Strength of Materials
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Lecture # 20
Torsion - III

Welcome to the 3rd lesson of module 4 which is on torsion III.

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In fact in the last two lessons on torsion we looked into some aspects of torsion moment in a bar. Now we are going to look into some more aspects of torsion in the bar.

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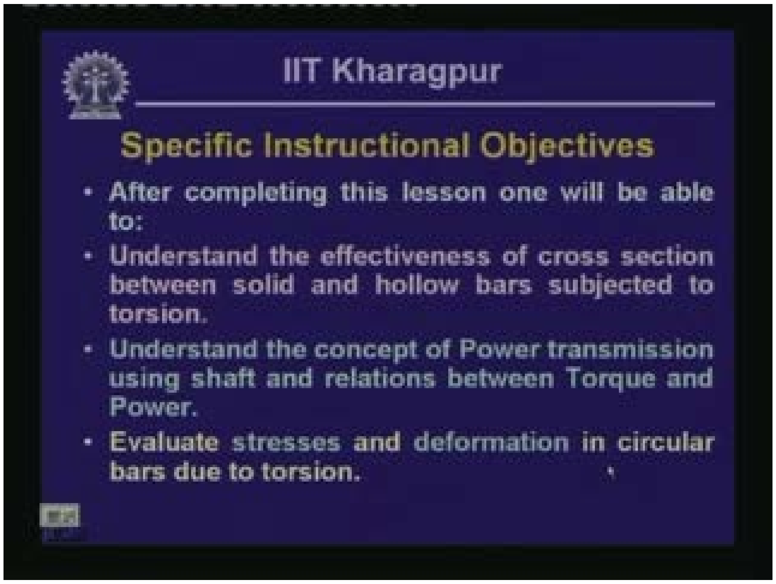

It is expected that once this particular lesson is completed one should be able to understand the effectiveness of cross section between solid and hollow bars subjective to torsion. In the last two lessons we discussed about the effect of torsion on a solid bar or a bar with a circular cross section that is a solid section and a bar which is having a cross section which is a hollow tube. In both the cases we have seen how stresses vary because of the twisting moment. Now let us see which section is effective; whether the solid one or the hollow one.

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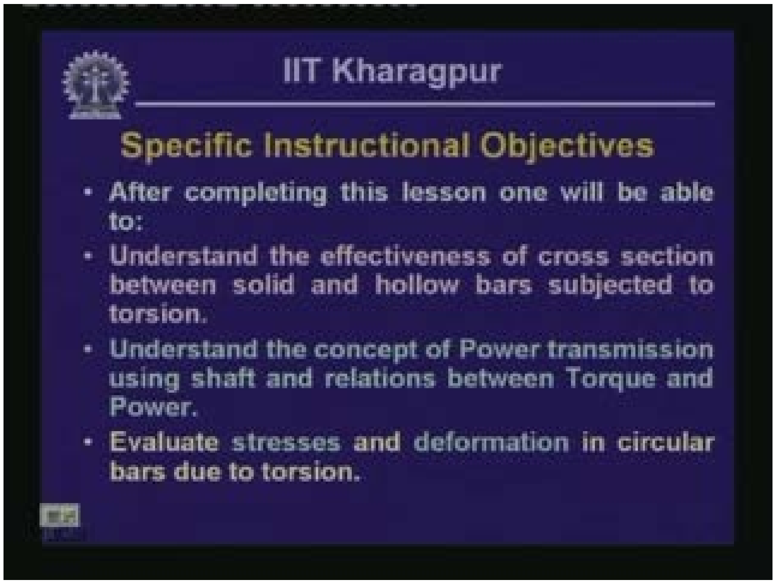

Also, one should be able to understand the concept of power transmission using shaft and relations between torque and power. Let us also look into the shaft or the bars which are used effectively in transmitting mechanical power and look into the aspects of how this power transmission is related to the torque.

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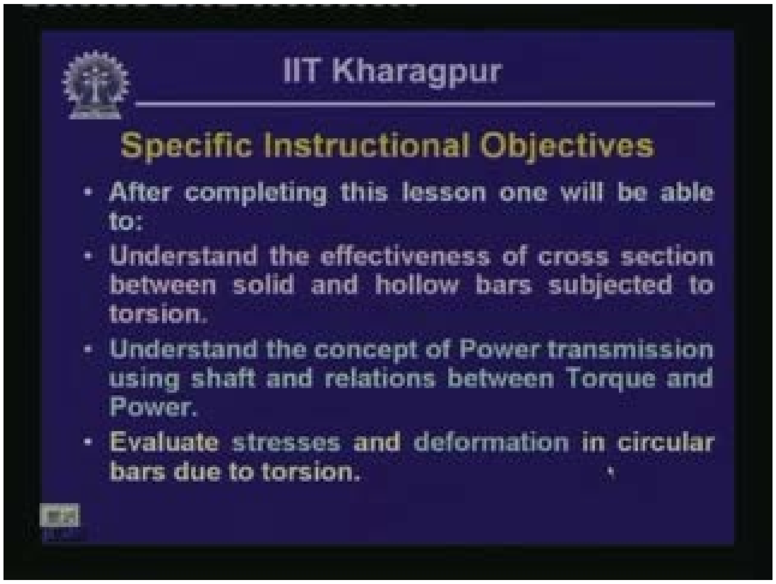

Also, one should be able to evaluate stresses and deformation in circular bars when they are subjected to twisting moment or the torsion.

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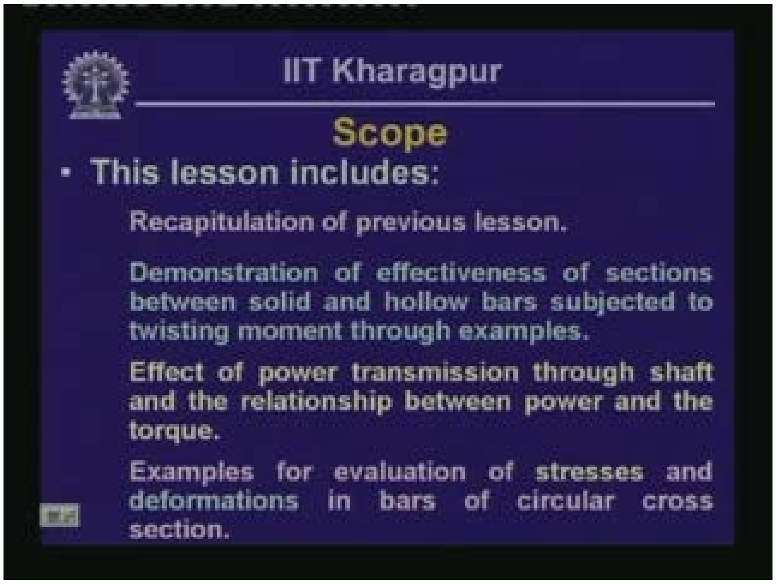

The scope of this particular lesson includes the recapitulation of the previous lesson where we will be looking into the question and answers and also we will demonstrate the effectiveness of sections between solid and hollow bars subjected to twisting moment which will be done through some examples.

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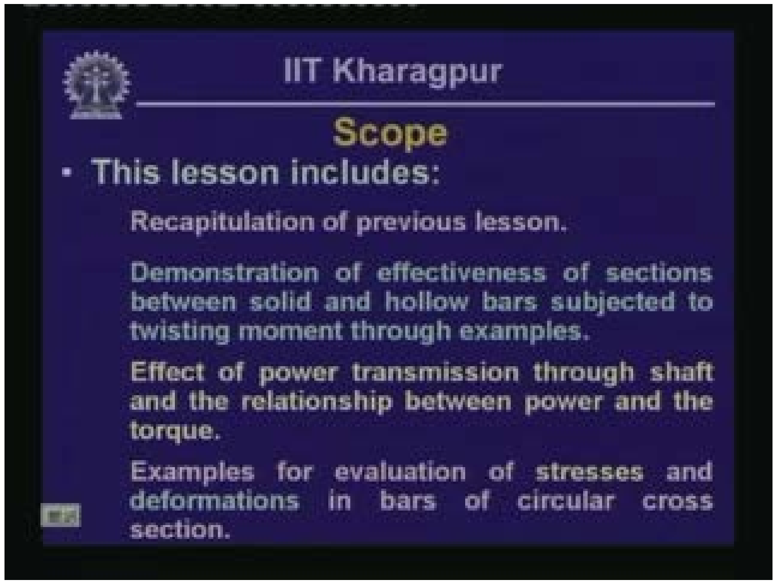

We will also look into the effect of power transmission through shaft and the relationship between power and the torque and examples for evaluation of stresses and deformation in bars of circular cross sections.

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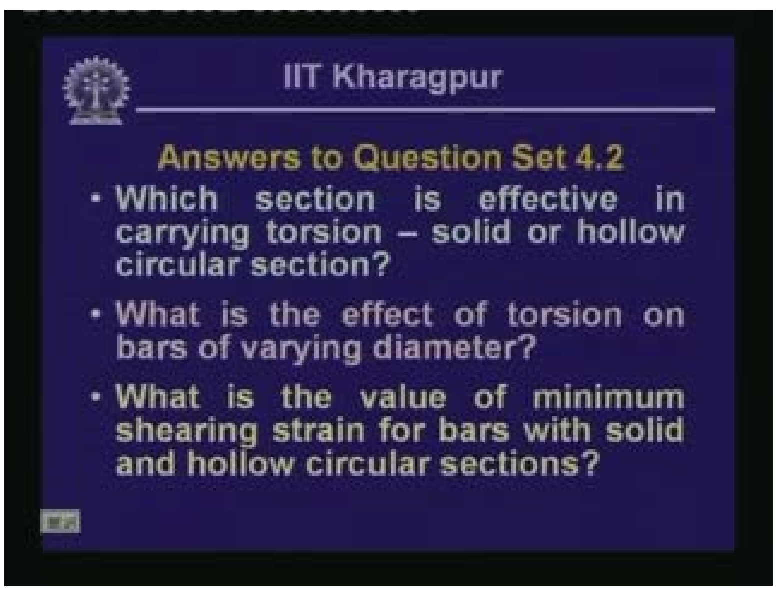

Let us look into the questions which were posed last time. The first question was which section is effective in carrying torsion. In the last two lessons we have discussed the effect of twisting moment on a bar which is having circular cross section but a solid bar. Subsequently, we have seen the effect of twisting moment in a bar which is a hollow tube and we could evaluate the stresses and the deformation. Now the question comes that out of these two, supposing if we have a bar which is subjected to may be the same twisting moment then whether a solid bar is more efficient than a tubular bar or it is the other way. Let us demonstrate that through few examples.

