Strength of Materials
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Lecture no 24
Bending of Beams- III

Welcome to the third lesson of the fifth module which is on Bending of Beams part three. In fact in the last two lessons we have discussed some aspects of the bending of beams. We have looked into the concept of shear force and the bending moment. In this particular lesson we will see how to evaluate the variation of shear force and bending moment along the length of the beam and we will discuss other related aspects.

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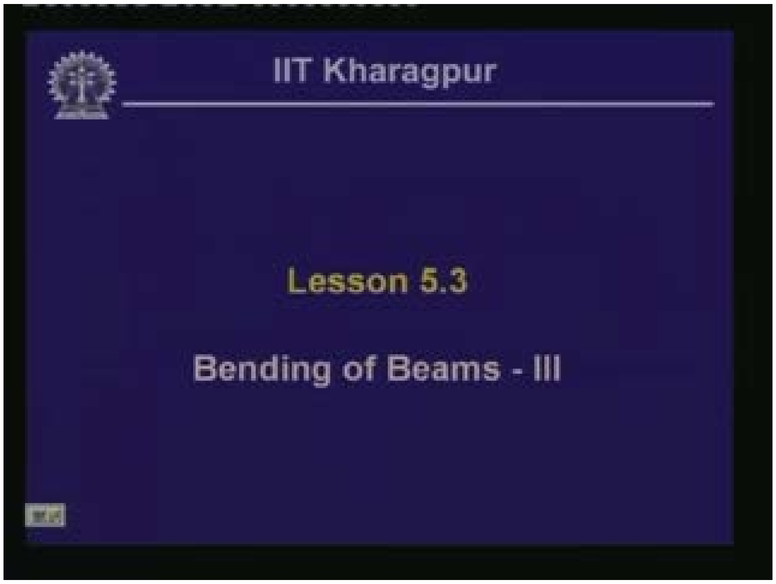

Once this lesson is completed one should be able to understand the concept of shear force and bending moment diagrams. We have calculated the shear force and the bending moment in a beam at a particular location along the length of the beam. Now to find out how the shear force varies along the length or in order to have the variation of shear force at any point along the length of the beam at a glance, we have to draw a diagram over the length of the variation of shear force and the bending moment.

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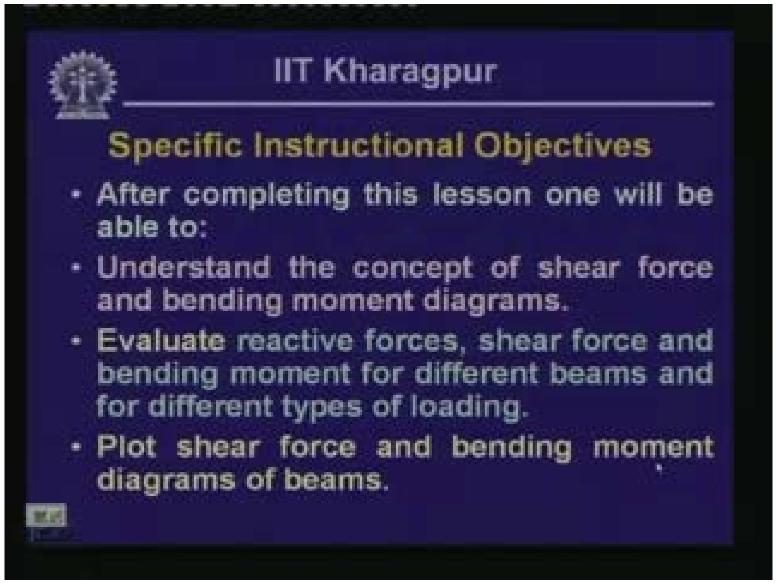

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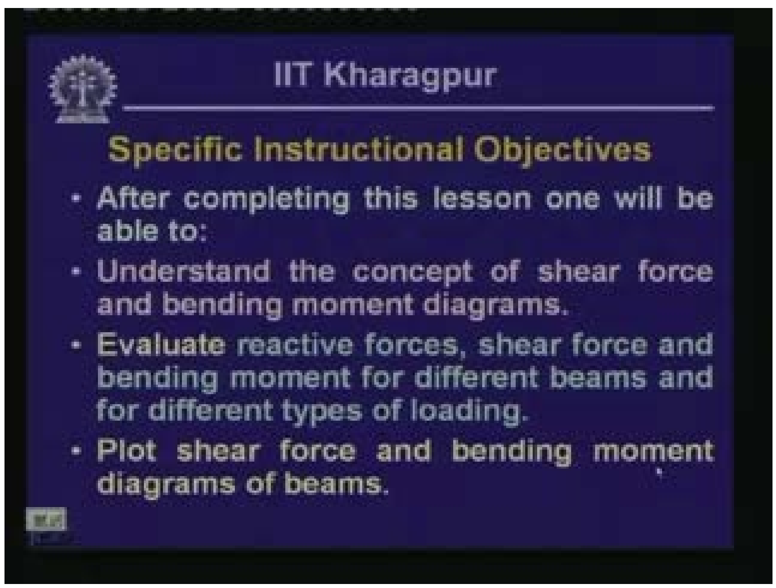

One should be able to evaluate the reactive forces, the shear force and bending moment for different types of beams which we have discussed earlier and for the different types of loading that the beam will be subjected to. Consequently, one should be able to plot the shear force and bending moment diagram of beams for different loading and different kinds of support conditions.

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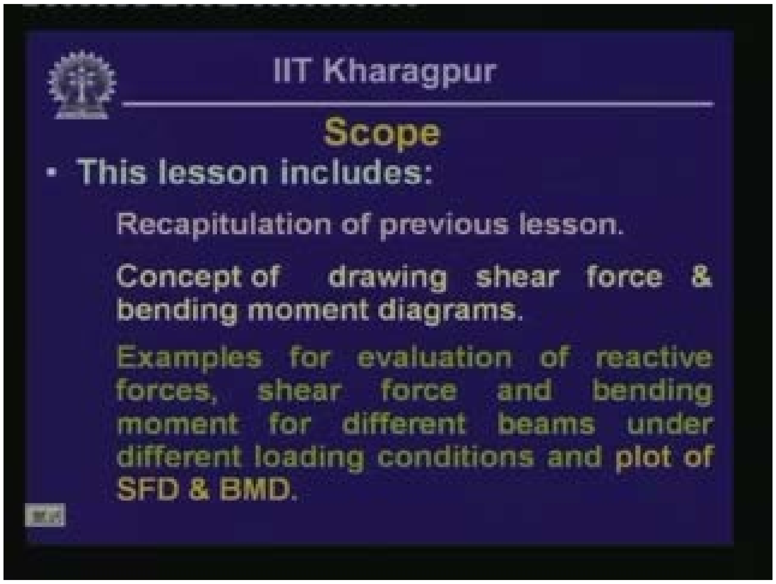

Hence the scope of this particular lesson includes recapitulation of the previous lesson and understanding of the concept of drawing shear force and bending moment diagrams. In this particular lesson we will see how to plot the variation of shear force and bending moment along the length of the beam which we defined as a shear force and bending moment diagram. We will also give examples for evaluation of reactive forces, shear force and bending moment for different beams under different loading conditions and plot of SFD (which is shear force diagram) and BMD (which is bending moment diagram).

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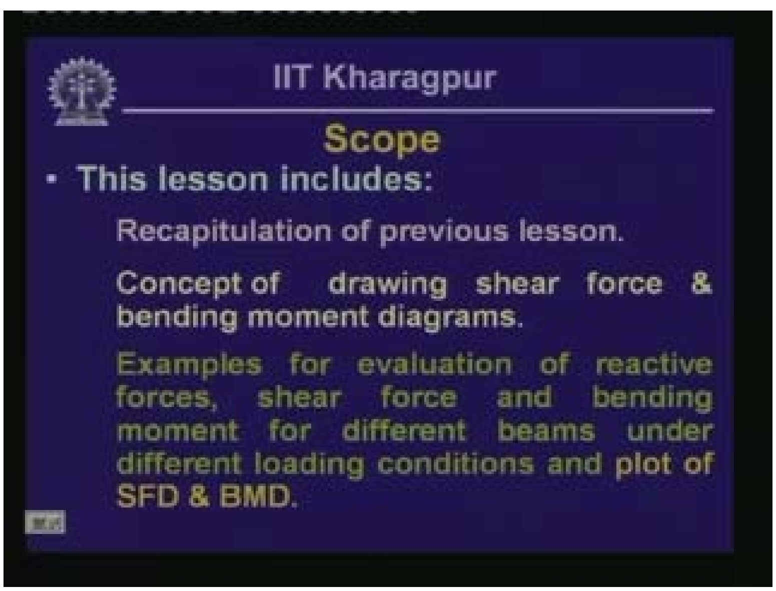

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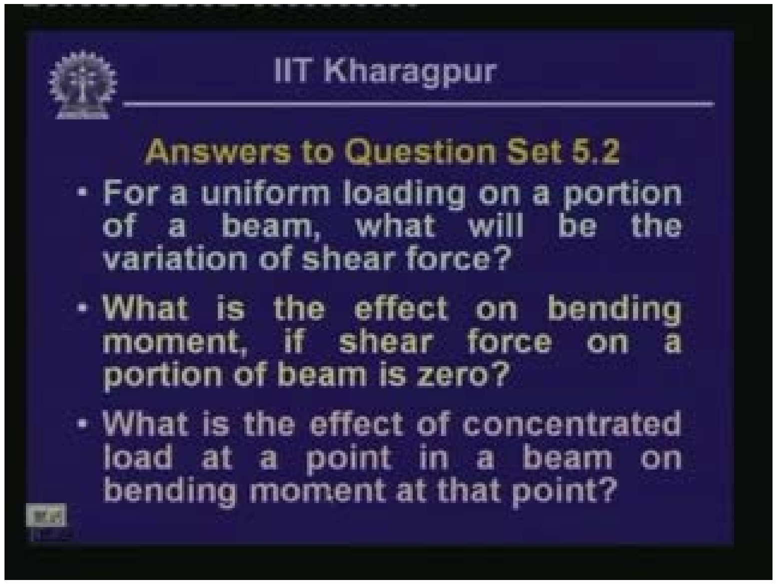

Before we go into the details of Shear Force and the Bending Moment Diagram, let us answer the questions which were given to you. The first question was that if a beam is subjected to a uniformly distributed load on a portion of a beam, what will be the variation of shear force in that particular part of the beam?

