# Strength of Materials

**Prof: S .K.Bhattacharya**
**Dept of Civil Engineering,**
**IIT, Kharagpur**

**Lecture no 28**

**Stresses in Beams- III**

Welcome to the 3rd lesson of the 6th module which is on Stresses in Beams part 3. In fact in the last 2 lessons, on this particular module, we have discussed aspects of the bending stresses in beams. In this particular lesson, we are going to look into some more aspects of the stresses and we will be looking into the effect of shearing stress in beams.

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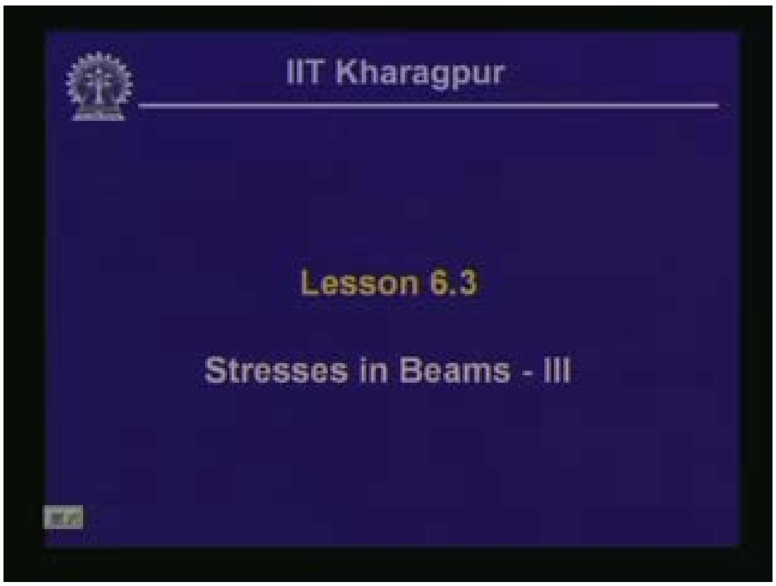


It is expected that once this particular lesson is completed one should be able to understand the concept of shear stress in beams of a rectangular cross section, one should be able to derive the shear formula for evaluating the shearing stress in a beam subjected to loads and one should be in a position to evaluate shear stress in beams of rectangular cross sections for different loading.

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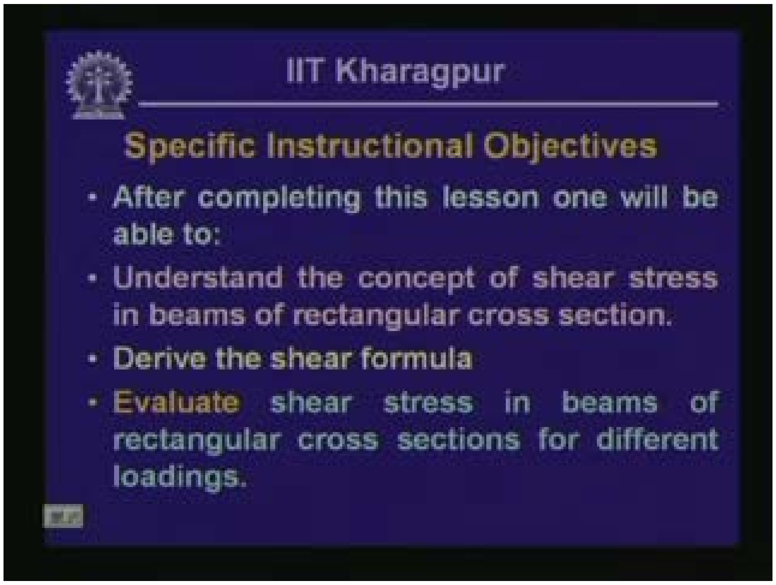


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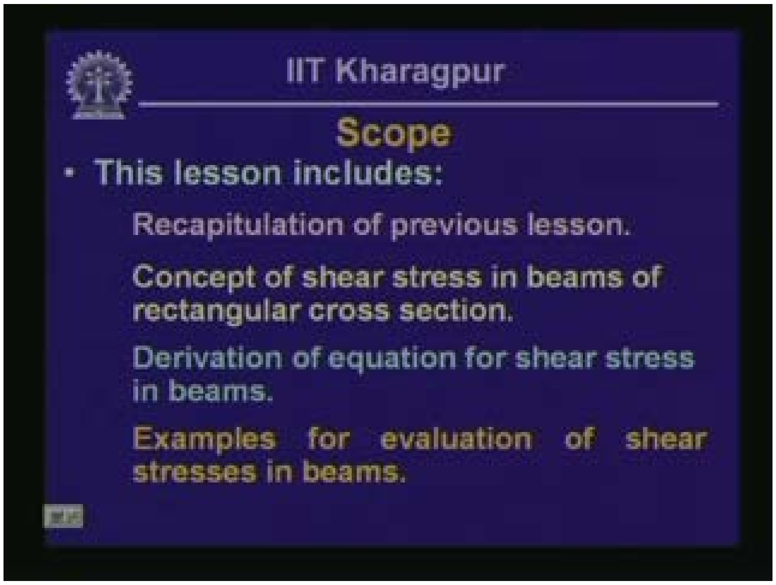


Hence the scope of this particular lesson includes recapitulation of the previous lesson, the concept of shear stress in beams of rectangular cross section, derivation of equation for shear stress in beams and examples for the evaluation of shear stress in beams. We will be looking at some examples which we know and we will see how to compute the value of the shearing stresses in a beam which is subjected to a load. Let us look into the questions which were posed last time.

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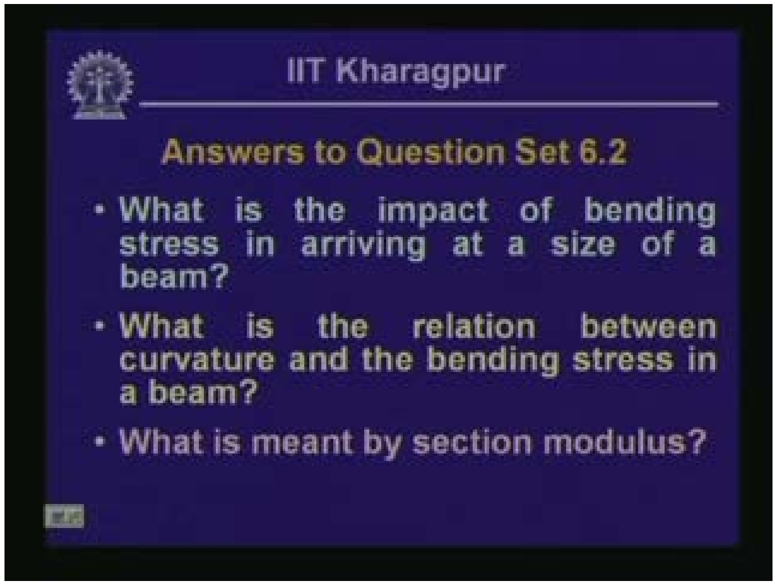


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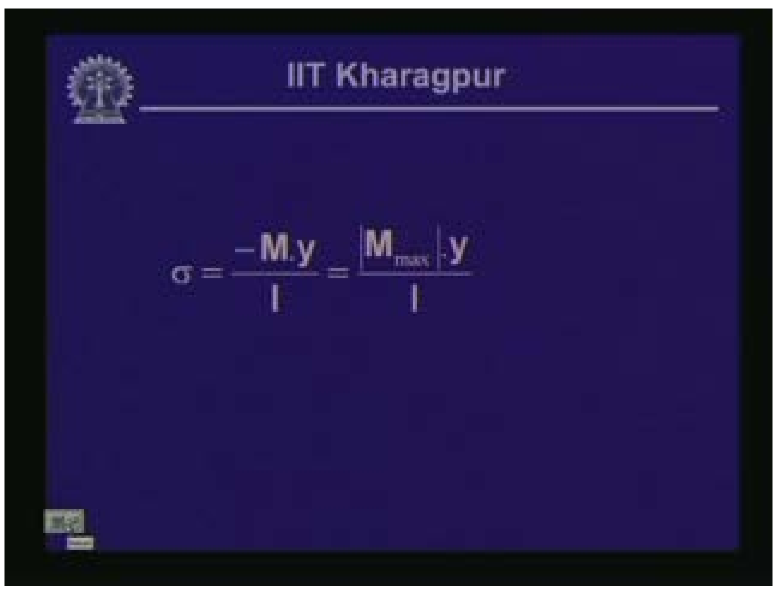


What is the impact of bending stress in arriving at a size of a beam? Last time, we had discussed that the flexible formula or the bending formula is given by this particular expression that Sigma = -MY/I. As you know M is the bending moment, Y is the point at which we are trying to evaluate the stress and the distance from the neutral axis; ‘I’ is the moment of inertia.

