Strength of Materials
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Lecture no 27
Lecture Title: Stresses in Beams- II

Welcome to the second lesson of the sixth module which is on Stresses in Beams part 2. In the last lesson of this particular module, we had discussed the effect of bending and the stresses in a beam due to bending and we have seen how to evaluate stress at any point due to the loads which are acting on the beam. Consequently, we have seen the stresses if it is acted on by pure bending and we have defined that as the bending equation of the flexure equation.

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We will be looking into some more aspects of the bending stresses if they act on a section which is symmetrical with respect to the vertical axis but could be unsymmetrical with respect to the horizontal x axis or z axis which we have defined. Once this particular lesson is completed, one should be able to understand the effect of bending stress in beams of different cross sections which are symmetrical about vertical axis but could be unsymmetrical with respect to the horizontal axis.

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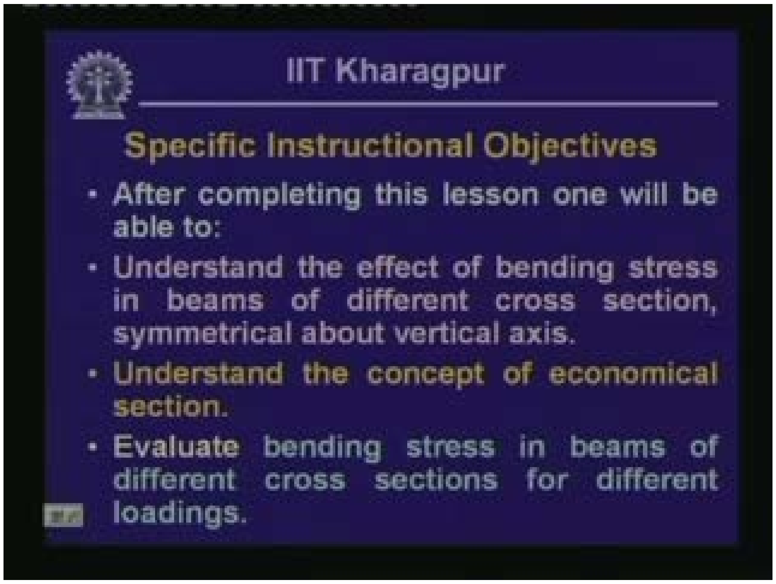

One should be able to understand the concept of economical section. We will study the sections which are more appropriate when they are subjected to bending which we can call as the most economical section and one should be in a position to evaluate bending stress in beams of different cross sections for different loadings.

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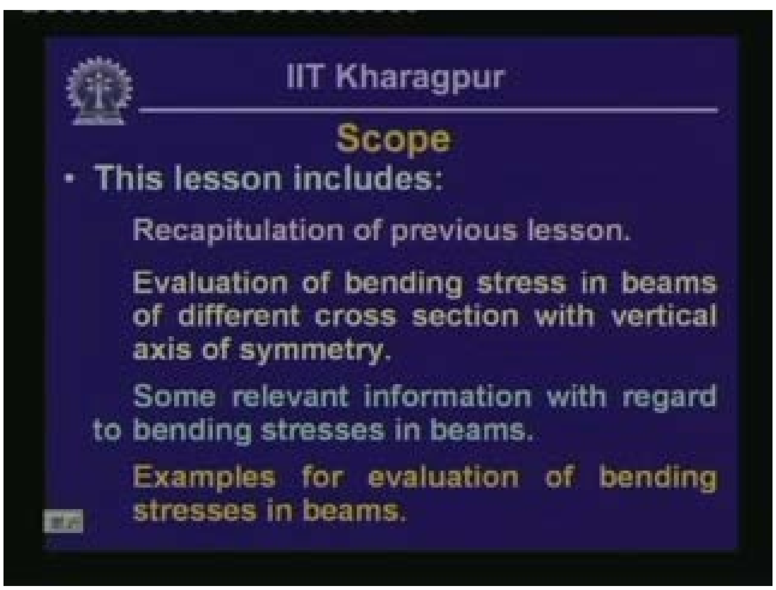

Hence the scope of this particular lesson includes the recapitulation of previous lessons which we have just discussed, evaluation of bending stress in beams of different cross sections with vertical axis of symmetry and some relevant information with regard to bending stresses in beams. We have derived the flexural equation of the bending equation. What are the consequences of bending stresses and what is the impact of bending stress on different types of cross sections? We will also be looking into some examples for evaluating bending stresses in beams.

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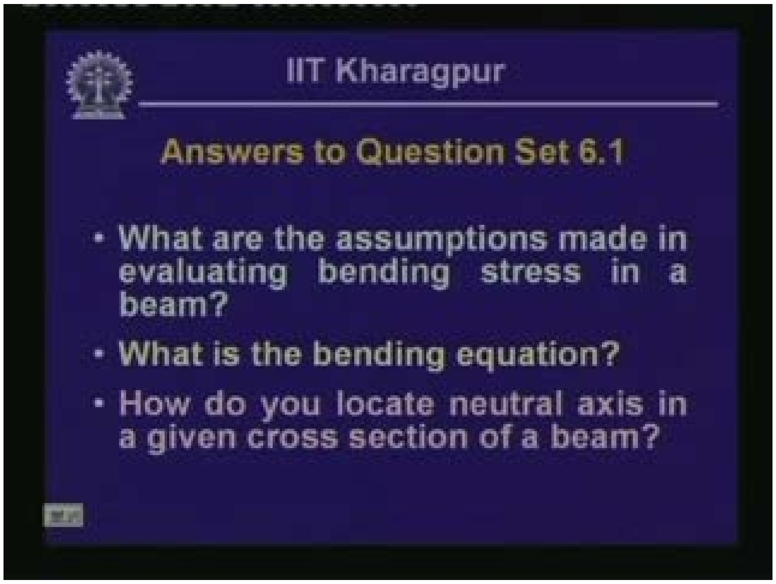

Let us answer the questions which I gave you last time. The first question was what are the assumptions made in evaluating bending stress in a beam? Now let us discuss this particular aspect with respect to the diagram which I had discussed last time in the last lesson. If you remember, this is a segment of a beam, the section of which is a cross section of the beam. It is like a trapezium which is symmetrical with respect to the vertical axis and unsymmetrical with respect to the horizontal axis.

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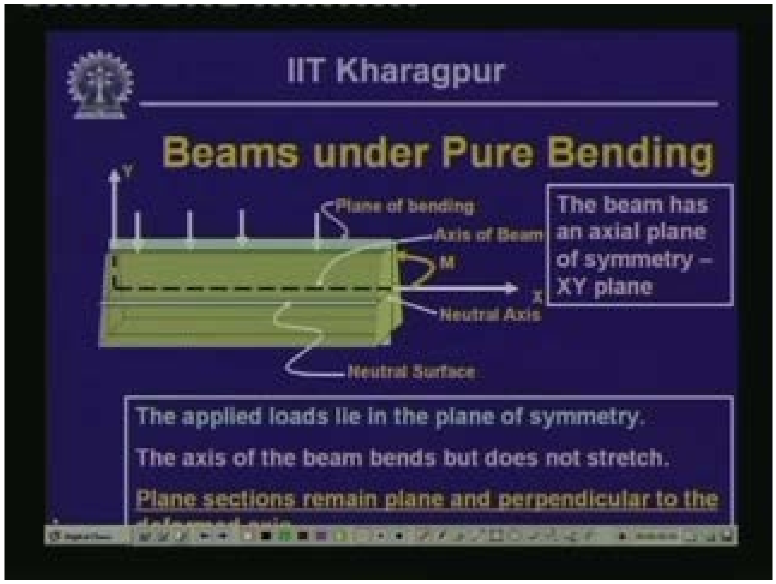

Here we assume that the beam has an axial plane of symmetry and XY plane is the axial plane of symmetry and the loads which act on the beam lie in the plane of symmetry. Also it is assumed that the axis of the beam bends, but it does not stretch, as we have seen last time while deriving the flexural equation. This occurrence is because of the load which lies in the plane of this symmetry where this beam undergoes bending and the axis of the beam undergoes bending, but does not stretch.

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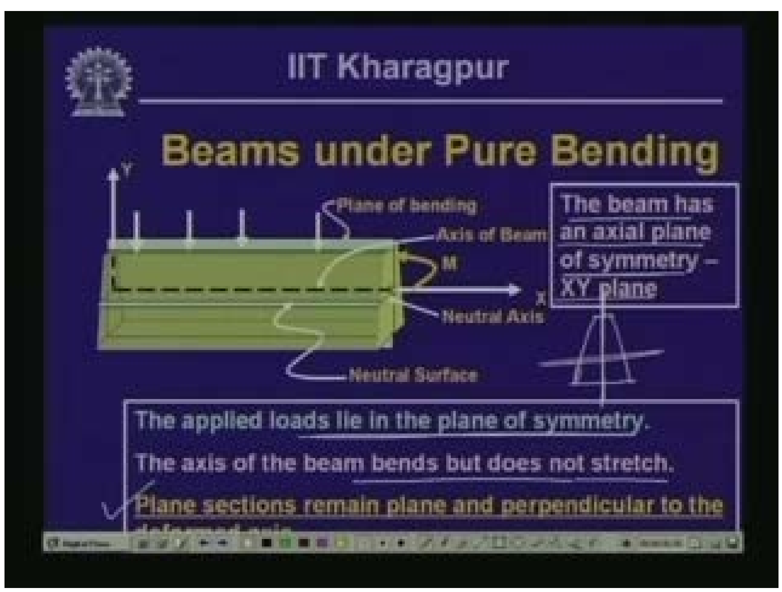

The most important point is that the plane sections remain plane and perpendicular to the deformed axis of the bar. The section remains plane at any cross section that we take and it does not deform. Rather it becomes perpendicular to the deformed axis and this is the main assumption while deriving the flexural equation.

