Strength of Materials

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Lecture - 31

Deflection of Beams - II

Welcome to the second lesson of the seventh module which is on which is on deflection on beams part II. In the last lesson we have introduce the concept of deflection of beams and also it has been introduced that how deflection of beams plays an important role in the engineering structures. We have derived the equation the differential equation for the elastic curve and subsequently we have derived the equation for the elastic curve and we have seen that how to compute the deflection of beam at any point for a particular kind of loading.

Now, in this particular lesson we will be discussing some more aspects of the deflection of beams for different types of beams and subjected to different kinds of loads. It is expected that once this particular lesson is completed one should be able to understand the concept of deflection of beams under different loading conditions or different types of beams.

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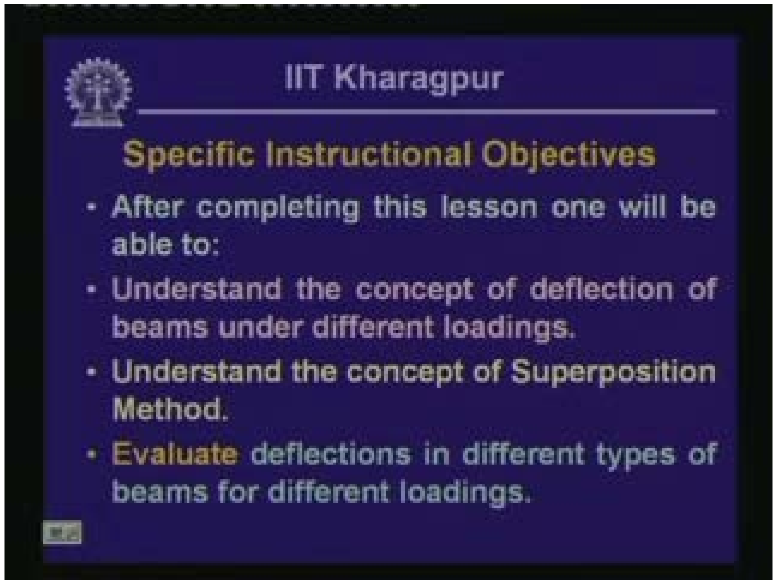

In fact, last time we have looked into simply supported beams subjected to loads. Here we will be looking into some more types of beams with different kinds of loading systems. One should be in a position to understand the concept of superposition method. We will discuss this particular method the superposition; how to evaluate the deflection if you have more number of loads which are acting simultaneously in a beam. Also one should be in a position to evaluate deflections in different types of beams for different loading.

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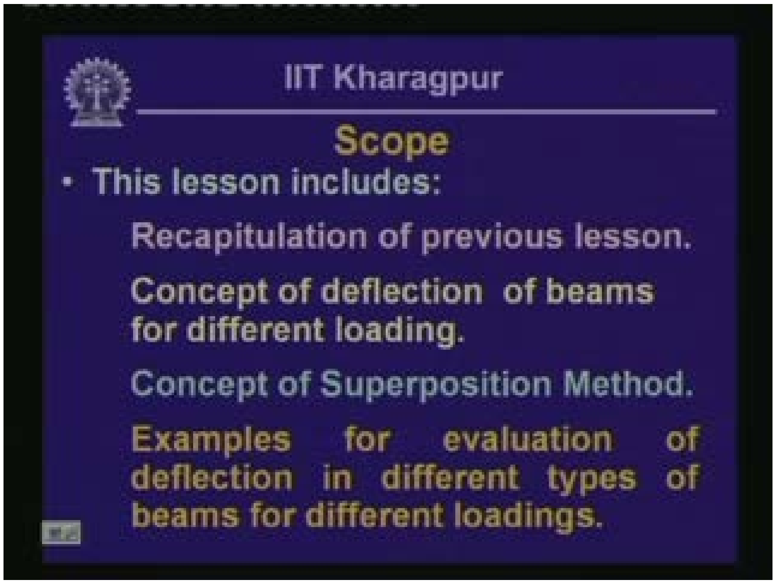

Hence the scope of this particular lesson includes recapitulation of previous lesson which we generally do through the question answer session. We will be looking into the answers of the question which I have posted last time and in the process we will be able to recollect whatever we have discussed in the last lesson.

We will look into the concept of deflection of beams for different loading, concept of superposition method and also it includes examples for evaluation of deflection in different types of beams for different loadings.

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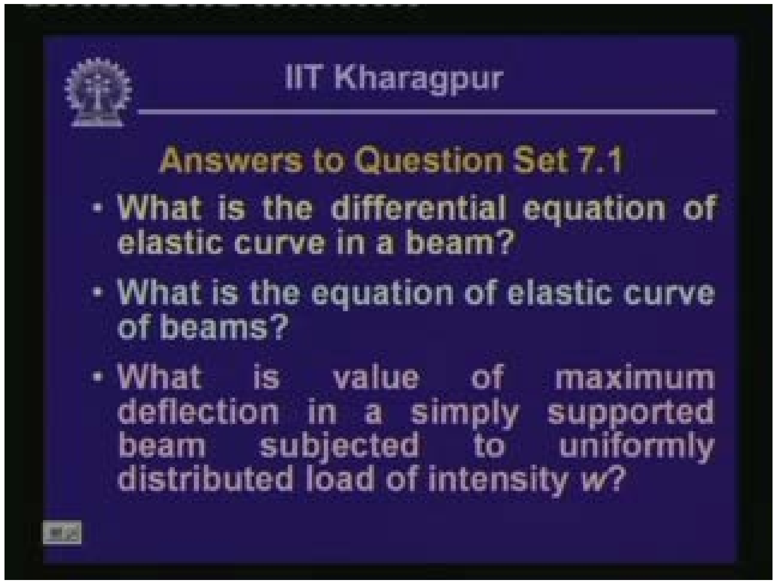

Well, the questions which were posed last time; the first question is that what is the differential equation elastic curve in a beam.

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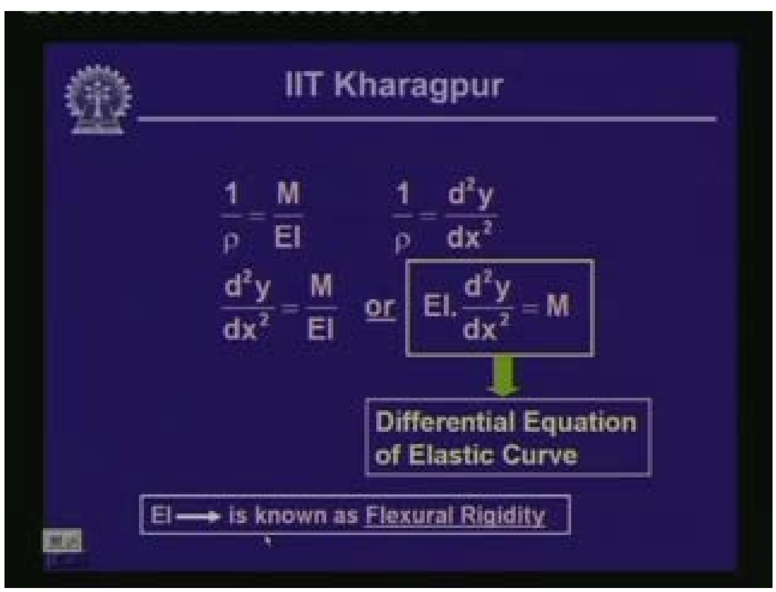

Now, if you remember, last time we had derived that if a beam is subjected to transverse loading then it undergoes bending and therefore the axis of the beam if you look into it deforms. And we have seen that in terms of curvature of this particular beam 1 by rho is called as curvature which you can write as d 2 y dx 2 where y is the deflection of the beam with respective is Avenal position.

And subsequently as we have seen, curvature is related to the bending moment at any cross section m which is a function of x hence the curvature can be related to the bending moment through this expression which is E I d 2 y dx 2 equal to m and this particular equation differential equation is called as the differential equation of elastic curve. So, when we mean that the differential equation of the beam bending then this is the expression which we use: EI d 2 y dx 2 equal to m and from this particular basic equation we go for the other derived units like as we know that V the shear force is equal to minus dM dX hence if we take the derivative of this particular expression we have EI d 3 y dx 3 this is equal to minus V.

And as we know that q is equal to dV dX where q is loading on the beam the uniformly distributed load q so if we take the derivative of this we get EI d 4 y dx 4 is equal to minus q and this we have seen last time while deriving the governing equations wherein EI is the parameter which you call as flexural rigidity.

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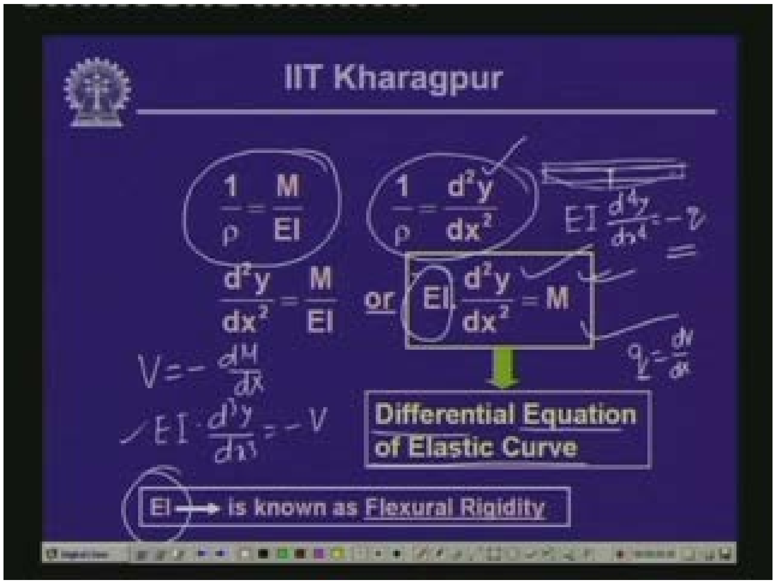

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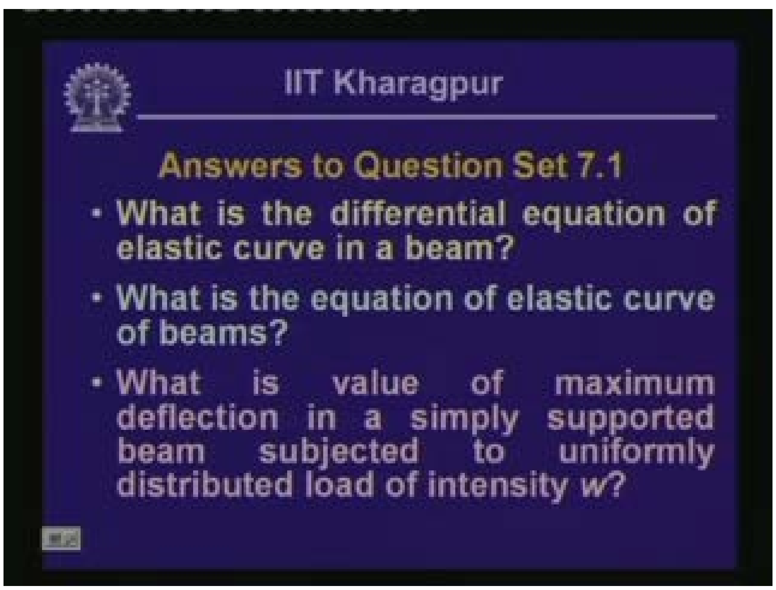

Now, the second question post was what is the equation of elastic curve. Now the first question is what is the differential equation of elastic curve and the second question is what is the equation of elastic curve of beams.

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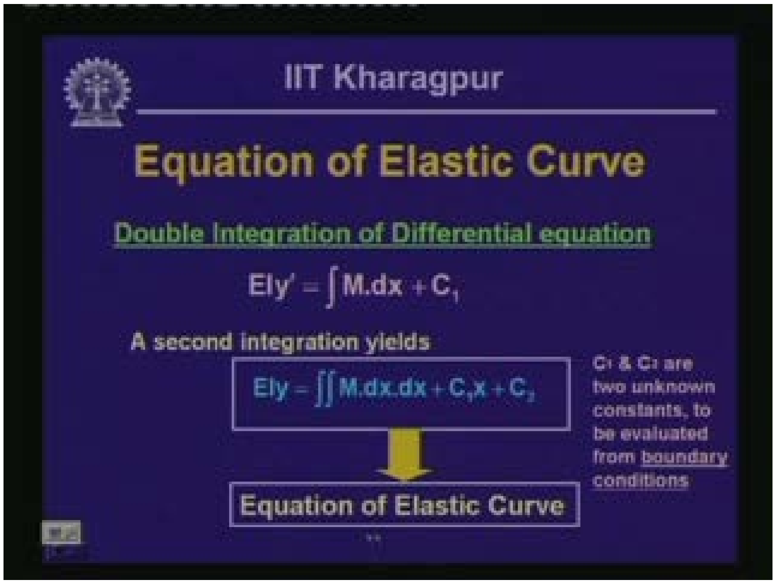

Now subsequently once we get the differential equation for the beam which is EI d 2 y d x 2 is equal to moment, now if we integrate it once then we get that EI dy dx y dash is dy dx is equal to integral M dx plus constant C 1 and further integration of this gives us EIy is equal to double integral M dx dx plus C 1 x plus C 2 where C 1 and C 2 are the unknown constants which are to be evaluated from the boundary condition. Now from these we can write that y is equal to 1 by EI integral (M dx) dx plus C 1 plus C 2.