When we are talking about the symmetrical section, we will be dividing it into two halves because the positive bending moment will be of the same magnitude. We have written the absolute value of the bending moments as xY/I. In this particular expression, Sigma is the stress which is allowed in this particular member which is a material characteristic.

It means that if we know the allowable stress of that particular material then we know the stress limits of that section. The maximum bending moment $M_{max}$ can be evaluated from the external loading that is acting on the beam member and we have seen how to calculate the bending moment at different sections. If we draw the bending moment diagram then we will know at which location the maximum bending moment occurs and therefore we will know $M_{max}$ and Sigma.

These two parameters I and Y basically are dependent on the cross sectional shape of the particular beam member. So in I/Y, 'I' is the moment of inertia and Y is the distance of the point where we are evaluating the stress from the neutral axis which is Y. This I/Y is nothing but $M_{max}$/Sigma. We know that from the loading Sigma and from the materially allowable stress, we can get the value of I/Y and from this particular numerical value of I /Y, we can select a particular section.

We generally designate I/Y by Z and if we can select a member of this Z value, then if we go for a section which has a higher Z value than the Z required, then the section will be able to withstand the bending moment that is occurring because of the load. That means if we select a member which has the I/Y value larger than we are getting from this expression $M/\sigma$, then the member will be in a position to withstand this particular load but if we provide a section for which the Z value is less than I/Y, then naturally the stress level will go beyond the capability of that particular material and the material will fail.

Let us suppose the member is unsymmetrical with respect to x axis as we had discussed last time. If we have a cross section which is symmetrical with respect to Y axis but unsymmetrical with respect to x- axis, then this Y value will be different from the neutral axis for two ends and we can designate it as $Y_1$ and $Y_2$. Correspondingly, we will have two values of I/Y, we can call 1 as $Z_1$ which is equal to $I/Y_1$; other one will be $Z_2$ which is $I/Y_2$ smaller than Y $Y_1$ and $Y_2$ which have the same I.

We will have the larger value of Z and if we choose a particular section then that particular section will be able to withstand the stress. We will have to decide whether it is a symmetrical section or an unsymmetrical section with respect to x axis and accordingly we have to find out the value of I/Y. Correspondingly, we have to select a particular section which satisfies this particular requirement. This bending stress has a great impact in selecting a particular sectional configuration of a member where we will have to find out the size.

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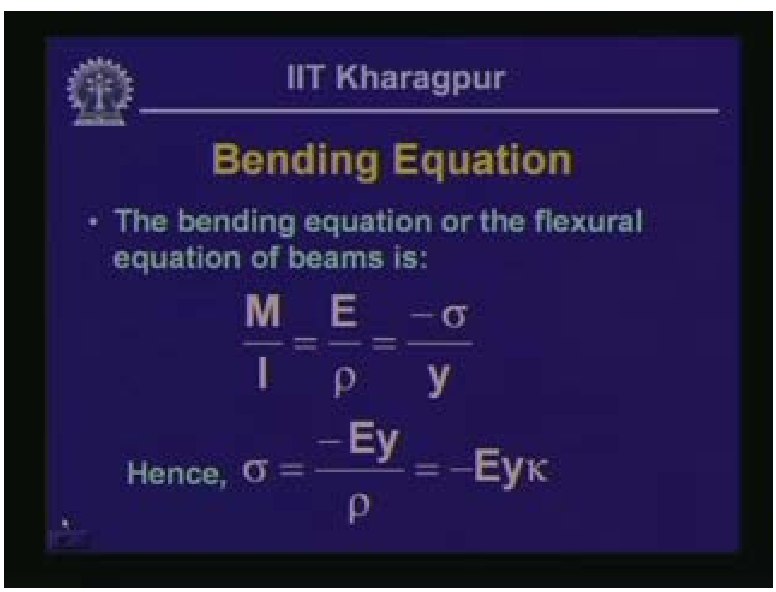

The next question is what is the relation between curvature and the bending stress in a beam? We have seen the bending equation which is M/I = E/ρ = Sigma/Y where M is the bending moment, I is the moment of inertia, E is the modulus of elasticity, Row is the radius of the axis of the beam, Sigma is the bending stress and Y is the distance of the point where we are evaluating the stress from the neutral axis and the normal stress gets generated because of the bending in the particular section.

If we take this particular relationship E/Row = -Sigma/Y, then we get Sigma = -EY/Row and Y/Row. This is how the stress is related to the curvature through this parameter E and Y. We can evaluate this at a particular point with respect to the neutral axis. The third question was what is meant by section modulus.

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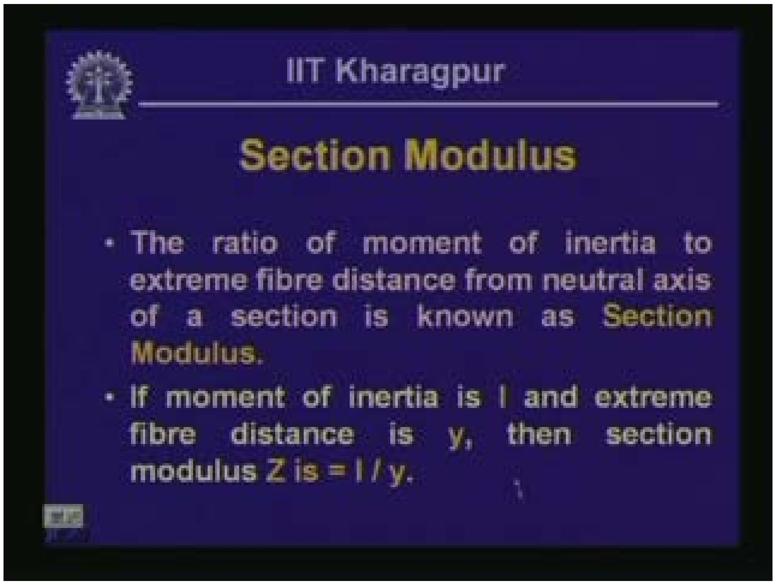


In fact, we have said that Z is a parameter which is I/Y and this is what we call section modulus. The ratio of moment of inertia of section to the extreme fiber distance from the neutral axis of the section is known as the section modulus. If I is the moment of inertia and $Y_{max}$ is the extreme distance from the neutral axis of the out of fiber then $I/Y_{max}$ = Z.

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In the previous two lessons we have computed the values of the bending stress. That means when beam members are subjected to a stress result which is a bending moment for that we could evaluate the value of the normal stress because of such a bending. As you have noticed that when beam members are subjected to loads, not only are they subjected to a bending moment but at location they are subjected to shear forces.

In some zones where the bending is associated with shear force we have to compute the value of the normal stress through the bending equation as we have done and we need to evaluate the value of the shearing stress as well. In this particular lesson, we are going to look into how to compute the value of the shearing stress at a particular location where we know the value of the shearing force for the external loading that the member is subjected to.

Let us look into this particular figure where we have taken a part of the beam segmented which is subjected to the loading. This is the positive shear force acting on this particular phase and this member is of a rectangular cross section having depth 8 and width 5 and we assume two aspects over here. One is that the shear stress is parallel with the shear force at that particular cross section and we also assume that the shear stresses are uniformly distributed across the width of the beam.

It is reasonable to assume that across the width of the beam, the shear stress is uniform otherwise there could be a variation of the shear stress along the depth of the beam but at a particular point along the width, we assume that the shear stress is uniform. We assume two aspects: one is that the direction of the shear stress is in line with the shearing force which is acting at a particular cross section and the distribution of the shear stress along the width is uniform. For example, if we take out a small segment from this particular beam, then we get the segmental size which is here.

