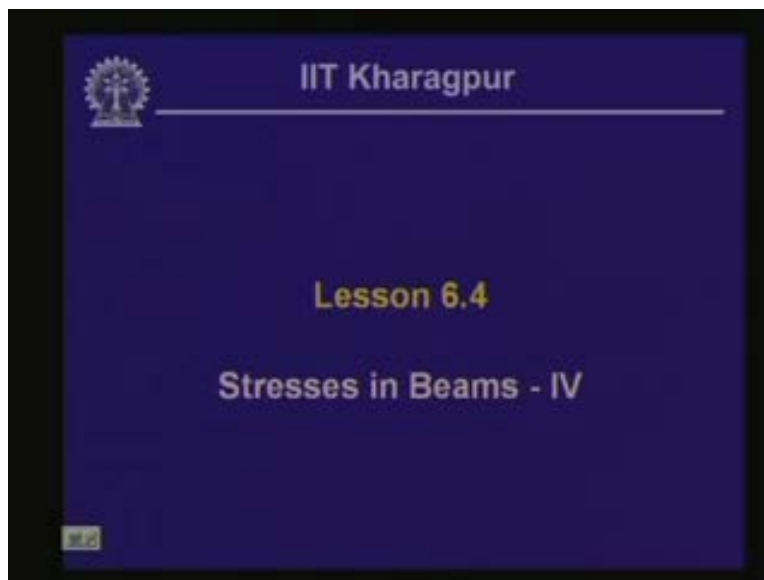


**Strength of Materials**  
**Prof: S .K.Bhattacharya**  
**Dept of Civil Engineering,**  
**IIT, Kharagpur**  
**Lecture no 29**  
**Stresses in Beams- IV**

Welcome to the fourth lesson of the sixth module on Stresses in Beams part 4.


In the last lesson we have discussed some aspects of shearing stress in beams and we have looked into the effect of shearing stress in a beam having a rectangular cross section. In this particular lesson we are going to discuss what would happen if shear stress acts in a beam having cross sections other than the rectangular one, like a circular one or the t section.

(Refer Slide Time: 01:00)



Once this particular lesson is completed one should be able to understand the concept of shear stress in beams of different cross sections and understand the effect of shearing strain on longitudinal strain in beams.

(Refer Slide Time: 01:28 - 02:16)



IIT Kharagpur


### Specific Instructional Objectives

- After completing this lesson one will be able to:
- Understand the concept of shear stress in beams of different cross section.
- Understand the effect of shear strain on longitudinal strain in beams.
- Evaluate shear stress in beams of different cross sections for different loadings.

1/23

In fact we have discussed the effect of the bending movement acting in a beam and we have calculated the stresses corresponding to that. We have also seen that the beam is subjected to the longitudinal strain in the longitudinal direction of the beam because of the effect of this bending. Now what is the consequence of the shearing stress which is acting in the beam on this longitudinal strain? We will be looking into that and also one should be in a position to evaluate shearing stresses in beam of different cross sections for different loadings.

(Refer Slide Time: 02:17)




IIT Kharagpur

### Scope

- This lesson includes:
  - Recapitulation of previous lesson.
  - Concept of shear stress in beams of different cross section.
  - Effect of shear strain on longitudinal strain of beams.
  - Examples for evaluation of shear stresses in beams of different cross sections.

1/23

(Refer Slide Time: 02:34 - 02:50)



IIT Kharagpur

---


**Scope**

- This lesson includes:
  - Recapitulation of previous lesson.
  - Concept of shear stress in beams of different cross section.
  - Effect of shear strain on longitudinal strain of beams.
  - Examples for evaluation of shear stresses in beams of different cross sections.

1/3

Hence the scope of this particular lesson includes answering questions that were posed in the previous lesson. We will recapitulate the aspects which we have discussed in the previous lesson and the concept of shearing stress in beams of different cross sections is included in this particular lesson. In this lesson we will give examples for the evaluation of shear stresses in beams of different cross sections.

(Refer Slide Time: 02:51 - 3:13)



IIT Kharagpur

---

**Answers to Question Set 6.3**

- What are the assumptions made in deriving the shear formula?
- What is the limitation of shear stress formula?
- What is the value of shear stress in a cantilever beam subjected to a moment at its tip?

1/3

Now let us study the answers to the questions which were posed last time. The first question given was what are the assumptions made in deriving the shear formula? In the previous lesson we have seen how to derive the shear formula and consequently we have seen that we have made some assumptions.

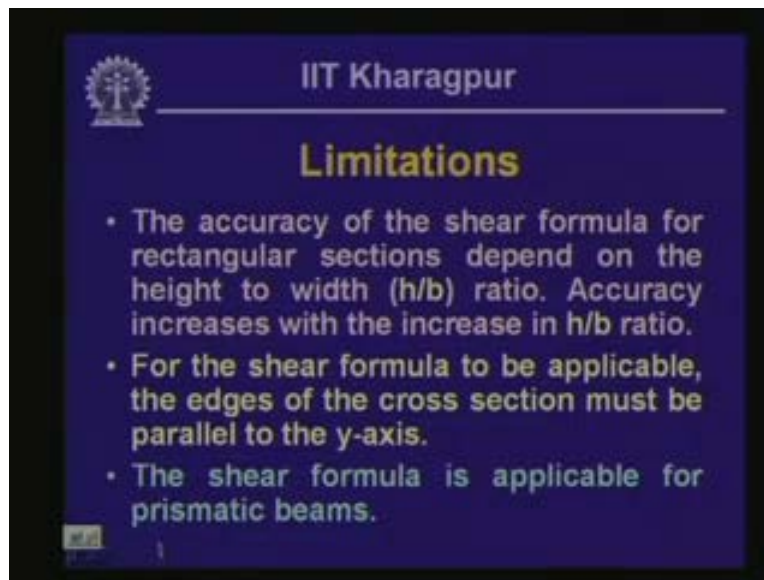
(Refer Slide Time: 03:14 - 04:24)



The assumptions are that at any cross section when there is shearing force acting, we assume that the shear stress is parallel to this shear force. That means in this cross section the direction of this shear stress is in the same direction as that of the shear force. Shear stresses acting on a cross section are parallel to the shear force acting in that particular section.

Also it is assumed that shear stresses are uniformly distributed across the width of the beam and that at any point along the depth, the distribution of the shear stress along the width is uniform and based on this assumption. We have derived the shear formula and as we have noticed that on a particular element, we have the vertical shear and the complimentary horizontal shear and this is the state of stress at a particular element based on which we have derived the shear formula. Basically these are the two main assumptions based on which we have derived the shear formula.

(Refer Slide Time: 04:25 -08:23)



What is the limitation of shear stress formula? While calculating the stresses in a beam due to the shear force we have used the shear formula which is  $VQ/ib$  and consequently we calculate the shearing stress at any point along the depth. Now is this particular formula applicable for all kinds of beams or are there certain limitations?

As we have seen the distribution of the shear stress across width, we are assuming that it is uniform. Let us suppose we are dealing with a beam of rectangular cross sections. This accuracy of this particular shear formula depends on the height to width ratio which means that if we have a beam with a rectangular cross section having the width  $b$  and height  $h$ , the accuracy of this particular formula depends on this ratio of  $h/b$ .

The accuracy of the shear formula increases which means that if  $h/b$  ratio is higher we get accurate results using the shear formula or in other words for a particular depth of  $h$ , for a constant value of  $h$ , if you have a lesser width of  $b$ , then the accuracy will be better using shear formula. You can understand from the concept that we are assuming that the shear stress is uniform over the width of the beam and hence if we have a smaller width, your accuracy of the shear stress formula will be more.

