Strength of Materials
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Lecture - 2
Analysis of Stress - I

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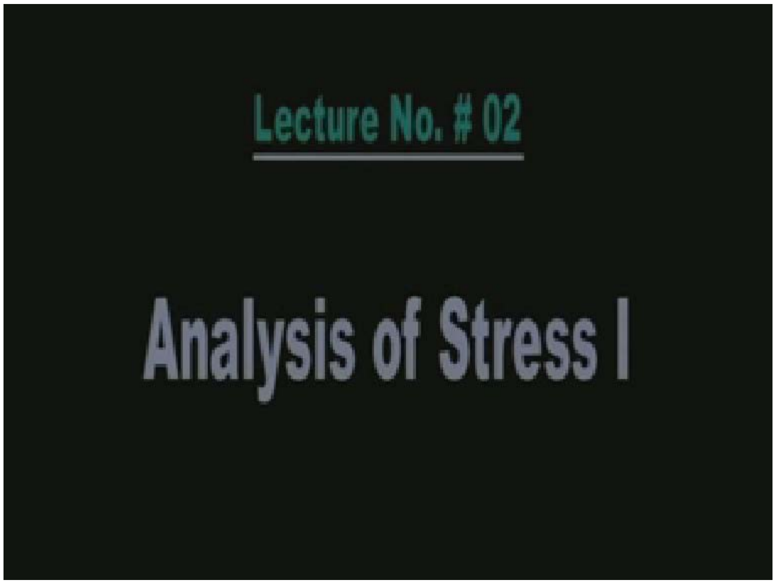

Welcome to the course on the strength of materials. In the last lesson I introduced the concept of stresses. In this particular lesson we are going to look into some more aspects of analysis of stresses.

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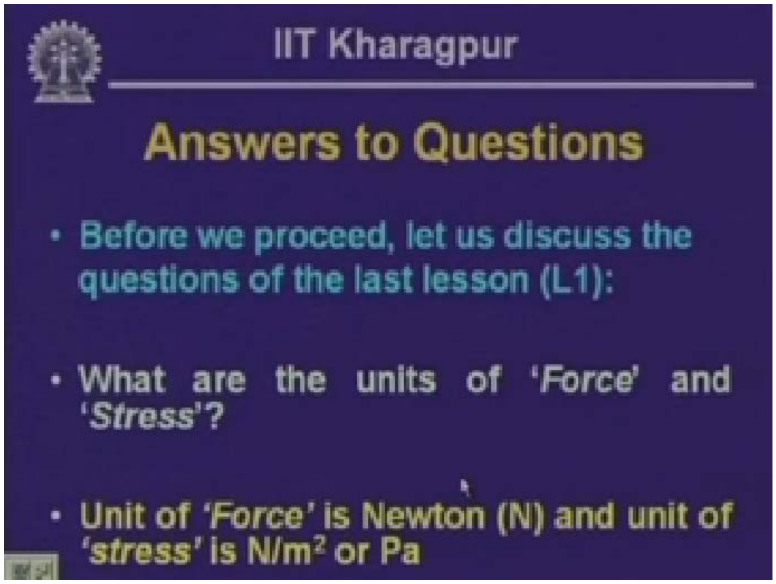

Here are some of the questions posed in the last session. The first question was; what is the unit of force and stress?
The unit of Force is Newton (N) and unit of stress is (N by m square) or Pascal (Pa). This can be represented in terms of also Mega Pascal (Mpa) or just Giga Pascal (GPa).

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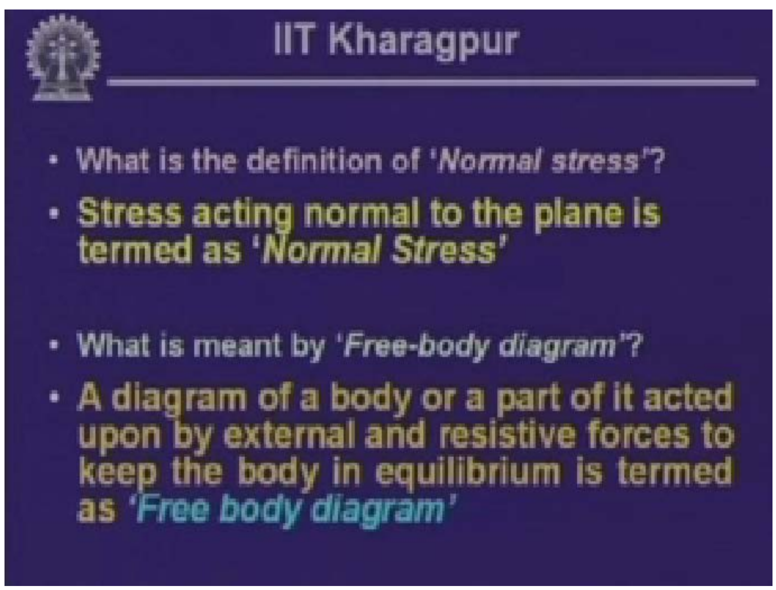

What is the definition of normal stress?
It can be defined as stress acting normal to the plane.

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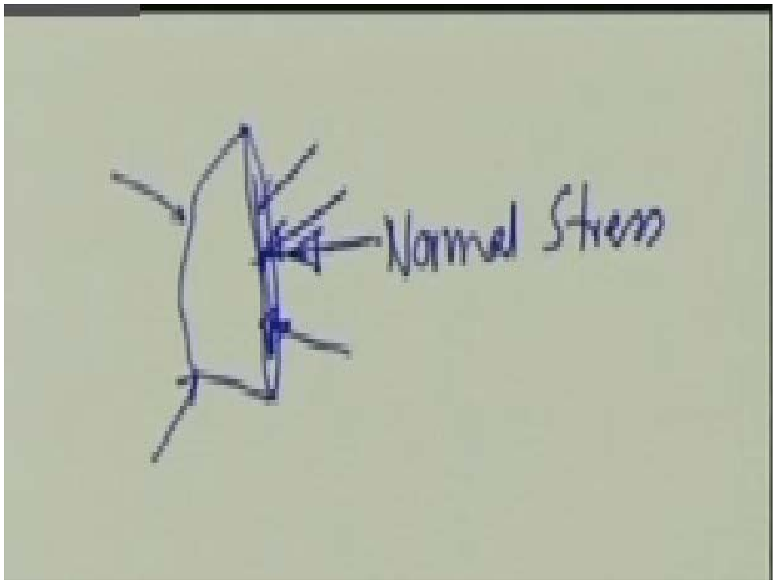

If you have a body which is a free body from a major body and is acted on by forces there will be resulting forces into it which will keep the body in equilibrium. At a particular small element if we take that this is a resulting stress then we can decompose this stress into two components, one is along the normal direction of this particular cross section which is this and another component along the plane of this particular section. The component which is acting normal to this particular cross section is normally known as the normal stress. Normal stress is the normal component perpendicular to the particular section.