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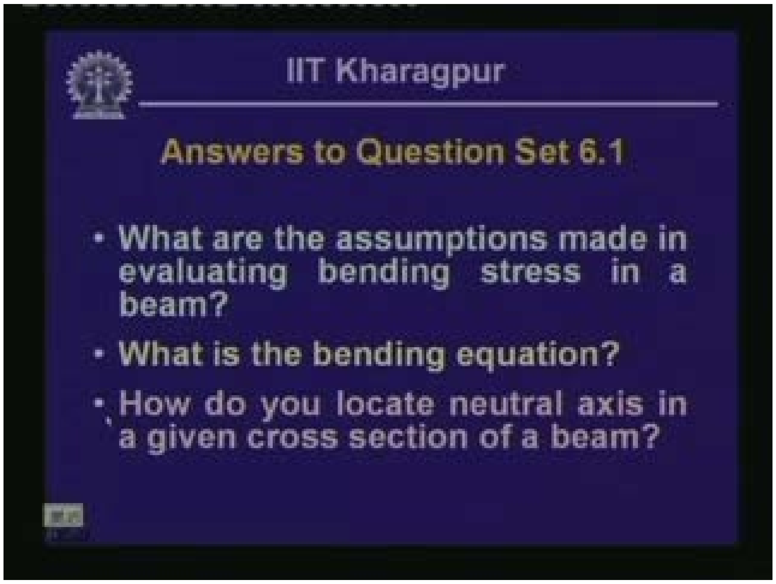

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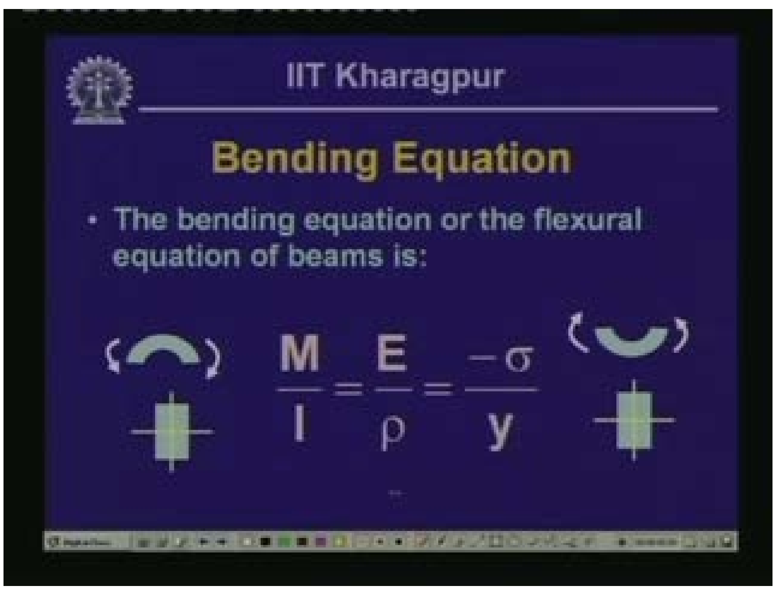

Now the second question was; what is the bending equation? We had derived the equation M/I = E/Row = -Sigma/y last time. We called this as the bending equation or the flexural equation. Where M is the bending moment, we call 'I' as the moment of inertia of the cross section, E as the modulus of elasticity, Row is the radius of curvature of the axis of the beam, Sigma is the bending stress and y is the point where we need to find out the stress.

From this particular equation, you can write M/I = - Sigma/y or this gives Sigma as -My/I. Now we call Sigma as the bending stress, M as the bending moment, I as the moment of inertia and y is the distance at which we are evaluating the stress. If you look into this particular equation, it has a similarity with the equation which we had derived when a bar was subjected to a twisting moment where the twisting moment was t and t/ j = Tao/Row = g Theta/l. Here, Tao was the shear stress, which corresponds to the sigma when we talk about bending.

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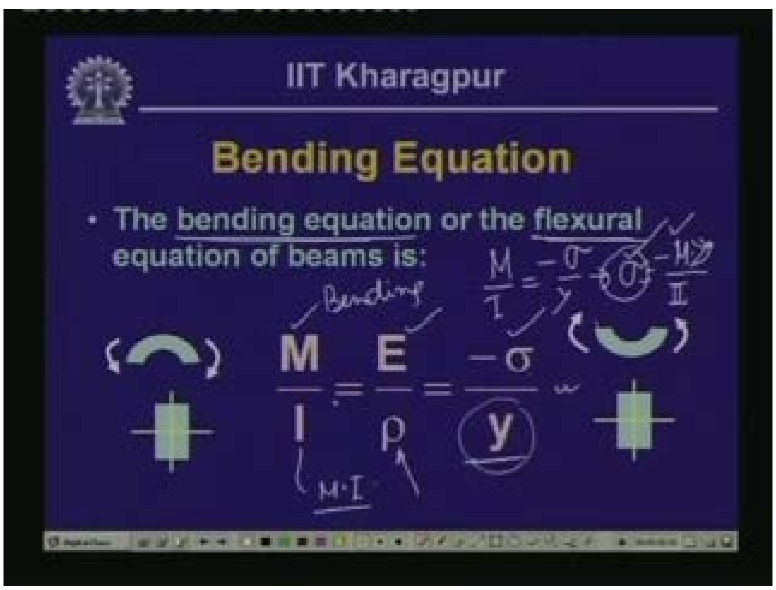

Now t, the twisting moment is comparable with the bending moment; shear stress is comparable with Tao and j the polar moment of inertia is comparable with the moment of inertia of the section. Now t/j = Tao /Row where Row was the radius of the shaft which is g Theta/l and that was the rotation. Here it is in terms of curvature 1/Row where Row is the radius of the curvature and we call 1/ρ term kappa which is the curvature of the beam.

There is a similarity between the torsion equation and the bending equation. The terms have different meanings and please note here that the sectional parameters in the case of twisting moment when it was acting on a shaft, we have used a polar moment of inertia j which corresponds to the sectional property. Now in this particular case when we are dealing with bending we are again dealing with sectional property, but we are computing the moment of inertia which is about the axis lying in the plane of the cross section.

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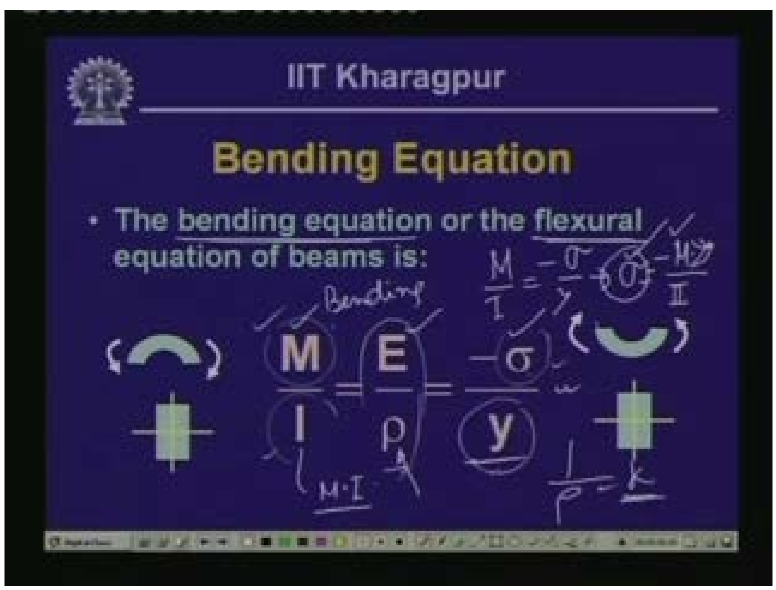

Now the question is, when the bending moment(y) is positive which have assumed in this particular derivation then stress is negative. Regarding the upward direction this is y and this is z and y is positive upward, at the upward point and the extreme point we have a stress which is negative and as you can see because of this kind of bending the top fiber undergoes a completion. So, the compressive stress is negative.

If M is positive and y is negative on the bottom side then the stress becomes positive, which corresponds to the stress at the bottom fiber. So when we get Sigma as negative then we call it compressive stress and when we get Sigma as positive we call that as tensile stress and this can be verified corresponding to this configuration where the moment M is negative because it is in the opposite direction.

Since M is negative Sigma becomes positive and according to our convention, when Sigma is positive it is tensile and physically. Now because of this kind of moment the top fiber undergoes stretching which is tensile in nature and correspondingly the bottom fiber undergoes a compressive stress. That is how we decide about the tensile or the compressive stress in a beam when they are subjected to a load depending on whether they have a positive or a negative moment. Remember that we have assumed the positive moment on the right side when it is in the anti-clockwise direction.

The last question was: how do you locate neutral axis in a given cross section of a beam? First of all we should know what we really mean by a neutral axis. When we were deriving the flexural equation we said that the summation of Fx is 0 for the equilibrium and that gives us -E/Row Integral Y d A =0.

Let us say that we have AY = 0 which indicates that Y is the distance of the centroid of the cross section with reference to the origin. If it is 0 then the summation of Fx will be 0. So in effect it means that the z axis must pass through the centroid of the cross section. Let us suppose that Y = 0 and as we have seen that Xi= -Y/Row and correspondingly Sigma is E Ksi and -EY/Row when Y = 0.

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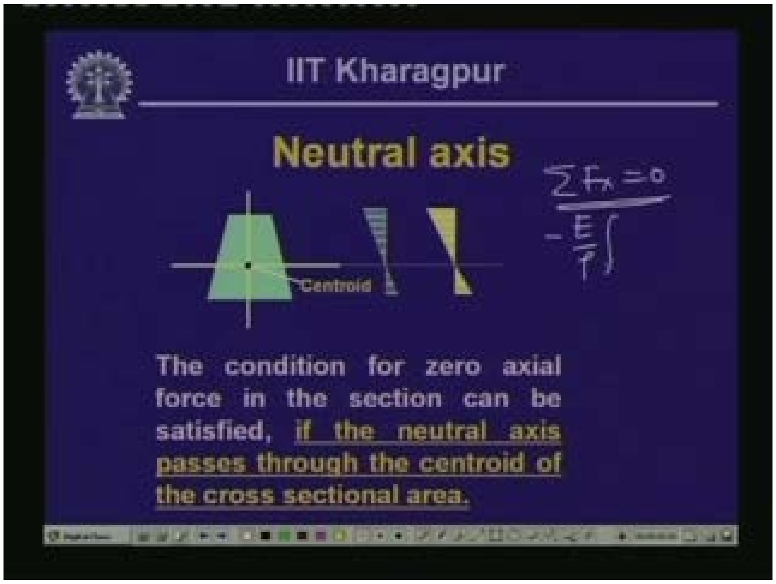

Correspondingly Sigma will be 0 at that particular point and along this axis if we called this as the strain diagram and this as the stress diagram, then we find that the strain and the stress is 0 and the strain and the stress linearly varies as a function of Y where it is linear with respect to 0 over here. So, this is the stress distribution and according to our positive moment which is acting about this particular axis, we have a compressive stress here and tensile stress over here.

