Strength of Materials
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Lecture #17
Application of Stress/Strain
Thin-walled Pressure Vessels - III

Welcome to the third lesson of module three, this is on thin-walled pressure vessels. We have looked into some aspects of thin-walled pressure vessels in the last two modules. In this particular lesson we are going to look into some more aspects of thin-walled pressure vessels. This is on thin-walled pressure vessels part III which is of course on the application of stress strain.

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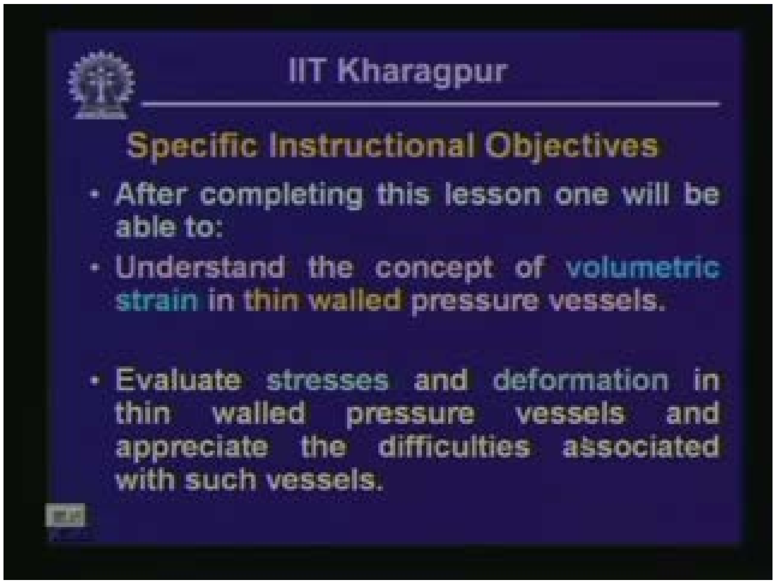

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It is expected that when one goes through this particular lesson they should be able to understand the concept of volumetric strain in thin-walled pressure vessels. In the previous lessons, we had looked into some aspects of cylindrical pressure vessels and spherical pressure vessels and how the strains are generated when it is subjected to internal pressure. Because of changes in the strain or change in the deformity, the cylindrical members of the spherical vessels undergo changes in their volume. We would like to evaluate the strain in terms of the volume. So change in volume over the original volume is known as the volumetric strain.

What is the value of those for different pressure vessels?
One should be able to evaluate stresses and deformation in thin-walled pressure vessels and appreciate difficulties.

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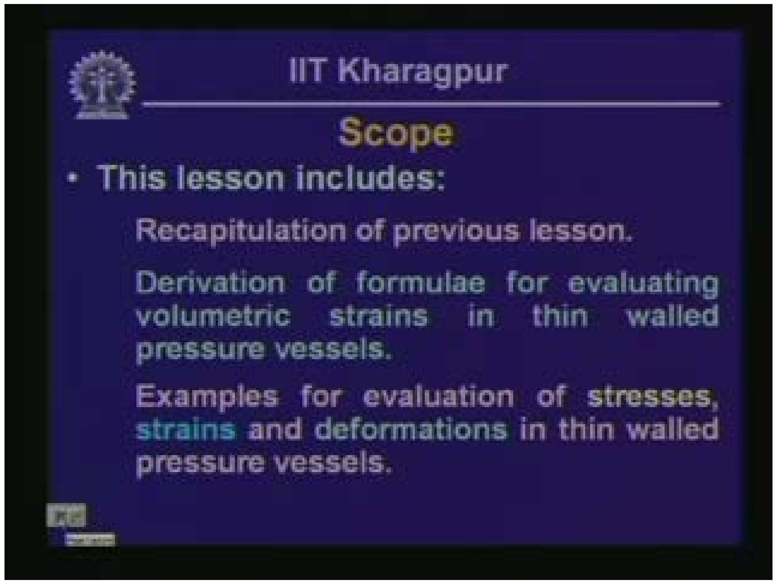