Along the width, the shear stress is uniform as we have assumed at the moment and we have the complementary. As we have noticed earlier, if we take a section and if we have verticals here then we have horizontals here as well which we call as a complimentary. This is the positive shear stress which we have on the particular phase and consequently we have the horizontal shearing stress which is the complementary shear stress.

Note that if you take this particular element closer to the top surface where there is no other element and the shearing stress on the top surface is 0, at the top point since the horizontal shear stress is 0, the vertical shear stress at the extreme also is 0. At the top and at the bottom the shearing stress is 0 at +h/2.

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Let us evaluate what is the magnitude of the shearing stress in terms of the sectional parameters. Before we really go into the evaluation of the shearing stress, you can think of a small experiment. Take a number of small beams which are supported on supports and these beams individually are racing one after another. We apply a load on to the top of this particular beam and it is expected that it is undergoing deformation in this particular format as it is shown over here.

Since they are not connected with each other, the bottom part of this particular top beam will be undergoing extension or tension and the top fiber of this particular beam will be undergoing compression and there is expected to be some amount of slippage. If we say that the frictional resistance is small then there will be a substantial change in the length between these two surfaces.

This indicates that if we could hold all these three segments together by applications of glue or nailing, then the whole system would have to act as one unit. This indicates that some amount of stresses are acting at the interface between this element and this is what is horizontal shear stress which retains these three segments together if they are tied together and they can resist the load in terms of the stresses.

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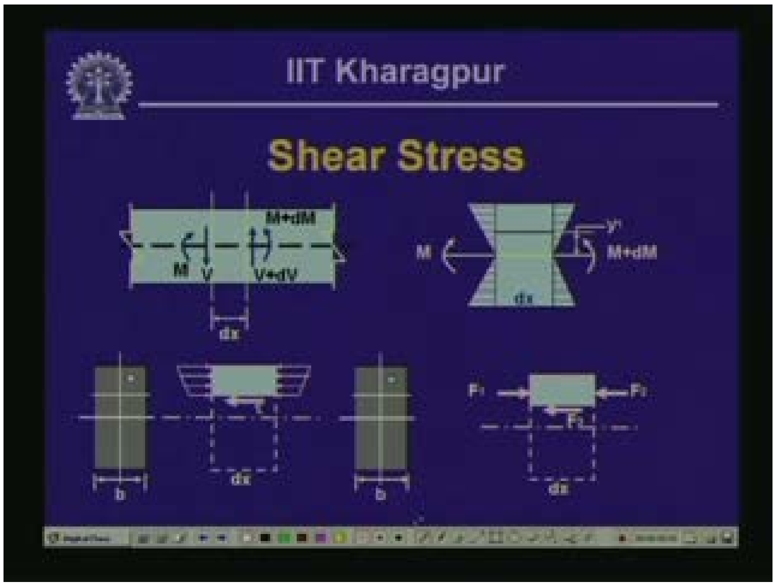


Let us see how to compute the value of the shearing stress in a beam. Consider a segment of the beam which is loaded and let us say these are two sections at a distance of dx. Let us call this section as ‘aa’ and this as bb and on phase ‘aa’, we have a shear force V and the bending moment M. On the right phase, we have the shear force V + dV and moment as M + dM. We expect that from ‘aa’ to bb over dx distance as we move, there is a change in the shear force and the bending moment.

Let us separate out this particular element and look into the stresses. This is the stress distribution which is shown, this is corresponding to the bending stress that the normal stress is acting in the beam and here we have considered only M and M + dM. There will be stresses because of the shearing action as well but since we are interested at this moment in computing the value of the horizontal shear stress, let us keep the vertical shear stress apart. So we are looking only at the stress because of the bending moment only.

Let us consider a small segment of this particular beam which is at a distance of say $y_1$ from this particular neutral axis and let us separate out this particular segment and look into the stresses. Here we have the normal stress which gets generated because of the bending moment M + dM. On this side we have the normal stresses which are getting generated because of the bending moment M and at this particular cross section of this particular member, we will have a horizontal shear which is acting at this phase.

There are verticals here on these phases, which we are not taking into account at the moment. Take the cross section of the beam at this particular section 'aa', where the width is b and depth is h and the distance from the neutral axis is h/2 and we are considering a section which is at a distance of $Y_1$ from the neutral axis. This is the section which we are looking into and the section at bb is a rectangular one having width b and depth h which is the same configuration. We are considering this particular segment.

If we consider a small area dA then the force which will be acting is Sigma (dA) and we can compute this Sigma from the expression as we have learned in terms of bending. Since we are considering only the bending moment acting in this particular section; the stress Sigma due to bending is MY/I which is loaded into the flex. This particular force $F_1$ will be the stress multiplied by the area of this particular segment and on this side the force $F_2$will be the stress which is generated because of the M + dM multiplied by this segmental area which will give us $F_2$.

In this particular element, we have force $F_1$, we have force $F_2$ and we have force $F_3$. If we take the equilibrium of these forces in the horizontal direction, we have $F_1 - F_2 - F_3 = 0$. How do we calculate $F_1$, $F_2$ and $F_3$? Here you note that the force $F_1$ gets generated because of the bending moment, My/I multiplied by the element area dA and the segment is from $y_1$ up to h/2 which gives us the value of $F_1$.

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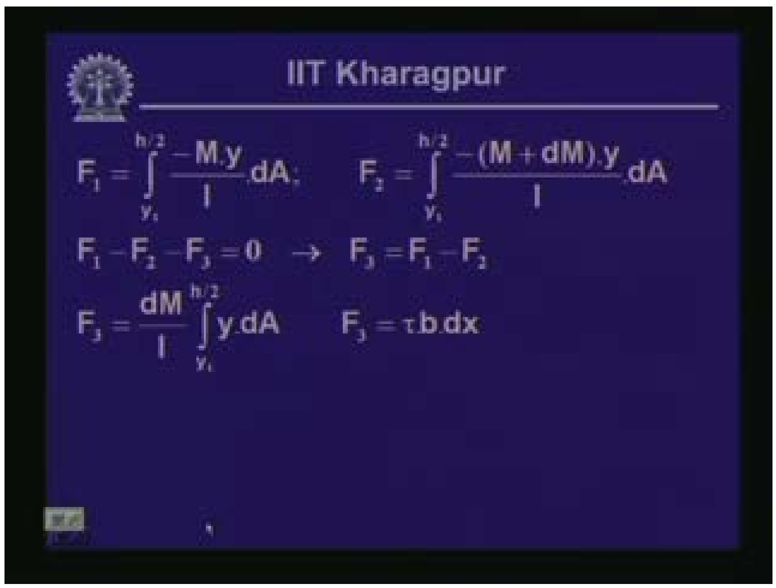


Likewise the value of $F_2$ = Integral$_{y1}$ to the power of h/2 - (M + dM) (Y)/I (dA)

Again it is integrated from $y_1$ to h/2 and we get the value of $F_2$. If we write down the equilibrium equation which is $F_1 - F_2 - F_3 = 0$ and thereby it gives us $F_3 = F_1 - F_2$ and $F_3$ is the force which we are trying to find out. If we substitute $F_1$ and $F_2$, we get $F_1$ = Integral$_{y1}$ to the power of h/2 – M(y)/I (dA) + Integral$_{y1}$ to the power of h/2 – (M + dM) Y/I (dA) and we get F3 and MY/I gets cancelled. So, we are left with dM /I $y_1$ to h/2 integral y dA and this particular parameter integral y dA we designate as Q Q as equal to integral y dA. In this particular case, it is from $y_1$ to h/2and $F_3$.

As you can observe from the previous one that $F_3$ is acting at the section of this particular beam segment wherein the width of the beam is b and the length we have considered as dx. We have taken the dx segment from the beam of width b; the area over which the shear is uniformly distributed is v (dx). If Tao is the shearing stress, then the force that is acting at that particular cross section is Tao(v) (dx) stress multiplied by the area $F_3$. That is why it is written as Tao (b dx) and hence if we equate these two we get a relationship and the moment Tao b dx is $F_3$, = dM/I Integral$_{y2}$ to the power of h/2 dA which we have designated as q and Tao is basically equal to dm/dx (I/b) (Q).

