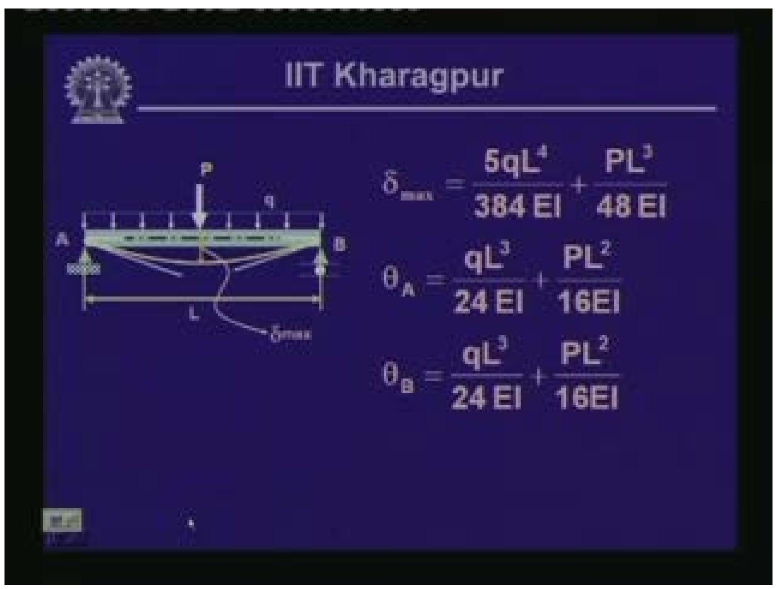

This is the final from of this result of this particular case that delta max............. this is for the uniformly distributed load and this (Refer Slide Time: 51:53) is for the concentrated load so the combined effect of this is the value of delta max over here, theta at A is the effect of the uniformly distributed load and concentrated load, again theta at B is the uniformly distributed and concentrated load, again these are with the magnitudes but not with the sign.

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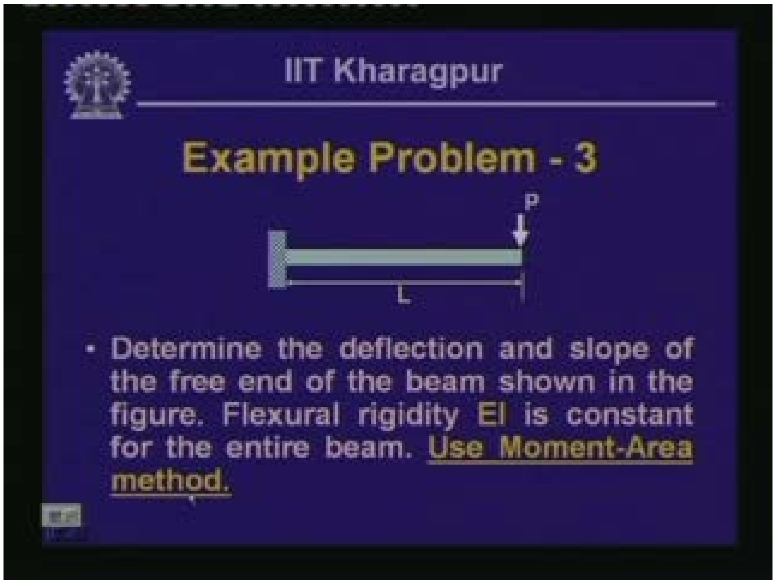

Now we have another example problem. Here we use the moment area method. In fact this particular problem we have already solved to find out the slope and deflection of the free end. Now we employ moment area theorem so that we can compute the values and check whether they are the same.