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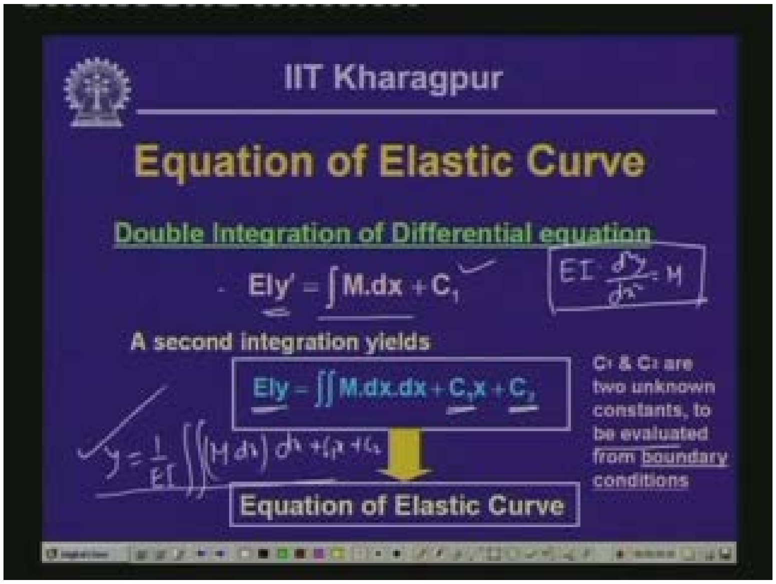

Now this particular expression is termed as the elastic equation of the elastic curve that why we are representing......... since moment M is a function of x, and y will get the function of x so that you can compute the value of y at any point along the length of the beam. Because if we consider the origin of the coordinate axis at the length of support then along the length of the beam as we go then at any x we can compute we can compute the value of y and from this particular equation which we call equation of elastic curve.

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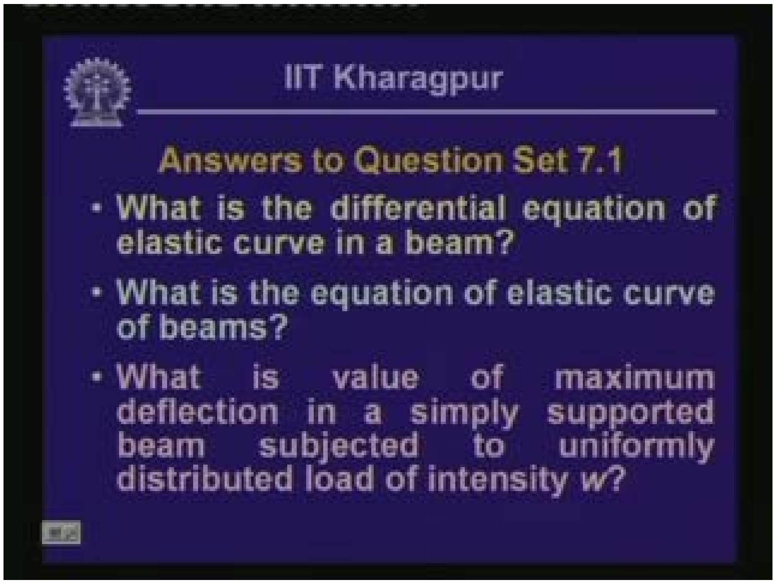

Now the third question the last question posed was what is value of maximum deflection in a simply supported beam subjected to uniformly distributed load of intensity w.

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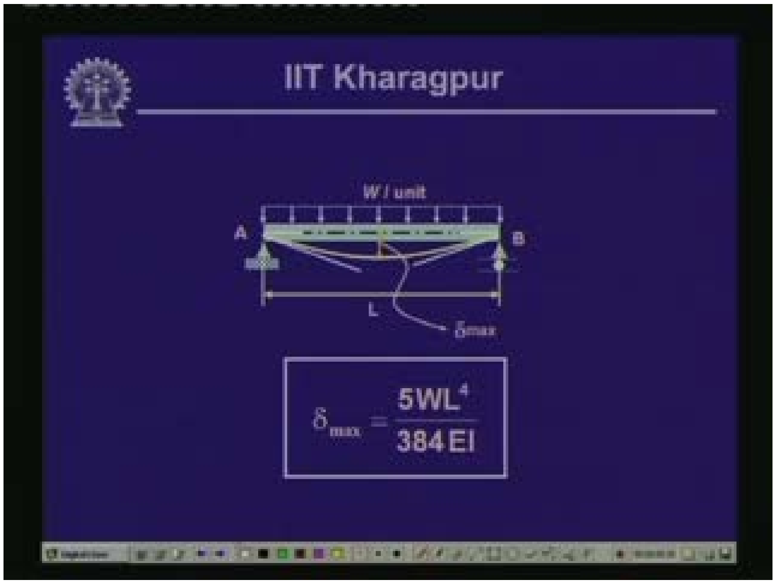

Now last time we had evaluated the expression for the elastic curve and subsequently the deflection at different points. Now what is the simply supported beam when the load is w for unit length along the entire span or what is the entire length of the beam?

The expression for the elastic curve as we have done from the expression of EI d 2 y d x 2 equal to moment if we do that we can get the value of y as a function of x and then if we evaluate the value of y at x equal l by 2 y at x equal to l by 2 we get the expression as y equal to 5WL 4 by 384 EI.

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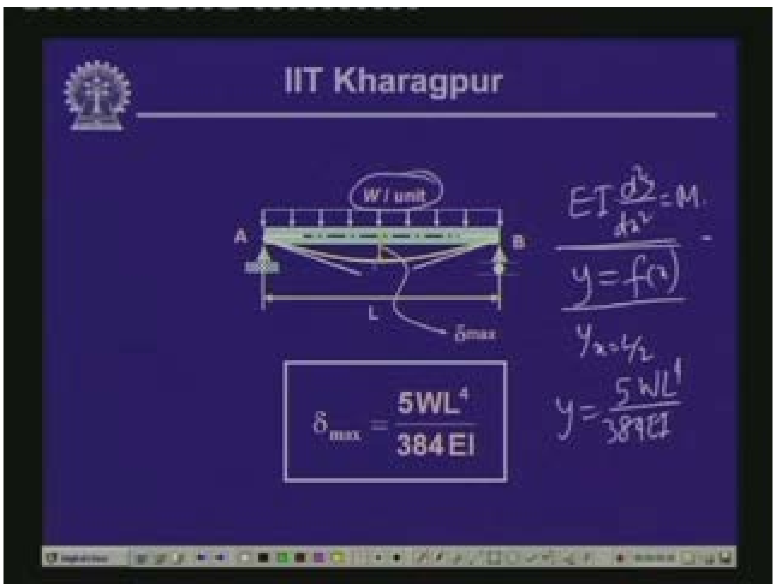

Now this particular beam is symmetrically loaded and hence as we do not have any horizontal reactive force over here the vertical reactive force R A and R B we have R A here and R B over here. Now, for the symmetrical reloaded beam the maximum deflection occurs at the center. In fact, for the elastic curve if we draw a tangent to an elastic curve at this particular point this becomes horizontal. So the point where the tangent to the elastic curve is a horizontal one that point the deflection we get is maximum. So the maximum deflection in this particular case is that x is equal to l by 2 which is equal to 5WL 4 by 384 EI. So this is the maximum value of the deflection for a simply supported beam subjected to uniformly distributed loads.

For a simply supported beam subjected to uniformly distributed load the maximum deflection occurs at the center of the beam and the magnitude of that is equal to 5WL 4 by 384 EI where EI is constant throughout the beam which we have termed as the flexural rigidity.

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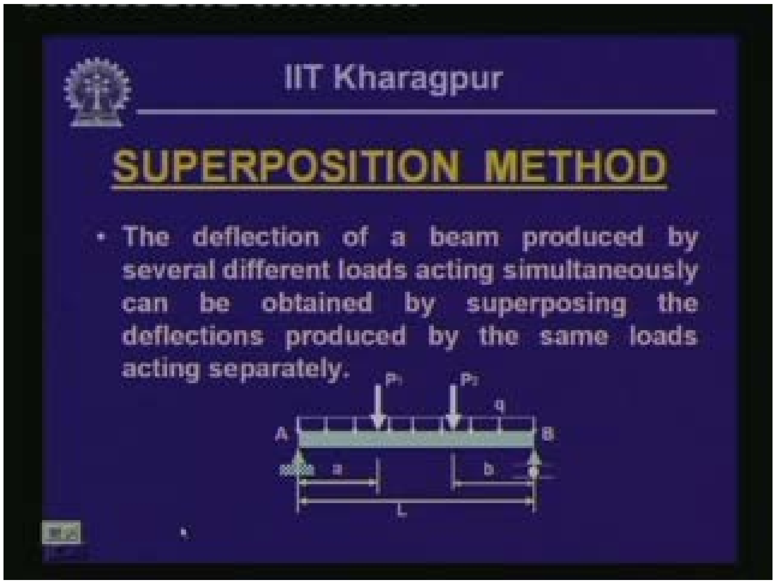

Well, after looking into these questions let us look into another aspect of this evaluation of deflection. Now this particular method we call as a superposition method wherein if you have a number of loadings on a beam................; now so far what we have seen is that a beam is subjected to uniformly distributed load or you have a concentrated load at any point in the beam then we can compute the expression for the elastic curve.

Now, supposing if we have a beam wherein there are several loads acting on a beam that could be uniformly distributed load and several concentrative loads acting, then if we like to compute the deflection of the beam at any point then of course we can use the methodology as we have discussed so far. That means at any section we can compute the bending moment and as usual we define the different segments over which that moment expression is valid and then we employee the differential equation of the elastic curve which is EI d 2 y dx 2 equal to moment and the validity of that moment expression segment-wise and we can compute the value of deflection with the appropriate boundary condition. So this particular technique we can employ and we can evaluate the deflection at any point in the beam.

Now, since we are dealing with the beams which are in the elastic stage it is a linear elastic condition of the beam. Also, if you look into the differential equations they are in a linear function of the loading hence we can compute the deflection at a particular point for individual such loadings and we can combine them together to obtain the deflection at a particular point or we can propose the effect of different loads at a particular point. In fact that is what we call as a superposition technique or superposition method.

So superposition method is the deflection of a beam produced by several different loads acting simultaneously can be obtained by superposing the deflections produced by the same loads acting separately. Now as an example if you look into this is a simply supported beam hinge at this end and this is on the roller on the other end and it is subjected to uniformly distributed load q per unit length and also it has two concentrated load p 1 and p 2 which are acting at a distance of A from left support and P 2 is acting at a distance of P from the right support.

Now when we have this loading, supposing if we are interested to evaluate the deflection of this beam at this particular point (Refer Slide Time: 12:05) let us call this point as C; now, if we like to find out the deflection delta at C; now as I said that we can employ the governing differential equation which is EI d 2 y dx 2 is equal to moment and based on this loading over here we have we can divide the whole beam in three segments: the segment 1 is from here to here just on the left hand side of this concentrated load, segment 2 is from here to or from this point to this point which is from a to L minus b and the third segment is after this loading to the end of this and at these three places we can take the free body diagram and compute the moment at these three places and consequently if we substitute in this particular expression for the differential equation we can obtain the value of y with appropriate boundary conditions for the different segments.

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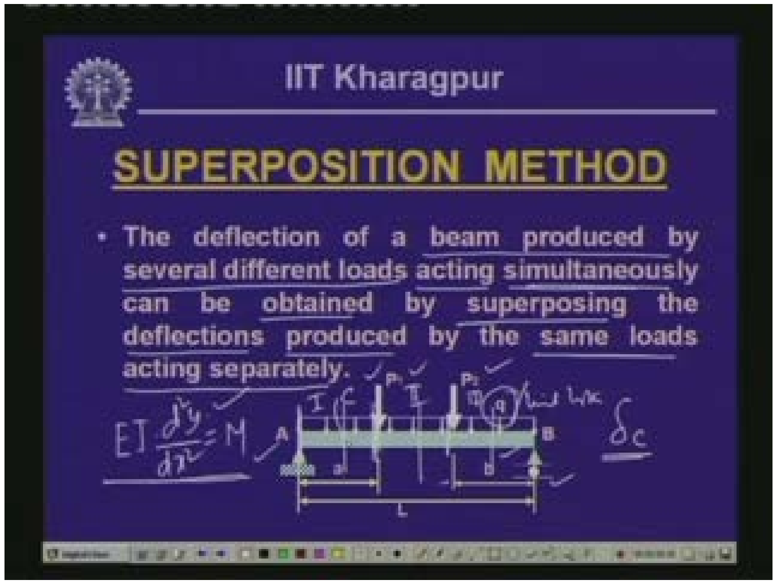

Also we can compute the value and accordingly at y we can compute from this expression for y which will get as a function of x so at a distance from here if we know this value of x we can compute the value of y. Now also what we can do is we can compute the value of delta c by taking these loads acting individually.

Now instead of these two concentrated load and the uniformly distributed load acting simultaneously if we assume; let us say that instead of the concentrated load, now only the uniformly distributed load is acting; now if the uniformly distributed load is acting we have seen how to compute the expression for the elastic curve. Now in that expression if we substitute the value of x we can get the value of deflection at point c which is delta c for uniformly distributed load. Let us call that as delta C 1. So, that is for the uniformly distributed load.