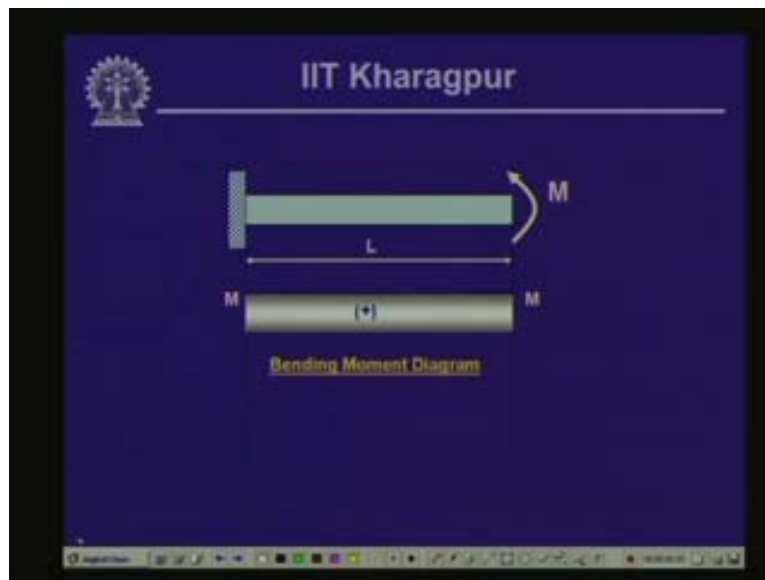
But if you have a wider beam then you will be using the accuracy if you use this shear formula. Hence this aspect should be kept in mind. This is one of the important limitations of shear formula. Also from this it appears that for the shear formula to be more effective or to be more applicable, the edges of the cross section must be parallel to the y axis.

Now for a rectangular or square section we have these edges which are parallel to y axis. Now if you have a section like a triangular one or if you have a section which is semicircular, for this kind of section, the edges are not parallel to the y axis. These sections or the shear stress formula cannot be used for evaluating stresses for such cross sections. So this is one of the prime limitations of the shear formula.

Thirdly the shear formula is applicable for the prismatic beams only, which means that if a beam has a taper and it is not uniform in each cross section then you cannot use this shear formula for evaluating the shear stress in the beam. So these are the main limitations of shear formula and we can use shear formula for evaluating the shear stresses in a beam when the edges of the cross section are parallel to y axis.

For the other section like a circular one for which the edges are not parallel to y axis, we resort to some means or the shear stresses for such sections can be evaluated by going for other rigorous theories which we are not going to discuss at this moment. The third question posed was; what is the value of shear stress in a cantilever beam subjected to a moment at its tip?

(Refer Slide Time: 08:24 -10:30)



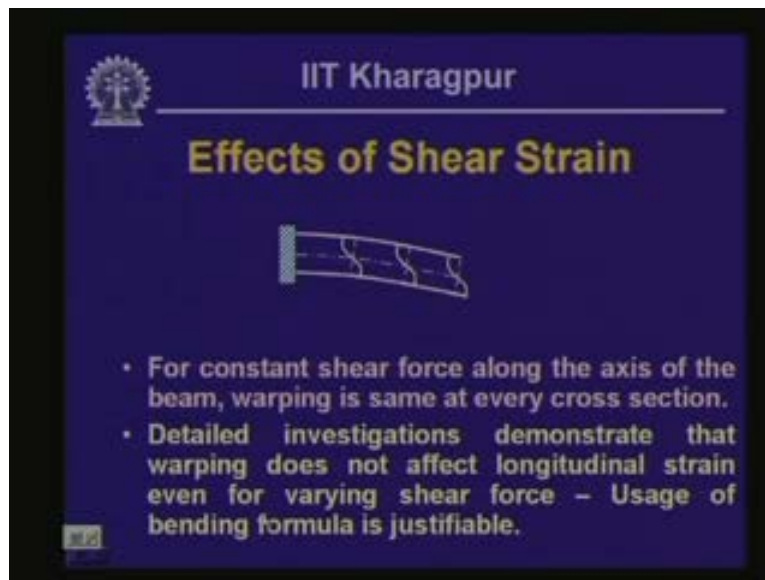
You have a cantilever beam, that means the beam which is fixed at one end and if a moment  $M$  acts at the end of this particular beam then what is the value of the shear stress in such a beam? Let us assume that the cross section of the beam is a rectangular one. If we draw the bending movement diagram as we have done in the past, we can remove the support and we can write down the reactive forces and thereby evaluate these reactive forces; the vertical force, horizontal force and the moment based on the external moment.

You will find that the vertical force will be 0, because the summation of the vertical force is 0 and there are no external vertical forces. So, the vertical reaction is going to be 0 and again the summation of the horizontal force being 0 will give the value of  $H = 0$  as there are no horizontal forces on the beam.

The moment will be the externally applied moment and at any section we take if we draw the free body diagram corresponding to that free body diagram, the value of the internal moment is equal to the external moment and that is how we get the bending moment diagram which is a rectangular one which means that every where we have a constant moment.

If the moment direction is reverse in a clockwise direction we will have exactly the same result but the moment will be negative instead of positive. Since the moment is constant everywhere you can make out the value of  $dM/dx$  which is nothing but equal to the shear force for the constant moment  $dM/dx = 0$ . Therefore the shear force is 0 and hence the shear stress will be 0 because the shear stress is  $VQ/Ib$  and since  $V$  is 0 the shear stress is going to be 0 and this answers the third question.

(Refer Slide Time: 10:31 - 14:57)



Let us see what the effect of the shearing strain on the longitudinal strain is. Let us consider a cantilever beam whose cross section is rectangular. The shear stress in such a section varies parabolically and since the shear stress  $\tau$  varies parabolically the shearing strain  $\gamma$  which is  $\tau/G$  also varies parabolically. So, at the neutral axis we get a shearing strain and since the stress at that neutral axis is higher, the corresponding shearing strain is also high.

If we consider a beam, a cantilever beam having a rectangular cross section and is subjected to loads so that every where you have the same shear force  $V$ , then the cross section is going to have a deformation because earlier when we have derived the bending formula we have assumed that the plane section remains plane even after bending and it becomes perpendicular to the axis of the beam.



Now because of this shearing strain, this distribution of the shearing strain varies and the shearing strain is 0 on the surfaces and has a maximum value at the neutral axis. The section is going to warp in this particular form and at this point since the shearing stress is 0 and the corresponding shearing strain is 0 there will not be any deformation.

This particular line will be perpendicular to the surface and maximum deformation will occur at the neutral axis point. For the constant shear force, all the sections are going to be in the same form and then the strain in the longitudinal direction will have no effect because of this shearing strain. Consequently, as we have seen, the bending formula which we have derived from the longitudinal strain criteria will also remain unaffected.

If you have a beam where the shear force is constant and consequently the warping which we are going to get is uniform in all cross sections, it does not have any affect as such on the longitudinal strain and consequently we can use the bending formula even if every moment is not uniform. It has been observed that when we go for detailed investigations for such warping because of the non uniform shearing stress where the shear force is not uniform along the beam, if there is a varying shear force, then the warping will not be constant in all cross sections and thereby there will be some amount of changes in the longitudinal strain.