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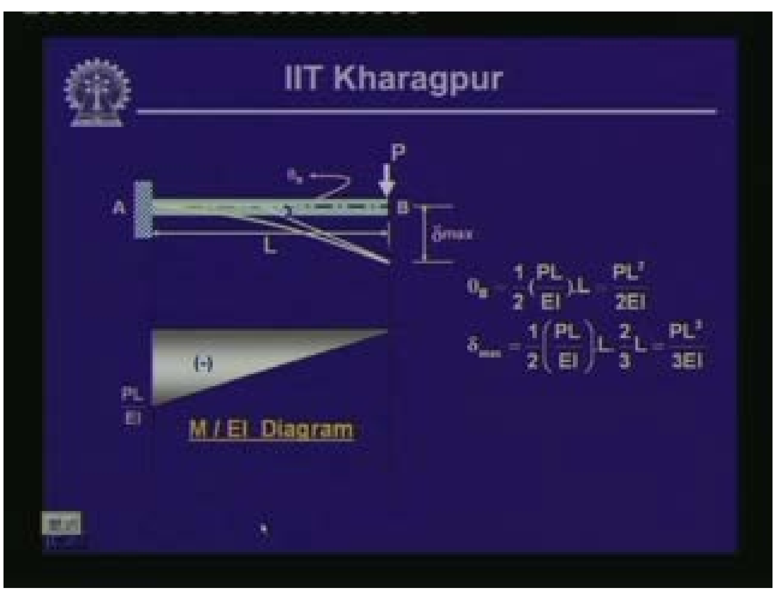

When we employ the when we like to find out the delta the deformation of this that means if we draw a tangent to the elastic curve at this and if we drop a perpendicular so this is the distance which is delta max or which we call as dv. Also, we are interested to find out the slope of this particular beam at this free end if we draw a tangent over here; if we draw a tangent over here this slope is 0 (Refer Slide Time: 53:03) so the difference between these two slopes is theta B again.

Now as per the moment area first theorem, if we are interested to find out the change in slope between these two points it should be equal to the M by EI diagram between these two points and that is what is considered over here. The bending moment diagram for the cantilever beam subjected to the tip load P will be PL by EI which is the magnitude over here and then gradually it will be coming to 0 linearly so we will have a triangular configuration for the M by EI diagram.

Now if we take the area of the M by EI diagram between these two points that is between A and B which is half times PL by EI times length that will give us the change in slope between these two points which is equal to theta B and that is equals to PL square by twice EI. Also, now when we draw a tangent over here this is the tangential equation of the elastic curve of this particular point which is equal to the delta max.

That means if we take the moment of the M by EI diagram with respect to this point we are going to get the value of the deflection. So delta max is equal to half PL by EI times L times the distance of the cg from here which is two third of L. So if we employ that that gives us a value of PL cube by 3EI. So these are the values which we have obtained earlier by employing differential equations. We can make use of these moment area theorems as well to get the vales of the slope and deflection.

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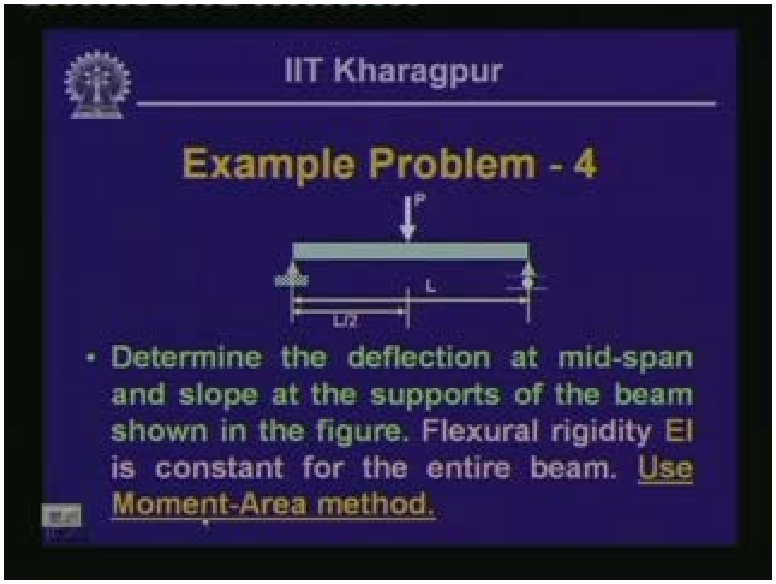

Well, we have another example problem. We have to employ the moment area theorem to find out the slope and deflection of this particular beam and this problem is set for you. We will discuss in the next class.