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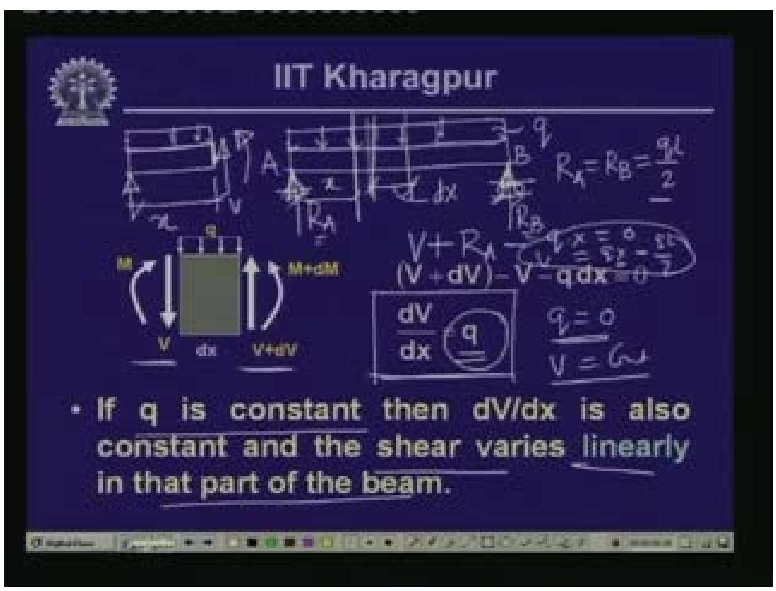

Let us suppose we have a beam which could be a simply supported one hinged at one end, supported on a roller at the other end and subjected to uniformly distributed load. If we take a part of this particular beam of length dx then according to our conventions we have placed the shear force and the bending moments on two sides. In this particular equation we have seen in the last lesson that dv/dx = q and the question is if q is constant and uniformly varying over the length of the beam then what is the state of the shear?

We know that if q = 0 then the shear force is constant. If q is constant and if we have uniform loading over a part of the beam or over the whole beam, then the shear varies linearly in that part of the beam. For a simply supported beam if q is the uniformly distributed load and since we do not have any horizontal force; the reactive force which we have at this hinged support is the vertical one.

If we call this as ‘A’ and this as ‘B’, then this is R_A and this is again a vertical support which is R_B and because it is symmetrical, we can write $R_A = R_B$ = q multiplied by L as the total load. Half the load will be carried by the reactive force A and half will be carried by the reactive force B and we have $R_A = R_B = q\ (L)/2$. Let us take a free body at a distance of x from A and take the equilibrium of the forces of this free body here. We have R_A; on this part of the beam we have q and this distance is x and the stress resultant at this particular end is V here (Refer Slide Time: 06:35) and moment M here.

If we take the vertical equilibrium $V + R_A - q\ (x) = 0$, then this gives us $V = qx - R_A$ where R_A is qL/2. Here, V is a function of x and x linearly varies in this particular equation. Hence the variation of V will be linear when we have a uniformly distributed load and we will discuss this further through examples. We will find out the value of the shear at different locations along the length of the beam, if we have a uniformly distributed load over it.

The second question was; what is the effect on the bending moment if the shear force on a portion of the beam is 0? In this particular equation when we take the equilibrium of the bending moment on the left edge of that particular segment then we get the resulting equation; dM/dx = V or the rate of change of the moment along the length of the beam is equal to the negative of the shear force V. If the shear force is 0 then M = constant and at that portion in the beam where the shear force is 0, the bending moment will be constant. This observation will be helpful while drawing the shear force and bending moment diagram.

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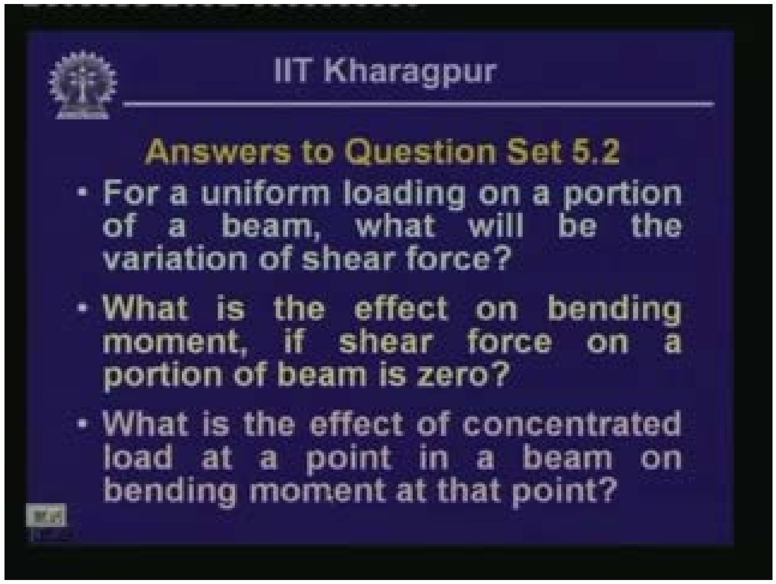

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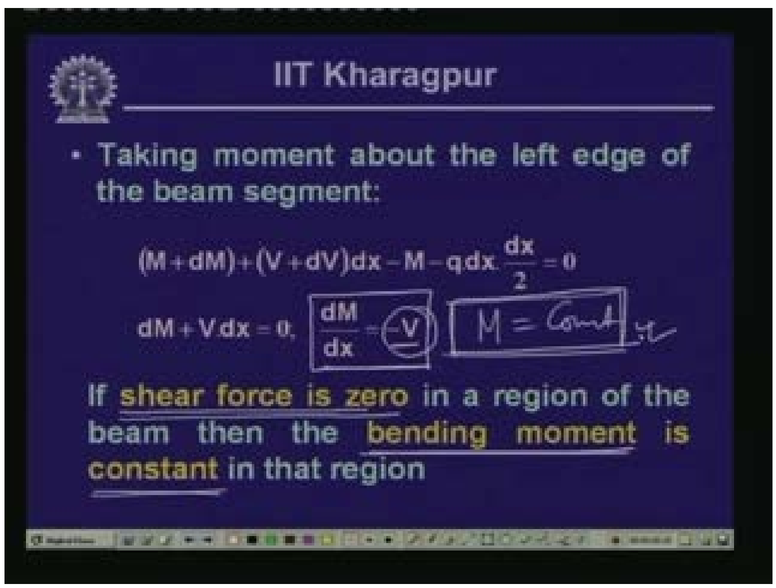

The last question was; what is the effect of a concentrated load at a point in a beam on the bending moment? In the last lesson we had discussed the effects of different kinds of loads on the sheer force and bending moment at a particular section and the loads which we considered were the uniformly distributed load, the concentrated load or the concentrated moment and the point where we had the concentrated load in a particular segment. We had evaluated the shear force on the left and the right of that concentrated load.

Let us suppose you have this concentrated load. On the left hand side of the load we have shear force V and moment M on the right hand side. The moment is M + M_1, where M_1 is given by this particular expression. Here, since the quantities are multiplied by this small value dx, the value of M_1 will be significantly smaller than M.

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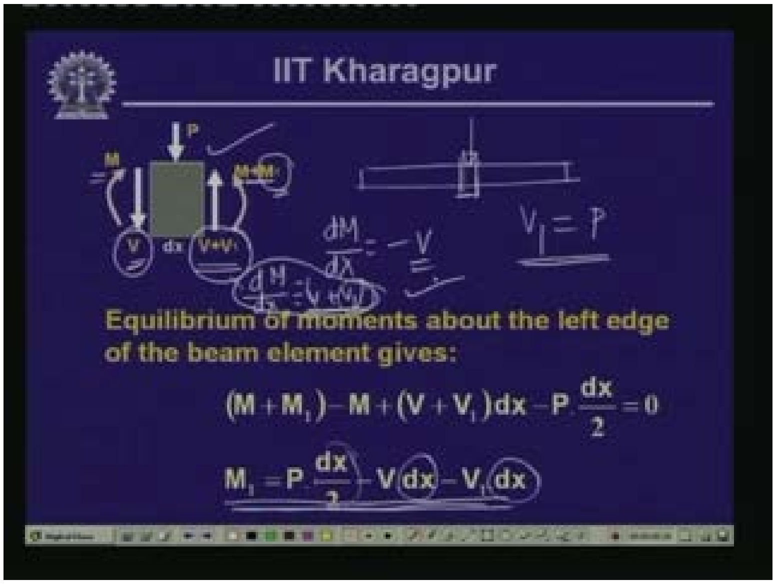

If we try to evaluate the bending moment on the left of this concentrated load and on the right of it we will find that the change is insignificant which we can neglect. But there will be a change in the rate of change of moment. On this side dM/dx = -V; on this side we have v and on this side we have V + V_1 and as we have seen from the vertical equilibrium, V_1 = P. On this side, the rate of change of moment dM/dx = V+V_1 or the negative of the whole equation. Here, there will be a change in the order of P from the left hand side.

