Strength of Materials
Prof. S.K.Bhattacharya
Dept. of Civil Engineering,
I.I.T., Kharagpur
Lecture No. 30
Deflection of Beams-I

Welcome to the first lesson of the seventh module which is on Deflection of Beams part 1. In the last two modules we have looked into several aspects of a beam bending, how to evaluate the bending moment and shear force in a beam which is loaded. Consequently we have evaluated the bending and the shear stresses in a beam. We have taken the curvature of the beam into effect and consequently we have derived the bending formulae.

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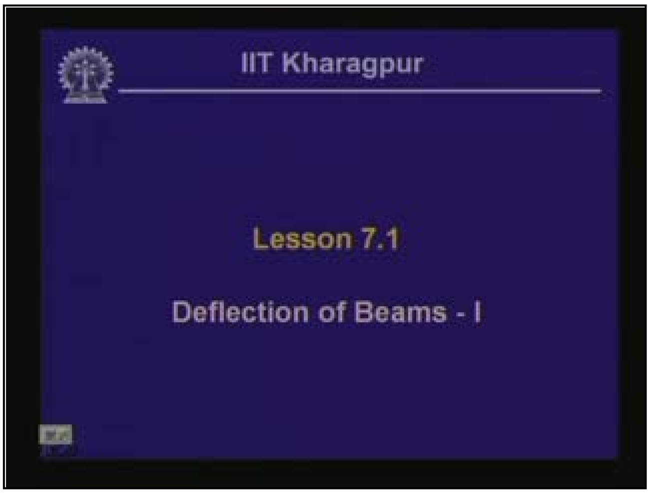

While looking into the bending formulae, we have taken the help of the curvature of the beam but we have not derived any equation for the curvature of the beam or for the elastic curve. In this particular lesson, we are going to look into the differential equation and consequently the equation of elastic curve. How do you find the deformation in a beam? In fact the deformation which we often call as a deflection of beam is of paramount importance in our engineering application.

When the beam members are subjected to load they undergo deformation which we are terming as deflection when the access of the beam moves in the vertical direction. This kind of moment of the beam creates problems in our engineering structures say for example the floor on which we are moving. Basically they are supported on beams, if for some loading the beam undergoes moment in the vertical direction.

If it depletes then the floor also deflects and as a result it becomes unusable. That means it becomes uncomfortable for the persons to use it or whoever will be moving over the floor. If you talk about any mechanical component, if it undergoes deformation then there is a possibility that it will lose its alignment and in the consequence there could be failure of the machine parts. Hence it is very important to evaluate the deflection in a beam member.

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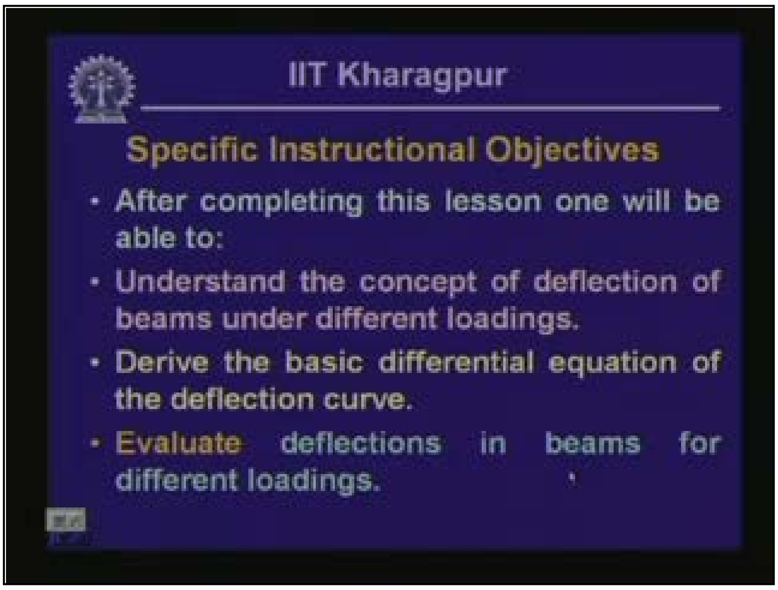

Once this particular lesson is completed one should be able to understand the concept of deflection of beams under different loading conditions; one should be able to derive the basic differential equation of the deflection curve and one should be in a position to evaluate deflections in beams for different loading conditions. Hence the scope of this particular lesson includes recapitulation of what we did in the previous lesson.

It includes the concept of deflection of beams for different loading; it includes the derivation of differential equation and consequently the equation of elastic curve for beams. It includes some examples for evaluation of deflection in beams for different loadings. Let us look into the answers of the questions, which were posed last time. In the last lesson we discussed aspects of shearing stresses and hence we have the questions pertaining to that aspect and the questions are related to that.

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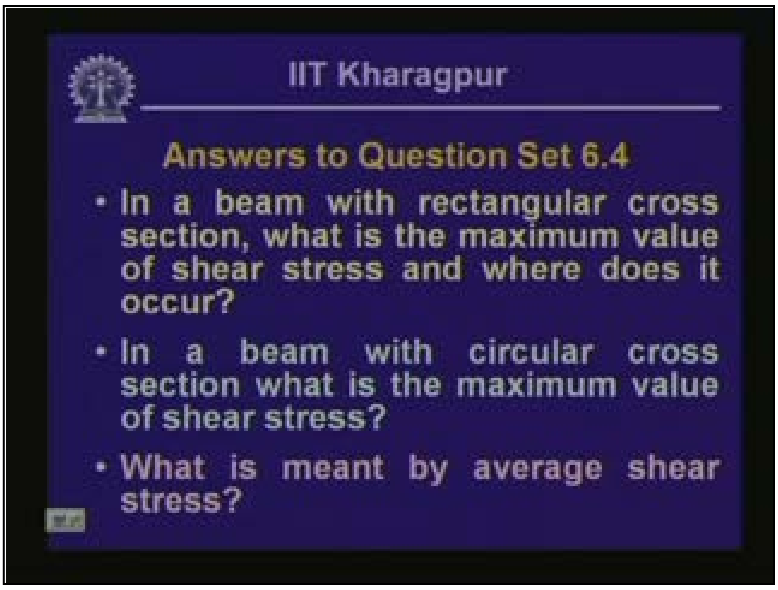

The first question is in a beam with a rectangular cross section what is the maximum value of shear stress and where does it occur? A beam having a rectangular cross section of width b and height h, when subjected to loads and thereby shear forces, are subjected to the shearing stress and the distribution of the shear stress. The parabolic distribution having maximum shear stress at the neutral axis, which we call as Tao_{max} and the shearing stresses at the outer surface is 0 and it varies in a parabolic manner.

The maximum shear stress is given by this expression which is 3/2 'V/A' and now we calculate yf. We remember that Tao = Vq/Ib where V is the shear force in the section where we are calculating the stress; q is the first moment of the area where we evaluate the stress. If you are evaluating the stress at this particular section which is at a distance of y_1, then q is the moment of this particular text area with respect to the neutral axis; 'I' is the moment of inertia of the cross section with respect to the neutral axis.