You can recognize this particular parameter dM dX dM dX as = V which is the shear force. The rate of change of the bending moment along the length is equal to the shear force which is I (b). So Tao from this particular expression is V (Q/I) (b). This is the expression for the horizontal shear stress where Tao is equal to the vertical shear force V, where Q the area is the moment of the area which is above the section where we are trying to calculate the shearing stress. We are trying to calculate the shearing stress in the section which is at a distance of $y_1$ from the neutral axis and the area above $y_1$ from $y_1$to h/2 is the segmental area and the moment of that particular area with respect to the neutral axis will give us the value of Q.

We know the vertical shear force V, we can compute the value of Q, we can find out I as the moment of inertia of the section with respect to the neutral axis V which is the width. We can compute the value of the shearing stress and as you can observe from this particular expression that at a particular cross section VI and b these three parameters are constant or the same. The only parameter which is varying is Q because Q depends on y which is the distance from the neutral axis. Hence along the depth of the beam the shear stress will vary. Shear stress depends on Q and Q in turn is dependent on the distance y.

We can compute the value of the horizontal shear stress at a particular section. As we have seen that the horizontal shear stress is complimentary to the vertical shear stress. The vertical shear stress also will be equal in magnitude of that horizontal shear stress. So once you compute this expression VQ/ Ib it is the same for the vertical shear stress as well.

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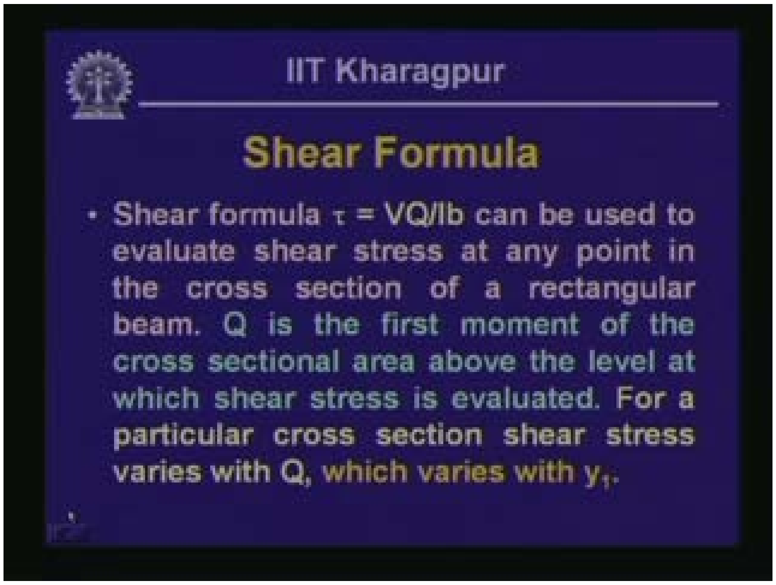


From this particular shear formula which we have just derived, we find that the expression for the shearing stress Tao = VQ/Ib and this can be used to evaluate shear stress at any point in the cross section of a rectangular beam. A rectangular beam is a beam having a rectangular cross section. The term Q is the first moment of the cross sectional area above the level at which we are looking for the shear stress and for a particular section.

The shear stress varies with the parameter Q because this parameter is constant for a particular section. The only parameter which varies is Q and Q is varies with respect to $y_1$. Hence we have the shear stress along the depth which varies and we get the distribution of the shear stress on the depth.

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Let us see how the shear stress varies across the depth. We have obtained the value of the Tao shearing stress which is VQ/Ib. Now Q for this particular beam as we have seen is integral. Supposing in this particular rectangular section where the width is b and height is h for this beam, the neutral axis divides the section into two hubs. We would like to find out the shear stress across this particular section which is at a distance of $y_1$ from the neutral axis. Hence this is a segment which is under consideration, which is above the point where we are computing the shearing stress.

If we compute the value of Q for this particular section, Q is equal to integral. For dA, if we take a small strip over having width di = b (y) and y varies from $y_1$ to h /2. This is b/2 (y) to the power of 2 and y dy is y to the power of 2/2 and y to the power of 2 is h to the power of 2/4 - $y_1$ to the power of 2. So, this is the value of Q and Tao = V/Ib b/2 × h to the power of 2/4 - $y_1$ to the power of 2 and ‘b b’ gets cancelled and we get V/2 I (h) to the power of 2/4 - $y_1$ to the power of 2.

Note that the shearing stress Tao varies parabolic-ally across the depth; it varies with $y_1$ to the power of 2. When $y_1$ = 0, we get the maximum value of Tao. That implies that at the neutral axis, we get the maximum value of the shearing stress and $y_1$ becomes h/2 + h/2 or – h/2 and the value of the shearing stress becomes 0.

As we have noticed earlier that at the top and the bottom of the beam, the value of the shearing stress is 0. Let us plot the shear stress distribution where at the top and bottom, the shear stress is 0 and over the depth it varies parabolic-ally and at the neutral axis, we have the maximum value of shearing stress. What is the magnitude of this maximum value? Let us say $Tao_{max}$ = V/2 I (h) to the power of 2 /4 and let us write down that Tao = V h square/8I. Now 'I' for this particular rectangular section, is bh to the power of 3/12.

After substitution we get Vh to the power of 2/8 bh to the power of 3(12). So, h to the power of 2 gets cancelled and h is 43 and we have 2 and 3/2 V/b (h) which is nothing but the cross sectional area. We write this as 'a' and the maximum value of the shearing stress in a rectangular beam is equal to what we call as $Tao_{max}$ corresponding to the maximum shear force $V_{max}$ and 3/2. Here $V_{max}$/a will give us the value of $Tao_{max}$ and as you can see that across the depth, the shear stress varies for the rectangular beam in a parabolic manner.

At the neutral axis, we get the maximum shear stress and at the extreme ends, the shearing stress is 0 and the magnitude of the maximum shear stress is 3/2/a. As we assumed in the beginning that although the shear stress varies across the depth, along the width we assume that the shearing stress is constant. That means that at this particular point, we get the value of the shearing stress along the width.

This particular value is constant, as we had assumed in the beginning two aspects: one is that the shear stress acts along the direction of the shearing force and the shearing stress at the particular point is uniformly distributed across the width. Here one aspect to remember is that we really do not bother about the sign of the stress. As such we consider that the direction of the shearing stress is along the direction of the shear force. From the bending moment and on the shear force diagram, we know the direction of the positive or negative shear. Accordingly, we take the direction of shearing stresses as well with this background.

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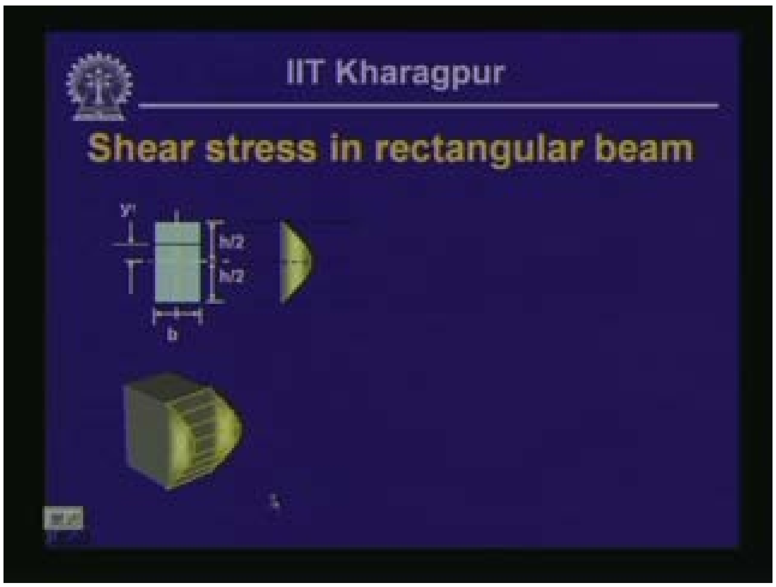


What are the steps that we need for evaluating the shearing stress at a particular point? First of all we need to find out the vertical shear force V in the cross section where we are trying to find out the particular shear stress. That means that at a particular point where we are evaluating the shear stress, we will have to find out in which section that particular point is and at that particular section, we have to calculate the value of the shear force and we can carry these out by drawing the shear force diagram.