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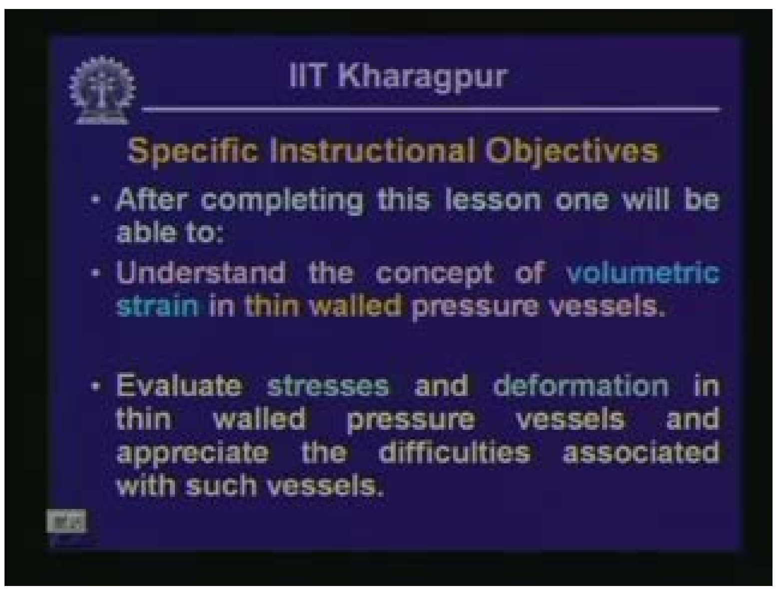

We will look into some aspects, especially when we discuss about the combination of spherical and cylindrical vessels. What are the problems related to deformation and what are the aspects to be looked into, when one goes for the detail analysis or design of such vessels. Hence, the scope of this particular lesson includes the recapitulation of previous lesson which we do generally through the question and answer session.

We will also look into the answers of the questions which I had posed last time. Thereby we will be able to recapitulate the aspects which we have discussed in the last lesson. We will be deriving the formulae for evaluating volumetric strain in thin-walled pressure vessels, both cylindrical and spherical type.

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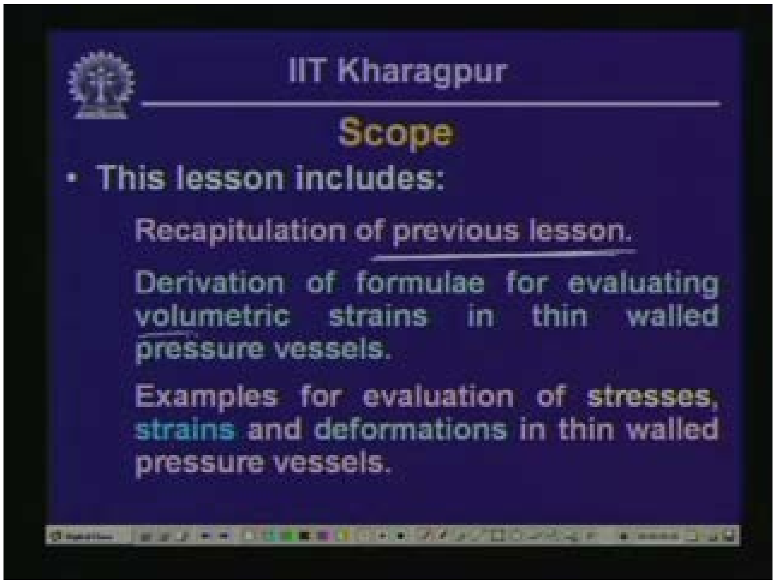

Then, we will be looking into some examples for evaluation of stresses, strains and deformations in thin-walled pressure vessels. These examples also will be related to cylindrical or spherical type pressure vessels.