Although there is insignificant or no change in the moment value from the left hand support of the concentrated load point to the right hand part of the point, there is significant change in the rate of change of moment. So, dM/dx will undergo change when you have a concentrated load and we will give some examples to illustrate it. In general as we have observed, dV/dx = q and dM/dx = -V and if q = 0 then the shear is constant. If q is constant then the shear will vary linearly along the portion of the beam. We have seen that the change of shear between two points is equal to the area of the loading diagram.

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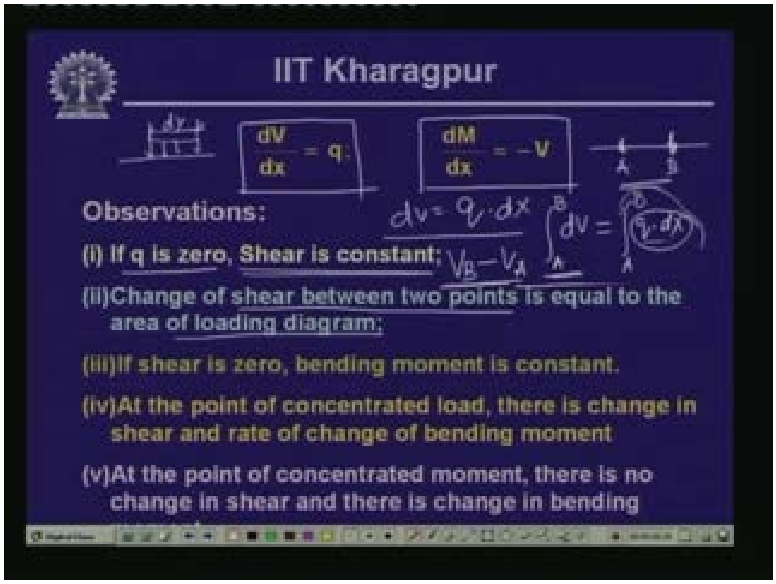

From this particular equation we can write dV = qdx. Let us integrate this expression between two points. Let us take two points on the beam, A and B at a distance apart and let us suppose we want to find out the change in the shear force between A and B. So, integral dV from A to B is equal to the integral from A to B qdx. From this we understand that integral dV from A to B is basically the shear force at B minus the shear force at A.

In the differential shear force between B and A, qdx represents the area of the loading. You have the distributed load q over the length dx and this gives the area of the loading integrated between A to B which is the area of the load between the two points and this area is equal to the difference in shear between them. Let us suppose we have a uniformly distributed load on the beam; then the difference in shear between the two points B and A will be equal to the area of the loading diagram between B and A.

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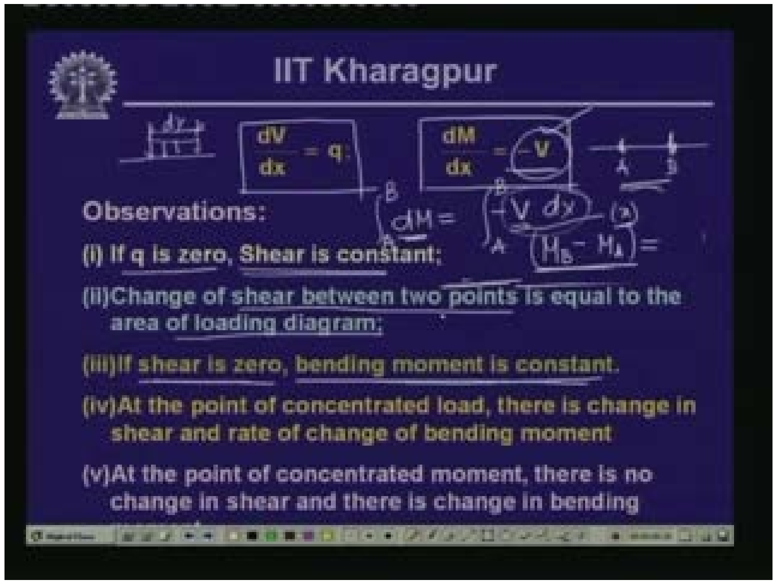

If we calculate the change in bending moment between the two points B and A as we have done for the shear force then what do we get? If we can integrate dM over the region A to B this is equal to integral –V dx from A to B. Here integral dM from A to B is the bending moment at B minus bending moment at A. Now, V which is a shear force could be a function of x as well. When we try to plot the variation of the shear force, we can have a diagram of the shear force and that integral Vdx precisely indicates the area of that shear force diagram and we will study that impact as we get to the shear force and bending moment diagram.

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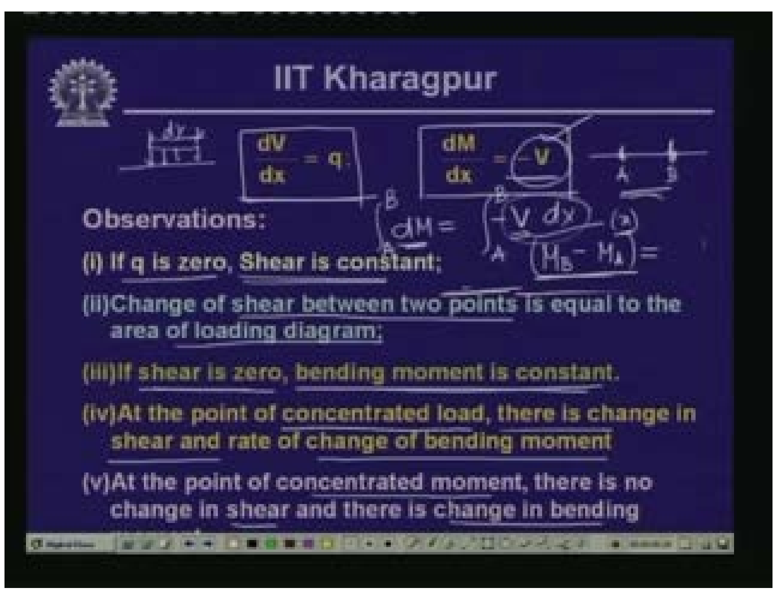

At the point of concentrated load there is a change in shear and we have observed the rate of change of the bending moment. At the point of concentrated moment there is no change in shear but there will be change in the bending moment. These are the different kinds of loading and their relationship with the shear and the bending moment can be summarized in this form.

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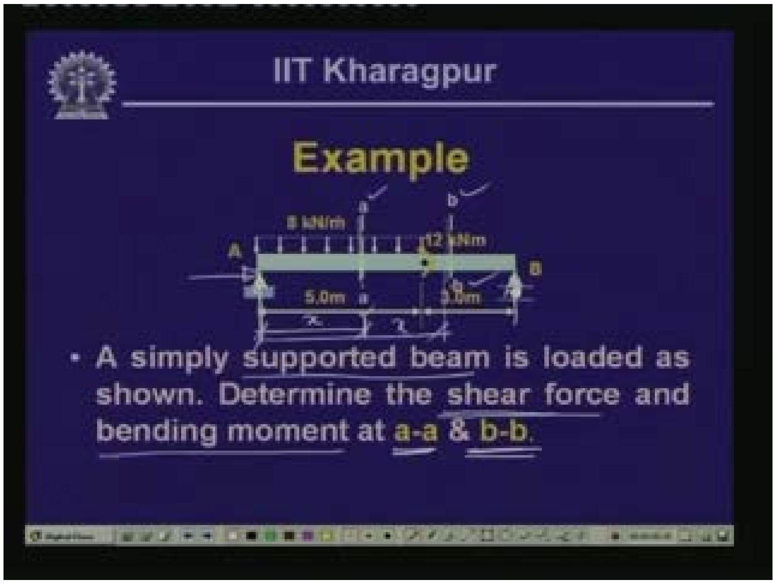

Let us study this example that I had given last time which provides a solution. A simply supported beam A B is loaded as indicated and you need to evaluate the shear force and bending moment at ‘a-a’ and b-b. Two locations ‘a-a’ and b-b are marked and at these locations we will have to find out the shear force and bending moment. Please note over here that ‘a-a’ and b-b are not fixed locations as such in the sense that we have not given any fixed dimension for this. (Refer Slide Time: 16:38) Let us call this, a variable dimension which is x and let us also call this variable dimension x.

The question is we will have to find out the value of the bending moment and shear force at ‘a-a’ and at b–b. Here, as we do not have any horizontal loading, the reactive force at A which is horizontal and vertical will be equal to 0. We have the vertical component and the horizontal component and this horizontal component will be equal to 0, since we do not have a horizontal force.