Once we draw the shear force diagram in the length of the beam at different points, we know the values of the shearing force and the point where we wanted to evaluate the shearing stress. We would like to find out where that particular point lies and what the magnitude of the shearing force is.

Secondly in the cross section, we will have to locate the neutral axis and we will have to compute the moment of inertia about this neutral axis. Then we will have to compute the value of Q which is the first moment of the area of the section above or below the point where we are trying to find out the shear stress.

Let us suppose we want to find out the sheer stress in a particular section. We concentrate on the area where we are trying to find out the shear stress either above or below that particular segment and we take the moment of that area with respect to the neutral axis which gives us the value of Q. Once we know Q we get I V and b is the width of the beam which gives us the shear stress from this particular expression which is VQ/Ib. Then we can find out the value of the shear stress at that particular point with this background.

Let us see how to evaluate the shear stress and the bending stress at a particular point in a beam having different cross sections. This is the example which I had given last time where the cross section of the beam is like an inverted u; this particular section is symmetrical with respect to y-axis but is unsymmetrical with respect to the horizontal axis. Hence we will have to compute or locate the position of the centroid of this particular section so that we can find out the position of the neutral axis.

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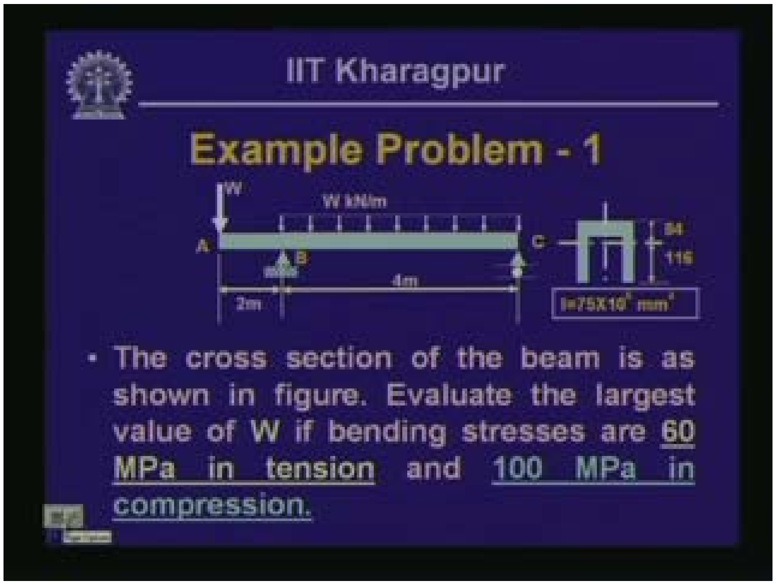


Fortunately in this particular problem, the values are given, the location of the neutral axis is indicated or the location of centroid is known which is 84 mm from the top of the cross section and the moment of inertia of the neutral axis is given as 75(10 to the power of 6) mm to the power of 4. This particular beam is subjected to a concentrated load at this part and over this length there is a load of W. We need to evaluate the value of W so that the bending stresses are 60 MPa in tension and 100 MPa in compression.

We will have to find out the maximum value of load that we can apply in the beam so that the tensile stress does not go beyond 60 MPa and the bending compressive stress does not go beyond 100 MPa. If we have to satisfy these two criteria then what is the value of W that we can apply on this beam? To do that let us first find out where the maximum bending moment and the maximum shear force occur. Here we are dealing with only bending stress, so we can know where the maximum bending moment is and we can compute the bending stresses.

This particular beam is supported on a hinge over here and roller on this particular end. So, we will have a vertical reactive force $R_5$ and a horizontal reactive force $H_B$ and a vertical reactive force $R_C$. Since we do not have any horizontal force $H_B = 0$, we will be left with $R_B$ and $R_C$ and the values of $R_B$ and $R_C$. If we compute $R_B + R_C = W + 4$ plus four times (+4W) we get 5W. If we take the moment of the courses with respect to B, we can get the value of $R_C$ and consequently, B.

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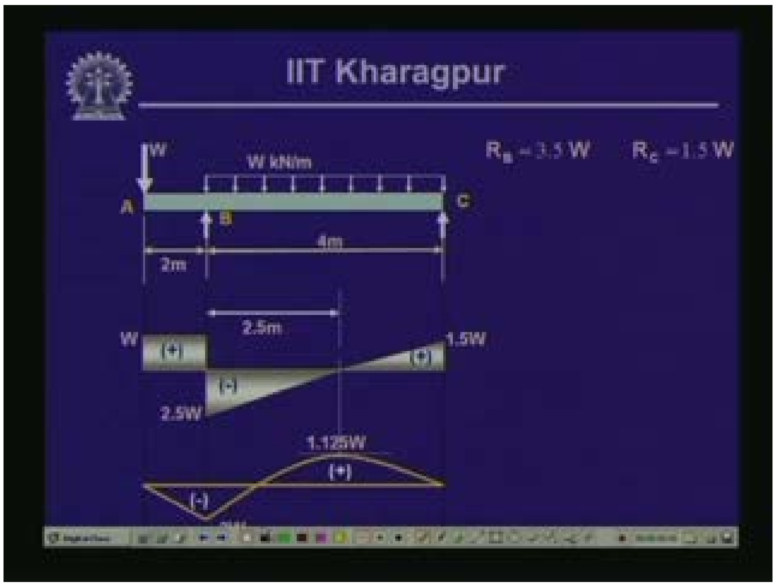


If we compute the values of $R_B$ and $R_C$, we get $R_B$ as equal to 3.5 W and $R_C$ = 1.5 W. Let us plot the shear force diagram and if we take a cross section over here and take the free body of the left part, then we have the shear force and the bending moment. Here we have the load W and since at this particular segment we do not have any other load V is equal to W. We have a positive shear from A to B and that is what is indicated over here and the magnitude of that positive shear is equal to W V which is equal to plus W over here.

Let us take a free body diagram of this beam and cut the beam of this particular section and take the free body of this left part. We have a reactive force $R_B$, we have the load W and we have the shear force V and the bending moment M. The load is distributed over this particular segment and let us call this particular distance as x. So, $V + R_B - W - W(x) = 0$ and $V = -R_B + W + W(x)$ and $R_B = 3.5$ W. Let us try to find out the shear force at any segment end beyond B.

Let us say that at distance C when x is 4 M, this is 4 W + 5 W and $R_B = 3.5$ W. Here V = 1.5 W and this is the shear force at the end C and at point B. If we take Wx = 0, $R_B = -3.5$ and W is W. This is -2.5 W, at point B, it is -2.5 W and at point C, it is 1.5 W. Since it changes from (-) to (+) somewhere along the length, the shear force is 0. If the expression for shear is 0,
$V = -R_B$ (+) (-) W + Wx V = 0 and it gives us $R_B = -3.5$ W. So, -3.5W + W = -2.5 W and that gives us -2.5 W= 2.5 W(x) = 2.5 W and W gets cancel and x = 2.5. It indicates that at a distance of x = 2.5, the shear force is 0 and it changes sign from negative to positive and here the value is 1.5 W.

As we know that when the shear force is 0 at the corresponding point, we will get a value which is maximum in the bending moment. Let us compute the value of the bending moment for different segments and take the moment from this particular free body diagram. At this particular free body we see that M = W(x) and when x = 2 M = M and W are in the same direction, M = -W(x) where x is equal to 2 meters and M = -2W and the magnitude of this moment is 2W.

At this particular point again, the shear force is 0 and we get a change in the shear from a positive to a negative sign. Here we expect some higher value of the bending moment, as we have seen that the maximum bending moment occurs either where the shear force is 0 or the point where shear force changes its sign from the negative to the positive or the vice versa.