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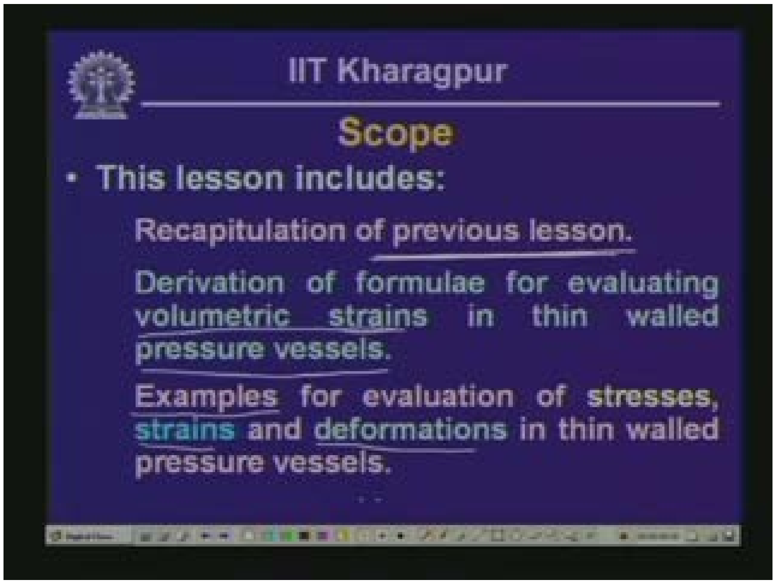

Questions asked last time is as follows:
What is the value of the maximum strain in spherical vessels? In the last lesson we had discussed aspects of the spherical vessels. If a spherical vessel is subjected to internal pressure, how does

the deformation take place and consequently what are the stresses and strain that occur in the spherical vessel. Now let us look at, what is the value of maximum strain.

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We had evaluated the values of the stresses. For a spherical system or the spherical vessel of this kind the stress that exists is sigma in both the x and y direction. The resistive force which is sigmaA is equal to 2pi rt where r is the internal radius of the sphere or spherical vessel and t is the thickness of this particular vessel.

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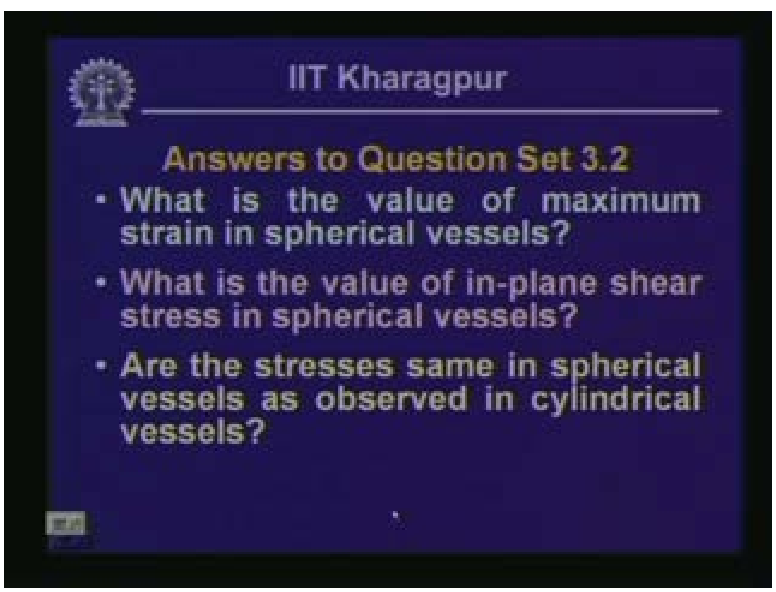

The pressure that is being exerted by the content inside the container P is equal to P(pi r) whole square. When we equate these two we get the value of the stress. This gives us the value of sigma is equal to $\frac{pr}{2t}$. This being a spherical vessel wherever we take a section through the center we get identical type of stresses. Hence on this particular element we have the value of normal stress as sigma both in the x and y direction. Based on this stress if we like to compute the strain in the x direction it is equal to $\frac{\sigma_x}{e} - \mu(\frac{\sigma_y}{e})$. Since sigma_x and sigma_y both are sigma, sigma_x is equal to sigma_y is equal to sigma. Hence, if we substitute over here we get $\varepsilon x = \frac{\sigma_x}{e} - \frac{\mu\sigma}{e} = 1 - \frac{\mu}{e}\sigma$. Corresponding to the maximum value of the stress which is equal to $\frac{pr}{2t}$ we can compute the strength ε which is the function of stress sigma. Therefore this is the maximum strain that occurs in a spherical vessel.

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The second question posed was; what is the value of in-plane shear stress in spherical vessels? For the in-plane stress in cylindrical pressure vessel if you have the values of sigmax and sigmay as sigma_1 and sigma_2, where sigma_1 was the circumferential stress and sigma_2 is the longitudinal stress then if we take these two stresses we can compute the strain and in-plane shear stress. In terms of these stresses now sigma_1 of the cylindrical pressure vessel was $\frac{pr}{t}$ and sigma_2 was $\frac{pr}{2t}$.