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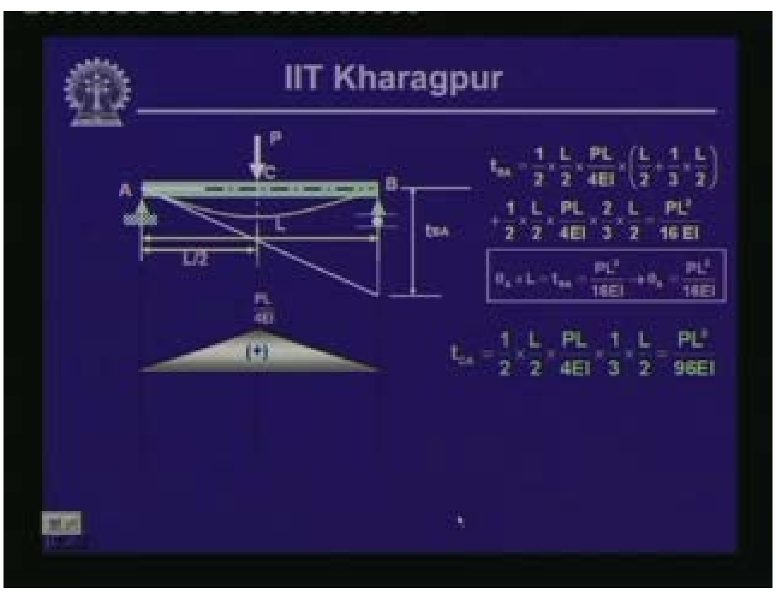

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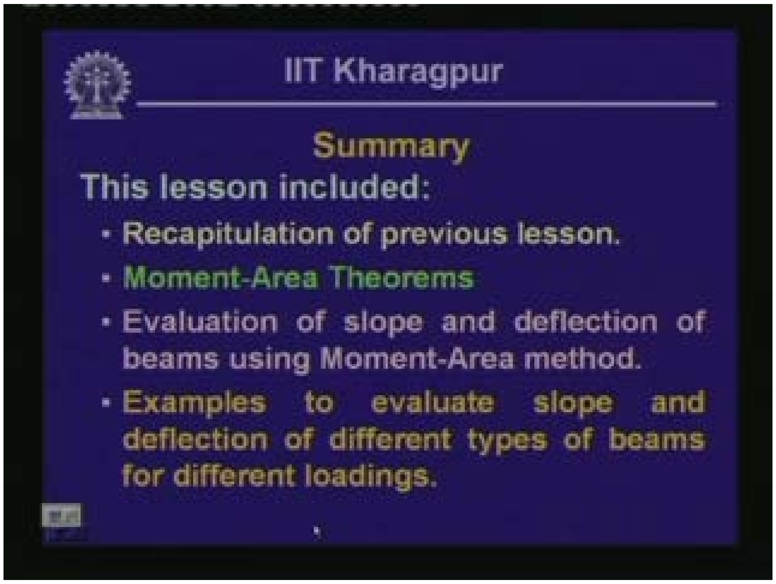

Now to summarize them, in this particular lesson we have included the recapitulation of previous lesson. We have looked into the moment area theorems. We have evaluated the slope and deflection of beams using moment area method and also we have demonstrated some examples to evaluate slope and deflection of different types of beams for different loadings.

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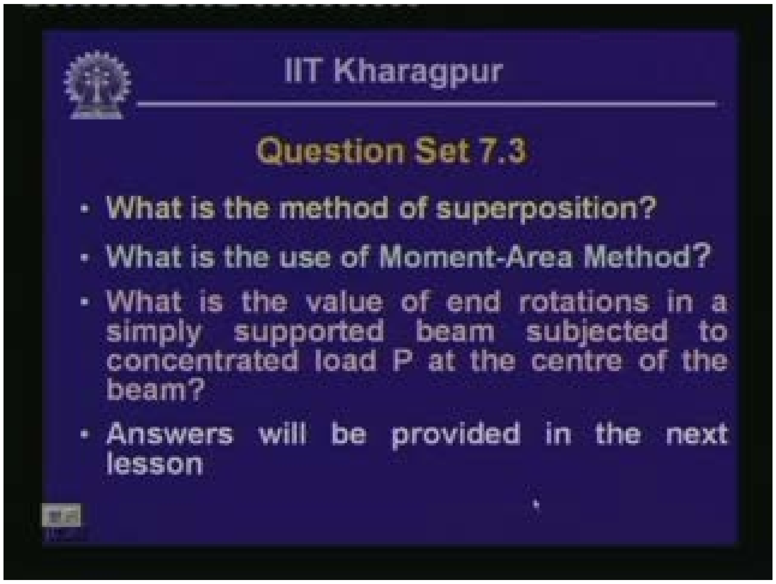