At this particular point M + W is acting in the same direction and W (x) + 2 – $R_B$ (x) + W (x to the power of 2)/2 = 0. The bending moment M = $R_B$ (x) - W x to the power of 2/2 - W (x) + 2 and if we substitute the value of x = 2.5,where the shear force is 0 we get the magnitude of M as 1.125 W and this is the maximum positive bending moment that occurs in this particular beam. We have 2 maximum values of the bending moment, 1 is the positive maximum, another is the negative maximum, the maximum positive bending moment = 1.125 W and the maximum negative bending moment is 2 W along the length of the beam.

Once we plot this bending moment diagram, it is clear that at two locations one is the maximum positive bending moment and at the other point we have the maximum negative moment. We have to compute the values of the stresses in the beam corresponding to these two values of the positive and the negative bending moment and then we will have to take a decision on what maximum load we can apply on this particular beam. Let us compute the value of the stresses corresponding to these two bending moments.

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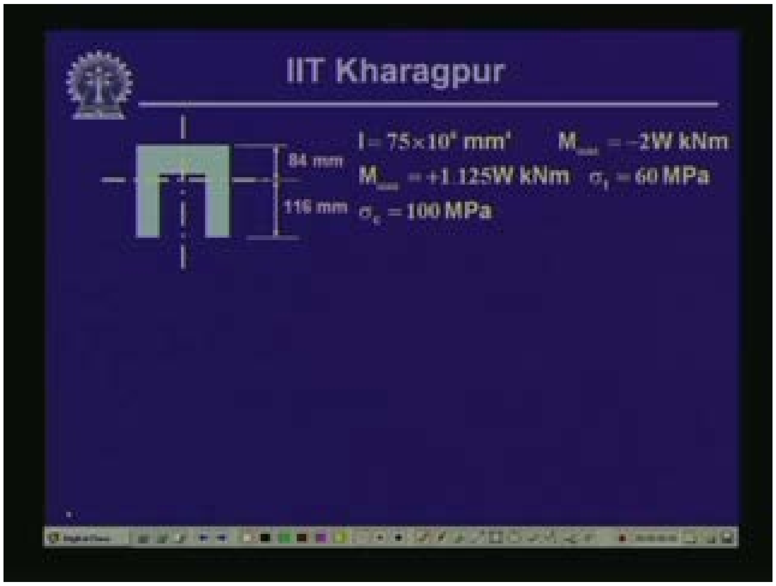


We have obtained that the maximum positive bending moment is 1.125WkNm and the maximum negative moment is 2 W kNm. If we have the negative bending moment or the negative bending moment sign, this causes tension at the top and compression at the bottom. Let us calculate the bending stress Sigma which is MY/I and I of this particular section is 75(10 to the power of 6) mm to the power of 4. The stress which will be acting at the top for this negative moment is the tension and tensile stress is 60 MPa.

Thereby for the first case we have 60 = M2W kNm(10 to the power of 6) Nm and Y is the neutral axis from the top distance which is 84mm because maximum tensile stress will occur at the top fiber and divided by I it is 75(10 to the power of 6) MPa. From this, if you compute the value of W, the value of W is 76.8kN and this is one value of W which you get corresponding to the maximum tensile stress corresponding to the maximum negative bending moment.

The correspondingly allowable compressive stress is 100 MPa and 100 corresponding to this moment is 2W (10 to the power of 6)(116) which is the bottom fiber. The fiber distance from the neutral axis is 116/I which is 75(10 to the power of 6). From this, if you compute the value of W, we get W = 32.33 Mpa. Corresponding to this value of the bending moment which is 2W, the maximum negative bending moment, we get the tensile stress at the top and the compressive stress at the bottom and corresponding to this limiting value of the tensile stress and the compressive stress we get the two values of W which is 26.8 kN and 32.33 kN.

If we have this particular load applied on this beam, the stress level intension will exceed 60 and as a result, the beam will fail. So, we will have to limit ourselves to this load value of 26.8 kNm. Before we take a decision on this particular loading, we will have to check up with respect to the stresses that develop with respect to this positive bending moment. Let us compute the value of the stresses that developed because of the positive bending moment which is of magnitude 1.125W.

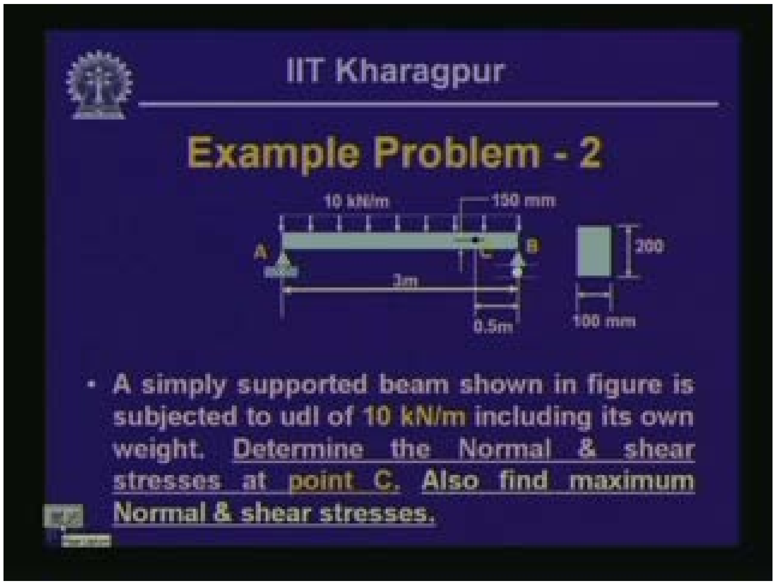
If we calculate the value of the stress, the positive bending moment which is of this kind will cause a compression at the top and tension at the bottom. If we compute the stress corresponding to the positive bending moment, the compressive stress at the top is 100 = 1.125 W (10 to the power of 6) Nm multiplied by the distance of the top fiber from the neutral axis which is 84/I and we have 75(10 to the power of 6).

From this, if we calculate W the value of W comes as 79.4 MPa, much of kN. Correspondingly to this particular moment, if you compute the stress at the bottom which is the tensile stress which is 60 = 1.125W $\times 10^6$ into the distance of the bottom fiber from the neutral axis which is 116/I = 75 $\times 10^6$. From this, if you compute the value of W, comes as 34.5 kN. Out of these 2 values, we find 34.5 kN is the low value and because if we apply 79.4.

Naturally the stress level here will exceed and the member will fail. We have obtained four values; one is 26.8 kN and 32.33 kN. In the earlier case, corresponding to the maximum negative moment and corresponding to the positive moment, we have 79.4 kN and 34.5 kN. Out of these four values, we find that 26.8 is the lowest value and this is the value which we will have to apply on the beam member so that all the other stresses are within limits.

Let us suppose that we apply a load of more than 26.8 kN. Corresponding to the other cases the stress level in other values will exceed hence we will have to apply the minimum possible W which we have evaluated out of these four cases and that is the maximum load that we can apply on the beam so that the beam functions safely and carries the load as indicated in the beam.

(Refer Slide Time: 47:36 - 50:24)




Take the example which corresponds to the one which we have discussed. The example deals with the aspects of shear stress that a simply supported beam is subjected to on a uniformly distributed load of 10 kNm including its own weight. Determine the normal and shear stress at point C and at this particular point, we will have to find out the value of the normal stress which is generated due to bending and the value of the shear stress.

The point C is at a distance of 150mm from the bottom fiber that means, this is the position of the neutral axis because it is symmetrical. The point C is somewhere along this length which is 50mm from this neutral axis. We will have to compute the normal stress and the shearing stress and also we have to find out the value of the maximum normal and the maximum shear stress. Naturally to compute the value of maximum normal and the maximum shear stress, we will have to know at which point along the length of the beam you have the maximum bending moment and the maximum shear force. For the value of the maximum bending moment and the maximum shear force we will have to draw the bending moment and shear force diagram.

Correspondingly you can find out where the maximum bending moment and the maximum shear stress is occurring. This particular end is on hinge and we have the vertical and the horizontal reaction which is $R_A$ and $H_A$. You have the vertical reactive force $R_B$ over here and you have to compute the value of $R_A$ and $R_B$ and the $H_A = 0$ because you do not have any horizontal force in this particular member.