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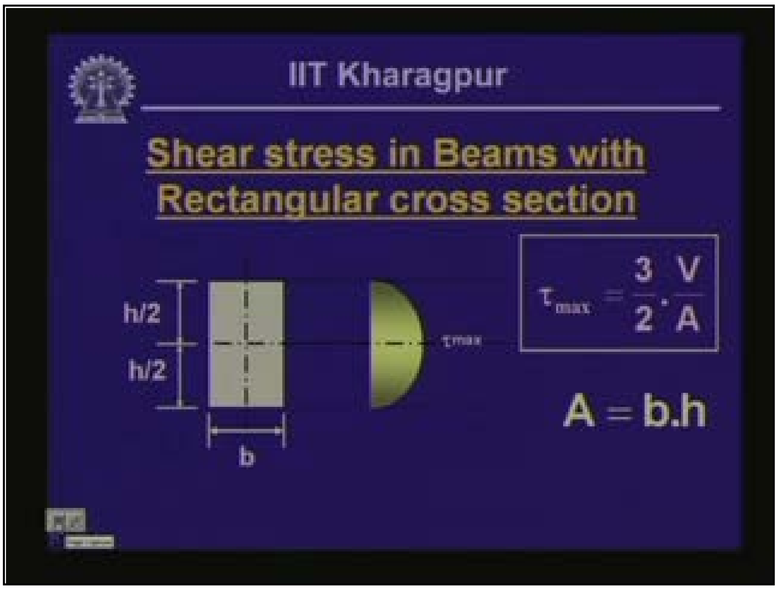

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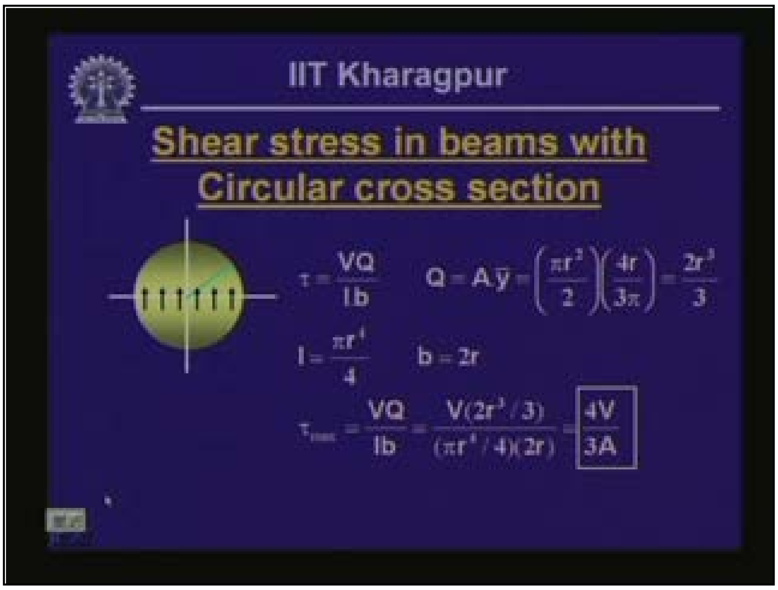

Here b is the width of the section and this gives the maximum value when $y_1 = 0$, which is at the neutral axis and thus we have the value of the Tao_{max}. Thereby if you compute it, it becomes 3/2 V/A where 'A' is the cross sectional area. So, the maximum shear stress occurs at the neutral axis and for the rectangular section the maximum value is 3/2 V/A.

The next question posed was that in a beam with a circular cross section, what is the maximum value of shear stress? Now this aspect also we have discussed in the last lesson wherein we have said that we cannot use Tao = VQ/Ib for the inter cross section of the circular cross section of the member.

However, we can apply at the neutral axis wherein the stress distribution is parallel to the y-axis and thereby we can apply this expression VQ/Ib. Consequently, if you compute the maximum shear stress, which we get at the neutral axis it is = 4/3 V/A. This we compute from VQ /Ib and Q is the moment of the area about the section, where we are considering the shear stress, which is a neutral axis in this particular case. The maximum value of the shear stress for the circular cross section is 4/3 V/A, while again A is the cross sectional area of the circular section.

Lastly, we had the question; what is meant by average shear stress? Over the cross section, when we are dealing with a rectangular cross section we have seen that the distribution is a parabolic one and I mean the maximum value of the shear stress is at the neutral axis. Often, to evaluate the value of the shear stress for engineering applications we need to get first-hand information about the stress.

To evaluate quickly instead of calculating this parabolic distribution we need time to assume that the stress received is uniformly distributed around the entire depth. Let us consider this particular stress as the average stress and call this as Tao_{avg} which is its value uniform over the entire depth. When we do that we take the area of this particular rectangular configuration which is uniform over the entire depth equal to the area corresponding to the distribution, which we get from 0 to the maximum value. If we say the Tao_{avg} is this width, we have the area which is Tao_{avg} (h) = the area under the curve with the actual distribution of the shear stress as 2/3 (h) Tao_{max}.

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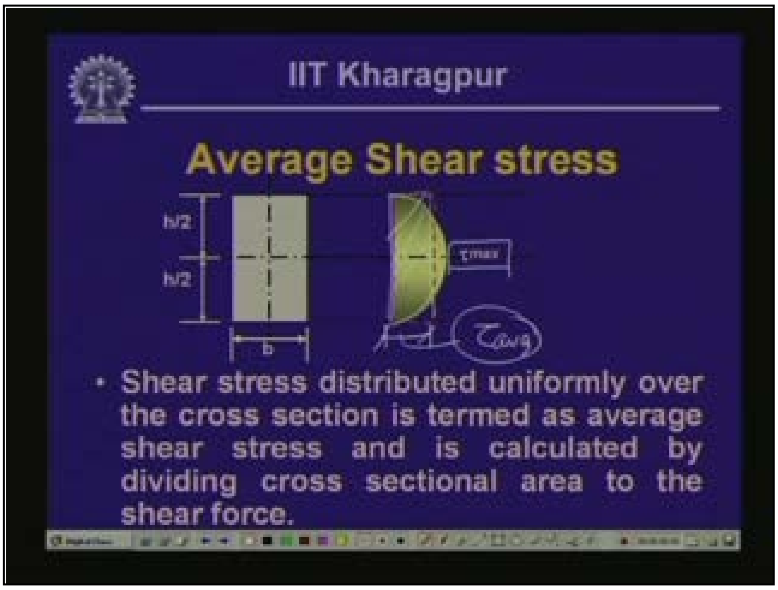

As we have seen earlier Tao_{max} is 3/2 V/A where V is the shear force and A is the cross sectional area and we have Tao_{avg} (h) = h (V/A) and Tao_{avg} gives us a value of V/A Tao_{avg} = V/ A where V is the shear force and A is the cross sectional area. We are assuming that the stress is uniformly distributed over the entire cross section and that is why it is called as average. So, the shear stress distributed uniformly over the cross section is termed as the average shear stress and the shear force divided by the cross sectional area will give us the value of Tao.

Let us look into the aspects of the elastic curve based on which we arrive at the differential equation. Let us consider the segment of the beam, say this is the origin O, along the member axis beam and this particular point is A, which is at a distance of X, from the origin. Let us assume that this particular beam undergoes deformation in the positive direction.

