Strength of Materials
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Lecture no 23
Bending of Beams- II

Welcome to the second lesson of the fifth module which is on Bending of Beams part 2. In the last lesson of this particular module we have discussed some aspects of the bending of beams. We have introduced what is meant by beam and then we have introduced the concept of shear force and bending moment. In this particular lesson, we are going to look into the relationship of shear force and bending moment with the different types of loadings.

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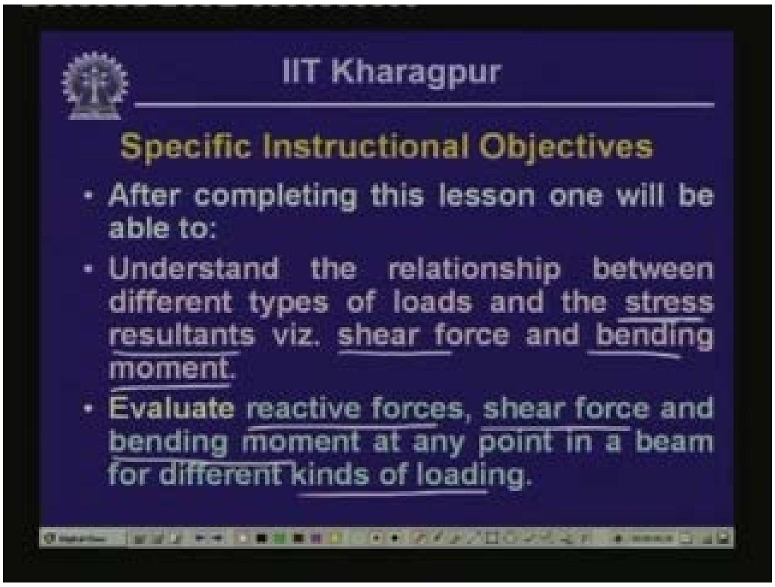

Once this particular lesson is completed one should be able to understand the relationship between different types of loads and the stress resultants, which are shear force and bending moment. One should be able to evaluate reactive forces, the shear force and bending moment at any point in the beam for different kinds of loading, or in the length of the beam at any section.

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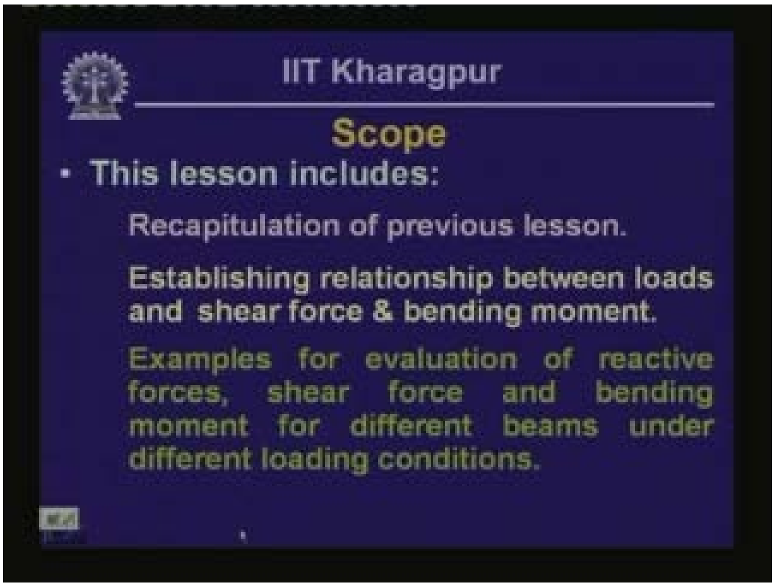

When we talk about loads, it could mean uniformly distributed load, concentrated load, or it could be concentrated moment and this includes the relationship of these loads with the shear force and bending moment. We will give examples for the evaluation of reactive forces, shear force and bending moment for different beams under different loading conditions. Let us look into the answers of the questions which were posed last time.

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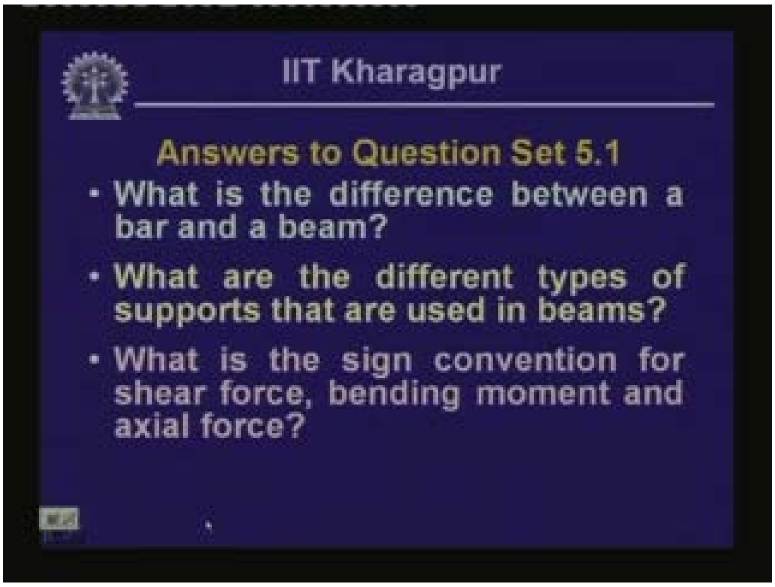

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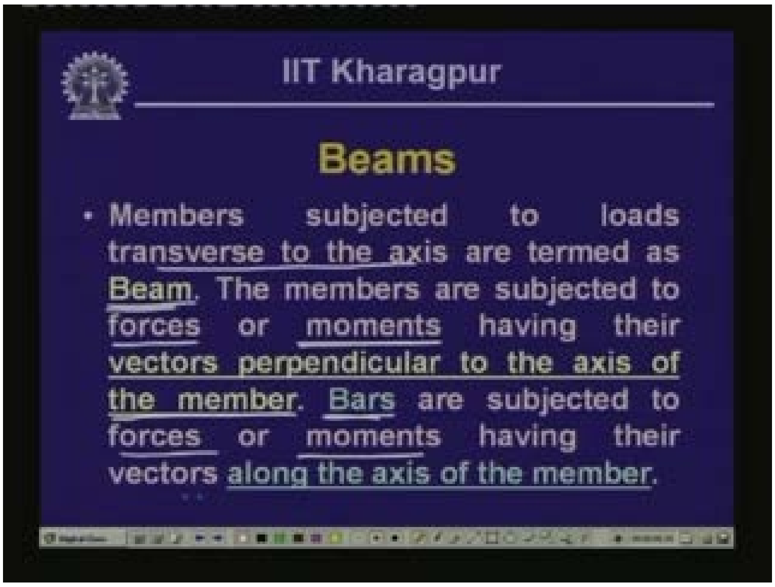

What is the difference between a bar and a beam?
In the last lesson we had discussed the concept of a beam. As we said that when a member is subjected to loads in its transverse direction, which is perpendicular to the axis of the member, it is designated as a beam. The members are subjected to forces or moments having their vectors perpendicular to the axis of the member and in this way they are different from bars. Bars are subjected to forces or moments having vectors along the axis of the member.

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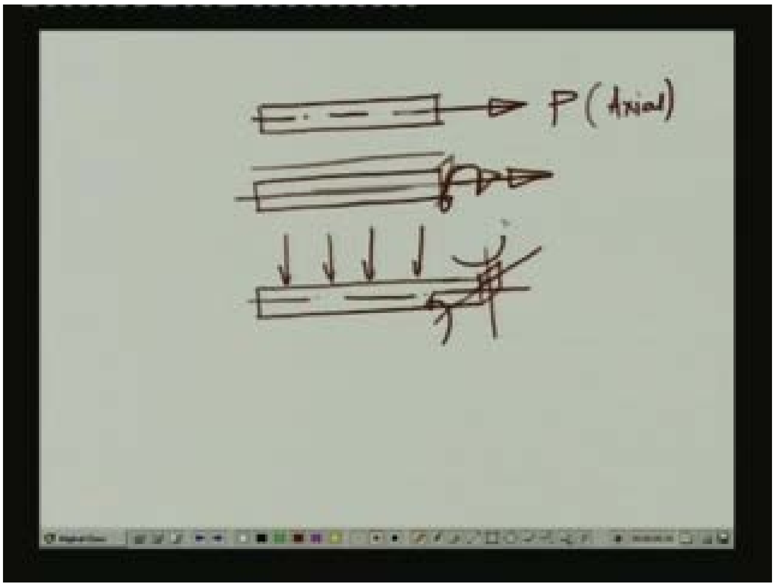

Let us take an axially loaded member which has a member and a member axis. Then the force acting in the direction along the axis of the member is what we have called the Axial Force. Consequently, we have seen that if a bar is subjected to a twisting moment, which is about the axis perpendicular to the cross section of the bar, the vector direction of this twisting moment will be along the axis of this bar.

In the case of members which are designated as beams the loads act in the transverse direction which is perpendicular to the axis of the member. If the moment acts at this particular axis, which is in the plain of the cross section of the vectorial direction, the vectorial direction will be perpendicular to the axis of the bar.

If the moment acts about the vertical axis, also, the vectorial notation of this particular moment will be perpendicular to the axis of the bar. The members that are subjected to the loads or moments, with their vectorial notation perpendicular to the axis of the bar, are designated as beams. This is the difference between a bar and a beam.

