**Strength of Materials**
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**Lecture no 25**
# Bending of Beams- IV

Welcome to the 4th lesson of module 5 which is on Bending of Beams part 4. In the last lesson we had looked at the aspects of shear force and bending moment. We had studied how to draw the shear force and bending moment diagrams. Now, in this particular lesson, we are going to look into some more aspects of shear force and the bending moment diagram.

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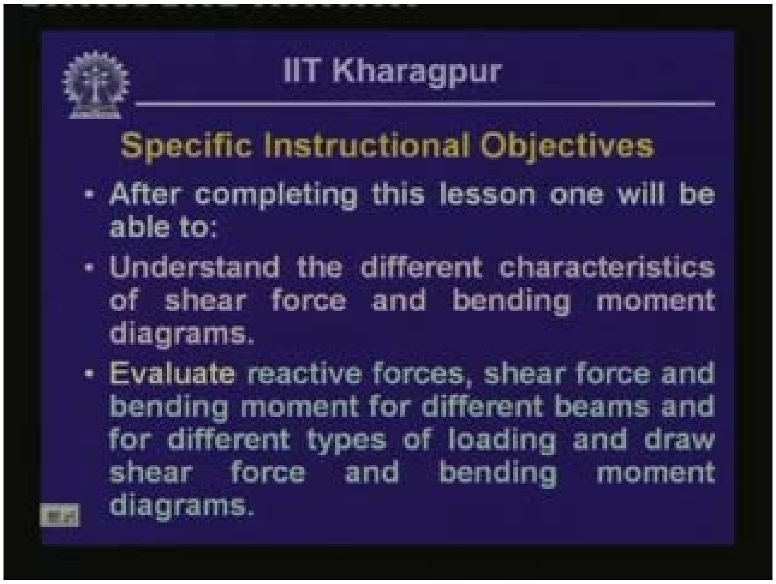


It is expected that once this particular lesson is completed, one should be able to understand the different characteristics of shear force and bending moment diagrams for different kinds of beams subjected to different kinds of loads. We will analyze those beams for which we have drawn the shear force and the bending moment diagram.

We will look into the different salient features of shear force and bending moment diagram and then we will look into some more examples of different kinds of beams which are subjected to different kinds of loading and how to plot the shear force and bending moment diagram for those beams. One should be in a position to evaluate the reactive forces, shear force and bending moment for different beams and for different types of loading. Also one should be able to draw shear force and bending moment diagram for such beams.

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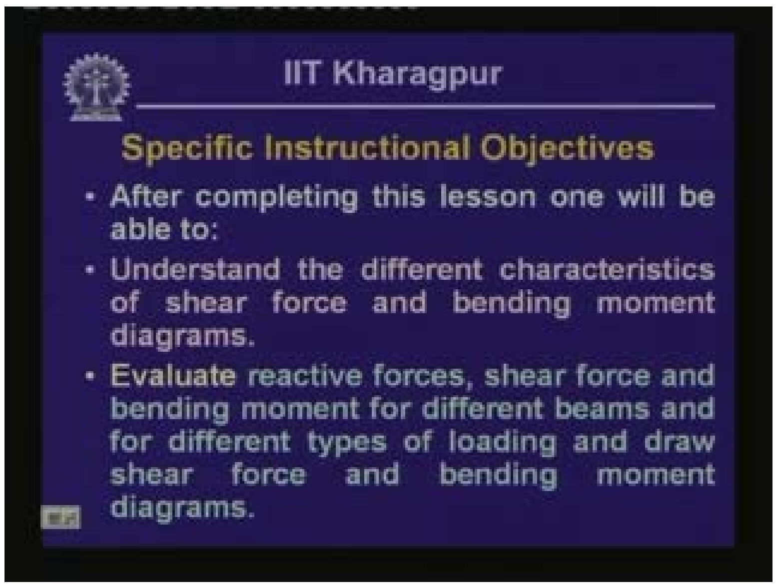


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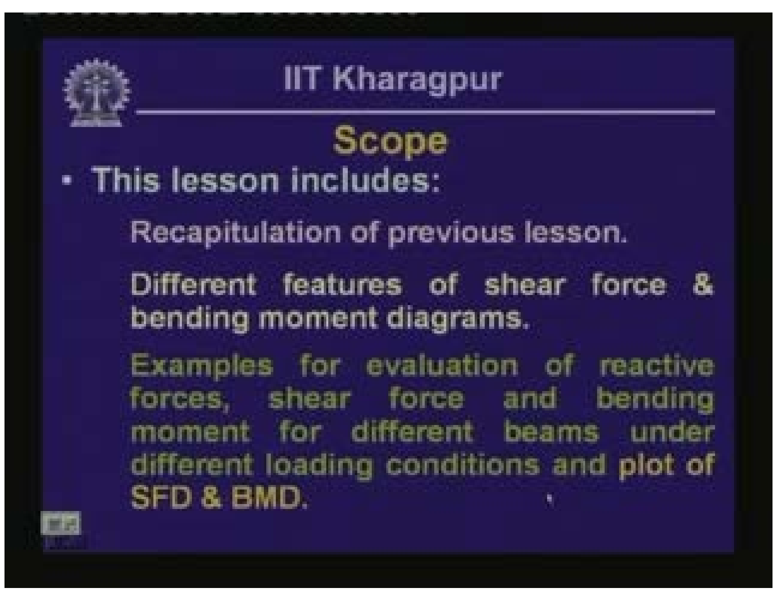

As a part of this lesson we will recapitulate what we did in the previous lesson. We will look into the aspects which we have discussed about the shear force and bending moment diagram and we will study their different features in a more elaborate way. Also we will give some examples for the evaluation of reactive forces, shear forces and bending moment for different beams under different loading conditions. We will plot the shear force and the bending moment diagram for those beams.

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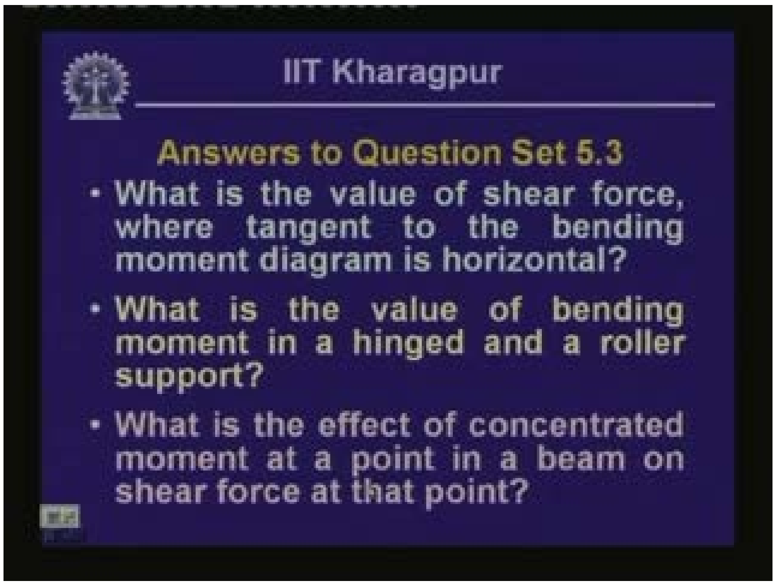


Let us look into the answers of the questions which were posed in the last lesson. The 1$^{st}$ question which was posed was what is the value of shear force where tangent to the bending moment diagram is horizontal?

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In the last lesson, we had seen that the differential equation dV/dx along the length of the beam is equal to q where V is the shear force and q is the uniformly distributed load. If we have q = 0 then what happens to V? Subsequently, if we take the differential of this moment dM/dX this is equal to - V. If the shear force V is 0 then we had said that dM/dX = 0 and over that particular zone moment is shown as a constant.

What if dM/dX is 0? We have solved this particular problem last time wherein one end of a simply supported beam is on a hinged support, the other end is on a roller support and if this beam is subjected to a uniformly distributed load then what will be the shear force and the bending moment diagram? We had looked into how to plot the shear force and bending moment diagram which are shown over here. (Refer Slide Time: 04:36) This is the shear force diagram and this is the bending moment diagram.

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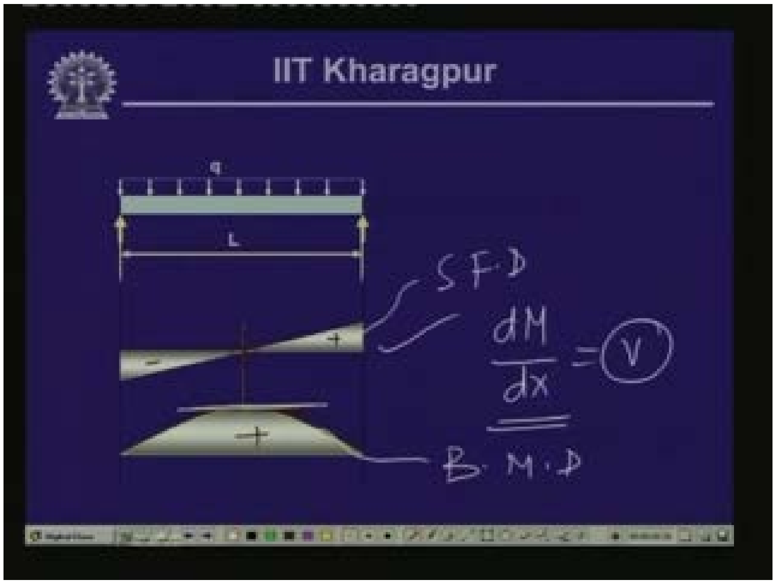


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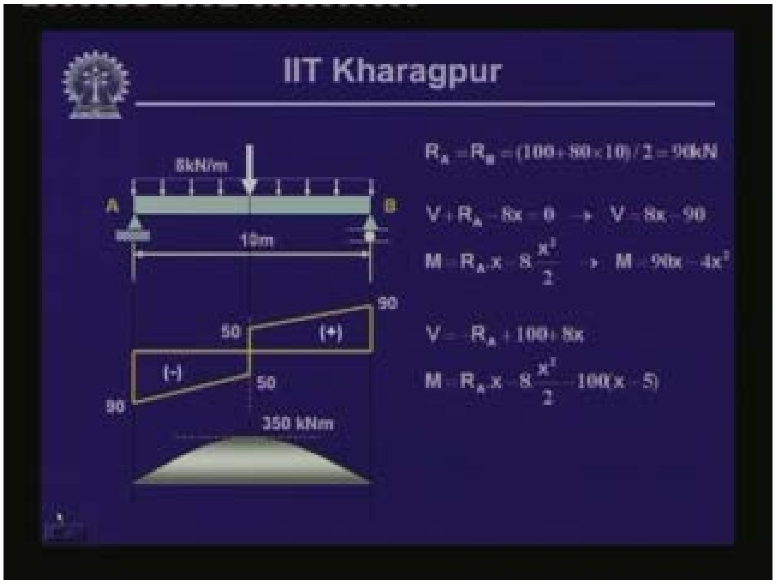


As you can observe from this bending moment diagram that at this particular point, the tangent to the bending moment diagram is a horizontal one. It indicates that the derivative at that point dM/dX is 0. Corresponding to this, as you can see the shear force at this particular point is 0. Let us look into another problem to describe what happens if the tangent is horizontal at a point in a bending moment diagram.