The free body diagram of this particular beam will look like this and let us call this as H_A, let us call this as R_A and the reactive force as R_B. Here, summation of (horizontal force) H = 0, $H_A = 0$; summation of (vertical force) V = 0 which gives us $R_A + R_B$ = 8 kNm multiplied by the length 5m and we get 40 kN as the value for $R_A + R_B$. Now we need another equation for evaluating these two unknown quantities R_A and R_B. If we take the moment of all the forces with respect to this point ‘A’, the summation of moment with respect to A is 0.

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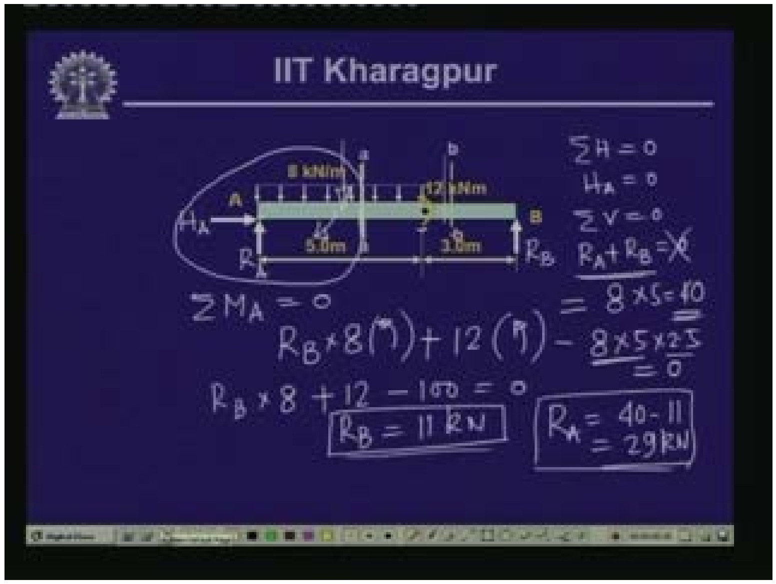

Let us take the moments of all the portions. Now R_B multiplied by 5 + 3 = 8 M acting in an anticlockwise direction. Then we have an external moment which also acts in an anticlockwise direction which we have considered as positive. So this is 12 kNm which is in an anticlockwise direction and then we have this loading which acts at the center of this length and this causes a moment in the clockwise direction. So, -8 multiplied by 5 multiplied by 2.5 is the moment which is equal to 0.

This gives us R_B (8) + 12 - 100 = 0 and we get the value R_B = 11 kN. Since $R_A + R_B$ = 40 kN and R_B alone is 11 kN R_A = 40 - 11 = 29 kN. We get the reactive values R_A = 29 kN and R_B = 11 kN. Now we need to evaluate the value of shear force and bending moment at section 'a-a' and section b–b. Let us get the beam at section 'a-a' and let us take the free body diagram of this particular part and see what are the values of shear force and bending moment that we get.

Let us take the free body diagram of the left hand part. We have R_A which is 29 kN, we have the uniformly distributed load which is 8 kN per meter and this is the distance which we call as a variable distance x and here we have the stress resultants V and bending moment M and this is the section 'a–a'. If we take the equilibrium of the vertical forces, we have V + 29 - 8x (over the length) x = 0 and so the shear force V = (8x – 29).

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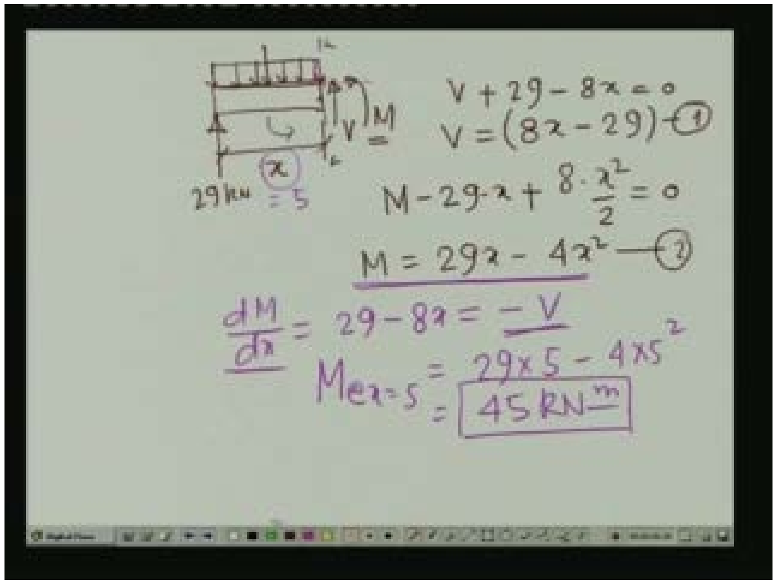

Let us calculate the bending moment here. If we take the moment of all the forces with respect to this point or section 'a–a', then we have moment M which is in an anti-clockwise direction. Now R_A will cause a moment in the clockwise direction. So, -29(x) and the load will cause a moment in the anti-clockwise direction and we get +8 (x square/2) = 0. So, the bending moment, M = 29x - 4x square. This is the value of the shear force (1) and this (2) is the value of the bending moment.

From this particular expression, if we take the derivative of this moment dM/dx = 29 - 8x = -v; this conforms to the expression which is the relationship between the shear and the bending moment and the loading between the two. We may try to calculate the value of the bending moment at the limiting point in a sense that the bending moment at this particular point is where the uniformly distributed load comes to an end.

There we have a concentrated moment and on this part we do not have any loading. Let us see what its consequence is. If you substitute the distance x as equal to 5 where the uniformly distributed load ends, then the moment at the distance x = 5 = 29 (5) - 4 (5) square which gives us a value of 45 kNm. The moment at that particular point is 45 kNm. Let us calculate the value of the bending moment and shear force at section b-b. Then you can compare the two and see what happens to the shear force when there is no loading into it.

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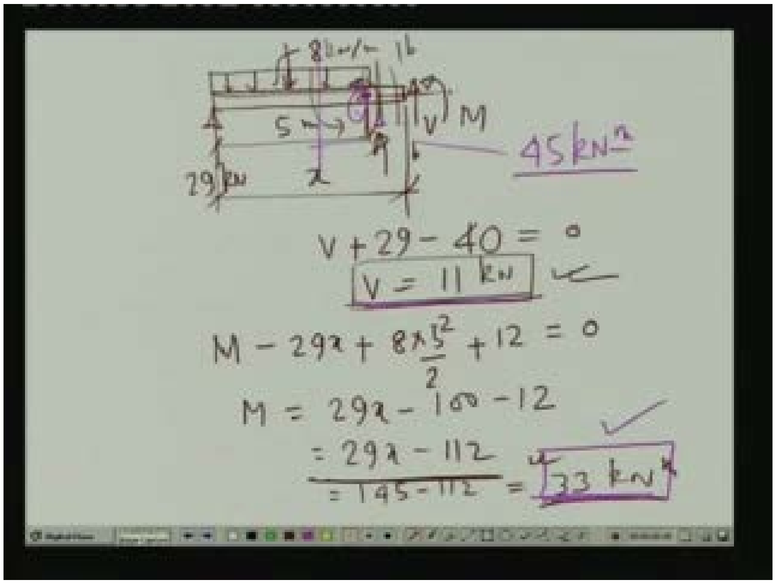

Let us take the free body diagram of the part which is at section b-b. Here again we have the shear force V, the bending moment M and we have the uniformly distributed load up to a distance of 5M. Here we have the reactive force which is 29 kN. This is 8 kN per meter and this is at a distance of 5M. This is a distance which we have chosen as x which is a variable distance and this particular section is b-b.

If we take the vertical equilibrium of all the forces, we have V + 29. Here the whole loading which acts is 8 kN per meter over the length 5M and we have 40. At this point we have a concentrated moment which is 12 kNm which is equal to 0 and we get V = 11 kN. This is the constant quantity; so beyond this particular loading point up to the end of the beam the shear force is constant and please note that over this particular part of the beam we do not have any uniformly distributed loading and as we have seen if dV/dx = q and if q becomes 0 then v is constant.

We are observing over here that the shear force V = 11 kN and whichever section we take up to the end of the right support, we will get the same value of the shear force everywhere. If we compute the value of the bending moment at this particular section then after taking the moment of all the forces with respect to this particular section b–b, we have M which is in anticlockwise direction; then the contribution of the moment from R_A is in the clockwise direction, so we get M - 29x. This is also in an anticlockwise direction so +8 (5) (5/2) gives us $5^2/2$ and we have a concentrated moment which is also in an anticlockwise direction and we get 12 = 0.

From this we have M = 29x – 100 - 12 and this will give us a value of 29x - 112. There is an immediate change in the value of the bending moment as we go from this end to this end because we have a concentrated moment over here. Let us substitute the value of x as 5 = 145 and 145 - 112 = 33 kNm. On the left hand side which we had calculated from the previous expression, the bending moment at this point was 45 kNm. That was the limiting point from this end.

We had considered a section which was valid up to this particular end where the concentrated moment is applied. When we take the effect of this concentrated moment the moment value comes down to 33 kN. There is a jump in the bending moment value but there is no change in the shear force value and this is what we have observed when we looked into the effect of the concentrated moment in the span of the beam. There is no change in the shear force in the effect of that concentrated moment but the value of the bending moment changes immediately from its left part to the right part.

