Strength of Materials
Prof S. K. Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture - 18
Torsion - I

Welcome to the first lesson of Module 4 which is on Torsion I.

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In the last three modules we discussed about the aspects of stress, strain, and application of stress and strain on thin-walled pressure vessels. If a bar is subjected to an axial pull it is subjected to a normal stress which we have seen as load divided by the cross-sectional area. Consequently, we have seen that if the bar is subjected to axial pull it undergoes deformation to its original length which we have defined in terms of strain. In this particular lesson we are going to look into, that if this particular bar is subjected to another kind of force which we term as a twisting moment or torsion then what are the effects of torsion on such bars?

It is expected that, once this particular lesson is completed or once someone goes through this particular lesson one should be able to understand the concept of twisting moment and its effect on bars of circular cross-sections. Now we will be concentrating on the bars which are of circular cross-section and in this particular lesson we will be looking into the bars of solid circular cross-sections.

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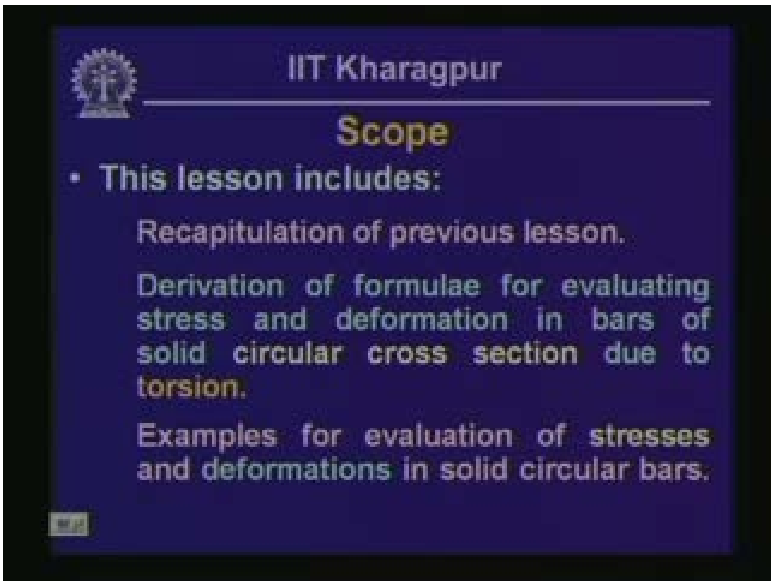

Also, one should be able to understand the concept of shear stress which gets generated due to torsion, and one should be in a position to evaluate stresses and deformation in solid circular bars which develop due to torsion.

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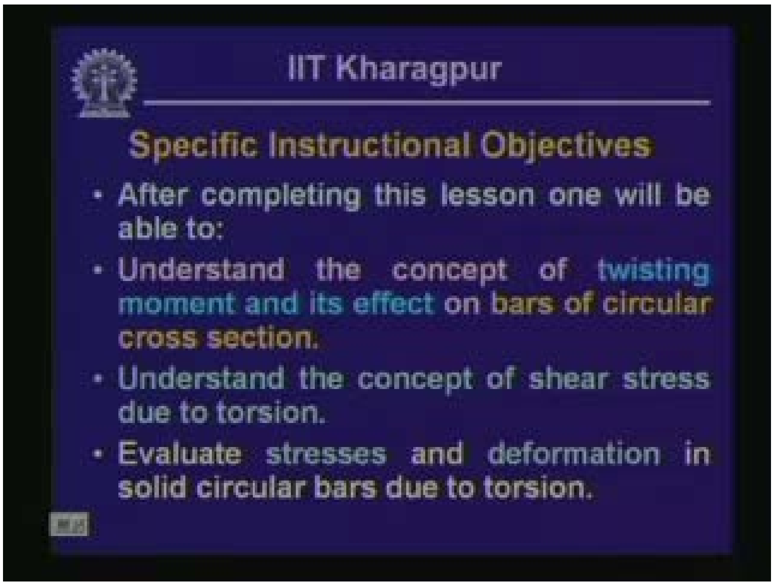

Hence, the scope of this particular lesson includes the recapitulation of previous lesson. We will be discussing the questions posed last time in the form of answers. We will be deriving the formulae which are necessary for evaluating the stress and the deformation in bars of solid circular cross-sections which arise due to the application of the twisting moment or the torsion and then we will be looking into some examples for the evaluation of stresses and deformations in solid circular bars. In the previous lesson of the third module we were discussing the aspects of thin-wall pressure vessels and we have seen some questions related to that. The first question was to define a thin-walled pressure vessel. That is an important question because the previous module was devoted to this. You should know how to define a thin-walled pressure vessel.

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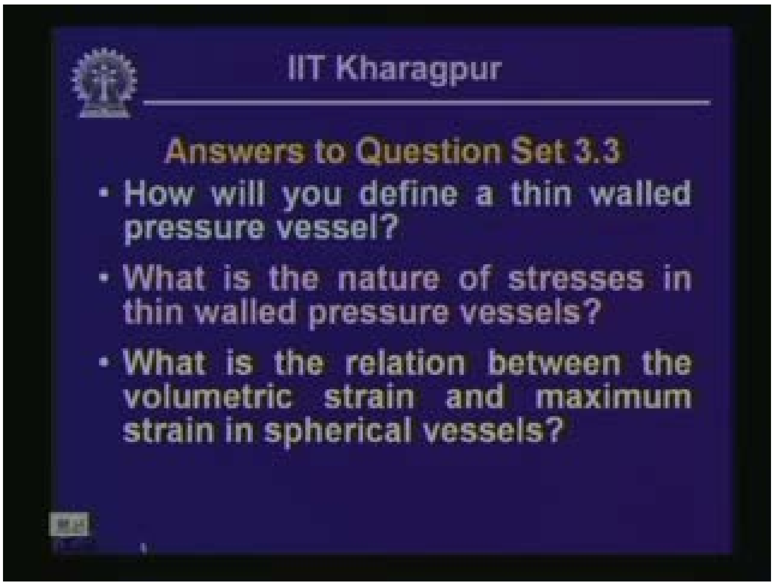

A thin-walled pressure vessel may be defined in this particular form. Basically, thin-walled pressure vessels are the closed pressurized containers and when these pressurized vessels are subjected to internal pressure they are subjected to stresses. We classify these pressure vessels as thin-walled pressure vessels. When the thickness of such a vessel is sufficiently small compared to the radius of the container the stresses are almost uniform throughout the thickness.

Across the thickness of the pressure vessel there is no change in stresses. If you consider the thickness the change in the stresses is significantly small and that is the reason we call these particular types of vessels as thin-walled vessels. It has been observed that, if the radius to thickness ratio, $\frac{r}{t}$ ratio ≥ 10, then the stress distribution across the thickness of the wall is significantly small. That means we can presume that, the stress will be the same or uniform and we consider that as a thin-walled pressure vessel. So, to define it precisely, we should say that when the thickness of the wall is sufficiently small compared to the radius so that the stresses are almost uniform throughout the thickness. These vessels are termed as thin-walled pressure vessels and the criteria is that, if $\frac{r}{t}$ or the radius by thickness ratio ≥10 then we call those kind of vessels as thin-walled pressure vessels.

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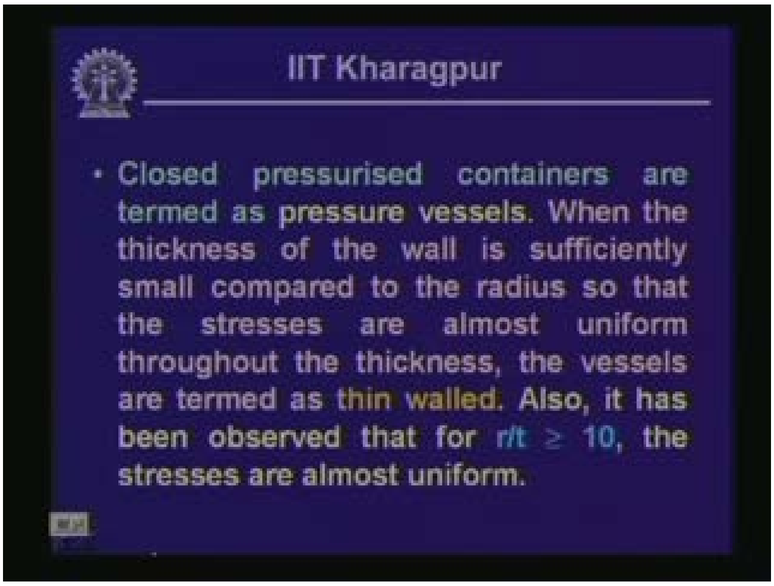