These are the questions set for you. What is the method of superposition? What is the use of moment area method and what is the value of end rotations in a simply supported beam subjected to concentrated load P at the center of the beam. Now answers for this will be given in the next lesson, thank you.

Preview of next lesson

Strength of Materials

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Lecture no - 33

Deflection of Beams - IV

Welcome to the fourth lesson on the seventh module which on the deflection of beam part IV.

In fact in the last three lessons in this particular module we have discussed how to evaluate the slope and deflection in beams for different loading situations considering the differential equation of elastic curve and also we have looked into the method of superposition, how to employ superposition in evaluating slope and deflection of beams and subsequently we have looked into the theorem of moment area or moment area method to evaluate the slope and deflection in beams.

Now in this particular lesson we will be concentrating on the evaluation of slope and deflection of beams employing moment area method again. But in the previous examples we have looked into for the beams where the EI value or the flexural rigidity is uniform throughout the beam. Now here we will be considering the beam where EI may not be uniform throughout the beam there could be variable moment of inertia or if there is a non-prismatic beam where it is varying continuously then what will be the values of slope in such beams.

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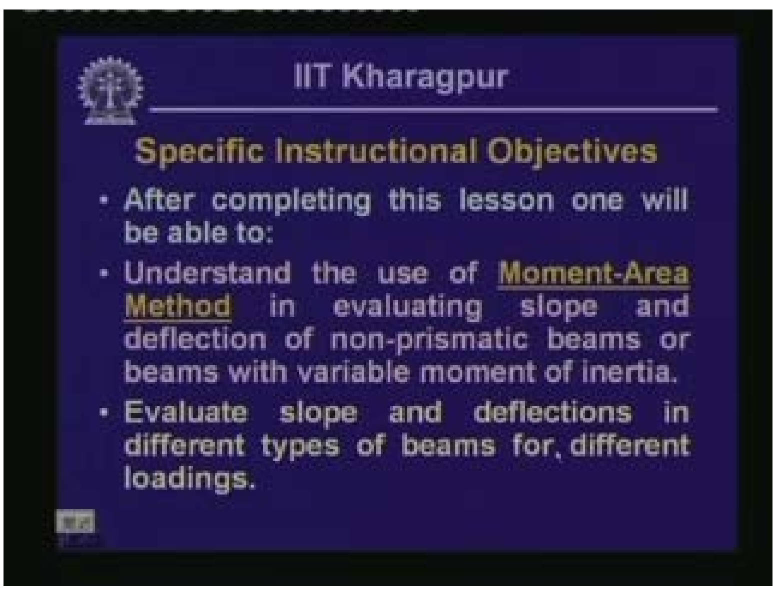

Hence it is expected that once this particular lesson is completed one should be in a position to understand the use of moment area method in evaluating the slope and deflection of non-prismatic beams or beams with variable moment of inertia. One should be in a position to evaluate slope and deflections in different types of beams for different loadings.