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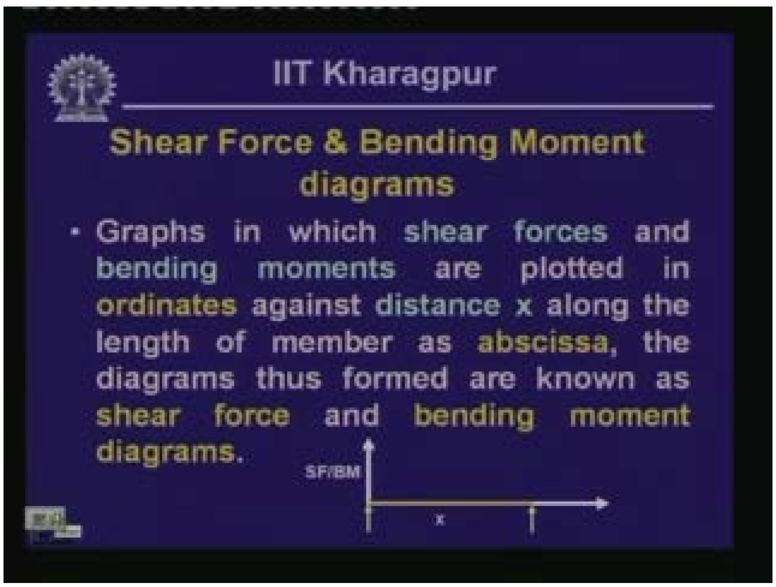

Having looked into the aspects of the relationship between the different kinds of load and the shear force and the bending moment, our objective for these kinds of members which are subjected to the load is to finally evaluate the stresses. In order to find out the values of the maximum stress that occurs in the beam for such a loading we must know the maximum magnitude of the bending moment or the shear force that is acting.

If we try to find out the value of the bending moment of the shear force which is largest in magnitude, we must know the variation of shear force and the bending moment along the length of the beam and we can plot the variation of the shear force or the variation of the bending moment along the length of the beam.

At this particular point we have the largest value of the bending moment or the largest value of the shear force based on which we can start evaluating the stresses. Now to find out the variation of the bending moment or the shear force along the length of the beam, we call the diagram which we plot as the shear force and the bending moment diagram.

It is defined here (Refer Slide Time: 30:15) as the graphs in which shear forces and bending moments are plotted in ordinates against distance x along the length of the member as abscissa. Now we need to place the magnitudes of the shear force or the magnitude of the bending moment in the ordinates and compute any distance x from the beam.

If we have to compute the value of the shear force or the bending moment and place it in the abscissa, then the graph which we get gives us the plot of the shear force or the bending moment or the variation of the shear force and the bending moment along the length of the beam and we call such diagrams shear force and the bending moment diagram.

If we have a simply supported beam, we choose our coordinate axis in such a way that abscissa lies along the axis of the beam and the ordinate is perpendicular to the axis of the beam. Here if we plot the shear force or the bending moment ordinates along the length of the beam then we will get a shear force diagram or bending moment diagram if we plot the bending moment ordinate into it. This is how we can arrive at the variation of shear force and bending moment along the length of the beam. We will look into how we can evaluate and plot the shear force and bending moment diagram.

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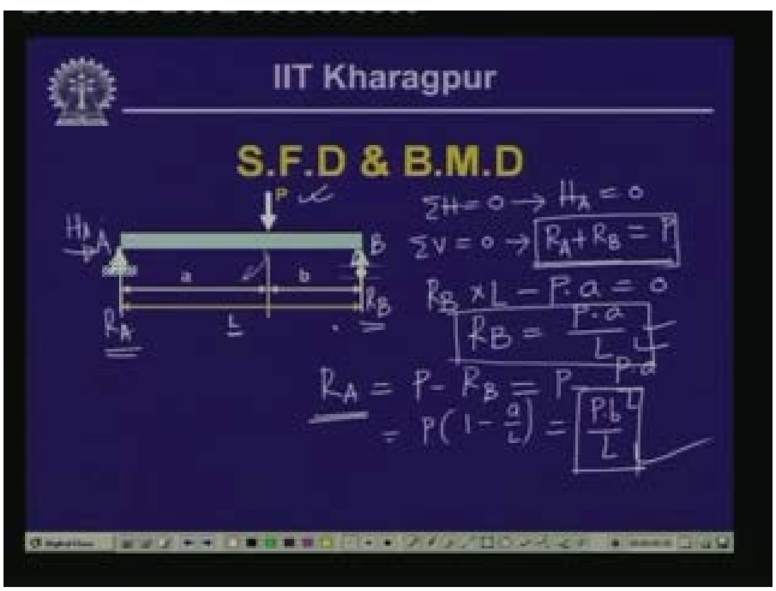

Let us start explaining this concept of shear force and bending moment diagram or how to plot the shear force and bending moment diagram through this particular example. We have taken a simply supported beam where the end is hinged which we shall call as A; the other end is on a roller which we call as B and is subjected to a concentrated load P which is at a distance of A from the left support and the length of the beam which we call as pan is equal to L.

Let us draw the free body diagram of the whole beam. Since this particular beam does not have any axial or horizontal force the horizontal reaction is; $H_A = 0$. So, we will have the vertical reactive force R_A and the vertical reactive force at B which is R_B. The summation of H = 0 gives us $H_A = 0$. The summation of V = 0 gives us $R_A + R_B$ = the vertical load P which acts downward and R_A and R_B act upward. If we consider this as positive then this is negative. So, $R_A + R_B - P = 0$ gives us $R_A + R_B = P$.

If we take the moment of all the forces with respect to support a, then we have R_B (L). This is in an anticlockwise direction and the load P causes a bending moment in the clockwise direction. We have -P (a) = 0 which gives us a value of R_B = P (a)/L. If R_B is this, then the value of R_A from this particular equation is $R_A = P - R_B = P - Pa/L$ and this gives us P (1) - a/L and L - a/L which is nothing but b which is equal to Pb/L. The value of R_B is Pa/L and the value of R_A is Pb/L. So these are the two values of the reactive forces R_A and R_B.

Let us suppose we need to plot the variation of the shear force and the bending moment along the length of the beam. We will have to look into the different segments such as what is the value of the shear force and bending moment in those segments and if we can place the shear force values or the bending moment values in the ordinate along the length of the beam, the diagram that we get will give us the shear force or the bending moment diagram.

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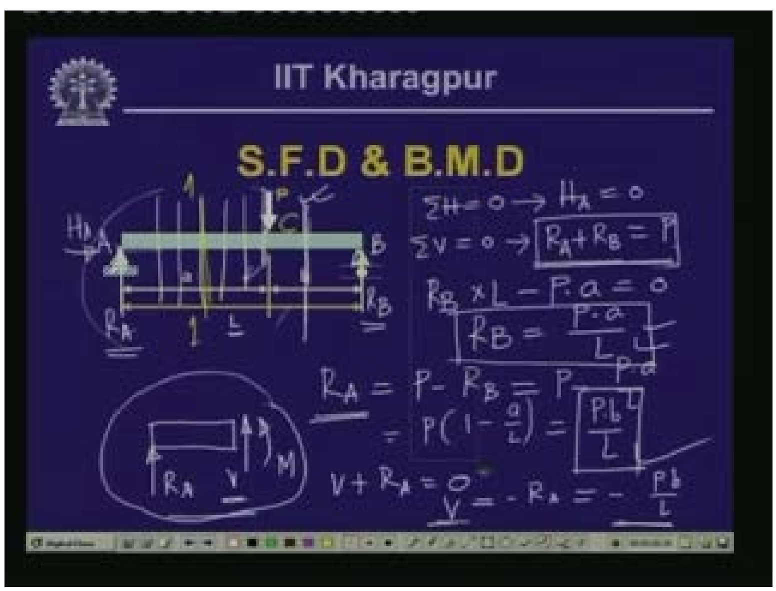

Let us calculate the values of the shear force at different segments. Let us say we take section 11which is between A and point C. Between A and C we cut the beam over here and if we take the free body of the left hand of this particular part then the free body will look like this (Refer Slide Time: 36:13). Here we have the reactive support R_A and on this part there is no external load but on this section there will be a stress resultant V and a moment M.

If we take the vertical equilibrium of the vertical forces, then $V + R_A = 0$. This gives us a value of shear force as equal to $-R_A$ and $R_A = -Pb/L$. This is the value of the shear force. If you notice that wherever you take the section up to the left edge of this particular load, we will get identical types of free bodies.

This indicates that in the segment from A to C or just to the left of this particular load, the value of the shear will be $-Pb/L$. It indicates that we have a uniform shear force from A to the left of this particular concentrated load. Now let us find out the shear force just after this particular load. If we take a section somewhere here and cut the beam and take the free body of the left part then what do we get?