Now next let us consider that it is acted on y the concentrated load P 1 and no other loads are acting. So now first we are considering that it is subjected to uniformly distributed load, second we are considering that there is neither uniformly distributed load nor the other concentrated load P 2 but it is subjected to only concentrated load P 1. Now, for this particular condition again we can solve the differential equation, we can get the expression for y in the beam and accordingly we can compute the deflection at point c for these loads alone and let us call that as delta C 2 the deflection at c for the concentrated load P 1 is delta C 2.

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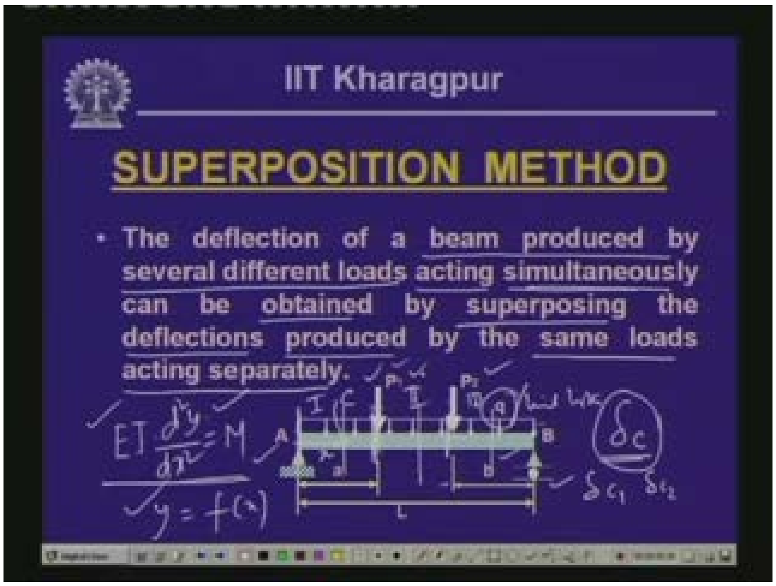

Now thirdly we assume that this particular beam is subjected to a concentrated load P 2 and no other loads are there; like neither the uniformly distributed load nor the concentrated load P 1. So if the beam is subjected to only concentrated load P 2 again we can solve the differential equation and compute or arrive at the expression for y as a function of x and then we can compute the value of deflection at c from this differential for the equation of elastic curve that what will be the deflection at point c and let us call that deflection as delta c 3.

So, if we add these three deflections to together delta C 1 plus delta C 2 plus delta c 3 we will get the total value of the deflection delta c which otherwise we have obtained from the expression of the elastic curve had we taken the loading all the loading simultaneously. So this is what we call as the method of superposition where we can compute the deflection at any point in the beam when we take the simultaneous or the many loads which are acting on a beam, individually we take the loads and their effects on a particular point we consider and if we sum them up we can get the deflection of that particular point for all the loads when they are acting simultaneously on the beam and this is what we call as the superposition method.

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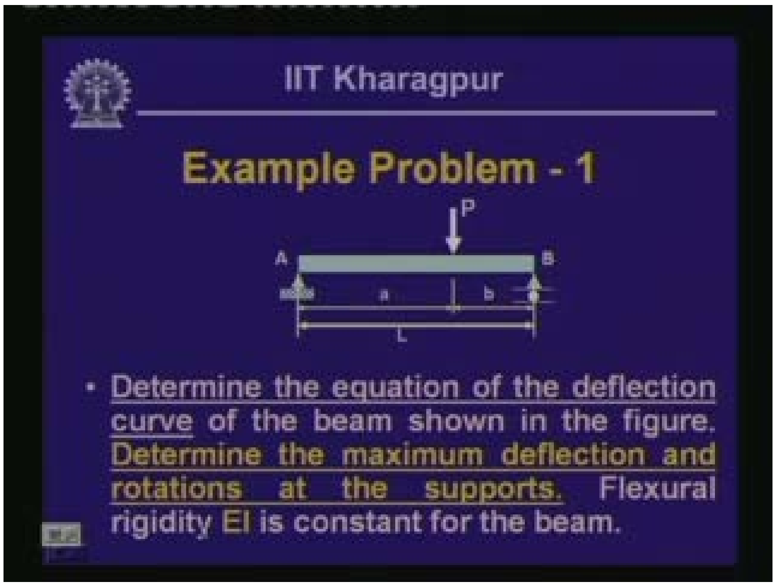

Well, now let us look into the examples for evaluating the deflection at different points. Now, the last lesson we started with this particular problem wherein we had a simply supported beam which was subjected to a concentrated load P at a distance of A from the left support or B from the right support. Now what we need to do is that we will have to find out the expression of the elastic curve for this particular loading on this beam and also we will have to compute the value of the maximum deflection and rotation at the support.

Now the last time we started with evaluating the moment at different points. Now as I said the beam we can divide into two segments segment 1 and segment 2. now from 0 to A this will be guided by one expression of the bending moment, from A to L it will be guided by another expression of the bending moment but before that what we need to do is that we need to evaluate the reactive forces and they are say R A and R B and since there are no horizontal forces in the beam so the value of the horizontal force is equal to 0 and we can compute to value of R A and R B; if we take moment about B we get the value of R A and from R A class R B equal to P we can compute the value of R B.

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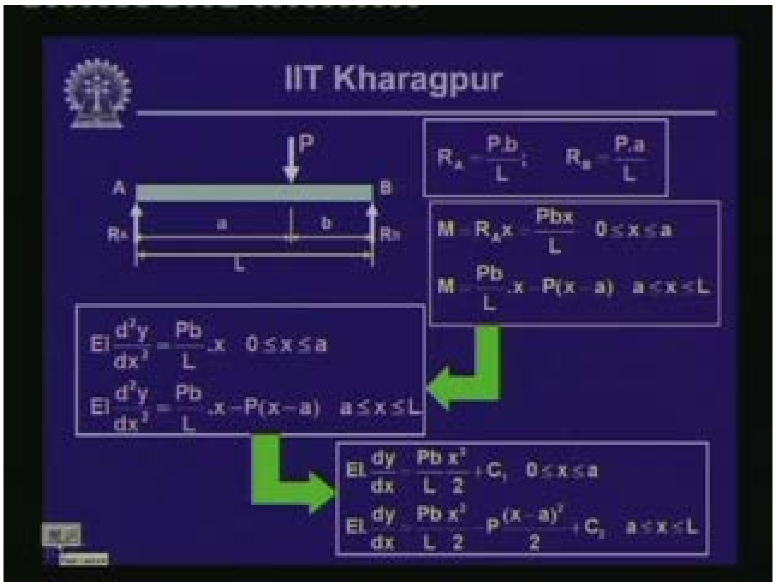

Now if you compute the value of R A and R B there come like this that R A is equal to Pb by L and R B equal to Pa by L. This in fact we have seen last time. Also for these two segments: segment 1 and segment 2 we can compute the value of the bending moment. Now, for the segment 1 from 0 to a if we take the free body over here if we take a cut and draw the free body of the left support the free body will be that we have the reactive forces over here and on this cut we have the balancing shear force and the bending moment M and V and this is R A.

So, if we take the moment of all the forces with respect to this particular point which is at a distance of x from left support so M is equal to R A times x and this is what is written over here and R A being Pb by L so M is equal to Pbx by L and this is valid between 0 to

A. This expression for the bending moment is valid between 0 to A. But as soon as we go beyond this loading now this expression of bending moment is no longer valid because the contribution of this load will come into picture.

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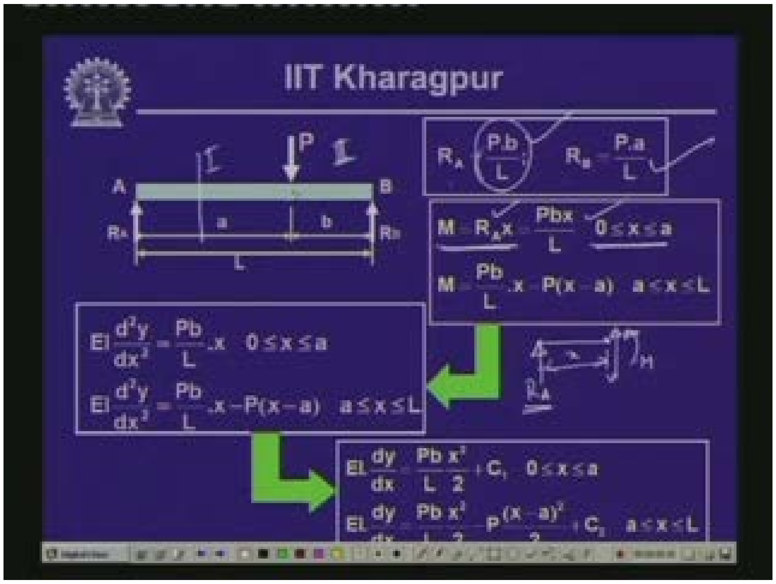

Now, if we take the segment 2 and if we take a cut over here and draw the free body diagram then the free body of this is going to be like....... we have the reactive force R A over here (Refer Slide Time: 19:27), we have the concentrated load P over here and then on this cut we have the shear force and the bending moment and this is the distance x from here now and this being a so this particular distance is x minus a.

Now if we take the moment of all the forces with respect to this particular point the point b the point c let us call this cut point as c, now moment at c will be equal to R A times x. Now R A being Pb by L so Pb by L times x minus P times(x minus a) so p times(x minus a) so this is what is the expression for the bending moment and this expression of the bending moment is valid between this point to the end of this. So from a to L is the range of x wherein this particular expression will be used.

Now you see that over these two segments we have two expressions for the bending moment. Now our next step is we substitute these values of the bending moment in our differential equation which is EI d 2 y dx 2 is equal to moment and moment is equal to Pbx by L. This is the range of the validity of this particular expression and subsequently the other bending moment equation which we have obtained for the other segment we substitute over here again in this differential equation which is EI d 2 y dx 2 is equal to moment and the range of validity is between a and L.

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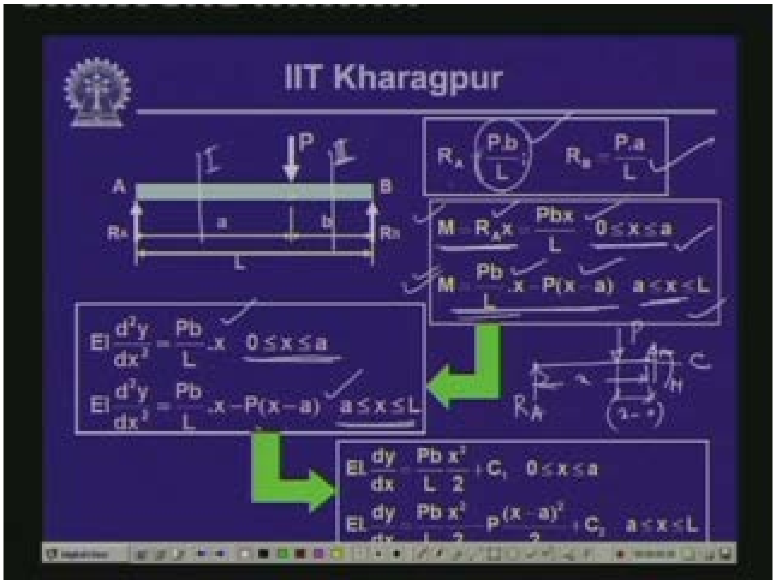

Now, once we integrate these expressions we come up with this that EI dy dx is equal to Pb by L; now x will give us x squared by 2 by two with the constant of integration C 1 and again the validity is between 0 to a.

And for this particular expression if we integrate we have EI dy dx is equal to Pb by L x squared by 2 minus P times(x minus a) square by 2 plus another integration constant which is C 2 and the validity of this particular expression between a and L. These are the expression for d y dx.

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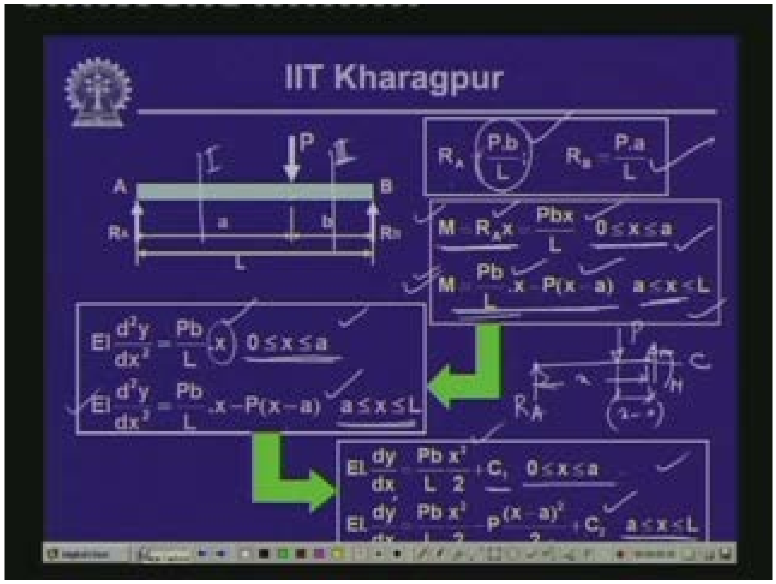

Now if we integrate this expression further then we will get the values of y or the expression for y.