Let us look into this particular example where or again the beam is a simply supported one having a hinged support over here and a roller support over here. It is subjected to a concentrated load and let us say that this concentrated load is 100 kN. If we try to draw the shear force and the bending moment diagram of this particular beam then there is no horizontal force in this we will have the value of $H_A = 0$ from the summation of horizontal force 0.

Here we have the vertical reactive force $R_A$ and for the roller support, we have the vertical reactive force $R_B$. Summation of horizontal force = 0 will give us $H_A = 0$, $R_A$ and $R_B$ are the 2 vertical reactions. Since the beam is symmetrical and we have load of 100 kN which is concentrated at the center it is at the center of a beam at distance of 5 M from here and we have 8 kN per M uniformly distributed load over the entire span. If we write down the reactive values as $R_A + R_B$ = the vertical force 100+80 (10) and then if we take the moment of all the forces with respect to a, we can evaluate the value of $R_B$.

Therefore you will find that $R_A$ and $R_B$ will be 90 kN which is half of the total load that has been carried by the beam because $R_A$ and $R_B$ are symmetrically supporting the whole beam and the load is also symmetrical. The reactive values of $R_A$ and $R_B$ are same and the magnitude is 90 kN.

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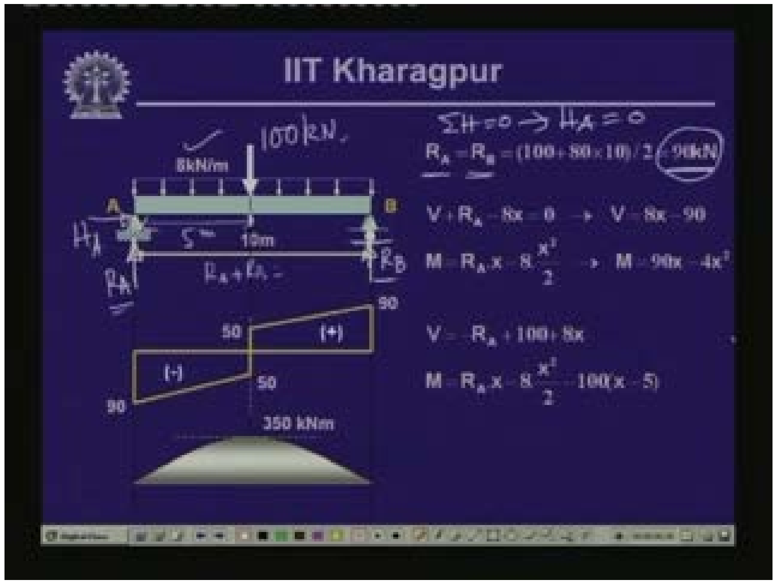


Let us suppose that we cut the beam at this location. Let us say this is ‘a a’ and take the free body of this particular part then we have the reactive force $R_A$ over here. We have the distributed load which is at 8 kN per meter. We have the shear force P and the bending moment M at this cut section. If we take the vertical equilibrium, the equilibrium of the vertical forces then V + $R_A$ are in the same direction - 8 kN per meter.

This section is at a distance of x and so – 8 (x) = 0. This gives us a value of 8 x - 90 and that means this particular expression is valid because the load is constant from a up to the point where the concentrated load acts so just before the concentrated load everywhere the loading situation is the same. So, the shear force will be guided by this particular expression. At x = 0, we get a value of shear force which is - 90 and this is what is reflected over here that as x = 0 the ordinate is -90. Now at x = 5, before this concentrated load if we try to calculate the value of the shear force for x = 5 V = 5(80) which is 40 - 90 = -50.

The value at this particular point is – 50. Precisely, the concentrated load acts at this particular point. We are evaluating the value of the shear force a little left to this load. Basically we have the coordinate over here as 50. Likewise, let us assume for the time being that we have this variation from here to here which is linear with respect to x. If we like to find out the value of the shear force immediately after this 100 kN load, let us take a section here at this point which is again at a distance of variable distance x where we include this concentrated load as well.

If we draw the free body diagram of this particular portion then we have the reactive force $R_A$. We have the concentrated load which is at a distance of 5M and we have uniformly distributed load over the entire length of x and as usual we will have a reactive force V and the bending moment M so this is V and M. If we compute the value of shear force V and if we take the equilibrium of the vertical forces then V + $R_A$ -100 kN - 8 (x) = 0. So this gives us - v = - $R_A$ +100+8x.

Here, if we substitute the value of x next to this load point which is approximately 5 then this becomes 8(540) which is 140 and $R_A$ = 90. So this becomes 50 and this is +50, on this side we have -50 and on this side of the loading we have +50. We get the ordinate over here. Basically if we enlarge this particular part, it will appear to have a variation like this but as this distance is very small we take this as straight line.

There is a jump of the shear force from -50 to +50 immediately after this load point. If we compute the value of shear force at this end where x = 10 M in this particular expression, if we substitute the value of x is 10 meter then 8xx = 1080, so 180-90 gives us a value of 90 which is the shear force at this end. It linearly varies and comes down to this place. This is the variation of the shear force for this particular beam where it is subjected to a uniformly distributed load of intensity 8kN per meter and has a concentrated load at the center of the beam which is 100 kN.

If you look into the values of the bending moment, correspondingly in the first case when we have taken this particular free body here, we had the bending moment as M and if you take the moment of all the forces with respect to this particular section then we have M = r(x) – 8(x square)/2 and this gives us the value of $R_A$ as equal to 90(x – 4(x square). If we substitute again the value of x as 0 then the value of the bending moment becomes 0 which is reflected here.

At x = 5 just on the left side of the load, if we substitute the value of x = 5 this becomes 450 - 100 which is 350 kN meter on the left side of this particular concentrated load. If we take the other free body diagram which is a section beyond the load point and take the free body of this left part and if you compute the value of the bending moment there we have bending moment as $R_A$ (x), we have the reactive force $R_A$-8(X square)/2-100(X)-5 and this is at a distance of 5M.

This is the expression for the bending moment. If you substitute the value of x = 5 again which is just after the concentrated load point then this term becomes 0 $R_A$ = 90. So, 90(5) = 450 - x is again 5. This is 100 and this is again 350. From both the expressions we get the value of bending moment at this point as 350. When we tried to establish the relationship between the loading and the shear force and the bending moment, we had seen that when we have considered a small segment of the beam in which a concentrated load acts, the change over from the left end of the load to the right end of the load has little variation of the bending moment which is negligibly small and thereby the moment at that particular point because of the concentrated load does not change at all.

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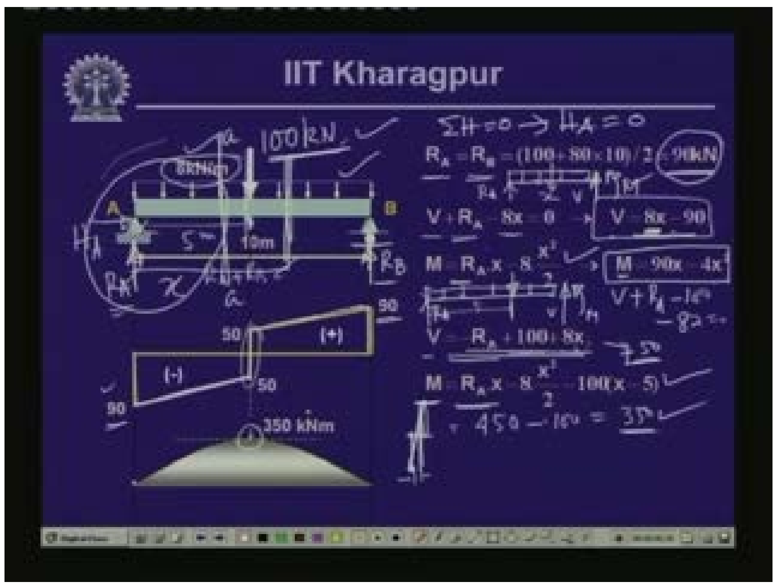


This is the magnitude of the 350 kN meter and if you compute the bending moment with this expression again when x = 10M, you will find that M = 0; this is 10 - 5 = 5, this is 500 and this is 10 square. This is 400, this is 900 and this gives us 0. This is a parabolic distribution and this is the bending moment diagram for this particular loading. At this point, this is the maximum bending moment which is 350 kN per meter and this is the point. As we have seen that, where we have a concentrated load though there is no change in the bending moment but the rate of change of the bending moment from the left of the concentrated load to the right of the concentrated load changes immediately and this is what is happening here.

There is a positive slope whereas from here, this slope is negative. At this particular point, if you take the tangent to this bending moment diagram, this tangent is a horizontal one and thereby the dM /dx = 0 for this particular case. Correspondingly, in this case the shear force changes its sign at the point of the concentrated load from negative to the positive value. In fact in the bending moment diagram at a particular point the tangent drawn is a horizontal one as we have seen from this 2 problems that either we get the value of the shear force as 0 or the shear force changes its sign from the negative to the positive or vice versa. If you go back to that expression that dM /dx=-v if shear force is = 0 then we can say dM/ dx = 0 or moment is constant or if dM/ dx= 0 then either V = 0 or V changes its sign from the negative to the positive or positive to the negative.