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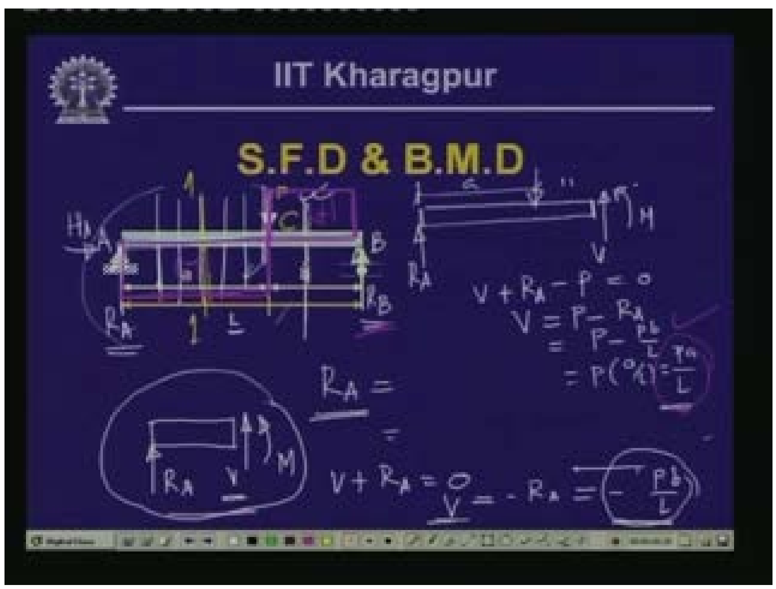

Let us take the free body of this particular part. Here we have the reactive force at the end A which is R_A which we have seen as Pb/L. Here we have the concentrated load which is at a distance of A. At this particular section of the free body, we have the stress resultant V and the bending moment M. If we take the vertical equilibrium of the forces then $V + R_A - P = 0$ or the shear force $V = P - R_A$ and $R_A = Pb/l$; so this is $P - Pb/L$. If we take out P this is $L - P/a$ and we get $a/L = Pa/L$.

Please note here that on the left hand section of this concentrated load P, we had a value of shear force which is minus Pb/L and on the right hand section of the load we have a value of the shear force which is equal to +Pa/L. This is the length of x and at this point, everywhere we have -Pb/L where we plot minus on the downward direction on the negative of y.

Everywhere we get the same value; so, it is a constant value over here. Now, Pb/L is from here up to this left edge of the load. After the concentrated load point there is a change and we get a value of Pa/L. So from here to here, there will be a change in the shear force value and again wherever you take a section over here, you are going to get the value as Pa/L. This again is constant up to this particular point where R_B is given as Pa/L.

If you look into this variation of the shear force along the length of the beam which looks like this, this is the abscissa and these are the ordinate values and this is the negative value and this is the positive value. This is nothing but the shear force variation along the length of the beam because of the concentrated load which acts at a distance of ‘a’. From here immediately we can find out that the magnitude of the shear force acting at any section along the length of the beam.

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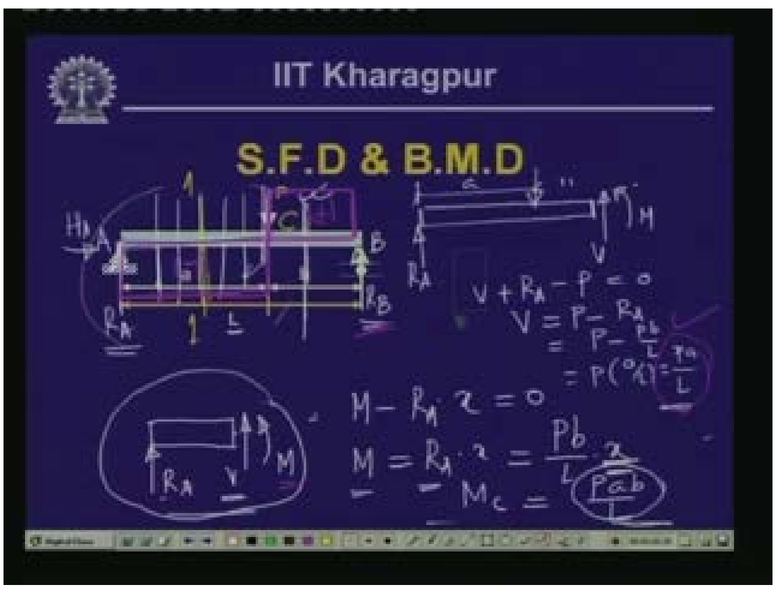

Consequently, let us compute the value of the bending moment at this particular section. If we take the moment of all these forces on this particular free body with respect to this particular section then we have the bending moment M which is in the anticlockwise direction and then we have the clockwise moment because of R_A. So, this is minus R_A multiplied by x and there are no other loads so this is equal to 0. This gives us a value of the bending moment M = R_A (x) and we have obtained R_A which is equal to Pb/L (x).

Here the moment varies linearly with x and when x = 0 the bending moment is 0 at this particular point. When x = a, at the left of the load point the magnitude of the bending moment becomes Pb/L (a). Thus the value of the bending moment at point C is M_c = Pab/L. This is the value of the bending moment and it varies linearly as the value of x increases, the magnitude of the moment also increases from 0. When x =0 bending moment is 0 and gradually as the value of x increases, the value of the bending moment linearly varies and at a distance which is ‘a’ from left end the magnitude of the moment = Pab/L.

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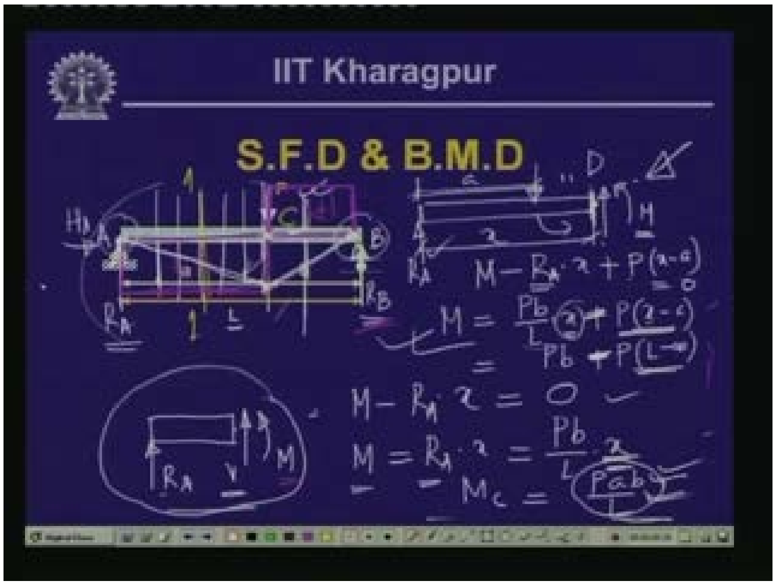

What happens to the bending moment which is on the right hand side of this particular load? Let us calculate from the free body of the other part. If we take the moment of all the forces with respect to this particular point, D, the general moment M at this section is in the anticlockwise direction. The R_A causes a moment which is in the clockwise direction. So, we have - R_A (x) and load P which acts at a distance of ‘a’ also causes a moment which is in the anticlockwise direction and so we get +P.

The load is at a distance of ‘a’ and we have taken the section from here which is at a distance of x. The distance of this load from this particular point where we have taken moment is ‘x - a’ and this is P (x - a) = 0. The value of the bending moment M = R_A which we have seen as Pb/L (x) + P (x - a). This particular expression is valid for x = ‘a’ to L. Now when x = a, this part is 0 this becomes Pba/L or Pab/L which matches with the value of the bending moment over here. That means from this side we have computed the value of the bending moment over here which we get as Pab/L.

The expression which we have obtained from this point to this point from this particular free body diagram; we find that the bending moment at this particular point C substituting the value of x = a gives us the value of Pab/L. Now what happens between these is there are no loads, now if you substitute the x = L, then what happens now this is equals to when x = l = Pb + P × x.

From the expression which we have obtained we find that the bending moment at this particular point C substituting the value of x = a gives us the value of Pab/L. Between these there are no loads. But what happens if we substitute x = L? Let us take x = l = Pb + P (x). Now we have substituted x here as L, this is L - a. We have computed the bending moment over here which is M - R_A (x) + P (x – a). So, this is going to be
-Pb/L (x) - P (x - a). Here ‘L - a’ is nothing but P and this is equal to 0. Now in L - a, L is the whole length, ‘a’ is this length and ‘L – a’ is b and so Pb - Pb = 0.

At this particular point the bending moment is 0; it linearly varies up to this point with the value of Pab/L; and again from this general expression of this segment we have here the value of the bending moment as Pab/L, when we substitute ‘x = a’ and when we substitute x = L then we find the bending moment here as 0. At the roller support and the hinged support the moment carrying capacity of the roller support and the beam supports are 0.

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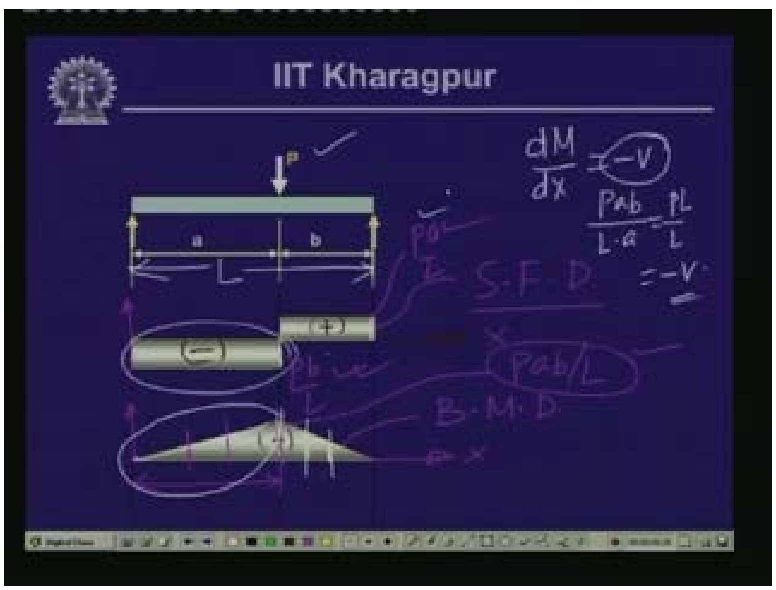


The variation of the bending moment along the length of the beam as we can see is in this triangular form and this is what has been indicated over here. This is the shear force and the bending moment diagram of this particular beam which is loaded with the concentrated load P over the span L. If we have a concentrated load which is placed at a distance of ‘a’, we have the shear force variation over here which is negative and this is positive.