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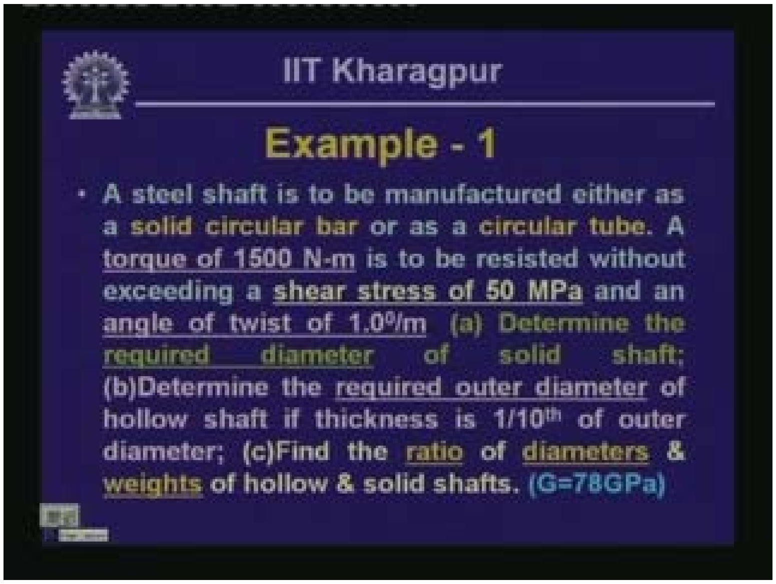

First example which I am going to demonstrate is that a steel shaft is to be manufactured either as a solid circular bar or as a circular tube. We will compare these two cases. We will consider a solid bar and a circular tube and a torque of 1500Nm is to be registered without exceeding a shear stress of 50 MPa and an angle of twist of 1 degree by m. Here we are going to consider that, if a bar is subjected to a twisting moment of 1500Nm and if the maximum stress in the material is not allowed to go beyond 50 MPa and the angle of twist cannot go beyond 1 degree by m and now both our criteria has to be satisfied then what could be the effectiveness of a solid bar which have a tubular section? This is what we are going to compare now.

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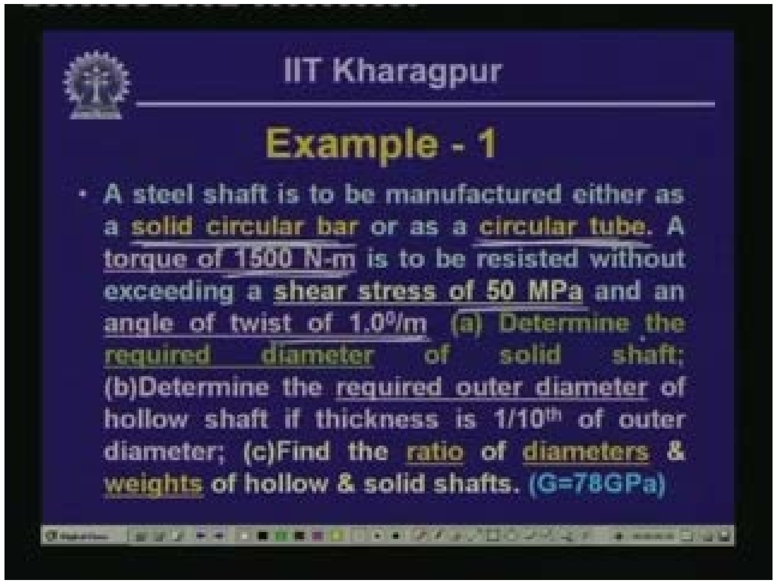

So what you need to determine is the diameter of the solid shaft in the first case, this is the part A. Part B) Determine the outer diameter of the hollow shaft if thickness is 1 by 10 of outer diameter. So the inner diameter is given in the process, and if the outer diameter is d the thickness is 1 by 10 of the outer diameter hence if we deduct twice the thickness we will get the inner diameter. The outer diameter minus twice the thickness of the hollow tube will give you the inner diameter. And also, you have to find out the ratio of diameters and weights. In fact this will give us a comparison between the hollow and the solid shaft.

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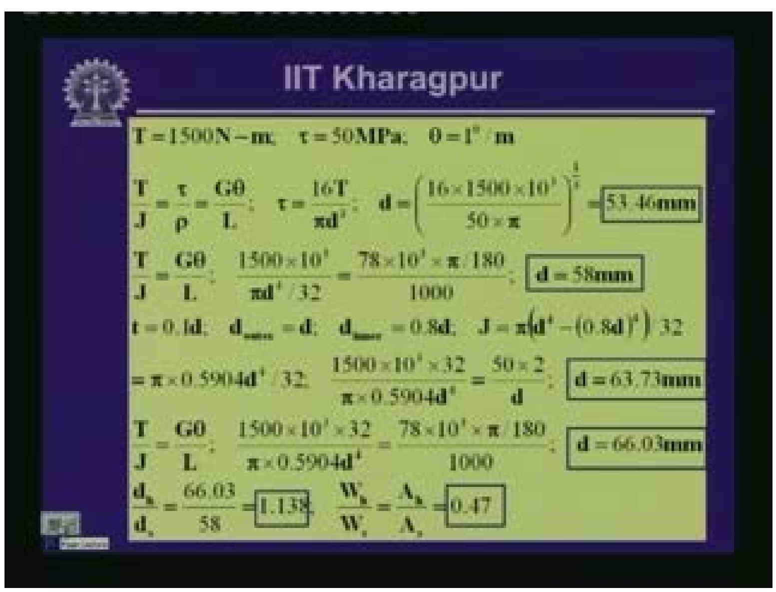

This is how it appears here. Twisting moment t is given as 1500Nm, the shear stress not to be exceeded beyond 50MPa and also the angle of twist is 1 degree by m. These are the limiting values and it is subjected to a twisting moment of 1500Nm. Now as we know the expression that $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ is equal to $\frac{G\theta}{L}$ is applicable for both circular shaft as well as the tubular shaft. Out of these if we take the first part of it that is $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ then we get the relationship.

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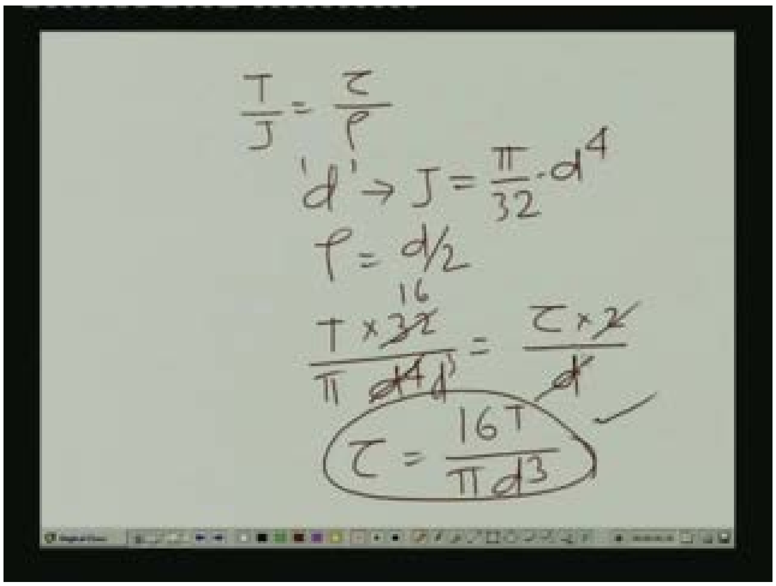

If we look into that $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ then for a solid shaft of diameter d the J is equal to $\frac{\pi}{32}d^4$ and $\rho = \frac{d}{2}$. So $\frac{T \times 32}{\pi d^4} = \frac{\tau \times 2}{d}$ so this d cancels with d cube so tau is equal to $\frac{16T}{\pi d^3}$ because these two cancels this 16 and this 16 over here. This is the value of tau as we have seen here.

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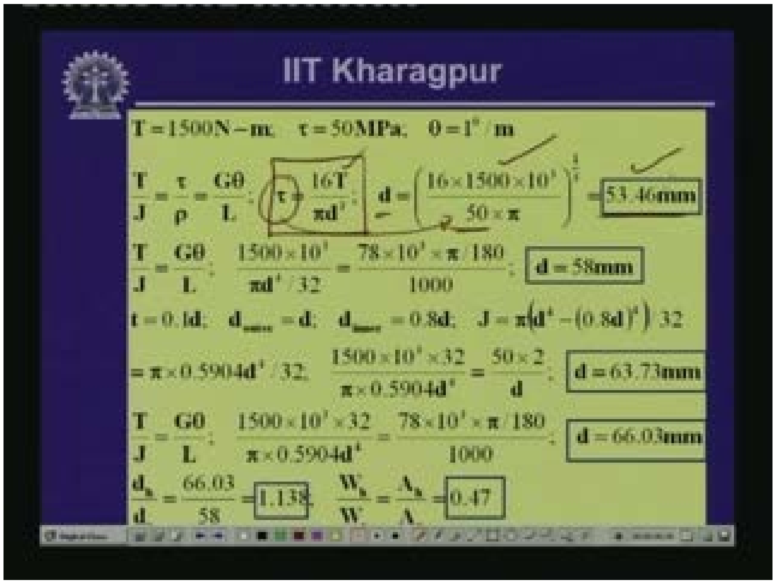

So tau is equal to $\frac{16T}{\pi d^3}$ and if we substitute the value of twisting moment as 1500 $\times$ 10 cube and then shearing stress d is limited to 50 as substituted here. So this to the power 1 by 3 gives us the value of d which comes out to be 53.46 mm. since we have to satisfy both the criteria that shear stress cannot exceed beyond 50 MPa and the angle of twist cannot be greater than 1 degrees by m length so if we consider 1m length of this shaft then if we go through this expression $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ so t again is 1500 into 10 cube Newton mm and J is equal to $\frac{\pi d^4}{32}$; g is given as 78 $\times$ 10^3 GPa which has to be converted into Mega Pascal into theta is equal to 1 degrees so 1 degrees is $\frac{\pi}{180}$ radian. This is what is used over here divided by 1000 is equal to L and this gives us the value of d 58 is equal to 58 mm.

Out of these two diameter if we look into that this much of diameter is necessary so that shear stress is kept within a limit of 50 MPa. This much of diameter is necessary for theta or the rotation to be 1 degree by m length. Now the question is you have to satisfy both the criteria, you will have to select the diameter in such a way that it satisfies both the criteria. If we select the lower diameter then it satisfies the stress criteria but it will fail in the rotation and if we take the higher diameter which is necessary for the twisting angle then it will satisfy both the criteria. Hence the higher diameter is to be selected to satisfy both the aspects. The selected diameter in this case will be 58 mm.

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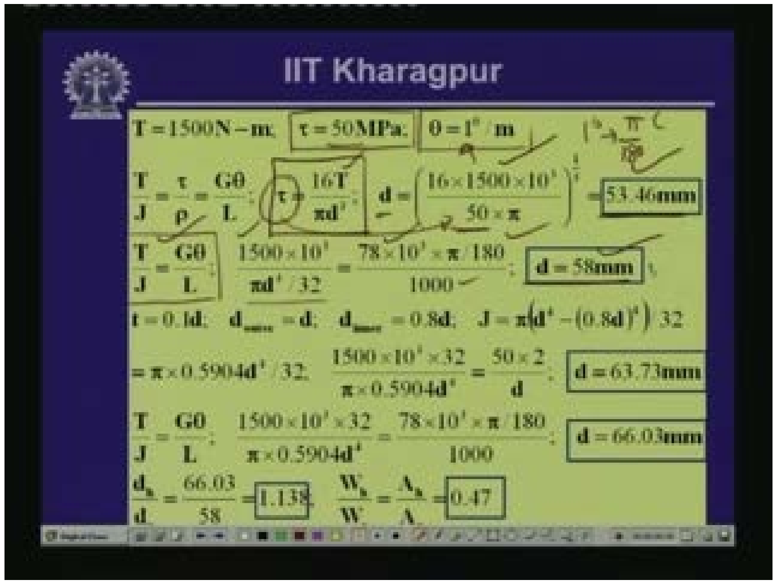

Secondly, if we look into the second aspect that when you have a hollow shaft the thickness of which is 1 by 10 of the outer diameter d, so thickness is 0.1 into d so the inner diameter is going to be 0.8 into d. Again the same expression should be applicable over here, the polar moment of inertia j is $\frac{\pi d^4}{32}$ and here d to the power 4 is d outer to the power 4 minus d inner to the power 4 and this comes out as pi $\times$ 0.5904 d to the power 4 by 32. So if you substitute again $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ this is d and this is J is equal to tau is equal to 50 and rho is equal to d by 2 and this gives us a diameter of 63.73 mm. This is from the criteria of the satisfaction of the stress. If you try to satisfy the rotational angle criteria, the angle of twist should not exceed 1 degrees by m length and then the twisting moment is 1500 and j and again we have calculated here pi x 5904 d power 4 by 32 and this is equal to g which is 78 GPa into 10 cube so much of MPa $\times$ $\frac{\pi}{180}$ radian is the rotation divided by 1000 and here the d comes out to be 66.03 mm.