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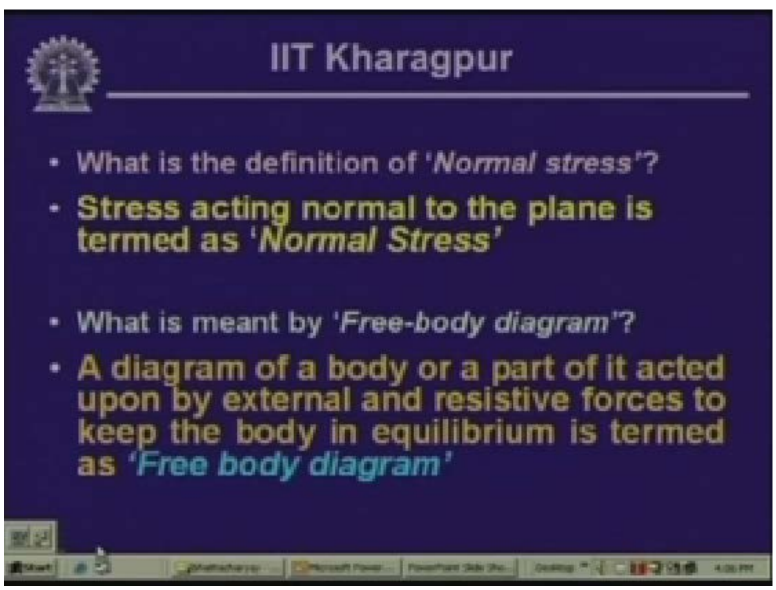

What is meant by free body diagram?

Free body diagram is a diagram of a body as a whole acted on by external resistive forces or a part of it. It can be the whole body acted on by active forces and resistive forces or if we cut any section from the body and if they are also acted on by the same forces and the resistive forces which will keep the body in equilibrium then we call that as the free body diagram of that particular configuration.

Let us say we have a body which is supported at these two points and acted on by forces. If we have to draw free body diagram of this, we will draw a diagram subjected to the external forces. This may not be the right diagram, the right diagram will be, if you take a body acted on by active force and also at the support point we have the resistive forces which will keep the body in equilibrium. Then this is the true free body diagram of the body.

Now, supposing if we want to cut the free body clearly and separate it into two halves then we have this half which is acted by active and resistive forces and this section will have resistive force related which will keep this part of the body in equilibrium. Then this particular part also is a part of the free body of whole structural form. So, free body diagram is essentially, when they are acted on by the active forces and the resistive forces of the whole body or part of a body subjected to the external active and resistive forces and the internal resistive forces.

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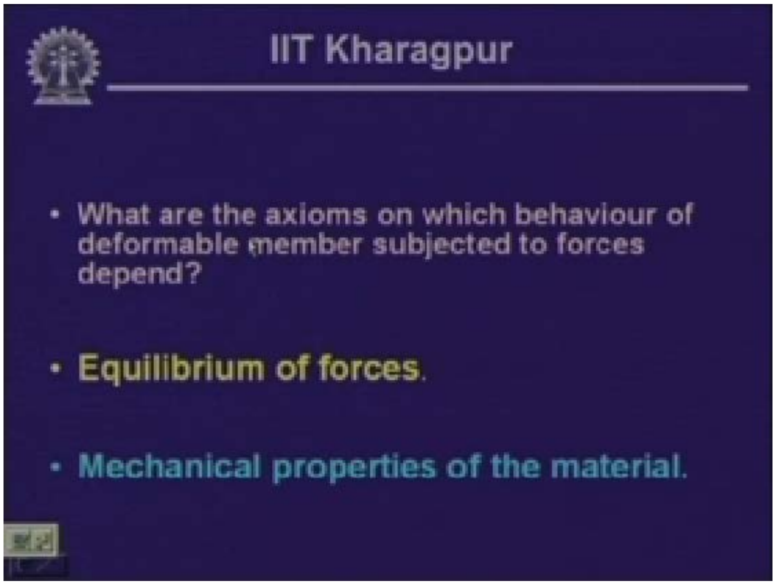

What are the axioms on which behavior of deformable member subjected to forces depend?
We have the first thing which is equilibrium of forces. Fundamental laws of Newtonian mechanics are used for the equilibrium of forces and for the body should be such that it must be having the forces in x direction. Summation of forces in x direction should be equal to 0, summation of forces in y direction should be equal to 0, and summation of forces in z direction should be equal to 0. These equations must be satisfied to fit for the body which is in space. In a two dimensional form, these equilibrium equations reduces to summation of forces in x direction equal to 0, summation of forces in y direction equal to 0 and summation of movement of z is equal to 0. Also the forces must satisfy the parallelogram of forces.

Supposing we have two forces in the plane which is normal and plane forces in the direction of the plane this must satisfy the law like the resultant should pass through the diagonal of the parallelogram. Or, if we are talking about forces or stresses in three dimension, if we look into this parallelepiped the forces acting in the x direction or the stress acting in the x direction the y direction and the z direction the resultant of this must be acting along the diagonal of this parallelepiped. This is the resulting stress of all these stress components.

Therefore either in two dimensional plane it should be in this configuration or in a three dimensional plane it should be in this configuration which is the parallelogram of forces that must be satisfied. So these are the two basic axioms based on which the forces act on the deformable body are guided.

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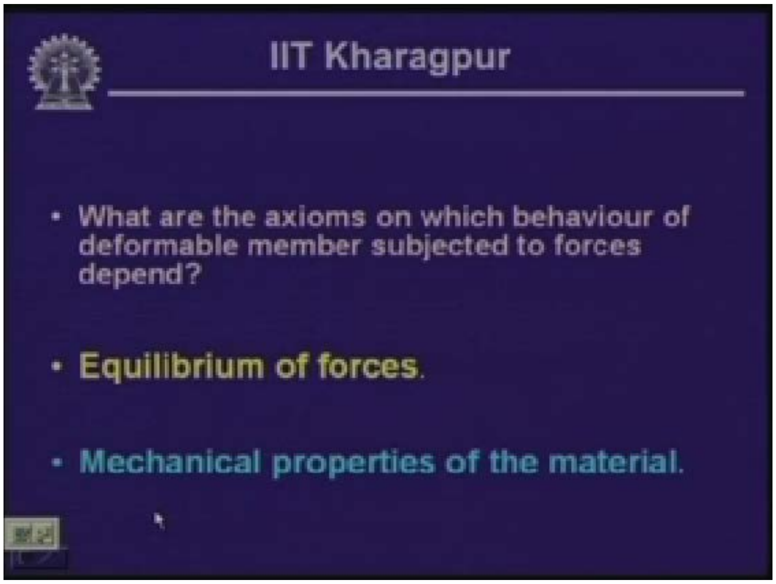

Also, the mechanical properties of the material are essential to be satisfied. These properties of materials are to be established through laboratory experiments. Hence the equilibrium of forces and mechanical properties are the main aspects of behavior of the members subjected to forces.