This is the Y-positive direction and we assume that the member undergoes deformation in the positive y direction and after deformation this is the deform configuration and point A moves to A-. If you remember we have said that when the beam axis undergoes deformation it does not stretch, which means that while deriving the bending formulae the strain at the neutral axis level is 0.

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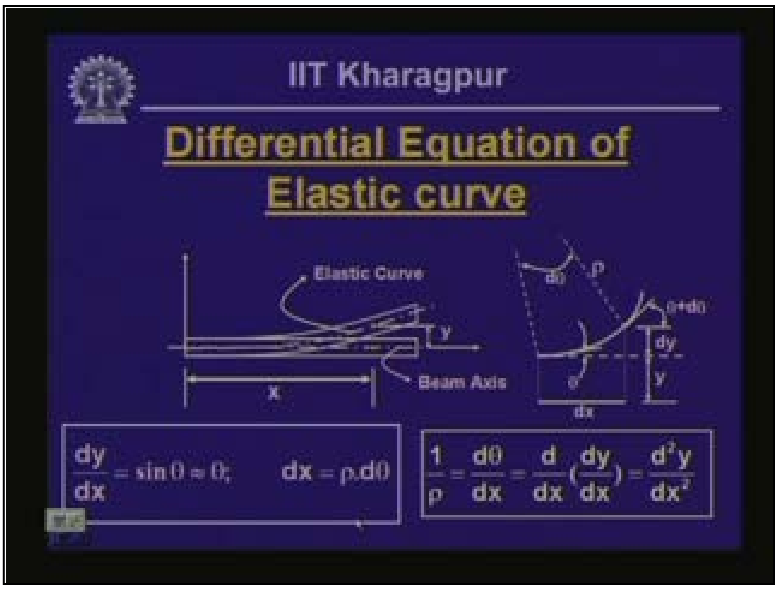

We assume that there is no stretching of the axis of the beam. If we consider a small segment after A as distance AB having a length dx and if we exaggerate this particular figure and plot it over here after deformation we have A- and B-. Point A has undergone a deformation which is Y as indicated over here.

Consequently, over the small distance D x point B has undergone a deformation which is y + dy. This particular stretch is dy and also the deformation at this point and in the slope of this particular axis if we call theta at this point over the length dx it undergoes a small change. We call this as Theta + d Theta and the radius of curvature of this beam axis is Row. Over this particular segment the angle dx at the centre is d Theta. Based on our assumptions that the beam axis does not undergo any stretching, this particular curved part also is dx.

From this particular rectangular configuration, we can say that dy/dx = sin Theta and Theta being small as we have assumed that the beam undergoes a very small deformation and thereby the slope is very small. Here Theta is small; hence sin Theta can be represented in terms of Theta and dy/dx = sin Theta = Theta. This particular arc length dx can be written in terms of Theta = Row (d Theta); dx = Row (d Theta).

From these we can write that 1 /Row = d theta dx and in place of theta substitute dy/dx; ddx (dy d) = d to the power of 2 y / dx to the power of 2; 1/Row = d to the power of 2 y/dx to the power of 2. This is the second order differential of y with respect to x and this is the curvature. In the previous modules we had a curvature of 1/Row.

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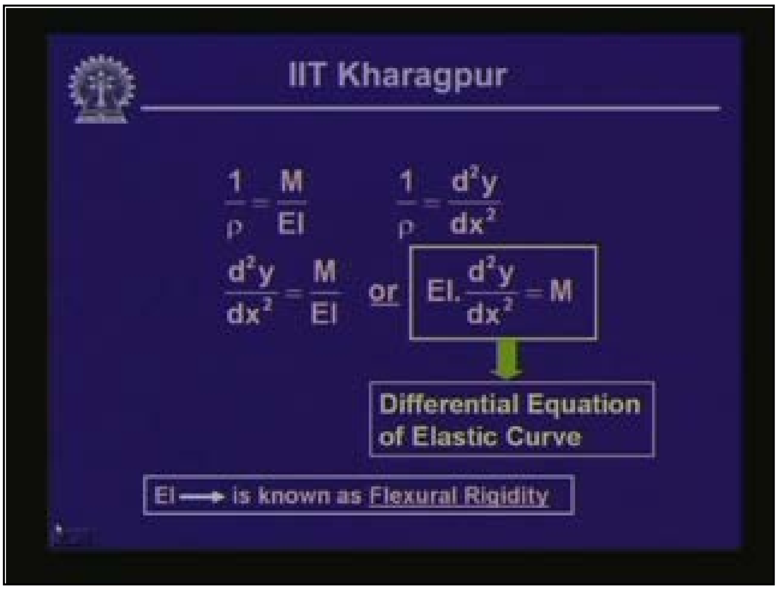

If we write down 1 /Row = d to the power of 2 y/dx to the power of 2. Earlier while evaluating the beam bending formulae we had obtained that 1/Row = M/EI and thereby if we equate these two we get d to the power of 2 y/dx to the power of 2 is = M/EI or M = EI d to the power of 2 y/dx to the power of 2 = M.

This is what we call the differential equation of the elastic curve. This particular differential equation gives us the deflection curve and the beam axis which was stated before bending; after bending it deforms and takes shape. From this particular equation we can arrive at what will be the shape of this particular deformed axis after it has undergone the deformation. This particular parameter EI is termed as the flexural rigidity of the beam.

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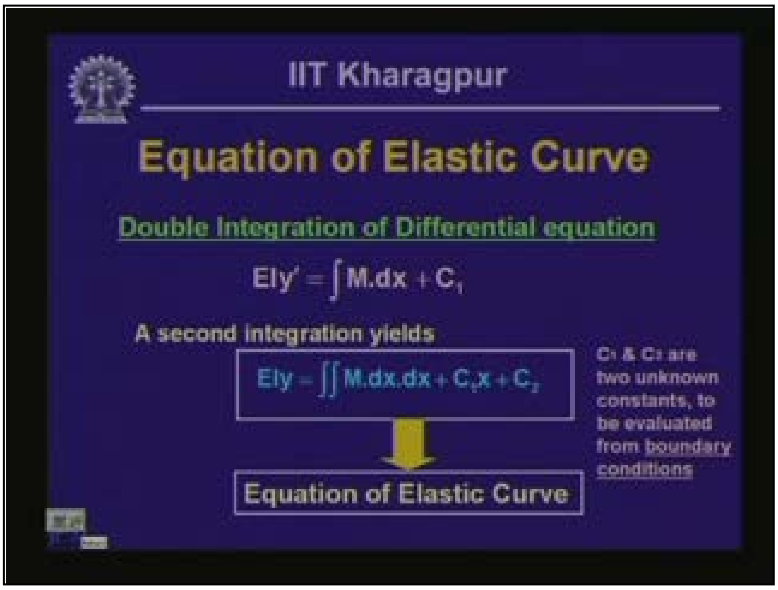

The differential equation which we have obtained is EI d to the power of 2y/dx to the power of 2 is equal to the bending moment M. If we integrate this, we get this EI; y dash indicates dy/dx, EIy- = Integral M dx + C_1 (the constant of integration C_1). If we integrate this equation once again the second integration EIy = Integral M dx dx + C_1x+ C_2; where C_1 and C_2 are the unknown constants and these constants are to be evaluated from the given boundary conditions of the beam.