The bending moment is positive for the whole region. We have taken the direction of the beam as the abscissa; these are the ordinates, this is the positive direction and this is the negative direction. Here this is the length of the beam and this is the ordinate of the bending moment. We call the diagram over the length of the beam which we get as the bending moment diagram, BMD and we call this the shear force diagram, SFD.

The value of this is equal to the reactive force which is Pb/L and the value of this shear here is Pa/L (Refer Slide Time: 48:30). This is the diagram with the ordinate value given as the shear force diagram and the value of the bending moment as we have seen over here is Pab/L and thus the largest magnitude of the bending moment is over here and in all other places the values of the bending moment is less than this value of Pab/L.

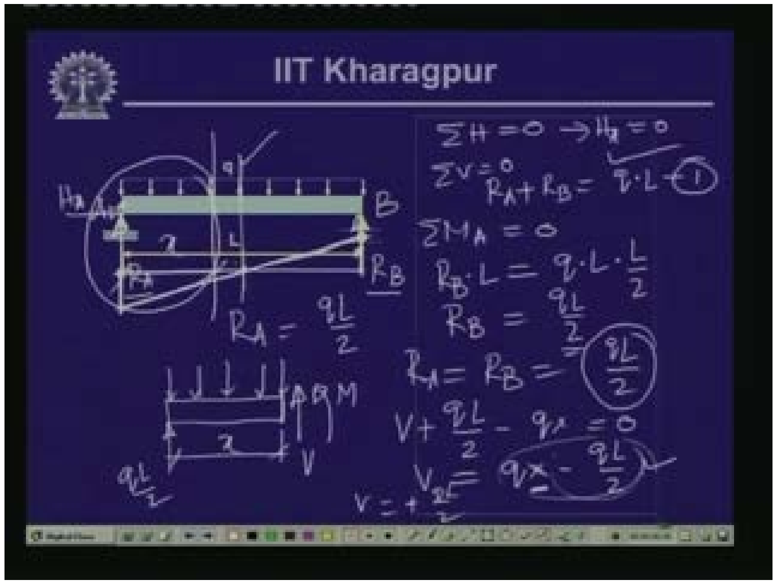
Along this particular segment between 0 to a, the slope of the beam is positive because the larger value minus the lower value will give a positive value over this length and as it is expected dM/dx = -V. Here the moment is Pab/L and here it is 0 over the length L. The value of dM/dx is Pab/L (a) = Pb/L. P here is the shear force and Pb/L is -V over here.

This particular segment indicates that the slope of the bending moment is equal to the negative of the shear force. On this side, the slope will be negative since we have a lower value on the right hand side with respect to the left hand side and we will get -dM/dx which is equal to -V. Eventually it will become +V and so the value over here is equal to Pa/L.

In fact from this particular diagram, you can observe that when a simply supported beam is subjected to a concentrated load which is at a distance of ‘a’ from the left support, the shear force and the bending moment vary and the relationship between the shear force and bending moment as we have obtained through those differential forms can also be satisfied through this particular example.

Where there are no loads on the beam the shear force is constant and the bending moment varies linearly. From the concentrated load point there is a jump in the shear force from the left hand to the right hand support. From the bending moment diagram you can see that at the concentrated load point there is no change in the bending moment but there is a change in the slope. On the left hand side we have a slope which we call as a positive slope. On the right hand side we have a negative slope and there is a jump in the rate of change of the bending moment value.

(Refer Slide Time: 51:26 - 55:25)



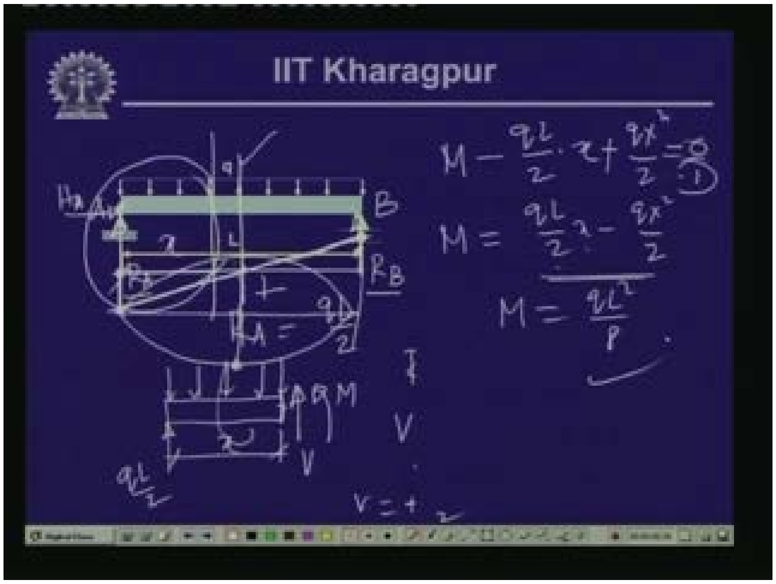
If a simply supported beam instead of a concentrated load is subjected to a uniformly distributed load of intensity q over the length L then what happens to the shear force and the bending moment diagram? Here we have the reactive forces acting. Let us call this end as A, this end as B and here the reactive force which is acting is R_B, this is R_A and we have the horizontal force H_A.

The summation of H = 0 gives us that $H_A = 0$. Now the summation of V = 0 will give us $R_A + R_B = q (L)$ which is the vertical load acting in the opposite direction of R_A and R_B. Let us call this equation as 1. Let us take the moment of all the forces with respect to A. If we say that the moment at 'a' is 0, we have R_B (L) which is in the anti-clockwise direction as equal to q (L) which is in the clockwise direction to L/2. This gives us a value of R_B as equal to qL/2 and from equation 1 we can write $R_A = qL - qL/2 = qL/2$ and from this particular diagram also we can visualize that $R_A = R_B = qL/2$.

Now the total load is q (L) and half of the load gets distributed or shared by R_A and R_B. If we take a section here which is at a distance of x and draw the free body diagram of the left part of this particular beam then we have this particular beam with the reactive force which is qL/2 we have the distributed load over this particular segment and let us call this distance as x and on this particular side we have the stress resultant shear as V and the bending moment as M.

If we take the vertical equilibrium of all the vertical forces then we have V + qL/2 - q (x) = 0. This gives us V = qx - qL/2. Now here please note that if x = 0 which is the left support then V is equal to a value of -qL/2. Now as x goes along the length of the beam when x becomes L/2, the value of b becomes 0. Now as x increases naturally this particular quantity keeps on increasing from - qL/2 and then at x = L/2 it becomes 0 and at x = L this becomes V = +qL/2.

(Refer Slide Time: 55:29 - 56:32)



If we take the center over here as this particular point then over the point x = 0 we have the value of an ordinate as -qL/2. At L/2, we have a value of an ordinate as 0 and at L we have the value of an ordinate as qL/2. If we join these three, since it varies linearly with x, we get the shear force diagram. This is the shear force diagram for this kind of loading and here we do not have any change in the loading. This will give us the value of the shear force. If we take the moment of all the forces with respect to this particular section so M which is anticlockwise qL/2 is clockwise so -qL/2 × x and this is also anticlockwise. So $-qx^2/2 = 0$ this is +. So M = qL/2 x $-qx^2/2$.

Here let us substitute the value of x. At x = 0, the moment is 0 which is expected at the hinged support; at x = L again the moment is 0 and at x = L/2 you get a value of qL square/4 - qL square/8 = qL square/8 at the mid span. If we plot the variation of the bending moment we get a parabolic distribution. (Refer Slide Time: 56:32) So, it will be qL square/8 here, 0 here and 0 here and this will be in the opposite direction because this is positive everywhere.