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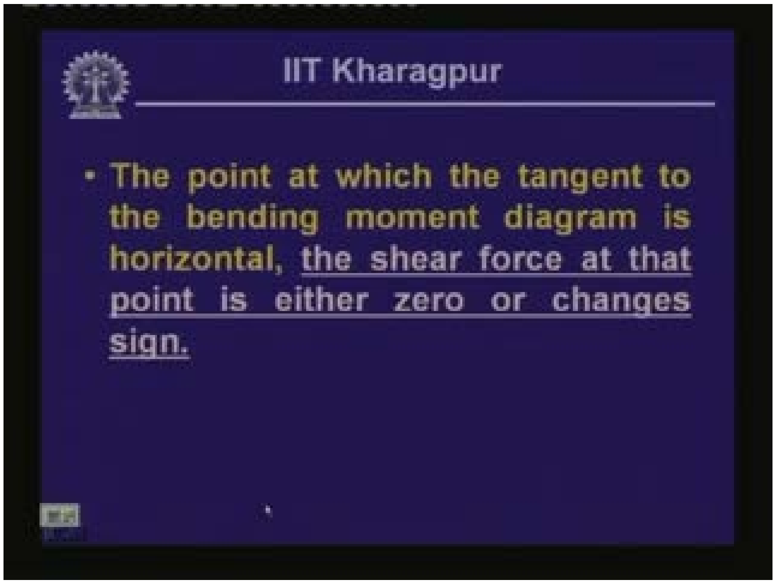


This is the consequence of the particular question. Now the answer to that question then is the point at which the tangent to the bending moment diagram is horizontal, the shear force at that point is either 0 or changes its sign. Let us look into the second question.

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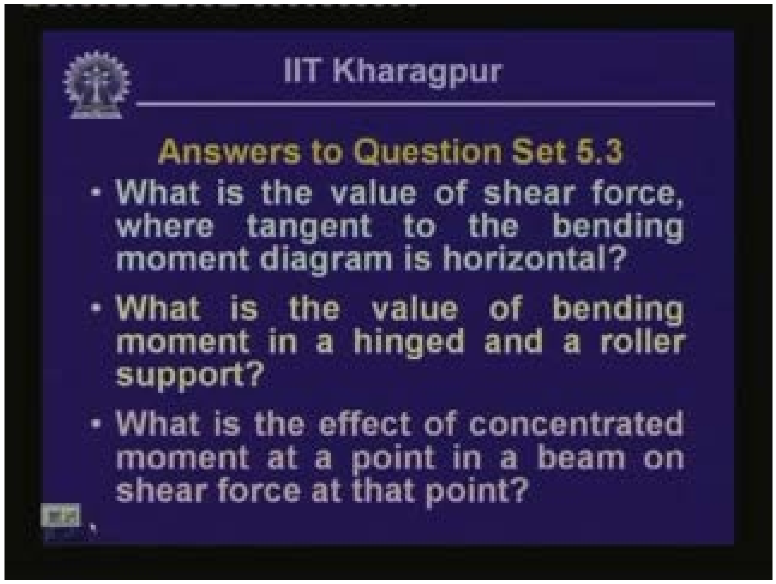


What is the value of a bending moment in a hinged and a roller support? This question is straightforward as we have seen while evaluating the reactive forces at the hinged support and the roller support. In the case of a hinged support it is constrained to move against the horizontal and the vertical direction but it is allowed to rotate, thereby there is restriction on the rotation and thereby it cannot sustain any moment.

As we have seen that it is allowed to move in the horizontal direction because of the rollers and thereby it cannot have any restraint in the horizontal direction but in the vertical direction we apply restriction, which means that it is not allowed to move and thereby there will be reactive forces in the vertical direction at the roller support again the beam is allowed to rotate. This means that there is no restraint on the rotation or it cannot resist any bending moment. The values of the bending moment both at the hinged and the roller support are 0.

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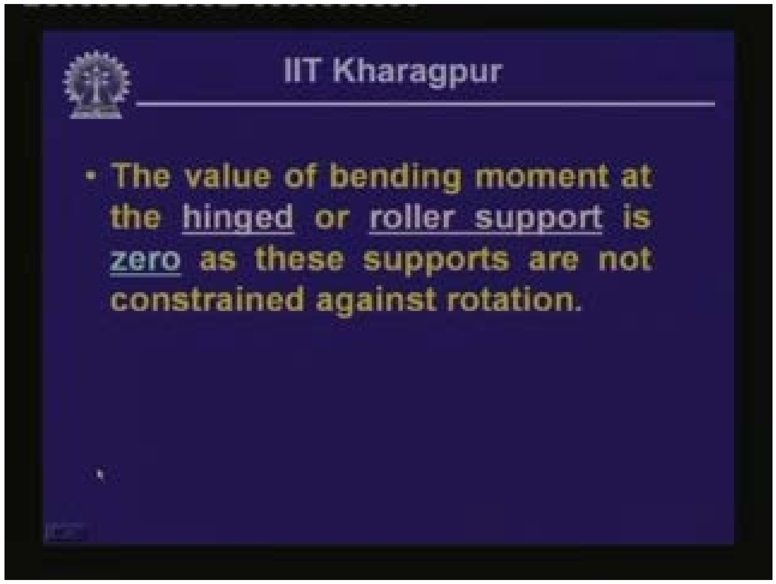


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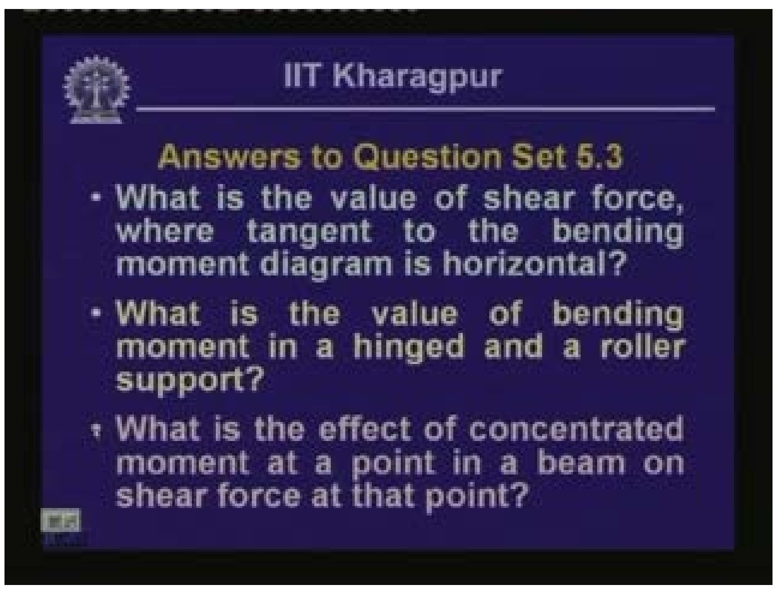


Let us look into the last question which is; what is the effect of a concentrated moment at a point in a beam on shear force at that particular point? We have seen the effects of the concentrated distributed load in the beam. What are the effects of a concentrated load at a point and how do they affect the shear force and bending moment on either side. In a beam at a particular point, if there is a concentrated moment, how does that affect the shear force at the particular point? To answer that question let us look into this particular diagram once again.

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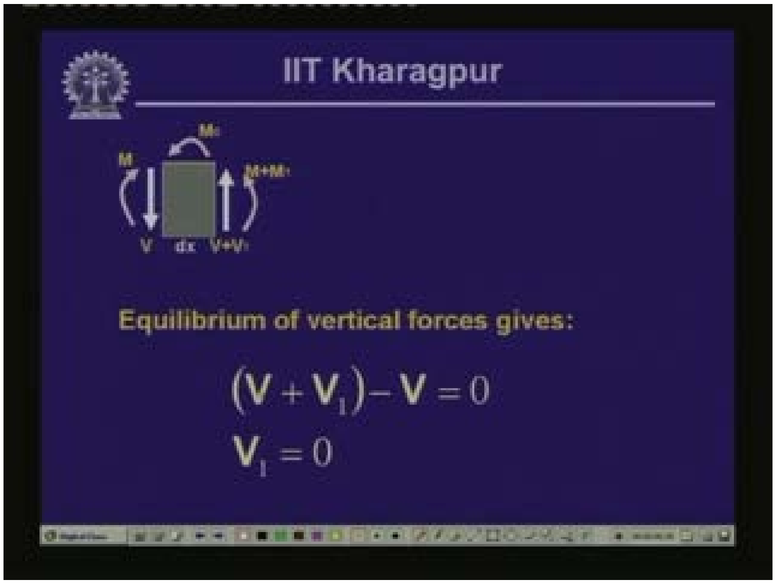


A segment of a beam of length dx is subjected to a concentrated moment M 0 and these are the values of the shear force and bending moment on either side of the section. If we take the equilibrium of the vertical forces then we have this particular expression that V + $V_1$ which is acting upward - V which is downward this is = 0, thereby this gives as $V_1$ = 0. That means in a beam at a point where there is a concentrated moment it does not affect the shear force so there will be not any changes in shear force level when you plot the shear force diagram. Wherever you have the concentrated moment that it does not affect shear force diagram there will not be any change in the shear force value.

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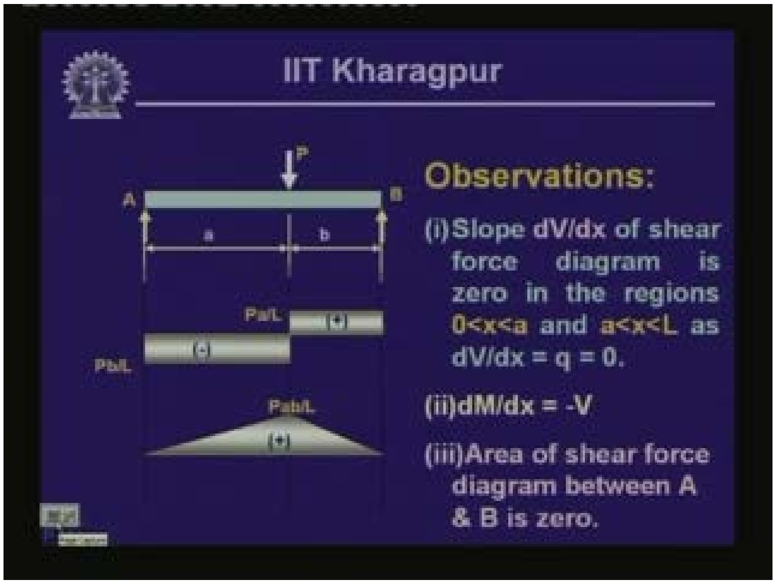


In the last lesson we had discussed the features of a shear force and bending moment diagram and how to plot them. We have defined a shear force and a bending moment diagram based on the example which we had seen last time both for the concentrated load, simply supported beam to a concentrated load or a simply supported beam subjected to a uniformly distributed load. Let us look into some of the salient features of shear force and the bending moment diagram.