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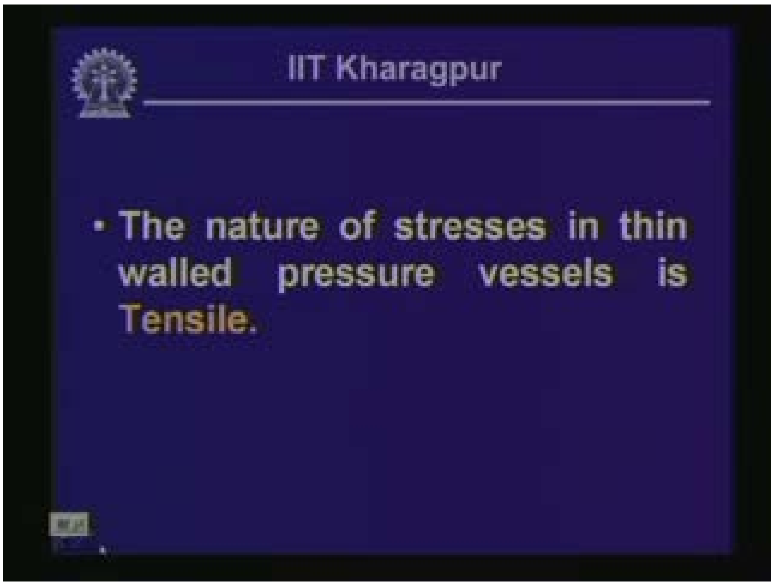

The second question was: what is the nature of stresses in thin-walled pressure vessels?
In the case of thin-walled pressure vessels the nature of the stresses is tensile. We have already discussed the type of stresses that are generated in pressure vessels. They are basically the tensile stresses which occur on the surface. In the case of cylindrical pressure vessels it is tensile stress along the circumference. We have called it as circumferential stress or hoop stress, and in the longitudinal direction we have the stress called longitudinal stress. Both the stresses are tensile in nature. So, as in the case of spherical vessels we have seen that the stresses which get generated in any direction are tensile in nature. So the stresses which develop in thin-walled pressure vessels are tensile in nature. The last question was what is the relation between volumetric strain and maximum strain in spherical vessel?

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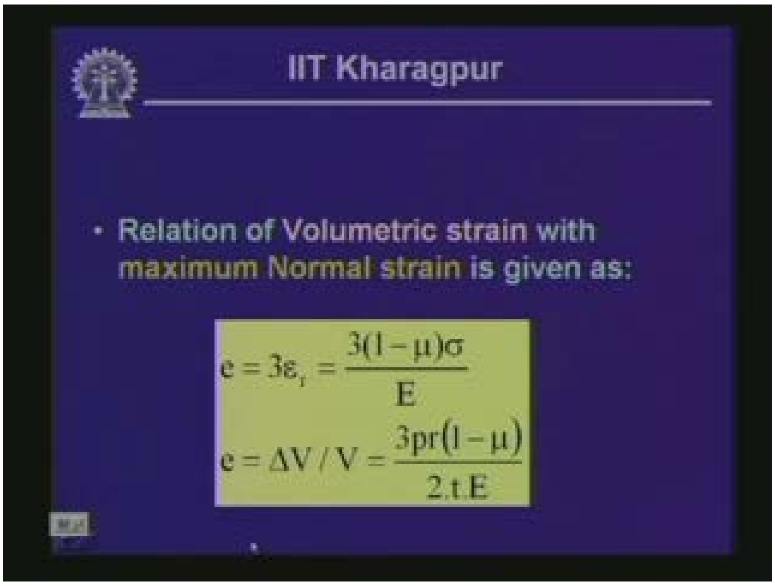

Here is an example in relation to the derivation of the strain. We had defined the volumetric strain e as the change in the volume to the original volume which is defined here as, e is equal to $\frac{\Delta V}{V}$. Now we had derived this as the radius r and because of the pressure it undergoes deformation and becomes r plus deltar. Thereby the change in the volume because of r plus deltar over the original volume gives us the expression for the volumetric string epsilon which we get as 3 into epsilon$_r$ where epsilon$_r$ is the normal strain which is developed because of the stresses.

In the spherical vessels if we take a small element we have the stresses developed into it in any direction which is equal to sigma. Hence the strain at any point can be written as $\frac{\sigma}{E} - \frac{\mu\sigma}{E}$ which is bi-axial in nature. So this becomes epsilon is equal to $\frac{(1-\mu)\sigma}{E}$. In the place of epsilon this is what has been substituted over here and this is what you see over here $\frac{3(1-\mu)\sigma}{E}$. So this is 3epsilon$_r$ and sigma being $\frac{pr}{2t}$ if we substitute that then it becomes e is equal to $\frac{3pr(1-\mu)}{2.t.E}$ so basically the relation between the volumetric strain and the normal strain is e is equal to 3epsilon$_r$. This is the basic relationship and finally we arrived at this expression in terms of the pressure, the radius of the vessels, Poisson's ratio, thickness of the vessel, and the modulus of elasticity.

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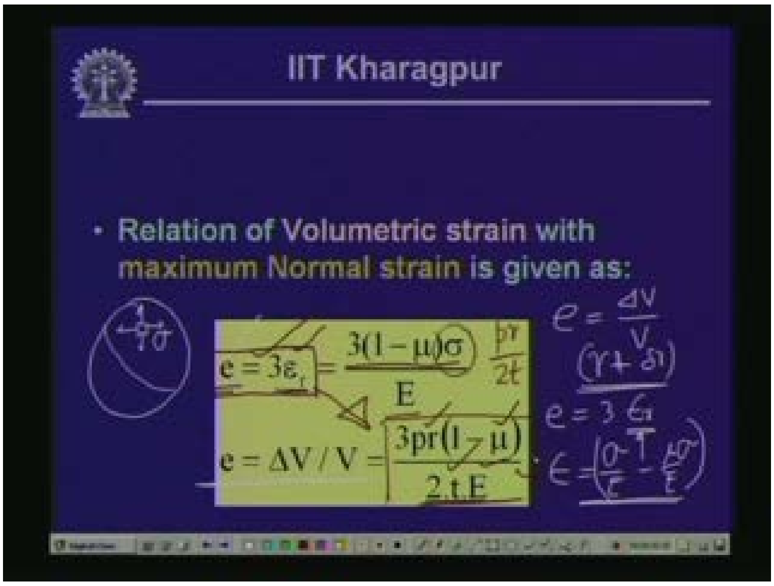

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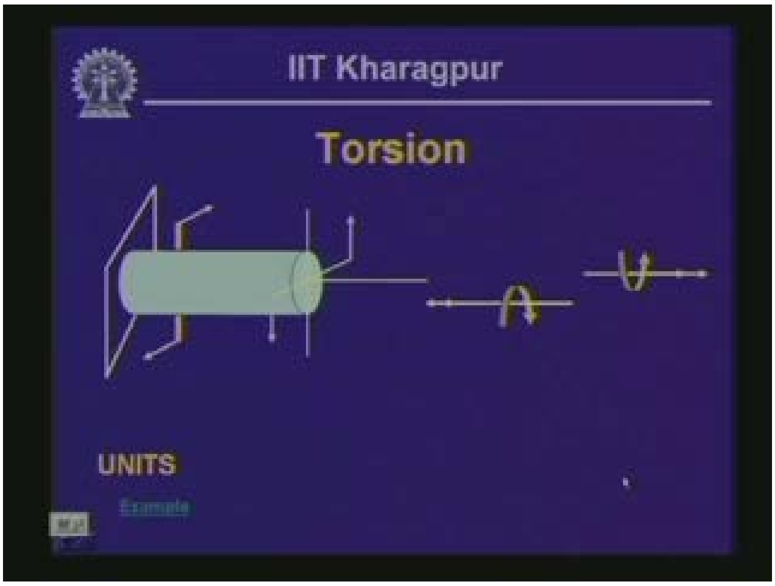

Having discussed about the pressure vessels, the nature of the stress we generally encounter and how do we define a thin-walled pressure vessel, and then the relationship between the volumetric strain and the normal strain in a spherical vessel now let us look into another example. If a bar is subjected to an axial pull, it is subjected to a stress which we have defined as normal stress which is nothing but equal to the load applied divided by the cross-sectional area.

Likewise, in the second module, we saw that if a bar is subjected to a load it undergoes straightening elongation or compression or shortening which we have defined in terms of strain which is deformation to its original length. Suppose we have a bar which has a circular cross-section and it is a solid bar as shown here, now if this bar is subjected to a load of a particular form which we generally call as a couple, these are the forces which are acting at different directions. Now if we say that the difference in the distance between these two being d, and if we call the force applied as p, then this particular bar which is fixed at one particular end will be subjected to a moment, the magnitude of which will be p(d). This particular moment is acting about the axis which is perpendicular to the cross-section, and is acting about the x-axis, the other two axes being y-axis and the z-axis.