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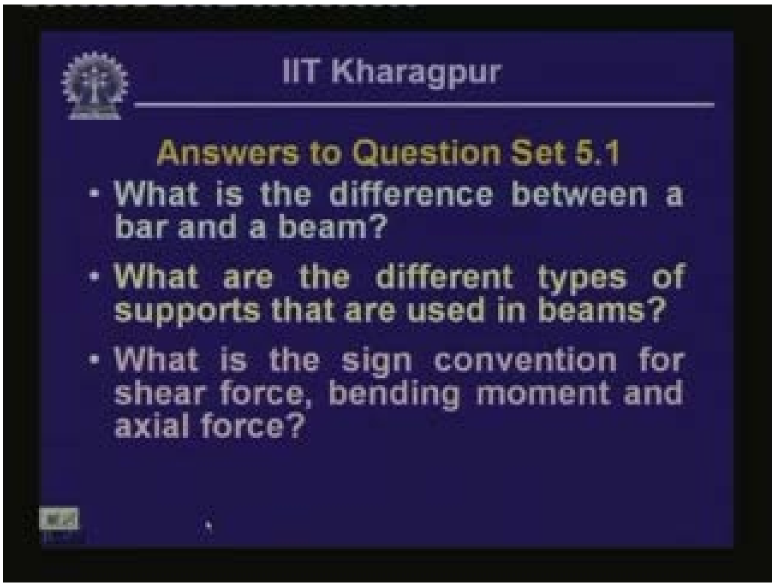

What are the different types of supports that are used in beams?
We had looked into the aspects of supports like the pinned support, hinged support, roller support and the fixed support. These are the kinds of support that we encounter in beam members. We have also discussed the types of beams where the first one is of a pinned category or hinged category and in a diagrammatic notation we generally indicate this kind of diagram.

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This is the roller support where the horizontal moment is allowed and this is a fixed support where no moments like the horizontal moment, vertical moment or the rotation are allowed. Here (Refer Slide Time: 06:25) the vertical moment and the horizontal moment are restricted, but the rotation is allowed. Here, the horizontal moment is allowed, the vertical moment is restricted and the rotation also is allowed and here all the three quantities are restricted.

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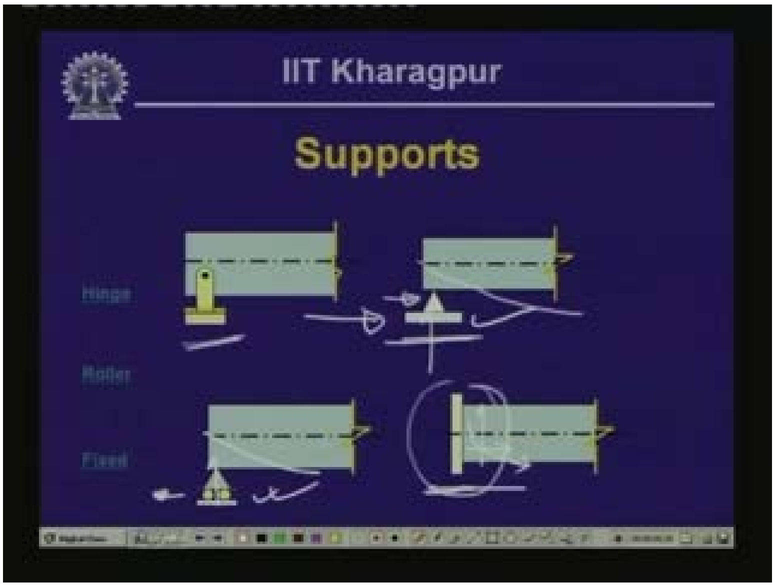

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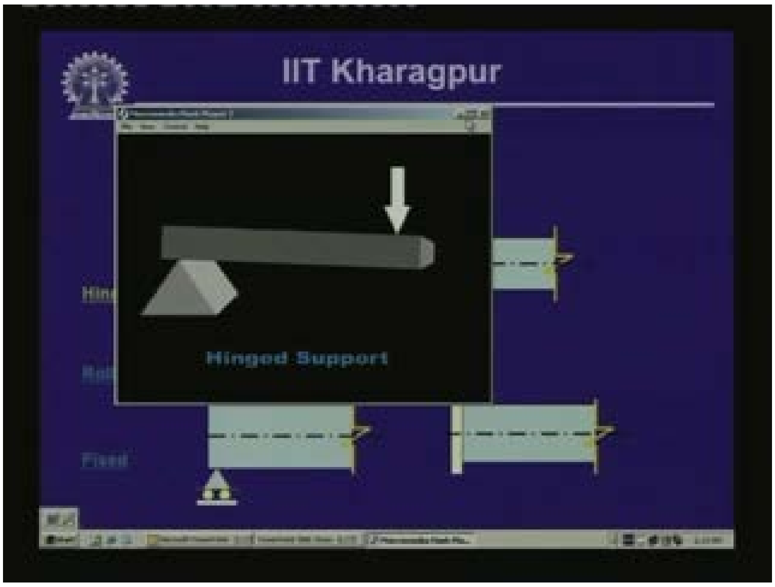

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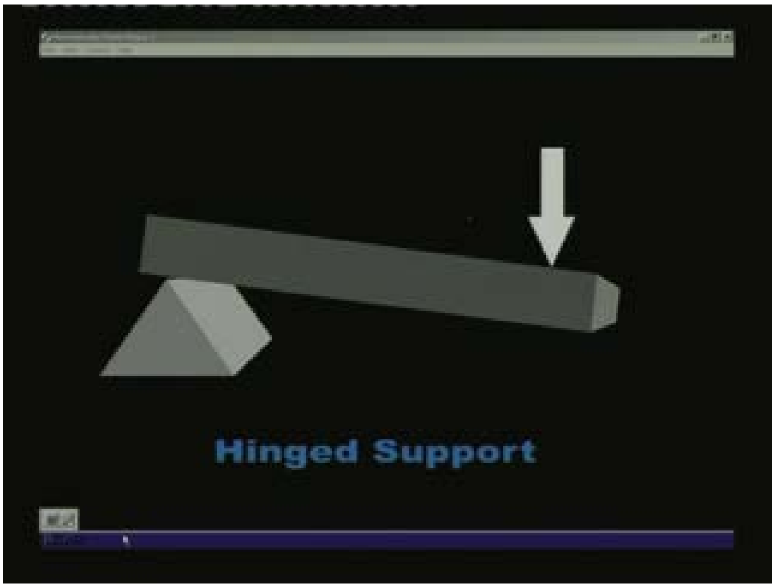

Let us look into each type of connections. When a part of the beam is loaded, it undergoes rotation. If we call this as the axis of the bar, then with reference to its original axis it undergoes rotation. Whereas this particular point, which was originally at this position, remains here. That means no horizontal or vertical moment is allowed except the rotation. So, this is a hinged type of connection or support.

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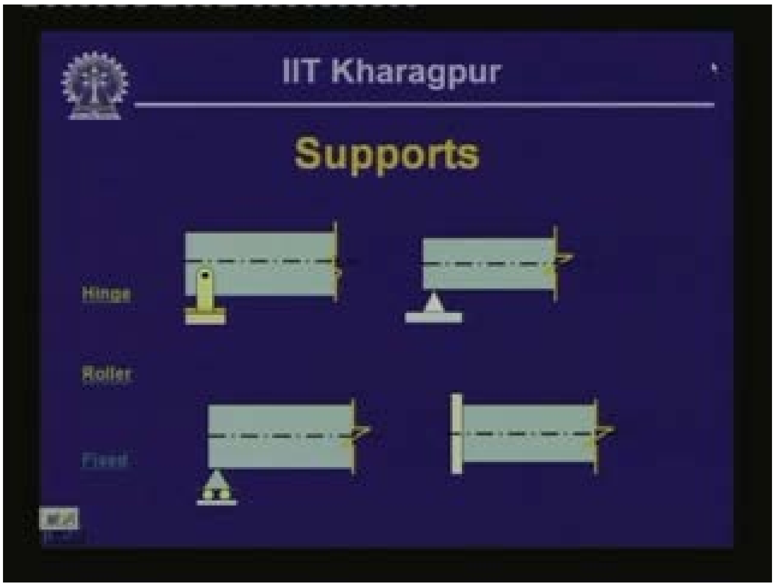

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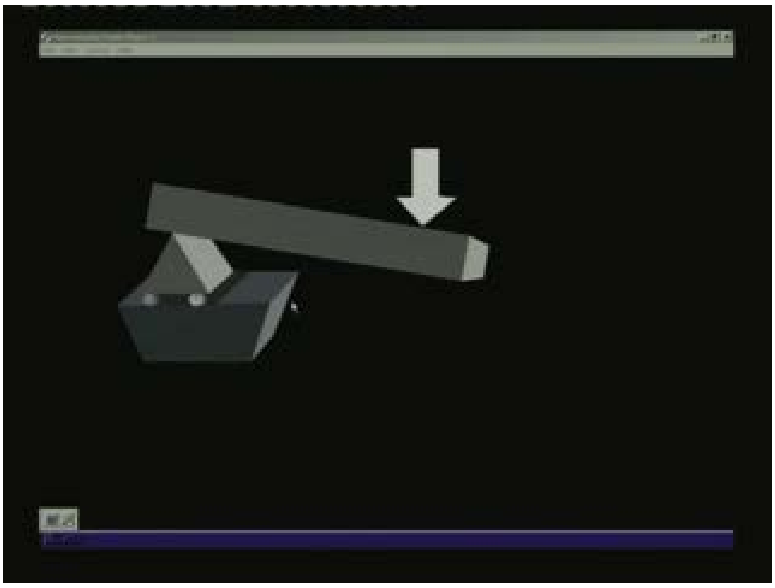

If we look into a roller kind of a support, under the action of the load, it moves in the horizontal direction. It cannot move in the vertical direction, but it can have rotation. Please note over here that it moves in the horizontal direction and it has rotation. With respect to the original position of the beam, it has moved in the horizontal direction and we call this as delta horizontal. Then we have rotation theta, but the vertical movement is restricted for the roller joint. Note that we have movement in the horizontal direction as well as rotation.