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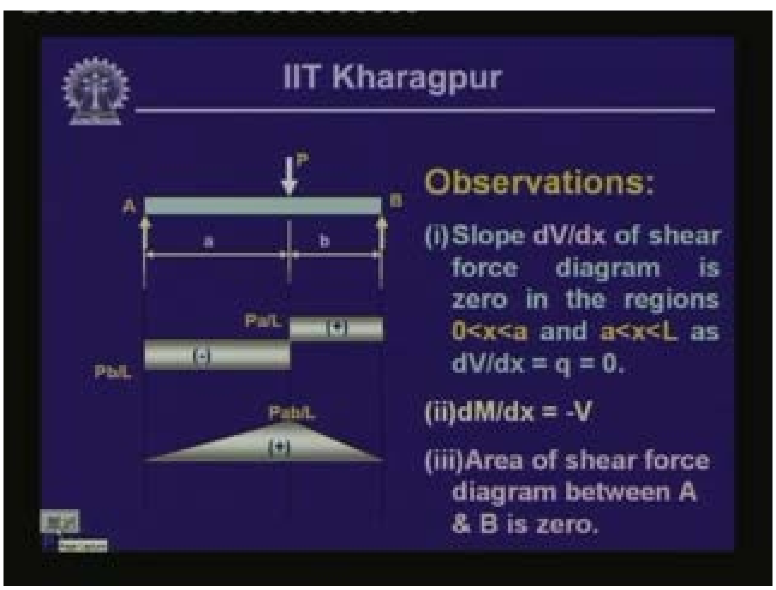
IIT Kharagpur
P
A
B
a
b
Pa/L
(+)
(-)
Pb/L
Pab/L
(+)
Observations:
(i) Slope dV/dx of shear force diagram is zero in the regions 0<x<a and a<x<L as dV/dx = q = 0.
(ii) dM/dx = -V
(iii) Area of shear force diagram between A & B is zero.

If you notice that the shear force and the bending moment diagram which we have plotted for a beam which is A B subjected to a concentrated load P at a distance of ‘a’ and this is on a hinged support and this on a roller support. Since there are no horizontal forces, we say h = 0 and we have these 2 vertical supports. Correspondingly, we have a shear force diagram and a bending moment diagram.

From this if you look into the slope of slope dV/ dx, that means the rate of the change of shears along the length or this part as well as this part which is x is between 0 to a or x is between a to L, we find that the dV/ dx value of the shear force diagram is 0 which is the case because dV /dx = q and if q = 0 expectedly dV /dx = 0.

As you have seen that dM/dx = - V is the expression which we have derived. If you look into this expression over here, at this particular point this is the bending moment diagram. Here, we have the magnitude which is = Pab/L and at this point of the bending moment value is 0. The dM /dx slope over here is equal to this moment minus this moment divided by this length L, which is why Pab/L - 0/a gives us Pb/L and the negative of Pb/L is the shear force at this particular location.

This equation again matches diagrammatically over here and on this particular part also. If you look into the slope here Pab/L this is 0, dM dx at this particular zone is 0 - Pab / L / b = - Pb/L and if you divide by p this Pb / L is equal to Pa /L, which is the value of the dM/dx of the slope which we can see from the shear force and bending moment diagram. If we integrate the dM between ‘a’ and b which gives us the moment value at b - moment value at ‘a’ we get Integral V dx.

From this expression we get Integral dM = b (dx). If we calculate the bending moment values at these two points $M_B$ - $M_A$ this is equal to Integral V dx. If we choose these 2 points V and ‘a’ then the difference in the bending moment values are 0 because the bending moment here is 0 and $M_B$ - $M_A$ value is 0 between these 2 points.

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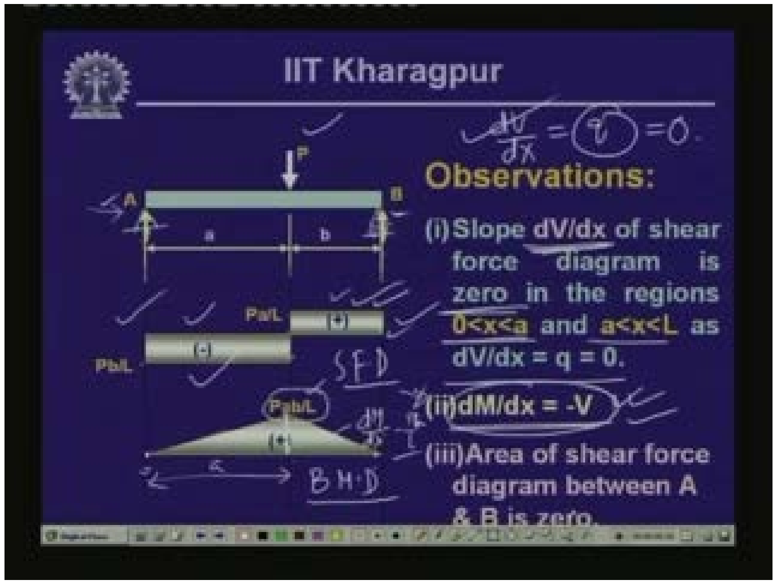


If we take the shear force diagram between these 2 points, this Pb/L which is negative multiplied by ‘a’. This is -Pab/L and over this region we have Pa/L (p) which is plus. We have Pab/L plus and if we sum them up we get 0. Again this satisfies the condition that the area of shear force diagram between ‘a’ and b = 0 and these are the observations from this particular diagram.

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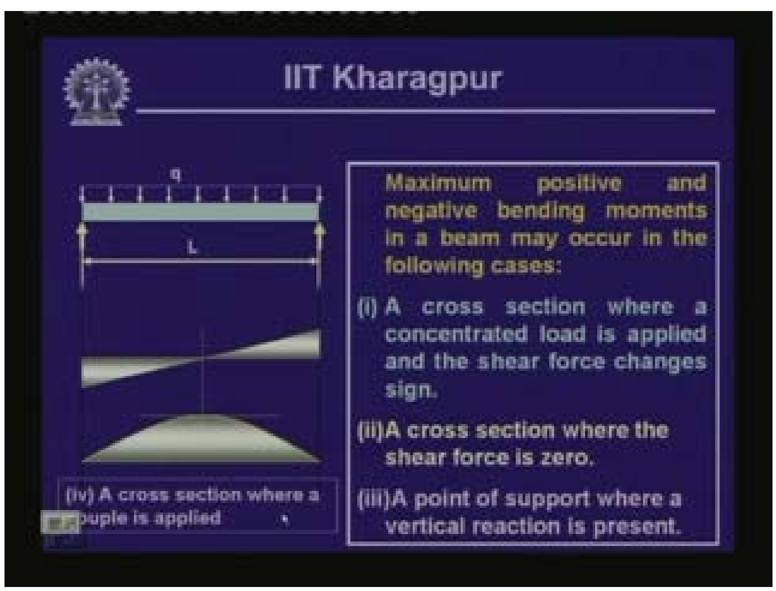

Let us look into some more aspects on this. The maximum positive and negative bending moments in a beam may occur in the following cases: Number 1:- a cross section where a concentrated load is applied and the shear force changes sign. We have solved one example where in on a uniformly distributed beam where a uniformly distributed load is there, we have a concentrated load as well as and we have seen in the shear force diagram the sign changes from the negative to the positive. Wherever we have a concentrated load, the shear force changes its sign.

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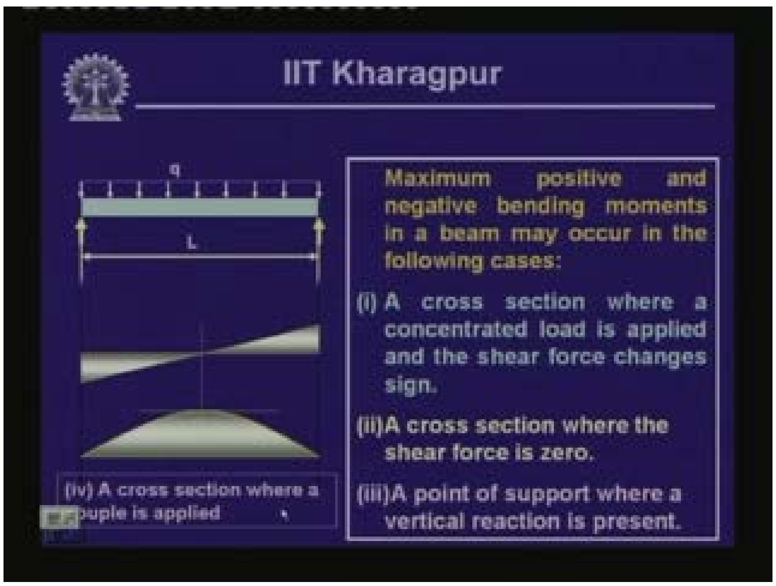


We have seen that the maximum bending moment occurs at that particular point. Also the maximum bending moment occurs at a point for a section where the shear force is 0. The bending moment is the maximum wherever the shear force is 0. We will also look at an example wherein if you have a beam that is simply supported or hinged on one end and supported on a roller at the other, you have an overhang. Therefore we have a vertical reaction at the roller support. In this particular point, you may get a moment at a support point which could have maximum value.

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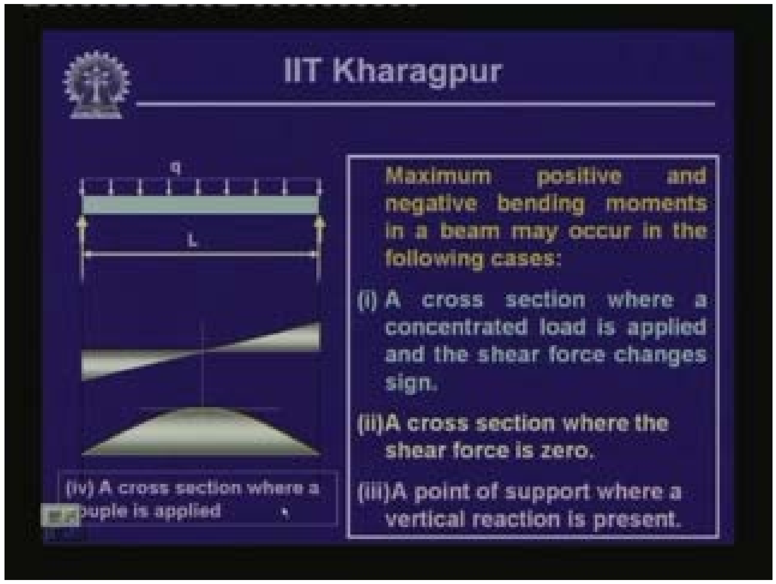


The maximum bending point may occur at a support point where you have a vertical reaction and a cross section where a moment is present. These are the situations where you can expect that the value of the bending moment could be maximum at those points. This could be positive maximum or a negative maximum. We talked about the magnitude of the bending moment where it could be on the negative side as well as the positive side.