Now, suppose a section or a particular member is subjected to a moment about an axis normal to the cross-section and is acting about the axis perpendicular to the plane of this board then the moment is acting about this particular axis called as the twisting moment or torsion. Now we derive here the relevant formulae on certain assumptions, and these assumptions are that the circular section (the cross-section) remains as it is. It does not get deformed and thereby there are no strains in the cross-section, there is no strain along the length and the axis of the member, and hence there are no deformations. Only because of this application of twisting moment the cross-section undergoes rotation and that deformation is defined in terms of rotation of this cross-sectional part. If this particular member is fixed at this end and if we take one length on this particular surface then let us see how it rotates because of the application of this twisting moment and let us find out the consequence of that.

Now let us look into another kind of loading which we have applied. Here we have another load z applied in this particular form. This is also coupled which is acting at a distance d again. This is also going to cause a twisting moment or a moment about the axis perpendicular or along the length of the member in this particular form acting in a clockwise direction. Hence this is also a twisting moment but the direction of this is different from the direction of the first one. Now let us look into these two directions and analyze it.

In the first case, when we had the moment which was acting about the x axis, it was acting in an anticlockwise direction. If you look into this particular figure here this is the axis and the twisting moment is acting in an anticlockwise form. Here it behaves in the form of a right hand screw and as the moment is acting in this form the direction of axis is towards the thumb. This is called the positive twisting moment where the vectorial direction is in the x direction. So the moment is acting in the bar in the form where the thumb is directed towards the positive x axis, and this we call as the positive twisting moment. Whereas when we consider the moment acting in a clockwise form the thumb goes opposite to the positive x direction called as the negative twisting moment and the corresponding rotation as a negative rotation.

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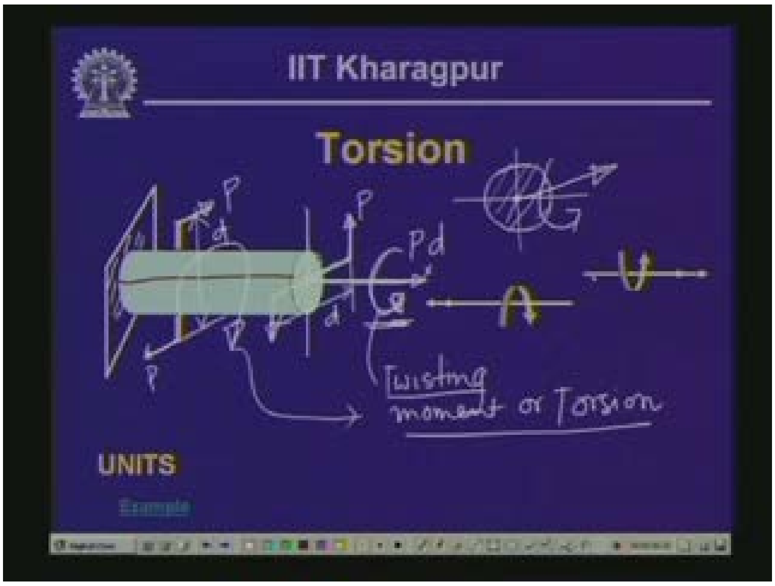

Hence, if you look into this particular figure where we have the twisting moment acting in a clockwise form and the vectorial direction of this twisting moment is opposite to the positive x direction. This is the negative twisting moment and that is the positive twisting moment. That is how we define the convention of these particular twisting moments of the torsions. We can either represent this twisting moment in this form negative twisting moment or in terms of vectorial direction notation where both the notations represent the positive torsion also defined as a negative twisting moment. Consequently, for this we can either give a twisting moment in this form known as negative twisting moment or a vectorial direction notation in this form also known as negative twisting moment.

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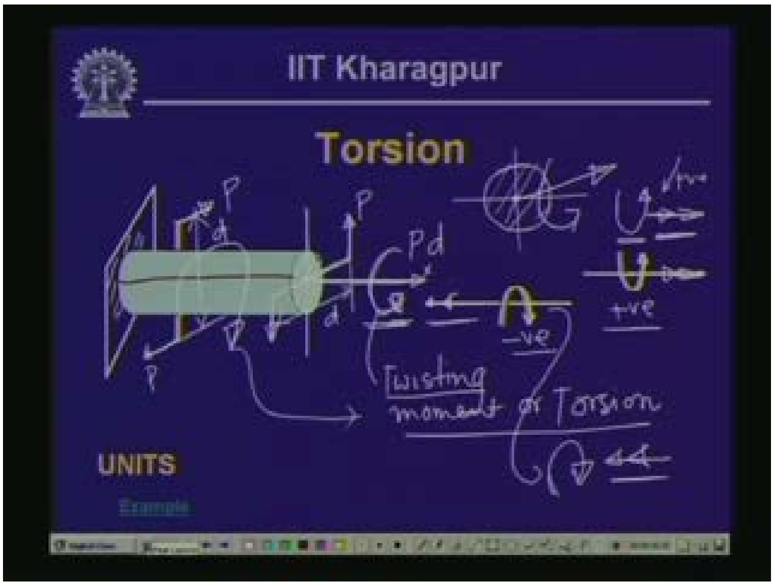

This is how it gets rotated when a twisting moment is applied on the bar which was initially straight. This is how it gets rotated. Here this end being fixed it moves and then forms an angle over here. This particular original radius is getting changed or there is an angular change over here, and this is what we will be looking into while deriving the formulae for torsion.

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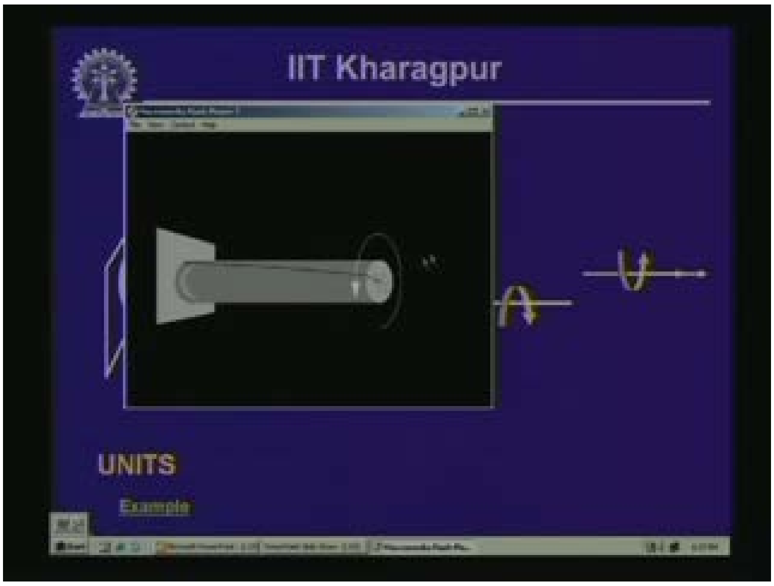

Coming to the units of moments here the unit of the force is Newton and the distance is written in meters. Hence the twisting moment or the torsional moment which you have has the unit Newton meter Nm.

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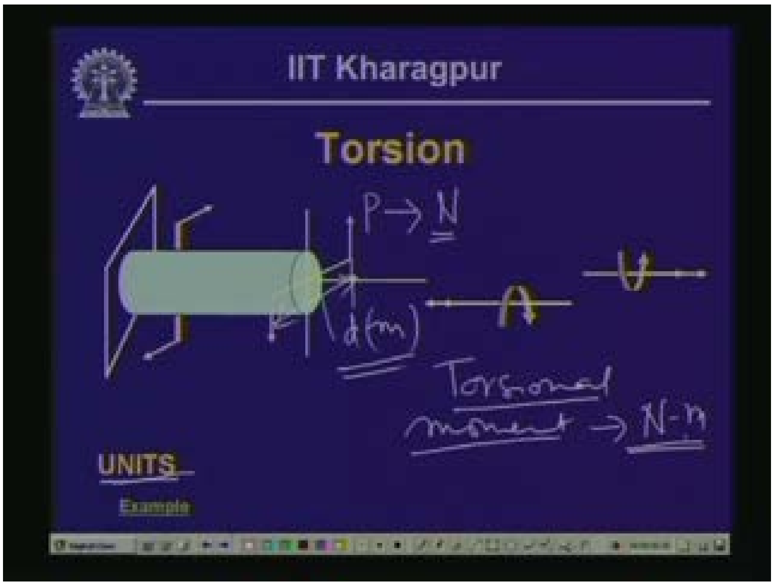

Let us look into this particular configuration which is again a solid circular bar, the cross-section is a circular one and it is solid. It does not have any hollowness in such a system. If this particular bar is subjected to a positive twisting moment then what happens to this particular system?