From the impact we can write y as a function of x because moment is a function of x in the beam. As y can be evaluated along the length of the beam at different points we can get the value of y. Let us suppose we plot the value of y along the length of the beam that gives us the deflection curve for the moment of the elastic curve which is an expression.

As we have seen that we have first derived the differential equation of the elastic curve and subsequently, we are computing the equation of the elastic curve. Making use of this differential equation which returns as the function of the beam bending moment, we can compute the value of the equation of the elastic curve or the deflection at any point along the length of the beam.

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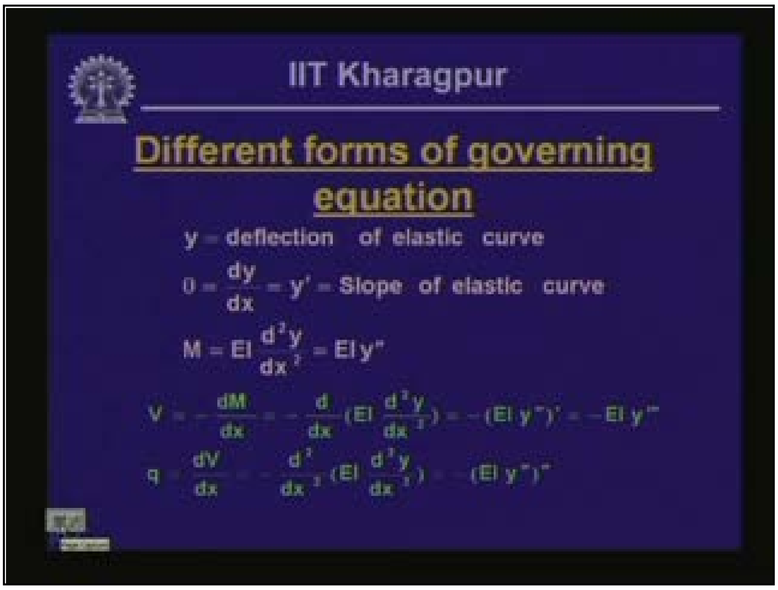

If we look into the aspects whatever we have evaluated so far; y gives us the deflection of the elastic curve; Theta is the first derivative of y, which you call as y- gives us the slope of elastic curve. This means if we have the beam and the deflection then it will take a tangent at any point on the elastic curve. This gives us the value of the slope Theta which is dy/dx the derivative of the deflection curve; the second derivative subsequently give us the value of the moment which is the differential equation for the elastic curve that EI d to the power of 2y/dx to the power of 2 = the moment.

In the previous lesson, we have seen that the shear force V is equal to the rate of change of the bending moment with the negative sign and v = -dm/dx and M, as we have seen in terms of these differential form is EI d to the power of 2 y/dx to the power of 2. So, we get -d dx EI d to the power of 2 y/dx to the power of 2 and this gives us -EI d to the power of 3y/dx to the power of 3 is the value of the shear or EI d to the power of 3 y/dx to the power of 3 = -V.

We have seen that q = dv/dx and it will substitute v which is the second derivative of this moment. This gives us the value -EI d to the power of 4y/dx to the power of4 = the loading q; or EI d to the power of 4y/dx to the power of 4 = -q. These are the different forms of the governing differential equation which we have obtained and EI d to the power of 2y/dx to the power of 2 = moment. From these we can derive the other relationships as well.

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As we have seen that EI d to the power of 4 y/dx to the power of 4 = -q which is the differential equation. We have seen that the differential equation in terms of the bending moment is EI d to the power of 2 y/dx to the power of 2 = m. Also we can write EI d to the power of 4y/dx to the power of 4 = -q. If we integrate this, we get EI d to the power of 3y/dx to the power of 3 = -q dx + C_1 and EI d to the power of 3y/dx to the power of 3 = -v; -v = -q dx + C_1 and if we write v = q dx - C_1, this is constant.

We can write this in the modified form C_1 v = qd x + C_1 and we have seen these expression earlier when we had evaluated the shear force and the bending moments. We have seen that q = dv/dx and thereby v was integral q dx plus a constant term. We had derived this particular expression from the equilibrium of a small section which we had taken out from the whole beam.

This particular expression represents the equilibrium of the loading system. Subsequently in fact we have seen that the moment is dm/dx; the derivative of the moment is equal to shear and v = -dm / dx. If we compute it from this particular expression we can get m as integral of q dx + C_1 x + C_2. This is our moment EI d to the power of '2 y/dx' to the power of 2 and hence these particular sets of equations represent the equilibrium of the beam segment and further integration of this gives us the value of the slope and corresponding deflection.

If you look at the constants which are associated with these particular sets of equations like C_1 and C_2 these can be evaluated from the boundary conditions of beam, from the known values of the shear in the moment. These particular constants can be evaluated from the static boundary conditions and we can compute C_1 and C_2 from the static boundary conditions.

Now for this dy/dx and y we have the constants C_3 and $C_{4.}$ These constants C_3 and C_4 are evaluated from the relationship of the deflection and the slope of the beam and these boundary conditions are called dynamic boundary conditions. Here C_3 and C_4 are evaluated from the dynamic boundary conditions and these are the two sets of boundary conditions that we need.

In fact for the evaluation of these unknown constants they are the static boundary conditions or the dynamic boundary conditions and it is dynamic in the sense that after the deformation of the beam it undergoes moment and it has some slope. We can evaluate the values of C_3 and C_4 for clarity.

Let us look at the boundary conditions that we come across. If we have a fixed support for a beam, at this support as we have seen in the earlier lessons that it does not allow any moment, so the deformation of the displacement of this particular point is 0 and consequently it does not allow any rotation as well at this point. Hence the dy/dx Theta = 0 and both the conditions are dynamic.

In a hinged support or a roller support we know that the vertical displacement is a constraint and thereby y is 0. At the hinged support it cannot register any bending and thereby it gives rise to the slope and the bending moment at this point = 0. At the hinged or a roller support the vertical displacement is 0 which is a dynamic condition and the moment is 0 which is a static boundary condition.