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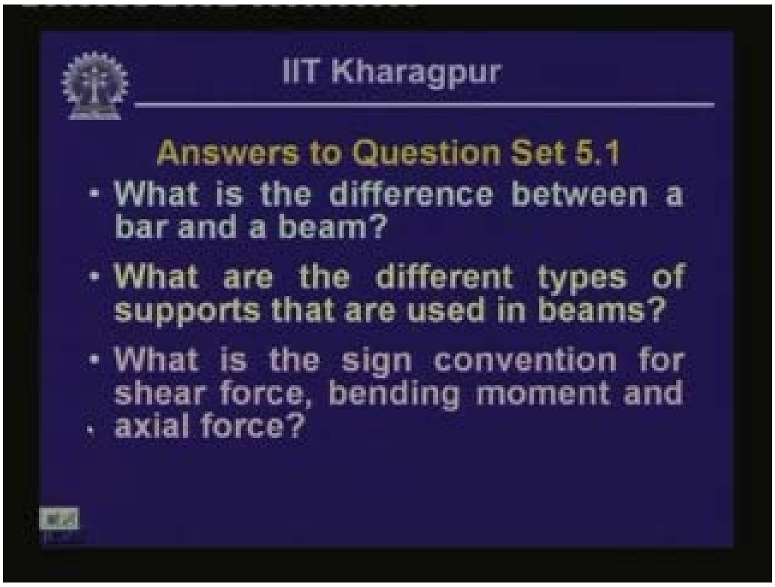

These are the different kinds of supports that we encounter and now let us take the last question.

What is the sign convention for shear force, bending moment and axial force?

For shear force and bending moment when we compute at any cross section of the beam, we must place them with an appropriate sign and this sign convention can be of different types. The sign conventions which we will be following over here are based on the initial sign conventions as we had seen in the stress diagram.

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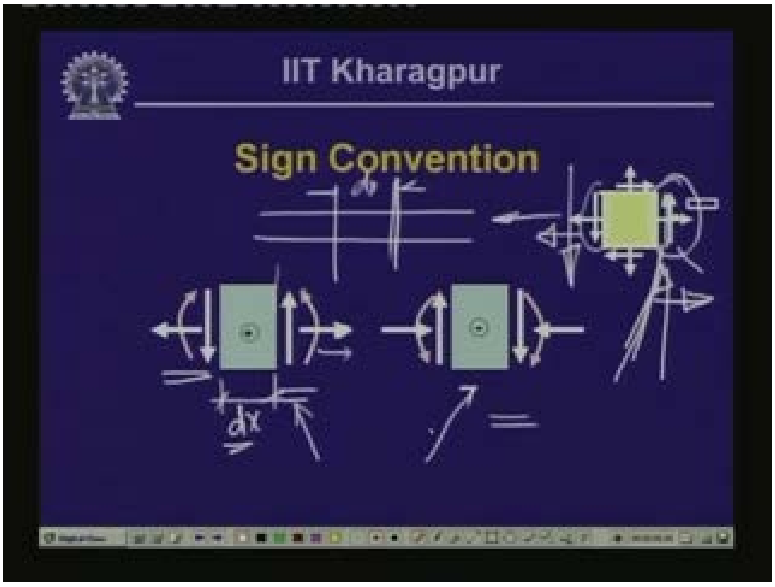

If we look into a 2-dimensional element of a rectangular configuration, or a square configuration, the stresses act in the x positive direction. The shear, which is positive, is in the positive y direction. On the negative phase, the axial force acts in the negative x direction, and the shear force in the negative y direction. Here the convention which we have used is the shear force on this right hand side.

Let us suppose dx is the length which is taken out from a beam. On this the shear force which is on the right phase acts upwards, the moment which is anti-clockwise is positive and axial force which is directed towards the x direction is positive and the reverse acts on the left hand support which are the negative conventions. So, these are the sign conventions that we will be following. We have used these conventions in the evaluation of stresses at a point with respect to the axial stress.

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The basic objective of this kind of analysis on beams, when a beam is subjected to the transverse loads, is to evaluate the stress that this particular member undergoes because of such loading and the deformation in the member. To know precisely what the stress level is at any point along the length of the beam, we need to know how the stress resultant, the shear force, the axial force, and the bending moment vary at a different cross section along the length of the beam, based on the loading it is subjected to. To do that, let us look into some of the aspects of how different kinds of loads can be related with the stress resultant quantities, which are only the shear force and the bending moment.

Let us consider a small segment of the beam. Let us say that we have a beam over here, which is supported at this particular end from which we would like to take a small segment, which is of length dx and this particular segment is drawn over here. This particular segment on its sides is given the positive shear force and the positive bending moment. Here it is in the opposite direction and it is subjected to uniformly distributed load of intensity q per unit length.

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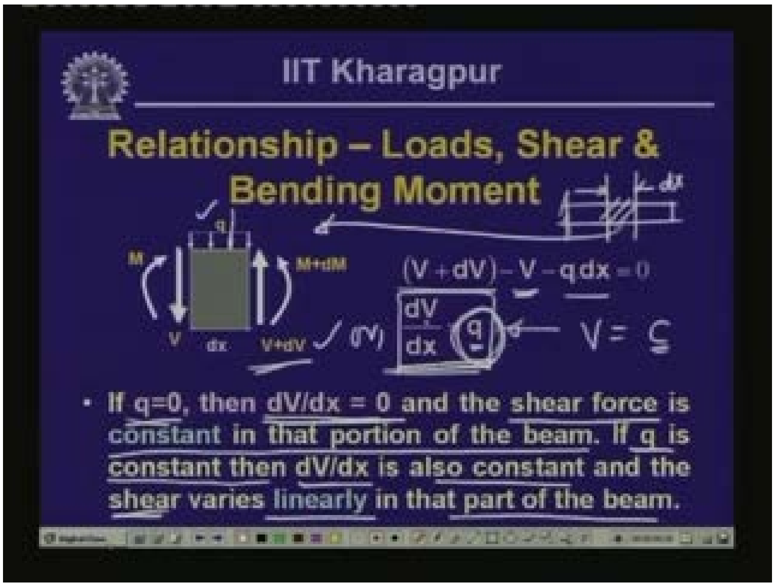

If we write down the equilibrium of the vertical forces then we get v + dv, which acts upwards in the positive direction, minus the force which acts on the left hand side which is downwards as a negative beam and q is the load per unit length. Here q (dx) again acts downwards which is negative and we get q (dx) = 0. From this particular expression we get dv/dx = q. The meaning of this particular expression is that the slope of the shear force is equal to the load q.

Let us look into this particular expression. If this term q = 0 then dv/dx = 0 and if dv/dx = 0 then the shear force gives us a constant value. Wherever in a beam, if we do not have any load and if we would like to compute the shear force over that particular portion, then the shear force will be constant over that region where there is no load. So, this is what is interpreted from this equation, which is that if dv/dx = 0 then the shear force is constant in that particular portion of the beam.

If q is uniformly distributed over the length of the beam, or a portion of the beam which we are considering, then dv/dx = q, which is constant. If we integrate it then we get v which varies linearly over that particular portion. If we have q which is constant or uniform over the portion then dv/dx is constant and consequently, the shear force will vary linearly in that part of the beam. This is what we get from this particular diagram.

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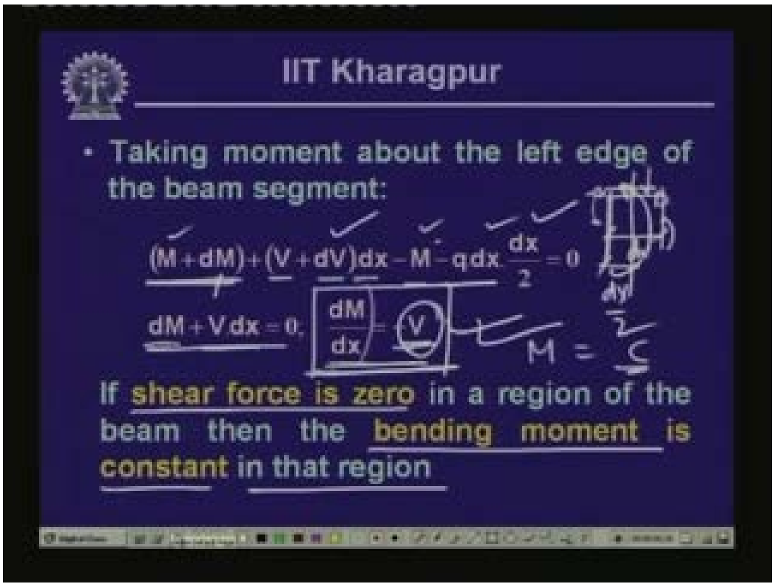

Supposing we take the moment of all these forces with respect to this left edge, then what do we get?

If we take the moment of all the forces with respect to the left edge of the beam, then we have m + dm in an anti-clockwise direction if we consider that as positive. On the right hand side we have v + dv multiplied by the distance dx which is the moment.

So, (v + dv) dx which also causes an anti-clockwise moment is positive and on the left hand side we have a clockwise moment which is -m and the load q over the length dx again causes a clockwise moment with respect to the left phase and we have q multiplied by dx, which is the total load multiplied by half the distance which is dx/2. So, this gives us the moment equilibrium.

If we eliminate the terms which is the product of the small quantities dv, dx q dx square/2, then m and m get cancelled and we are left with dm + v (dx) = 0. The rest of the terms gets cancelled out and from these we get dm/dx = -v.

This expression tells us that the rate of change of the moment is equal to the negative of the shear force along the length of the beam. That means along the length of the beam when they are subjected to uniformly distributed load if you take any cross section, the rate of change of the moment will be the negative of the shear force.

Here note that, if this particular quantity f force becomes 0, then the bending moment in that particular region is going to be constant because m is going to be a constant quantity c. So, if shear force is 0, then the bending moment is constant in that particular region.