We have one length of this particular bar as AB. Here this particular bar is fixed and let us call this point as B. Now this is fixed at this particular end and it is subjected to a twisting moment T at its end. Because of the application of the twisting moment it will undergo rotation and this helix line will be formed and thereby the radius of the cross-section r will simply change and there will be a deformation angle which we call theta. Because of the application of the twisting moment, the radius vector changes, the angular section moves and there is a change in the angle which we call theta. There are some assumptions which we make.

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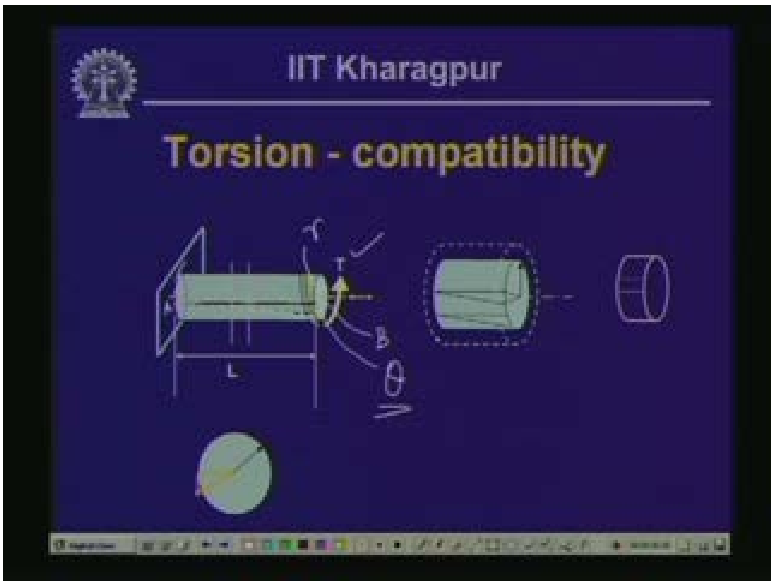

The circular cross-sections remain plane and perpendicular to the axis of the shaft. There is no change in the circular cross-sections and they remain plane before the application of twisting moment. Since there is no change in the cross-section, there is no deformation and hence there is no strain developed into the cross-section so the cross-sections do not deform.

Secondly, the distance between the cross-sections does not change and hence the axial strain is absent. Since there is no change in length of the member, there is no deformation, and thereby there is no strain in the axial direction. These are the assumptions we make based on which we derive the relationship between the twisting moment and the deformation.

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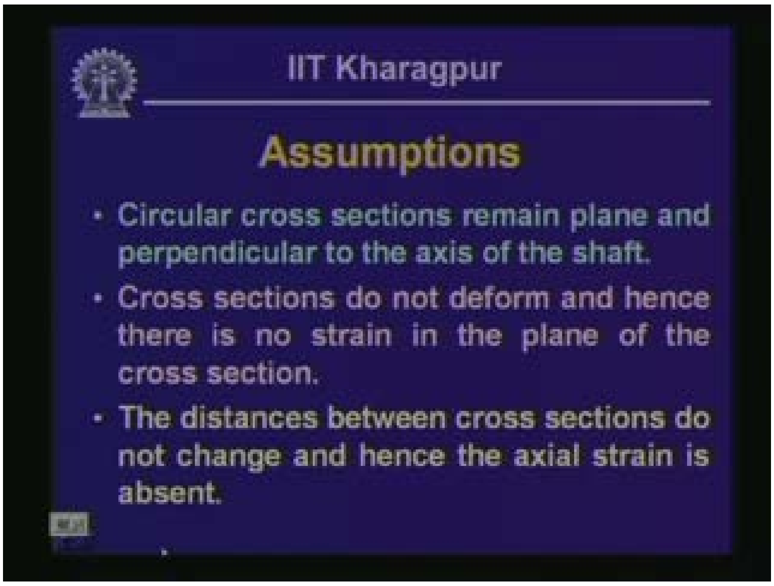

Now let us look into how we derive the relationship between the twisting moment and the angle of rotation. Because of the application of this twisting moment, the length which was originally straight forms this helix, and thereby there is a change in angle called as theta. If we cut a segment from this particular length, let us call this as length dx, also what we do is, out of the whole circular bar we cut off some top portion and take a smaller core. Let us assume that the smaller core is having a radius rho rho. Originally this particular bar has a radius r and the core which we are looking into has a radius rho rho.

Now, if we concentrate on this segment which we call as AB and this as CD or AB and DC it is an element on the surface of this particular cylindrical part or the circular body. When this is subjected to a twisting moment then the radius which is the line joining from center to B undergoes rotation. Since this is a small distance dx, let us say that the change in angle between these two segments of the cross-section is also small and let us call that as dtheta. If we look into the distance on the surface of this particular body, B moves to this particular point which is called as Bb prime. That is, if we look into distance the Bb prime from the cross-section point of view the radius is rho rho and the angle is dtheta so Bb prime is equal to rho(dtheta).

The segment ABDC undergoes change on the surface in this particular form. Hence there is a change in the angle. Because of the twisting moment the ABDC element has undergone deformation and there is a change in the angle. As we have seen in the past, this change in angle on the surface is called strain angle gamma. If this is the strain angle gamma and the distance is dx, distance Bb prime is equal to gamma(dx). When we equate these two gamma(dx) is equal to rho dtheta. Then, we can write gamma is equal to rho $\frac{d\theta}{dx}$. This is the shearing strain written in terms of the rotation and $\frac{d\theta}{dx}$ is called as the rotation per unit length.

Hooke's law states that the stress is proportional to the strain, tau, the shearing stress is related to shearing strain through the shear modulus G so tau is equal to Ggamma (the Hooke's law). So, if we substitute for gamma which is rho $\frac{d\theta}{dx}$, then the expression for tau becomes G rho $\frac{d\theta}{dx}$ or we can write this as (G $\frac{d\theta}{dx}$) rho. If you look into this quantity G$\frac{d\theta}{dx}$, it is independent of radius and as a result the shear stress is linearly dependent on the radius rho. It varies linearly from the center to the outer surface and the stress becomes the maximum on the surface of the cylindrical bar and is 0 at the center. Since it is varying linearly with rho along the section if you look into it, when rho becomes r, it goes to the surface and thereby you get the maximum stress.

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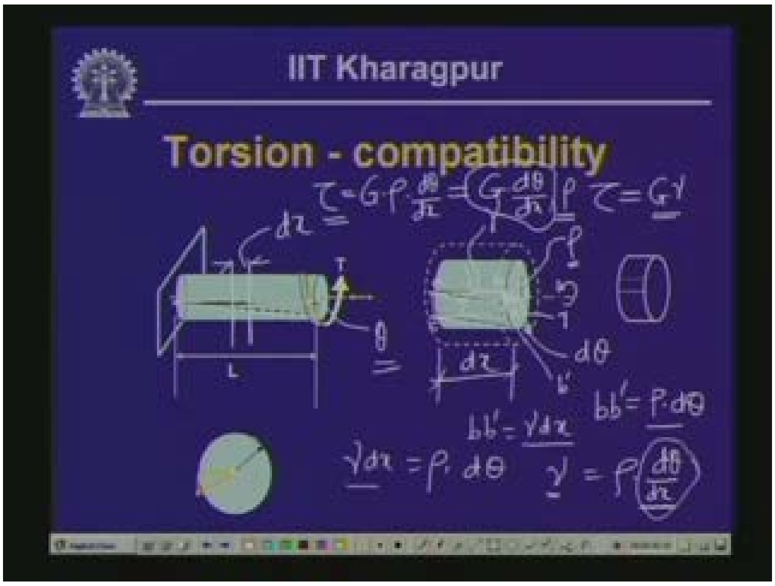

If you look into this particular figure, this is the radius vector here r and now at a distance of rho rho as considered in this particular configuration as rho, the shear stress is tau given by tau is equal to G $\frac{d\theta}{dx}$* rho. Now when this one goes to the surface it becomes tau_{max}, tau_{max} is equal to G $\frac{d\theta}{dx}$r, when rho becomes r which is the maximum shear stress.

Now if we look into an element which is taken out from here, this particular element as we have seen has undergone change in the angle which we have called as strain angle gamma and consequently it will be subjected to the stresses which we have named as shearing stress tau and the complementary will be in the perpendicular direction. This is the stress that gets generated on the surface of the cylindrical specimen or a bar which is of circular configuration.

What we need to do is to find out the relationship between the twisting moment and the stress so that we can evaluate the stress in a bar when it is subjected to a twisting moment called as the torsion formula. The torsion formula is; the relationship between the torsional moment which is acting on the bar and consequently the stress that gets developed on the bar.