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Having known these aspects of the bending moment and shear force diagram, we are in a position to summarize the steps necessary for drawing the shear force and the bending moment diagram. Given a beam with certain kinds of loading and support conditions, the reactive values of the supports should be such that they can be evaluated using the equation of Statics. That means they are statitically determinate systems.

If we have such a system and if we have to draw the shear force and the bending moment diagram, then what are the different steps involved? Firstly, we draw the free body diagram of the whole beam and thereby represent the supports with the reactive forces and we try to evaluate those reactive forces based on the equations of equilibrium.

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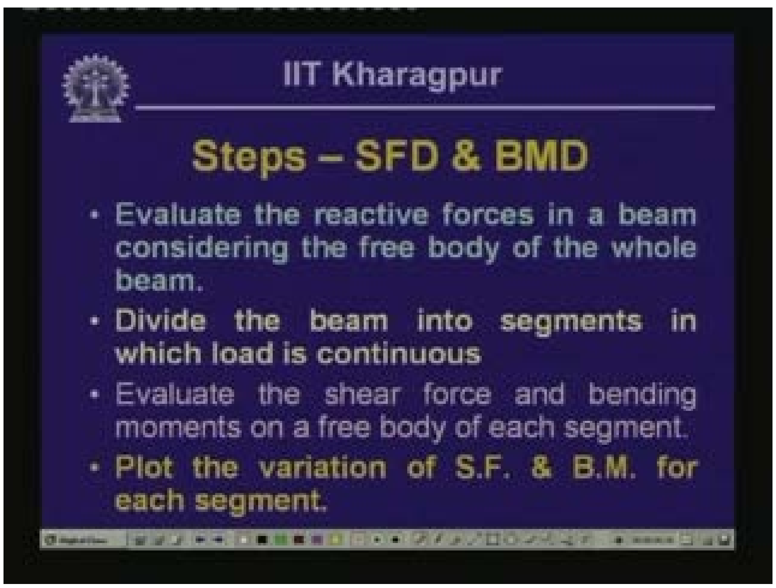


The first step will be to evaluate the reactive forces considering the free body of the whole beam. Then you will have to divide the beam into segments in which the load is continuous. Let us suppose we have a beam which is subjected to a uniformly distributed load with several concentrated loads. If we take this particular segment from here up to the load point they have a uniform kind of a loading. From this point to this point we can see that uniform load with one concentrated load; from this point to this point we have a uniformly distributed load with two concentrated load, then from this point to this point and we have a uniformly distributed load with three concentrated load from this left edge.

We can divide the whole of the beam into individual segments and then corresponding to those segments we try to draw a free body diagram taking a cut in the beam. At the cut we will have the stress resultants in shearing force and the bending moment for that particular segment which can be evaluated depending on the loads and the reactive forces we have. Corresponding to each segment if we evaluate the values of the shear force and the bending moment and if we plot the ordinates along the length of the beam, the diagram which we get will give us the diagram of the shear force or the bending moment as the case may be.

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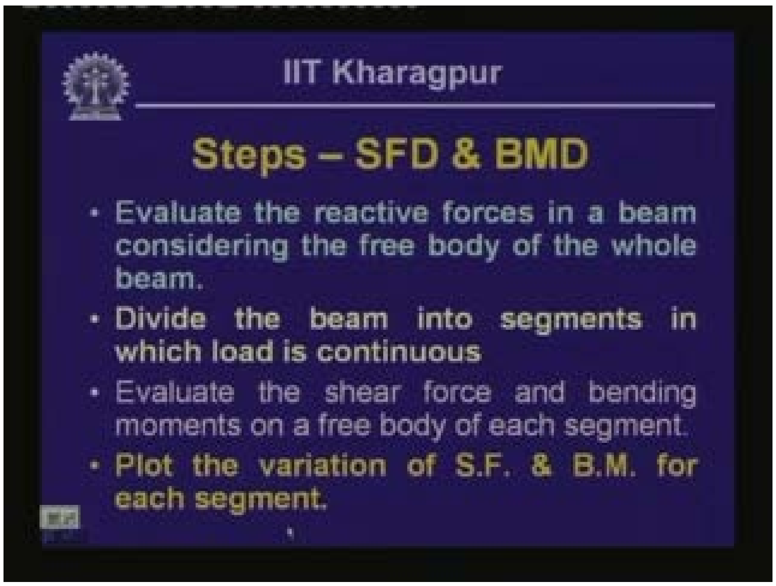


If we evaluate the shear force and bending moment on a free body of each segment and plot the variation of shear force and bending moment for each of such segments then we will get the shear force diagram and the bending moment diagram of that particular beam. These are the steps one has to go through to draw the shear force of the bending moment diagram.

Let us take some more examples on how to evaluate the reactive forces and draw the shear force and bending moment diagram. Here we will have to draw the shear force and bending moment diagram for the beam in which the two concentrated loads p are placed symmetrically with respect to the beam and at a distance of ‘a’ from the left support and this at a distance of ‘a’ from the right support and the length of the beam is L. We need to evaluate or draw the shear force and bending moment diagram.

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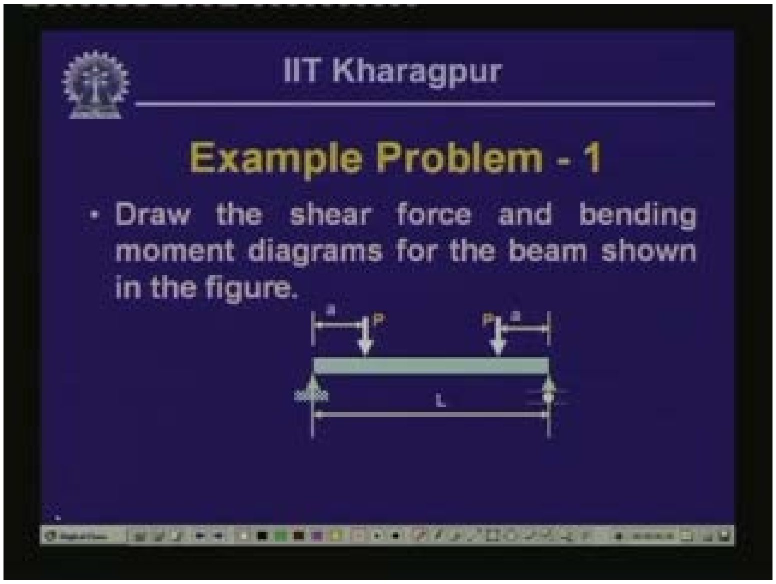


As we have done in the past, let us draw the free body diagram of the whole beam where we have a vertical reaction and a horizontal reaction. At the roller support, we have a vertical reaction and we call this as $R_A$ and this as $R_B$ for the beam a b and the horizontal force H is $H_A$. As we have seen in this particular beam the summation of horizontal force which is 0 gives us the values of $H_A$ as = 0.

The summation of vertical force is 0. Here $R_A + R_B = 2$ P. If we take the moment of all the forces with respect to 'a' then we have $R_B$ (L) which is anti-clock wise and both the loads cause a clockwise moment with respect to this particular point. So, $R_B$ = P (a) + P (L – a) = P (L) and this gives us $R_B$ = P. Since $R_A + R_B = 2$ P $R_A$ = P and the reactive values $R_A$ and $R_B$ = P. The whole of the beam can be divided into 3 segments, this is segment 1 from 'a' to let us call this point as C from here to here is another point which is D and this is B. We have clearly 3; segments 1, segment 2 and segment 3.

In the first segment if we take a section and draw the free body diagram we have the left part wherein we have the reactive force $R_A$ = P, horizontal force is 0 and on this we do not have any other load and at a cart we have shear force V and a bending moment M. If we take the vertical equilibrium that V + P = 0, this gives us the value of V = - P. In the segment 1 from the point 'a' up to this particular point, everywhere shear force is = - P.

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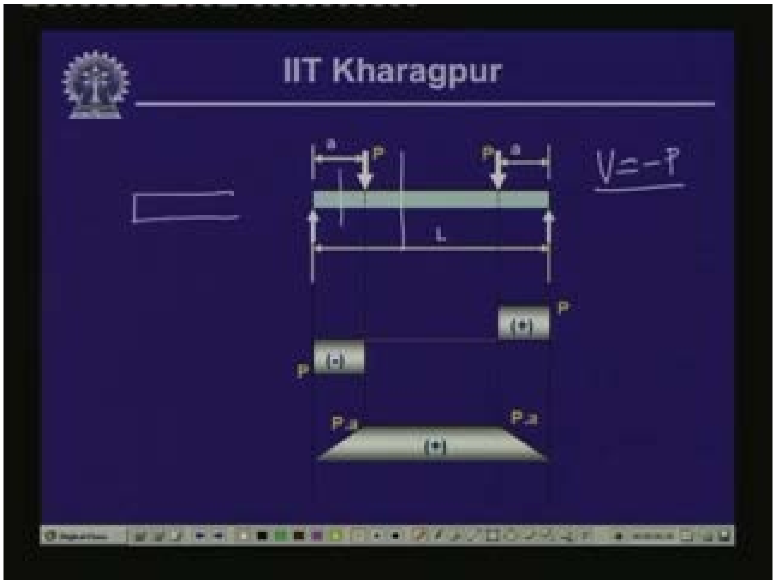


What happens to this diagram if we take the segment over here? As we have seen that on the left segments, on the segment 1 after taking the free body, we have V = -P. Let us take a segment here where there is a free body diagram and the free body diagram looks like this. (Refer Slide Time: 33:02 - 40:22) Here we have the reactive force $R_A$ = p, we have the value of force P which is at a distance of ‘a’ and at this point we have the shear force and the bending moment as V and M.

If we take the equilibrium of the vertical forces here, it gives us the V + the reactive force P - the load P = 0 which gives us V = 0. From here to here in the segment 2 wherever you take a section, you get identical body and hence on this point to this point the shear force is = 0. If we take the segment 3 and cut it here and take the free body of the left part then we have a free body diagram in which we have the reactive force P. Here we have the loading applied P which is at a distance of ‘a’. We have another loading applied where at a distance of total length is L this is L - 2 a.

Here you have the shear force V and the bending moment M. If you take the equilibrium of the vertical forces here we have V + P, the reactive force here then the loading P and P which is - 2 P and this gives the value of V = P. So between this loading point to the end wherever we take a section we get the identical free body diagram and hence V = P over the entire region.

As you can see over here that on this segment 1 we have a shear force which is negative and constant in this particular region. Between this point and this point the shear force is 0 and from this loading point up to the end again we have a shear force which is uniform having a magnitude P. This is the shear force diagram which is this particular beam.