Let us call this stress as Sigma compression and Sigma tensile. The condition for 0 axial force in the section can be satisfied only if the neutral axis passes through the centroid of the cross section. Now we call this a neutral axis because along this axis the stress and the strain are 0 in the beam cross section and these can be satisfied if the neutral axis passes through the centroid and then there will not be any axial force in the beam. This is one of the requirements of the equilibrium criteria. Hence the neutral axis around the axis where the stress is 0 must lie in the centroid to satisfy the equilibrium criteria.

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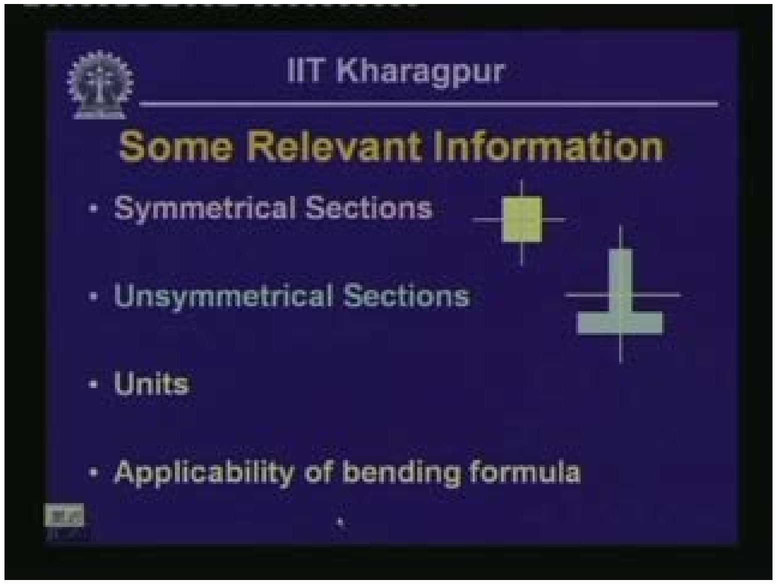

Let us study some of the aspects on the bending equation which we have derived last time. Please keep in mind that we are using the cross sections which are symmetrical with respect to the vertical axis. With Giga to that horizontal axis it could be unsymmetrical and let us see what happens when if it is unsymmetrical. If the cross section is symmetrical with respect to the horizontal axis as in the case of a rectangular or a square section wherein the extreme distance of the fiber with respect to this is equal, the two axes pass through the centroid and the centroid divides the whole depth equally.

The top distance also is Y and this is – Y and the stress which we get at the top for the symmetrical Sigma is - MY/ I where M is positive and we get a negative stress which is compressive at the top. If Y is negative at the bottom then this is going to be MY/I and this is a positive quantity which is a tensile at the bottom. But interestingly whether the stresses are compressive or tensile, in terms of magnitude the value will be the same for M because Y is identical. So, the bending compressive stress and the bending tensile stress for the symmetrical section will be the same. The section which is symmetrical about both the axes is the same.

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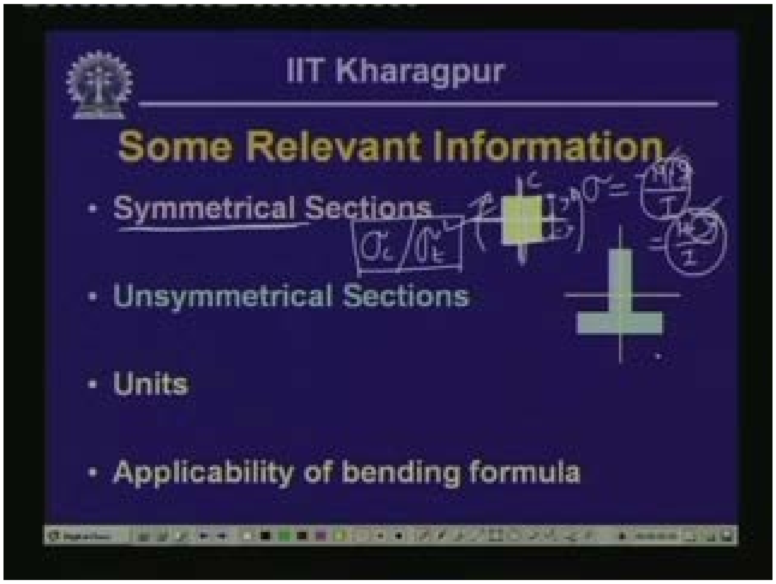

If we consider an unsymmetrical section where the section is symmetrical with respect to the vertical axis but not with respect to the horizontal axis such as in this particular case you can see that the distance of the fibers from here whose distance is different. So this is the location of the centroid of the cross section and we first locate the position of the centroid for a section. As we have seen that if there is no axial force in the beam then the neutral axis must pass through the centroid.

Once we locate the position of the centroid then the position of the neutral axis is known to us and then the distribution of the strain and the stress with respect to this will be 0 and because the distribution is linear the distribution of the strain or the stress will be linear with respect to the strain which is 0 and the stress along the neutral axis.

So, Y_1 and Y_2 the distances of the extreme fiber from the neutral axis are different. Corresponding to the bending moment this is our positive bending moment and the stress Sigma will be MY/I which is negative. For the top fiber y is Y_2 and this is $-MY_2$ / I and if M is positive then this quantity is negative and this stress is compressive. For Sigma this is equal to -MY/I and Y will be negative with respect to this which is $-Y_1$ and this is going to be a positive quantity MY_1/I and this stress according to our nomenclature is tensile.

But the magnitude of this compressive stress at the top and the magnitude of the tensile stress at the bottom will be different with the magnitude of the bending moment which is acting in the beam at a particular cross section. Since the section is not symmetrical with respect to the horizontal axis the compressive and the tensile stresses which we will be obtaining for a particular moment will be different.

Thirdly, let us look into the units of the stress from this expression where Sigma = MY/I is in Newton meter, Y is in meter and 'I' is the moment of inertia and the second moment of area which is m to the power of 4 gives a value in Newton/m square. This is nothing but Pascal, Pa and we represent Sigma in terms of Pascal or mega Pascal (MPa) or Giga Pascal (GPa).

Lastly, the most important aspect is the applicability of this bending formula. While deriving this bending equation, we had assumed that the segment of the beam is subjected to a pure bending. In a normal situation when a beam is subjected to loading, it will be subjected to a bending moment. In some zones there could be a bending moment associated with the shear force when we derive the bending formula which is subjected to a pure bending and we had also assumed that the plane sections remain plane.

Along with the bending the shear force may also act at a particular section. The assumption of the plane section remaining plane may not be truly satisfied. If you compute the stresses they will not be right using this bending equation. But in general when we talk about the beam it has its length as larger in compression to its cross sectional dimension and hence the errors which we get in evaluating the stresses with this particular assumption is insignificant and we can neglect it.

Hence we can apply this bending equation even if we do not have a pure bending in a particular segment of the beam if the bending is not uniform. But even then we can employ this equation without losing too much accuracy in the analysis of the stress. Although we derive the equation in terms of the pure bending moment, we can apply this equation in other areas as well where the beam is not truly acted on by pure bending.

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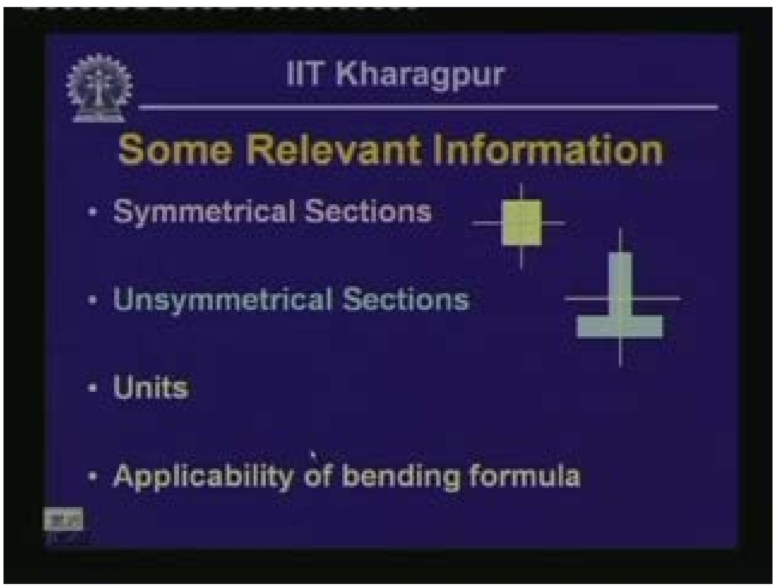

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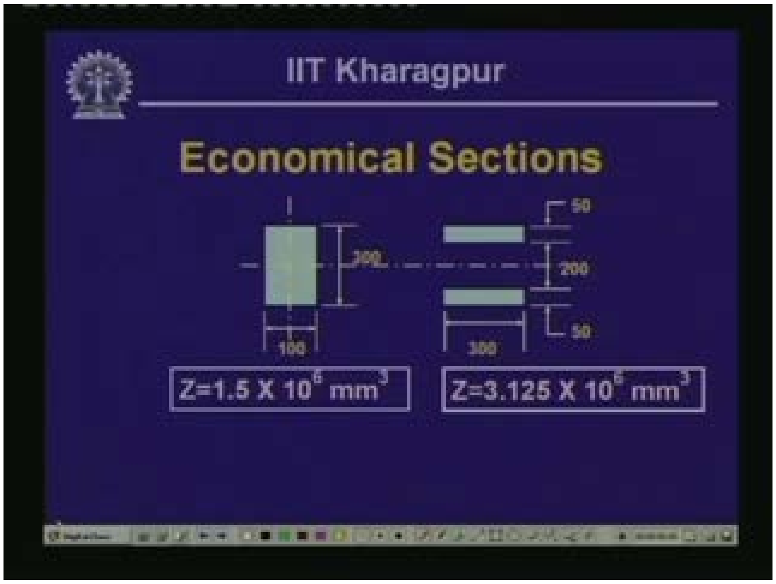

We were dealing with the section which is symmetrical with respect to the vertical axis and could be unsymmetrical or symmetrical with respect to the horizontal axis. Also we have seen the distribution of the stress. If you noticed we get 0 stress at the neutral axis and then it has a linear variation. So the maximum intensity of stresses that occur is towards the outer fiber. So this particular part of the beam really is not utilized because it is not fully stressed. Hence this it is not going to be utilized to the extent and naturally it becomes uneconomical that is point one.