Once again the diameter which is required from the satisfaction of the stress is 63.73 mm and the diameter which is necessary for satisfying the twisting angle is equal to 66.03. So higher of these two again will be governing which will satisfy both the rotation as well as the stress. Since the higher diameter will be satisfying the rotation as well as the stress so that is the diameter we will have to select, so the selected diameter in this particular case is 66.03.

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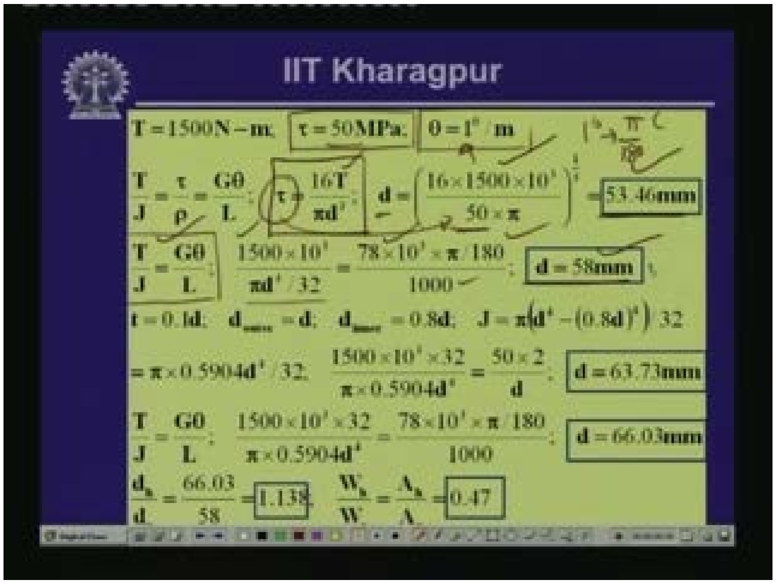

From this particular comparison we have a solid shaft which is having a diameter of 58 mm which will satisfy both the stress criteria as well as the rotational angle criteria and a tubular shaft which is having a diameter of 66.03 mm. Now if we try to take the ratio of these two the diameter of the hollow shaft to the diameter of the solid shaft then we find that the diameter of hollow is 66.03 and diameter of solid is 58 so the ratio gives us a value of 1.138. If we take the ratio of their width, the width is nothing but equal to the cross sectional area times the length times the density. Now the length and the density being common for both we take the ratio of the areas so ratio of the widths is equal to the ratio of the areas and that if we compute, the area of the hollow shaft is pi by 4 into d outer square minus d inner square divided by pi by 4 into d square as the solid shaft then we get a value of 0.47.

Please note that these two values that the ratio of the diameter is 1.138 and the ratio of the area is 0.47. This means the hollow area is equal to 0.47 times the solid area and diameter of the hollow bar is equal to 1.138 times the diameter of the solid bar. These comparisons state that the diameter in case of the hollow bar, to satisfy or to resists the 1500Nm of twisting moment, satisfying both the criteria of stress and the rotation angle, the outer diameter of the hollow bar will be 14% more than the diameter of the solid bar. Since we do not have any material in the core in the hollow bar naturally width wise there is a reduction and the ratio of the width between the hollow bar and the solid bar is almost ½.

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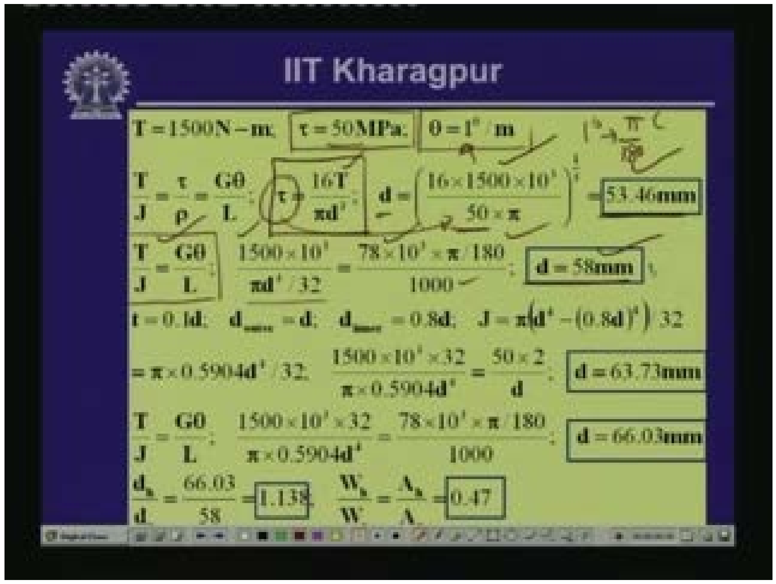

The weight of the hollow bar is 0.47 times the weight of the solid bar. So though we are utilizing 14% extra in the diameter we are gaining in the weight which is in the tune of half the weight of the solid bar. Evidently the hollow tube is more efficient than the solid shaft. The next problem explains it in more clear terms.

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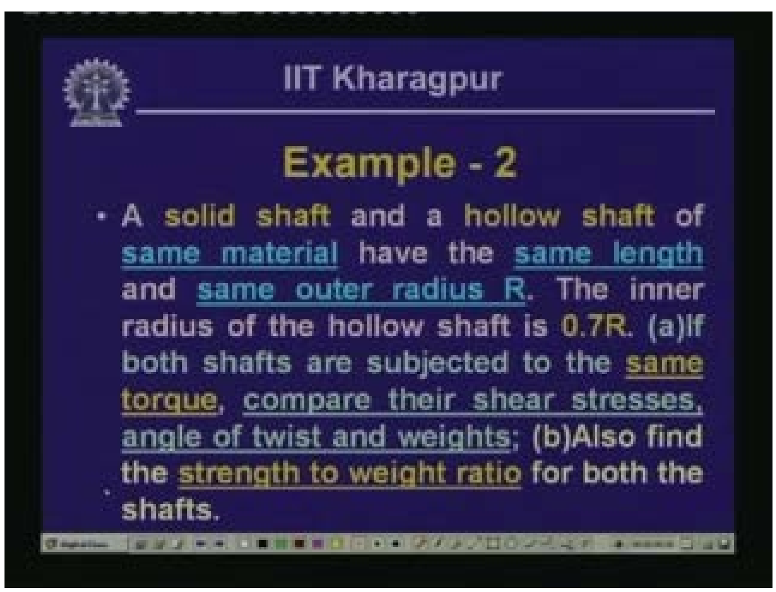

The next problem states that a solid shaft and a hollow shaft of the same material, please note that we have the same material, same length and same outer radius, these are identical as far as the material is concerned, and is identical as far as the external radius is concerned or the external diameter is concerned. Whatever be the diameter of the solid shaft will be d and the outer diameter of the tubular shaft also is d, so from external appearance they are identical but the only thing is that one is a solid shaft and the other is a hollow one.

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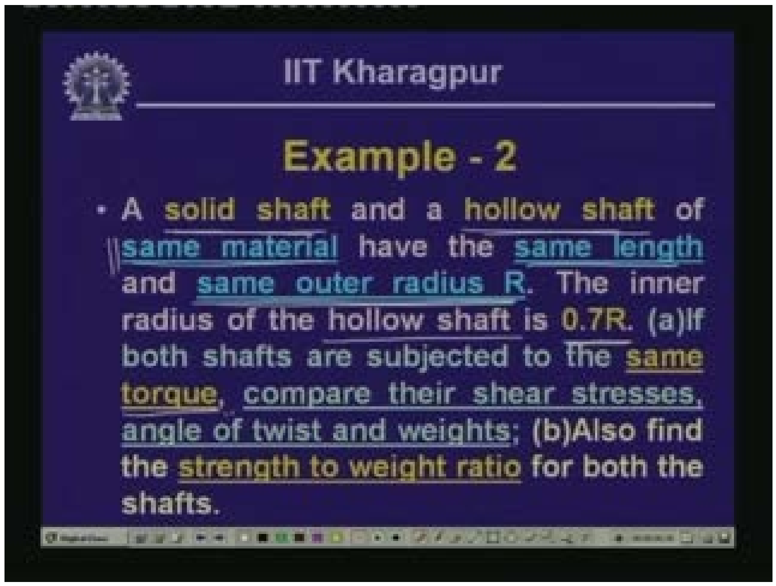

The inner radius of the hollow shaft is given as 0.7 into r. Now if both the hollow and the solid shafts are subjected to the same torque then compare their shear stresses, the angle of twist and weights. So three aspects are to be compared; their shear stress, the angle of twist and their weight. Subsequently, we will have to find out the strength to weight ratio for both the shaft. This is one important aspect to find out the strength to the weight ratio. The term strength, we understand that, if we take a bar or a shaft which is subjected to a twisting moment now how much resistance it has to take that twisting moment that is what strength is. And weight is the cross sectional area multiplied by the length multiplied by its density. Now, strength-wise we calculate how much twisting moment it can resist to its weight and that is the strength to weight ratio. This is what we will compare for both the hollow tube as well as for the solid shaft.

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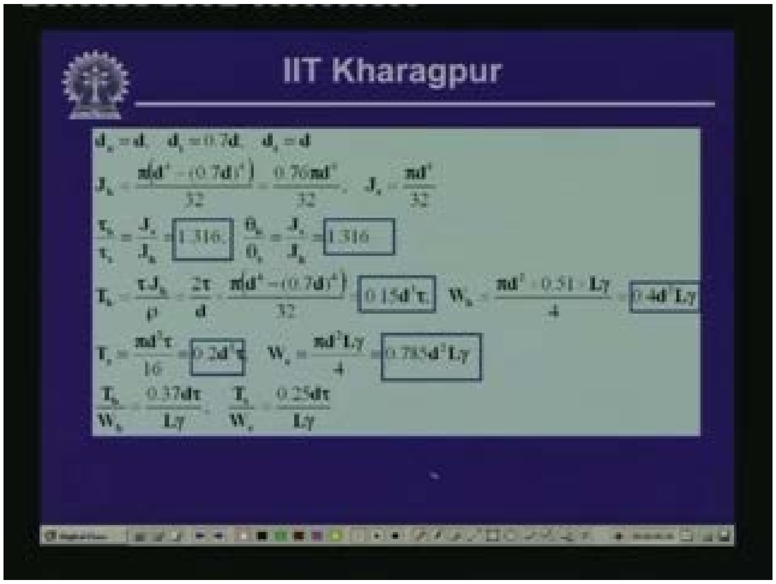

If we look into the calculations of this the outer diameter for the hollow tube is given as d the inner diameter is 0.07 into d given in the form of radius and we are writing in the form of diameter and the solid diameter is also d and suffix s stands for solid and suffix h stands for the hollow. If this is the external and internal diameter for the hollow one then the polar moment of inertia for the hollow one will be pi by 32 into d outer to the power 4 minus d inner to the power 4 and d inner is equal to 0.7 into d so this gives us a value of 0.76 into $\frac{\pi d^4}{32}$. This is the value of the polar moment inertia for the hollow section. Likewise the polar moment of inertia for the solid bar is $\frac{\pi d^4}{32}$. Now if we compare their shear stresses, the shear stress in the hollow to the shear stress to the solid, now as we know the $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ so tau the shear stress is equal to $\frac{T\rho}{J}$.