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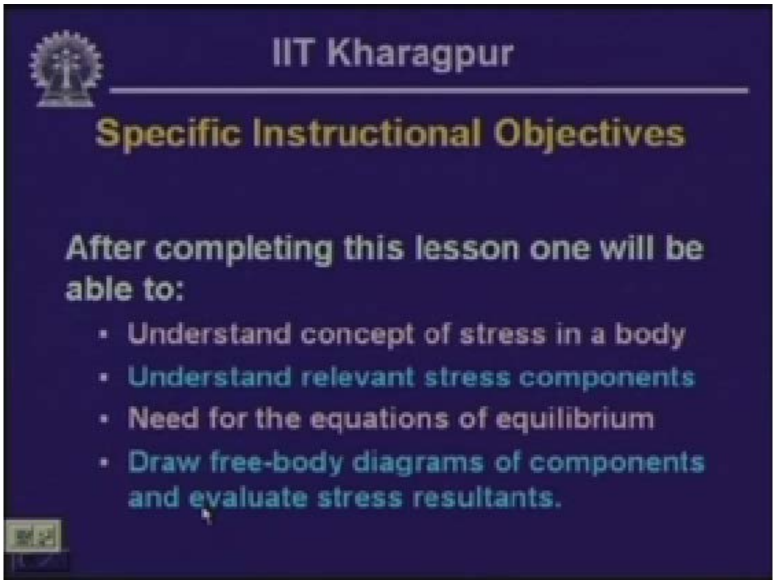

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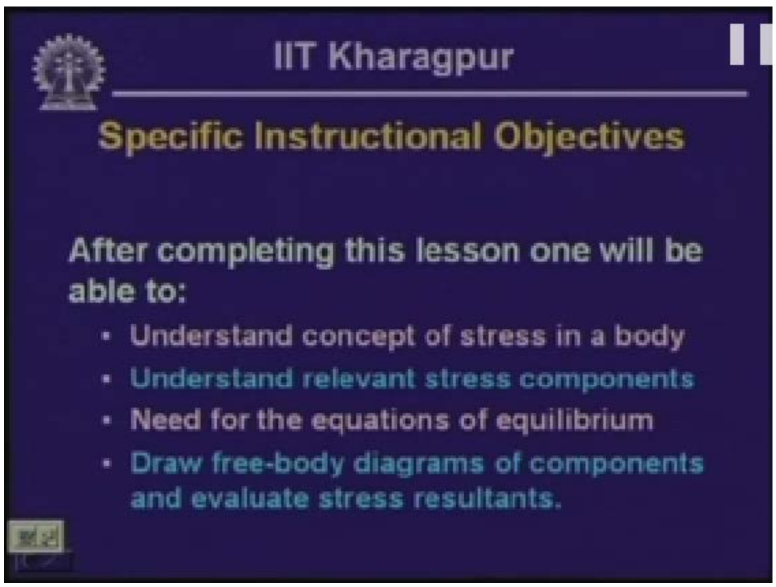

Here are some of the aspects. Now in this lesson we are going through the aspect of stress.

One should be able to understand the concept of stress in a body. Understand relevant stress components, and then one should be able to understand why we need to go for the equations of the equilibrium, for a given problem. One should be able to draw the free-body diagram and evaluate the stress resultants from these diagrams and thereby compute the stresses.

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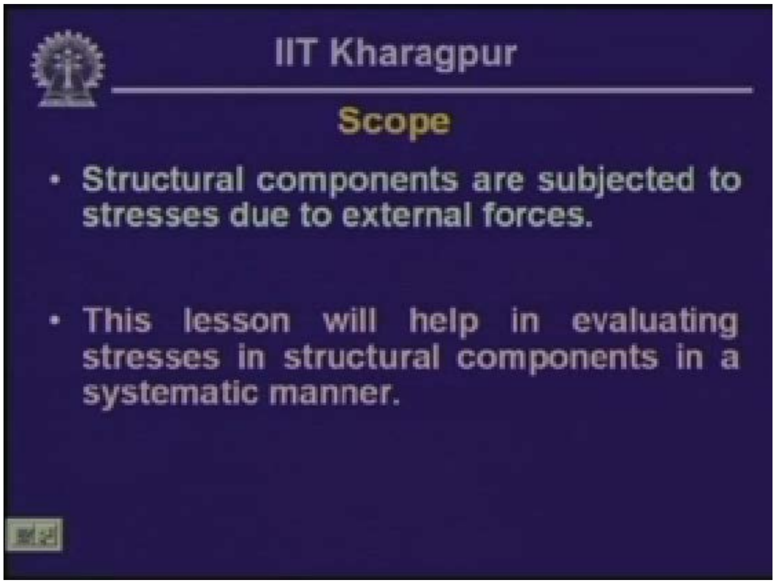

The components are subjected to stresses due to external forces. Now, we will be evaluating stresses in structural components in a systematic manner. How do you go? How do you solve given a problem and show the particular free body diagram of the problem considering that the external forces and the resistive forces acting on the member and finally once we know the stress resultant from the equations of the equilibrium. We should be able to calculate the stresses at different sections as we desire.

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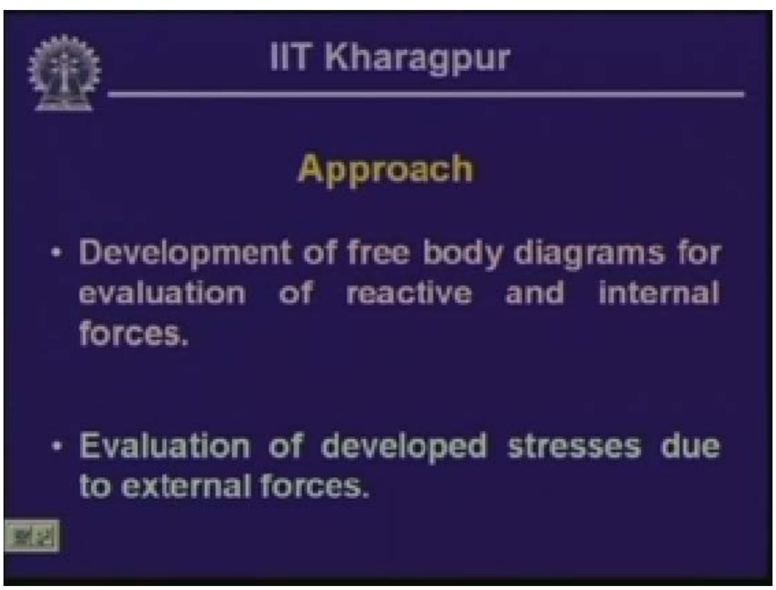

The approach for analysis of external forces should be through the development of free body diagram for evaluation of reactive and internal forces and thereby evaluation of developed stresses due to external forces.

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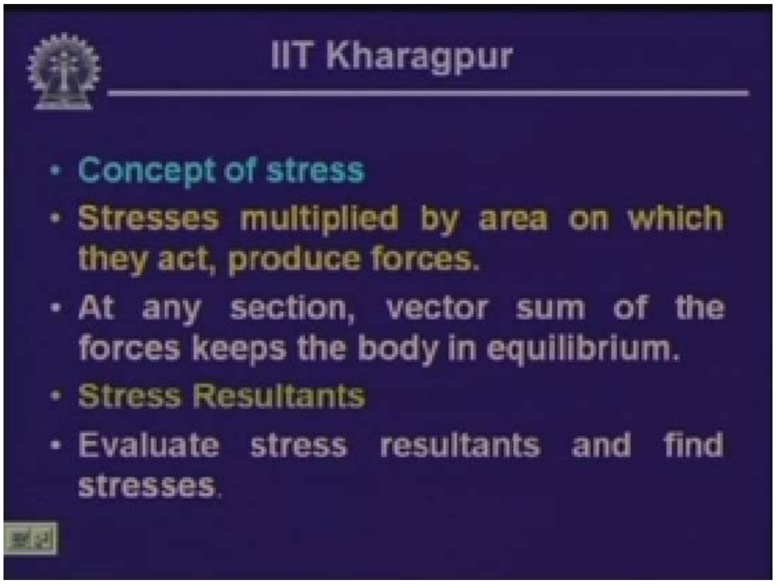

The concept of stress has been already discussed which is at a particular smaller element and is the intensity of the forces acting on the area. We had defined that force for smaller unit area as the stress.