But the change in the longitudinal strain is not to the extent where the accuracy level is jeopardized and hence the bending formula can be used for beams which are subjected to moments along with the shear. You should be in a position to appreciate that when we have evaluated the bending formula we have considered only pure bending and consequently we have calculated the bending stress.

When a beam is subjected to loads it is not only subjected to bending but the shear force is also associated with that. If you have such a kind of bending, it is called non uniform bending and when you have this bending along with shear force despite the bending formula being evaluated exclusively for the bending moment alone, we can use the bending formula for such beams. We also have non uniform bending where the bending is associated with the shear force and the accuracy level is not jeopardized to a great extent and hence it is justified to use the bending formula, even if there is warping due to the shearing stress.

(Refer Slide Time: 14:58 - 21:03)

IIT Kharagpur

### Shear stress in beams with Circular cross section



$$\tau = \frac{VQ}{Ib}$$

$$Q = A\bar{y} = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3}$$

$$I = \frac{\pi r^4}{4} \quad b = 2r$$

$$\tau_{max} = \frac{VQ}{Ib} = \frac{V(2r^3/3)}{(\pi r^4/4)(2r)} = \frac{4V}{3A}$$

If the cross section of the beam is circular instead of rectangular then what is the consequence of this particular shear formula which we have derived earlier which is  $\tau = VQ/Ib$ ? Let us consider a circular section whose radius is  $r$  and the shear force which is acting in the cross section is in the vertical direction. As we have assumed earlier that the stress direction of the shear stress also will be in the direction of shearing force and the shear stress distribution across the width is uniform along the depth of the beam.

On the surface of this particular beam there will not be any stress and it will be 0. The stresses which we will have are in the tangential direction and hence they are no longer going to be parallel to the y axis. It is very difficult to use this particular formula as it cannot be applied in evaluating the shear stresses in this particular location.

At this particular point or along this diametral width the shearing stress is in parallel with y axis and we can compute the shear stress only at this particular location. Now as we have seen in the case of the rectangular one, the shear stress is maximum at the neutral axis, which is true for the circular section as well.

At the neutral axis position which is along the diameter the maximum shearing stress occurs. So, if we compute the shearing stress value at the neutral axis we will get the maximum value of the shearing stress in the circular cross section. Along that particular diametral width we assume that the shear stress is uniform over the diameter and parallel to the y axis.

Hence we can use the shear formula exclusively for that particular location and we cannot use the shear formula in the upper and the lower part using this particular expression to evaluate the shearing stress. We can use the shear formula for evaluating the maximum value of the shearing stress in this circular cross section. Hence the value of  $\tau_{\max}$  is  $VQ/Ib$  for which Q is the section above this particular cross section where we are evaluating the shear stress.

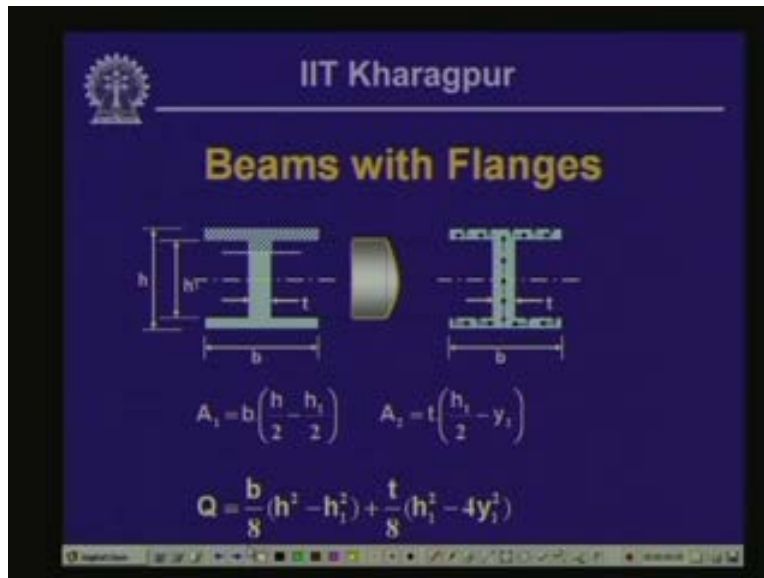
We take the upper half of the area for evaluating the value of Q and  $Q = \{Ay\}$  and A of the upper half is  $\frac{\pi r^2}{2}$  and the distance of cg from the neutral axis is  $\frac{4r}{3\pi}$ . The value of I is  $\frac{\pi r^4}{4}$  and V is the diameter which is 2r. So if we substitute the value of Q, I and b in this expression for  $\tau_{\max}$  we get the value of maximum shearing stress as  $\frac{4}{3}$  multiplied by  $V/A$  and we have  $V/A$ .

Here,  $\tau_{\text{average}}$  is  $V/A$  and that multiplied by  $4/3$  will give us the maximum shear for the circular cross section. Hence though we cannot use the shear formula for evaluating the shear stress over the entire cross section which is quite complex, we can evaluate the maximum shear stress at the neutral axis position using this shear formula.

The cross section is an annular section or a tubular section where one part is open. Let us assume that the outer radius of this is  $r_2$  and the inner radius is  $r_1$ . As we have seen that the moment of inertia  $I$  is  $\pi/4r$  to the power of 4. If we subtract the internal circular part from the external one we get the 'I' of the annular part. So this is  $\pi/4(r_2$  to the power of 4  $- r_1$  to the power of 4). Here  $Q$  is  $2r/3$  and in place of  $r$  we are substituting  $r_2$  to the power of 3  $- r_1$  to the power of 3(b) which is  $= 2r$  and we are substituting  $r$  with  $r_2 - r_1$  the external radius - the internal radius and let us substitute this value of  $I$ ,  $Q$  and  $b$  in this expression for the maximum shear stress.

In this case also we compute the shear stress at the neutral axis position which is the maximum shear stress and which is given by this particular expression  $4V/3A$ . It is a solid circular cross section which gets modified by this particular parameter where  $r_2$  and  $r_1$  are the two radii of the external and internal core of this particular circular section. So the values which get modified are  $4V/3A (r_2$  to the power of 2  $+ r_2 r_1 + r_1$  to the power of 2) / ( $r_2$  to the power of 2  $+ r_1$  to the power of 2) where area  $A$  is  $\pi (r_2$  to the power of 2  $- r_1$  to the power of 1).

(Refer Slide Time: 21:04 - 28:53)



Take another type of cross section where the cross section of the beam is similar to that of 'I'. If you remember while discussing the bending stress, we have discussed that if we take a beam and if we have two rectangular sections placed at a distance apart, then that contributes more to the value of the moment of inertia and thereby we get maximum effect in the bending stresses.

Now the question was how are two rectangular strips placed apart going to be utilized as cross sections? Subsequently we have seen that if these two particular sections are connected by a vertical strip, then we can use that section as a beam cross section and that turns out to be the most economical section when we talk about the transfer of the bending stress. Now if we use those kinds of sections for a beam where they are subjected to load and thereby they are subjected to bending moment and shearing force, then what is the consequence of shearing stress on such sections?

The section 'I' is shown over here. We call this part of the beam as Flange and consequently this is also a Flange and the part which connects these two flanges is the web. Now the width of the flange is  $b$  and the thickness is  $h - h_1/2$ . Now this distance is  $h/2$  and this distance is  $h_1/2$ . Thereby this distance is  $h/2 - h_1/2$ . If we are interested in evaluating the shearing stress at any

cross section which is at a distance of  $y_1$  from the neutral axis, then we need to calculate the moment of this area which is above the section, where we are considering the shearing stress.