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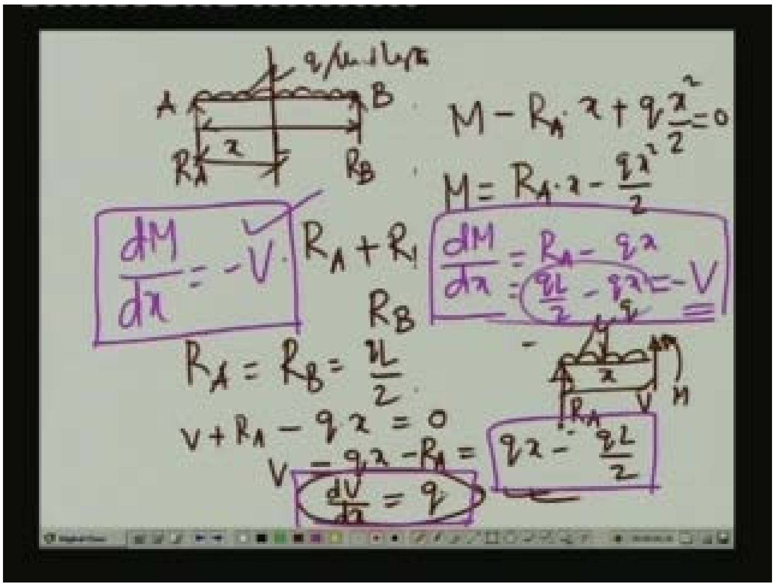

We can verify it readily, if we have a beam which is simply supported. Since, here we do not have any axial force the horizontal force at this end is 0. This is the hinged end and this is the roller end, and this is subjected to a uniformly distributed load, say of q/unit length and the length of this beam is L. If we call these two ends as A and B, the reactive forces are R_A and R_B.

We can take moment about B and we can say that R_A (L) = q (L) L/ 2, and thereby, R_A = qL / 2. Since, R_A +R_B = q multiplied by L from the vertical equilibrium, R_B = qL/2. The reactive values R_A = R_B = qL / 2. If we take the free body of the beam at a particular section which is at a distance of x from this left end then we have this reactive force here, which is R_A and then we have a uniformly distributed load over this particular portion and we have the shear force, the positive shear force and the bending moment. So this is v and this is m and this is q/unit length.

As we have noted this distance is x. If we take the vertical equilibrium of the forces we have v + R_A – q(x) = 0 and from this we get v = qx - R_A and R_A as we have seen, is:
qL/2. V = qx - R_A = qx - qL/2. Let us take the derivative of this dv/dx = q which conforms to the derivation as we have seen and if we compute the value of the bending moment that is if we take the moment about this particular point, then we have moment M in an anti-clockwise direction.

Moment contribution of R_A which is in a clockwise direction is - R_A (x) and the moment contribution of this downward load also in an anti-clockwise direction, which is positive, is q (x square) /2 = 0. So, this gives us m = R_A (x) - q x square/2. If we take the derivative of this, then dM/dx = R_A – qx and R_A as we have seen is qL /2 and so this is qL /2 - qx = -V.
Here V = qx - qL /2 and we get qL /2 - qx = -V and we have obtained the expression,
dM /dx = - V. So, it can be verified from this particular example that we can get dM/dx = - V and dv/dx = q.

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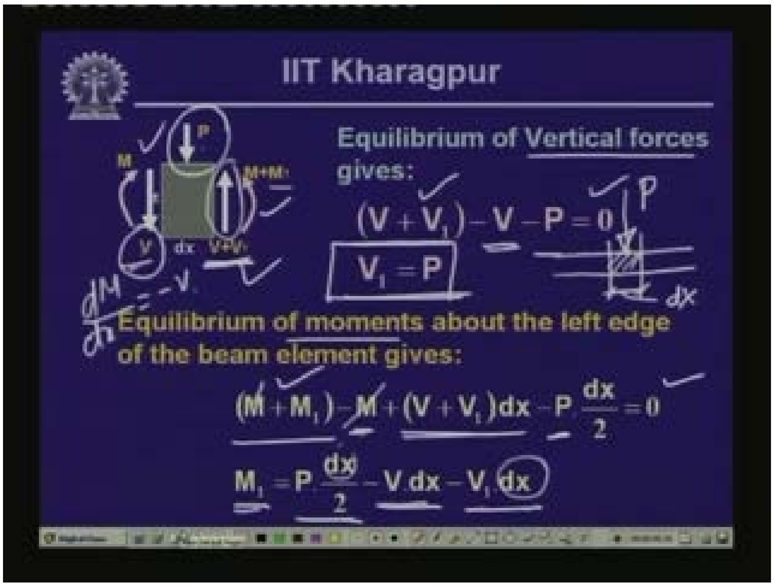

Let us suppose that the beam is subjected to a concentrated load. We have shown earlier that we have a part of the beam; we take out a small segment, the length of which is dx and over this dx, a concentrated force or a concentrated load acts on the beam, which is p. In the previous case, we had the change between V over the length dx as V + dv and here we have noted $V + V_{1.}$

In the previous case we had uniformly distributed load over the small segment. So, the variation was comparatively less, but in this particular case we have the concentrated load acting on the beam at that particular point and the change over from the left of the beam to the right of the concentrated load is drastic. Since there is a finite change we have designated the right hand shear by a change over a finite value. So, on the left hand side we have the shear as V and on the right hand side we have $V + V_1$ where V_1 being a finite value can give us the change due to the concentrated load and, so is the case with the moment $m_{1.}$

If we take the equilibrium of the vertical forces, then $V + V_1$ which acts in the upward direction, - V which acts in the downward direction and – P which acts in the downward direction, is equal to 0. So, we get $V_1 = P$ and this particular expression gives us the information that on the right hand side the change in the shear from the left hand support is equivalent to the addition of this concentrated load.

From the left hand support or the left hand side of the concentrated load the shear force gets changed by the quantity of that concentrated load over the right side. So, on that left side we have the shear of V and on the right side we have the shear of V + P, which we have called as V_1 and so, V_1 in this case is P.

If we take the equilibrium of the moments about the left edge of the beam element, then again we have $M + M_1$ which is an anti-clockwise moment, which we have considered as positive, - M which is a clockwise moment acting on the left end, plus the moment due to $V + V_1$ is V + V1 multiplied by dx and the contribution in the moment due to P is P multiplied by dx /2. Here, V does not have any contribution because we are taking the moment at this edge which is equal to 0. After cancelling the terms, we get M_1 = P (dx/2) - V (dx) - V_1 multiplied by dx.

Since this quantity dx is small there is not going to be a significant change of moment at the point of the concentrated load between the left hand side of the load and the right hand side of the load. While computing the bending moment at different points along the length of the beam, if we compute the bending moment on the left side of the load and then the right side of the load, due to the concentrated load there would not be any change in the bending moment.

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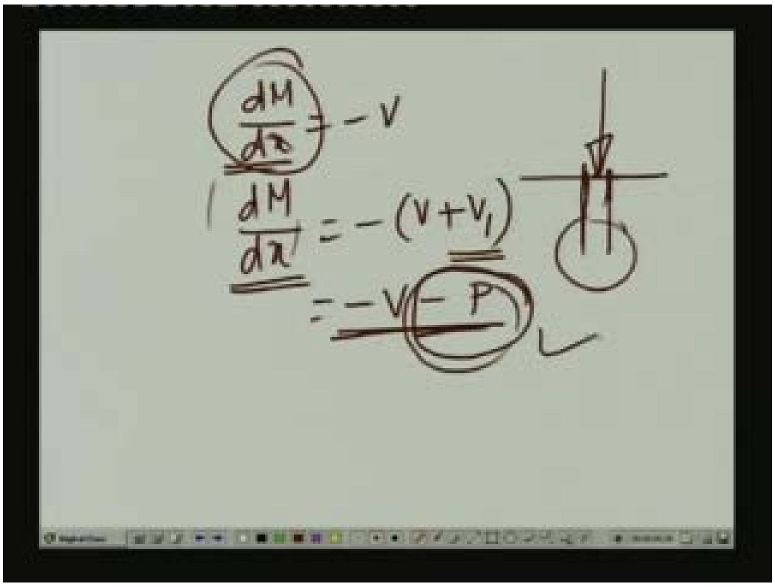

But, from the previous expression we have seen that dM/dx which is the rate of change of the moment is equal to the negative of the shear force. On this side we have V, and on this side we have $V + V_{1.}$ So, on the left side, the value of dM/dx = - V and the value of dM/dx on the right hand side is - ($V + V_1$).

As we have seen V 1 = P = -V - P. There is a drastic change in the value of the rate of change of moment from the left side of the concentrated load to the right side of the concentrated load. Taking the left side of the concentrated load and the right side there will be a difference of - P over the dM/dx value on the left support. So, there will be a drastic change in the rate of change of moment because of the concentrated load.

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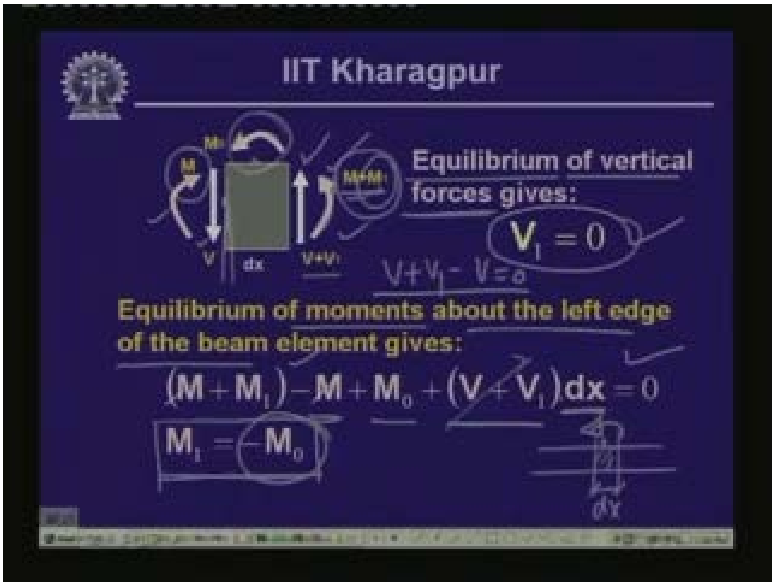

As we have seen, in a beam we can have a concentrated moment. Consider a small part of the beam which is of width dx and at this point there is a concentrated moment M_0, which acts over here. Again the positive shear and positive moment are indicated over here and consequently the negative shear and the negative moment on this phase.