The second point is that if you have a larger concentration of area away from that neutral axis the contribution in the moment of inertia for that area will be larger because we are calculating the second moment of area which is AY square. The larger the distance from the neutral axis the larger will be the contribution in the moment of inertia and instead of having the masses concentrated at the neutral axis; if you can take away the mass from the neutral axis we can have a larger effect.

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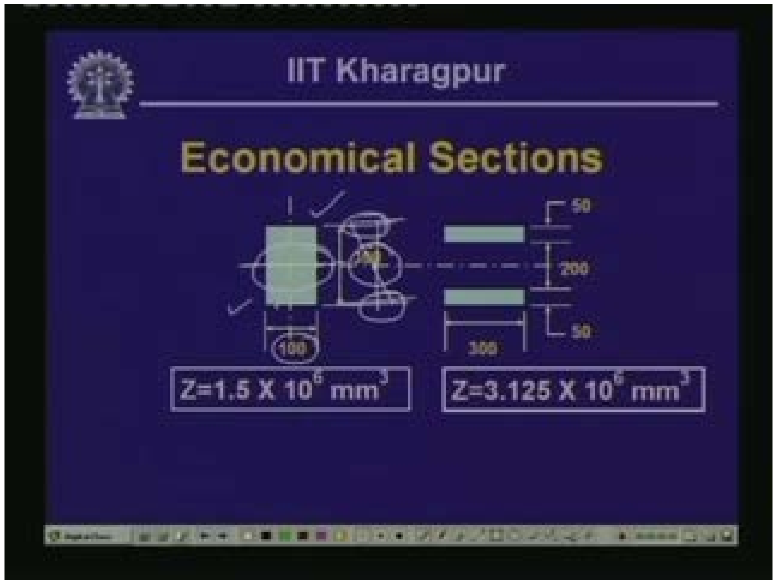

In this particular rectangular section, we have a dimension of 100mm width and 300mm depth, so the cross sectional area of this is 300x 100mm square. Now let us say we utilize the same area but we place the masses in such a way that they are at a distance away from the neutral axis. This is the neutral axis where the width of this particular segment is 300mm and the width and the depth is 50mm and we have two size segments which are away from the neutral axis. When we have 2x 300x 50 the area is identical as that of the first one. When we compute the moment of inertia I and then consequently if we divide this I by the extreme distance which is Y I/Y, we call this quantity as Z which we generally define as sectional modules.

Let us suppose you compute I in this particular case and divide by the distance Y I/Y. If you look into the value I /Y which we get in this particular case and the value of I /Y which we get in the other case you will see that when the segments of distance are away from the neutral axis, we get a value of Z which is almost more then twice this value. This shows that if we can concentrate the mass away from the neutral axis, then we can get a larger contribution in the moment of inertia and as we know that the stress Sigma is MY/I or M/Z the larger the value of Z the lower will be the stresses.

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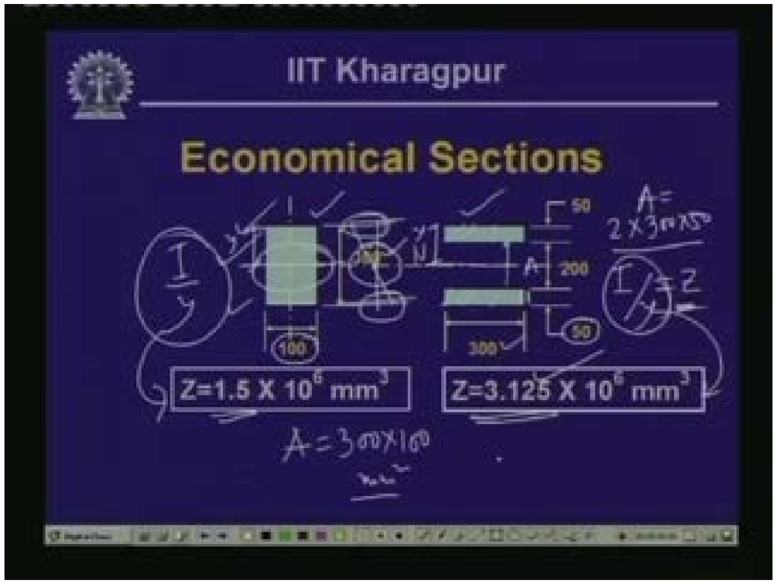

If the intensity of the stress of the particular member is subjected to the stress in the material and if we can keep the stress level lower then the allowing limit then that particular member will be more efficient. Larger the value of Z the larger will be the moment carrying capacity because the stress level will be lower and we can put more loads in such beams.

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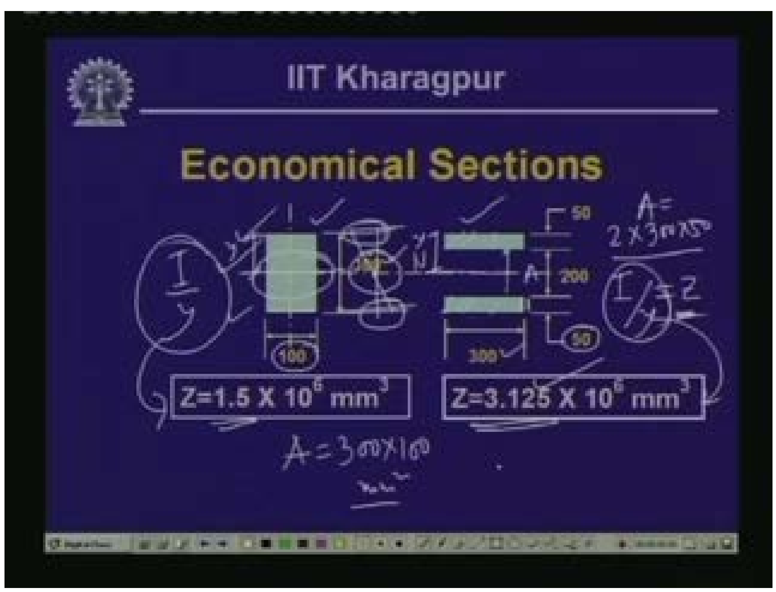

In a section if we lump the mass away from the neutral axis then that kind of section becomes more efficient. But then we cannot keep these two segments just fully at a distance apart without having any connection between the two because to act integrally we need to have some kind of connection between the two and that means some material is necessary to interconnect the two units.

If we can have this kind of a section which is similar to an ‘I’ section this becomes more efficient in comparison to a rectangular section having the same area. This means that if we have the same area for a rectangular section and the ‘I’ section has a similar area with the same depth we find that the ‘I’ section will be more efficient in comparison to a rectangular section as far as the bending stress transfer is concerned.

Let us now look into some examples related to this. How do you evaluate the bending stresses? We have an example in which this is a simply supported beam which is hinged at end A and is supported on a roller at end B subjected to a uniformly distributed load of 3kilo Newton per meter over the entire length and as concentrated load P which is at distance of 4M from support A.

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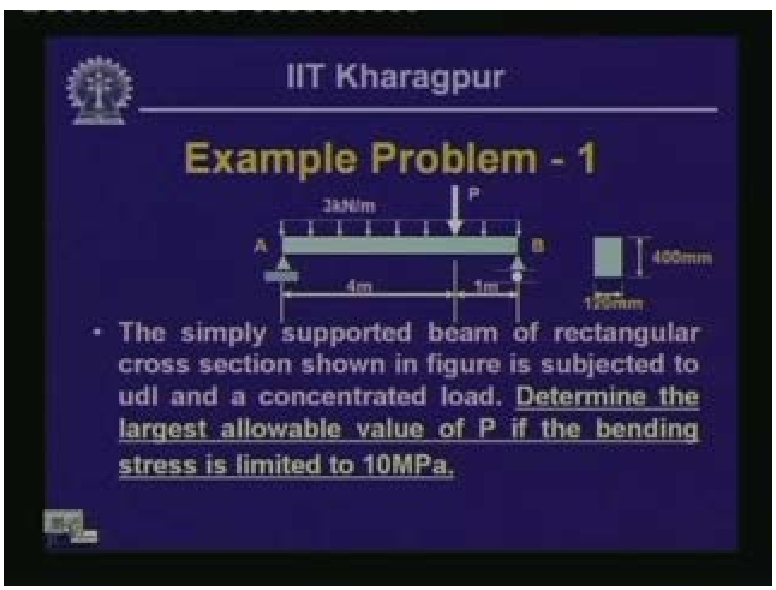

In fact this example was given to you last time. Let us see how we can solve this particular problem. We will have to determine the largest allowable value of P if the bending stress is limited to 10 MPa and here the cross section of the beam is a rectangular one with a width of 120mm and depth of 400mm.

Let us evaluate the reactive forces. As we know ‘a’ will have a vertical reaction R_A and will have a horizontal reaction H_A because the end and the roller support will have a vertical reaction R_B but since there are no horizontal forces, H_A will be 0. So, we will have only R_A and R_B and if you look into this particular example it has a uniformly distributed load over the entire span and a concentrated load P.

We are dealing with the stresses within the elastic limit hence we can individually calculate the effect of these two different loads and can superimpose them to get the same effect. If we take the combined loading the reactive values will be 1/2 of the total load and for the concentrated load we can take this in proportion with the distances. So if we compute the values of the reactive forces then R_A will be 3x 4 x4 + 15 where the total length is 5m. So, 15 by 2 which is 7.5 because of a uniformly distributed load plus P gives us the value of R_A.