Please note over here that when we have computed the shear formula we have assumed that the shear stress is uniform across the width. In this kind of section if the width becomes larger then the shear stress is no longer uniform over the width and if you use this formula it is expected that the results which you are going to get will be erroneous. Since the width of the flange is substantially large in comparison, in addition to the vertical shear which we have in the flange we get the horizontal shear also.

Now because of this horizontal shear the shearing stress distribution in the flange part is not uniform and consequently if we apply the shear formula for evaluating the shear stress in the flange zone then it is going to be ineffective or erroneous. When we evaluate the shear stress in the cross section we are calculating the shearing stress for the web part only and in fact you will notice that the maximum percentage of the shear force is being carried by the web and in the process the whole section becomes effective in carrying the bending and shear.

In the case of bending we have observed that if we have two rectangular components at the top and bottom, they are effective in carrying the bending stress. The whole section in combination now is effective in carrying the bending and the shearing action. In the web part which is similar to that of a rectangular section we compute the shearing stress.

We calculate the distribution of the shearing stress which is a parabolic distribution. We will have the maximum value here which is  $\tau_{o_{max}}$  and  $\tau_{o_{mm}}$ . Since we are evaluating the shear stress at this cross section which is at a distance of  $y_1$  from the neutral axis, we take the moment of this particular part of the area and divide it in two segments and we call them rectangle 1 and rectangle 2.

Now for rectangle 1, the width is  $b$  and thickness is  $h/2 - h_1/2$ . Hence this is the area  $a_1$  for the second part area  $a_2$  is the thickness  $t$  multiplied by  $h_1/2 - y_1$ . Consequently, as you know  $Q = Ay$  Bar. So in the first rectangle  $y$  Bar we have  $h/2 - h_1/2$  which is the thickness and half of that,  $1/2$  of  $h/2 - h_1/2$  is the distance of cg from the top or the bottom.


If I add this particular distance we will get  $y$  Bar of the first rectangle and for the second rectangle we get the distance. So this distance is  $h_1/2 - y_1$  half of that is the cg distance, plus if we add  $y_1$  to that this will give me  $y$  bar for the second rectangle. When  $a_1$  is multiplied by  $y$  bar and  $a_2$  is multiplied by  $y$  bar we will get the value of  $Q$ .

In this particular expression note that  $y_1$  to the power of 2 is constant. Hence the shear stress varies again with respect to  $y_1$  and in a parabolic manner in a square quadratic form of  $y_1$  if  $y_1$  becomes 0 then we get the maximum value of the shearing stress and  $y_1 = h_1/2$  gives us the minimum value of the shearing stress.

We will get the maximum shear stress over here and this will give us the value of the minimum shearing strain. At this particular section the shearing stress is 0 and at this particular section shearing stress is minimum. Hence at this particular cross section there is a complex distribution of the shearing stress and as a result we cannot apply the shear formula for evaluating the shearing stress in the plane zone.

Let us restrict ourselves to the web part and calculate the shearing stress and we will see consequently that the maximum shearing stress is being carried by the web part only. Hence in order to compute what is the value of the shearing force, the web is going to carry the area of the stress diagram. The web part that is multiplied by the thickness of the web will give us the value of the shearing force that the section will be subjected to.

(Refer Slide Time: 28:54 - 29:32)




IIT Kharagpur

### Beams with Flanges

$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3)$$
$$\tau = \frac{VQ}{It} = \frac{V}{8It} [b(h^2 - h_1^2) + t(h_1^2 - 4y_1^2)]$$

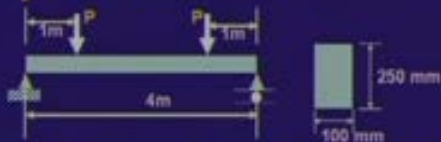
Well these are the values of  $I$  of the cross section which is  $bh$  to the power of  $3/12$  for the whole section - the 2 sides of it  $(b-t)h_1$  to the power of  $3/12$ . So this is the value of the moment of inertia  $I$  and thereby the shearing stress  $t$  and this is the  $Q$  of the section above the neutral axis and this is the value of the shearing stress that we are going to get and since  $y_1$  varies we get the variation from the bottom part to the top part.

(Refer Slide Time: 29:33 - 31:21)



IIT Kharagpur

### Example Problem - 1



- A beam AB is subjected to loads as shown in the figure. Determine the maximum allowable value of  $P$ , if limiting value of bending stress is 60 MPa and that of shearing stress is 10 MPa.

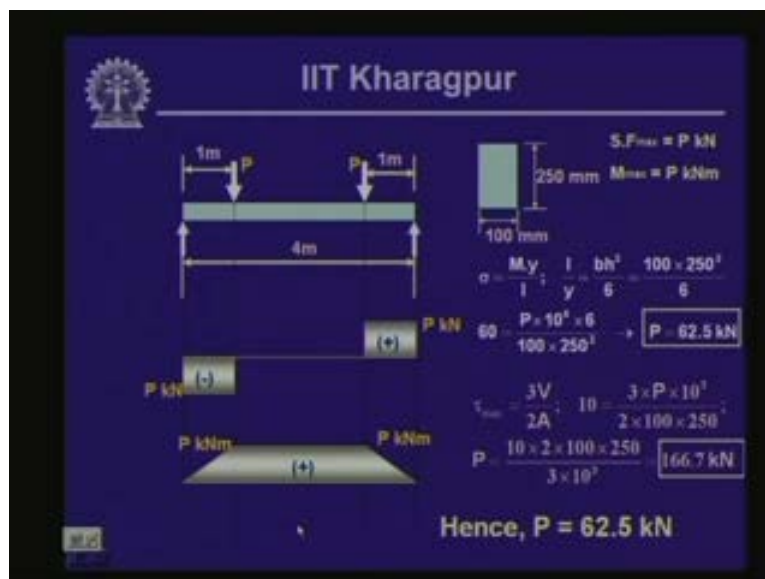


Let us look at some examples I had given you in the last lesson. The beam is subjected to loads P as shown over here where it is a two point loading and they are placed equidistant from the two sides at the distance of 1 meter. Now you will have to determine the maximum allowable value of P if the limiting value of bending stress is 60 MPa and the limiting value of shearing stress is 10 MPa and the cross section of the beam is a rectangular one.

Now please note over here that you will have to find out the maximum value of P in such a way that the bending stress does not exceed its allowable limit and consequently the shearing stress also does not exceed the allowable limit. You will have to satisfy both the criteria and satisfying both the criteria you have to prescribe how much load you can apply in this particular form so that the stresses are not exceeded.

Since we are going to deal with the maximum possible value of the bending stress and the shear stress that will be generated on this section, we need to know the variations of the bending moment and the shear force along the length of the beam. Thereby we need to compute the bending moment and shear force diagram so that we can find out the value of the maximum bending moment and the maximum shear force that is occurring at any point in the beam. Consequently we can also compute the value of the stresses.

(Refer Slide Time: 31:22 - 36:29)



Let us compute the bending moment and shear force diagram. The first step is to evaluate the unknown reactive forces. Let us call this beam as A and B and end A is on hinge so you have a vertical force  $R_A$  and a horizontal force  $H_A$  and the vertical force at B as  $R_B$  as the reactive forces. The summation of horizontal forces = 0 will give you  $H_A = 0$  and you can compute the value of  $R_A$  and  $R_B$ .