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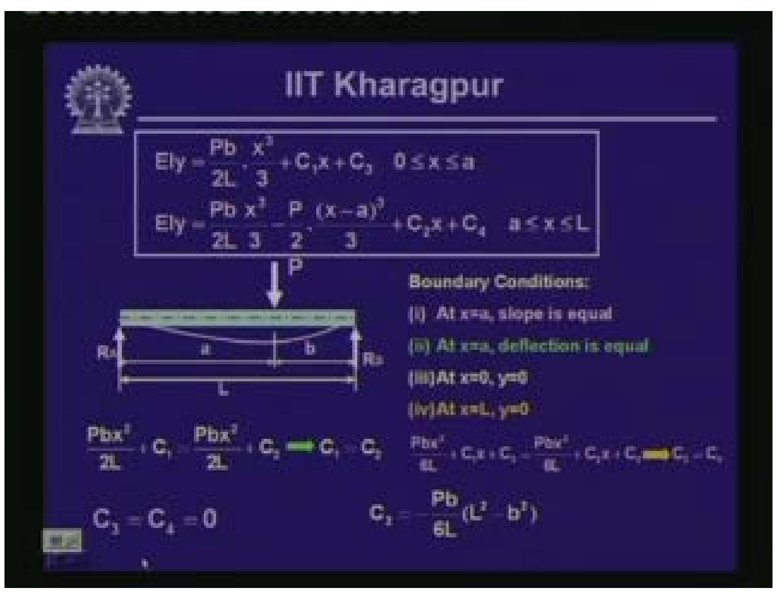

Now if you look into that the one integration of first equation we get EIy is equal p by Pb by twice L x cube by 3 plus C 1 x plus another constant which we call as C 3 and the validity range x between 0 and a. Subsequently, the second expression yields: EIy is equal to Pb by twice L x cube by 3 minus P by 2(x minus a) cube by 3 plus C 2 x plus C 4. So we have another two constants C 3 and C 4 have come in after integration and the validity of this particular expression the value of x ranges between a and L.

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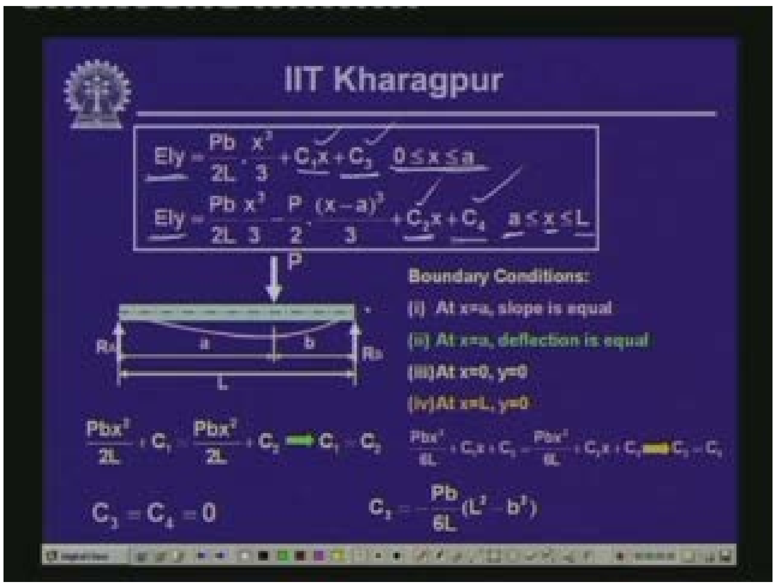

Now there are four constants: C 1 C 3 C 2 C 4 and these four constant are to be evaluated from the given boundary conditions. Or we know the support conditions of the beam, the loading conditions of the beam; now we will have to substitute the appropriate boundary conditions so that we can evaluate the values of C 1 C 2 C 3 C 4. And as we have discussed in the previous lesson you must be remembering that we can have the boundary conditions either the static boundary conditions or the kinematic boundary conditions and that could be imposed we can evaluate the unknown constant.

Now let us look into what are the boundary conditions that we can impose for evaluating this unknown constant.

Now here (Refer Slide Time: 23:24) if you look into the deflection curve now this is the axis of the beam and this is the specked deflected profile of the beam. Now at this point at the load point; on the left hand side of the load as we have seen that one expression is given for the deflection, on the right hand side of the load there is another expression for evaluating the deflection.

Now at this particular point of the load where this is the limiting or the boundary for the two expressions to be employed now at this particular point now from the left hand side and from the right hand side the value should be equal. Or what I mean is the deflection of this beam at this particular point or the slope of the deflection curve at this particular point (Refer Slide Time: 24:11) from whichever side we compute either from this side or from this side this would be the same because the elastic curve has to be continuous; it is not discontinuous though; using two different functions for evaluating moment and correspondingly the deflection curve but physically the beam segment is not a discontinuous one so when it deflects the deflection curve has a continuity at that particular point or the boundary point between the two expressions which we are using.

So, at that particular point if you compute the deflection or the slope whichever expression we use from either side this would give us the same value. So we can use this particular condition that at x equal to a, slope on either side or at that particular point is equal whichever expression you use at x equal to a, the deflection at that point should be equal whichever expression you use and of course we have these two kinematic condition that at x equal to 0 y equal to 0 and at x equal to L y is equal to 0.

Now if we substitute that at x equal to a, slope is equal, then from the expression of dy dx......... in fact if you look into the expression of dy dx as we have obtained (Refer Slide Time: 25:30) as Pb by Lx square by 2 so x is equal to a so Pb by twice L a square and this particular expression since x is equal to a, this goes off so they also we have Pb by twice L x square.

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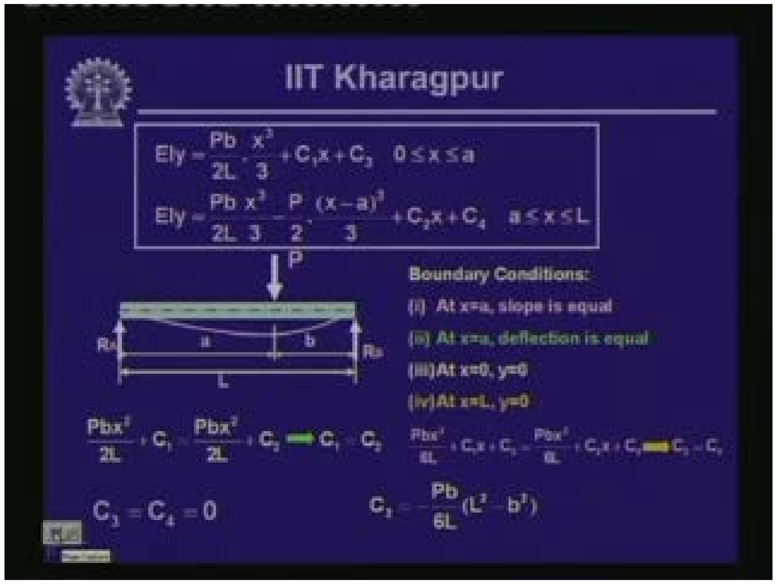

So if you substitute that then we get that Pbx square by twice L at C 1. Of course this is a, because we are substituting x equal to a, and so this x equal to a, this is a square so this gives us the value of constant. That means relationship between C 1 and C 2 we get C 1 is equal to C 2.

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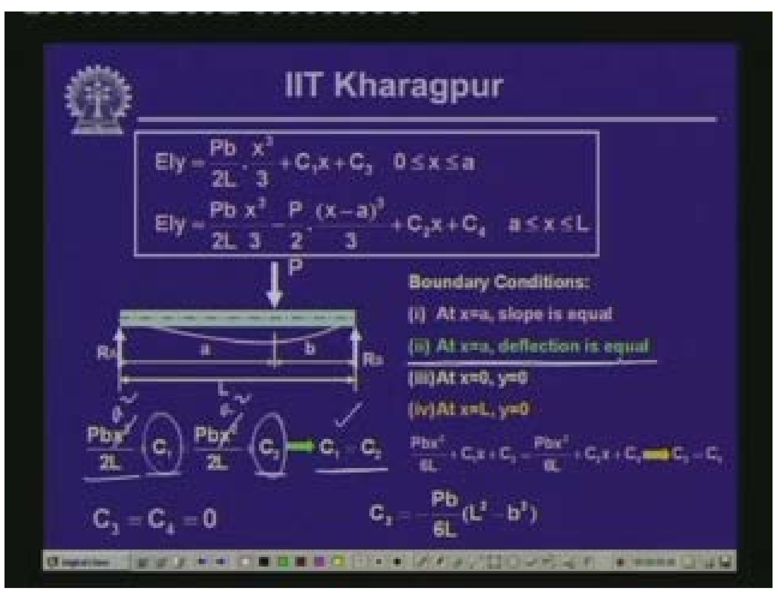

Now if we consider the second of the boundary condition that x equal to a deflection is equal; that means if we employ these two equations now, now whichever equation we use, at this particular point the deflections would be same from either side. So if we substitute x equal to a y R so this Pb a cube by 6L plus C 1 times a plus C 3 is equal to Pb a cube by 6L plus C 2 a plus C 4 now these two terms being same identical gets cancelled. Now C 3 is equal to C 4 and C 1 is equal to C 2 we have already seen earlier so this C 1 a and C 2a also gets cancelled so this gives us C 3 is equal to C 4.

Now if we substitute this particular boundary conditions that are x equal to 0, y equal to 0; at x equal to 0 y equal to 0 means we will have to use the first of the expression since it is valid between 0 to a and that gives us that C 3 equal to 0. Now since C 3 is equal to C 4 so C 3 and C 4 both are equal to 0. From this expression we get C 3 equal to 0 and since C 3 equal to C 4 so C 3 equal to C 4 equal to 0.

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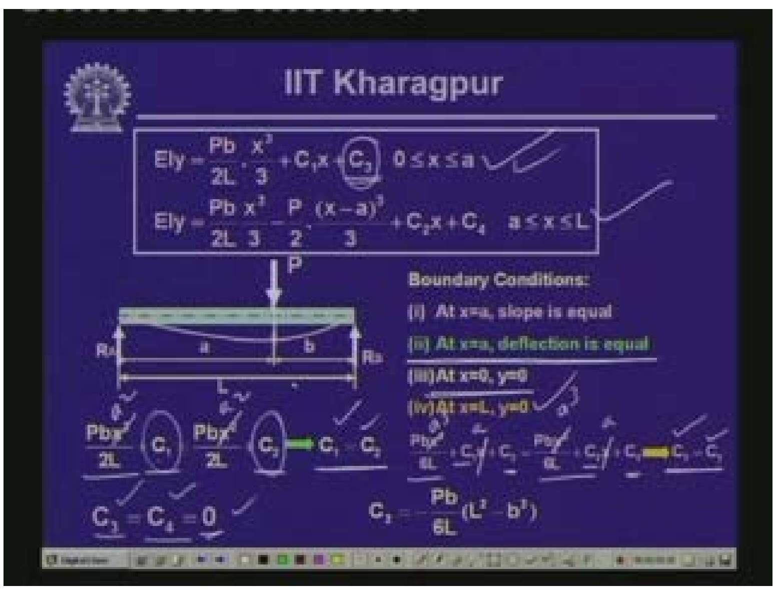

Now if we employ the last boundary condition which is at x equal to L, y is equal to 0 now again the validity of this particular boundary condition will be with respect to this particular expression because this expression of y is valid between a to L. So if we substitute the value of x as L in the second expression and since we have already seen that C 3 and C 4 equals to 0 so we are left with C 2 only and if we substitute the value of x as L we get the value of C 2 as equal to minus Pb by 6L times l square minus b square. This is the expression which we get once you substitute the value of x as L.

So all the unknown constants now are evaluated except C 2 and C 1; since C 1 is equal to C 2 so C 1 and C 2 will be existing and C 3 C 4 are 0. Now once you substitute this value of C 2 then the expression for the deflection curve comes like this.

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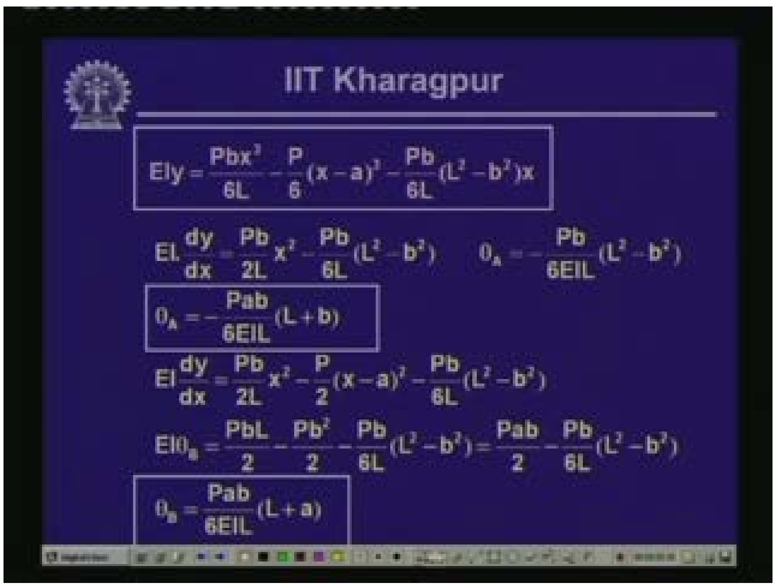

It is that EIy is equal to Pbx cube by 6L minus p by 6(x minus a) cube minus Pb by 6L (L square minus b square) x. This is the value of C 2 this is for the second part and for the other part we substitute the value of C 1 and we get the expression for EIy.

So we can now get the expression for the deflection curve, so y as a function of x and at any point now from these two sets of the deflection curves which we have or the expression equation of the deflection curve as we have, from 0 to a and a to L we make use of these two expressions and we compute the value of y at any point in the beam.