Now if we say tau for hollow the twisting moment is the same, also diameter rho is the same for hollow and the solid one. So, if we keep the rho parameter same for both then we have 1 by J this is J for hollow and likewise τ_S will be τ_ρ which is constant for both 1 by J_s. So τ_h by τ_s will be J for the solid divided by J for the hollow and this is what has been done over here, and if you compute this to take the ratios of js to jh we will find that this value comes to 1.316. Again if we take the ratios of the angle of rotations which is theta hollow to θ_s and this is again $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ hence theta is equal to $\frac{TL}{GJ}$. All other parameters TL and G are common for both shafts; hollow as well as for the solid.

The parameter which is varying is J so theta$_h$ by theta$_s$ is equal to 1 by J_h into J_s by 1. Again this is J_s to J_h. So if you compute that then it is 1.316. So both the ratios of the shear stress as well as the angle of twist are dependent on the polar moment of the inertia of the shaft and the value corresponding to that is 1.316 in both the cases because both shear stress and the angle of twist depends on the ratio of the polar moment of inertia.

Now if we compute the twisting moment, that is because we have to compute the strength to weight ratio, we can compute the value of the twisting moment that can be resisted by the hollow section and the twisting moment that can be resisted by the solid section. The twisting moment that can be resisted by the hollow section is equal to $\frac{\tau J}{\rho}$ this is from this particular expression $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ and this is $\frac{\tau J}{\rho}$ so rho is equal to d by 2 and J_h is pi by 32 d to the power 4 minus 0.7d to the power 4 the inner diameter so this gives us the value of 0.15 d cube tau. And the weight of the hollow bar is equal to the cross sectional area pi d square by 4 times the length. Since this is the hollow bar our cross sectional area will be equal to pi by 4 into d_o square minus d_i square and this gives us the value of 0.51 pi d square by 4 because d_i is equal to 0.7 into d and the length times the unit weight gamma so this gives us a value of 0.4d square by L_{gamma} this is the weight. Subsequently, the twisting moment that can be carried by the solid shaft is equal to; and again $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ so from that we get pi d cube tau by 16 is equal to 0.2 into d cube by tau$_d$ into tau and the weight of the solid shaft is equal to pi d square by 4 is the cross sectional area times length times the unit weight which comes as 0.785d square L_{gamma}.

Now if we take the ratios that $\frac{T_h}{W_h}$ that means twisting moment that can be carried by the hollow shaft divided by the weight of the hollow shaft is equal to $\frac{0.37d\tau}{L\Upsilon}$ and twisting moment that can be carried by the solid shaft to its weight is equal to 0.25 x$\frac{d\tau}{L\Upsilon}$. Now $\frac{d\tau}{L\Upsilon}$ this particular parameter is constant for both the shafts, they are identical because it is subjected to same amount of stress for the same length for the same unit wide and having the same outer diameter. Hence the values the strength to weight ratio is more in the case of tubular section than the solid shaft. But how much is the difference? If we look into the difference it is 0.37 minus 0.25 by 0.25 into 100 so this is 48%. As we can see, in case of hollow shaft we have 48% more strength to weight ratio.

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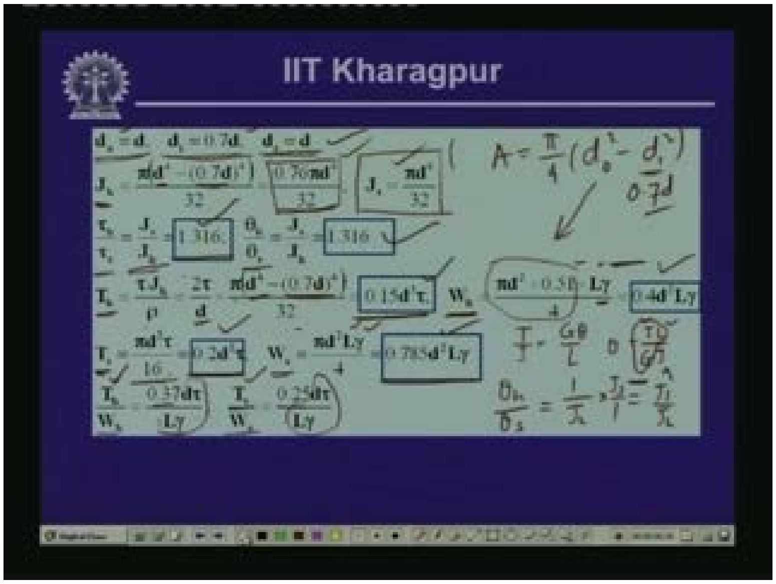

These two examples demonstrate clearly that tubular shaft is more efficient than a solid shaft. And as we can make out, from the center as we go away towards the radius we have maximum stress as well as the contribution of the material in terms of the polar moment of inertia is more in case of tubular section than the solid one because in the solid one the core part does not contribute much. So if we take off the particular material eventually the hollow section becomes more efficient. In fact if we go for thinner tube that means if the tubular section has lesser thickness then we will see that it will be more efficient in resisting the twisting moment.

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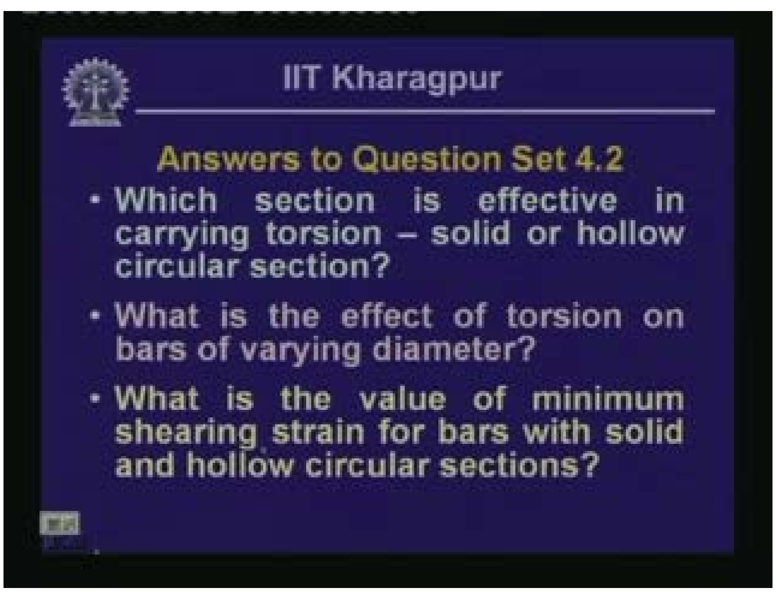

What is the effect of torsion on bars of varying diameter?

So long we were looking into, if we have a bar of uniform diameter and subjected to twisting moment then what is the effect of such twisting moment on the bars, on the stresses and the deformation. Also we have looked into that, if we have a step shaft if we have a varying a diameter then what will be the consequences. If we have a gradually varying diameter of a shaft then what will be the consequences if it is subjected to a twisting moment.

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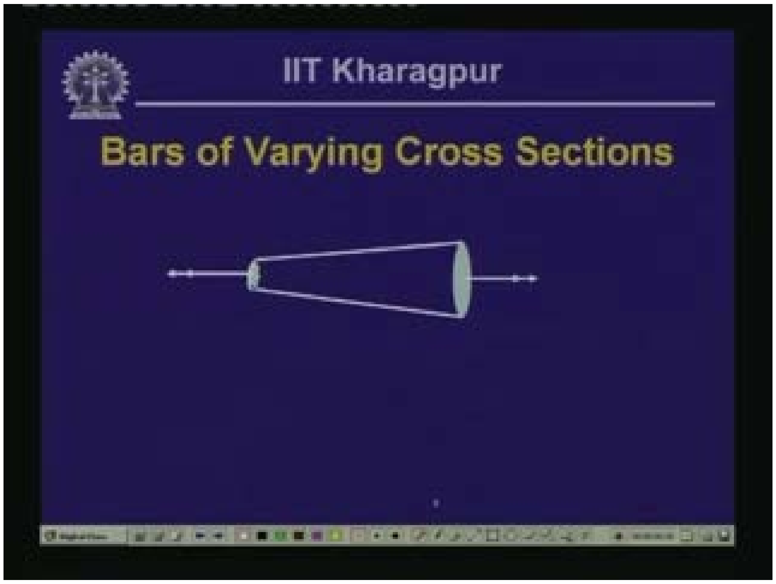

If we take a bar which is gradually varying, let us say we have a diameter here as d_1 and diameter here as d_2 and it is subjected to a positive twisting moment t. As we know that $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ so from this if we compute the shearing stress tau this is $\frac{T\rho}{J}$ and this in terms of diameter is 16T by pi d cube. Now this shows that the shearing stress tau will be higher if d is lower. So for a lowest diameter we will get the highest stress in such a kind of bar.

If this particular bar is subjected to a uniform twisting moment then the maximum stress will occur where the diameter is the least. In fact we do not have to bother for anything else. So wherever we have the least diameter corresponding to that if you compute the stress that will be the maximum possible stress in a bar of varying diameter. Now if we like to compute the value of the theta the rotation, let us say if we take a section which is at a distance of x and let us say this small length is dx then the change in angle in this small strip if we call this as dtheta then as we know $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ or for this small segment which is of length of dx and the rotation angle is dtheta this is equal to $\frac{T.dx}{G.J}$.

Here J is the parameter which is the function of the diameter and the diameter in this particular case is varying. Now if this is diameter d_1 and this is diameter d_2 and add this particular point if you say this diameter is d_x and the value of d suffix x will be if we draw this length. This is also d_1 so this balance length is d_2 minus d_1 by 2. So the length d_x is equal to d_1 plus d_2 minus d_1 by 2 into x by L, if l is the length of this bar. Since we will have to add this side plus this side so twice of that, so this 2 and this 2 will get cancelled so this is equal to d_1 plus d_2 minus d_1 into x by L. So this is the diameter d_x and correspondingly the value of polar moment of inertia will be pi by $32d_x$ to the power 4.