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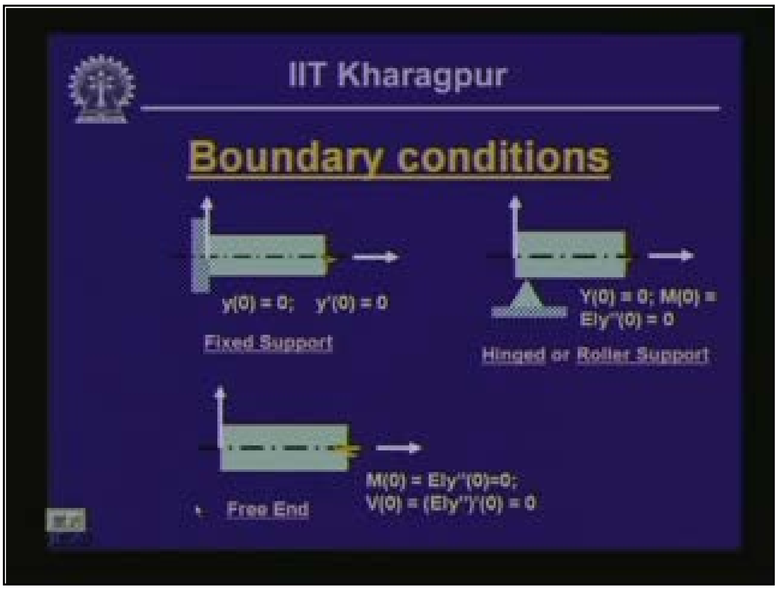

For a free end since it undergoes both displacement as well as rotation, here we have the boundary conditions which have basically static boundary conditions and the moment is 0 and the shear force is 0 at this particular point. These are the different support conditions which we have seen in the past that a beam is supported on some supports, this kind of support could be fixed support, or hinged support, or a roller support, or it could be free on one end and fixed at the other, as we have seen in the case of a beam.

Many combinations of these supports give us the appropriate configuration of a beam. When such beams are loaded they undergo deformation and because of those deformations we need to evaluate what will be the deflection and the slope because of the loading. In this particular case, while deriving the equation for the elastic curve we have considered that the deformation is a small one and thereby we said that when the axial axis of the beam undergoes deformation it does not stretch and the length dx remains dx.

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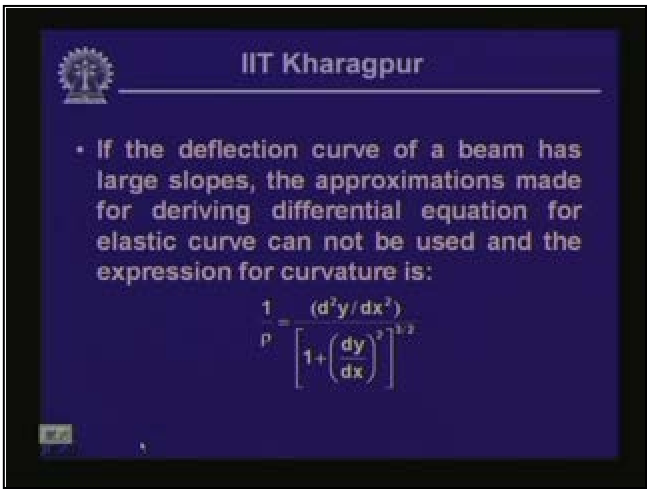

If the deflection is very large then we cannot make these approximations and thereby the curvature term that is 1 /Row takes this particular form and the whole form has to be taken into account for evaluating the differential equation for the elastic curve. Hence the expression which we have derived is suitable for a small deformation only.

For a large deformation this is not applicable we will have to use the curvature or in terms of these expression. We are not going to the detail derivation of this but for your information that for a large deformation of the beam the curvature term is to be used with this particular expression. Let us look into the sign convention that we get acquainted with it; then when we deal with the problems. We do not have problem as such, like we had considered for the beam segment and this is the beam segment, have been with dx and loading on this as q on the right hand side we have the shear force which is pointing upwards to the positive y direction.

On the left side we have the positive shear downward and the positive bending moment which is in an anti-clock wise form on the right hand side and a clockwise form on the left hand side. Consequently, we had obtained the relationship which is v = -dm/dx and dv/dx = q. We have used these relationships while deriving the differential equations and its different form of the governing differential equation. We have used EI d to the power of 2y/dx to the power of 2 = bending moment.

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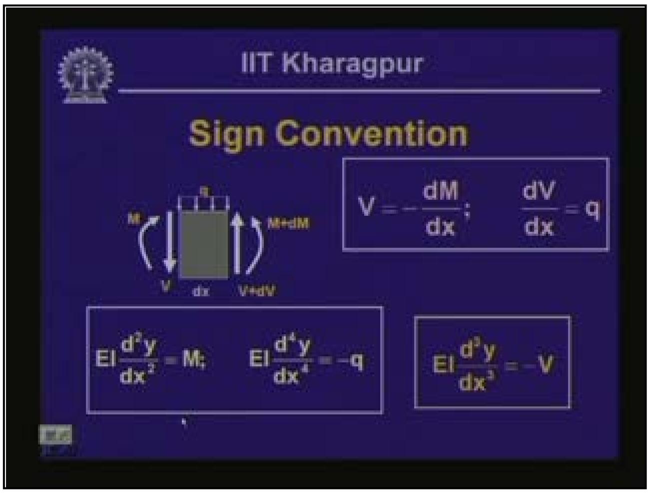

Subsequently, we have taken the derivative d m/d x as -v and dv/dx as q. Thus we have q while deriving the different forms of the governing equation. Here EI d to the power of 2 y/dx to the power of 2 = moment; considering the moment positive as anti-clockwise on the right hand side and EI d to the power of 4y/dx to the power of 4 = -q. Please note over here that the loading q is in fact in the opposite direction of the positive y axis which is positive upward.

The loading which is directed upward is positive and since this q is pointing downwards q is negative. Rightly we have EI d to the power of 4y/dx to the power of 4 = -q. We have EI d to the power of 3y/dx to the power of 3 = -v with the positive shear pointing upward. These are the expressions which we have used for evaluating the equation of the deflection curve y. Here EI d to the power of 2 y/dx to the power of 2 = m and EI d to the power of 4y/dx to the power of 4 = -q. These are the two expressions which we are going to use and we do not use this particular expression very frequently which is EI d to the power of 3y/dx to the power of 3 = -v.

However any of these three equations can be used for evaluating the equation of the elastic curve which is y as a function of x. Thereby at any point along the length of the beam member, we can compute the value of y and thereby the plot which you get will give you the elastic curve of the deformed shape of the particular beam after it is loaded.

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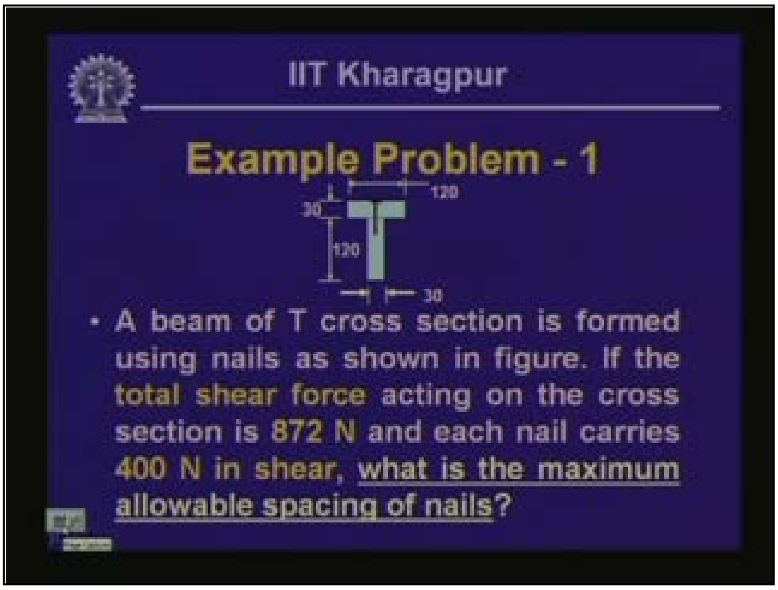

We have looked into the equations for the elastic curve. Let us look at some examples of problems. Before we go into the example of problems of this particular aspect, let us look into the example which I had given you last time which is related to the evaluation of the shearing stress. If you remember that a beam of t cross section is formed using nails.