If we take the equilibrium of vertical forces it gives us $V + V_1 — V = 0$, which tells us that, $V_1 = 0$. The meaning of $V_1 = 0$ is that along the length of the beam, if there is a concentrated moment then there is no change in the shear force. Along the length of the beam whenever we come across a concentrated moment at a particular point and if we compute the shear force on the left of the concentrated moment and the right of the concentrated moment, we will find that there is no change in the shear force.

Let us take the equilibrium of the moments at the left edge of the beam element. We have this anti-clockwise moment $M + M_1$; we then have the clockwise moment which is M, plus this anti-clockwise moment which is M_0 and then the moment contribution of this $(V + V_1)$ dx = 0 where the sum of all the moments with respect to the left edge equals to 0. If we neglect the terms which are multiplied with the small quantity dx, then we can neglect this particular quantity and M gets cancelled with this, so we are left with $M_1 = -M_0$.

This indicates that on the left hand side we have the moment M, on the right hand side of this particular small length dx we have $M + M_1$ and M gets added with this value M_1 where $M_1 = - M_0$. That means whatever moment we get on this left edge of the moment, on the right edge this quantity M_0 gets subtracted with this value and finally we get the moment over here. So, when we have a concentrated moment at a point we will find that there is no change in the shear force from the left to the right of that concentrated moment.

When you compute the bending moment on the left of the concentrated moment, whatever moment value we get again gets modified by the concentrated moment acting to the right of it. This observation will be necessary when we compute the values of the shear force and the bending moment in the length of the member.

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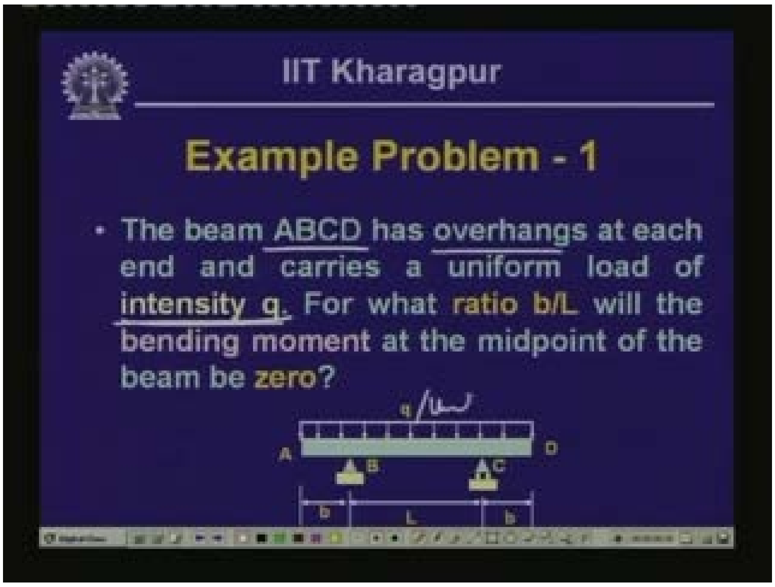

Before we go into the evaluation of those quantities, let us examine the previous examples. In the first example the beam ABCD has overhangs at each end and carries a uniform load of intensity q/unit length and this is uniformly distributed over the entire length. So, B is a hinge support and C is a roller support and the distance between B and C is L. From B we have an overhang of b and from C also we have an overhang of b and this is subjected to a uniformly distributed load q.

We will have to find out the ratio b/L so that the bending moment at the midpoint of the beam is 0. For what value of b/L will the moment at E be 0? Let us suppose this is the overhanging part which is of length b and we know that the intensity of the load is q per unit length. So, over this particular width b the load which acts is equal to q multiplied by b and the intensity q /unit length multiplied by b is the total load and that will act mid way between the distances which is b/2.

If we transform this beam into a beam which is supported on B and C with the effect of the overhangs, then the effect will be transferred from this concentrated force over the support. At this point we did not have any concentrated force and in the opposite direction there has to be one force. (Refer Slide Time: 30:54) So, this opposite force along with this, forms a couple and this gives a moment of this nature, which is anti-clockwise.

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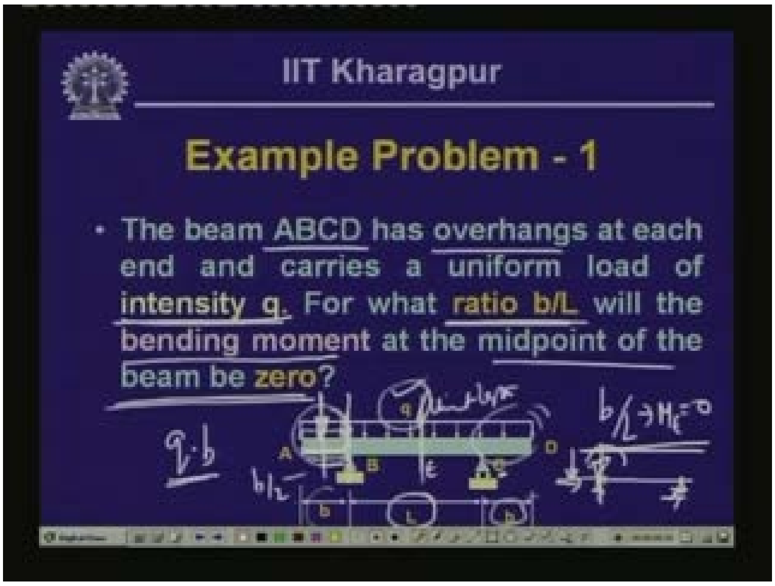

Similarly, if we transfer this concentrated force over the support we will have a concentrated force. If we make a reduced form of this particular diagram, then we have replaced the hinged support with the reactive forces. Let us call this as R_B, let us call this as H_B and let us call these reactive forces as R_C. This being a roller is constrained to move in the vertical direction only and it can have moment in the horizontal direction and thereby there would not be a reactive force generated in the horizontal direction. So, there would not be any reactive force at the roller support.

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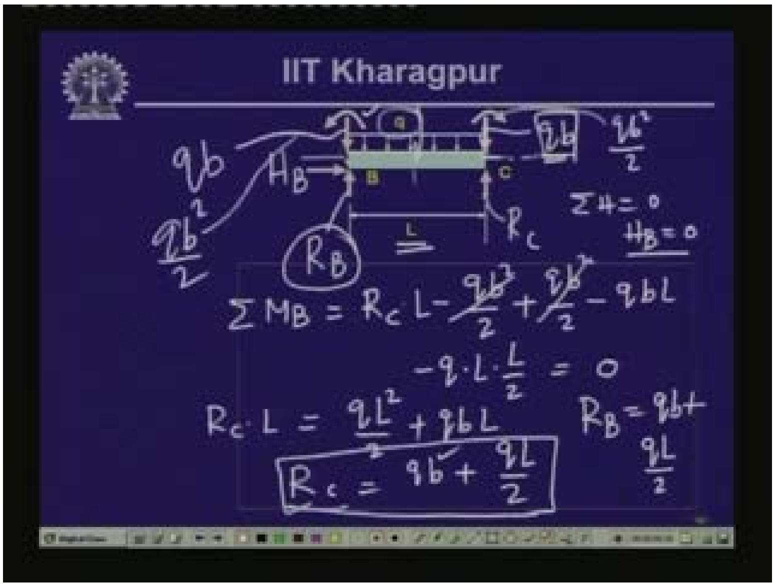

We have converted the effect of the overhang in terms of the vertical forces and the moment. The magnitude of this vertical force is equal to q (b) and the magnitude of this moment is qb square 2/2, qb (b) square/2 which is qb square/2. This is qb and this is qb square/ 2. Let us find out first the reactive values B and C. Let us take the moment of all the forces with respect to b.

We have this moment which is at c in a clockwise direction, -qb square/ 2. Then we have this moment which is in an anti-clockwise direction, + qb square/ 2. We have the moment of this reactive force from the overhang which is qb and this is going to cause a clockwise moment which is q multiplied by b multiplied by L and we have the uniformly distributed load which again causes a clockwise moment which is – q multiplied by L multiplied by L/2 which equals to 0.

This part gets cancelled and we have R_C (L) = qL square/2 + qbL. This gives us a value of R_C = qb + qL/2. You can readily evaluate the value of R_c also. This reactive force or the value qb acting at this support point which is the effect from the overhang gets transmitted directly to this support, which is qb. Over a length of L, we have a uniformly distributed load q per unit length. So, the total load is q multiplied by L and this being a symmetric half of the load goes here and half comes here, which is qL/2. So, we get this as the reactive force which is qb + qL /2.

Likewise, since we do not have any horizontal force, summation of horizontal force equals to 0 which will give us the value of Hb as 0. So, the value of R_B also from symmetry will be equal to qb + qL /2. We will have to find out the ratio between the length b to the length L in such a way that the load produces 0 bending moment at the mid span. If we compute the magnitude of the bending moment at mid span and equate that to 0, then we can find out the ratio between b and L.