If you take the moment of this we will get P x 1 by 5. Here 7.5+ P/ 5 will be the value of R_A and R_B will be 3x5 which is equal to 15 + P - 7.55 - P/5 and consequently it will be 7.55+4P/5. These are the values of the reactive forces R_A and R_B and in this particular case we will have to find out the value of P for which we have the maximum value of P wherein the bending stress is limited to 10 Mpa. It cannot exceed a value of 10 Mega Pascal when you have this kind of loading.

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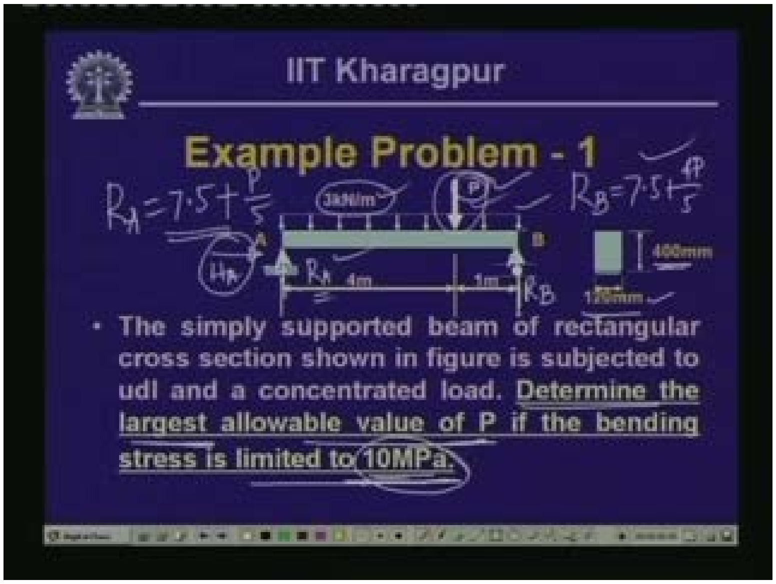

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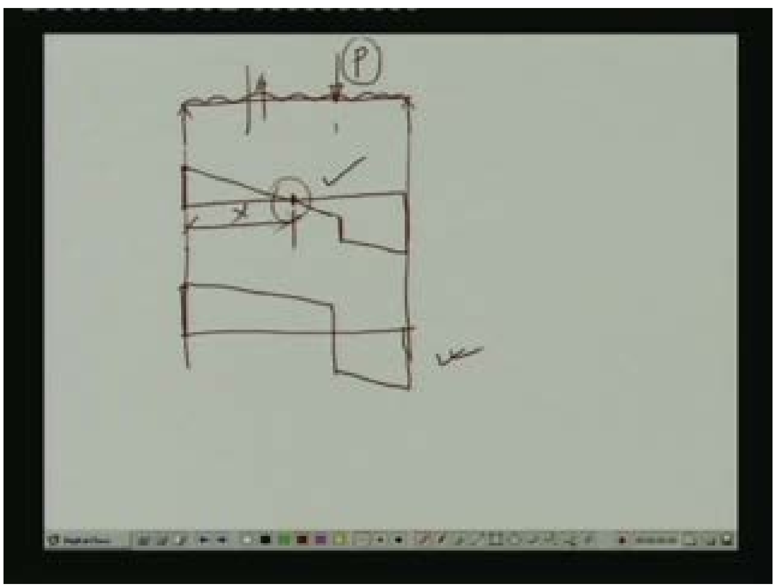

Let us suppose we have to first locate the position where we get the maximum bending moment. Let us see how to compute that the maximum bending moment. When you have this particular beam where you have the hinge support and the roller support and you have the uniformly distributed load acting over the entire span along with the load P over here, there could be two cases.

Since we do not know the value of P, the shear force diagram at this point goes over here. Then if you take a section, in fact, you can place the shear force b where b is equal to the load w x x. Since we have a uniformly distributed load it comes down and at the load point it comes down further and comes over here.

There could be a diagram wherein we will get 0 shear force here of some distance. Let us call this as x and we can have a situation where the shear force may not be 0 at this particular place. It will go here, then it comes down, at the load point it crosses and comes over here. (Refer Slide Time: 30:42 - 31:51)

The shear force diagram could be like this as well where at a distance x some where the shear force is 0 and we are trying to find out the point where shear force is 0 because of the fact that at that particular point, we know that the bending moment will be maximum. So, the bending moment could be maximum where the shear force is 0 or it could be maximum where there is a change over form the positive to the negative or negative to the positive sign and we have these two cases.

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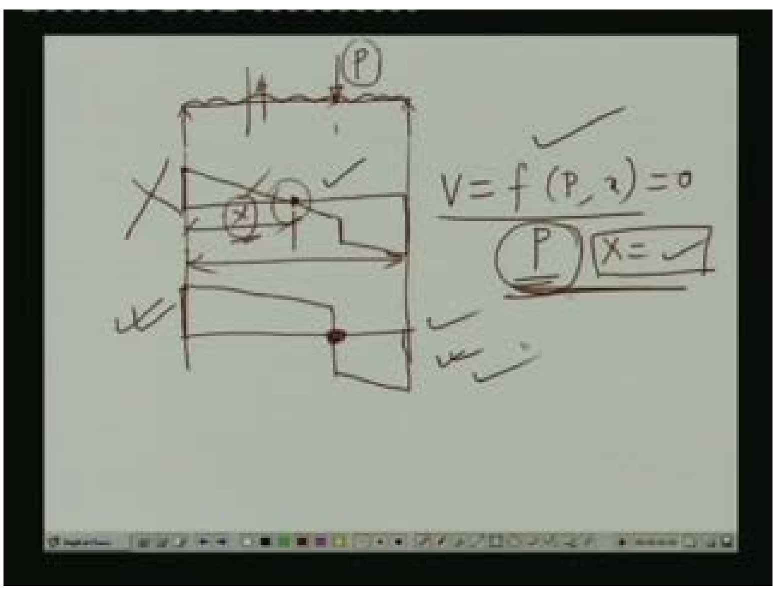

If we write down the expression for V = f (P, x) and if we get 0 correspondingly, we can get the value of P. If we substitute the value of P which we get in the expression for x, we find that it goes beyond the length of this particular beam. This indicates that this kind of diagram is not possible, which means that the shear force does not cross the base line at an intermediate point before this loading P.

Hence this particular diagram will be applicable in this particular case and hence this is the point where we expect that there will be the maximum bending moment in the beam and we compute the maximum bending moment at this particular point and correspondingly we find out the loading. If you compute the value of the bending moment at point c the bending moment at c is equal to what we have.

If you take the free body diagram we have the reactive force R_A and this uniformly distributed load of intensity 3 kilo Newton meter and this is at a distance of x and we have the shear force and the bending moment. If you call this as moment M_c = R_A x - 3x square/2, R_A as we know is (7.5+ P/ 5) x -1.5 x square and this is the magnitude of the bending moment. As we know that Sigma = - MY/I, here M is positive and Y is also positive and at the top we will have the compressive stress.

Since this particular cross section is a symmetrical section with respect to both the x and y axis, the top will have the compressive stress, the bottom will have the tensile stress and magnitude-wise they will be same. So, we disregard sign over here and we take the value of M. Here Sigma has to be restricted to 10Mega Pascal and the expression for the bending moment is (7.5+ P/ 5) x -1.5 x square.

The maximum bending moment will be occurring at the point of the load which is at a distance of 4m. So, if we calculate the bending moment at that particular point substituting the value of x =4M we get the bending moment as 6+4 P/ 5kilo Newton meter assuming that P is in kilo Newton.

Let us substitute the values in this particular expression M as 6 + 4 P/5x10 to the power of 6 Newton mm into Y and Y is that the distance from the natural axis. The neutral axis divides the whole section into two hours where this distance is 200 because the total depth is 400. This divided by the moment of inertia which is 120x400 cube by 12 gives us the equation MY/ I = stress. From this if we compute the value of P we get the value of P as 32.5 kilo Newton. This is the largest value of P that we can apply so that the level of stress is within 10Mega Pascal. It does not exceed the value of 10Mega Pascal if we limit ourselves to 32 kilo Newton load.

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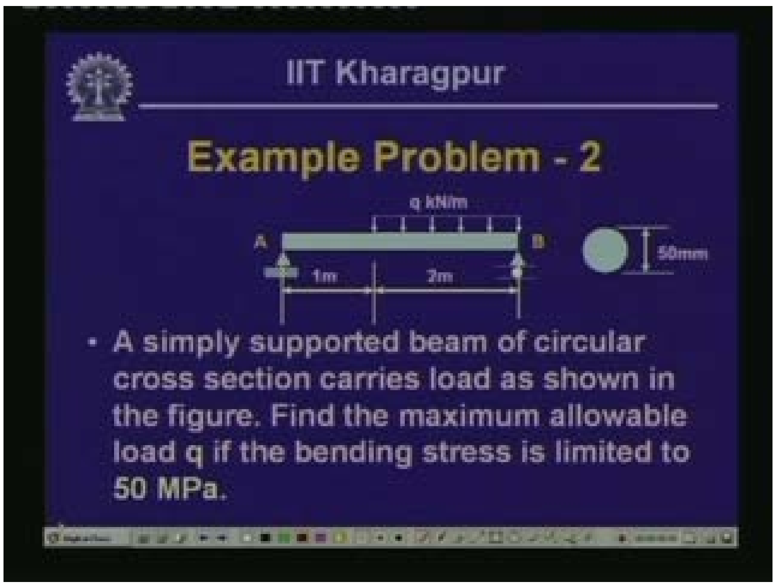

Let us look into another problem wherein the uniformly distributed load of intensity q is acting over the distance of 2M which is a partial loading and the cross section of the beam is a circular one having a diameter of 50mm. The maximum load q can be applied on this beam so that the bending stress is limited to 50mm Mega Pascal.

Here we will have to find out the value of the load q so that the stress does not exceed a value of 50 Mega Pascal. As we have done earlier we will have to find out the value of the maximum bending moment. We resort to the drawing of the shear force and the bending moment diagram so that we can get a clear picture of what is the maximum value of the pending moment and at which location it occurs.