Here there are two rectangular components; one is horizontal, another one is a vertical one and these two separate units are joined together providing a nail along the length of the beam and the nails are provided at an interval. The shear force which is acting on the cross section is 872 Newton and the nail carries 400 Newton in the shear. At this interface where two blocks are getting connected together by nails and they will be subjected to the shearing axis.

This nail has a regressive power and it can receive 400 Newton of shearing load. We will have to find out what spacing of these nails along the length of the beam we can provide, so that it can withstand that amount of load. That means the shear force in each nail should not exceed the allowed limit and if it is less than that then it can withstand the load. Thus the aspect is to evaluate what is the maximum allowable spacing of such nails.

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In this particular figure, we have separated out the wave part of the section and this particular part is shown over here. This is the phase of the beam cross section and here the positive shear is acting at this particular direction which is positive upward. We have the complement here which is acting on this perpendicular phase.

We have the shear in the other direction as well. Along this length of the beam, these are the nails which are spaced at a spacing of p. If you can compute the level of shear stress acting at this particular level then we know how much shearing stress will be acting at this phase.

Once we know the shearing stress on this phase, we know how much shear force these particular bolts can resist in terms of the shearing stress. To compute the shearing stress at this particular cross section we need to take this particular area which is above this section and take the moment of this area with respect to the neutral axis.

First we will have to find out the position of the neutral axis of the cross section and if we say that this is y_1 from the top, then take the moment of all the segments with the top part of it. The first segment is 120/30 and so 120/30 (15) is the moment of that area with respect to the top line; plus the area of this bottom part. The width is 120 (30) (60) + 30 = 90/ the area of the whole cross section which is 120 (30) + 120 (30) and this gives us a value of y bar = 52.5 mm from the top.

We go to evaluate the moment of inertia of this particular cross section with respect to the neutral axis. We divide the whole of the cross section into two rectangles and the first rectangle 120/30 has a moment of nrca about its own axis as 120(30 to the power of 3) / 12+120(30). It is at a distance cg from the neutral axis which is 37.552 as 15 to the power of 2 + for this is 30(120 to the power of 3)/12 + area 120(30) and 60+30 = 90 - 52.5 will give you 37 to the power of 2. This is the moment of inertia of the section which is 14.715 (10) to the power of 6 mm to the power of 4.

From the expression of Tao = vq / IB, v in the section given is 872 Newton. Now q is the moment of the area above the section where we are evaluating the shear stress. We are evaluating shear stress at this section and the area about that is 120 (30); 120(30) and its neutral axis c g is 15 from the top and distance of the neutral axis of it from the neutral axis is = 37.5; 52.5 - 15/ i and v here is 30.

This gives us a value of a shearing stress at this interface. Along this length, we have a shear stress of 0.27 Newton/mm to the power of 2. If these nails are spaced at p mm apart and if I consider this particular nail which has an area half of this length and half of this length, this length multiplied by this width is 30 multiplied by p.

Thus the area multiplied by the shearing stress that is acting at that interface will give the force that this particular nail can withstand since a nail can withstand a maximum shear force of 400 Newton and we get a value of p as 49.4 mm. That means this particular nail can be spaced at a maximum distance of 49.4 mm and we can keep a lower value than 49.4 mm.

We cannot exceed this particular value, which is the maximum value of the spacing of the nails that can be provided so that the shear stress in the nail does not exceed 400 Newton. If you go beyond that, that shear capacity of the ball will go beyond 400 Newton and as a result the bolt will not be able to withstand the cross section and the full section will fail. This is the value of the spacing of the nails that are to be provided so that the member becomes safe.

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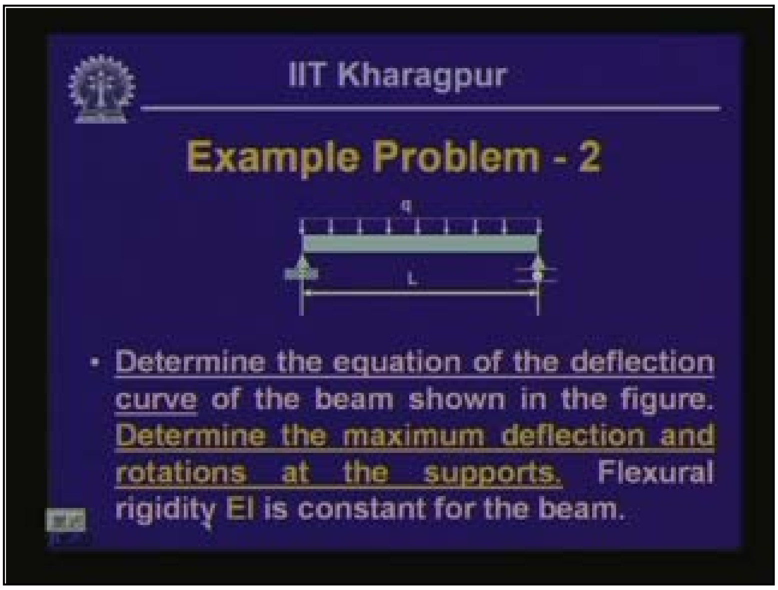

That was the problem for the shear stress. Let us come back to this equation of the elastic curve. Now that we have a simply supported beam which is subjected to uniformly distributed q for unit length, we have to determine the equation of the deflection curve.

Secondly we have to find out the maximum deflection and the rotations at the supports. The flexural rigidity EI is constant for the entire beam. We will have to determine the equation of the elastic curve and secondly the maximum deflection and the rotations at the support points. To do that let us first evaluate the reacting forces as we have done in the past.

As you know this is a hinged support and so you have the vertical support, vertical reactive force and the hardest centre reactive force. This being a ruler support you have a vertical reactive force. If you call this as a, and this as b, then this is ra, and this is rb and this is ha. Since there are no horizontal loads in this beam the summation of horizontal force is 0 and we will get ha = 0.

We will have to compute the value of ra and rb. As we have seen in the past, this is a symmetrical beam subjected to a symmetrical loading hence the value of ra and rb = q (L /2) which is the total load. This we can compute from ra + rb = total load ql and if we take the moment of all the four sides with respect to one of the supports, we get the value of ra and then you can compute from the vertical equilibrium the value of rb and you will get the value of ra and rb as = ql / 2.

Let us employ the differential equation of the elastic curve as we have derived, so we have the beam in which we have evaluated the value of the reactive forces ra and rb and we have seen ql /2 where q is the uniformly distributed load that is acting on the beam. If I take a segment at a distance of x, a free body of this particular part and you have the reactive force ra and q. On this cart we will have the shear force v and bending moment m. These are the positive directions of v and m as we have seen if we compute at a distance of x from a.