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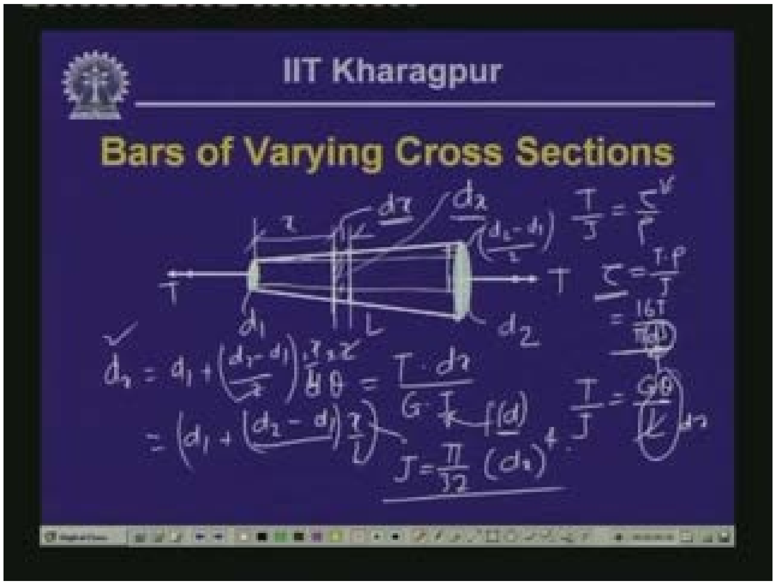

Now this is the function of x so if we like to calculate the theta now we have got the expression dtheta is equal to $\frac{T.dx}{G.J}$. If we like to compute the value of theta which is over the entire length so ∫ 0 to Ldtheta will give us the value of theta is equal to ∫ 0 to L $\frac{T.dx}{G}$. Let me write J as a function of x and J we have computed in terms of d and d is a function of x so J also is a function of x. We can compute the value of theta, if we evaluate this ∫ we can get the value of theta. So, for a bar of varying diameter we can compute the stress for the least diameter and we can compute the angle of rotation theta from this particular expression. Instead of having constant twisting moment in the bar if we have the varying twisting moment, if we have twisting moment which is non-uniform then between two such twisting moments we take the free body, find out how much is the twisting moment and in that particular segment again we compute the stress and calculate the maximum of the stresses in these zones corresponding to the twisting moments. This maximum will give us the maximum possible shear stress in the whole of the bar.

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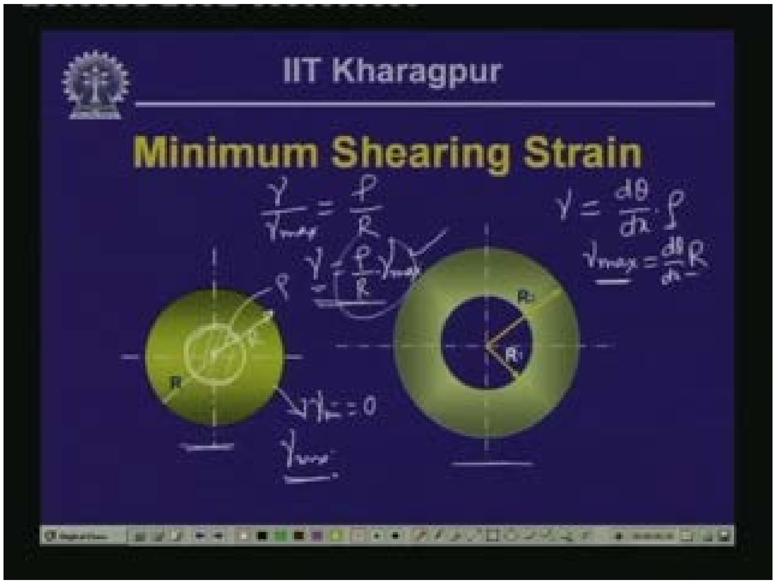

The third question was; what is the value of minimum shearing strain for solid shaft as well as for the tubular shaft?

We have seen the value of the shearing strength gamma is equal to dtheta d_x rho where rho is the radius of a segment which is taken out from the solid shaft. Now this shearing strain becomes maximum when it reaches to the outer periphery equal to r so gamma_{max} is equal to $\frac{d\theta}{dx}R$ and when rho becomes r then it becomes gamma_{max}. We can write that $\frac{\gamma}{\gamma \max}=\frac{\rho}{R}$ or gamma is equal to $\frac{\rho}{R}$. gamma_{max}. From this expression you can see that when rho is equal to r the outer one, r by r is one so gamma is gamma_{max} and when rho is equal to 0 then gamma is equal to 0. So in case of solid shaft the minimum strain gamma min is equal to 0 and gamma_{max} we can compute from this particular expression once we know the stress which is occurring at the outer periphery. In case of tubular shaft, here gamma again in terms of this if we write as rho at any radius divided by the outer radius here is R_2 so gamma $\frac{\rho}{R_2}$. gamma_{max}. Now when rho becomes r_2 then the value of the strain is the gamma_{max} which is on the outer periphery. So gamma_{max} in this particular case is equal to gamma_{max} at r_2 which is at the outer periphery. Since we do not have any material at the core the material starts at a distance of $r_1$from the center so the gamma minimum value in this particular case where rho is equal to r_1 by r_2 0.gamma_{max}. So in terms of the maximum strain of the periphery we can compute the strain at this level which is r_1 in their ratio of the radius. So $\frac{R_1}{R_2}.\gamma \max$ is the minimum value of the gamma in case of tubular shaft. This is the difference between the strain value in the hollow shaft and the strain in the solid shaft. In case of solid shaft the minimum value of the strain is 0 which is at the center of the shaft and its maximum at the outer periphery.

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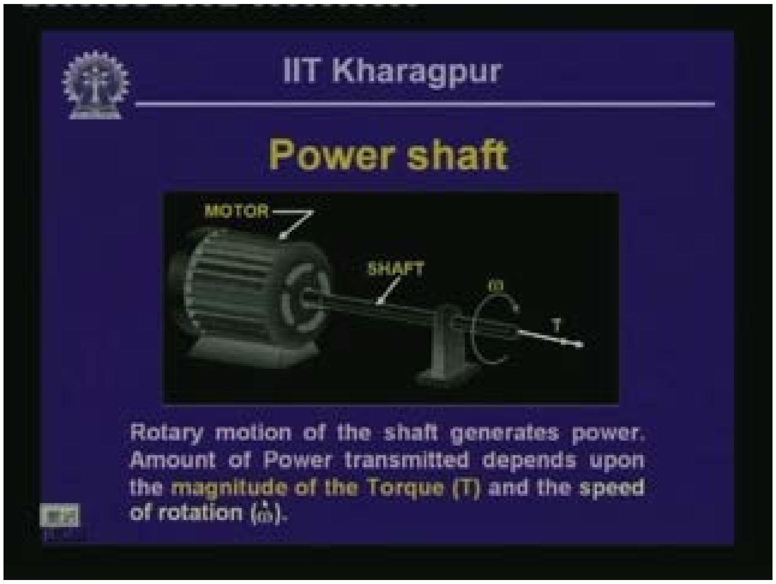

In case of the tubular one the minimum strain is at the inner periphery and the maximum strain is at the outer periphery. Having looked into the aspects, now we know that the tubular sections are more effective. One of the applications of these shafts is in transmitting the mechanical power from one device to another. The shafts are extensively used for transmitting mechanical power from one device to the other. If a shaft or a bar is rotated by a motor then the shaft moves and it transmits the shaft to another system so it undergoes rotation.

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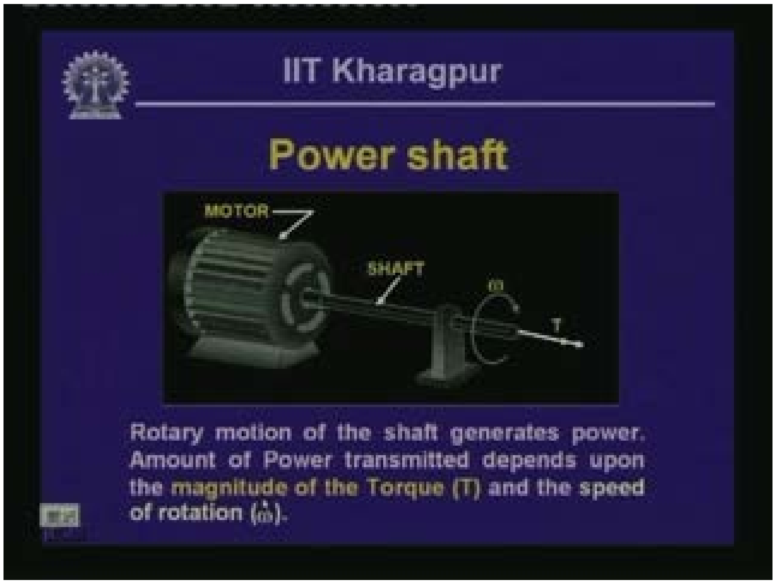

There is a rotational speed which is omega and it transfers a twisting moment which is t. The twisting moment which is transferred in the other device from this is exactly in the same direction as this angular rotation. That device gives a resistive twisting moment which is in the positive direction as shown here. This particular direction is the positive twisting moment which is acting in the transmitter shaft.

Now this particular shaft is transmitting the twisting moment to the other shaft and as a resisting moment it is acting in the positive direction as demonstrated over here. This rotary motion of the shaft generates power and this amount of power that is being transmitted through rotation of the shaft depends on the magnitude of the torque that is applied and the angular speed of rotation which we called here as omega. So it depends on the magnitude of the torque and the speed of rotation. In fact it is the product of the torque and the speed of the rotation.

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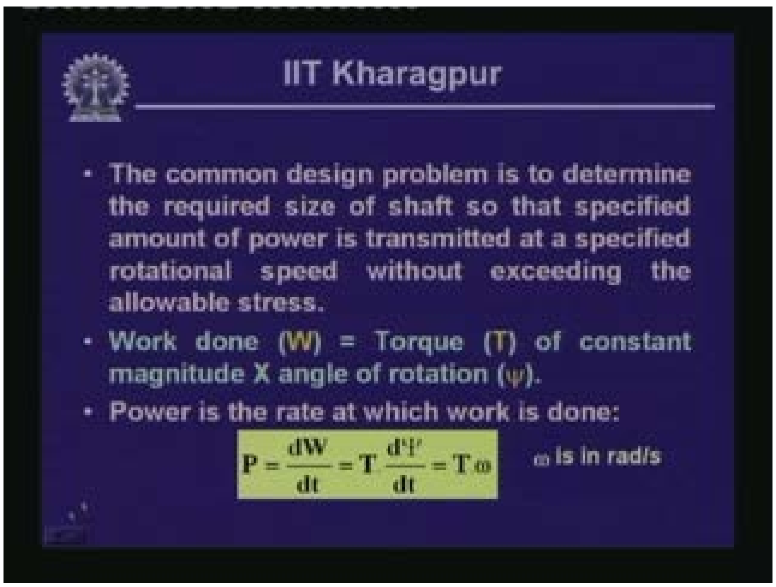

In such shafts the main problem which is essential to be determined is the size of shaft so that it can transmit the power at a specified rotational speed without exceeding the allowable stress. This is very important. From the design point of view we need to find out what should be the diameter of this particular shaft so that it can transmit the requisite power as we desire and a particular rotational speed without exceeding the stress level in this particular material. Hence we need to find out what should be the diameter of these kinds of shafts. Since it involved that amount of torque and a constant magnitude, so when it rotates the work done by rotating this shaft under angular speed omega is equal to the torque multiplied by this rotation. So the work done omega is equal to ψ the torque multiplied by the angle of rotation so t. ψ is equal to omega the work done.

Power is generally the rate at which this work is done so p is equal to dW dt which gives us, and t being a constant magnitude so dW dt gives us omega if you substitute as tψ, t is the time so this is equal to T which is a constant twist times d ψ d t and t again is with respect to time. And dψ dt is nothing but is a change of twist angle per unit time which is the angular speed and that we have defined as omega which is generally defined in terms of radians per second. So power p is equal to t into ω.