Since this is a symmetrical beam with symmetrical loading  $R_A = R_B$  will be = P. We can compute the value of shearing forces at any cross section and we can draw the shear force diagram over the left region between the support and the load. We will have a shear force of longitude P kN which is negative and between the second load point and last reaction we have again P kN which is positive.

The maximum value of the shear force occurring anywhere in the beam is P kN and in between the loads there are no shear forces at all. So, the maximum shear force is = p kN and consequently if you draw the bending moment diagram you will find that the bending moment at these two supports is 0. The moment value here is going to be  $p_1$  which is p kilo Nm and it is constant about this zone. Hence the shear stress is 0 over here.

The maximum value of the bending moment that you get is p kilo Nm and that is occurring at a load point and maximum shear force is occurring within the support, and the load point. So, the maximum shear force and the maximum bending moment  $\sigma$  which is =  $MY/I$  acting in the beam Y, the furthest point from the neutral axis, will give you the maximum stress either at the top or at the bottom and 'I' is the moment of inertia of the cross section with the neutral axis.


Now as we know that for rectangular section  $I = \frac{bh^3}{12}$  and  $Y$  for this distance is  $\frac{h}{2}$ , so this is  $\frac{2}{h}$  this is going to be  $= \frac{bh^2}{6}$  so  $I/y = \frac{bh^2}{6}$  and if you substitute the values you get the values of  $\frac{bh^2}{6}$ . Now the bending stress is limited to 60 MPa and  $m$  we know as kilo Nm. So if we substitute these values of stress the  $P$  and  $I/y$  which is  $100 \cdot 250^2/6$ , from these we get the value of  $P$  as  $= 62.5$  kN.

From the maximum bending stress criteria we get the value of  $P$  is  $= 62.52$  kN. Now let us look into the value of  $P$  which we get if we have to satisfy maximum shearing stress criteria, Now as we know that the maximum shear stress in a rectangular cross section occurs again at the neutral axis since it varies parabolically and the maximum value which we get is  $= \frac{3V}{2A}$ . Now maximum shear stress is  $= 10$  MPa  $V$  we have obtain as  $P$  maximum value of  $V$  and cross sectional area is  $100 \cdot 250$ .

If we compute it from this the value of  $P$  come out to be 166.7 kN. So now that you have two values of  $P$  and you have to choose the most appropriate one, now if you use the higher value of these two,  $P = 166.7$  N, if we use that value then the bending stress is going to be go beyond the 60 MPa. That is the limiting value because we have obtain the value of  $p$  corresponding to the bending stress as  $= 62.5$  kN and if we use 62.5 kN as the loading then shearing stress will be lower than 10 MPa.

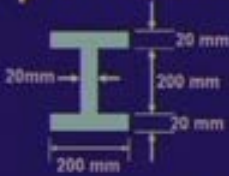
Since we cannot go beyond the allowable limit of the stresses we have to use the lower value of the beam so that both the stress criteria, bending stress and the shearing stress are satisfied and the member is safe against this loading. The value of the  $P$  which should be used is 62.5 kN. Now let us look into another example problem in which there is an application of the shearing stress. Now the maximum vertical shear force acting on this beam cross section is given as 100kN.

(Refer Slide Time: 36:30 - 37:55)



IIT Kharagpur

### Example Problem - 2

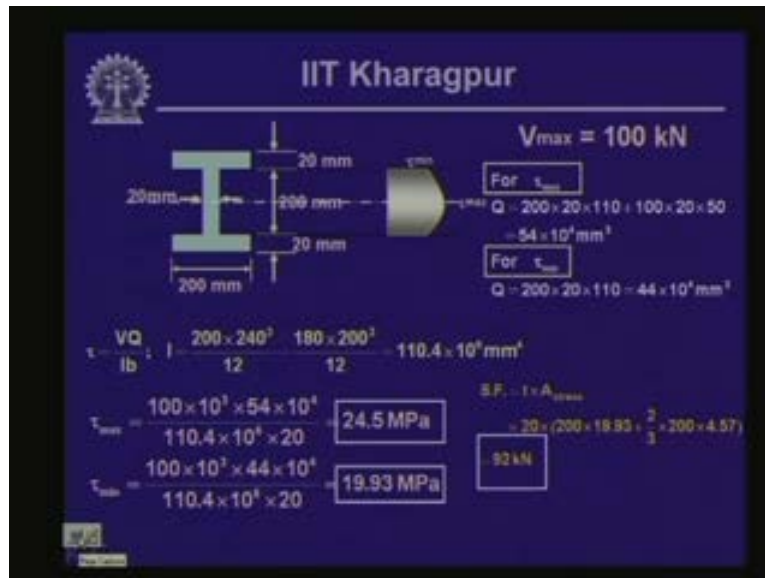


- The maximum vertical shear force acting in a beam of cross section as shown in the figure, is 100 kN. (i) Compute the maximum shear stress acting on the section; (ii) percentage of shear force carried by web.

Now we have computed the shearing force, the cross section of the beam and the value of the shear force is known. We have to compute what is the maximum shearing stress that will occur in this particular section and how much shear force is going to be carried by the web out of the total shear force in this particular cross section.

You have to compute the maximum shear stress acting on this particular section and then percentage of shear force that will be carried by the web of the beam. Now the cross section of this is an 'I' section; this is a symmetrical one so the position of the neutral axis will divide the section into two equal halves. This is the position of the neutral axis and we have to compute the maximum value of the shear stress which will be acting at the neutral axis location.

(Refer Slide Time: 37:56 - 45:40)



Let us compute the value of Q for evaluating the shearing stress at this particular location and since we are considering this particular section, the area above this is to be considered for evaluating the first moment which is  $Q = Ay$  bar. We also need to compute the shearing stress at this particular location because we need to compute the value of the shearing stress in the web as flanges.

We cannot calculate the shearing stress in the flanges using this shear formula, so when we will be computing the shearing stress at this particular location, the area to be considered is this particular part. So that is why the value of Q will be different for evaluating the shearing stress in different places. When we are going to compute the shearing stress at this particular location we have two rectangular components and width 200(20) into the distance of it is cg from the neutral axis.

The distance of cg from the top is 10 and this is 120 and from here to the neutral axis it is 120. So,  $120 - 10$  is 110 and for the second rectangle the area is  $20(100)$  which is the area and its cg distance from the neutral axis is 50. For this particular section when we are evaluating the stress where the shearing stress is maximum for  $\tau_{\max}$  this is the value of the cube and if you compute this we get  $54(10 \text{ to the power of } 4 \text{ mm to the power of } 3)$ .

Here  $Q$  is  $Ay$  bar and the unit of  $A$  is millimeter to the power of 2 and  $y$  bar is millimeter and hence this is mm to the power of 3. Now for computing the minimum shear stress to  $\tau_{\min}$  which is at this particular section, the area to be considered for  $Q$  is the area above this particular section which is this rectangle alone and the area of this rectangle is 200. Then 20 is the thickness and 110 is the distance of its cg from the neutral axis because while 20 is the distance from the central axis to the top -the cg distance from top to this cg which is  $10 = 110$ .

Here  $Q$  for the top rectangle only is this which is required for the evaluation of the minimum shearing stress and we know  $\tau_{\max}$ , the shearing stress  $VQ/Ib$  and  $I$  of this particular section. If we consider the whole thing as one rectangle and if we compute the moment of inertia with respect to this neutral axis, then 200 and height is  $200(240 \text{ to the power of } 3) / 12$  for the entire rectangle minus the width which is  $200 - 20 = 180$  multiplied by this height which is  $(180) (200 \text{ to the power of } 3) / 12$ .