Now if we like to find out the slopes at the two ends at end a and end b now theta a from the first half of the expression we can compute and we get the values of dy dx as Pb by 6 EIL (L square minus b square) noting that L minus a is equal to b and you know if we substitute we can get the value of theta a and theta b accordingly if we substitute or in the expression of dy dx as we have obtained earlier if we compute if we substitute for x equal to L we get the value of theta at b and we get Pab by 6EIL (L plus a).

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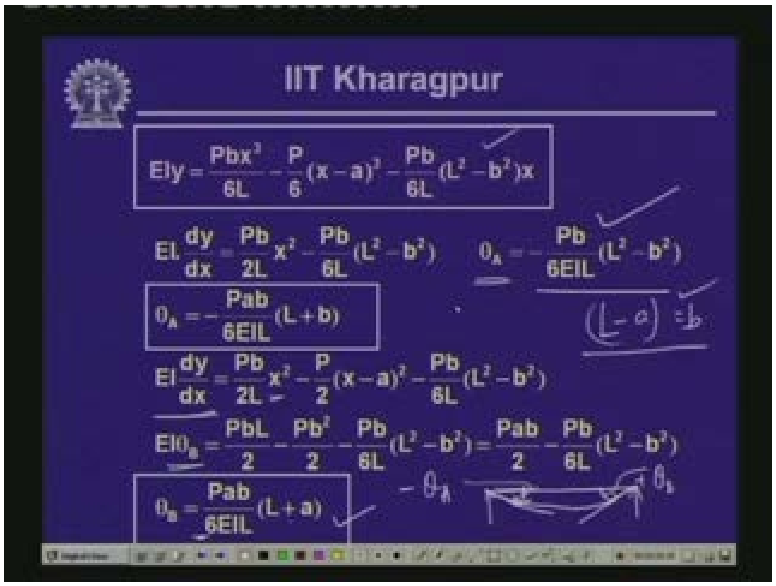

Now here, if you look into the sign of this, this theta b is positive whereas theta a is negative and as we have seen that we have earlier noticed it that if you have the deflection curve and if you take the tangent at this particular point this is moving in a clockwise fashion with respect to the original axis of the beam and because it is clockwise, according to our sign convention this is minus theta a and if we take tangent over here (Refer Slide Time: 30:56) this moves in an anticlockwise direction and according to our sign convention this is plus theta at b and the values of these are given by this particular expression.

And as we have seen, that the loading is placed at a distance of a from the left support or b from the right support so it is not symmetrically loaded. Now, as in the previous case or the first case where we have taken that a beam is loaded over the entire length of the beam with uniformly distributed load where we said that the beam is symmetrically loaded and hence we had obtained the maximum value of the deflection at the center where the tangent to the deflection curve is horizontal.

Now at this particular for this particular problem since the loading is not symmetrical hence we need to find out the position of the maximum bending moment. Now for evaluating I mean sorry to find out the position for maximum deflection in the beam; now for finding out the position for the maximum deflection we need the criteria which we adopt is that the tangent which we draw to the elastic curve must be horizontal.

Or in other words, the value of dy dx at that particular point should be equal to 0. So if we adopt the value of dy dx equal to 0 you can compute the value of x where the tangent is horizontal and once you substitute the value of x in the expression e for y, you can get the value of deflection curve which is y the deflection at that particular point.

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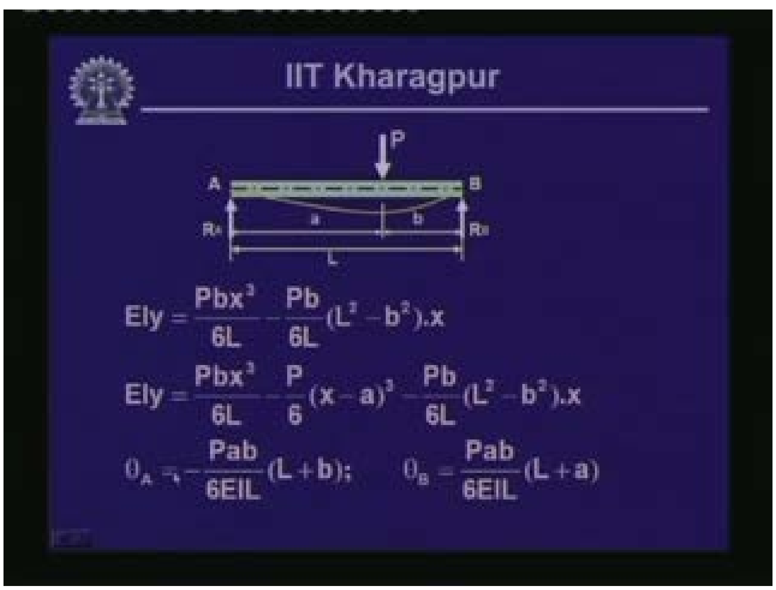

Now let us look into this particular aspect of this beam as we have evaluated that the deflection for the segment 1 is: EIy is equal to Pbx cube by 6L minus Pb by 6L into (L square minus b square) into x and for the segment 2 we have: EIy is equal to Pbx cube by 6L minus P by 6 into(x minus a) cube minus Pb by 6L into (L square minus b square) into x and consequently the values of the rotations at end a is theta a and at end b theta b and as we have seen that theta a is negative and theta b is positive as I explained already to you.

Now let us look into the case of a particular situation when this particular load which is located now at a distance of a, if we try to place this at the center of position means that the values of a and b if they are equal and is equal to L by 2 so if we substitute the value of a and b as L by 2 we get the situation where the beam is subjected to a load a concentrated load which is acting at the center of the beam and thereby the distances from the two ends is equal to L by 2. Hence that gives us a particular situation of this generalized loading situation.

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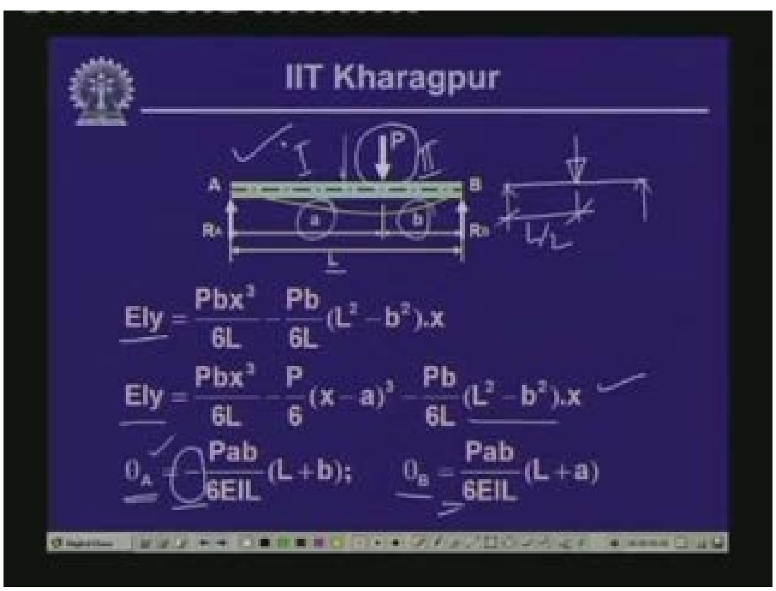

Now, if we substitute the values of b and a as L by 2 then we get the expressions like this that when the load P is acting at the center of the beam thereby a is equal to L by 2 and b is equal to L by 2.

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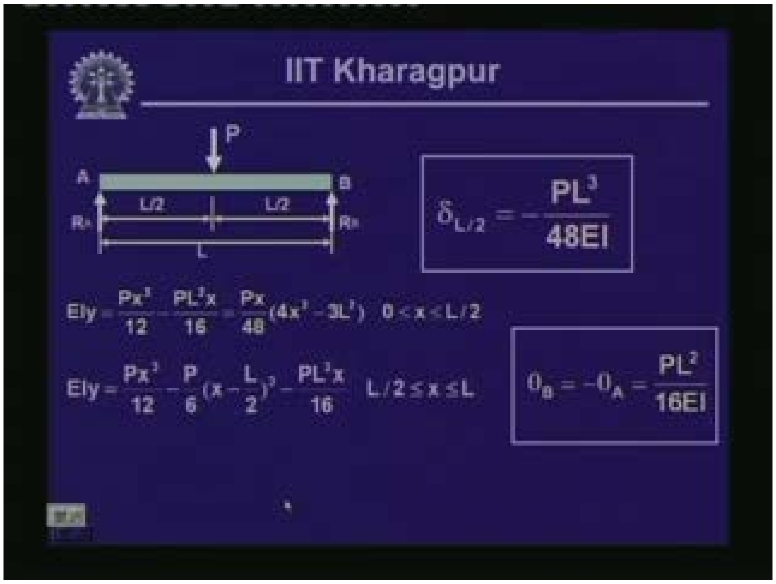

Hence the value of EIy which was general so long with the values of a and b now becomes a function of L alone and of course with the value of x around the length of the beam and value of EIy is equal to Px cube by 12 minus PL square x by 16 which if we take Px by 48 out it becomes 4x square minus 3L square.

Now as you can see here that this particular expression is little different than what we have evaluated in case of the general expression where P was located at a distance of a from the left spot. Now this particular expression is valid between 0 to L by 2. When the value of x lies between 0 to L by 2 this particular expression is used. Now consequently if we like to evaluate the deflection at any point between L by 2 to L then the second half of the expression for the general expression will be used where again for a and b we use a value of L by 2 and consequently we see that EIy is equal to Px cube by 12 minus P by 6 into x minus L by 2 cube minus PL square by 16 and the validity is between L by 2 to L.

Now fortunately the beam is being symmetrical, so if we compute on one side we can get the values for the other as well as we have seen earlier and for the deflection curve as it is known that at this point this will be, the tangent to the deflection curve will be horizontal and this will give us the maximum value of the deflection.

And consequently, if we substitute the value of x as L by 2 then we get the value of the deflection maximum deflection of the beam at center which is equal to PL cube 48 EI. And if you look into the sign of this it is negative which indicates that the beam deflects towards the opposite direction of the positive y. Hence the negative sign comes in. this is the maximum value of the deflection which is occurring at the center. And consequently, if we substitute the value of x, I mean, if we substitute the value of a and b as L by 2 in the expression for theta a and theta b we will get the values as PL square 16 EI and minus for theta a and plus for theta b as we have seen earlier that this is minus theta a and this is plus theta b (Refer Slide Time: 37:02) and the magnitude of this will be PL square by 16 EI.

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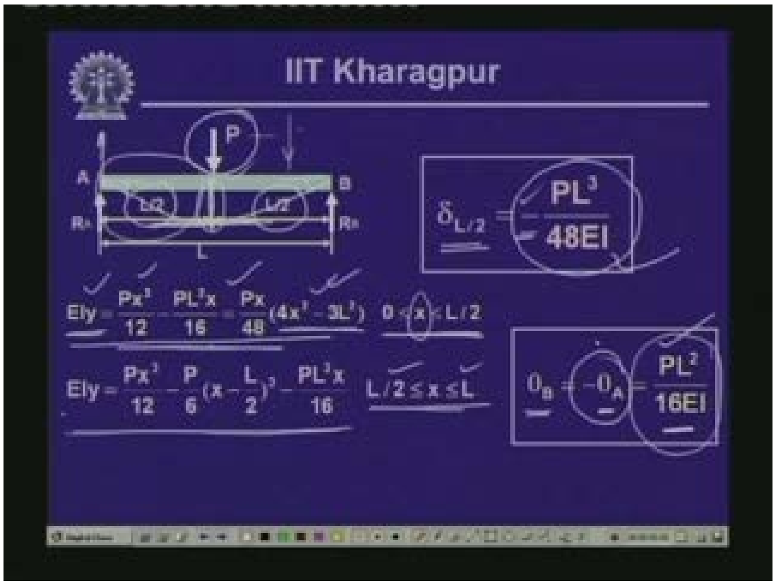

So, when we talk about a particular case then delta maximum is PL cube by 48 EI and theta a equal to theta b magnitude y that is PL square by 16 El.

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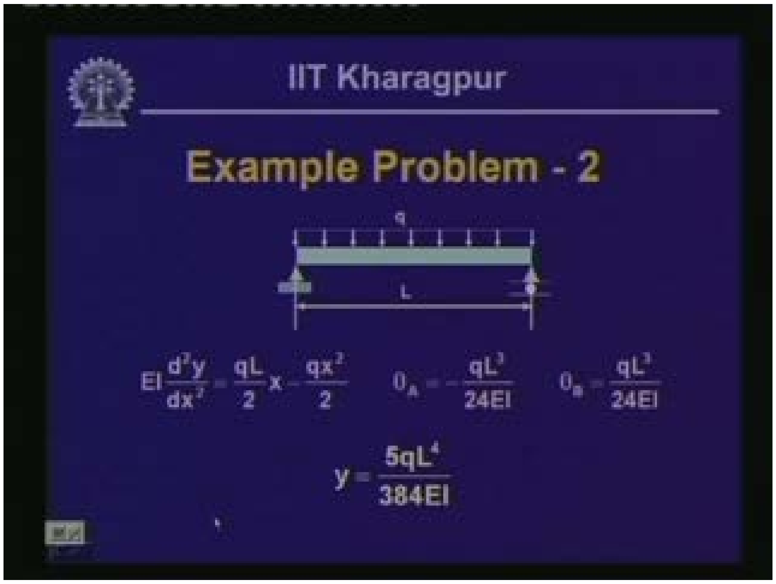

Now let us look into another example wherein we have........... in fact this particular example we have solved last time wherein we say that the loading is uniformly distributed over the entire span of the beam. And if you remember that we have started with the differential equation of the elastic curve which is EI d 2 y dx 2 is equal to the moment. And moment we had completed at any cross section which is at a distance of x from the left support and consequently we have evaluated the values of the deflection which is the...