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What we have seen now is the compatibility criteria. When the body is subjected to a twisting moment it undergoes deformation in terms of rotation and we have related the deformation to the stress. Let us write down the equilibrium equations so that we can write down the relationship between the stress and the corresponding twisting moment. The equilibrium criteria demand that the resultant shearing stress which will be acting on this surface should be equal to the twisting moment that is being applied in the section. The twisting moment that is applied is t. When we are applying this twisting moment the shearing stresses are generated. Here this is the center of the cross-section.

Let us take an element whose area is dA. So the force that will be acting which is perpendicular to this radius is equal to the stress times the area. Hence if we say that the force which is acting on the small element as dP is equal to the stress tau times area dA; dP is equal to tau dA. We have already seen that tau is nothing but, G $\frac{d\theta}{dx}$ rho. This is the stress on this element which is at a distance of rho from the center times dA. So the resistive moment that is being applied by this particular element will be the force multiplied by this distance rho. So the twisting moment T, which is acting on this particular cross-section, is nothing but the summation of all such forces over this particular cross-sectional area which is ∫dP into rho the distance. This is equal to integral over the area. If we substitute dP in terms of G $\frac{d\theta}{dx}$ rho, and rho into rho becomes rho square into dA so that is the twisting moment that is applied in this particular cross-section.

Now G $\frac{d\theta}{dx}$ being a constant parameter let us take that out and this is equal to ∫rho square into dA. You can recognize that this particular term rho square into dA is the second moment of this smaller area with respect to the axis which is passing through the center of this particular cross-section. And as we know, this is nothing but the moment of inertia of this particular section about the axis which is passing through the center of this cross-section which is perpendicular to the plane of this cross-section.

We generally designate this as J and call it the polar moment of inertia. This is the sum of moment of inertia in x and y axes so J is equal to I_x plus I_y, where I_x is the moment of inertia of the section about xx axis and this as yy axis and the perpendicular direction as z which we have designated as x here is the sum of the moment of inertia in the planar part which if we add up we get J the polar moment of inertia. So this is the expression for torsion T is equal to G $\frac{d\theta}{dx}$ J.

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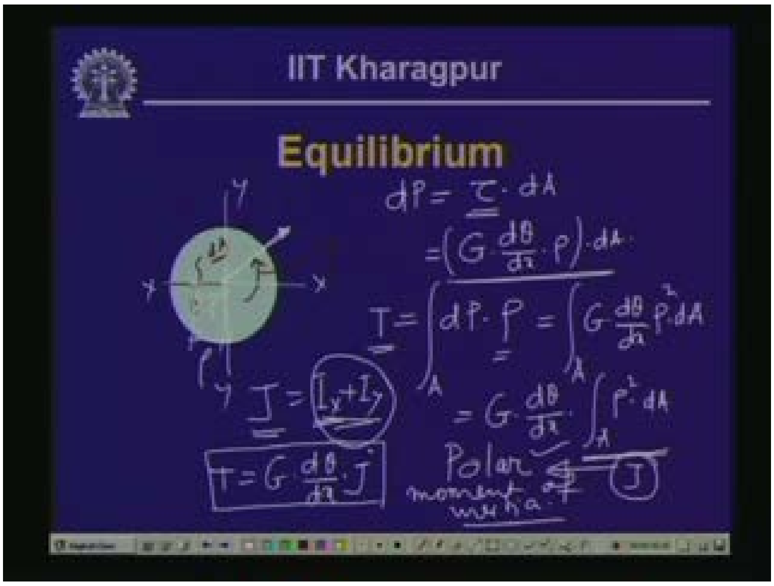

Now if we try to evaluate the rotation over the entire length of the bar, here the twisting moment which is applied in the bar is related to the rotation part in unit length in terms of the polar moment of inertia of the section and the shear modulus. If we like to find out that the bar which is fixed at one end and subjected to twisting moment on the other, what is the change in the angle or the change in rotation over the entire length of the bar?

We have seen T is equal to GJ $\frac{d\theta}{dx}$ or dtheta is equal to $\frac{T}{GJ}$ dx. If we integrate this over the entire length then we will get the rotation between the two ends and this is theta is equal to integral dx which gives us the x and the length l. So $\frac{T.L}{G.J}$ is the deformation of the member which is subjected to a twisting moment and the deformation is in terms of the angular change which is the rotation between two ends. The theta is the change in rotation between the two ends over the length l which is subjected to a twisting moment T.

Now if we compare this particular expression with the deformation that we had derived in module II wherein when the bar is subjected to a load p we have said delta is equal to $\frac{PL}{AE}$ which is the deformation. Here the deformation in terms of rotation is analogous to p where we have the twisting moment T, the length of the bar, and there we had the modulus of elasticity but here we have the shear modulus and in place of cross-sectional area here the polar moment of inertia J comes into the picture. So this is the rotation theta that we can have.

Now if we like to relate the shear stress with the twisting moment, because of the application of the twisting moment the shear stress gets generated and if we like to evaluate that, we had seen tau is equal to G $\frac{d\theta}{dx}$ rho. From this particular expression we can see that $\frac{d\theta}{dx}$ is equal to $\frac{T}{GJ}$.

So, if we substitute for $\frac{d\theta}{dx}$ as $\frac{T}{GJ}$ this gives us $G\frac{T}{G.J}\rho$ it gives us $\frac{\tau P}{J}$ or $\frac{\tau}{P}=\frac{T}{J}$. This is the expression of shear stress which is related to the twisting moment T. We have just seen that theta is equal to $\frac{TL}{GJ}$ and we can write down $\frac{T}{J}$ is equal to $\frac{G\theta}{L}$ and from here we get $\frac{\tau}{P}=\frac{T}{J}$ so by combining these two we can write down the torsion formula $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ is equal to $\frac{G\theta}{L}$ which relates the shear stress with the twisting moment and also the angle of twist over the length of the member. From this equation we can evaluate that; if a bar is subjected to a twisting moment then what will be the stresses in the bar in terms of the sectional parameters.

We can evaluate the strain once we know the shear modulus, we can find out the angle of rotation, angle of twist, and find out that because of this twisting moment how much angular deformation a bar undergoes. In the previous cases we were evaluating the stress when the bar was subjected to axial pull, subjected to axial force in the bar. Here we are computing the deformation and the stresses of the bar when it is subjected to a moment which is about the axis perpendicular to the cross-section.

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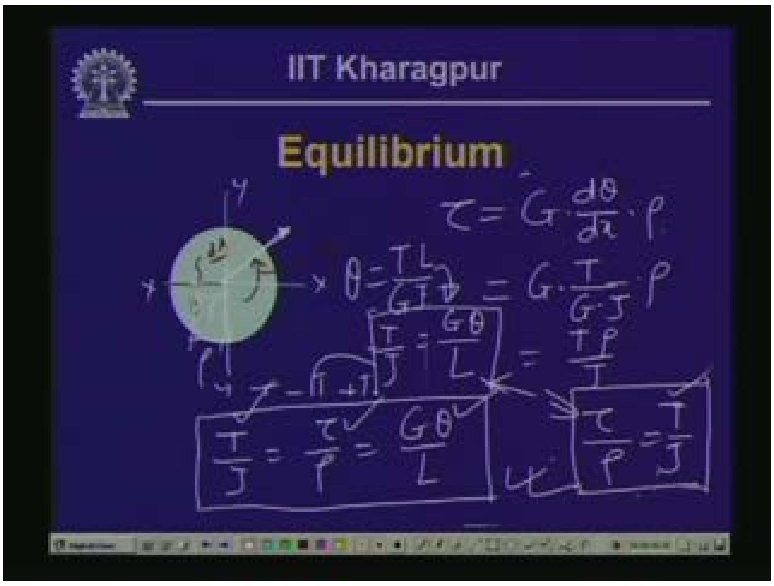

Let us look into some of the sample problems based on this discussion. Before we go to the sample problems for the twisting moments or the torsions, let us look into this example.