If we subtract that we get  $1110.4$  (10 to the power of 6 mm to the power of 4) .So this is the value of moment of inertia  $I$  and let us compute the maximum shear stress  $Q$ . The shearing value is  $100\text{kN}$  (10 to the power of 3)  $N$   $54(10 \text{ to the power of } 4)$  is  $Q$  and  $I$  is  $110.4(10 \text{ to the power of } 6(t))$  which is 20 and this gives us a value of 24.5 MPa.

Consequently, this is the maximum shearing stress at the neutral axis; this is the value of the maximum shear stress and this is  $\tau_{\max}$ . The minimum shear stress which is at this particular location is given by this expression  $VQ/Ib$ . There is only a change in the value of  $Q$  which is  $44(10 \text{ to the power of } 4)$  and thereby we get a stress of 19 MPa which is at a particular location. So this is the value of  $\tau_{\min}$ .

We compute the maximum stress and the minimum stress and this is the maximum value of the shearing stress that will be occurring in the beam section when they are subjected to the shear force of 100 kN and the maximum value of the shear stress is at the neutral axis and the magnitude of that is 24.5 MPa.

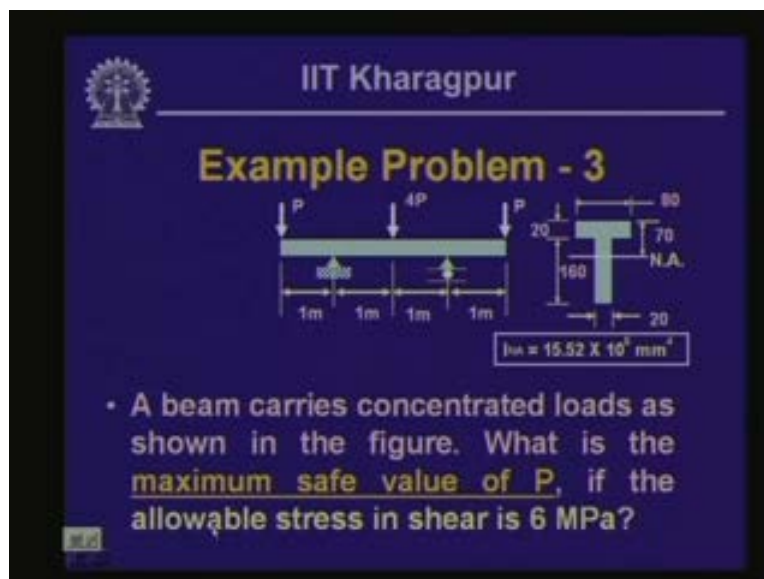
Let us look into the second aspect of it in which we need to find out the shear force out of 100 kN the web part of is going to carry. Since this is the shear stress distribution over the web of the beam, the force as we know is stress times area. If we take the area of this shear stress multiplied by the thickness, it will give us the shear force that will be carried by web. The shear force which is being carried by web is equal to the thickness of the web multiplied by the area of the stress diagram.

We can divide this area of the stress diagram into two parts; one is the rectangular part and another one is this parabolic distribution. Now this value is  $\tau_{\min}$  and this is height  $h$  which is = 200. So,  $200(19.93)$  is the minimum value of the shearing stress which is this particular rectangular area and this parabolic area will be  $2/3$  of this height times this distance.

Now this distance is  $\tau_{\text{max}} - \tau_{\text{min}}$  which means  $24.5 - 19.93$  will give us the value of  $4.47$ . So,  $\frac{2}{3} \cdot 200(4.47)$  is the area of this parabolic path. So this gives us the area of this inter stress diagram and with the thickness, we get the value of the shearing force and this turns out to be  $92$  kN. Please note over here that out of the  $100$  kN shear force that the entire cross section is subjected  $92$  kN shear force is being carried by the web alone. Hence the  $92\%$  of the stress total shearing force is being carried by the web.

We have called the top and the bottom rectangle flanges. This flange is effective in carrying the bending stress whereas the web is quite effective in carrying the shearing stress because  $92\%$  of the shear force is being carried by this particular web. Hence in fact in normal design or in an engineering design where we do not really need to compute the stresses, we assume that the shearing stress is being carried by the web alone and the bending stresses are being carried by the flanges of this section and that simplifies the calculation to a large extent.

(Refer Slide Time: 45:41 - 48:05)



The slide features the IIT Kharagpur logo and title at the top. Below it, the text 'Example Problem - 3' is displayed in yellow. The main diagram shows a beam of length 4m with three concentrated loads:  $P$  at 1m,  $4P$  at 2m, and  $P$  at 3m from the left end. The beam is supported by a roller at 1m and a pin support at 3m. To the right, a T-section cross-section is shown with dimensions: top flange width 80, web thickness 20, bottom flange width 20, and total height 160. The neutral axis (N.A.) is located 70 units from the top. A box below the cross-section states  $I_{NA} = 15.92 \times 10^8 \text{ mm}^4$ . At the bottom, a bullet point asks for the maximum safe value of  $P$  given an allowable shear stress of 6 MPa.

IIT Kharagpur

### Example Problem - 3

$P$   $4P$   $P$

1m 1m 1m 1m

80 20 70 20 160 N.A.

$I_{NA} = 15.92 \times 10^8 \text{ mm}^4$

- A beam carries concentrated loads as shown in the figure. What is the maximum safe value of  $P$ , if the allowable stress in shear is 6 MPa?



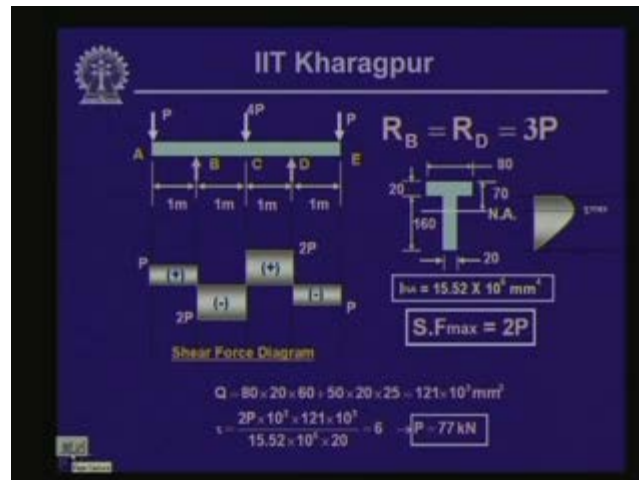
Now let us look into another example where we have a beam and is loaded in this particular form and the loads are  $P$  at the two ends and  $4P$  at the center .Now this particular beam is hind at this particular end, let us call this as A, and this end as B ,this point as C ,this D, and this E, so it is hinged at B, and roller support at D, over hang between B and A, and D and E ,and this the loading it has on the cross section. If this beam here is similar to that of T section.

Also we have discussed as in the case of the 'I' section that we cannot use a shear formula for evaluating the shear stress in the flanges of the 'I' beam. For T section also, if we compute the same shearing stress in flange using the shear formula it is not going to be accurate or correct. Hence we restrict ourselves to the evaluation of shear stress only in the web part of the beam. We call the top part the flange of the T beam or the T cross section and we have the web part of the T cross section.