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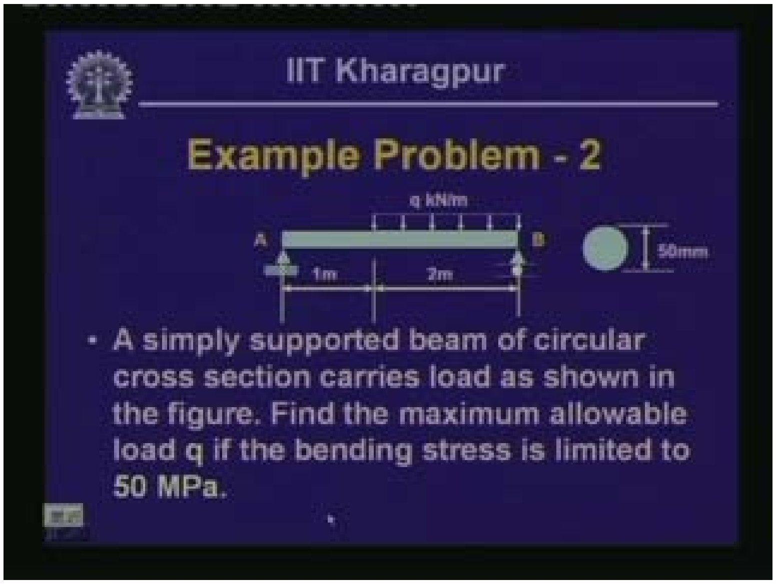

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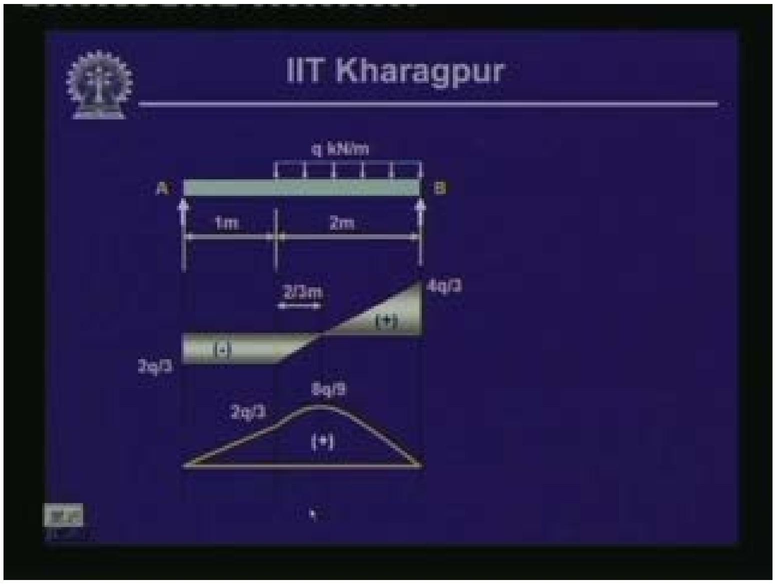

We need to compute the value of the reactive force as usual. Let us do that with respect to this particular beam. Take the free body of the beam H_A as = 0 and if we compute the value of R_A and R_B, R_A + R_B = the vertical load which is 2 q. If we take the moment of all the forces with respect to P we can get the value of R_A which is 2q x1 divided by 3 and we get 2q/ 3.

Correspondingly, R_B = 4q /3 and these are the values of the reactive forces. If you take the free body of this left segment from 0 to 1 then we have the reactive force R_A. Over here we have the shear force and bending moment V and M. Now V + R_A =0 and this gives you V = - R_A and R_A is again =2 q / 3.

From 0 to 1, we have the constant value which is 2q/3 but negative. At this section in this segment between 0 to 1 the value of the moment in M is RA x. It is linear with x when x =0. The bending moment is 0 when x =1 and the value of the bending moment is 2 q/ 3 kilo Newton M.

Let us take another segment which gives us a value from 1 to 3 and let us take the free body diagram of that particular part. We have the reactive force over here and we have the uniformly distributed load which is 1m away from this n and at this section again we have V and the bending moment n. Let us call this distance as x and here the value of the shear force V, if you take the vertically equilibrium V + R_A - w (x -1), is 0. So,
V = w (x - 1) + R_A and R_A = 2 q / 3 q which is the uniformly distributed load. When x = 1 and V = 2 q / 3 as we have seen this is a - 2 q/ 3.

When x = 2 this is q and when x =3 it is 2 q and this is also 2 q / 3. So, this gives you 2q - 2q/ 3 which is = 4 q / 3. The shear at this end is -4 q /3 and at this point we had - 2 q / 3; at this point we have +4 q / 3, when x =3 we have 2 q - 2 q / 3 which is P = 4 q by 3. It changes from minus to plus. Somewhere we will have a zero value and if we substitute the shear force value as 0 then we get the value of x.

With P =0 we get q x - q - 2 q /3 and 5q/3 = q x which gives us x = 5 /3. At a distance of 2/3 from here we will get the shear force as 0. From a distance of 2/3m from here we get the shear force as 0 and it is expected that the bending moment at that particular point will be the maximum.

This particular moment will be a continuous one without any kink from this point to this point since we have a uniformity distributed load. Take the equilibrium of the bending moment and carry out the calculation over here. Then we get the bending moment equation as M and then - R_Ax + q (x -1) square /2 and this is 0. Since a square of x is the parabolic distribution and as we have seen that when x = 5/3 then the shear force is 0. We get x as 5 /3 and we get the value of bending moment M as 8 q/9. So, we get the value of the bending moment as 8 q /9 and this is a positive bending moment right through.

In order to find out the bending stress in a beam we have two clear parts. The first part is that we will have to draw the bending moment diagram to know how the bending moment varies over the entire length, whether there is any change over in the sign of the bending moment or not and then it will give us the maximum value of the bending moment and at which location that maximum bending moment occurs. Once we have that particular information then we can compute the stress based on that cross sectional information that we have done and we have know that the maximum bending moment = 8q/ 9.

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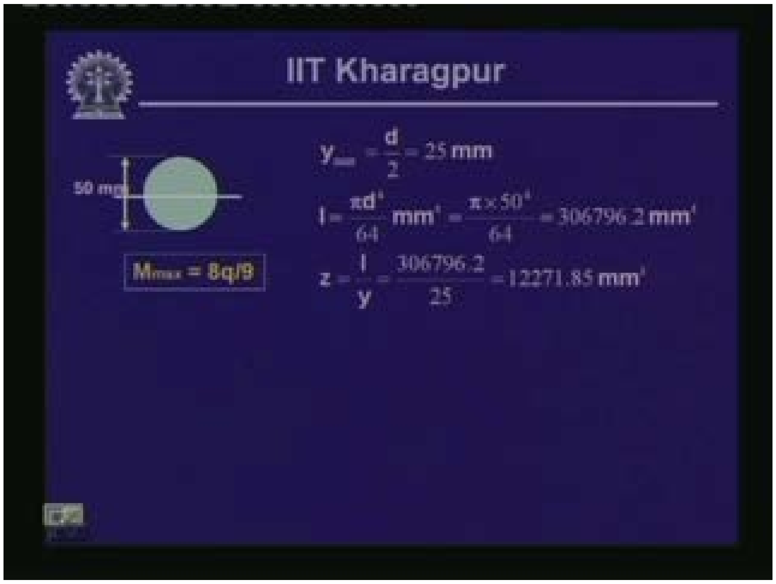

Let us suppose that in this particular configuration the cross section of the beam is a circular one and the neutral axis lies at the centroid of the cross section, which divides the section into two halves. There by the maximum distance y = d / 2 which is =25 mm and the moment of inertia and the cross section with respect to the neutral axis is pi d / 64 where d is 50. This is the value of the moment of inertia that we get and we have the information that the maximum bending moment is 8 q /9 which is occurring at a distance of 5/3 from the left support.

Let us suppose we have to calculate the bending stress Sigma. We disregard the sign because we know that in the positive bending moment the stress is compressive at the top and tensile at the bottom and we know the nature of the stress so what we are dealing with is only the magnitude.

So, Sigma = MY/I. Here M is a positive moment and we 8 q/9 x 10 to the power of 6 Newton mm because q is in kilo Newton per meter. Here we have 8 kilo, 8 q / 9 kilo Newton M multiplied by 10 to the power of 6. Now Y is 25 and I = 306796.2 mm to the power of four.

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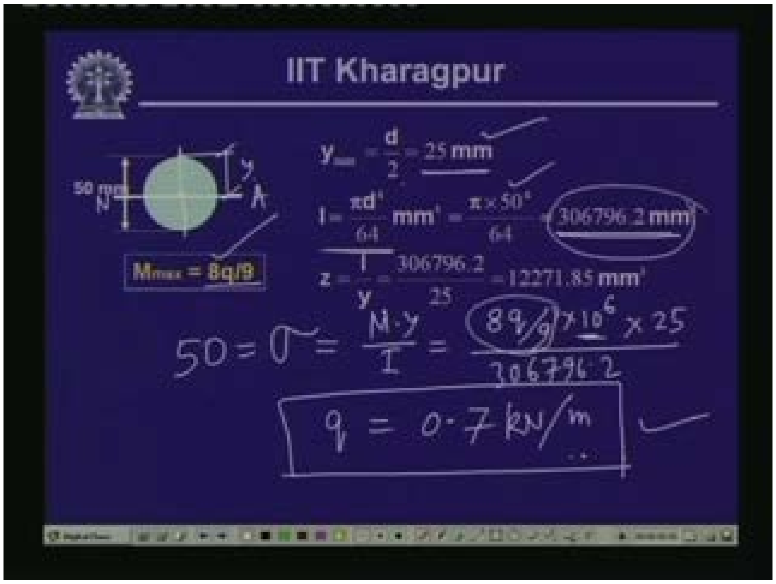

The stress as we have seen is limited to 50 Mega Pascal and from these expressions if we compute the value of q we will get the value of q as 0.7 kilo Newton per meter. This is the maximum intensity of the load that we can apply on the beam so that the stress is within 50 Mega Pascal.