Let us look into the values of the bending moment. For this particular segment where the reactive force is P, we have P and M and this M = P (x) this distance being x and M is anti-clockwise. This p causes a clockwise moment and so P (x) is the moment. When x = 0, the moment is 0 and this is the value which is shown over here. At a distance of 'a' just before this load point, the value of bending moment = P(x). So this is the magnitude of the bending moment over here.

Corresponding to this particular session in this segment, if we take a section here this is the free body diagram and here we have the bending moment M. If we take the moment of all the forces with respect to this particular point, we have M which is anti-clockwise. Then this reactive force causes a clockwise moment and this load causes an anti-clockwise moment. So, this is + P(x – a) = 0 and this gives us the value of M = P(x) - P (x - a).

If we substitute the value of x as equal to 'a' which is just after this load, then we find that again the contribution of this is 0 and so, M = P(a). Wherever we take section between this 2 point, we have the value of bending moment is P(a) which is reflected over here in this diagram and from here to here the bending moment is constant. Corresponding to this particular segment, when we take a section over here, we have the shear force as V and the bending moment at this section as M.

If we write down the moment equation here, the moment equation comes as M which is again in the anti- clock direction - P (x). Then we have these two loads 1 where this distance being x causes a moment again in the anticlockwise direction which is positive. This is P(x - a) + P x - this distance which is equal to L – a. This distance is x - a - L -2a + L + a so this gives us. The value of this is equals to + P(x - a) - (L -2a) = 0. We get M = P(x) - P(x – a) and - P (x - L + a) and this is the value of the bending moment.

At this particular point if x = L then this is L - a and this is 0. At this point again x = L; the bending moment is 0 and also if we substitute x as L – a, we get the value of bending moment as P (a) which is at this point. From this free body of the different segment, we get a bending moment which varies like this. We had seen that wherever the shear force is 0, the bending moment is expected to be constant because dM/dx = -V. If shear force V = 0, then the moment at that portion of the beam is expected to be constant and that is what is reflected over here.

In this particular zone, the value of the shear force is 0 and hence the value of the bending moment in this particular zone is constant and here it linearly varies from 0 to P2a and from P to ‘a’ to 0. If you take the slope here you see the P X a -0/a is P and the negative of P is the shear force here. Again here -P - of - P = + P which is the shear at this particular point which satisfies the criteria as we have seen earlier.

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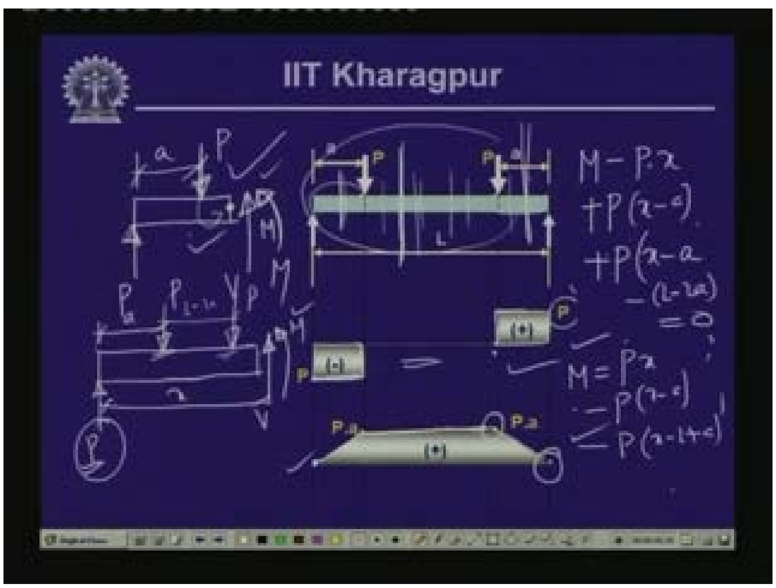
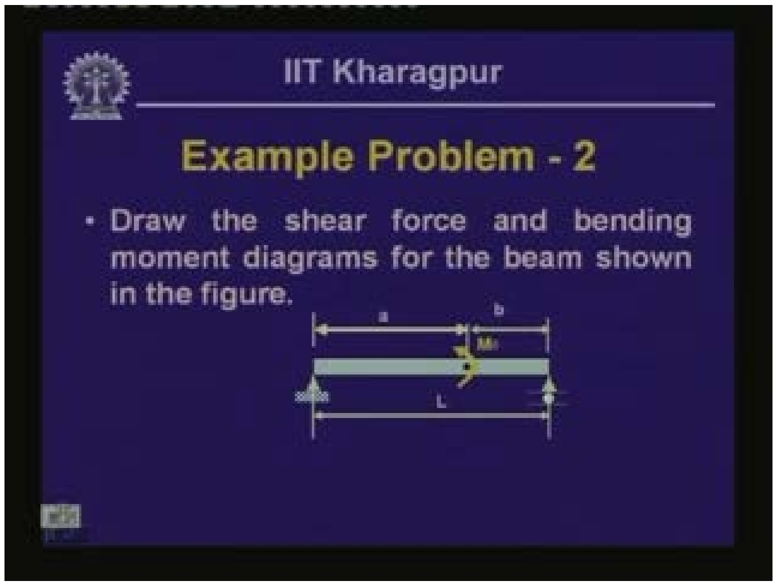



Let us look at another example where we have a concentrated moment. We have to draw the shear force and the bending moment diagram for this particular beam where we have a concentrated moment which is at a distance of A from the left hand A this is B and the span of the beam or the length of the beam is = L. As we have seen earlier, this is supported on a hinge and this is supported on a roller.

As usual, we will have a reactive force here which is $R_A$, and a horizontal force which is $H_A$ and a vertical reactive force which is $R_B$. Again the summation of the horizontal force will give us 0 and so $H_A = 0$ and the summation of vertical force is 0 which means $R_A + R_B = 0$. Here there are no vertical loads, hence $R_A + R_B = 0$.

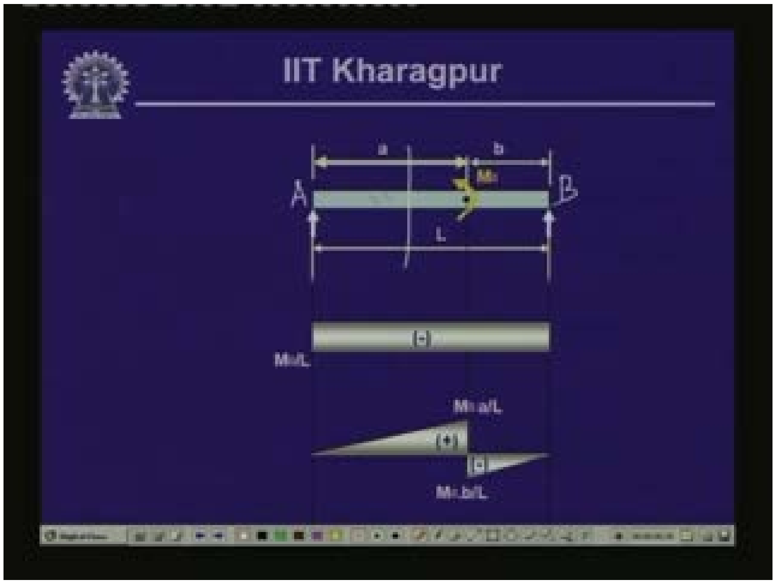
(Refer Slide Time: 41:20 - 43:50)




If we take the moment of all the forces with respect to ‘a’ and if we say that the moment with respect to a = 0 then we have $R_B$ (L) which is in the anti-clockwise form. Also we have the concentrated moment which acts in an anti-clockwise manner, L + $M_0$ = 0. The value of the reactive force $R_B = - M_0/L$. Since RA + RB = 0, the value of the reactive force $R_A = R_B = L + M_0 / L$.

We have got RB as negative, which is in this particular direction which is $- M_0 /L$. We have two segments where one segment is from here to here and another segment from this point to this point. Let us take the free body diagram in these two segments keeping in mind that $R_A = M_0/ L$ and $R_B = - M_0 /L$. Let us see what happens to the shear force and the bending moment diagram.

(Refer Slide Time: 43:50 - 47:06)




Let us draw the free body diagram of those 2 segments and you will get the bending moment and the shear force. If I take the free body of this particular diagram at this segment on the left side, we have this as the reactive force, the other forces at the segment which is V and M and we have seen this as $M_0/ L$. The summation of vertical forces will give $V = - M_0/ L$ and the value of the shear force at any point between the segment from here to here is not a function of x which is a constant.

Wherever we take this section, we will have the value of shear force as $M_0 / L$. If we take a free body at this particular segment, then we have 'A' the reactive force which is $M_0 /L$. We have the concentrated moment in an anti-clockwise direction and we have the shear and the bending moment. If we take the moment of all the forces with respect to this particular section, then we have bending moment m which is anti-clockwise, then we have $M_0$ again which is anti-clockwise and $M_0/L$ and this distance is x. and so, $- M_0/ L(x) = 0$ and $M = (M_0 / L) x -M_0$.

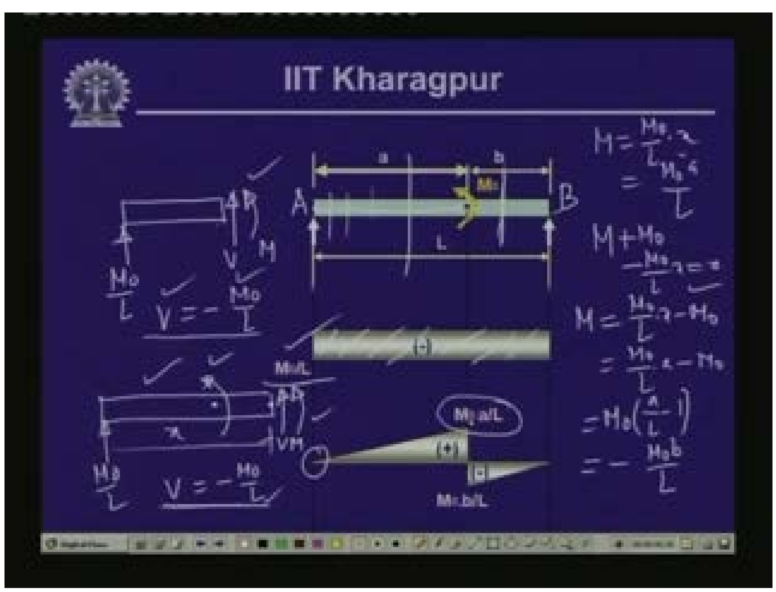
When x = a, the value of bending moment is $M_0$ / L (a). Let us look at the shear force where V = - $M_0$ / L. This moment does not have any effect on the vertical force and hence the shear force is constant from here to here which is - $M_0$ / L. Regarding the entire length of the beam, the value of the shear force is

-$M_0$ / L. Looking into the bending moment over here for this segment M = ($M_0$ / L) x from here to here and up to this particular point x varies linearly. When x = 0, the bending moment is 0; when x = ‘a’ the bending moment is $M_0$ a/L. This is the value of the bending moment over here.