Here the neutral axis position is given which is at a distance of 70 mm from the top of the beam and the moment of inertia of this particular cross section also is given over here. We would like to know the maximum sheer force value of  $p$  so that the shearing stress does not exceed beyond this allowable stress limit of 6 Mpa but is silent about the maximum allowable bending stress.

Since it is not indicated with regard to the bending stress we compare our actual shearing stress that is going to happen in the beam because of such loading and if we equate it to the maximum allowable shearing stress then we can find out what is the value of  $P$  that we need so that the shearing stress does not go beyond the 6 MPa value.

(Refer Slide Time: 48:06 - 50:36)

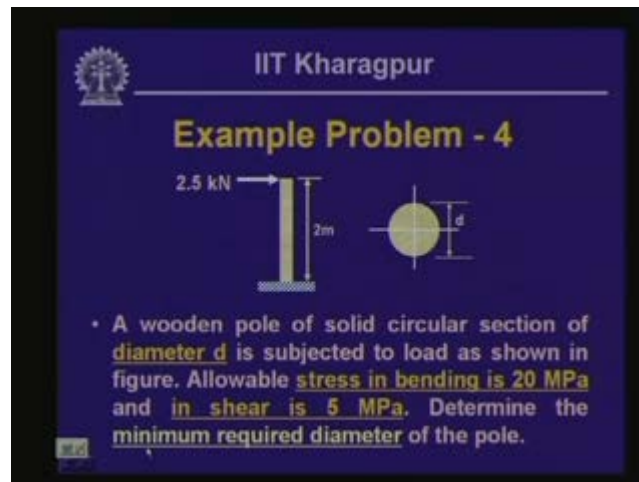


Let us look into the shear force diagram of this particular beam and as usual let us compute the value of the reactive forces  $R_B$  and  $R_D$ . Here  $R_D$  is going to be the shear force diagram and the maximum value of the shear force is  $2P$ . This is the maximum value of the shearing force which is indicated over here and the value of the moment of inertia is already given. In the case of a rectangular section we get a maximum amount of shear stress at the neutral axis. For this rectangular component now at this particular section our shearing stress is not 0 but you will have some value which is called as  $\tau_{\text{min}}$  because you have some area for which you get the  $Q$  value which is  $A_y \bar{y}$ .

While taking the rectangular section above the rectangular part, we did not have anything and that was the value of the 0 shear stress. Above the web we have the flange area which is contributes to the first moment of area  $Q$  but we are calculating the shearing stress from the interface and downward so we have some value of the shear stress at the interface between the web and the flange and we are calling that the minimum shearing stress which is  $\tau_{\text{min}}$ .

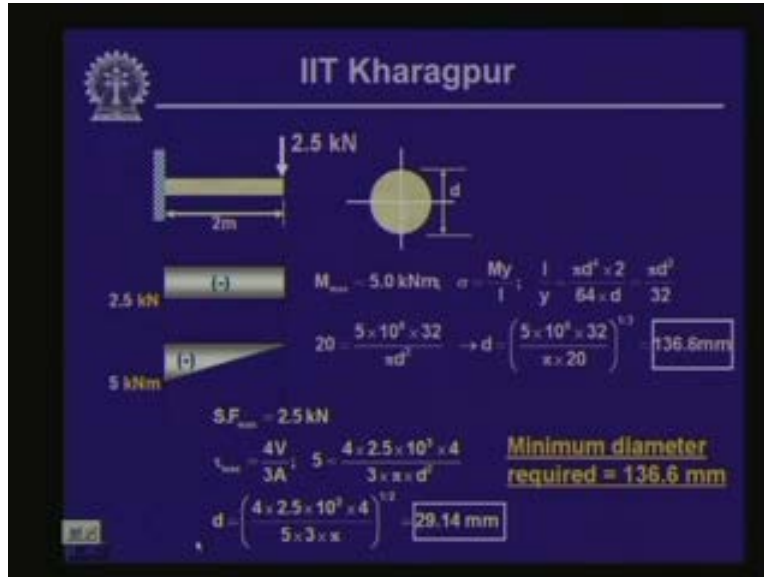
If you compute the value of this maximum shearing stress at the neutral axis then again from  $\tau_{ao}$ ,  $VQ/ITV$  is known which is  $2pQ$  and that is  $A_y \bar{y}$  which is above this neutral axis and this is what  $Q$  is. Here we have two rectangles one is  $80(20)$  and the distance of  $cg$  from the neutral axis is  $70 - 10$  which is  $60$ . The second rectangle is  $50(20)$  and that distance of  $cg$  from neutral axis is  $25$  and this gives us a value of  $120(10 \text{ to the power of } 3)$   $\text{mm to the power of } 3$ . If we substitute these values, we get the value of  $P$  as  $77 \text{ kN}$ . So the maximum value of  $P$  which can be applied in this beam is  $77 \text{ kN}$  so that the shearing stress does not exceed a value of  $6 \text{ MPa}$  as stipulated over here.

(Refer Slide Time: 50:37 - 51:31)



Let us look into this particular problem which has relevance with the discussion we had today and it is that the cross section of this particular pole is a wooden pole having a solid circular section of diameter  $d$  and this is subjected to a lateral load of  $2.5 \text{ kN}$  at the top. Now as you can make out, this particular member is subjected to this load and you can orient this and take this as a cantilever beam subjected to a load at its tip which is of value  $2.5 \text{ kN}$  and this length is equal to  $2 \text{ meters}$ . We have to determine the diameter of this particular pole so that the stress in bending does not exceed  $20 \text{ MPa}$  and the stress in shear does not exceed  $5 \text{ MPa}$ .

(Refer Slide Time: 51:32 - 53:37)




If we draw the bending moment and shear force diagram for this particular pole then the bending moment and shear force diagram is going to be like this, which we have already done earlier. The maximum shear force is going to be 2.5 kN and the maximum bending moment is going to be 5 kilo Nm. Now if we compute the bending stress from this particular expression which is  $\text{Sigma} = My/I$  and the maximum bending moment is 5 kilo Nm.

The moment of inertia with respect to the neutral axis is  $\pi d^4/64$  and the extreme distance which is from the neutral axis is  $d/2$  and thus the point where you are going to get the maximum bending stress. So  $I/y = \pi d^4/32$  and we have  $\pi d^3$  to the power of  $3/32$  and if you substitute these values limiting the bending stress to 20 MPa, we get the diameter of this particular pole as 136.6 mm and corresponding to the maximum shear force of 2.5 kN. We need to evaluate the maximum shearing stress as we cannot compute the shearing stress at any other point in this particular circular cross section.


So,  $\tau_{\text{Omax}} = 4V/3A$  and if we substitute the limiting value of the shearing stress from this particular expression we get the value of  $d$  as = T 29.14mm. Now out of these 2 values where corresponding to bending we have  $d$  as 136.6 mm corresponding to shear, we have 29.14 mm. Naturally we will have to go for the larger diameter so that it can withstand both the stresses and when we provide the lower diameter it will be safe but not against bending. However if you go for the larger diameter then it will satisfy both the criteria hence the diameter of the pole has to be minimum as 136.6 mm so that it can withstand both the stress or withstand this amount of load, so that the bending stress and shear stress are within limits.