This particular member is symmetrically loaded with symmetrical supports as we have seen earlier. That mean you have a uniformly distributed load and the reactive values will be equal. Here $R_A$ is equal to $R_B$ and that is equal to the uniformly distributed load 10 multiplied by the length of the beam by 2 = 15 kN.

The reactive values will be equal to 15 kN and in fact we can evaluate as we have done earlier. So, $R_A$ and $R_B$ = 10(3) and if we take the moment of the forces with respect to one of the support, we can get the value of one of the reactions. These are the reactive values; $H_A = 0$ $R_A$ = 15 and $R_B$ =15. Let us look into the shear force and the bending moment diagram and this particular beam along the length so that we can compute the value of the stresses.

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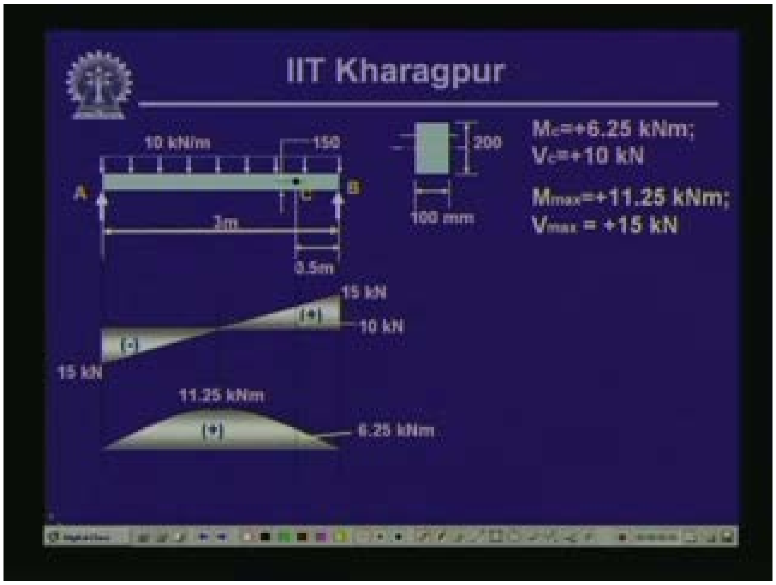


We have obtained $R_A$ as 15 and $R_B$ as 15. If we take a free body diagram of this particular member on the left hand segment we have the support reaction $R_A$ and on these we have the shear force V and the moment M. So, $V + R_A = 0$ and $V = -R_A$ and we have 15 kN negative as the shear force over here. Since there are no other loads acting on this particular beam member we have V- at this particular segment which is at a distance of x and we have a uniformly distributed load also. So, we have $-R_A + 10kN(x)$; so at x = 0, V is $R_A$ and we have a shear force.

As we go along the length the value of x changes and consequently there is a change in the shear. When x = 1/2 length which is 1.5 then the shear force becomes 0 and at the central point the shear force cross is the base line and goes to the other side and becomes a positive shear over here. If you take the value of x as 3m then you get the value of V = +15 and from -15 to +15, at the central point it becomes 0 which is because of the symmetrical loading.

We have seen it earlier that if you have a simply supported beam subjected to a uniformly distributed load then 1/2 the total load is on one support and gradually it becomes 0 at the center and goes to the other end. Also if you compute the bending moment M the value of the bending moment M = $R_A$ (x) – W x to the power of 2/2, $R_A$ =15. So, 15(-W) is 10 and 5 x to the power of 2. At the center where x = 1.5 if you compute the value of the bending moment it is 11.25 kNm. That is the maximum bending moment that we get corresponding to the 0 shear force. We will have to find out the maximum bending stress and the maximum shearing stress.

From this particular diagram we can see that the maximum value of the bending moment is 11.25kNm and the maximum value of the shear force which is acting at the support is 15kN and we have to compute the value of the bending stress and the shear stress at point C. The maximum value of the bending moment at point C = 6.25kNm and the value of the shear stress at point C is 10kN.

First let us calculate C at C and the bending stress is My/I and at C the moment is 6.25kNm. We get 6.25 (10 to the power of 6) (y) which is at a distance of 50 from the neutral axis because this is 150 from the bottom. So, we have 50/I where I = 100 (200 to the power of 3)/12 and if we compute the value of the Sigma we get 4.7MPa. If we compute the value of the shearing stress at this particular point Tao = VQ/Ib. Now we know that v is 10kN at this particular section Q = a y bar or integral ydy ydA.

We will have to take the segment above the place where we are evaluating the shear stress. We have to compute this particular segment and the segment area is 100(50) and Q = 100(50) which is the area and it is own cg is 25 from the top. From the neutral axis it is 75 so this multiplied by 75 is Q and I again is 100(200) to the power of 3/12 and b is 100. If you substitute this, you get the value of Tao = 0.5625 MPa and this is the value of the shearing stress that you get at point C.

Corresponding to the maximum value of the bending moment and the maximum value of the shear force if we compute the stresses, we get the bending stress Sigma again as My/I and $M_{max}$ = 11.25(10 to the power of 6) and y is the extreme point where the maximum value is 100. Correspondingly, 100(200 to the power of 3)/12 gives us the value of the bending stress. If we compute this we get 16.875 MPa for a rectangular section.

The maximum shear stress $Tao_{max}$ = 3/2 $V_{max}$ / area and $V_{max}$ here is 15 kN and the area is 100(200). If we substitute the values, the maximum shear stress comes to 1.125 MPa and at point C we can compute the value of the bending stress, the normal stress and the shearing stress from this particular diagram.

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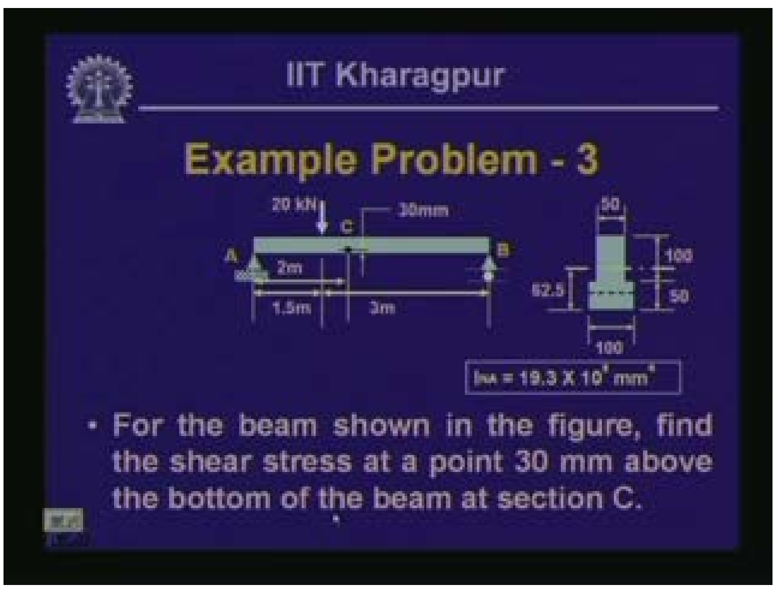
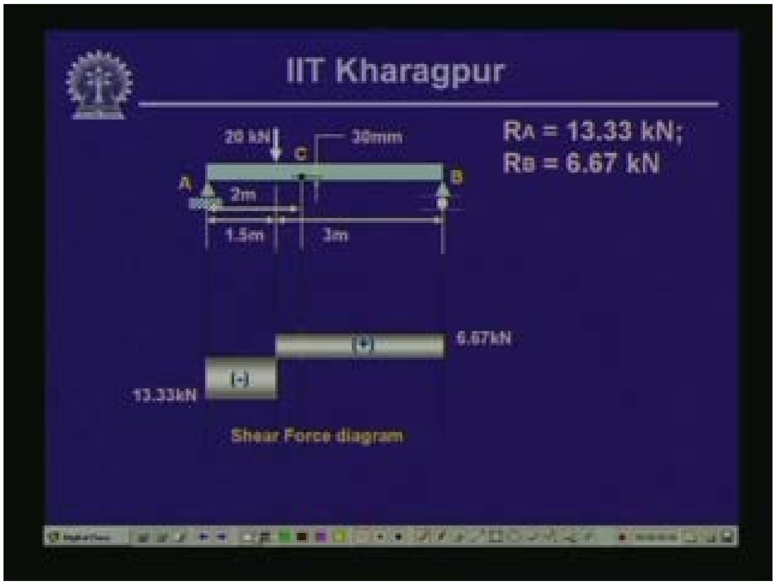



We have another problem where in the beam which is shown over here, we will have to find out the stress at a point which is 30mm above the bottom of the beam at section C. That means that we will have to compute the value of the shearing stress at this particular point and at this location which is at a distance of 30mm from the bottom.