If we take the cut here and draw the free body diagram and you compute the value of the bending moment, then we get the expression for the bending moment as this. If we compute the value of the bending moment at a distance of ‘a’, we have $M_0$ (a / L) - $M_0$. If we take $M_0$ common then we have a/L - 1. So, a - L / L and a - L is nothing but - b which is equal to - $M_0$ b /L.

On the left hand side we had a bending moment of $M_0$ a/ L. On the left hand side of the beam where we have the concentrated moment, we had a positive value of $M_0$ a/L and immediately next to that point of the concentrated bending moment, we have the bending moment which is -$M_0$b/ L. So there is a certain jump in the bending moment from the left to the right where we have a concentrated moment.

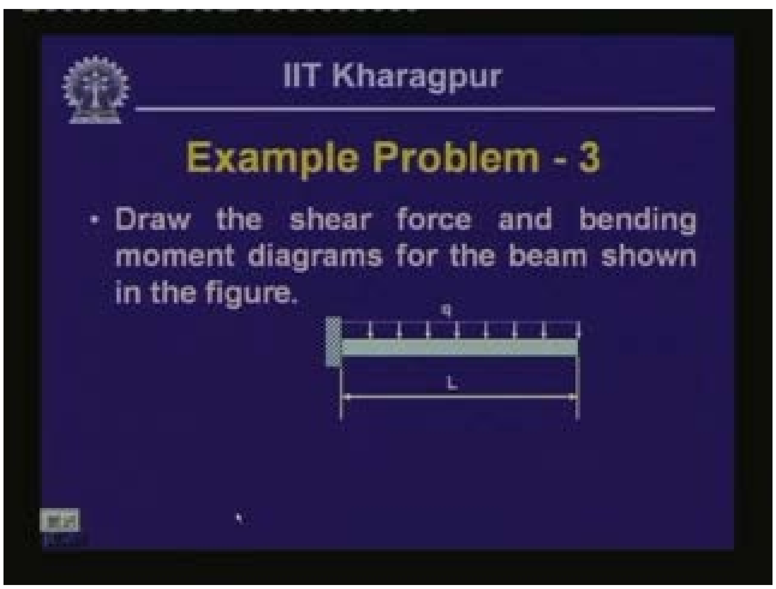
(Refer Slide Time: 48:24)



This is what is reflected in this particular diagram at this point. Again if you substitute x = L, you get $M_0$ - $M_0$ as 0 and this is 0. At this particular segment we have drawn this straight line and let us suppose I enlarge this part since this particular point is a concentrated moment. We are computing on the left hand side of this particular concentrated moment, we are computing the moment value on the right hand side of this concentrated moment; on the left hand side, we have got a positive $M_0$ / L, on the right hand side we have got a negative $M_0$ b/ L and we get a variation.

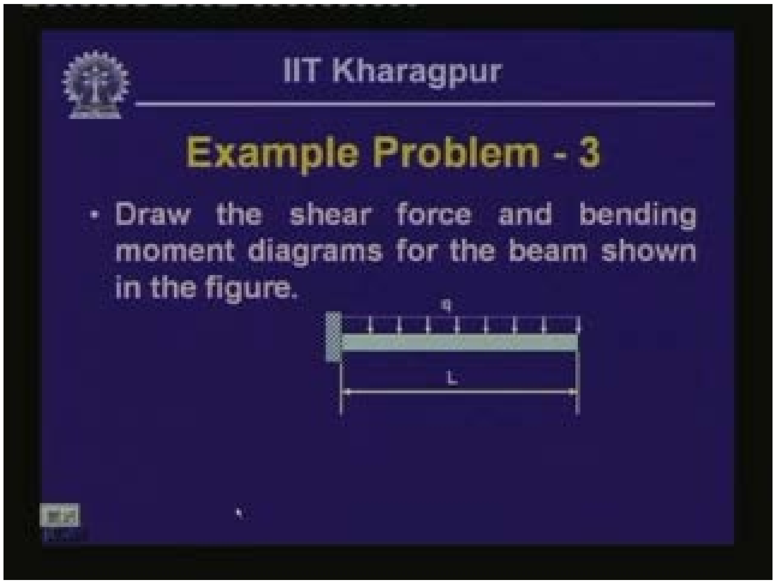
It goes from 0 to a value; then after varying it moves from this to 0(Refer Slide Time: 48:24). We approximate this particular distance, which is very small, as a straight line instead of that inclined line and this is what is reflected in this particular diagram. This is the shear force diagram and this is the bending moment diagram with the magnitude of the moment and the shear force values written over here. These are the bending moment and shear force diagram for this kind of beam where the beam is subjected to a concentrated moment.

(Refer Slide Time: 48:51 - 49:08)




Let us look into another kind of beam. As you can recognize that this particular beam is supported at this edge; edge A and end B. End B is unsupported or free and we have defined this kind of beam which is supported at one end and free at the other as a cantilever beam and this particular cantilever beam is subjected to a uniformly distributed load.

(Refer Slide Time: 49:08 - 51:11)

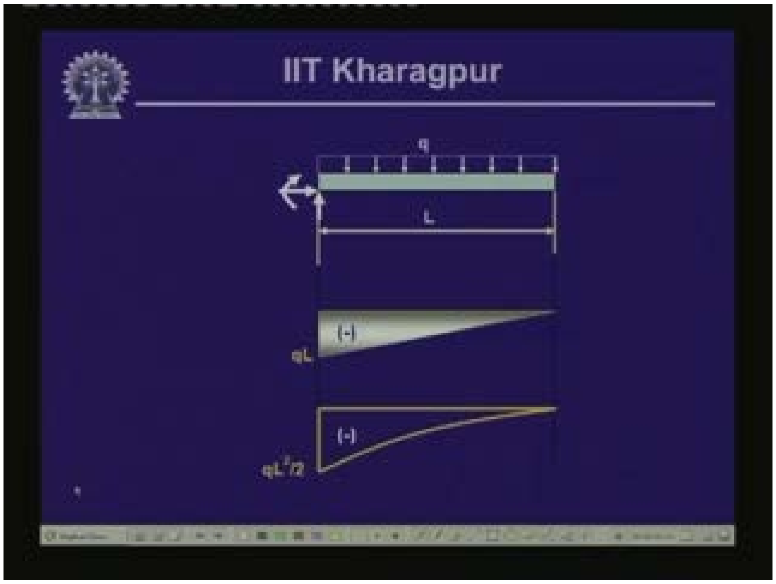



We will have to draw the shear force and the bending moment diagram for this particular beam. Let us draw the free body diagram of the whole beam and thereby let us compute the reactive forces first. The reactive forces are the vertical reactive force which is RA, the horizontal force which is $H_A$ and a bending moment which is $M_A$.

The summation of the vertical forces is 0, which gives us $R_A$ – q (L) = 0 and the reactive force $R_A$ = q (L). The summation of the horizontal force will give us $H_A$ = 0 because there are no other horizontal forces. If you take the moment of all the forces with respect to this point A, we have $M_A$ which is acting in an anti-clockwise direction and q is the load, q (L) is the total load and at the moment of this load with respect to support 'a' is q (L) (L /2) which is again in the clockwise direction.

So, - q (L) (L/2) = 0 and moment at $M_A$ = q L square/2. We have the reactive force $R_A$ = q (L) and the moment as qL square /2 and these are the values of the reactive forces. Let us take the free body diagram of this segment of the beam so that we can find out the values of the shear force and the bending moment at that particular segment.

(Refer Slide Time: 51:11 - 53:29)

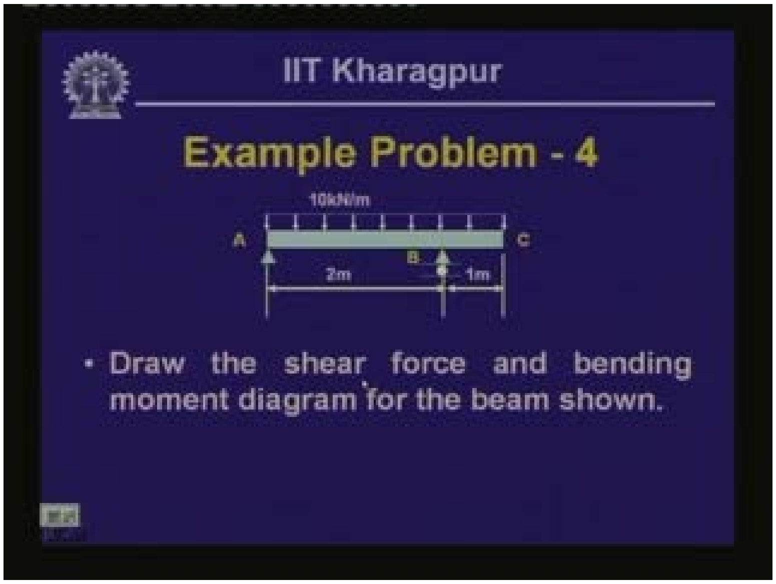



This is site A, this is site B and we take a section over here which is at a distance of x and this is $R_A$ and moment A. As we have seen, $R_A$ = q (L) and moment as A = qL square/2. If we take the free body diagram of this particular part, we have this as $R_A$, we have $M_A$, we have the uniformly distributed load which is q, here we have load V and here we have load M and this is at a distance of x. As we can see over here that the vertical force V = V + $R_A$ - qx = 0 and this is qx - $R_A$ and we have qx – qL.

When x = 0, V = - qL and when x becomes L then V becomes 0. So, it linearly varies between qL to 0 and this is the shear force diagram of this particular section. If you take the moment of all the forces with respect to this, we have a moment which is in an anti-clockwise direction, $M_A$ is also in the anti-clockwise direction, the load q is also in the anti-clockwise direction which is qx square/2 and $R_A$ is in a clockwise direction which is $R_A$ (x) = 0.