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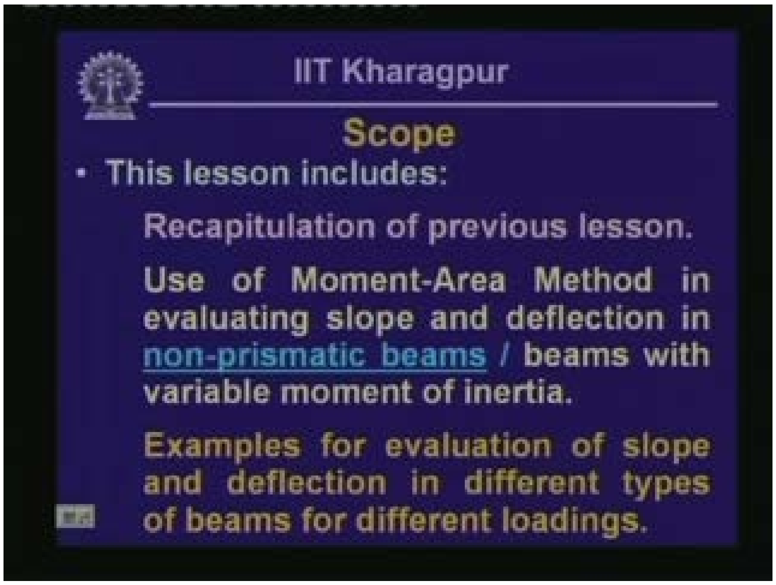

The scope of this particular lesson includes recapitulation of previous lesson and we will be doing that through the question answer session; we will be answering the questions which were posed last time and thereby recapitulate the previous lesson.

Use of moment area method in evaluating slope and deflection in non-prismatic beams or beams with variable moment of inertia. Also, this lesson includes the examples for evaluation of slope and deflection in different types of beams for different loadings.

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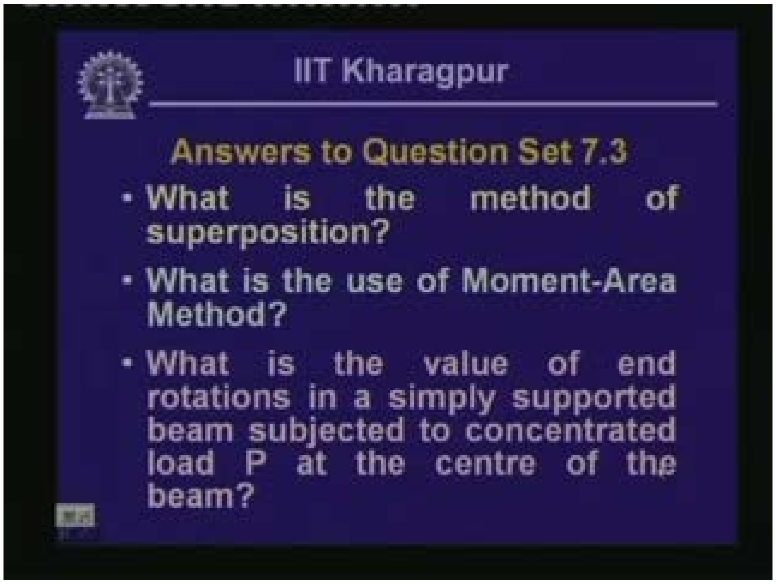

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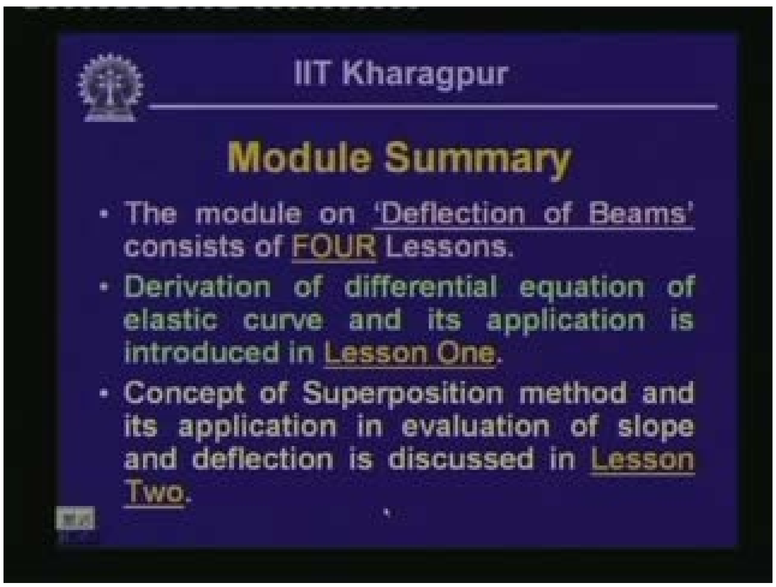

Well, let us look into this particular module and in this module we have seen it includes four lessons and the first lesson we have discussed the derivation of differential equation of elastic curve and its applications. Then subsequently we have introduced the concept of superposition method in the lesson 2.