This is the expression for differential equation and then subsequently we came up with that the value of y at x equal to L by 2 is this and correspondingly the value of theta a and theta b is this. Now, also you have seen that the derived equation of this governing equation is EI d 4 y dx 4 is equal to minus q the loading. Now we can start from this particular point as well and we can arrive at the value of y and theta a and theta b.

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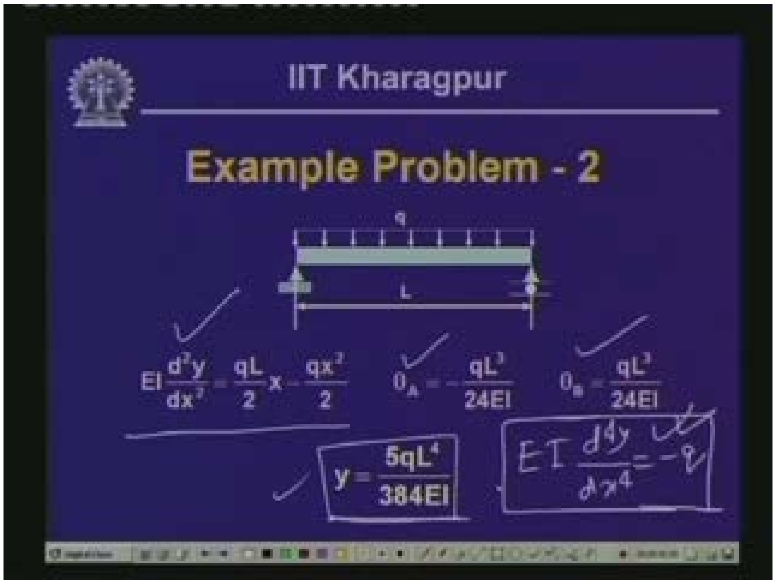

Now let us substitute this or let us start from this particular expression of the differential equation and see what we get.

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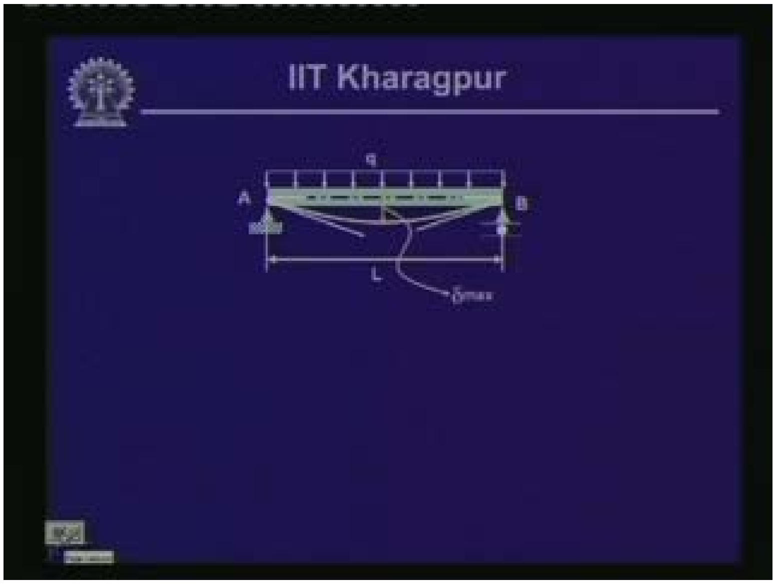

Now we have EI d 4 y dx 4 this is equal to minus q. Now if we integrate this we have EI d 3 y dx 3 this is equal to minus q x plus the constant C 1. Now if we integrate it further we have EI d 2 y dx 2 is equal to minus qx square by 2 plus C 1 x plus C 2. Now if we apply the boundary condition here itself; now since we know that C 1 and C 2 are the two unknown constants and EI d 2 y dx 2 represents the value of the bending moment, now for a simply supported beam the bending moment at supports for both hinge support and roller support is zero so bending moment at x equal to 0 at x equal to 0 d 2 y dx 2 is 0 and also at x equal to L d 2 y dx 2 equal to 0.

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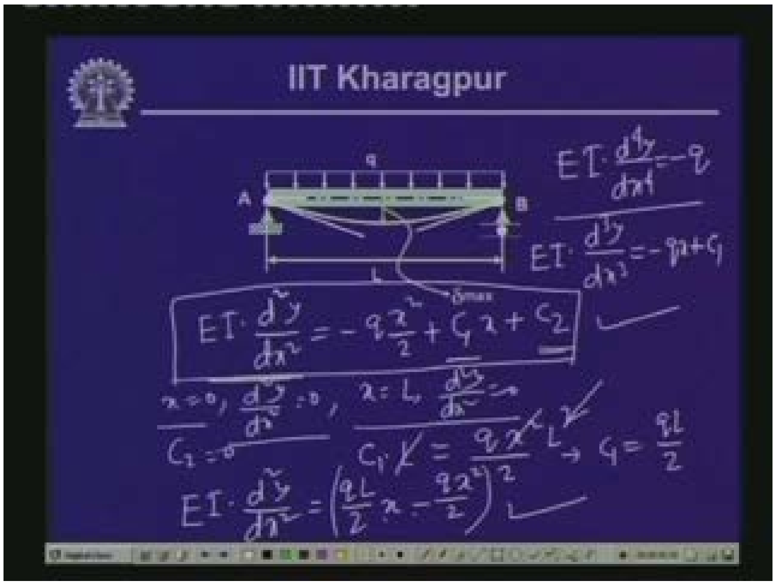

Now if we substitute that x equal to 0 d 2 y dx equal to 0 this gives us that C 2 is equal to 0. now if we substitute this that at x equal to L d 2 y dx 2 equal to 0 we get C 1 into L is equal to qx square by 2 and x is L so this is L square by 2 so L square gets cancelled so this gives us that C 1 is equal to qL by 2. Hence the expression here that we get EI d 2 y dx 2 this is equal to qL by 2 x minus qx square by 2.

Now this is the expression of the bending moment or the equation which we had started with in the previous case. In fact from this particular point onwards again if we follow the same steps as we have done earlier for evaluating the deflection curve equation of the deflection curve then it will follow the identical situation. so whether we can consider EI d 2 y dx 2 equal to the bending moment or we can start from even EI d 4 y dx 4 equal to loading, finally the solution which you will get or the expression for the elastic curve which you will get and consequently the values of the maximum deflections and the rotations at different points we compute they will be identical.

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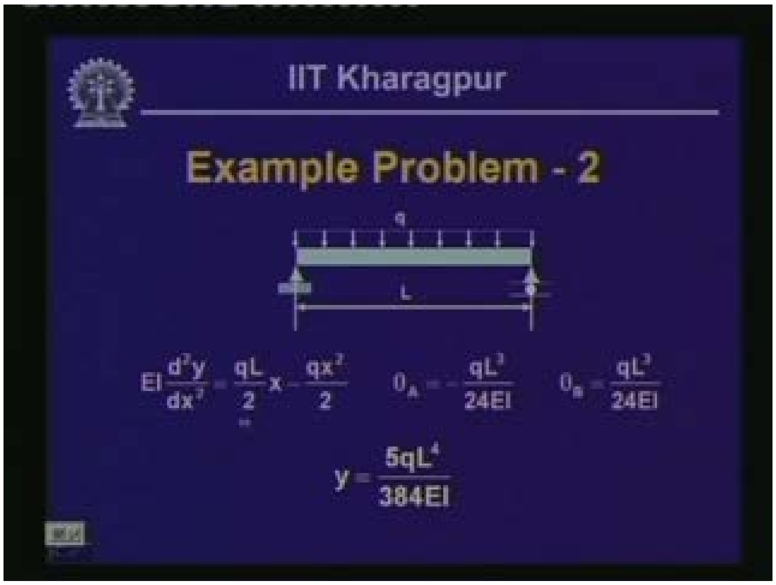

This is the expression which we had used. In fact if you look into that qL by 2 x minus qx square by 2 for the bending moment based on which we had obtained the values of deflection and the rotation.

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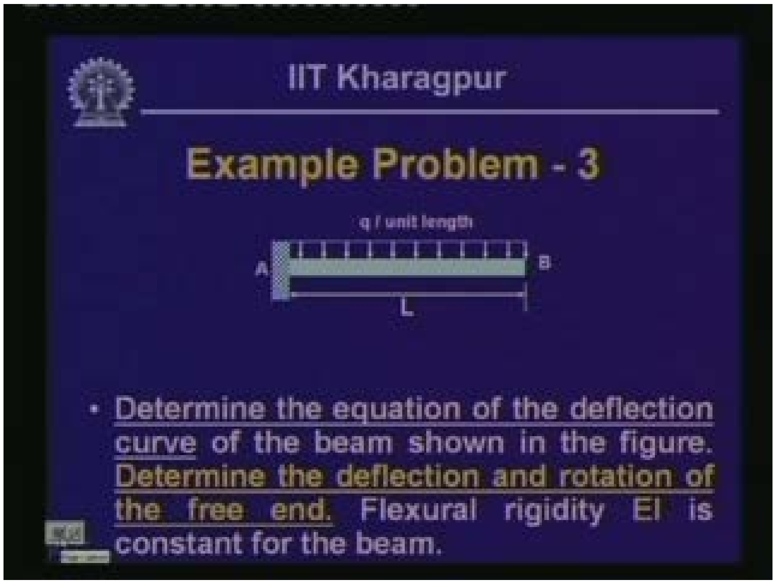

So we can adopt either EI d 2 y d x 2 equal to moment or EI d 4 y dx 4 equal to the minus of the loading. now let us look into another example where this is a beam; now this is a different type of beam, the beam is fixed in a and is free at the end b and thereby as we know that this is a cantilever beam and this beam is subjected to uniformly distributed load of q bar unit length.

Now what you will have to do is that you will have to determine the equation of the deflection curve of this beam and you will have to find out the deflection and rotation of the free end the end b. Now flexural rigidity EI of this particular beam is constant. So right through the EI value is the same.

Now for evaluating the deflection curve arriving at the expression for the deflection curve what we need to do as the first step is as we have done before we need to evaluate the reactive forces. now at this fixed end you will have the vertical force let us call this as R A, (Refer Slide Time: 43:21) we will have the horizontal reactive force let us call this as H A, we will have the bending moment which is equal to M.

Now if we take the horizontal and vertical and the moment equilibrium, summation of horizontal forces equal to 0 will give us h a equal to 0, summation of vertical forces equal to 0 will give that R A which is acting vertically upwards minus q times L this is equal to 0 so that gives us R A equals to qL and it will take the moment of all the forces with respect to this particular point; we have the moment value M let us call M dash and qL also will have the moment in the same direction so plus q into L into L by 2 so qL square by 2 this is equal to 0 so this gives us M dash is equal to minus qL square by 2. So you will have the value of R A as qL and M dash as equal to qL square by 2.

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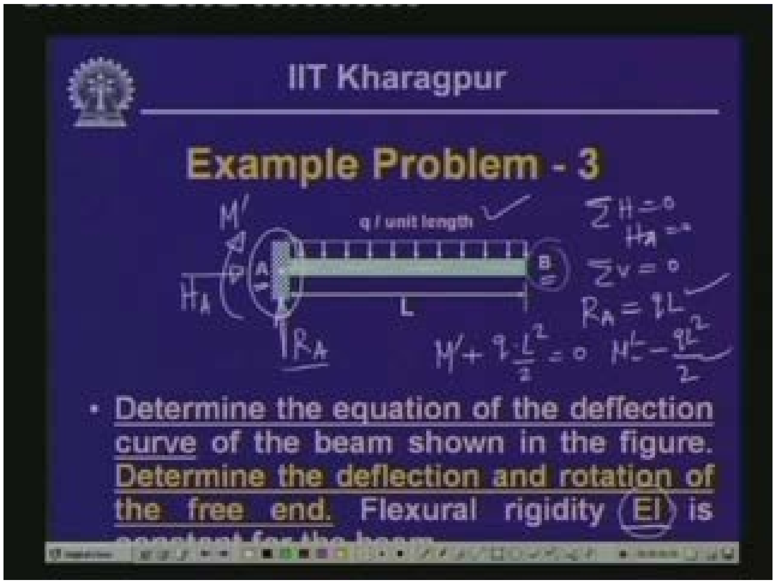

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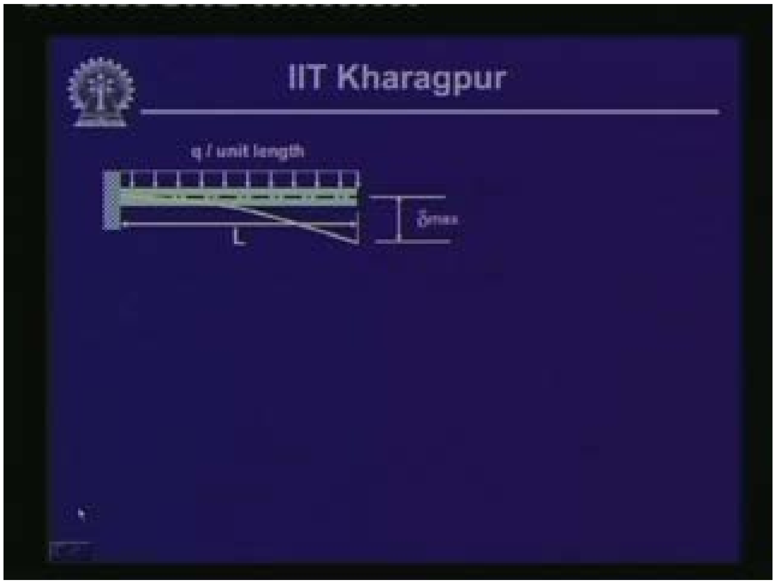

Now we follow the same steps as we have done earlier. that means now we have; for this particular beam the reactive value is known which is R A is equal to qL and we have the moment which is equal to that M dash is equal to minus qL square by 2 and R A is equal to qL.