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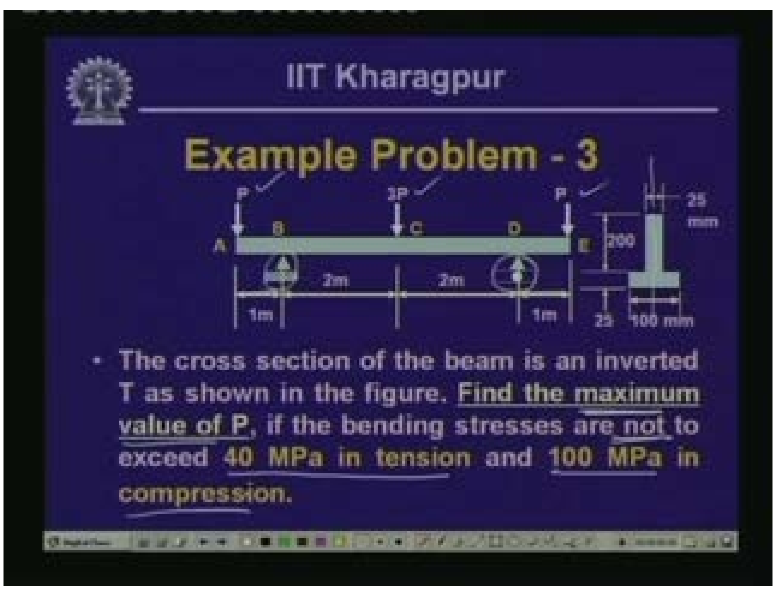

Let us take another example problem where in the cross section of the beam is an unsymmetrical one with respect to the horizontal axis but it is again symmetrical with respect to the vertical axis and the beam is subjected to three concentrated loads. So, we have P, 3P at the center and P and it is hinged at this particular end and is on a roller support at the other. Geometrically it is symmetrical and the loading is also symmetrical. We will have to find out the maximum value of P and it should not exceed 40 MPa in tension and 100 MPa in compression.

In the pervious occasions where we had depth with a rectangular cross section or a circular cross section both the sections were symmetrical with respect to both vertical and the horizontal axis and we did not bother about the stress limits like compressive and tensile stresses because many of the magnitude of the stresses that will be developed because of the bending moment at any section will be the same because the value of I and Y will be the same at the extreme points.

If we have a cross section which is not symmetrical with respect to the horizontal axis and thereby there will be a difference in the Y values with respect to the neutral axis of the extreme fiber at the top and bottom. Consequently, there will be a change in the bending stress. We will have to know precisely the stress in tension and compression so that we can check if the members can withstand the load they are subjected to. In this particular case we have the loads P, 3P and P which are acting.

We will have to find out the maximum value of p so that these stress limits are not exceeded. To do that again we will have to first find out the maximum value of the bending moment that can occur in this particular beam at any point along the length of the beam because of the loading and the location at which such a maximum bending moment occurs.

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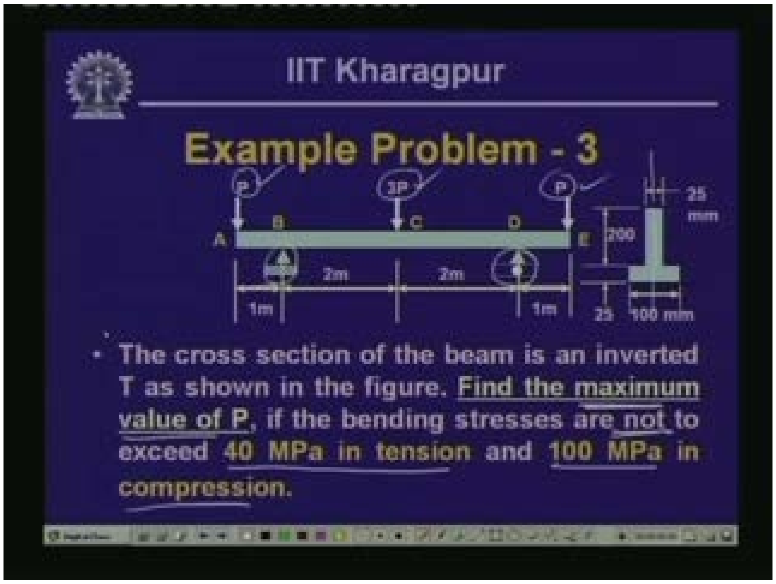

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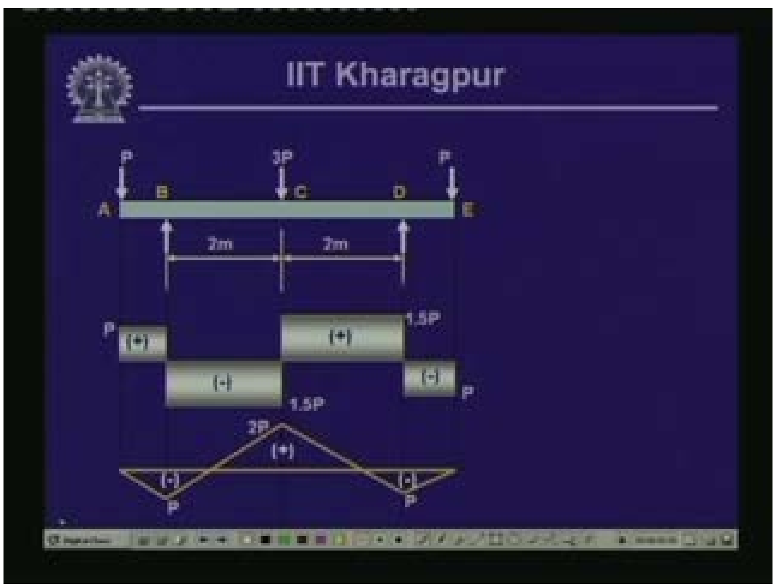

Let us again compute the values of the reactive forces. The vertical reactive forces have P, this is R_D and as usual since there are no horizontal forces, the horizontal force at the hinge support is 0. So, $R_B + R_D$ = the vertical forces, 5 P and since they are symmetrical, R_B and R_D will be equal in magnitude and this is 2.5 p.

If we look into the shear force diagram between A and B in the first segment and if we take a section over here the value of the shear force will be equal to P V = - P and from A to B there is no other load so the value of P is constant. If I take a section between B and C and draw the free body diagram we have the load here which is P.

We have the reactive value which is 2.5 and we have the shear force which is V and of course the moment now $V + R_A - P = 0$. This gives us V as $P – R_A$ and R_A is 2.5 and this is 2.5 P, so this is =-1.5 P and from + P 2 it goes to -1.5 P at this particular point. Since there are no loads the 1.5 P continues up to the load at C.

Let us take a free body diagram with a section over here. We have the concentrated load P acting here; we have the reactive force R_B over here, we have the concentrated load which is 3P over here and at this section we have again the shear force V. If we take the vertical equilibrium of the vertical forces we have $V + R_B$ and in the opposite direction this is 4 P, R_B is 2.5 P and V =+1.5 P.

Before C -1.5 P and immediately after C we have the concentrated load P. Since the shear force which is 1.5 P +1.5 P and again between C and D since there are no loads it continues up to 1.5 P and then again at reactive force R_D it is 2.5P. So, if you take a section over here we get immediacy after this support shear force which is – P. It comes down from +1.5 P +P and then again it continues and at this point we have the force P and we get the shear force diagram.

Along the length of the shear force diagram we have three places where the shear force has changed signs from positive to negative or negative to positive. At these three locations we expect that there will be the values of the bending moment which are maximum in nature.

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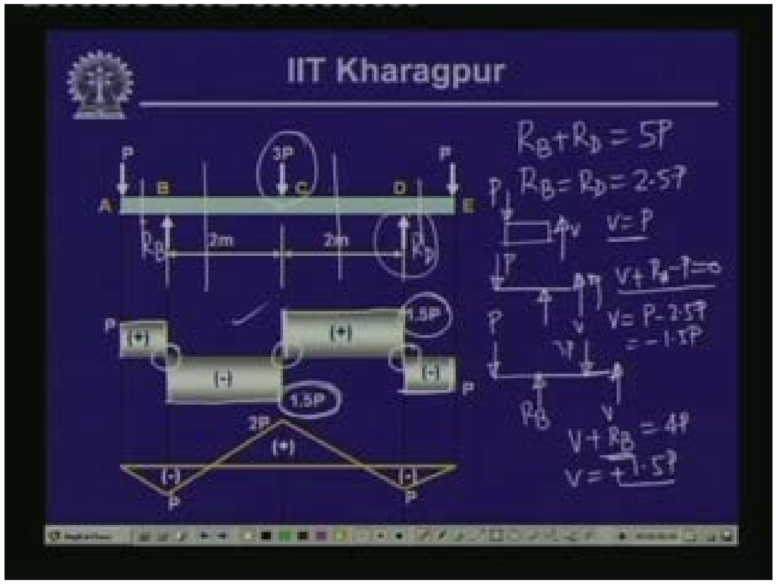

Now the question is: If we compute the values of the bending moment at these sections then you will find that we get a value of P kilo Newton meter. At this particular point, we get a bending moment of 2P kilo Newton meter at the central point and at this point again we get p kilo Newton meter which is negative at this particular point. From this bending moment diagram we will find that we had a value of bending moment P which is negative and here the value of the bending moment is positive.

Along the length of the beam at three locations during the magnitude this is symmetrical. On either end at the support we had a negative P kilo Newton meter and at the center we have 2P kilo Newton meter as the moment and note that the moment which we are getting at the center is a positive moment and at the support it is a negative moment. Hence the maximum positive moment is 2 P kilo Newton meter and the maximum negative moment is P kilo Newton - P kilo Newton meter. Let us deal with this two moment values.