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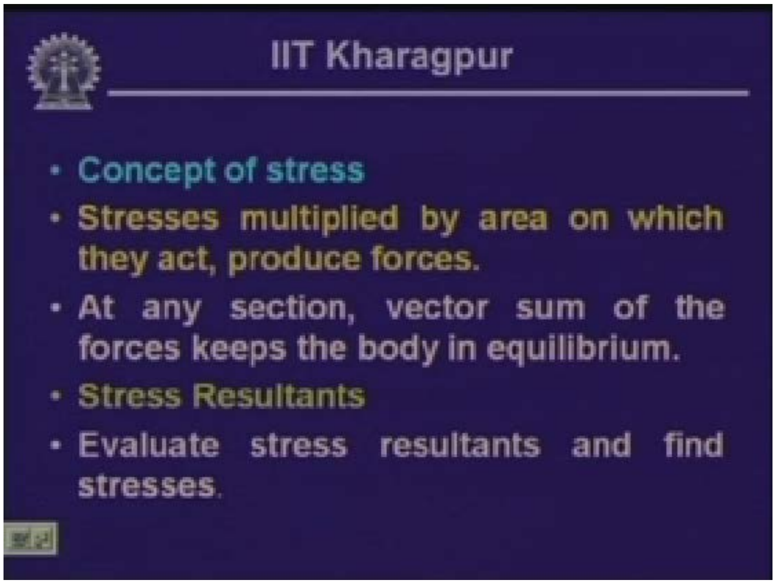

Now this stress when it is multiplied is acted on by external forces. If we take a small element in which a stress is acting this stress multiplied by the area gives the force which we call as the stress resultant and the total stress resultant is the stress multiplied by the elemental area integrated over the whole of the area is the resulting force which we call as stress resultant. And thereby, we assume that at any point it has the same behavior.

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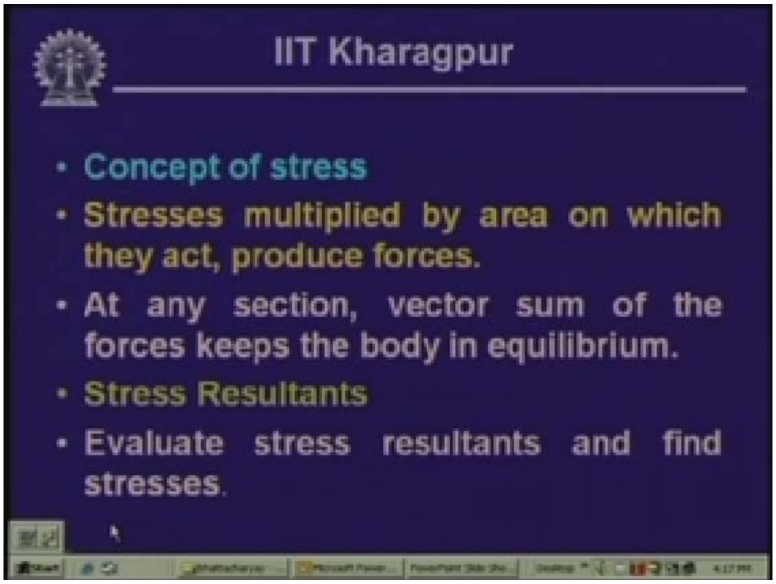

So at any section the vector sum of the forces keeps the body in equilibrium and that is how the stress resultant will be obtained for that particular section. So our job is to evaluate this stress resultant. And once we know the test resultant we can compute stresses at that particular section.

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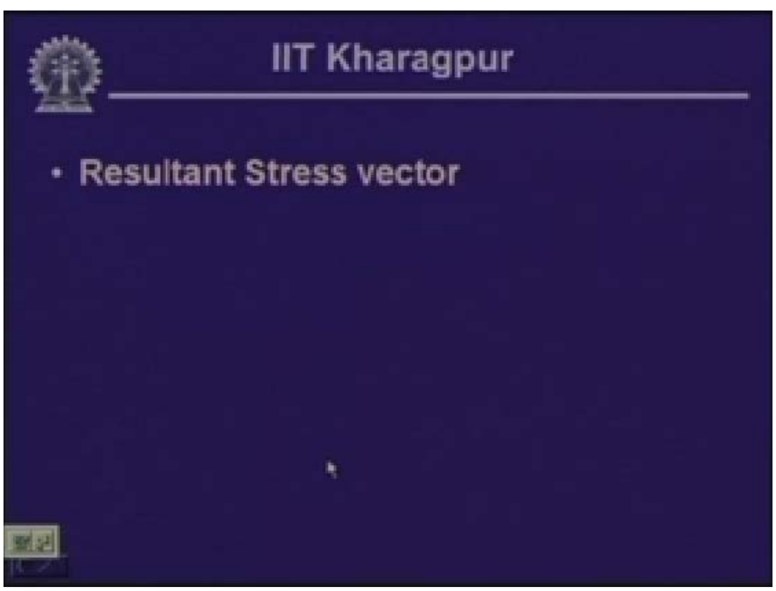

We are interested in knowing the resultant stress vector in a particular section. Let us look into a body.

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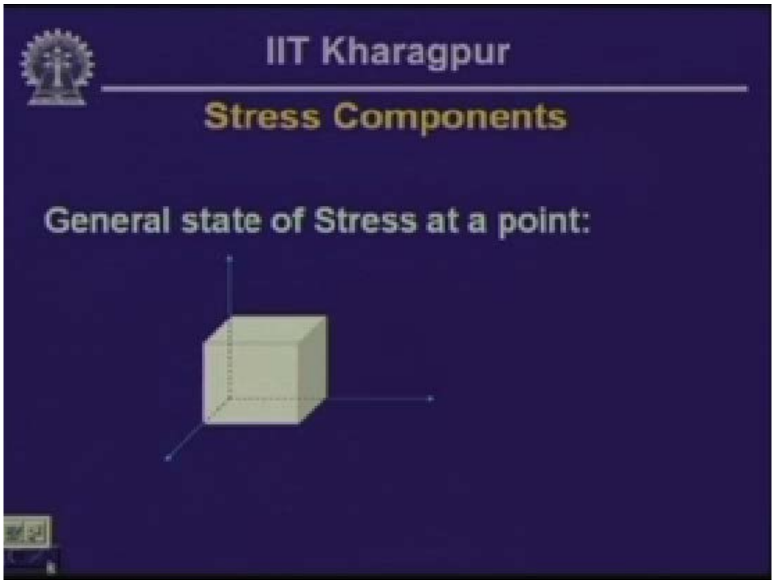

This is x-axis, this is y-axis and this is z-axis. This is a body which is acted on by external forces. If we cut this particular body to a plane which is perpendicular or rather parallel to the y, z plane then the normal drawn on this particular plane will be parallel to x-axis. For any plane when you draw the normal to that particular plane and if that particular normal coincides with any of the axis we designate that plane with the name of that particular axis.