Now if I take a section here and if I take a free body of that particular part so here you have the reactive force, you have the moment and you have the uniformly distributed load (Refer Slide Time: 45:08) this is at the distance of x and on this cross section you have the equilibrating forces the shear force V and the bending moment M.

So, if we take the moment of all the forces with respect to this particular point at the cross section then we have M which is in an anticlockwise direction then we have moment at this supporting dash which is in a clockwise direction so M minus M dash, then we have moment due to R A which is again in a clockwise direction so minus R A into x and the loading again which is in the anticlockwise direction so minus q into x times x by 2 which is q x square by 2 this is equal to 0. so this gives us that value of the moment M is equal to M dash which is minus qL square by 2 plus R A into x so qL into x plus q for this minus....... this would be plus (Refer Slide Time: 46:26) because qx square by 2 has a moment in the same direction the clockwise direction so this is going to be equal to minus so minus qx square by 2 this is equal to 0 sorry this is this is the moment expression this is equal to the moment.

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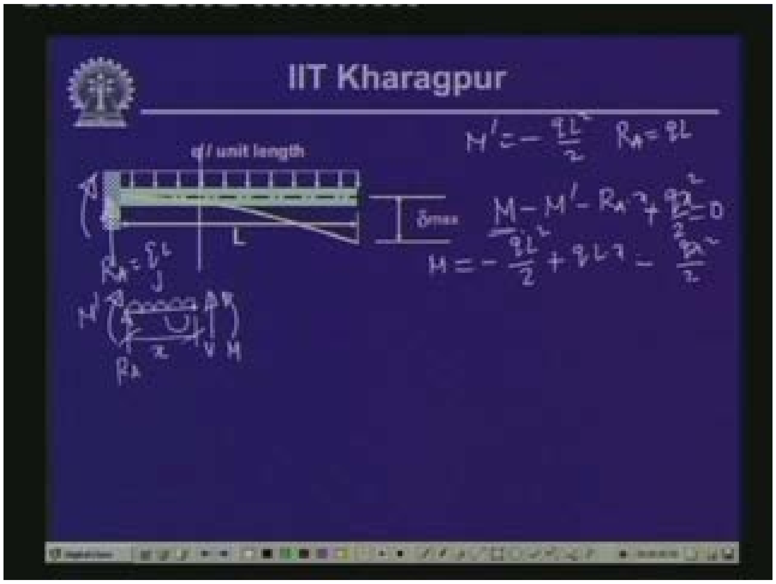

Now if I substitute the value of the moment in the differential equation so we have EI d 2 y dx 2 is equal to moment which is qL x minus qL square by 2 minus qx square by 2. Now if I integrate this we have EI dy dx this is equal to qL into x square by 2 minus qL square by 2 x minus q by 2 into x cube by 3 plus you will have the constant C 1.

Now, on further integration we will have EI y this is equal to qL by 2 and x square will give you x cube by 3 minus qL square by 2into x square by 2 minus q by 6 x cube will give us x 4 by 4 plus C 1 x plus C 2.

Now the boundary conditions what we have now C 1 and C 2 are the two unknown constants which are to be evaluated from the two boundary conditions and the boundary conditions here are at x equal to 0, y equal to 0 and also at x equal to 0 dy dx equal to 0.

Now if we substitute this that at x equal to 0 dy dx equal to 0 that gives us that C 1 equal to 0 and at x equal to 0 if we substitute y equal to 0 then we get that C 2 also is equal to 0. So we get that both C 1 and C 2 are 0 and hence the expression for the deflection curve y is written in this form that EI y is equal to qL by 6 x cube minus qL square by 4 x square minus q by 24 x to the power 4.

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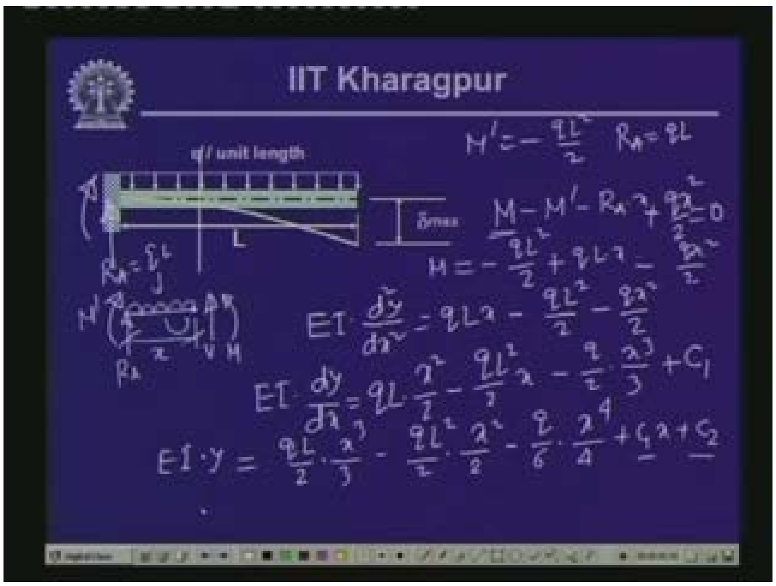

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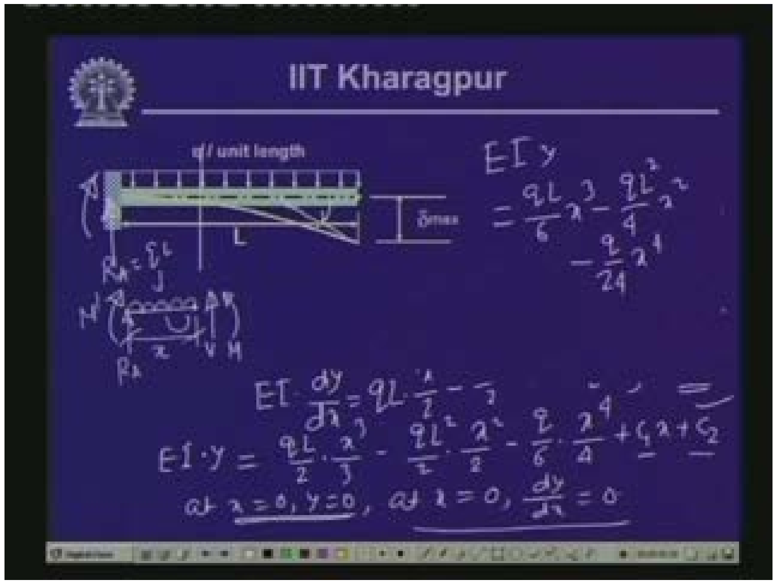

Now what we need to do is that we need to compute the value of the deflection at this free end and also the slope of this elastic curve at this free end. That means if we take the tangent of the elastic curve at this particular point then what is the rotation of this particular point of the elastic curve. So, if we substitute the value of x as L then we get the value of y from here so this gives us that EI y is equal to qL by 6 into this x is L so L cube minus qL square by 4 times L square minus qL 4 by 24. So if we combine this together then we have this as equal to qL 4, (Refer Slide Time: 50:40) this we have 24 then 4 minus 6 minus 1. So 6 minus 1 is 7 so 4 minus 7 is minus 3. So this is equal to minus qL 4 by 8 which will give you the value of EI y.

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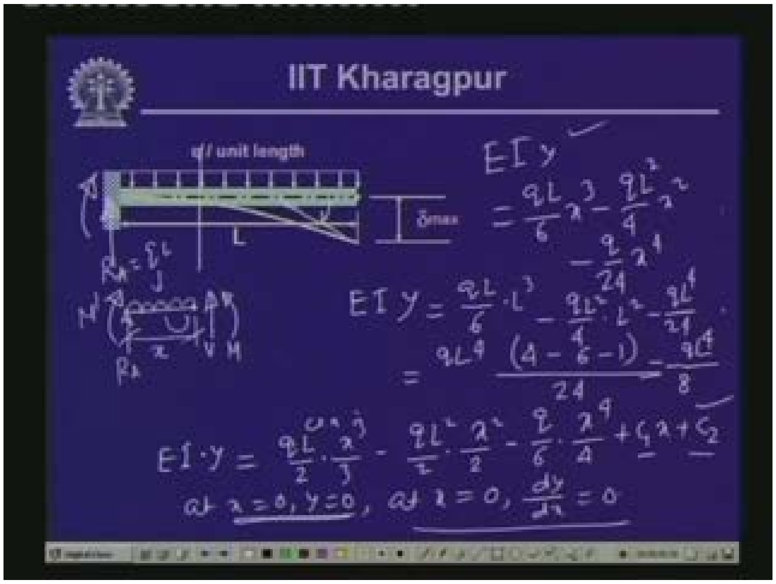

Now let us remove this part (Refer Slide Time: 51:04) so we get the value of y is equal to minus qL 4 by 8EI. Now again this minus sign indicates that it is in the opposite direction of the positive y so it is deflecting downward and the magnitude of the deflection at the end if you know the value of intensity q and the final length L then qL 4 by 8EI is the value of the deflection at the tip point or at the free end.

Now we can compute the value of dy dx. In fact we could substitute in the expression for dy dx the value of x as L or in fact we can take the derivative of this expression y from which we can compute the value of dy dx or the slope at the free end. So EI dy dx is nothing but equal to qL by 6 into 3 x square minus qL square by 4 twice x minus q by 24 4x cube. Now in this if we substitute x equal to L this gives us qL cube by 2 minus qL cube by 2 minus this is qL cube by 6 so this is what so qL cube by 2 qL cube by 2 this gets cancelled so we have minus qL cube by 6 is the............ and again as you can see that it is minus it indicates that the rotation is clockwise and rightly as we have seen since the deflected set is in the negative direction so at this point if you draw the tangent this gives us a clockwise rotation which is minus qL cube by 6.

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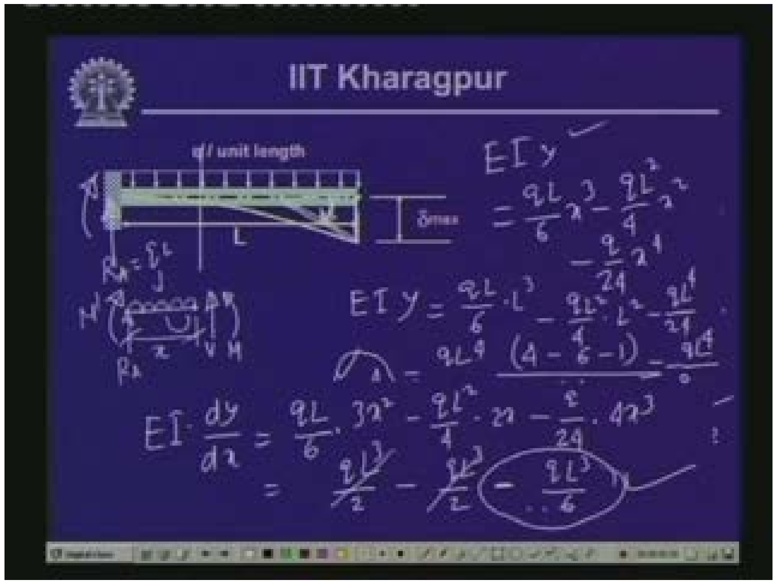

So dy dx at x equal to L is minus qL cube by 6EI dy dx is equal to minus qL cube by 6EI. This is the value of the slope at the free end and the deflection at the free end is equal to qL 4 by 8EI.

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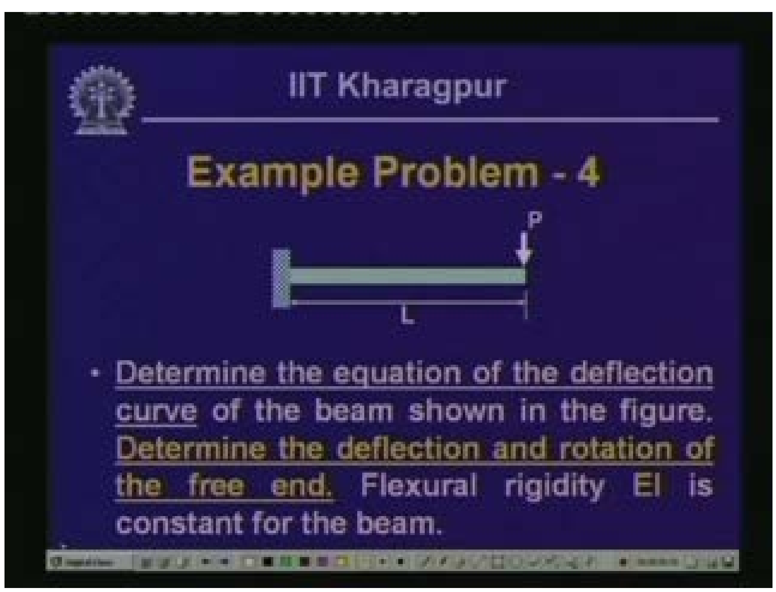

Well, we have another problem wherein the load is acting at the free end which is a concentrated load instead of the uniformly distributed load and exactly in the same way we compute again the reactive forces the value. So if we call this beam A and B we have the reactive force R A, we have the horizontal force H A and we have the moment M A. Now here again if you take the horizontal equilibrium we get H A is equal to 0, if we take the vertical equilibrium we get R A is equal to P and if we take the moment then we get that M A is equal to minus P into L. So these are the values of the reactive forces.