(Refer Slide Time: 53:38 - 54:40)



IIT Kharagpur

### Example Problem - 5

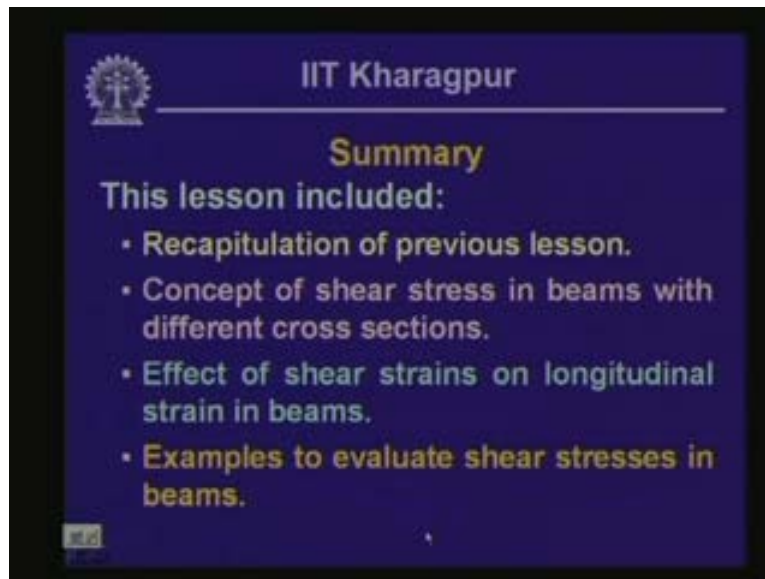


- A beam of T cross section is formed using nails as shown in figure. If the **total shear force** acting on the cross section is 872 N and each nail carries **400 N in shear**, what is the maximum allowable spacing of nails?

Another problem which is again the cross section of beam is just like a T. Here two rectangular components are there. One rectangle is on the top, one is at the bottom and these two rectangular sections are interconnected along the length using nails and these nails are spaced. We will have to find out the particular spacing so that we know the shearing capacity of these bolts.

These bolts are subjected to shearing load and 872 N is the shear force that the particular cross section is subjected to and the nail carrying capacity is 400 N. Then you need to evaluate the spacing so that you know that these particular bolts do not fail against the shearing stress. This particular problem is given to you and we will discuss next time and let us try to solve this problem and see whether you get the result.

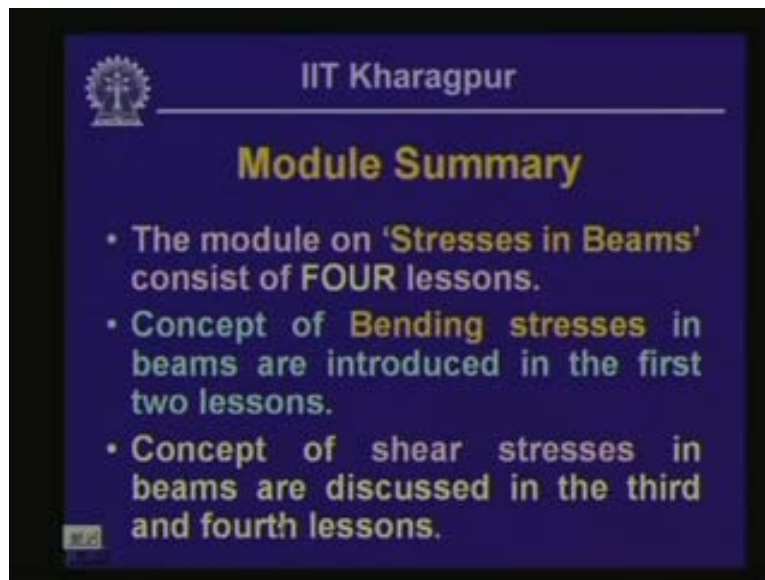
(Refer Slide Time: 54:43 - 55:14)



To summarize what we have discussed in the particular section, we have recapitulated the previous lesson, we have introduced the concept of shear stress in beams with different cross sections other than the rectangular one in the previous case and we have discussed about a cross section where a cross section is a rectangular one. We have talked about the circular or T or I section and then the effect of shear strains on longitudinal strains in beams. Also we have looked at some examples to evaluate the shear stress in beams.

In this module of stresses in beams which consists of four lessons we have divided it into two parts. The first two lessons are focused on the aspects of bending stresses, the last two lessons were focused on the aspects of shearing stresses and thereby we have seen the effects of loads in terms of bending and shearing stresses in beams.

(Refer Slide Time: 55:15 - 55:41)



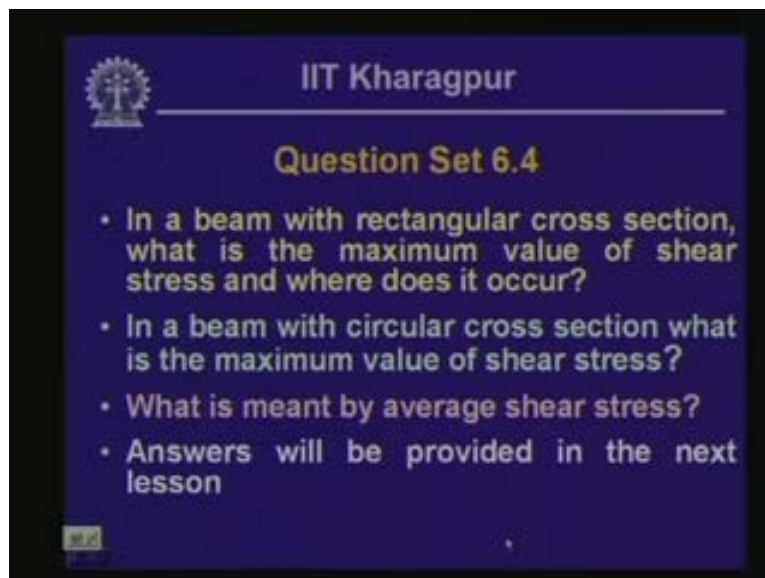
The slide features the IIT Kharagpur logo in the top left corner. The text is centered and reads: "IIT Kharagpur" followed by a horizontal line, then "Module Summary" in a larger font. Below this, there are three bullet points: "The module on 'Stresses in Beams' consist of FOUR lessons.", "Concept of Bending stresses in beams are introduced in the first two lessons.", and "Concept of shear stresses in beams are discussed in the third and fourth lessons." A small navigation icon is visible in the bottom left corner.

IIT Kharagpur

### Module Summary

- The module on 'Stresses in Beams' consist of FOUR lessons.
- Concept of Bending stresses in beams are introduced in the first two lessons.
- Concept of shear stresses in beams are discussed in the third and fourth lessons.

(Refer Slide Time: 55:42 - 56:01)



The slide features the IIT Kharagpur logo in the top left corner. The text is centered and reads: "IIT Kharagpur" followed by a horizontal line, then "Question Set 6.4" in a larger font. Below this, there are four bullet points: "In a beam with rectangular cross section, what is the maximum value of shear stress and where does it occur?", "In a beam with circular cross section what is the maximum value of shear stress?", "What is meant by average shear stress?", and "Answers will be provided in the next lesson". A small navigation icon is visible in the bottom left corner.

IIT Kharagpur

### Question Set 6.4

- In a beam with rectangular cross section, what is the maximum value of shear stress and where does it occur?
- In a beam with circular cross section what is the maximum value of shear stress?
- What is meant by average shear stress?
- Answers will be provided in the next lesson

The questions that I have set for you are that in a beam with a rectangular cross section, what is the maximum value of shear stress and where does it occur? In a beam with a circular cross section what is the maximum value of shear stress and what is meant by average shear stress? While we were discussing, we had looked into a term called average shear stress but what do we really mean by average stress? Look into these questions and try to find answers for them. We will discuss these questions in the next lesson.