For example, here we have cut this body through a plane which is parallel to y, z plane. So, if we draw a normal on to this section this normal is going to be the parallel to x-axis and thereby this particular plane we designate this plane as x-plane. Now on this x-plane we have at a particular point the resulting stress which we call as R. If we take the component of this stress R in three perpendicular axis direction then we have the stress acting in the direction of x which is normal to this particular section and as per the definition of normal stress this is the normal stress which is acting in the direction of x. If we take the component of R along y-axis or parallel to y-axis then the plane we get as stress is acting in y direction.

Also, we have the component which is acting in z direction. The stress which is acting parallel to y or in the y direction we designate this as the stress tau acting in the plane x in the direction y which we call as tau x, y.

Or if we designate this particular stress which is acting in the plane x along z direction then we call these as tau_{xz}. Thereby in this particular x-plane we have three stress components where one is the normal to the plane and the other two are in the plane in the direction of y and z. The normal stress we call as σ_x and the other two components which are in the plane are called as shearing stresses which are τ_{xy} and τ_{xy}.

Likewise, if we cut this body into a plane which is parallel to this z plane we cut along this then on this plane on a particular point we can get three components of the stresses and they are σ_y , τ on the y plane in the x direction as τ_{yx} , the shearing stress tau in the y plane in the z direction as τ_{yz} . Also, if we cut this body with a plane which is perpendicular or parallel to x, y plane and if we plot the three stress components the stress which is normal to the plane gives out the normal stress sigma$_z$ and two stresses which are in the plane are on the z plane in the x-direction and stress in the z-plane in the y-direction.

These are the nine stress components that we are going to get at a particular point. Now if this particular body is cut in such a way that you take another plane which is at an infinite small distance away from here if we cut it off by two parallel planes then we can get small a cubical element on which you can plot the stresses.

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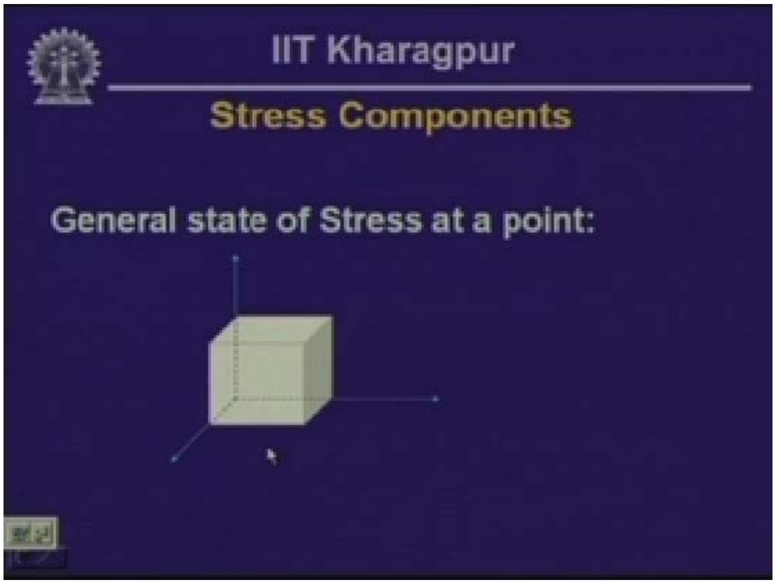

Thereby the three faces which are visible to us are in front of the board, if we look into that this particular plane let us say this is x-axis, this is y-axis, and this z-axis. The stress on this particular plane the direction of x is the normal stress sigma$_x$ is in the x-plane, on the x-plane the stress acting in the y-direction is tau$_{xy}$ and on the x-plane the stress acting in the z-direction is designated as tau$_{xz}$.

Likewise, on this plane we will have stress acting in the y-direction the normal stress which is sigma$_y$ and for the positive direction of x on the y-plane we have the shearing stress which is designated as tau y-plane in the x-direction and on the y-plane in z-direction we have the stress which we designate as tau$_{yz}$. This particular plane the normal to z we call it as the z-plane and on this we have the stresses acting the normal to the z as sigma$_z$.

On the z-plane in the x-direction we have tau_{zx} and then this is the z-plane with the y-direction which is in this particular direction the positive direction of y so this is tau_{zy}. Likewise on the other three faces which are hidden from this side this face, this face and this face also have the three components of the stresses which are sigma tau and in the two planes we have tau.

In this particular plane normal to this which is acting in the negative x-direction will have the stress $sigma_x$ in this particular direction. The face will have the shearing stress component this being the x-plane and in the opposite direction of y we will have the shearing stress that is tau_{xy}, in the x-plane but in the z-direction opposite to the positive z-axis we will have x, z so this is tau_{xy} and this is tau x, z.

Likewise on the y-plane we will have the stress acting perpendicular to it which is $sigma_y$. The y-plane in the x-direction will have the shearing stress but it will be in the opposite direction of x which is being the other end. On this y-plane in the z-direction we will have stress tau_{yz}. This is tau_{yx} and this is tau_{yz}. Likewise on the other side of the z-plane we will have the normal stress acting which is $sigma_z$ and on the z-plane we have stresses in the x-direction which is tau_{zx} and in the z-plane we have tau_{zy}. These are the stress components that will be acting on this particular body at a point.

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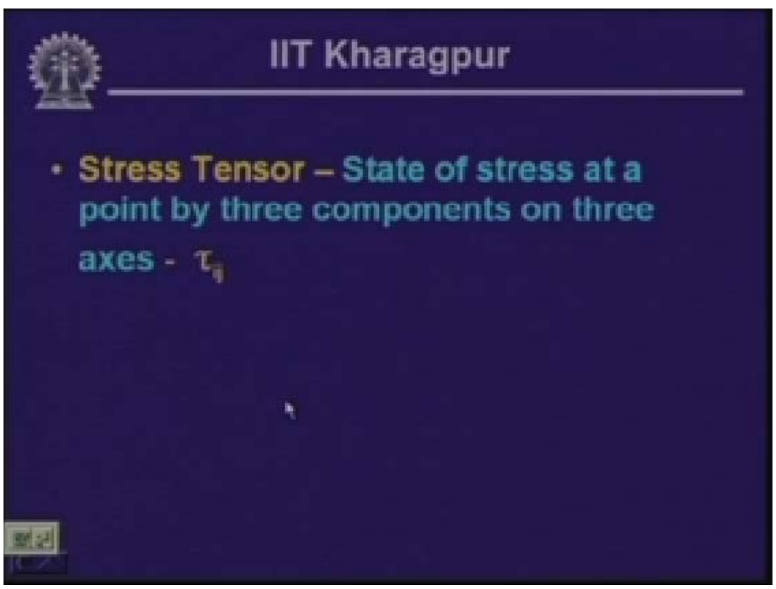

Let us look into the state of stress at a point. As we have seen, the state of stress at a point is represented by those nine stress components.