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Here are the units that we use for this system. Power generally is given in Watt and torque is in Nm; angular speed is radian per second. What is 1Nm by sec?
That is the unit of the power.

What is the unit of the power and Newton meter as we have seen is the unit for the torque and angular speed is defined in terms of radian per second?
Many a times we defined the angular speed of the shaft which is the transmitting power in terms of the frequency that how many number of revolutions it has. So, one revolution per second is called as the frequency or Hz, we call this as cycles per second. One cycle moves over 360 degrees is equal to 2pi radian. We write this angular speed omega is equal to 2pi f radian per seconds, so f is cycle per second or one revolution per second. This gives one revolution is equal to 2pi fT radian, hence if we substitute for omega we get the expression for power p is equal to 2pi fT and this is the relationship between the transmitted power and the twisting moment t where f is the frequency at which the shaft is rotating. And f is the frequency t is the torque that is being applied in the other device and p is the power that is being transferred through the rotation of the shaft. Sometimes frequency is used as the number of revolutions per minute and in short we call this is as RPM the revolution per minute and is commonly designated with the notation n, since it is revolution per minute so $\frac{n}{60}$ is equal to f which is cycles per second. So, n is equal to 60 into f and if we substitute for f is equal to $\frac{n}{60}$ we can get the relationship between the power and the revolutions per minute which is p is equal to 2pi nt by 60. So in terms of if the rotation of the shaft is defined in terms of the revolution per minute which is n then power is equal to 2pi nt by 60 or if it is defined in terms of the frequency f which revolution per second then it is twice pi fT.

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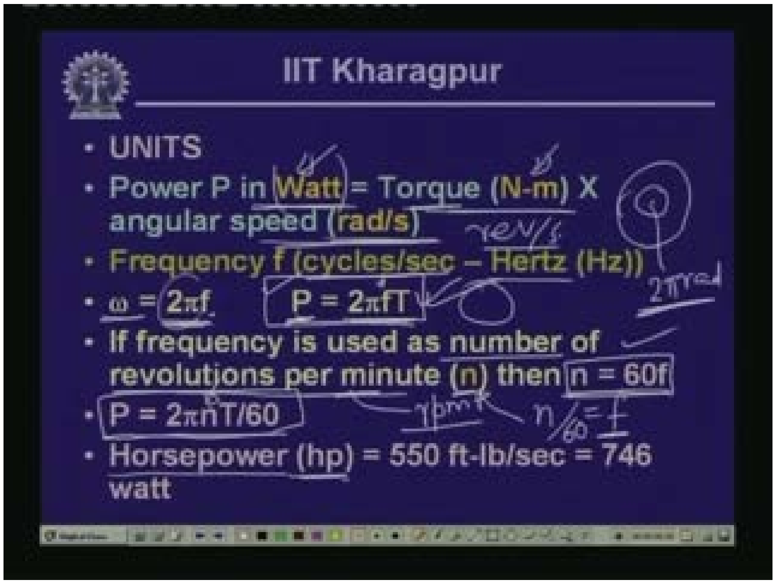

Sometimes in fps unit we define the power as the horse power, so one horse power 1(hp) is equal to 550 ft-lb by sec. So, if you write power in the terms of horse power then t we write in terms of foot bound and we get an expression that p is equal to 2pi nt by 60 into 550 so much of p is in hp horsepower and t is in foot bound. Approximately 1hp is equal to 746W. If the power of a motor is defined in terms of horse power we can convert that in terms of watt and we can use avordilation to see whether that is equal to Newton meter times power multiplied by the radian per second as the frequency or the angular speed. So this is helpful in relating the transmitted power and the torque.

So when we like to find out the diameter of a shaft which is necessary for transmitting power from one device to the other without exceeding the level of the stress, then first we compute the amount of twisting moment that is getting generated in the shaft for transmitting that amount of power. And for that particular twisting moment as we have seen in the previous sections how to compute the value of stresses and the angle of the rotations that we compute and if we know the allowance stress for that particular material then we can safely say that what diameter is necessary for that shaft or for a particular diameter whether that particular shaft will be able to withstand that amount of stress or not.

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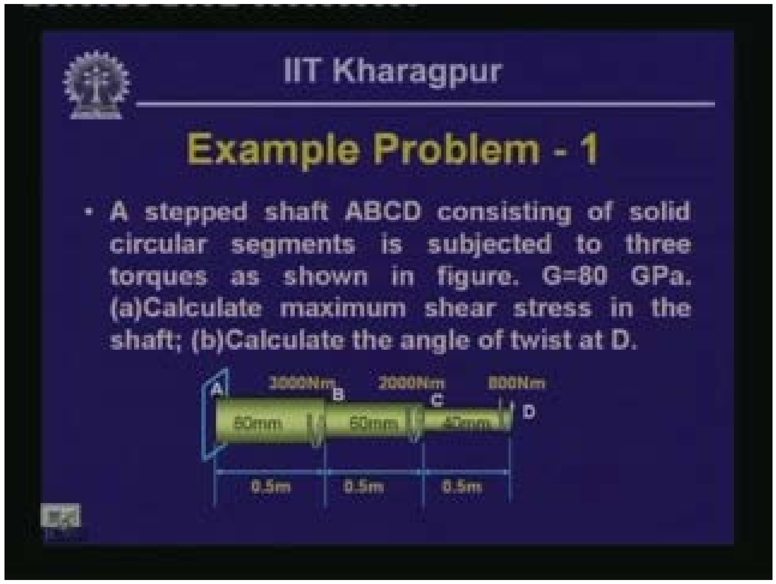

Now let's look into some of the examples. In fact these are the examples which I have said last time and asked you to look into, I am sure you have done this. Now in this we have a stepped shaft A B C D is consisting of solid circular segments, now there are 3 segments over here the diameter of which are given here. This is 80 mm this is 60 mm and this is 40 mm. They are subjected to a torque which is 3000Nm.

In this particular case that is according to our notation it is positive is subjected to a twisting moment of 2000m here at c and at d we have a twisting moment of 800Nm which is also positive. Now incidentally all three twisting moments in this particular case they are all positive which are acting at A B C and D. Now what we need to do is that we will have to find out the value of the maximum shear stress in the shaft and the angle of twist at point d. As you know that we have looked into earlier that to evaluate the twisting moment when they are subjected to a non uniform torsion that we compute, we take free body diagram and compute the values of the twisting moment. Now if we divide these segments in three parts lets call this segment as segment 1 this part as segment 2 and this part as segment 3 and then we compute the values of the stresses in each of the segment. Since three segments are subjected they are having three different diameters so they will be having different polar moment of the inertia.

We compute the value of the shearing stresses corresponding to these three segments and the maximum value of the shearing stress in either of these sections will be the maximum possible value of the shearing stress. But when we like to compute the angle of the rotation, because of the twisting moment that is acting in different segments of different magnitudes all will contribute to the rotation. Here the sense of the twisting moments is all positive therefore all will be contributing to the same angle of rotation. When we try to rotate in different form, if one rotates in an anticlockwise form then it will have anticlockwise rotation. If you have a negative twisting moment in a clockwise form then it will have negative rotation. So the sum effect of these two rotations will be the algebraic sum of the rotational values. What will be the value of the stresses in each of these segments?

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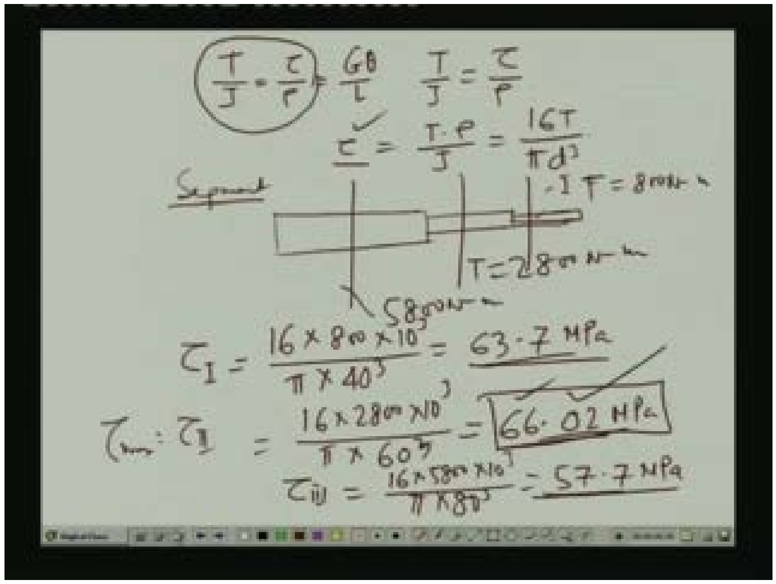

If we compute the values $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ is equal to $\frac{G\theta}{L}$. If we take the first of these two values that $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ then $\tau = \frac{T.\rho}{J}$ and this gives us this value as $\frac{16T}{\pi d}$ which we have seen earlier. Now we need to compute the value of stress in three different segments. If we look into the values of twisting moment the shaft is a stepped one. For segment 1 if we cut a section over here and if we take the free body diagram then we will have a twisting moment as 800Nm, for segment 2 we will have twisting moment as 2800Nm and for segment 3 we will have 5800Nm.

Now if we compute the values of the stresses corresponding to these three segments, segment I, segment II and segment III, tau for segment I is equal to 16 times t which is 800Nm into 10 cube by pi into d cube and d in this particular case is 40 cube and this is eventually 63.7 MPa; τ_I is equal to $\frac{16 \times 800 \times 10^3}{\pi \times 40^3} = 63.7 MPa$. Now for τ_{II} is equal to $\frac{16 \times 2800 \times 10^3}{\pi \times 60^3} = 66.02 MPa$ as the diameter of the second segment is 60. Likewise the shear stress for the third segment comes as 16 into 5800 into 10 cube by pi into 80 cube where diameter is 80 and this comes as 57.7 MPa; so τ_{III} is equal to $\frac{16 \times 5800 \times 10^3}{\pi \times 80^3} = 57.7 MPa$. Now out of these three, as you can see at three different segments we have three twisting moments and correspondingly we have three stresses, the maximum stress occurs in the second segment which is 66.02 MPa. So out of these three this gives us the maximum value. Therefore this is the maximum value of the shearing stress, this is τ_{max}. Now let us look into the value of the rotational angle theta.