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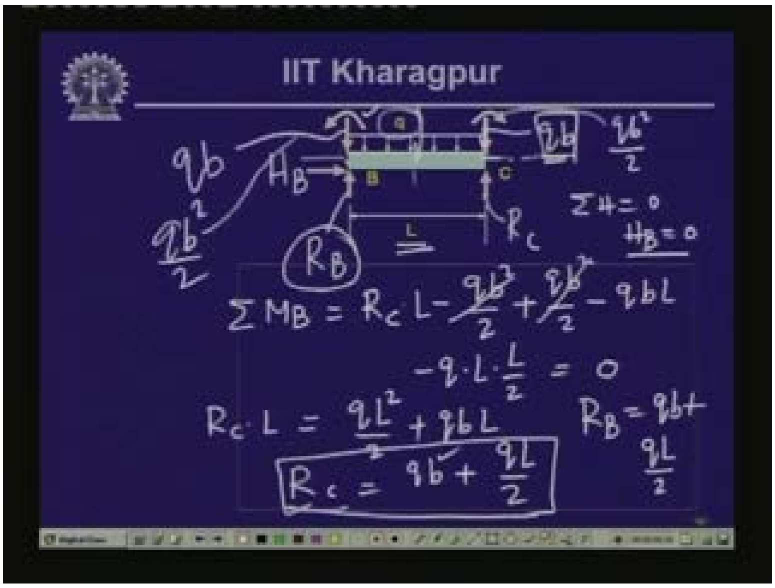
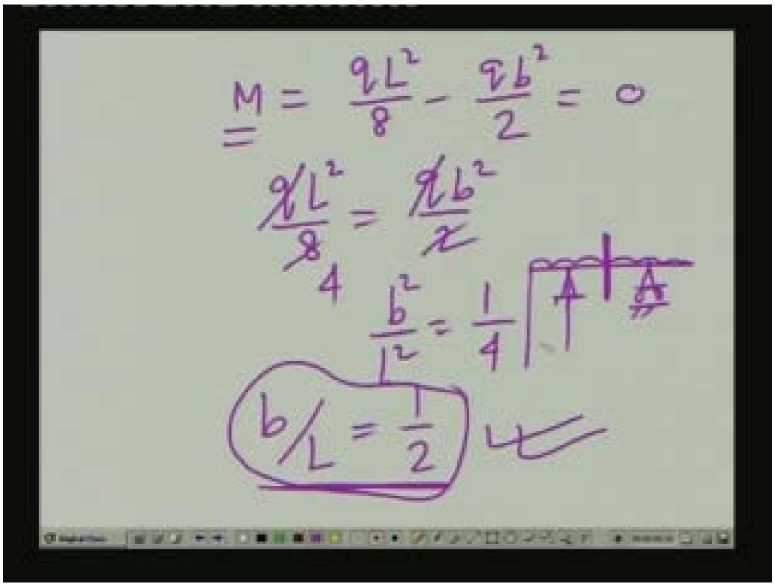


Let us compute the value of the bending moment. Let us keep in mind that R_B = qb + qL/2 and RC = qb + qL/2. Let us calculate the value of the bending moment at E and take the pre-body part of the left end of the beam. Here we have the vertical reactive force which is R_B and we are omitting H_B which is 0. We have the vertical force which is qb and we have an anti-clockwise moment which is qb square/2. We also have a distributed load, which is of intensity q, and with this being the mid span, its length is L/2.

On this particular card there is a stress resultant, which is the positive shear, which is V and the bending moment M, which is anti-clockwise. If we take the moment of all the forces with respect to this particular midpoint, then, we can write M; R_B causes a clockwise moment and we have M - R_B L/2. Here, qb causes an anti-clockwise moment and we have q (b) (L)/2 and the uniformly distributed load will also cause an anti-clockwise moment that is positive which is q multiplied by L /2 multiplied by L /4 which is half the distance.

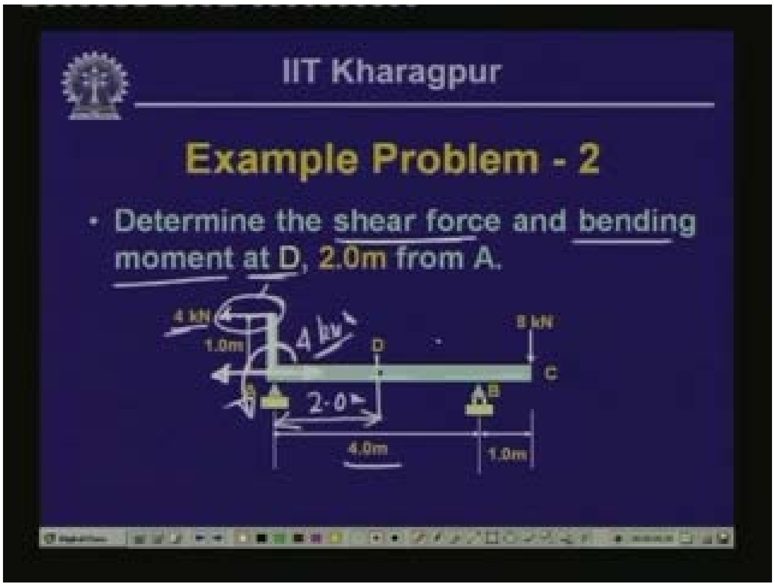
We have a moment which is in an anti-clockwise form, which is qL qb square/2. These are the moments which we have and we get 0. We have seen the value of R_B = qb + qL /2 and if we substitute that, then we get the value of M - R_B = qb + qL /2 multiplied by L /2.
Here, - qbL / 2 – qL square/ 4 + qbL /2 + qL square/8 + qb square/2 = 0 and the two terms get cancelled. So, - qL square/4 and + qL square/8 gives us – qL square/8. We get M – qL square / 8 + qb square/2 = 0 and hence, the moment value M = qL square/8 – qb square/ 2.

(Refer Slide Time: 39:06 - 40:12)



The value of the moment, m = qL square/8 – qb square/2 and since the moment has to be 0, we write qL square/8 = qb square/2. So, q and q get cancelled and we have 4 and we get b square/L square = 1/4 and b/L= 1/2. Let us suppose we maintain this particular ratio. This part hinged on a beam and this on a roller with an overhang of quantity b with a uniformly distributed load will produce a 0 bending moment over here. Then the value of this particular part has to be equal to the half of this length. If this is L then this has to be L/2 and if you have this distribution, then the moment at this particular point will be 0.

(Refer Slide Time: 40:15 - 42:26)



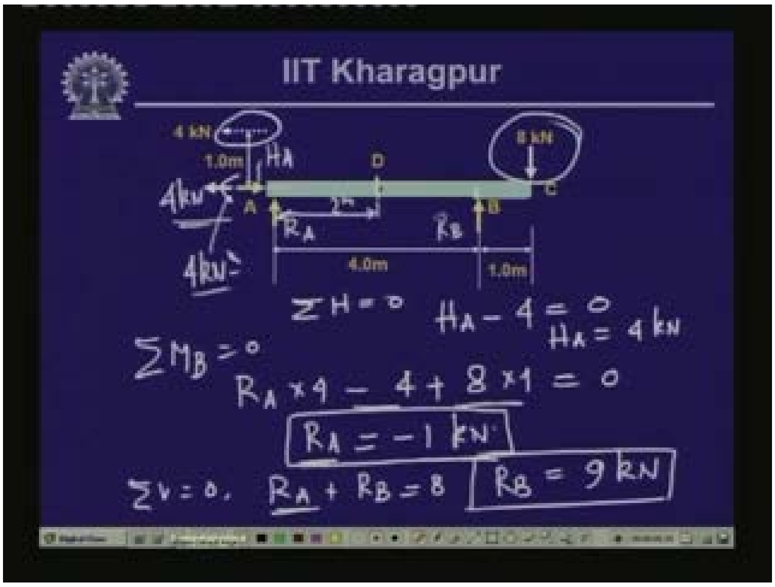
The second example determines the shear force and bending moment at d, which is 2 meters from ‘a’. This distance between ‘a’ and ‘b’ is 4 meters. At a distance of 2 meters, we have to find out the values of shear force and bending moment. Here, we have a load of 4kN at a distance of 1m vertically above this particular support point. We neglect the width and take up the centre of the beam and this particular force can be transferred to the centre of this beam.

Since we did not have any forces we need an opposite force. So, this force along with this forms a couple, which is anti-clockwise in nature. The magnitude of this moment will be equal to 4 multiplied by 1, which is 4kNm. The effect of this particular 4kN force, acting at a distance of 1m from support ‘a’, is equivalent to a 4 kN load at this particular point along with a moment of 4kNm.

Let us suppose we transform this beam by taking the effect of this load in terms of the load and the moment and remove the supports to have the reactive forces. For example; if we have the vertical and horizontal force at the hinged end and the vertical force at the roller end and if we take the free body of the whole beam, then we can find out the values of the reactive forces and thereby compute the shear force and the bending moment.

Let us take the free body of the whole beam. The 4kN load which was acting at a distance of 1m from the support is now transformed to this place where we have a horizontal force of magnitude 4kN and the moment is 4kNm because 4 multiplied by 1 will give you 4kNm as the moment. Now the reactive forces which act at A and B are $R_{A,}$ H_A, R_B and there is an overhang of 1m from b. We need to evaluate the bending moment and the shear force at this particular point d, which is at a distance of 2 meters from the left end.

(Refer Slide Time: 42:29 - 46:03)



Let us compute the values of the reactive forces R_A and R_B. The summation of horizontal forces is 0, H_A is in the positive x direction, - 4kN axial force is equal to 0 and H_A = 4kN. Let us take the moment of the forces with respect to this point b. Since 4kN and H_A act in the axial direction, they do not contribute to any moment at point b and hence we take it with respect to the centre of this beam. So, the moment contribution of R_A is R_A multiplied by 4, which is a clockwise moment.