Now three stage state are represented by the three stress components like $sigma_x$, tau_{xy} and tau_{xz} and likewise in other planes. So all these three stress components are written on three mutually perpendicular axis and they are represented by a term which we generally written as tau_{ij} which is called as a stress tension.

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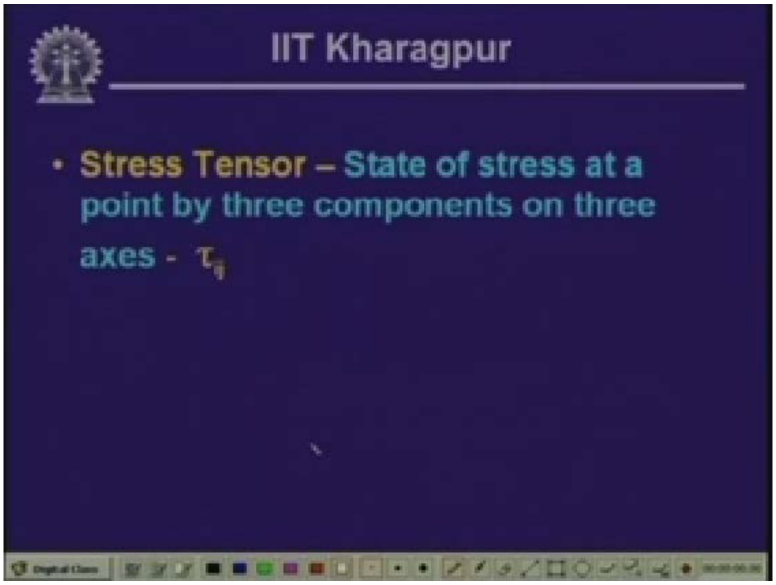

So stress tensor is tau_{ij} where i represents the (x, y, z) directions or the (x, y, z) plane and j represents the three stress directions. So if we expand these on the x-plane ad when i is x we have j as xyz, so the stress components are tau xx, tau_{xy} and tau_{xz}. When i is the y plane we will have three components in x, y and z direction. So we will have tau_{yx}, tau_{yy} and tau_{yz}. Likewise when i is in the z-plane we will have three components in the x, y and z-direction. We have tau_{zx}, tau_{zy} and tau_{zz}. Now the components which are tau_{xx}, tau_{yy} and tau_{zz} are normal to the plane. Let us call them as sigma_{x}, this as sigma_{y} and this as sigma_{z}.

So, we have the stress components sigma_{x}, tau_{xy} and tau_{xz} which are acting in the x-plane, we have tau_{yx}, sigma_{y}, tau_{yz} in the y-plane, we have tau_{zx}, tau_{zy} and sigma_{z} which are acting on the z-plane or in the z-direction. These are the nine stress components we have at a point.

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Now let us look into another aspect of it. Out of the stress components which we have obtained, again let us call these as x-axis as usual, we call as y-axis and this as z-axis.

Now, if we take the movement of all these forces, now let us assume that this distance which we have taken at a particular point of the body is dx, the vertical height being dy and along the z direction tau with dz. Now if we like to take the movement of all the forces about z-axis, now in this particular figure only the forces which will have relevance while taking the movement about z-axis has been taken into account. Now this particular plane being x plane and the force which is acting in the y-direction as per our nomenclature we call this as tau_{xy}.

Accordingly this particular component of the stress which is acting in the direction of x and acting on y-plane we call this as yx. Likewise, this is also tau_{yx} and this is tau_{xy}. Along with this we have the other stresses like normal stress $sigma_x$, $sigma_y$ and $sigma_z$ and shearing components as well in the other plane. Since only these forces or these stress components are going to cause the movement other forces have not been shown here.

If we take the movement of all the forces about z-axis then the movement expression can be written as tau_{yx} which is the stress acting on the area dx by dz, so tau_{yx} into dx into dz is the force. The movement about the z-axis is the distance dy so this multiplied by dy is the movement about the z-axis as tau_{yx} which is clockwise in nature minus tau_{xy} which is acting on the area dz(dy). So tau_{xy} into dy into dz is the force.

If we take the movement of this force with respect to z- axis then this is multiplied by the distance dx. Assumingly that there are body force components in the x and y-direction, this x is the body force per unit volume then along with this we have plus, but for the time being we are neglecting the body force components because that is also not going to cause any moment as such with respect to the z-axis.

Therefore here it is z is equal to 0 for equilibrium. This produces tau_{yx} is equal to tau_{xy}. In effect this means that the cross term tau_{yx} and tau_{xy} are equal. Likewise if we take the movement of forces about x and y-axis and take the relevant forces we can see that tau_{zx} is equal to tau_{xz} and tau_{yz} is equal to tau_{zy}. This gives us that the cross hearing terms are equal. So if we look into stress and strain which we had tau_{ij} the tau_{ij} is equal to $sigma_x$, tau_{xy}, tau_{xz}, tau_{yx}, $sigma_y$ and tau_{yz}, tau_{zx}, tau_{zy} and $sigma_z$ if we write in the matrix form. This is the stress tensor.

Now for the equality of the shear we have obtained tau_{xy} is equal to tau_{yx}, tau_{xz} is equal to tau_{zx}, tau_{yz} is equal to tau_{zy}. Thereby stress tensor can be written as tau_{ij} in the matrix notation as $sigma_x$, tau_{xy} and tau_{xz}. Now tau_{xy} is equal to tau_{yx} we will write this as tau_{xy}, $sigma_y$ and tau_{yz} and zx and xz being the same we write this as tau_{xz}, tau_{yz}, tau_{yz} and $sigma_z$ and thereby it reduces to the six stress components $sigma_x$, $sigma_y$ and $sigma_z$, tau_{xy}, tau_{xz} and tau_{yz} and this we find is symmetrical in nature so the stress tensor has a symmetric form.

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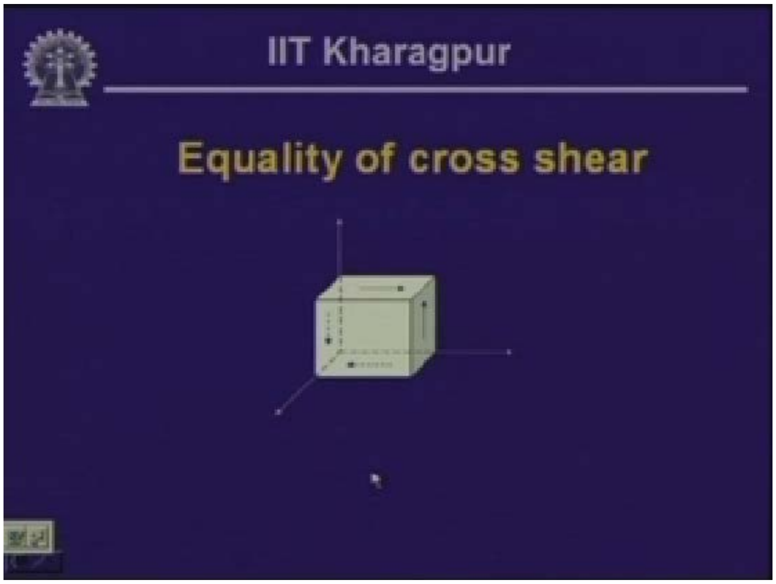

If we write down the stress components in a two dimensional plane, then we call those stresses as the elements in the plane stress.