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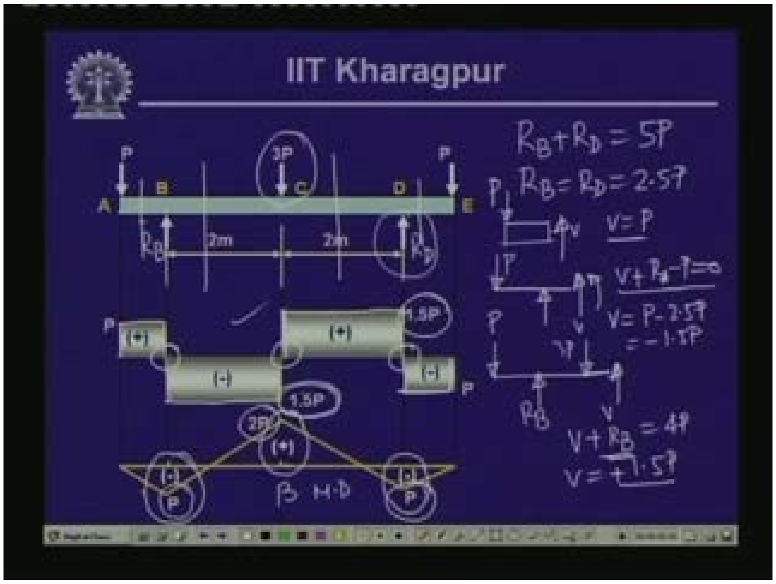

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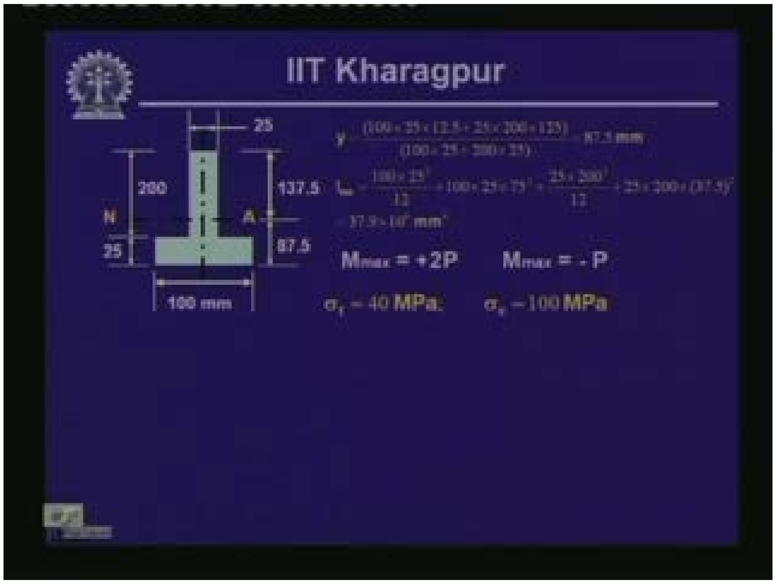

We have identified that a maximum positive moment is 2 P the maximum negative moment is P and when we have the positive moment it is in this particular direction causing compressive stress at the top and tensile stress at the bottom. When we are dealing with the negative moment the tension is at the top and compression is at the bottom. Since the section is unsymmetrical the value of Y1 and Y2 will be different and consequently the values of the stresses will be different.

Since we have tensile and compressive stress we will get four values of P corresponding to all these four criteria. First, try to find out the centroid of this particular section through which the neutral axis will pass through and take the moment of all the areas of with respect to the top. You will find that the distance Y will be 87.5mm from this bottom base.

Consequently this is 225 so 225 - 87.5 and we will get 137.5 over here and if we calculate the moment of inertia of this neutral axis we will get a value of 37.10 to the power of 6 mm to the power of four. Let us try to compute the stresses corresponding to these two moments.
First let us take the value of 2P. Here Sigma as we know is - MY/I and by being negative we know that for the positive moment we will have the compression at the top. This is 2 P(10) to the power of 6 Newton mm. When we are computing the stress, at the top y is 137.5 divided by I = 37.9(10) to the power of 6 mm to the power of 4. This is the stress at the top which is compressive in nature and we have this as 100 Mpa. Then correspondingly the value of P which we get is 13.8 kilo Newton.

Now consequently we calculate the tensile stress at the bottom corresponding to this moment and it is 640 Mpa. So, 40MPa is 2 P(10) to the power of 6 (87.5) /I which is 37.9(10) to the power of 6. Now from this if you compute the value of P we get 8.66 kilo Newton. These are the two values of P which we have obtained corresponding to the compressive and tensile stress.

Likewise if we compute the value of the stresses corresponding to this negative maximum which is P, this will produce tensile stress at the top and compressive stress at the bottom. Let us try to compute the value of the stresses from which we can get the value of P where again the tensile stress at the top is 100 Mpa which is P (10) to the power of 6 (y) which is at the top is 137.5/37.9 (10) to the power of 6 and this gives us a value of P as 11.03 kilo Newton.

Let us compute the stress at the bottom which is a compressive stress which is 100 Mpa. The tensile stress was 40 Mpa and if you take the compressive stress at the bottom we get 100 Mpa = P (10 to the power of 6) (87.5)/37.9(10 to the power of 6). This gives us a value of P as equal to 43.31 kilo Newton. We have four values of P and we get P = 8.66 kilo Newton. This is the value of P which can be applied so that the stresses will be within the limit of this particular cross section.

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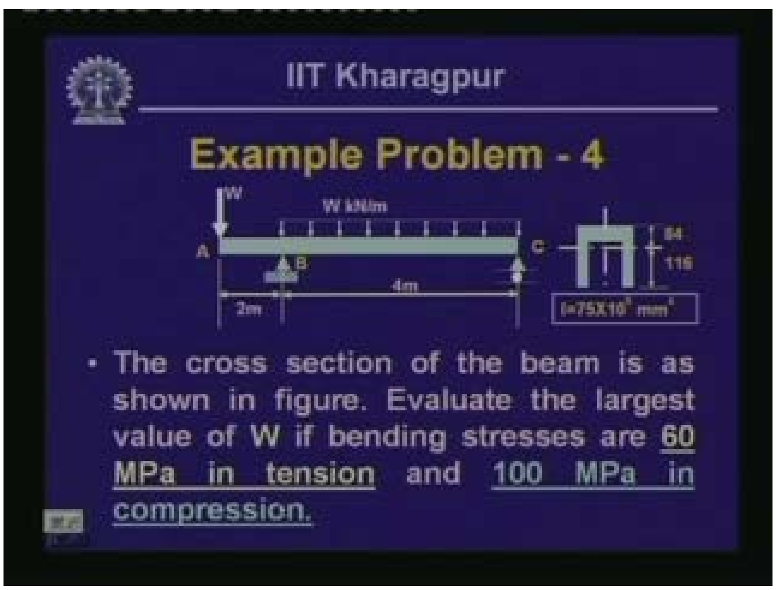

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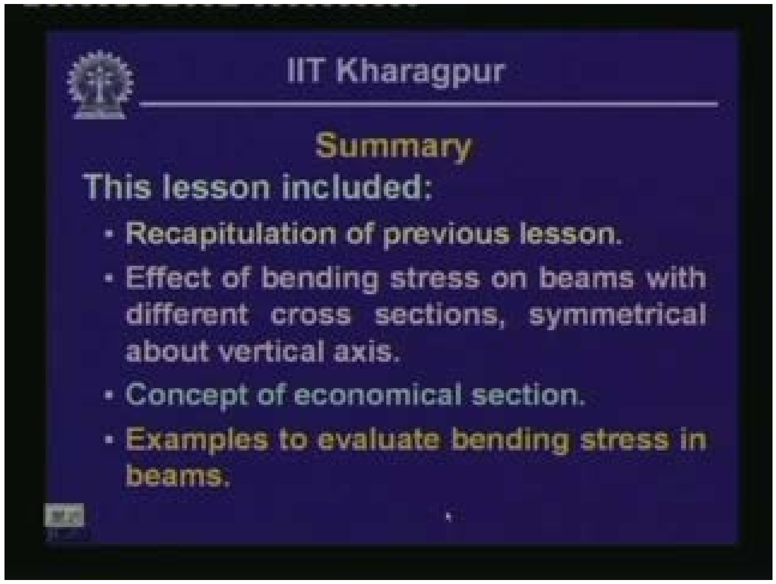

We have another example problem for you which you can try. This is a beam which is subjected to a uniformly distributed load of w kilo Newton per meter and at this point we have a load of w. There has been a cross section for which the distances from the neutral axis are given and based on the moment of inertia with respect to the neutral axis given we will have to compute the value of w so that the stresses do not exceed the limits.

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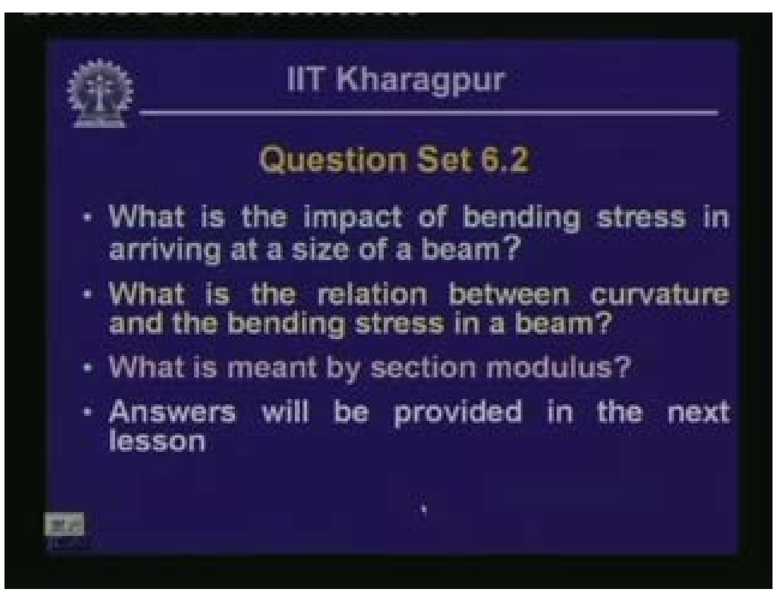

In this particular lesson we have recapitulated aspects of the previous lesson; we have looked into the effect of bending stress on beams with different cross sections. The sections are symmetrical with respect to the vertical axis but they could be unsymmetrical with respect to the horizontal axis and then we have looked into the concept of economical section.

We have also solved some examples to evaluate the bending stress in beams. The questions for you are: what is the impact of bending stress in arriving at a size of a beam? What is the relation between curvature and the bending stress in a beam and what is meant by section modulus? We will answer these questions in the next lesson.