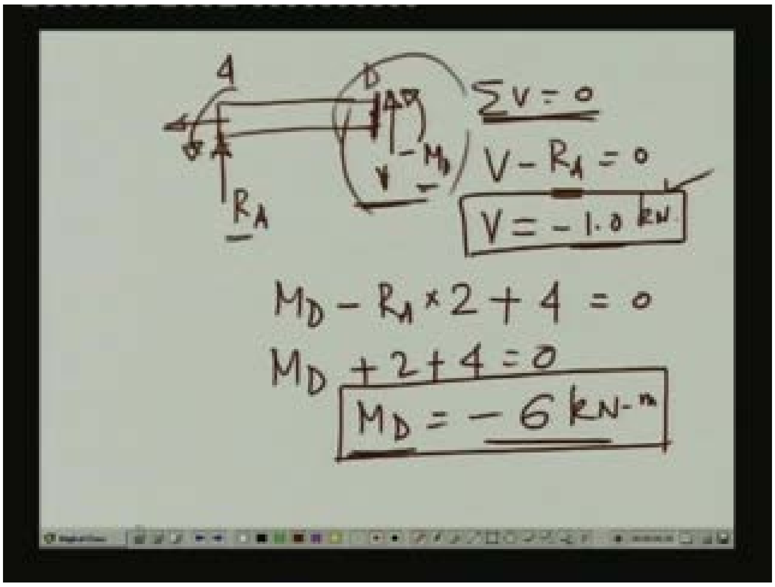
If it is positive, then we have an anti-clockwise moment which is 4kNm, which is – 4 and then, we have 8kN load which causes a moment over here which is again a clockwise moment, + 8 multiplied by 1. This is the total moment contribution of all the forces and this is 0. We have taken the summation of moment at b as 0. Now this gives us R_A multiplied by 4 - 4 + 8 which is + 4 = -1kN. So, the value of the reactive force at ‘a’ is -1kN.

If we take the other equilibrium where the summation of vertical force is 0, then we have R_A + R_B and the only vertical force which we have is 8kN = 8. We have obtained R_A as -1, so R_B = 8 - R_A which is -1 and we get 9kN. The reactive forces which we have are R_A = -1 and R_B = 9 kN. Let us calculate the bending moment at this point d. Let us take the free body diagram of this particular part from the left hand support and substitute all these forces so that we can evaluate the shear force and the bending moment.

Let us take the left part of the beam up to point d and on the right hand side the positive shear is V and the positive bending moment is M. Here, we have the reactive force which is R_A and here, we have the 4kN force and the bending moment 4kNm. If we take the vertical equilibrium of the forces, the summation of vertical forces which is equal to 0 will give us the value of the shear. So, V is positive upward and -R_A = 0. We have obtained R_A as —1 and V = -1.0kN, which is the shear force, at this particular point.

Let us take the moment of all the forces with respect to this particular point at the midpoint which is d and the moment at d is anti-clockwise. The contribution of the reactive forces, R_A, is in a clockwise direction. So, - R_A is multiplied by the distance which is 2M from point 'A'. We have this moment 4kNm, which is in an anti-clockwise direction, in line with M_D and we get 4kNm = 0.

(Refer Slide Time: 46:11 - 48:46)



We have obtained the value of R_A as -1 and we have M_D. So, -1 + 2 + 4 = 0 and moment at d is -6 kNm. If you look into the values of the shear force and the bending moment as we have obtained at this particular point, it shows us that V = -1kN and moment is equal to -6kNm. This indicates that the direction of the shear force, as we have assumed at this particular cross section, is on the opposite direction and hence, the magnitude of the shear at this particular point is 1kN, and the magnitude of the bending moment at this point is 6 kNm.

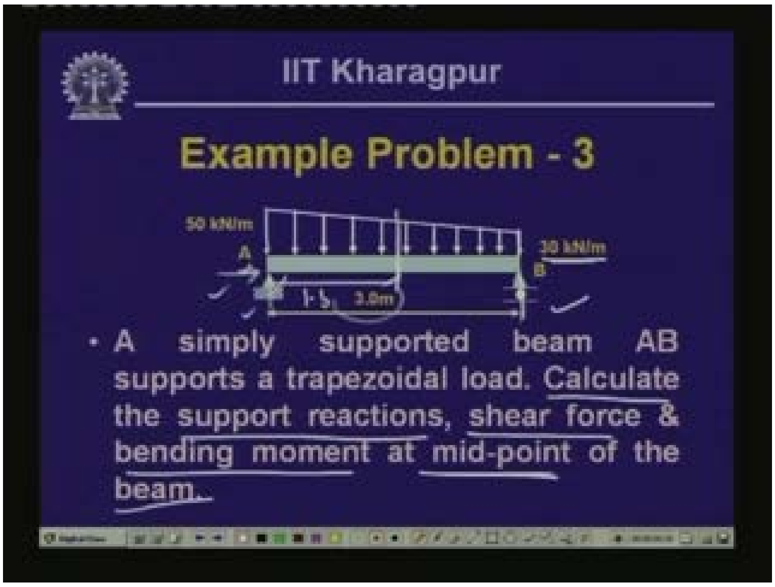
Let us look at another example problem where the load varies linearly, but as we had seen earlier, it is in the form of a trapezium that we have 30kN/m as the load, and here, we have a load of 50kNm and the length of the beam is 3m. This is point 'A' which is hinged and the hinged point will have a vertical reaction and a horizontal reaction and this is the ruler end where we have a vertical reaction only.

We need to calculate the support reactions, shear force and bending moment at the midpoint of the beam. So, we will have to find out the values of R_A, H_A, R_B and we will have to compute the value of the bending moment, which is at the centre of this beam, at a distance of 1.5m from the left support or the right support.

Let us look into the free body diagram of this particular beam. In the free body diagram let us call this particular reactive force as $R_{A,}$ let us call this as H_A and this reactive force as R_B; the intensity of the load at this point is 30 kN/m and the intensity of the load here is 50 kN/m. Let us first find out the reactive forces $R_{A,}$ R_B and $H_{A.}$

If we take the equilibrium of the forces where the summation of horizontal forces is 0, we get $H_A = 0$, as there are no horizontal forces acting in the beam. If we take the moment of all the forces with respect to point A, then we can compute the reactive force R_B.

(Refer Slide Time: 48:50 - 49:47)



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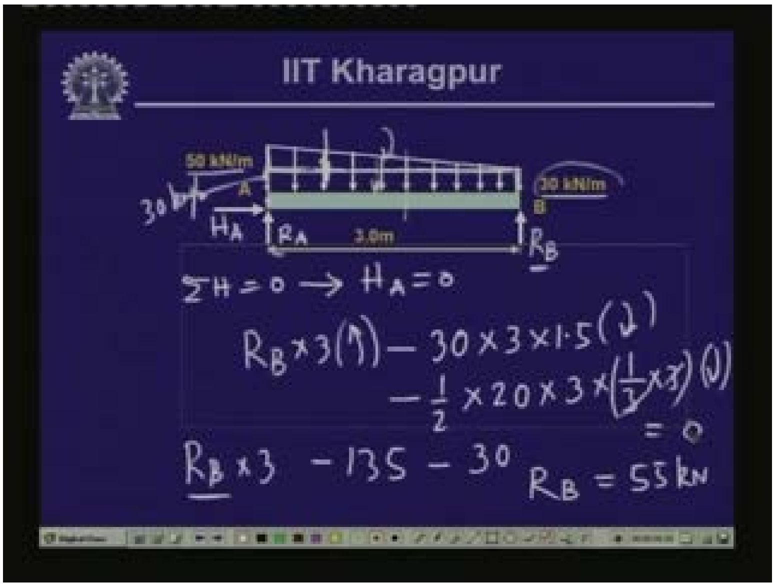
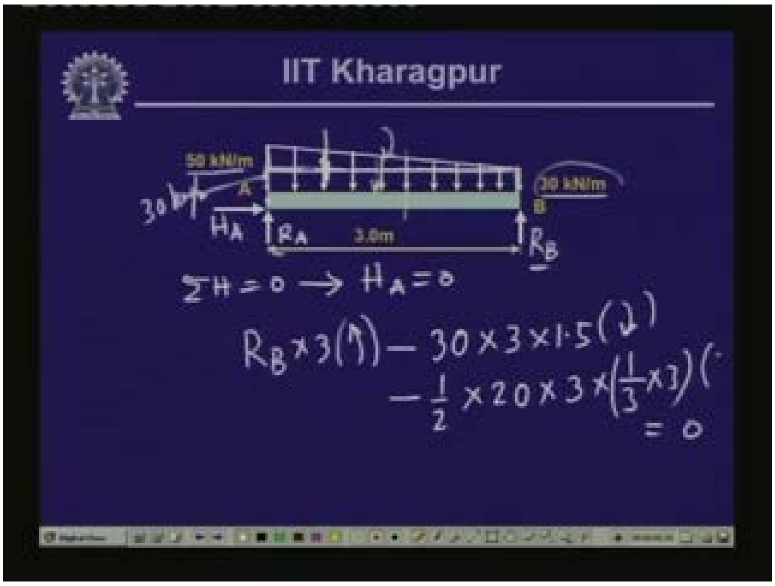


So, R_B multiplied by 3 is the moment with respect to A, which is in an anti-clockwise direction. Let us give the notation that is in the anti-clockwise direction. Then H_A R_A obviously will not have any contributions because we are taking the moment at this particular point. We can distribute the load part and the trapezoidal load in 2 parts.