The stress components which we will have here are $sigma_x$ the normal stress this being the x-direction, this being the y-direction we have the stress $sigma_y$ so we are concentrating on the xy-plane and this is also $sigma_y$ and this is $sigma_x$. This being the x-plane the stresses acting in y-direction is tau_{xy}. This being the y-plane and this is acting in the x-direction according to our designation nomenclature we call this as tau_{yx}.

But since tau_{yx} is equal to tau_{xy} we call this as tau_{xy}. And so are these stresses which are tau_{xy} and tau_{xy}. Since all the shearing stress components are xy we can call this simply as tau. Therefore we have normal stress components $sigma_x$, $sigma_y$ and shearing stress tau.

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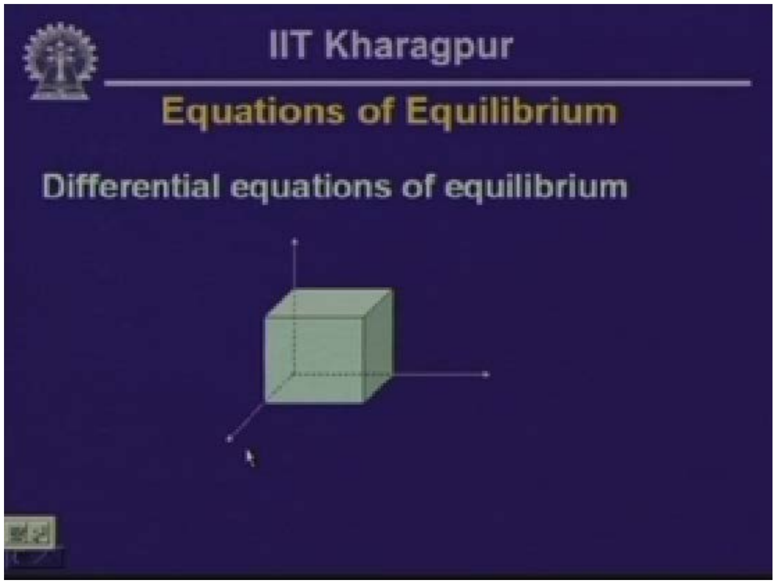

Having known that the stress at particular point is acting which are combinations of normal stresses and shearing stresses let us look into that if we have a body and if we are interested to find out the change in stress from one point to another then the change of the stresses is from point to point, we need certain equations to be solved and those equations are called as equations of equilibrium.

Coming back to the body here, for example we have a body which is stress and we like to find out the change in stress from this point to this point. So we need these changes to be evaluated through these equations of equilibrium. Now as usual we call this as x-axis, this as y-axis and this as z-axis. Now on this particular plane which is normal to the x-plane we have normal stresses known as sigma$_x$.

We have two shearing stress components in the x-plane acting in y-direction called as tau$_{xy}$. We have stress in the x-plane in the z-direction which we call as tau$_{xz}$. When it comes to this particular plane which is at a distance of dx from this plane and likewise let us assume that this length is dy and this is dz so the stress which will be acting in this which is the normal stress will have a component as sigma$_x$ plus del sigma$_x$ del x which is acting over the length dx. Likewise we will have the shearing stress component tau$_{xy}$ which is varying from this end to this end we will have tau$_{xy}$ plus del tau$_{xy}$ del x by dx.

We will have x in the z-direction that is tau$_{xz}$ and tau$_{xz}$ in on this particular plane so when it is coming to this plane there is a change over the length dx so tau$_{xz}$ plus del tau$_{xz}$ del x by dx. Likewise the stress in this particular plane normal to this which is the y-plane will have sigma$_y$, the stress acting normal to this is sigma$_y$ plus del sigma$_y$ del y by dy the length. The shearing stress component on the y-plane acting in the direction of x will have tau$_{yx}$, plus del tau$_{yx}$ del y by dy the length. On this plane we will have sigma$_z$ and the normal stress on the front z-plane is sigma$_z$ plus del sigma$_z$ del z over the length dz and so on.

Now if we take the forces which are acting in the x-direction and sum them up as for the equations of equilibrium the summation of all the forces in the x-direction must be equal to 0. If we write down the forces in the x-direction we have sigma$_x$ plus del sigma$_x$ del x by dx. So in the equation we have sigma$_x$ plus del sigma$_x$ del x by dx and acting over the area dy and dz minus sigma$_x$ acting over the area dy and dz.

Also, in this particular direction we have plus (tau$_{yx}$ plus del tau$_{yx}$ del x(dx)) and delta yx by del y by dy acting over the area dx and dz minus tau$_{yx}$, dx dz plus we have a term in the z plane acting on the x direction which is tau$_{zx}$ plus del tau$_{zx}$ del z by dz into dx and dy the area minus tau$_{zx}$(dx and dy) plus if we assume that X is the body force per unit volume then this multiplied by dx, dy and dz is equal to 0. So from this we will get del sigma$_x$ del x, if we cancel out these terms and divide the whole equation by dx, dy and dz we have del sigma$_x$ del x plus del tau$_{yx}$ by del y plus del tau$_{xz}$ by del z plus x is equal to 0 where x is the component of the body force.

Likewise if we take the equilibrium of the forces which are acting in the y and z-direction we get two other sets of equations and they are, del tau$_{xy}$ by delx plus del sigma$_y$ by del y plus del tau$_{yz}$ by del z plus y the body component force is equal to 0 del tau$_{xz}$ by del z plus del tau$_{yz}$ del y plus del sigma$_z$ del z plus z is equal to 0. These are called the equations of equilibrium.

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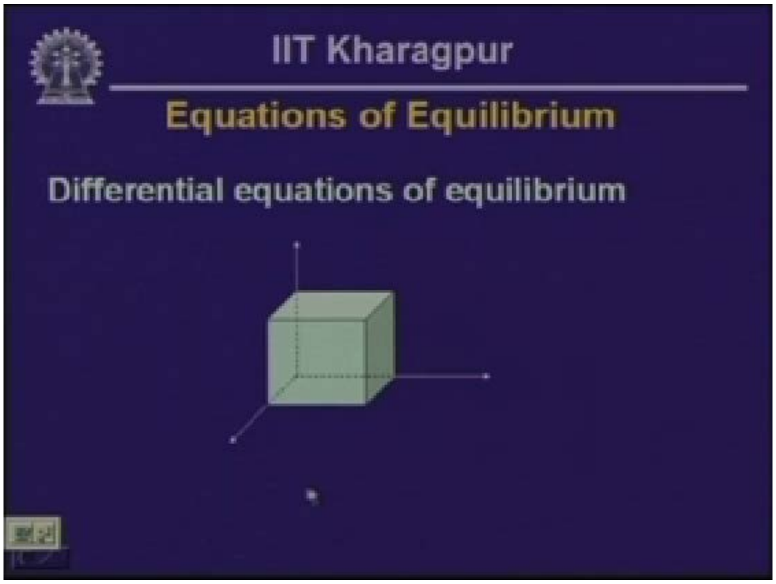

These equations of equilibrium can be written down in a two dimensional form as well. You can designate these in xy-plane, here we have x and here we have y, this is sigma$_x$ and the variation along the x is sigma$_x$ plus del sigma$_x$ del x by dx the length where this is the length dx and this is dy, this is tau$_{xy}$ plus del tau$_{xy}$ del x by dx the length. This is sigma$_y$ plus del sigma$_y$ del y by dy the length, then we have tau this is tau$_{yx}$ and this gives the variation of tau which is tau$_{yx}$ plus del tau$_{yx}$ by del y(dy).