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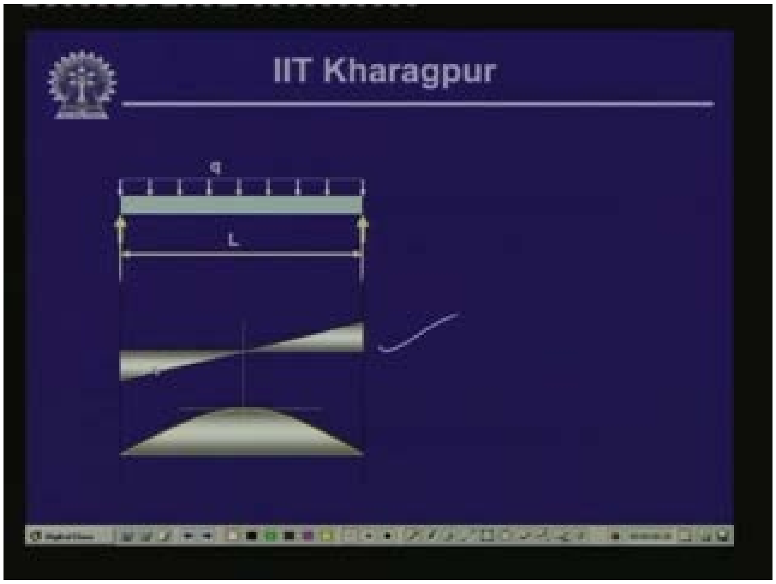

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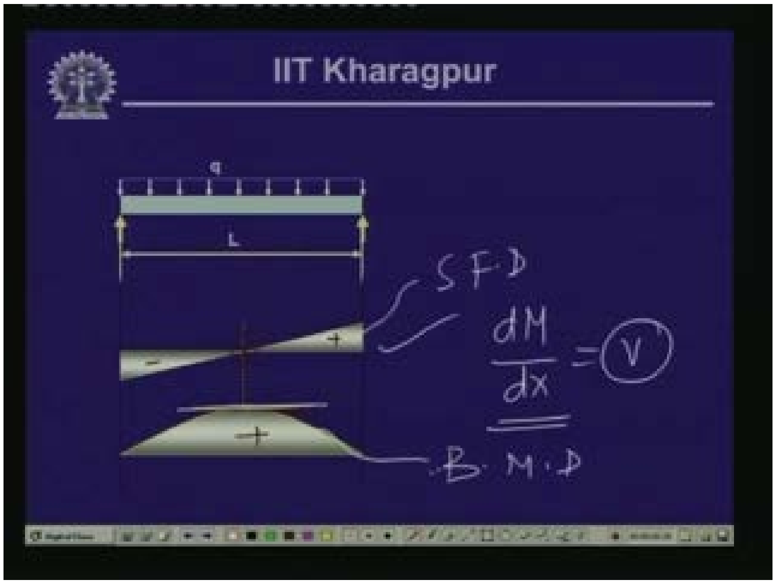
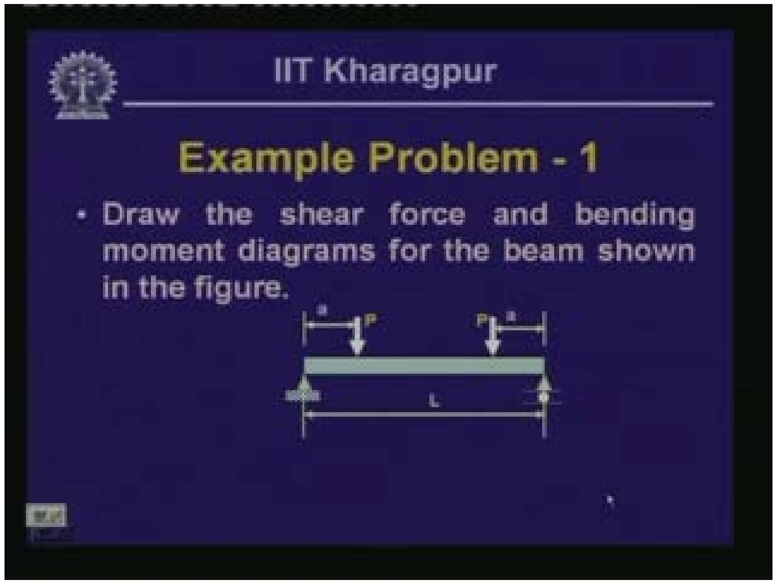


This is the diagram of the shear force and the bending moment as we have evaluated. This is the shear force diagram which is negative here and positive here and this (Refer Slide Time: 56:49) is positive. Please note that where the slope of this bending moment diagram is at L/2, the tangent to the diagram is horizontal and correspondingly the shear force value here is 0 and that is what we get if we have dM/dx.

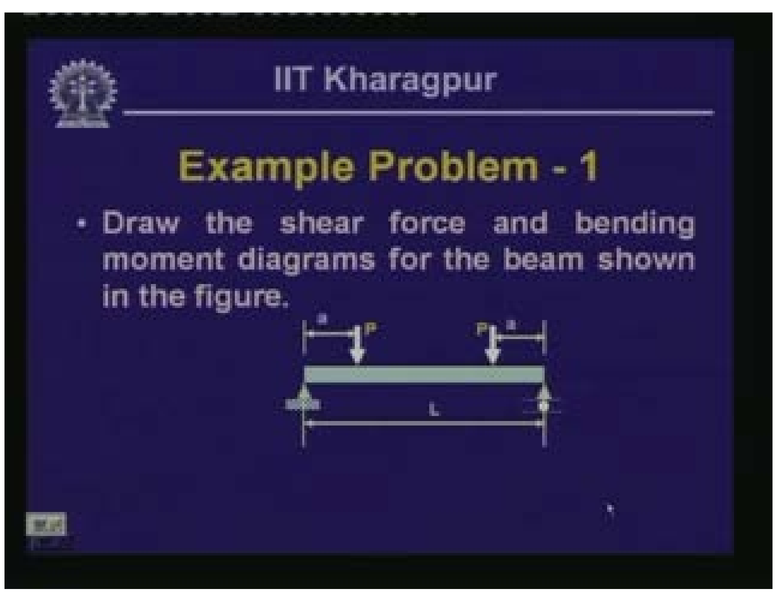
If V=0 then dM/dx and the moment are constant and the slope over here is the horizontal one. So, we call this diagram the shear force diagram and we call this diagram (Refer Slide Time: 57:22) the bending moment diagram.

(Refer Slide Time: 57:24 - 57:29)



There are some example problems set for you. Please look into this and we will be discussing them in the next class. They are to draw the shear force and bending moment diagram for this kind of a loading, to draw the shear force and bending moment diagram for a kind of loading where if we have a simply supported beam subjected to a concentrated moment then what will be the values of the shear force and bending moment over the length of the beam? Then if we have a cantilever beam subjected to a uniformly distributed load then what will be the shear force and the bending moment diagram for this particular beam?

(Refer Slide Time: 57:48 - 57:56)



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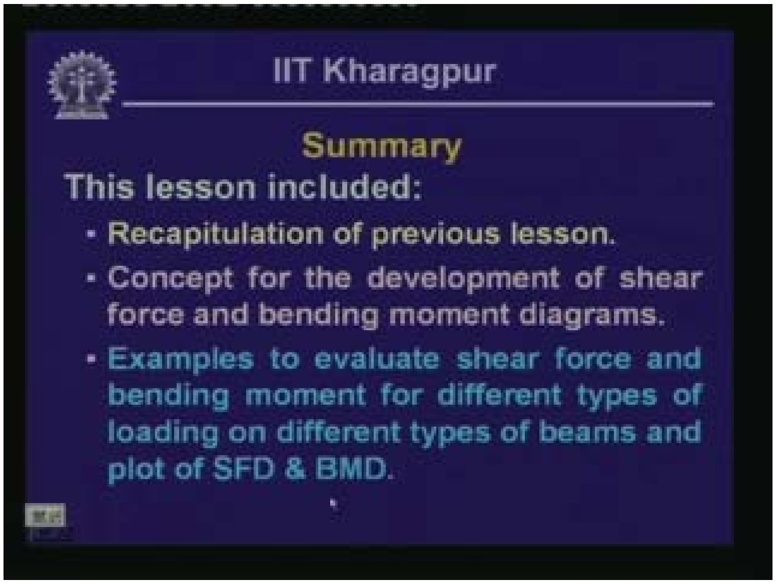
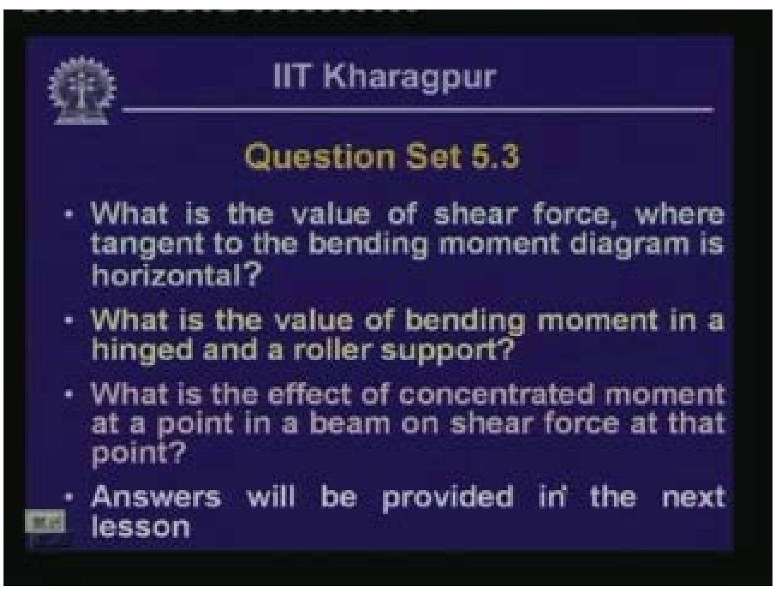


In this particular lesson, we have recapitulated the aspects of the previous lesson. We have discussed those aspects again and we have looked into the concept of drawing shear force and the bending moment diagrams. Also we have looked into some examples on how to plot the shear force and the bending moment diagram if we have a concentrated load or a uniformly distributed load over the length of the beam.

(Refer Slide Time: 58:26: - 58:42)



These are the questions given for you. What is the value of a shear force where tangent to the bending moment diagram is horizontal? What is the value of a bending moment in a hinged and a roller support? What is the effect of a concentrated moment at a point in a beam on a shear force at that particular point? The answers will be given in the next class.