If we take the moment of all the forces with respect to this particular point then we write m, which is in an anti-clock wise direction, then – ra(x) and q causes a moment which is anti-clockwise so + q(x) (x) /2 = 0. The value of bending moment m = ra = ql /2; q l/2 -q x to the power of 2/2. This is the value of the bending moment that we have and as we have seen that EI d to the power of 2 y/dx to the power of 2 = m (bending moment). This is our differential equation for the elastic curve and since we are going to find out the equation of the elastic curve, we start from this particular point as we have seen. Now m is q l /2 x – qx to the power of 2/2 = q l / 2 x - q x to the power of 2/2.

If you integrate this, we have EI dy/dx = q l / 2 x to the power of 2/2 - q/2x to the power of 3/ 3 + C_1 which is the constant of the integration. Further if we integrate this we have EI y = q l /4 x to the power of 3/3 - q/6x 4/4+ C_1 x + $C_{2.}$ This is the equation of the elastic curve with the unknown constant C_1 and C_2 and we have to evaluate the values of C_1 and C_2 with the help of boundary conditions. What are the boundary conditions we have for this pre-supported beam? If we look into the deflected profile of this the beam is going to the curve where the axis is going to deform.

Since this is hinged support and this is roller support, at x = 0, the deflection is 0, and also at x = l y is = 0. These are the two conditions we have. If we substitute these conditions you see the first of the conditions at x = 0, y =0, if we substitute in this equation, we find that C_2 = 0. If we substitute the second condition that at x = l; y = 0 if we substitute over here for x, we substitute the value of l and y = 0.

Consequently, we get 0 = q l / 12 and we have l, so l to the power of 3 - q / 24 l to the power of 4 + C_1l. This gives us the value of C_1 and if we compute it we get C_1 = -q l to the power of 3/24. The equation of the deflection curve is EIy = q l / 12 x to the power of 3 - q / 24x to the power of 4 – ql to the power of 3 /24 x. This is the value of the expression of the elastic curve for the beam. At any point x, we can get the value of y and now we have the expression for y and at any x, we can compute the value of y.

This particular beam is symmetrically loaded with uniformly distributed load. It is expected that we will have the maximum deformation at the centre. If we compute the value of the deflection at the centre. That means if we compute y at x = l y 2 let us see what we get the value of y. so EI y = q l / 12; l ^{3} / 8—q / 24 × l 4 / 16—q l ^{3} / 24×l /2. If you compute this, this comes as an expression of q l 4 / 96—q l 4 / 384—q l 4 / 48 and if you compute this it comes as —5 q l 4 / 384. In fact, this is the value of the deflection y at l / 2 = —5 q l 4 / 384; now taking EI on the other side.

This is the value of the deflection at the centre of the beam. Here note that we have a sign which indicates that our positive y direction is upward and hence the deflection curve is towards the downward side of the beam axis. This is the deflected shape of the beam, this is the y at l / 2 and so it goes from 0 to l / 2 and from here again it goes here. The value here which we call as the maximum displacement Delta max is 5 q l f4 / 384 EI.

Once we have the expression for the deflection curve we can take the derivative of this and can compute the value of the slope at this point. We already have the expression for the dy/dx. We have already obtained the value of C_1 and if we substitute over here we can get the value of dy/dx. We can compute the value of the slope at x = 0 and at x = l. Let us compute the value of the slope at the support points and this is the value of the deflection that we have obtained.

If we compute the value of dy/dx this is EI dy/dx. We are taking the derivative of that and we have q l / 12. Here this is 3 x to the power of 2 - q/24 and this is 4 x to the power of 3 - q l to the power of 3/24 because x goes off. This is the value of EI dy/dx and dy/dx at x = 0. We have -q l to the power of 3/24. Here if you note it again, this is given as negative and this is the deflected curve as we have obtained.

Take the tangent of this elastic curve at this particular point with this rotation which is in a clockwise direction. As we have seen that in a beam segment, when we consider the positive bending moment, that is in an anti-clock wise direction, consequently since this is in a clock wise direction, this Theta is negative which is clockwise. If we compute the value of the rotation at the other end at x = l then in this particular expression substitute x as l.

Let us compute the value of dy/dx at x = l and we get EI dy/dx = l to the power of 2; q l to the power of ¾ - q l to the power of 3 / 6 - q l to the power of 3/24. We have 6 - 4 - 1/24xq l to the power of 3. Here we get q l to the power of 3/24 and we get a positive value. If you take a tangent at this particular point, the rotation is an anti-clockwise direction and according to our normal sign convention, this is positive. So, Theta b = +q l to the power of 3 / 24. and we have Theta computed, at support 'a' and at support b.

These are the aspects you required to compute and we have computed first the equation of the deflection curve. Consequently, we have computed the deflection at the centre of the beam which is the maximum deflection in this particular guess and we have computed the value of the rotation at 'a' and b at the support points.

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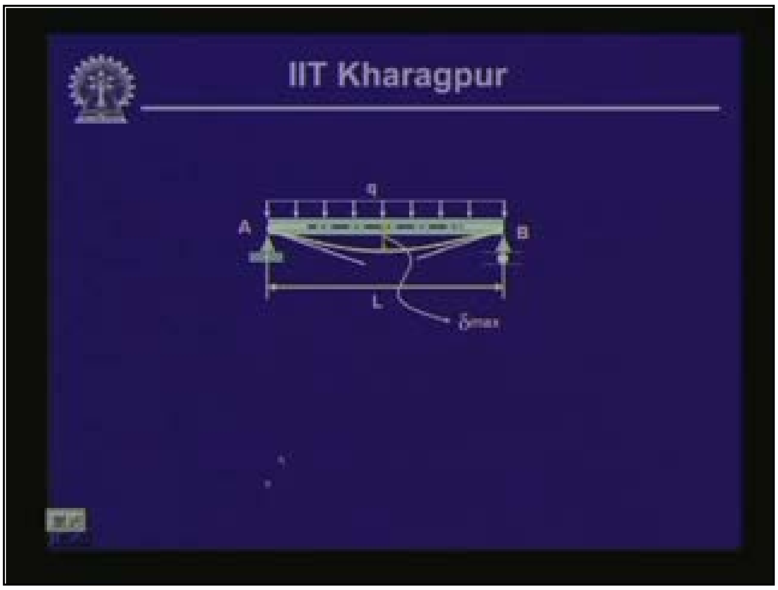

When the final form of the deflected form of the beam is loaded, you have the deflected curve and this is the maximum deflection which you have at the centre. From this rotation we have obtained Theta a, which is negative; this rotation is Theta b which is positive because it is counter-clockwise.