These are the stresses in the two dimensional plane and if we take the equilibrium of the forces in the x-direction then we can obtain the equations of the equilibrium in two dimensional plane which could be del sigma$_x$ del x plus del tau and as yx and xy being the same we can write this as tau$_{xy}$ del y plus the component of the body force x is equal to 0.

The other equations will be del tau$_{xy}$ by del x plus del sigma$_y$ del y plus y is equal to 0. So these are the equations of equilibrium in two dimensional planes. Now, having known these stresses at a point, equations of equilibrium and how to evaluate those stresses or how to write down those stresses at different planes if we have to evaluate the stresses in an axially loaded member, then we have a member or we have a body in which we have a force acting in the axial direction, so let us call this body acted on by force P.

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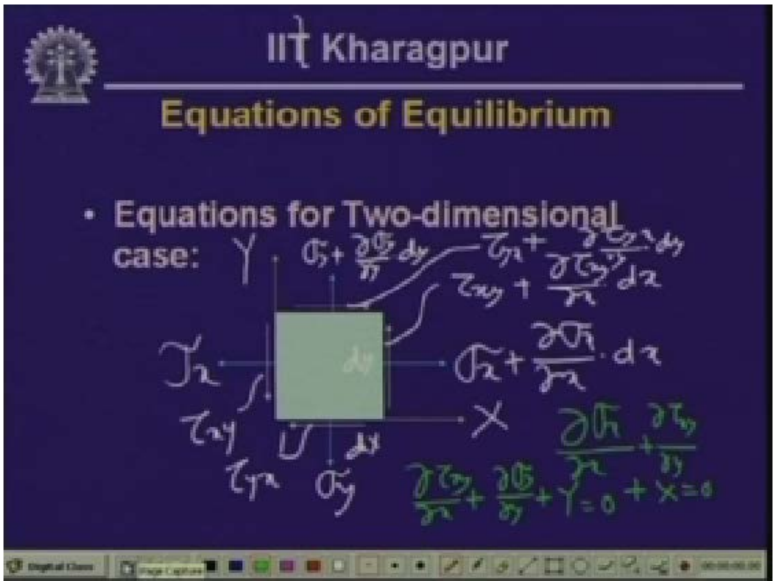

Now if we like to evaluate the stresses at any inclined plane let us cut this body by an inclined plane. And if we draw the free body diagram of this then we have the body in this form. Here we have the resistive force P which is acting. On this, the resulting force or the stress resultant is acting in this particular direction to equilibrate the body. Now we can take the components of this force in the normal direction, normal to this plane and along the plane which will give you the three forces of the stress component which is normal which we call as the stress corresponding to normal and the two shearing stress components tau.

If we concentrate on the two dimensional plane, if we take the axially loaded member the member is subjected to the load in the axial direction. Let us take a plane which is cutting this body in this form and let us assume that this plane is making an angle of theta with the vertical. If we take the free body diagram of this particular body this is angle theta, we have the force acting here as P, the resistive force acting on this body to keep the equilibrium is P so this will have two components one along the normal and one along the plane of this particular section.

Now this angle being theta, if we drop a perpendicular here this angle will also be theta hence this particular angle is also theta. So the force component along this is P cos theta and the force component along this direction is P sin theta. Now, if we say that the cross sectional area is A and the cross sectional area of this as A prime then the stress which is acting in this particular inclined plane if we say that normal stress sigma theta and theta being designated by this particular plane which has got an angle theta in the vertical, sigma theta equals to the normal force component which is P cos theta divided by this area A prime.

And A prime from geometrical property we can say A prime is equal to A by cos theta. Then P cos theta by A by cos theta is equal to P by A cos square theta. And the stress which is acting in the plane is P sin theta. The stress tau theta is equal to P sine theta by Acos theta so this eventually is going to give us P by 2A by sin 2 theta. So these are the two stress components on this inclined plane. The normal plane is P by A cos square theta and the stress which is parallel to the plane which is the shearing component is P by 2A sin2 theta. So the maximum value of sigma theta is when cos square theta is equal to 1 and is theta is equal to 0. And for this sin 2 theta as 90 degrees and 2 theta being 90 degrees so theta being 45 degrees is the maximum value of the shearing stress. So the maximum value of normal is P by A and the maximum value of shearing stress is P by 2A. Eventually the relationship between tau theta and sigma theta is that tau theta is half of sigma theta.

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We have seen the maximum normal stress which is P by A and maximum shearing stress is P by 2A. Shear stresses are at this particular body is acted on by the force P eventually the resistive force will be P by 2 and P by 2. At this level if we take the free body at this part if I cut here then this particular body will have P by 2 and this is also going to be P by 2. So at this particular section this is P by 2 and its resistive is going to be P by 2.

Now here there is little amount of eccentricity for this force to be transferred over here and this we ignore as the thickness is being smaller. This P by 2 is called as the shearing force and this shearing force divided by this area which is acted on between these two plates is called as the shearing stress. If we say this width is b and this width is t then the shearing stress is (P by 2) by b to the power t is equal to tau.

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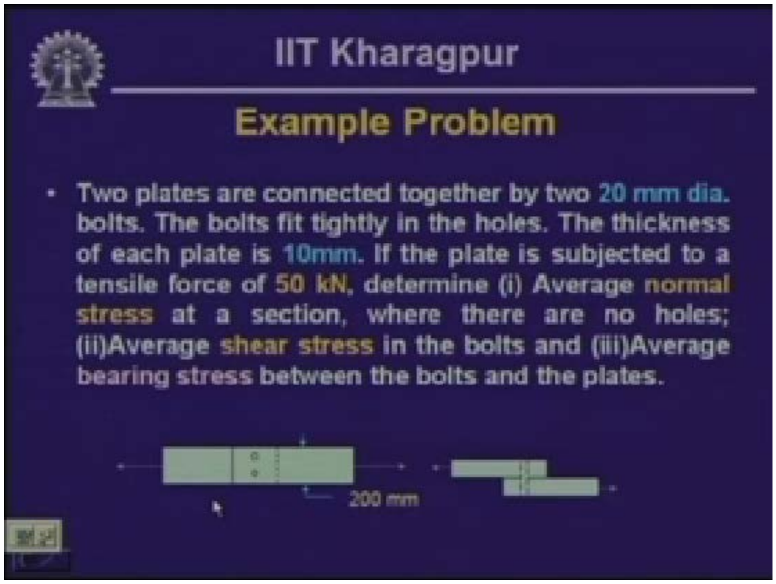

Now, here we are given a problem where two plates are connected by two bolts for which we have to evaluate normal stress, shear stress and the wearing stress.