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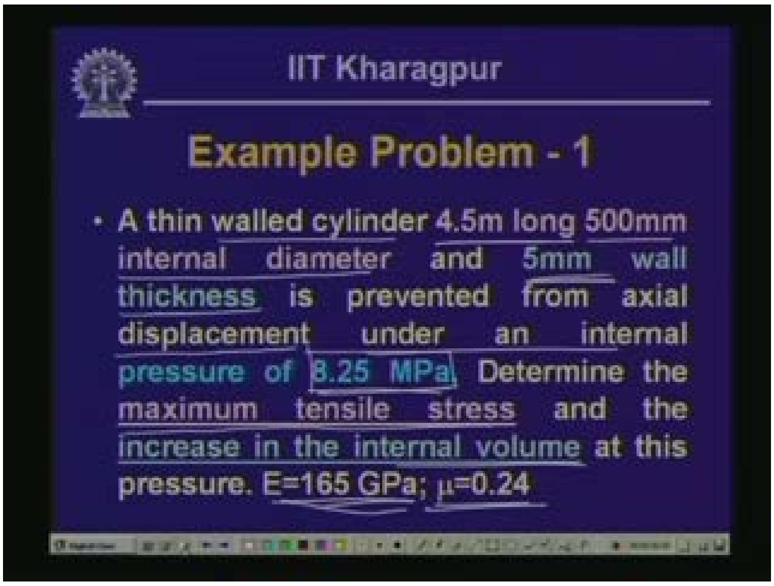

A thin-walled cylinder 4.5 meter long, 500 mm internal diameter, and 5 mm wall thickness is prevented from axial displacement under an internal pressure of 8.25 MPa. All we have to do is determine the maximum tensile strength and the increase in the internal volume at this pressure 8.25 MPa given the value of E is equal to 165 GPa and mu is equal to 0.24. Let us look into how you get the maximum tensile stress and then the change or increase in the internal volume.

Given length of the cylinder is equal to 4.5m; Internal diameter d is equal to 500 mm; the thickness of the wall t is equal to 5mm; the internal pressure is equal to 8.25 MPa; Modulus of elasticity E is equal to 165 GPa and mu, the Poisson's ratio is equal to 0.4. The maximum tensile stress that occurs on the surface of the cylinder on the cylindrical vessel is equal to sigma$_1$ is equal to $\frac{pr}{t}$ and p is equal to 8.25 MPa; diameter is equal to 500 mm, radius is equal to 250 mm, t is equal to 5 mm so sigma$_1$ is equal to $\frac{pr}{t}$ is equal to $\frac{8.25 \times 250}{5}$ is equal to 412.5 MPa.

Correspondingly the value of sigma$_2$ is equal to $\frac{\sigma_1}{2}$ is equal to 206.25 MPa. If we compute the value of the strain, which is epsilon$_1$ is equal to sigma$_1$ by E minus mu star sigma$_2$ by E is equal to 412.5 by 165 $\times 10$ cube minus 0.24 $\times$ 206.25 by 165 $\times 10$ cube is equal to 2.2 $\times$ 10 to the power minus 3. Consequently we can compute the strain: epsilon$_2$ is equal to sigma$_2$ by E minus musigma$_1$ by E is equal to 206.25 by 165 into 10 cube minus 0.24 into 412.5 by 165 into 10 cube so this consequently gives us a value of 0.5 into 10 cube.

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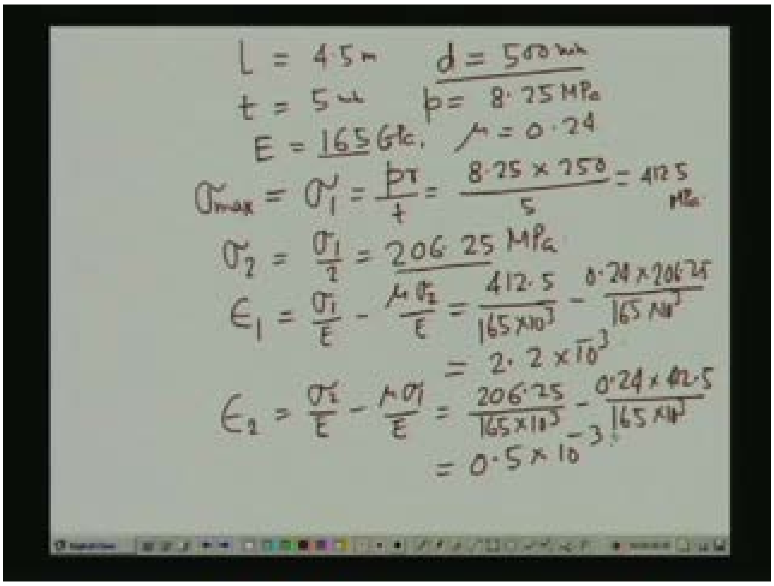

If you remember, for volumetric strain E is equal to deltaV by V. For cylindrical containers we have E is equal to 2 into epsilon$_1$ plus epsilon$_2$. epsilon$_1$ is equal to 2.2 into 10 to the power minus 3; epsilon$_2$ is equal to 0.5 into 10 to the power minus 3 is equal to 2 into 2.2 into 10 to the power minus 3 plus 0.5 into 10 to the power minus 3 (this is the volumetric strain) is equal to 4.9 into 10 to the power minus 3 so the change in volume deltaV which we will have to evaluate is equal to 4.9 into 10 to the power minus 3 into original volume is equal to pi into 250 square into 4500 is equal to 4.33 into 10 to the power 6 mm cube the change in the volume deltaV.

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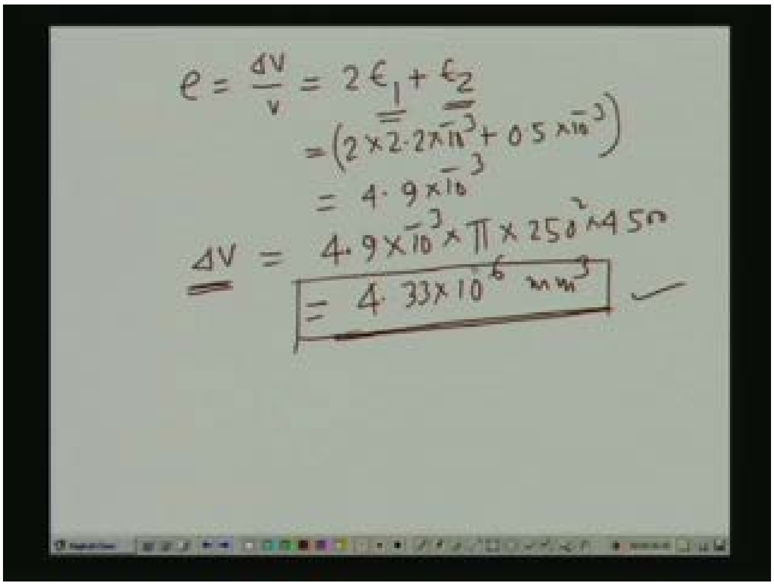

So the cylindrical container which was subjected to a pressure of 8.25 MPa is undergoing a change in volume which is 4.33 into 10 to the power 6 mm cube. Let us now look into the second problem based on the discussions we had today.

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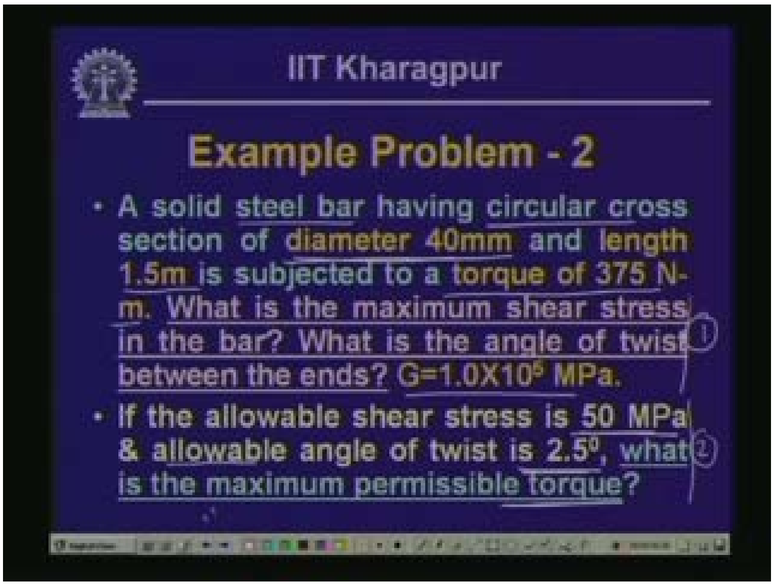

Here is a solid steel bar having circular cross-section of diameter 40 mm and length 1.5m subjected to a torque of 375 Nm.

What is the maximum shear stress in the bar? What is the angle of twist between the ends?
The value of G is given as 1 into 10 power 5 MPa .This is the first part. In the second part, if the allowable shear stress is limited to 50 MPa, and allowable angle of twist is 2.5 degrees then what is the maximum permissible torque?
In the second part, two aspects are defined. One is that the stress is limited to 50 MPa and the angle of twist also is defined as 2.5 degrees.

Constraints have been put on both sides. You will have to calculate the twisting moment, considering that the maximum stress is 50 MPa. You have to calculate the twisting moment also considering that the angle of twist cannot exceed 2.5 degrees.

Based on these two, which is the critical one?
That means the minimum of these will be the guiding twisting moment. If you go beyond that, one of them will fail. So let us look into the first part.