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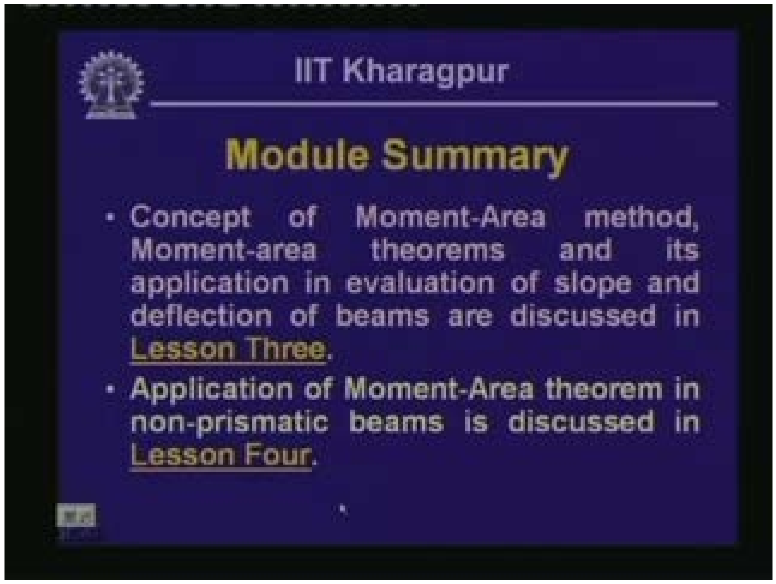

In the third lesson we have discussed about the concept of moment area method, the moment area theorem and its application in evaluation of slope and deflection. And in this particular lesson today we have seen application of moment area theorem in non-prismatic beams or in beams with variable moment of inertia.

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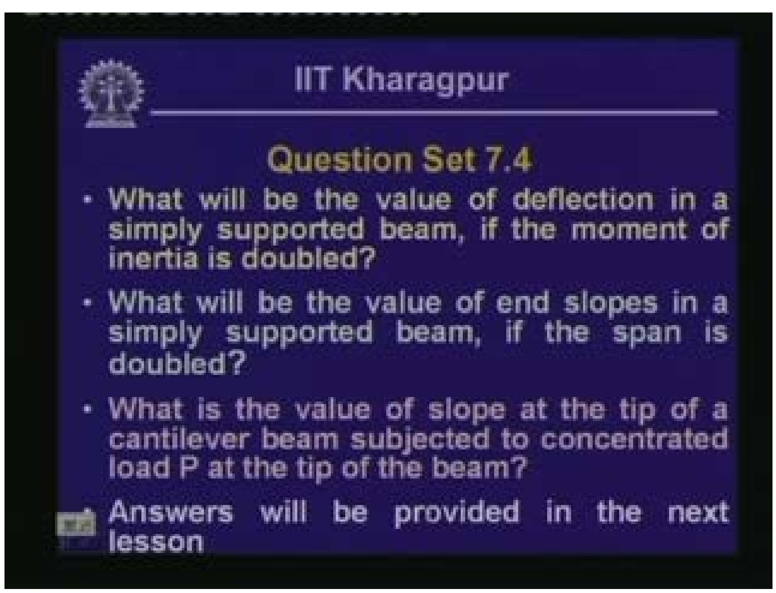

Now these are the questions set for you that what will be the value of deflection in a simply supported beam if the moment of inertia is doubled.

What will be the value of end slopes in a simply supported beam if the span is doubled?

What is the value of a slope at the tip of a cantilever beam subjected to concentrated load P at the tip of the beam?

Now answers for this will be given to you in the next lesson, thank you.