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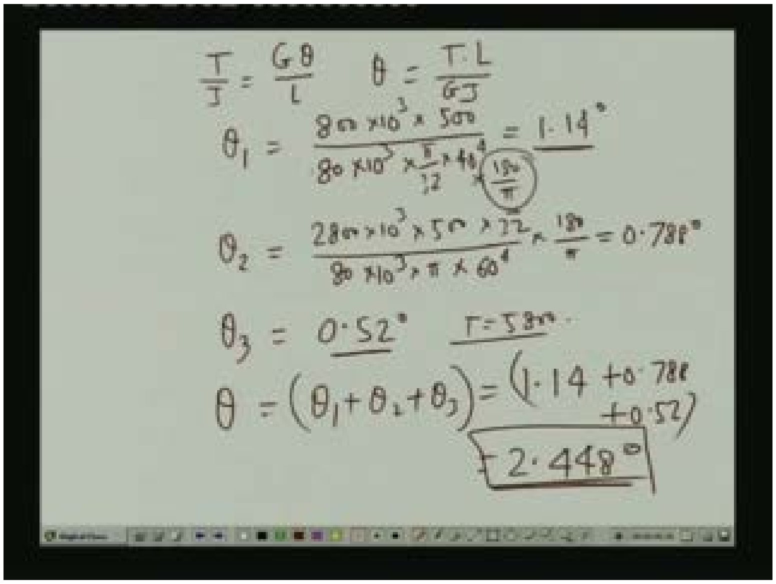

As you have seen $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ so theta is equal to gives us the value of $\frac{T.L}{G.J}$. Now for segment 1 if we compute theta$_I$ is equal to 800 into 10 cube into l which is 500 by g which is 80 GPa into 10 cube into pi by 32 into 40 to the power 4. So d to the power 4 by 32, this comes as 1.14 degrees; theta$_I$ is equal to $\frac{800\times10^3\times500}{80\times10^3\times\pi\div32\times40^4}=1.14^{\circ}$. This will be in radian and if you multiply that by 180 by pi that will give the value in degree so this comes out as 1.14 degrees. For the segment II that is theta$_2$ if you compute exactly in the same form then we have twisting moment as; theta$_2$ is equal to $\frac{2800\times10^3\times500\times32}{80\times10^3\times\pi\times60^4}\times\frac{180}{\pi}$ is equal to 0.788 degrees is what you get when you multiply for converting radian to the degree. And likewise for third segment theta$_3$ is equal to 0.52 degrees if we take t is equal to 5800Nm. So, the total rotation theta is equal to (theta$_1$ plus theta$_2$ plus theta$_3$) is equal to (1.14 plus 0.788 plus 0.52 degrees) is equal to 2.448 degrees because all twisting moments are in the same sense. This is the angle of rotation because of the twisting moment that is acting in the non-uniform section.

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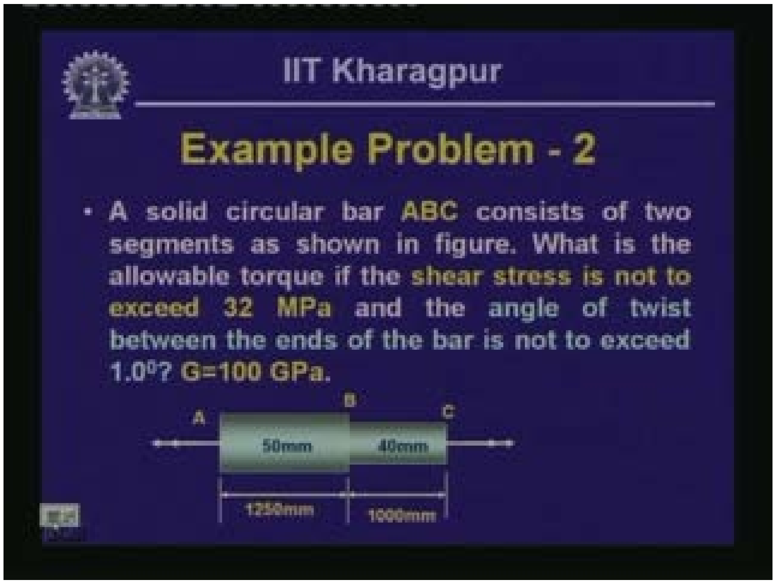

Here is the second example. A solid circular bar a b c consists of two segments as shown in figure. Now, what is the allowable torque if the shear stress is not to exceed 32 MPa and the angle of twist between the ends of the bar is not to exceed 1 degree, here g is given as 100 GPa. Here we have one constant torque t which is acting in this. This is the positive twisting moment and we have two segments which are diameter 50 mm and 40 mm. Hence we will have to compute the shear stress which will be definitely corresponding to the lower diameter but it should not exceed 32 MPa and angle of twist should not exceed 1 degree which will be a combination of theta for the two.

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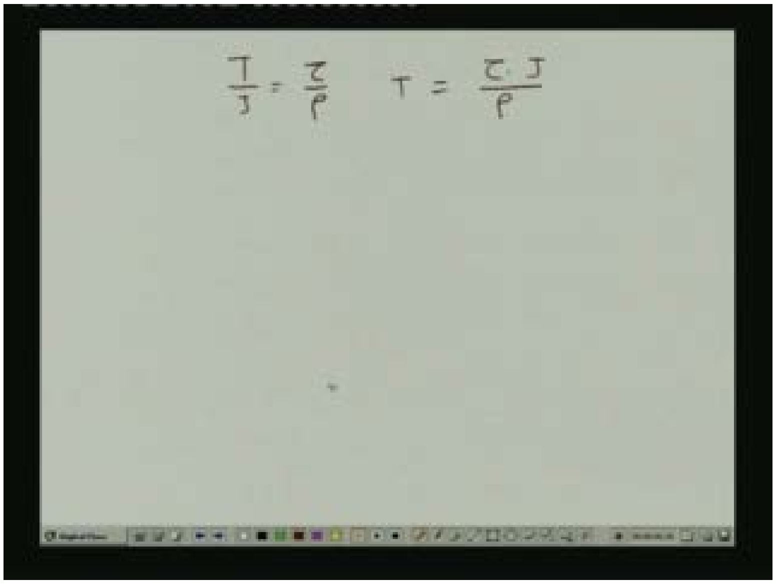

Now let us look into the value of this. Again $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ so we will have to compute the twisting moment and tau is given so this is $\frac{\tau . J}{\rho}$.

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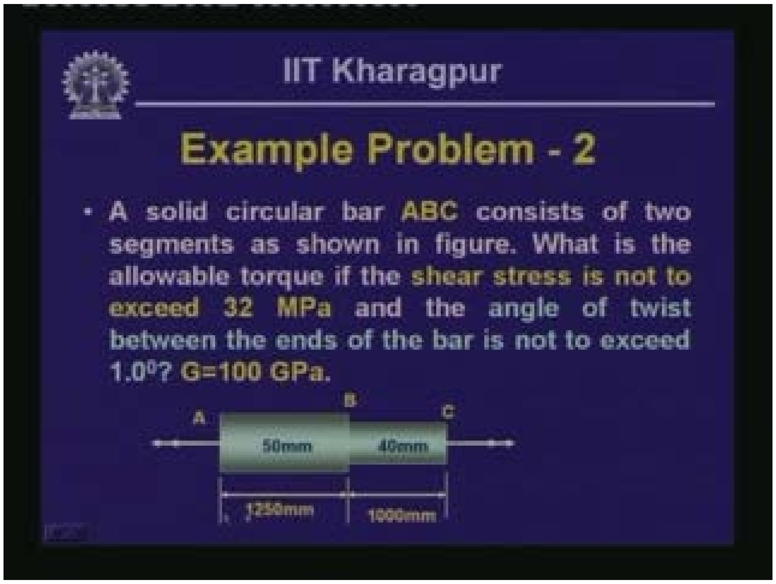

Here part AB is 50 mm diameter and for part BC is of 40 mm diameter.

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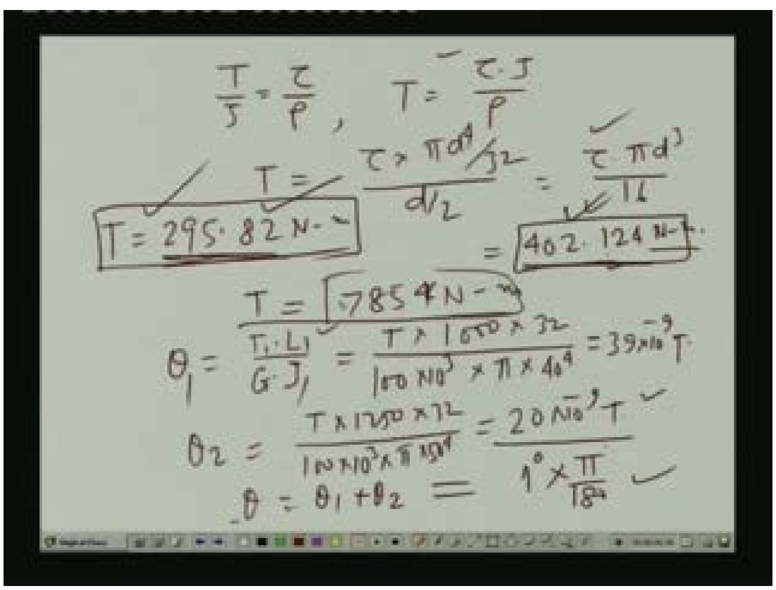

Now for part AB if we compute now as the twisting moment from $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ so T is equal to $\frac{\tau.J}{\rho}$ so when we need to satisfy the stress criteria T is equal to tau into pid$_4$ by 32 into d by 2 is equal to $\frac{\tau \pi d^3}{16}$ and now we know that it is to be limited to 32 MPa and that comes as 402.124N-m. For the second case when you have diameter as 50 then T is equal to 785.4N-m. So obviously you will have to apply this much of twisting moment so that it can resist both. Therefore this is the value of the twisting moment.

Now let us look from the angle of twist point of view. Now as we know theta is equal to $\frac{T.L}{G.J}$ so if we compute the value of the twisting moment, now we have two segments theta$_1$ and theta$_2$ so theta$_1$ is equal to $\frac{T_{1.}L_1}{G.J_1}$ is equal to and if we write T into L which is 1000 times 32 by g which is 100 into 10 cube and j will be pi into 40 to the power 4 pi d$_4$ by 32 and this comes as 39 into 10 to the power minus 9 T. Hence the equation is as follows:
theta$_1$ is equal to $\frac{T_{1.}L_1}{G.J_1}$ is equal to $\frac{T \times 1000 \times 32}{100 \times 10^3 \times \pi \times 40^4} = 39 \times 10^{-9} T$. Likewise theta$_2$ if you compute as t, t also is the same twisting moment but the only variable is L$_2$ GJ and that if we compute it will be t into 1250 is the length times 32 by 100 into 10 cube into pi into 50 to the power 4 so this comes as 20 times 10 to the power minus 9 T. so it is as follows:

theta$_2$ is equal to $\frac{T \times 1250 \times 32}{100 \times 10^3 \times \pi \times 50^4} = 20 \times 10^{-9} T$.

Now if we combine these two that will give us the value of the theta is equal to theta$_1$plus theta$_2$ and that if we equate to 1 degree which is equal to so much of pi by 180 radian and from this we can get value of the T. Now if we equate this theta which is the sum of this theta$_1$ plus theta$_2$ to this pi by 180 the value of T comes as 295.82N-m. Now, as you can see we have the three values of the twisting moment, this is one which is from the stress criteria and we have another from the rotational twisting angle criteria. So this being the small s this will be guiding twisting moment so that it can satisfy both the criteria of the stress as well as the twisting moment. So the minimum value of the twisting moment will be the governing moment in this particular case.