What is the maximum shear stress in the bar and what is the angle of twist between the two ends?
The diameter of the bar given is 40 mm. The length of the bar is 1.5m and the twisting moment that is acting is 375 Nm. The expression for the equation of torsion is $\frac{T}{J}=\frac{\tau}{\rho}=\frac{G\theta}{L}$.

For the given bar, the polar moment of inertia J is equal to; for pi d power four by 64 is ix plus iy pi d power 4 by 64 so twice of that is pi d4 by 64(ix plus iy) is equal to $\frac{\pi d^4}{32}$ the polar moment of inertia. And d is equal to 40; J is equal to ($\frac{\pi \times 40^4}{32}$) mm power 4; J is equal to (2.513 into 10 power 5) mm power 4.

Now we need to evaluate the shearing stress tau for the given twisting moment. So we take the first part of this equation where we write tau is equal to $\frac{T.\rho}{J}$. Now the twisting moment T is equal to 375 so Nm is equal to 3.75 into (10 cube)rho tau is equal to $\frac{3.75\times10^3\times20}{2.513\times10^3}$ is equal to 29.84 MPa tau the shear stress is equal to 29.84 MPa. Also, we need to evaluate the value of the angle of rotation theta; $\frac{\tau}{\rho}=\frac{G\theta}{L}$ and since the twisting moment is given let us take $\frac{T}{J}=\frac{G\theta}{L}$ so from this expression we get theta is equal to $\frac{T.L}{G.J}$; T is equal to 375 so $\theta = \frac{375 \times 10^3 \times1500}{1 \times 10^5 \times 2.513 \times 10^5}$ is so much of radiance and this comes as is equal to 0.0224 radiance and this is equal to is equal to 0.0224 into $\frac{180}{\pi}$ is equal to 1.28 degrees. This is the angle of rotation and this is the stress. These are the two parameters that we had to evaluate so shear stress and tau is given by this.

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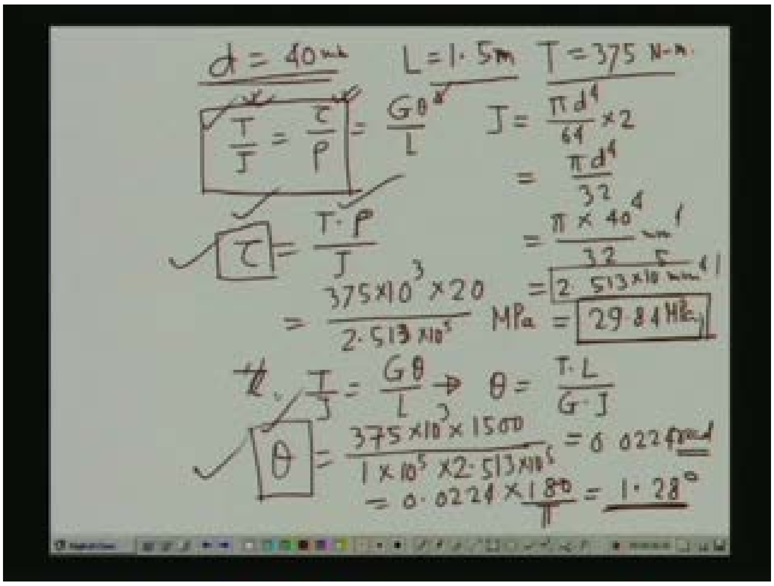

Now let us look into the second part wherein we have to compute the value of the twisting moment and the limiting stresses. The limiting stresses are that tau is limited to 50 MPa and the angular twist theta is 2.5 degrees. For these two cases we will have to find out T. Let us call the values corresponding to T as T_1 and corresponding to this as T_2. If we compute T_1 and T_2 lesser of these two will give us the value of the guiding twisting moment. We know that $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$. From these we compute the value of T where T is equal to $\frac{\tau.J}{\rho}$ and tau here is given as 50 MPa. So this is equal to 50 multiplied by J which is in terms of diameter pi into 40 power 4 by 32 is J and ρ is equal to d by 2 which is 40 by 2 is equal to 628.32 Nm. This is the amount of twisting moment that is necessary if we have to restrict the stress up to 50 MPa. If we go beyond this the stress level will exceed 50 MPa. So we cannot go beyond 628.32 Nm as the twisting moment because if we go beyond that then the stress level will go beyond 50 MPa. So if we have to restrict the stress in the bar up to 50 MPa then we will have to restrict the twisting moment to this particular value. If we have to restrict the angle of rotation to 2.5 degrees then what is the twisting moment that we need.

So for the second case we have theta is equal to $\frac{T.L}{G.J}$. From this, T is equal to $\frac{G.J.\theta}{L}$; G is equal to 1 into 10 power 5 and J we have pi into 40^4 by 32 and L is equal to 1500 and theta is equal to 2.5 degrees so 2.5 into pi by 180 so much of radian and this gives us a value of T is equal to 731.08 Nm.

Please note here that now we have got the two values of the twisting moment, one value of the twisting moment by restricting the stress up to 50 MPa. That means if we apply that amount of twisting moment the stress will be limited to 50 MPa. If we go beyond that value, the stress level will exceed 50 MPa.

In another case, we have got a twisting moment which is limited by the angle of twist. If we go beyond that, then the angle of twist will increase beyond 2.5. Since we will have to satisfy both the criteria, hence the limiting value of twisting moment will be 628.32 Nm. The maximum value of T which can be applied in this particular case is 628.32 Nm. So this is the answer for this particular example.

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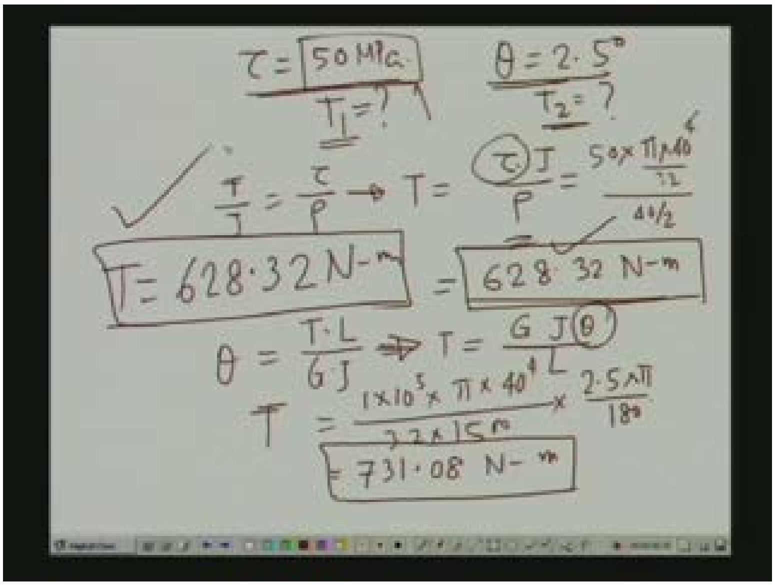

Now, let us look into another example, which is of a similar kind but a little difference is that here we are defining in terms of the strain. A copper rod of length 600 mm is to be twisted by torque T until the angle of rotation between the ends is 4 degrees. If the allowable shear strain in the copper is so much of radian, then what is the maximum permissible diameter in this particular rod?
This is the rod of length 600 mm subjected to a twisting moment. We will have to see that the strain does not exceed 0.008 in that particular rod. Then what is the value of the maximum diameter to resist this much of strain?

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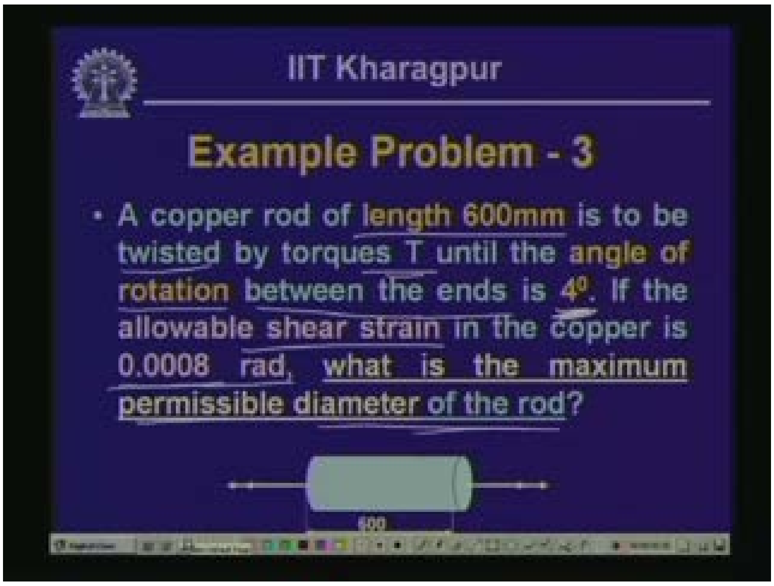

Here $\frac{T}{J}$ is equal to $\frac{\tau}{\rho}$ is equal to $\frac{G\theta}{L}$ so tau the shearing stress is equal to G gamma this is the relationship between the shearing stress and the shearing strain. So $\frac{\tau}{\rho}$ is equal to $\frac{G\theta}{L}$ or $\frac{\tau}{G}=\frac{\rho.\theta}{L}$ and $\frac{\tau}{G}$ is equal to gamma so gamma is equal to $\frac{\rho.\theta}{L}$ and rho is equal to $\frac{d}{2}$ so gamma is equal to $\frac{d\theta}{2L}$ here the diameter is to be evaluated d times theta which is limited to 4 degrees so this is multiplied by $\frac{\pi}{180}$ so much radian divided by 2 into L is equal to 600 and it comes to 0.0008 which is the limiting value of the strain and from this we will have to evaluate the value of the diameter d is equal to 13.75 mm.