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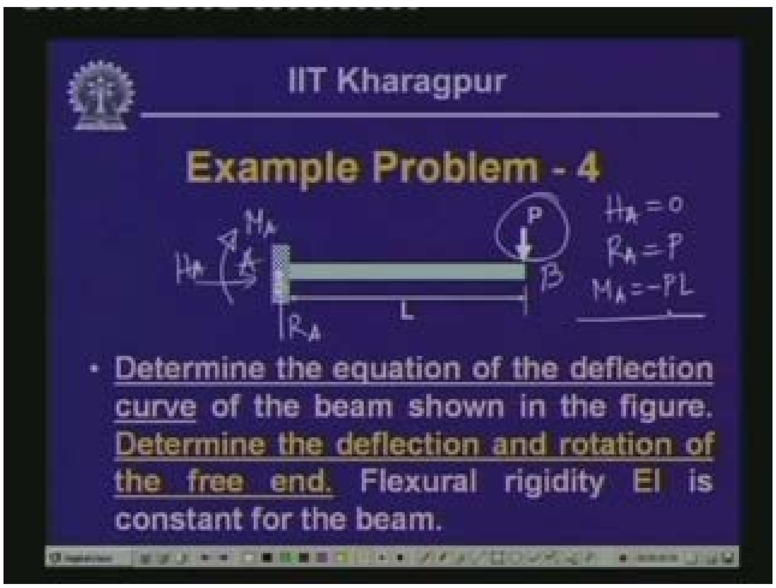

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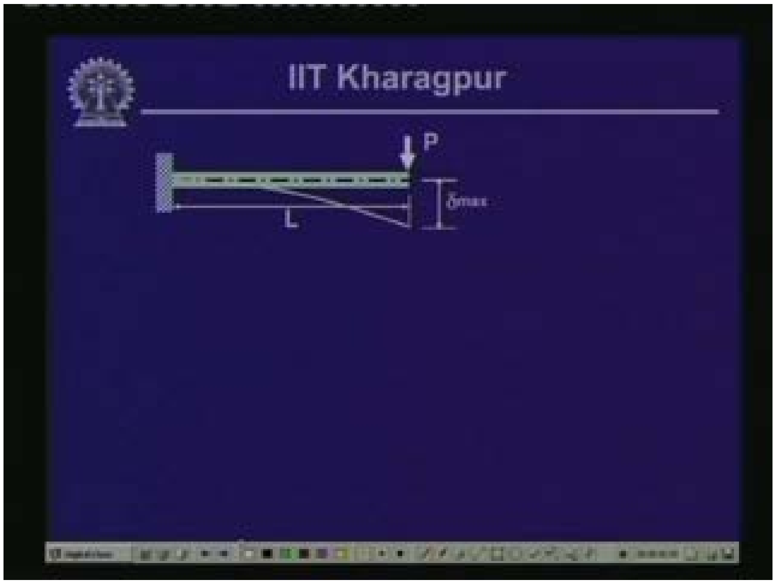

And if you take the moment at any cross section for evaluating the deflection curve; we have here R A is equal to P, the moment M is equal to minus P into L and R A is equal to P. Hence if we take a cut over here and draw the free body diagram then we have the reactive force this as R A, the moment here as M A and here we have to shear force and the moment M. So if we take the moment of all the forces here we have M, this is M A is minus, minus R A into x this is equal to 0 and hence M is equal to R A into x which is P into x plus M A which is minus PL.

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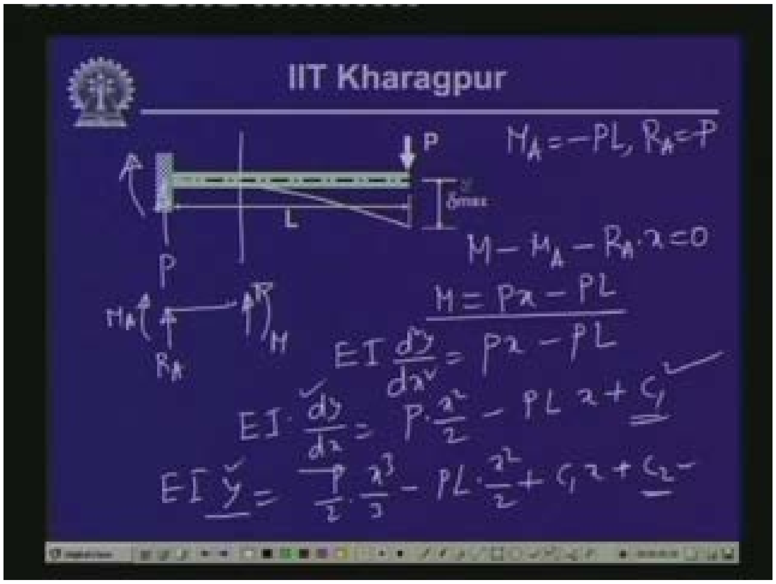

Now if we substitute this that means if we have EI d 2 y dx 2 this is equal to Px minus PL then we get EI dy dx this is equal to Px square by 2 minus PL x plus C 1 and EI y is equal to P by 2 into x cube by 3 minus PL x square by 2 plus C 1 x plus C 2 and C 1 C 2 can be evaluated from the boundary conditions again. At x equal to 0 dy dx 0, at x equal to 0 y also is 0, so at x equal to 0 dy dx 0 will give C 1 equal to 0 and at x equal to 0 y0 gives us C 2 equal to 0. Hence we have that EI y is equal to............ we get the values that EI y is equal to Px cube by 6 and minus Pl x square by 2. And at x equal to L if we substitute the value we will get the value of y so EI y is equal to PL cube by 6 minus PL cube by 2. This gives us that minus PL cube by 3. So PL cube by 3EI was the displacement and we can compute the value of dy dx; and by substituting the value of L we get the value of the rotation at the end.

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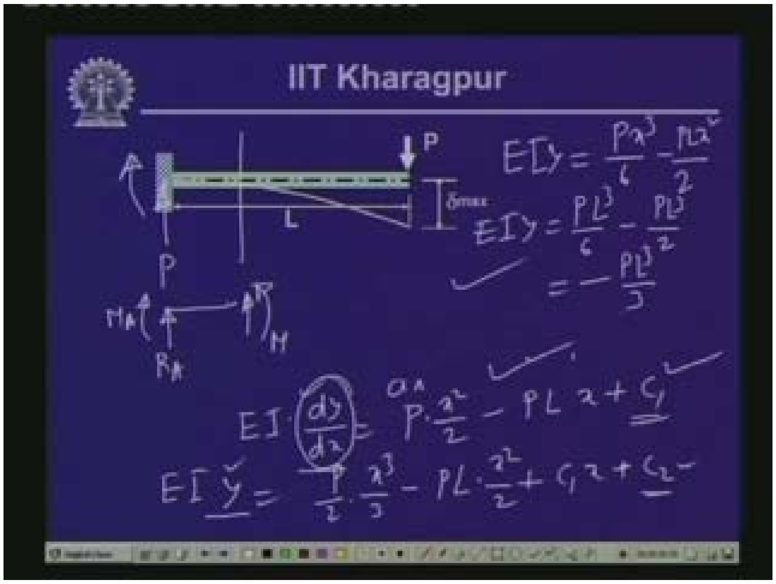

This is how we compute the value of the deflection and the rotation.

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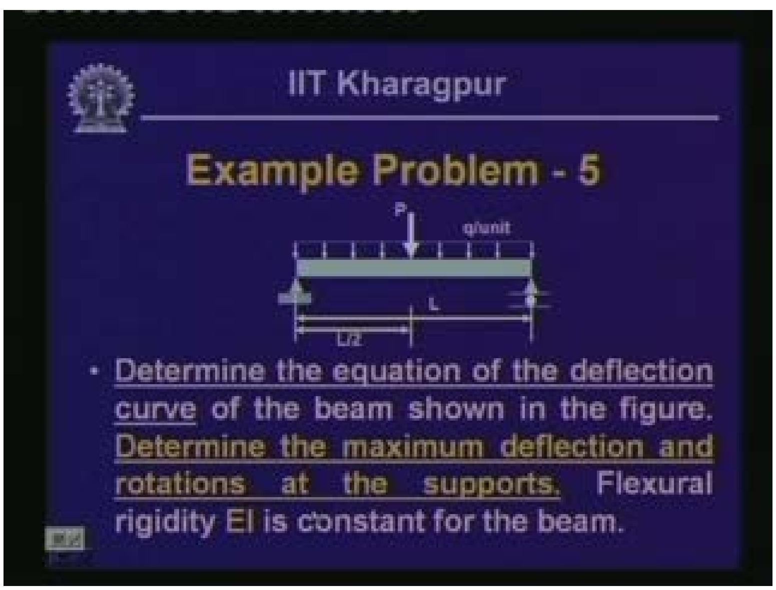

And this is an example which is given for you. In fact as we have discussed today that how to employ the method of superposition, we can employ the method of superposition in this particular case and you can compute the value of the deflection at any point. We will discuss this problem next time.

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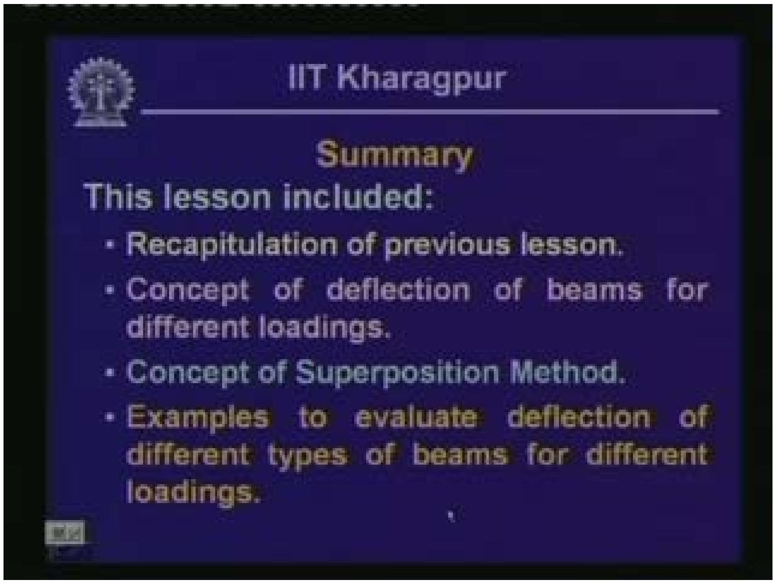

Now to summarize them, in this particular lesson we have included the following that we have recapitulated the previous lesson, we have introduce the concept of deflection of beams for different loadings and then the concept of superposition method we have discussed and some examples to evaluated deflection of different types of beams for different loading.

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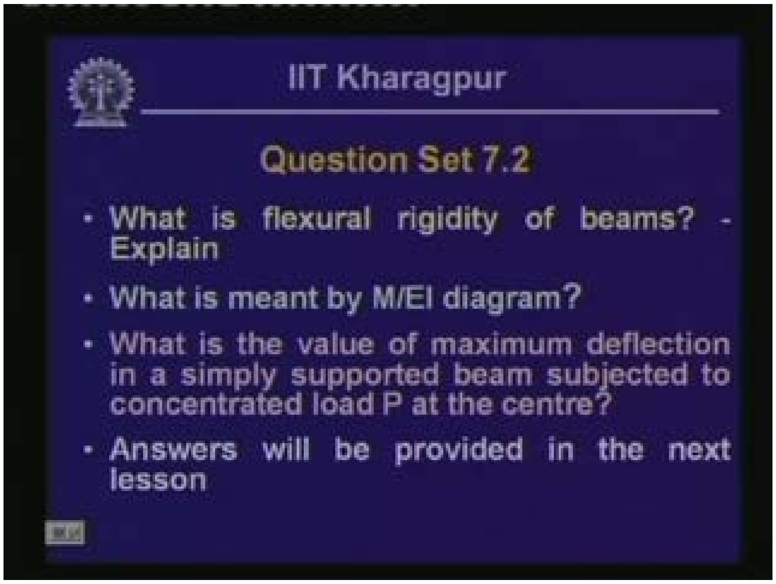

And these are the question set for you that what is flexural rigidity of beams please and what is meant by M by EI diagram and what is the value of maximum deflection in a simply supported beam subjected to concentrated load P at the center. We will discuss answers of this in the next lesson, thank you.

Preview of next lecture

Strength of Materials

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Dept of Civil Engineering

I.I.T Kharagpur

Lecture - 32

Deflection of Beams - 3

Welcome to the third lesson of the seventh module which is the deflection of beams part III. In fact on the last two lessons on the deflection of beams we have discussed how the deflection of beam affects in general and why we need to evaluate the deflection in beams and subsequently the slopes in the beams. Now, in this particular lesson we are going to look into some more aspects of..........