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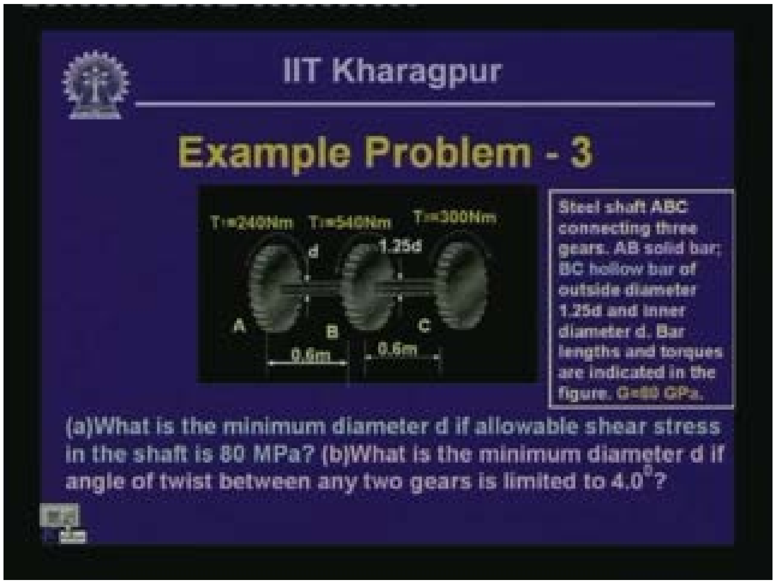

This is another example where we have discussed that if we have a shaft, earlier we had a non-uniform distribution of the torsion having varying diameters of the shaft. Now if we have uniform distribution of the uniform diameter and the torque are of different magnitude then we can compute the value of the stress.

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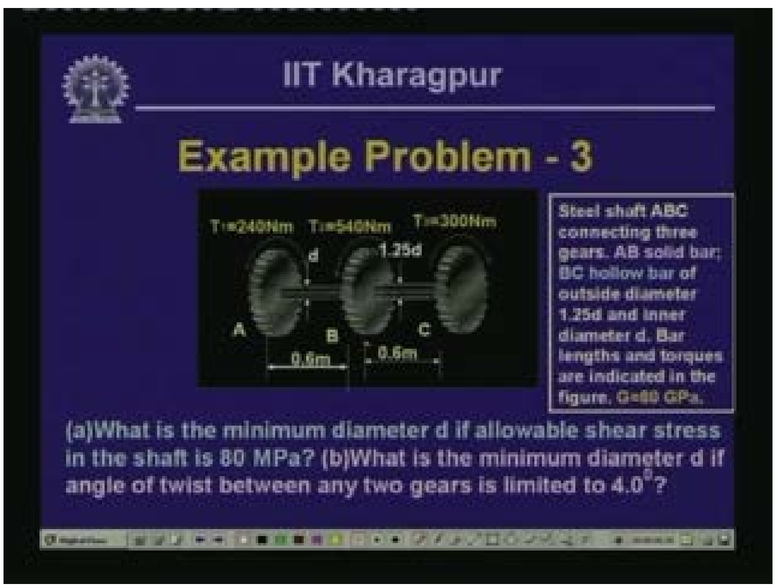

Here steel shaft ABC is connected with the three gears at A, B and C and these gears are transmitting moment of 240N-m, 540N-m and 300N-m and their directions are that 240N-m is acting in this form whereas 540 is acting in this direction and 300 is acting in this direction. So these are three directional values and AB is the solid bar and this is the hollow bar of outside diameter 1.25 into d and inner diameter is d, the bar lengths are given over here and G is 80 GPa. Now what you will have to find out is; what is the minimum diameter d if allowable shear stress in the shaft is 80 MPa and what is minimum diameter d if angle of twist between any two gears is limited to 4 degrees. Let us look into the value of that.

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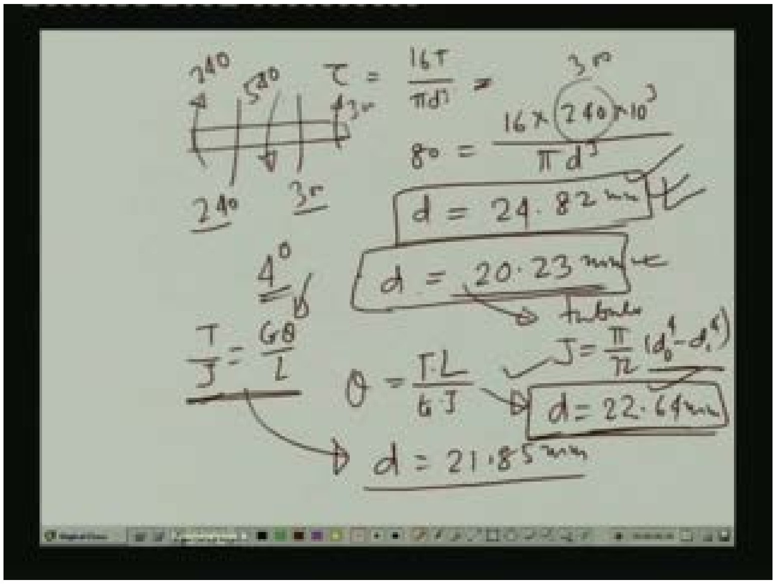

Now if we compute the value of shearing stress, tau is equal to $\frac{16T}{\pi \mathrm{d}^3}$ so if we look into the values of the twisting moment, here we have 240, here we have 540 and here we have 300. From equilibrium point of view you can see that these two sums balances this so at this we will have 240 and at this cross section we will have 300N-m. So, if we compute the value of the stress for the first one, the shearing stress is limited to 80 so 80 is equal to 16 into here is 240 into 10 cube by pi d cube that is 80 is equal to $\frac{16 \times 260 \times 10^3}{\pi d^2}$ so from this we get d is equal to 24.82 mm.

From the second case if we compute d from the same expression we get the value of d as 20.23 mm. Now the only difference here is that twisting moment is 300N-m in place of 240 so these are the two diameters we have and the higher of these two diameters will be the governing diameter. If we compute from the point of view of the rotation that $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ then again J is equal to pi d_4 by 32. In the second case when you compute the value of d keep in mind that here the shaft is a tubular one and for tubular one as we know J is equal to π by 32 (d_o to the power 4 minus d_i to the power 4) and this we will have to apply to compute the value of d.

Likewise, we compute the value of rotation theta is equal to $\frac{T.L}{G.J}$ and from this if we compute the value of d for the solid case we get d is equal to 22.64 mm and again for the hollow one if we apply d, d is equal to 21.85 mm keeping theta limited to 4 degrees. Now in this particular case as you can see the diameter which you get as 22.64 is again governing. So, in the first case we have

24.82 mm diameter which is to be provided and in this case we will have the provide 22.64 mm to resist the twisting moment to the gears as it is shown here.

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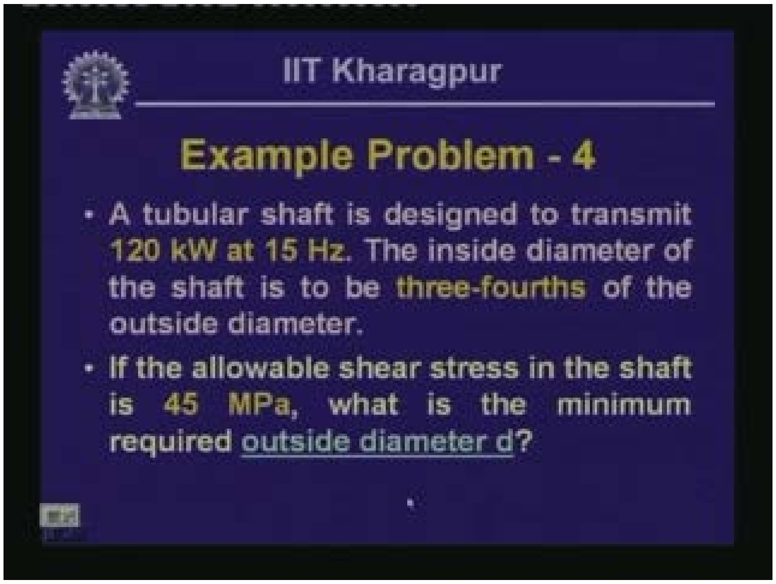

Here is another example problem. A tubular shaft is designed to transmit 120 kW at 15 Hz. The inside diameter of the shaft is to be 3 by 4 of the outside diameter. Now if the allowable shear stress in the shaft is 45 MPa what is the minimum required outside diameter d?
This particular problem is similar to the one which we have discussed today that, if we transmit the power then is the diameter of the shaft necessary.

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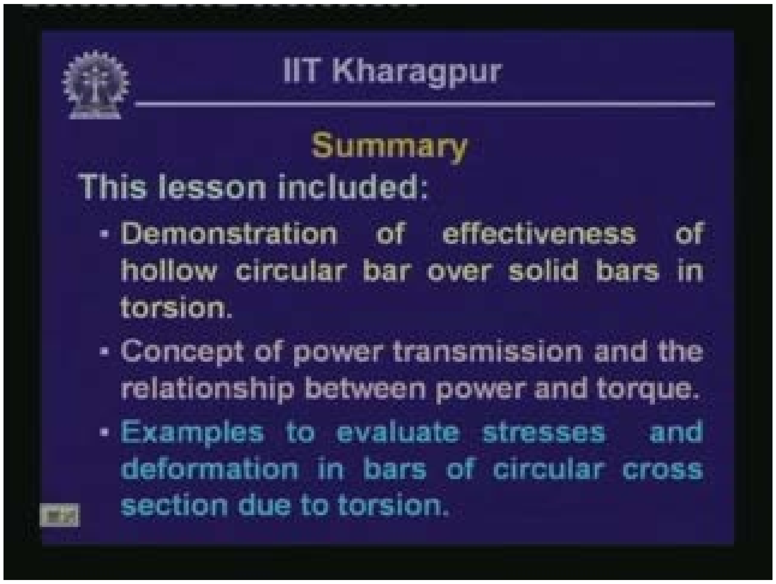

Summary of this lesson:

We have demonstrated the effectiveness of a hollow circular bar over a solid circular bar through the examples and we have looked into the concept of power transmission and the relationship between the power and torque. Then we have looked into some examples to evaluate stresses and deformation in bars of circular section due to twisting moment.

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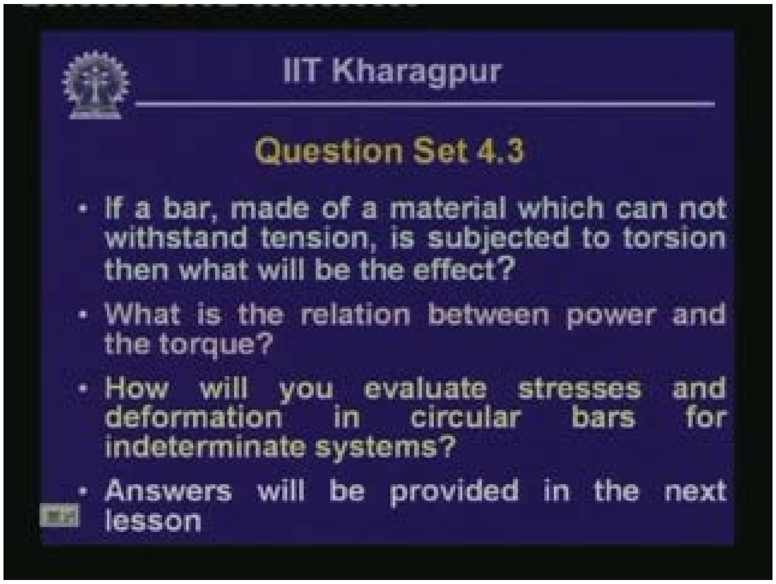

If a bar made of a material which cannot withstand tension is subjected to torsion then what will be the effect of such twisting moment. What is the relation between power and the torque and how will you evaluate stresses and deformation in circular bars for indeterminate systems.