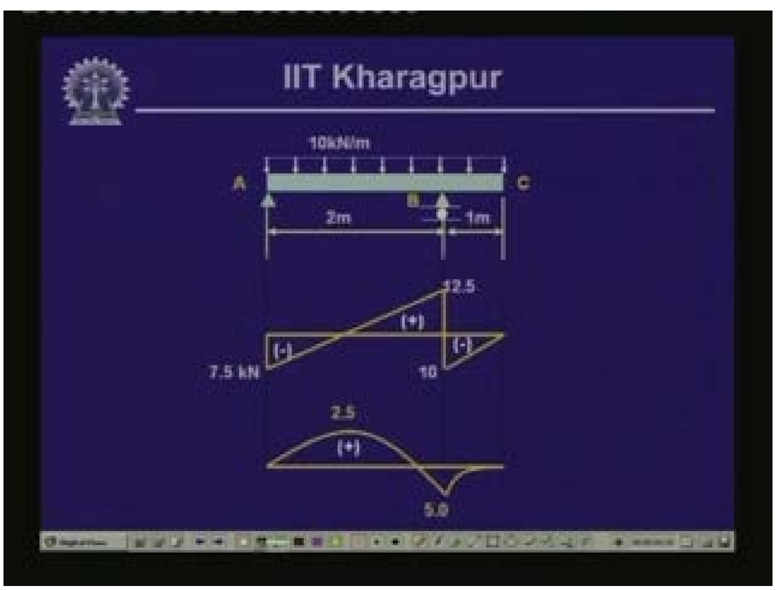
We get a value of the bending moment which is $R_A$ (x) - q x square/2 - $M_A$ which is equal to qL square/2. At x = 0, we find that the bending moment value is equal to - qL square/2 and x = L which we find as qL square/2 which is 0. So, it varies again in parabolic manner from – qL square/2 to 0 and this is the shear force diagram and this is the bending moment diagram for this particular beam.

(Refer Slide Time: 53:29 - 54:12)




We have another example problem where we will have to draw the shear force and the bending moment diagram for this particular beam and exactly in the same way for the hinged support. We write down the reactive force $R_A$, the horizontal force $H_A$ and the vertical force $R_B$ and as you can recognize $H_A = 0$, we can compute $R_A$ and $R_B$ from the free body and then we can take segments to find out the values of shear force and bending moment.

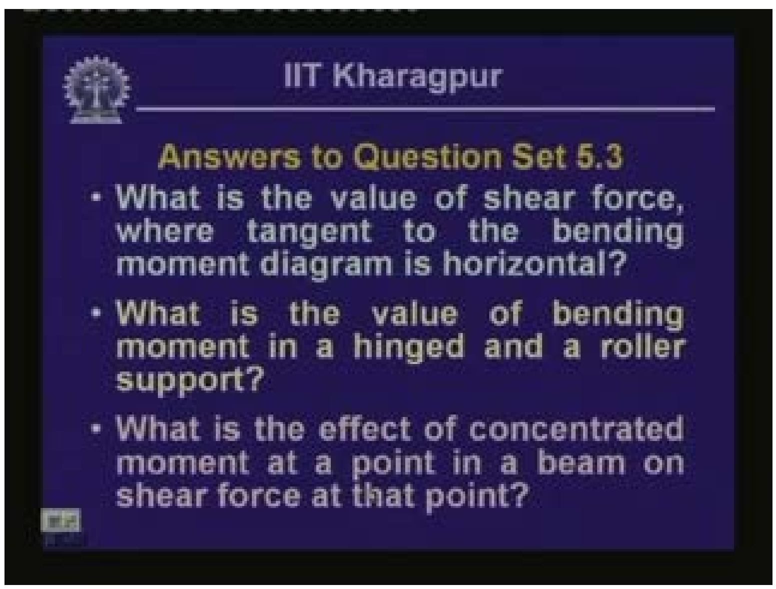
(Refer Slide Time: 54:46)



Let us draw the shear force and bending moment diagram where the sheer force is 7.5kN. At this particular point, since we have the overhang beyond this, it changes over to 12.5 and from 12.5 to reactive values here, it changes down to -10 and finally 0 over here. This is the shear force diagram for this particular beam and if we compute the bending moment we will find that the bending moment will vary again in a parabolic manner.

The magnitude at three different points is 2.5kNm, 5kNm and 0. This is the support point where we are getting the bending moment as maximum. You can have a maximum bending moment where shear force is 0 or where the shear force changes sign from the negative to the positive and positive to the negative. Where there is a reactive force at that particular point also you can have a maximum bending moment provided you have the support reaction at the particular point.

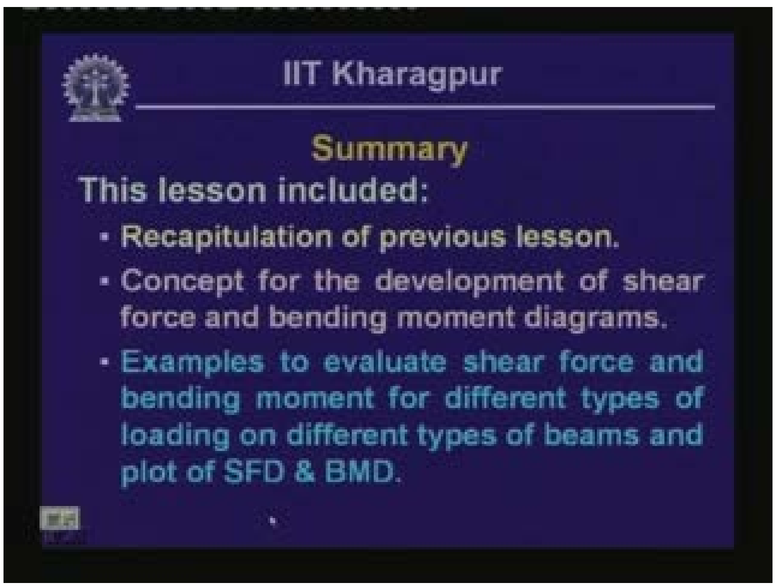
(Refer Slide Time: 55:18 - 56:35)




Often we do come across practical examples where we have two beams A B C D. This is one beam and we have another beam D E which are coupled together or joined together. We call this hinged and this particular hinge has a particular characteristic that it cannot resist any moment which is 0 but it can resist the shear force. We need to find out the maximum shear force in this particular beam.

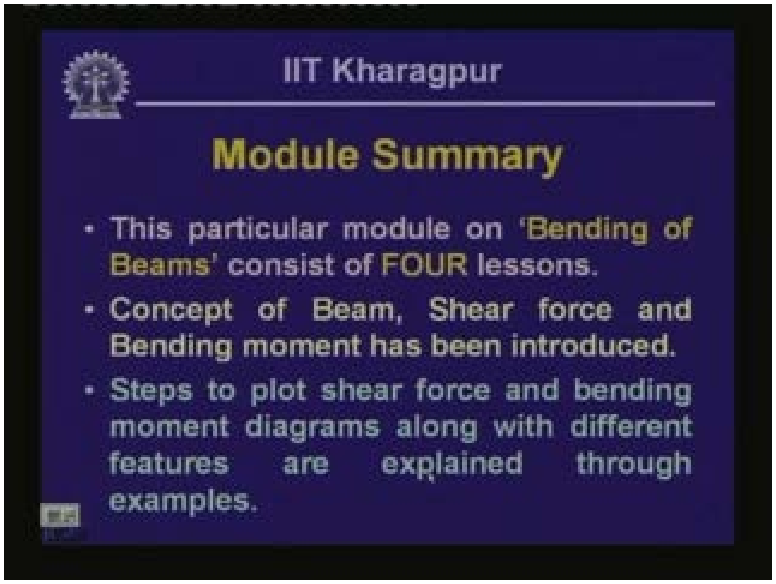
Here again we need to take the support reactions. We have a vertical force which is $R_B$ and the horizontal one which is $H_B$, we have $R_C$, and here we have $R_E$. At the internal hinge point, if you draw the free body we will have the force p over here $R_B$ here $R_C$ over here and we will have $R_B$ here. On the other segment we have $R_B$ which is downwards because these it should be in a balanced form and this will be $R_E$. If we evaluate the reactive forces and exactly in the same way if we compute the shear force and the bending moment, we can get the corresponding diagrams which we have termed as shear force and bending moment diagram. Do this particular problem which we will discuss in the next lesson.

(Refer Slide Time: 56:49 - 57:22)




Let us summarize this particular lesson. We have recapitulated the aspects that we had discussed in the previous lesson. We have introduced the concept for the development of shear force and bending moment diagram and we have looked into some more characteristic features of shear force and bending moment diagram and have seen they change at a particular point depending on the concentrated load or the concentrated moment. Also we have done examples to evaluate shear force and bending moment for different types of loading on different types of beams and plots of shear force and bending moment diagram.

(Refer Slide Time: 57:22 - 57:45)




This particular module comes to an end with these 4 lessons and here we have introduced the concept of beam, the shear force and bending moment diagram. Also we have learnt how to draw the shear force and bending moment diagram through several examples.

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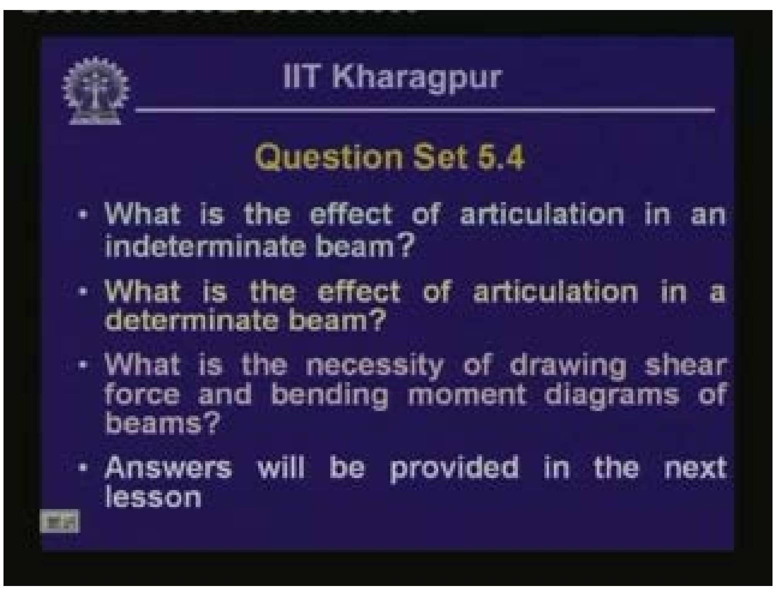

These are the question given for you. What is the effect of articulation in an indeterminate beam? What is the effect of articulation in a determinate beam? What is the necessity of drawing shear force and bending moment diagrams of beams? The answers to these questions will be given in the next lesson.