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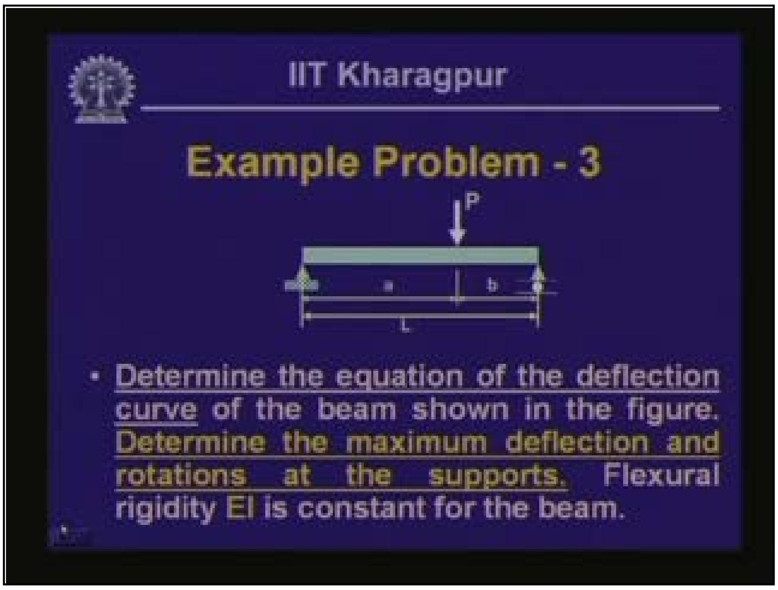

Take another problem where the beam supports are identical and that means we have a hinged support over here and a roller support here. Instead of a uniformly distributed load, we have a concentrated load p and to evaluate the deflection curve and to evaluate the deflection a general one, the load is placed not symmetrically but in an unsymmetrical form.

The load is located at a distance from support a; and it is at a distance of b from support b. The length of the beam or the span of the beam is l. We will have to evaluate the equation of the deflection curve and we will have to evaluate the maximum deflection. We will have to find out the rotation at the support and the flexural rigidity of the beam EI is constant throughout the beam.

As usual at the hinged support we will have the vertical reactive force and the horizontal force and at a roller support we will have a vertical force and we call this as ra ha and this as rb. Now ha = 0 as we do not have any horizontal load in this beam and if you take the moment of ra v and a load with respect to b then we have ra (l) which is in a clockwise direction. Here p (b) is in an anti-clockwise direction and therefore ra = p b / l and since ra + rb = p; rb = p - pb/ l = pa / l.

Here we have two segments. This is segment one and this is segment two and in these two different segments the bending moment expression will be different. Let us compute the value of the bending moment for these two segments. We have the beam which is ra; and this is rb and you have the load as p. For the segment one if I take a free body here, if I take a cut here and draw the free body diagram then this is the shear force, this is the bending moment m and this distance is at a distance of x from the support a.

The bending moment m = ra × x = p b x / l; the validity of this particular moment is from this particular point up to the load point and it is $0 < x < a$. If I take a free body of this particular segment over here, if we take the free body of this particular part then we have the reactive force here. The load here is at a distance of ‘a’ and the segment we have taken is at a distance of x from the support. As usual you have the shear force and the bending moment over here. Let us call this as moment m and this as p.

The moment expression which we get at this particular point is m = ra (x) - p x – a and ra being p b / l; p b x / l - p x – ‘a’ and the range of the validity of this is $a < x < l$. These are the two moment values that we have now. Once we get the expression for the moment then we can write down a differential equation which is EI d to the power of 3 y/dx to the power of 2 = moment. We write for the two segments differently; for the segment one, this is the expression for the bending moment and for the segment two this is the expression for the bending moment.

Consequently, we integrate 1/1 and then in the first type of integration we get dy/dx and in the second step we get y and that gives us the expression for the deflection curve. Once you solve this, we have two different segments and different equations so thereby you will have four unknown constants and these four unknown constants are to be evaluated from the four known boundary conditions. This problem is set for you to solve and we will discuss how to solve those unknown constants and how to arrive at the deflection curve in the next lesson.

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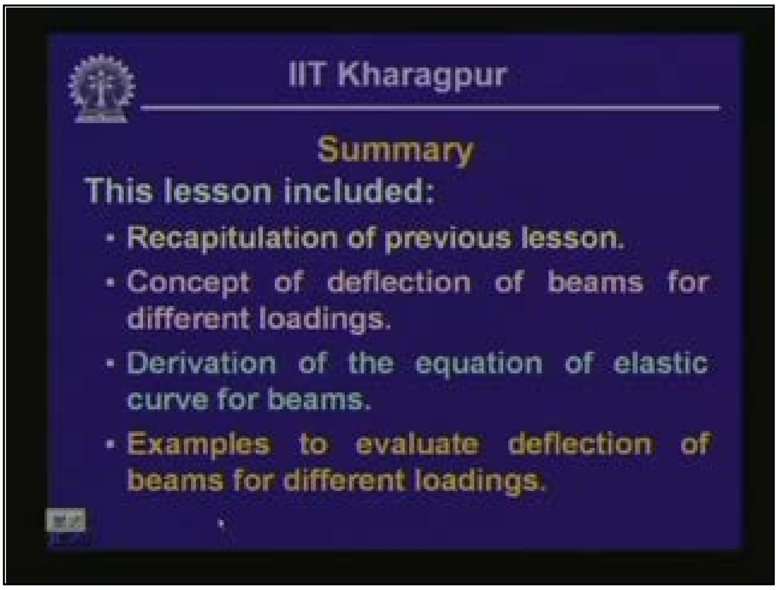

Let us summarize this particular lesson. First we had included the concept of deflection of beams for different loadings. We had discussed the previous lessons then we had looked into the derivation of the equation of elastic curve and the differential equation. Consequently, we have derived the elastic equation for the elastic curve. Then we have looked into some examples as to how to evaluate the deflection of beams for different loading.

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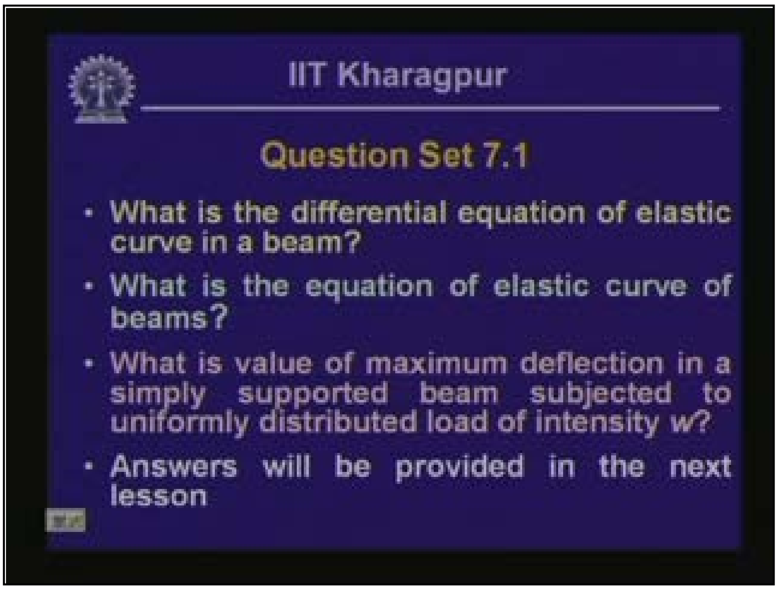

Now these are the questions set for you:

1. What is the differential equation of elastic curve in a beam?
2. What is the equation of elastic curve of beams?
3. What is the value of maximum deflection in a simply supported beam subjected to uniformly distributed load of intensity w?

We will answer these questions in the next lesson.