In this particular case, the strain of the bar was limited. Because of the application of the twisting moment the strain cannot exceed beyond a specific value. Now to limit to that particular value of the strain we have to find out the maximum diameter of the bar that can be used. Utilizing the relationship between the torque and the stress and the relationship between the stress and the strain we can find out the diameter of the bar that can be applied.

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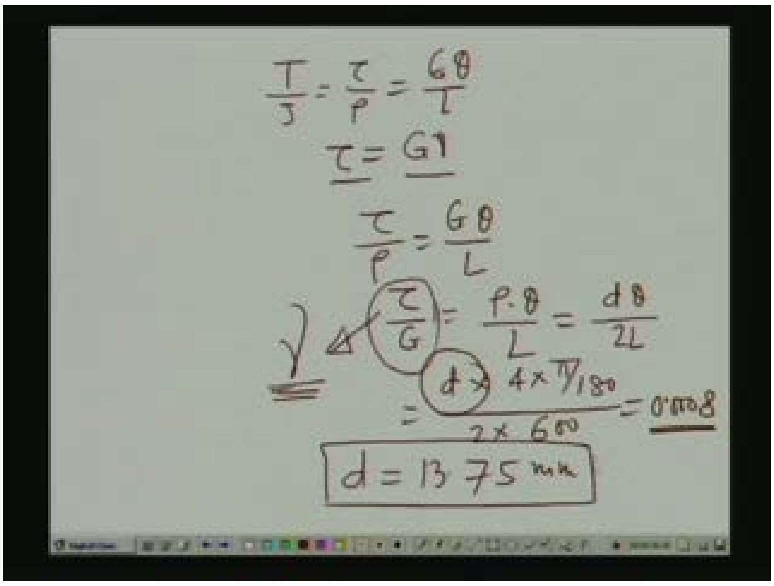

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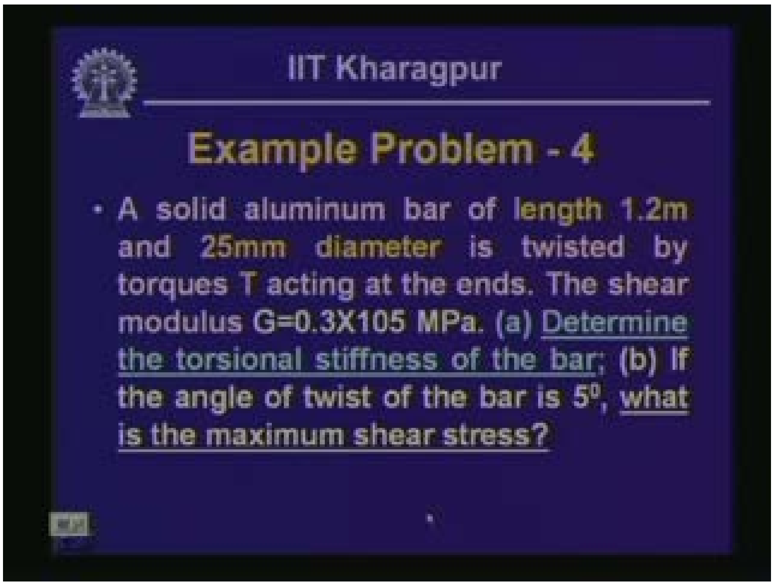

We have another example problem where a solid aluminum bar of length 1.2m and 25 mm diameter is twisted by torques T acting at the ends. The shear modulus is given and you will have to determine the torsional stiffness of the bar and secondly if the angle of twist of the bar is 5 degrees then what is the maximum shear stress?
These are the two parameters that we have to define.

We will now summarize this particular lesson: We have looked into the concept of torsion in a bar of solid cross circular section. We have taken a bar, the cross-section of which is a solid circular one with no hollowness. When such a bar is subjected to a twisting moment, then the consequences, the stresses and deformations that occur in such bar were learnt. We also saw the concept of stresses and deformation in a bar of circular cross-section due to torsion, and then we looked into an example to evaluate stresses, strain and deformation in bars of solid circular cross-section due to such torsions.

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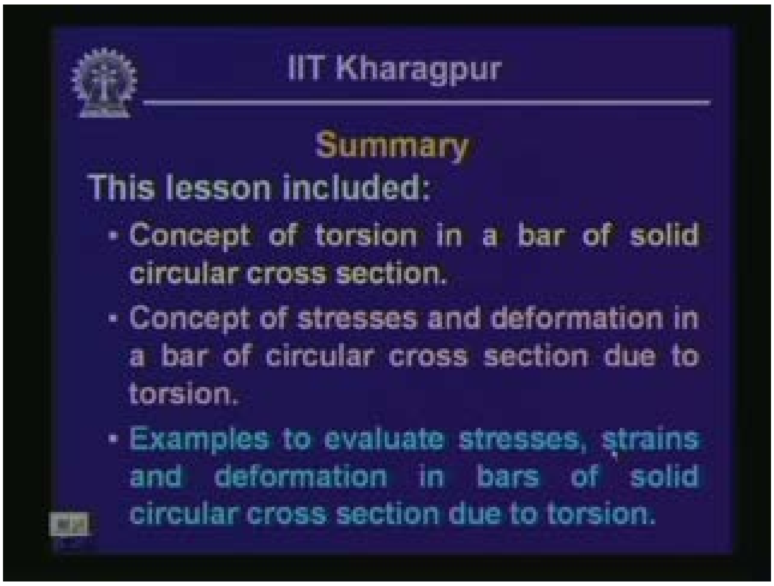

Questions:

What is the type of stress a bar encounters due to torsion?

Once you go through this lesson, you will be able to understand the stresses acting when a bar is subjected to twisting moment, the concept of torsional stiffness and so on.

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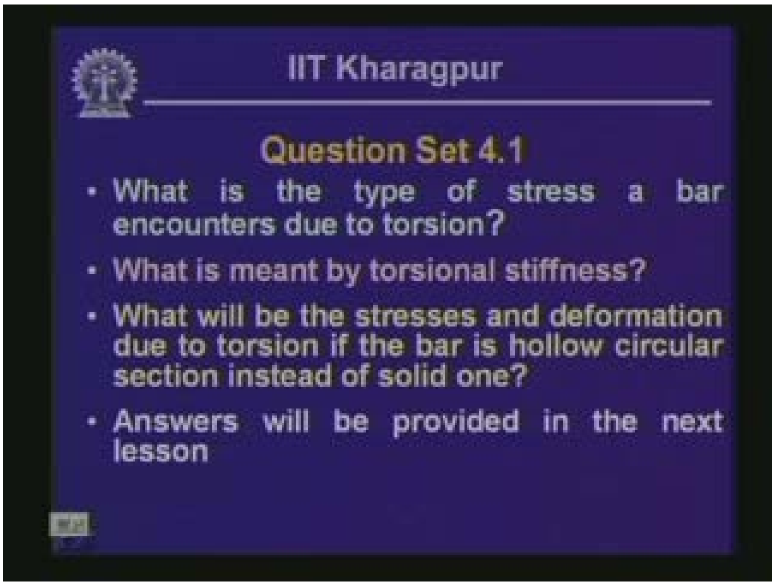

This aspect is important and the problem which is given as example is based on this. So you can find out what the torsional stiffness is and what will be the stresses and deformation due to torsion if the bar has a hollow circular section instead of solid one. The cross-section which we have considered here is a solid. If it is a tubular one (instead of solid) and if there is some hollowness within the bar, then we will have to see whether we can apply the same formulae which were derived here for solid section. Based on that, we have to find whether we can compute the stresses in a bar which is having the hollowness in the cross-section. The answers for these will be given in the next lesson.