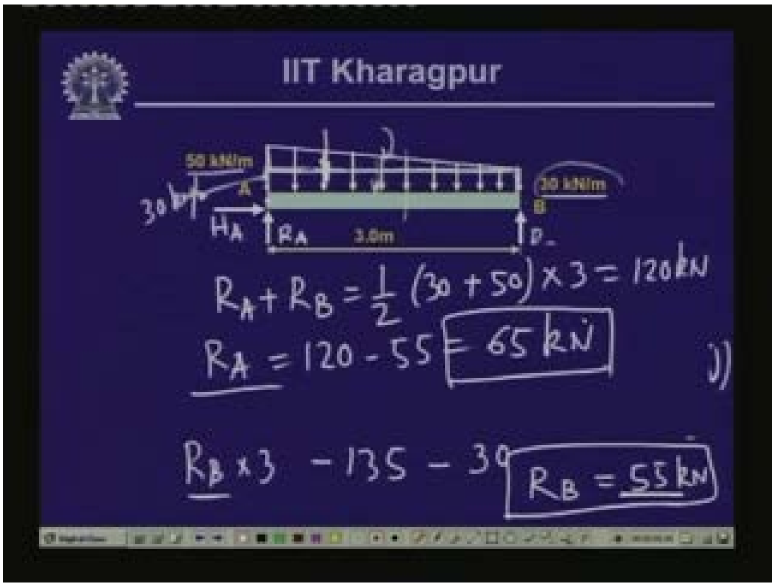
Let us say that this is of uniform intensity and this being 30kN/m, we get 20kN/m. We have two parts; one is the rectangular part which is 30kN and the other a triangular one, 0 to 20kN/m. If we take the moment of the uniformly distributed load over here, it causes a clockwise moment. This is negative and we get -30 (3) (1.5) which is the contribution due to the uniformly distributed load, which is 30kN/m. Here 30 multiplied by 3 multiplied by 3/2 which is 1.5 - 1/2 (20) (3) is the triangular load, acting at a distance of 1/3rd from here and so, the moment is 1/3 multiplied by 3 which is equal to 0. This moment (Refer Slide Time: 52:35) is also a clockwise moment and this moment also is a clockwise moment, and that is why they are negative.

(Refer Slide Time: 52:35)

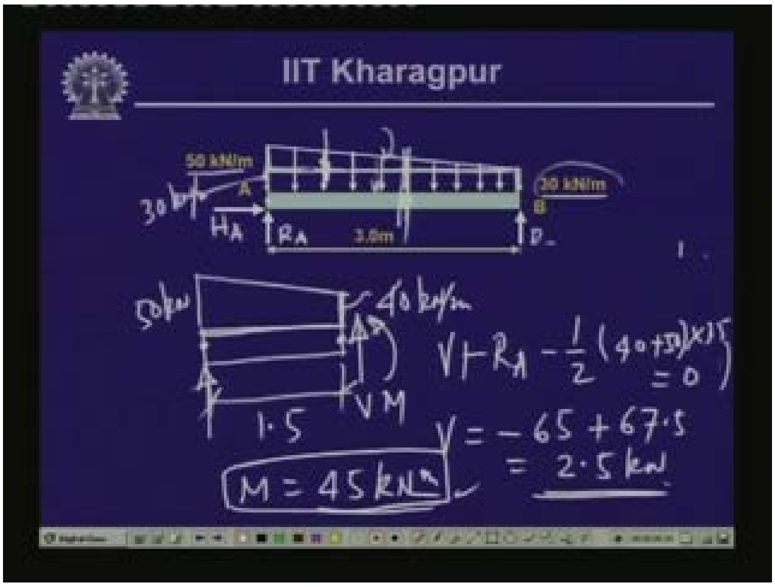


Here R_B multiplied by 3 minus this gives us a value of -135-30. From here we get a value of R_B = 165/3 = 55kN. Take the vertical equilibrium of the loads which is
$R_A + R_B$ = 1/2 (30+50) (3) = 40 (3) and this is 120 kN. This is the total vertical load that acts on this particular beam and, R_B being 50kN, R_A = 120 - 55 which is 65 kN. These are the reactive values of this beam where R_B is 55 kN, R_A is 65 kN and the value of H_A = 0.

(Refer Slide Time: 53:19 - 54:13)



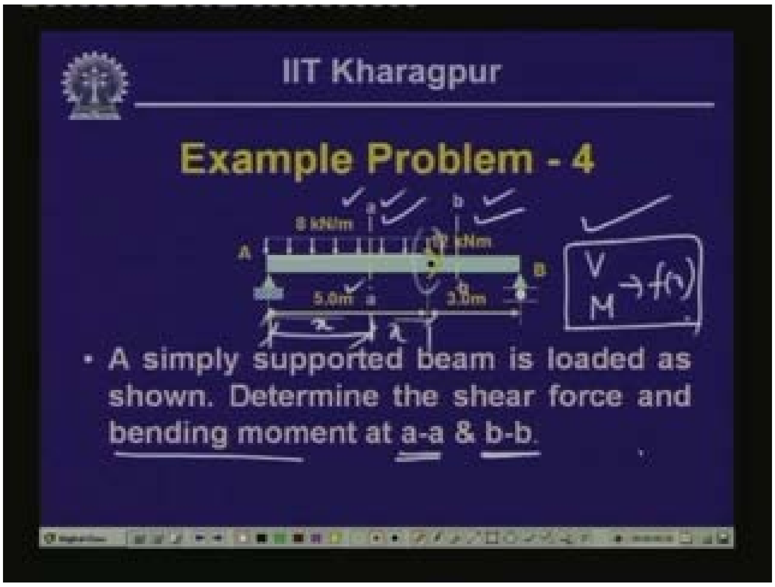
(Refer Slide Time: 54:15 - 55:58)



Let us take the free body of that particular part, so that you can evaluate the value of the bending moment and the shear force readily. Here, we have this particular beam, which is of distance 1.5m because this is at the midpoint. We have the R_A over here and here we have the trapezoidal variation. If you compute the value here, it comes as 40kN/m and this is 50kN/m. The positive shear is in this direction and the positive bending is in this form and this is V and M.

If you compute the value of shear,
$V + R_A - 1/2\ (40 + 50)\ (1.5) = 0$. The value of $R_A = 65$ kN; so this is -65 + 67.5 = 2.5 kN.
We can take the moment of all the forces and we get the moment as equal to 45 kNm. At the mid span because of this trapezoidal distributed load you have the shear force as equal to 2.5kN and moment as 45 kNm and both are positive.

(Refer Slide Time: 55:59 - 57:06)

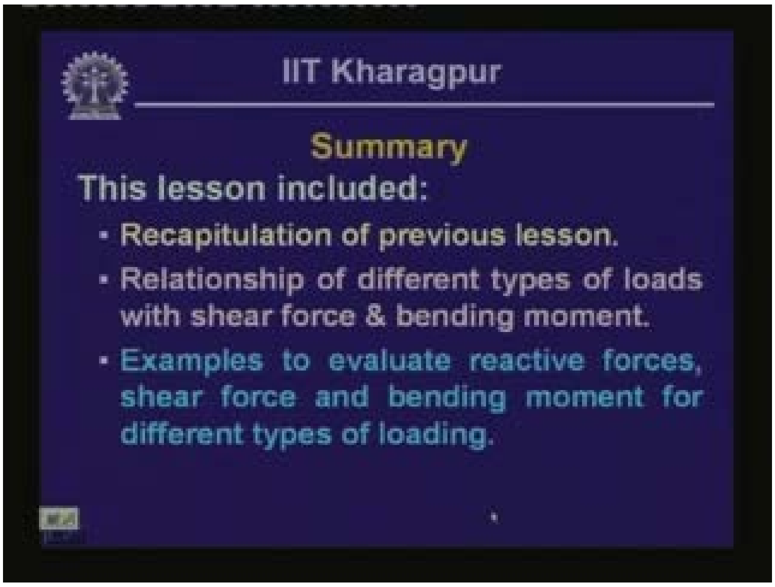


We have another example problem where a simply supported beam is loaded as shown. Determine the shear force and bending moment at the cross section 'a-a' and 'b-b'? This cross section 'a-a' and 'b-b', which are shown are not at any fixed points but we can assume that they are at a distance of x from 'a' and we can also call this as a distance x from a.

The idea behind this is that we can find out the effect of this concentrated moment or the effect of this uniformly distributed load, which is up to a distance of 5M and what happens to the shear force and the bending moment beyond 5M. If we compute the values again in the same form after computing the reactive values, we can write in a general form as the values of shear force and bending moment at the section 'a-a' and section 'b-b'. We can write down the values of V and M as a function of x. This problem is left for you to go through. Try to evaluate the values at 'a-a' and 'b-b' and we will discuss this problem in the next lesson.

In this lesson, we have recapitulated what we did in the previous lesson; we have looked into aspects that we discussed and the relationship of different types of loads with the shear force and bending moment. We have seen how the shear force and bending moment get related to the uniformly distributed load or the concentrated load, if you have a concentrated bending moment at any point along the length of the beam. Then we have looked into some examples to evaluate reactive forces, shear force and bending moment for different types of loading.

(Refer Slide Time: 57:13 - 57:46)



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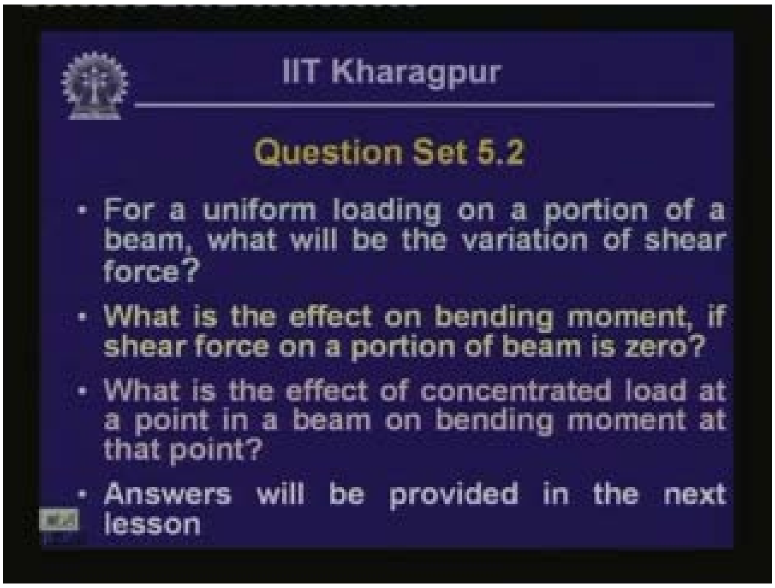

Some questions are set for you and they are the following:
For a uniform loading on a portion of a beam what will be the variation of shear force?
What is the effect on the bending moment if shear force on a portion of a beam is 0?
What is the effect of a concentrated load at a point in a beam on a bending moment?
We will give you the answers in the next lesson.