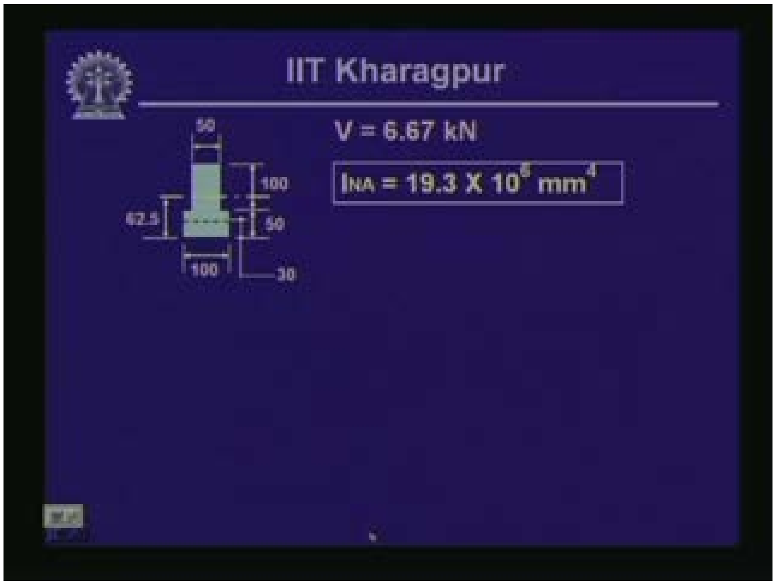
Here the cross section is symmetrical and with respect to the vertical axis. Here we have 2 rectangular components joined together and the neutral axis is located at a distance of 62.5mm from the bottom fiber. We need not calculate the shear force of the bending moment diagram as such.

(Refer Slide Time: 57:58 - 58:24)

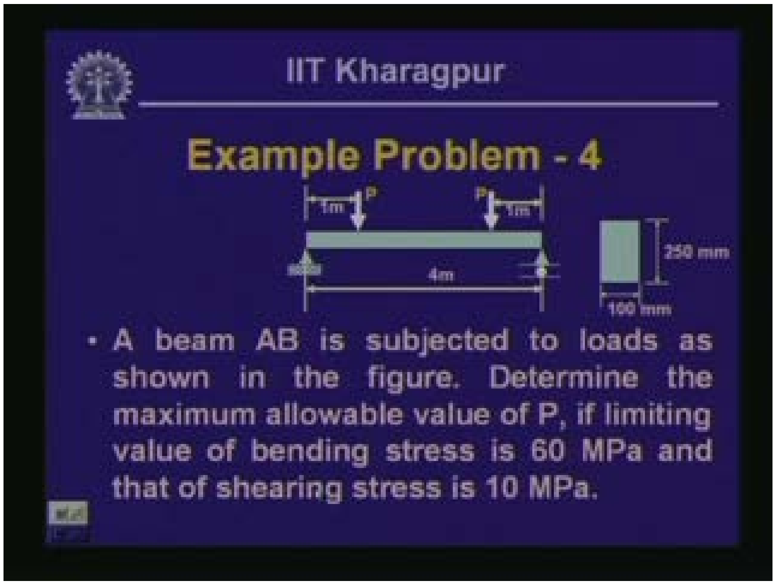



From the shear force diagram we can compute the value of the shear stress. At this particular location as the value of the shear force is equal to 6.67kN and correspondingly if you calculate the value of the shear stress which is 6.67(10 to the power of 3) this is V (Q)/Ib and if we look into b in this particular case it is 100. So, if we compute the value of the shear stress Tao = 6.67(10 to the power of 3) (Q) = 100 because this is the section which we are looking into. This is 100(30) and this is 62.5-15 which is 47.5/ I(b) which is 100 and if we compute this we get 0.5 Mpa.

(Refer Slide Time: 58:24 - 59:00)




(Refer Slide Time: 59:00 - 59:19)




There is another problem where a beam is subjected to two concentrated loads and you have to compute the maximum value of b so that the bending stress does not exit 60 MPa and shearing stress does not exit 10 Mpa. We will look into this particular problem next time.

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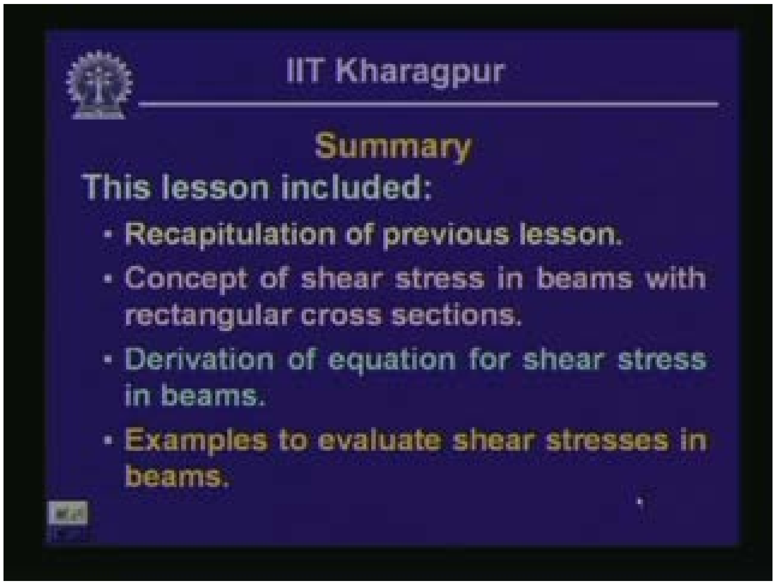


To summarize this particular lesson, we have recapitulated the concepts of the previous lesson, we have looked into the concept of shear stress in beams with rectangular cross sections, we have looked into the derivation of equations for the shear stress in beams and we have looked at some examples to evaluate shear stresses in beams.

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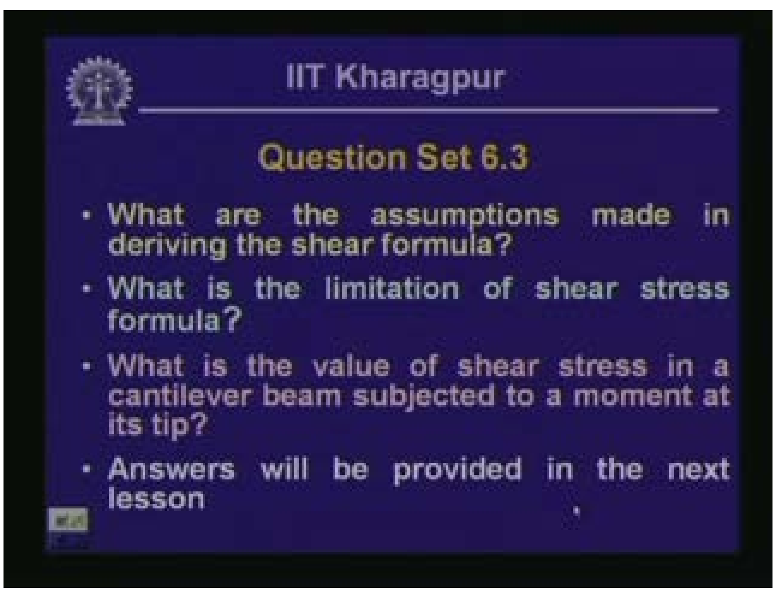

The questions given for you are: what are the assumptions made in deriving the shear formula? What is the limitation of shear stress formula? What is the value of shear stress in a cantilever beam subjected to a moment at its tip? We will provide answers for these questions in the next lesson.