As we have seen in case of spherical vessel sigma is equal to $\frac{pr}{2t}$. Here both the values are sigma_1 and sigma_2 which were in the longitudinal direction in case of a cylindrical member. In the case of a spherical vessel, they are the same which is $\frac{pr}{2t}$. As we had seen in the case of a cylindrical vessel the shear stress is sigma$_1$ minus $\frac{\sigma_2}{2}$ is equal to sigma$_1$ minus $\frac{\sigma_2}{2}$. In this particular case since both sigma$_1$ and sigma$_2$ are the same or sigma$_x$ is equal to sigma$_y$ is equal to sigma, hence the value of the in-plane shear stress in case of spherical vessel is 0.

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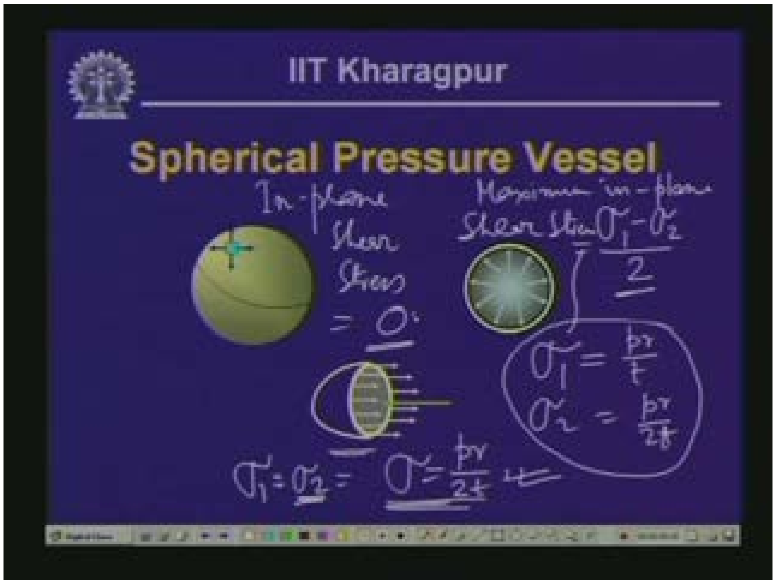

The third question was, are the stresses same in spherical vessels as observed in cylindrical vessels?
What we need to do is to look into the derivations we had done for both cylindrical as well as spherical vessels. If we look into these, we can immediately get the answer for the difference between the two. In case of cylindrical vessels, the componential stress which is sigma$_1$ and sigma$_1$ into t into l for the length L of this element was equal to P into 2r into L and here these L and L gets cancelled.

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Therefore sigma is equal to Pr/t and consequently the longitudinal stress sigma_2 we had obtained as $\frac{pr}{2t}$. In case of spherical vessel we have only one stress which is equal to sigma is equal to $\frac{pr}{2t}$. Please note that incase of cylindrical vessel, we have that circumferential stress, which we had called hoops stress is equal to $\frac{pr}{t}$ and the longitudinal stress is $\frac{pr}{2t}$ which is half of sigma. sigma_2 as we had seen earlier is equal to $\text{sigma}_1/2$ and the stress which we have in case of spherical vessel is equal to $\frac{pr}{2t}$ which is again half of sigma.

In case of the maximum normal stress which we get in cylindrical vessel, the maximum normal stress is equal to sigma_1 which is $\frac{pr}{t}$ and for the spherical vessel we get the maximum normal stress as is equal to $\frac{pr}{2t}$. Therefore the maximum normal stress in case of spherical vessel is half of that of cylindrical vessel. This aspect probably can be appreciated more through this particular diagram.

If we look into this stress corresponding to this sigma_2 which is the longitudinal stress, in case of cylindrical vessel, what happens is that, along the longitudinal axis the stress which exists really does not contribute to the equilibrium of the pressure which is being exerted on the curved surface by the content of the liquid. Only the stresses that resist the internal pressure is the circumferential stress and hence the value is larger whereas in the case of spherical vessel as shown here we get the stress from both the directions.

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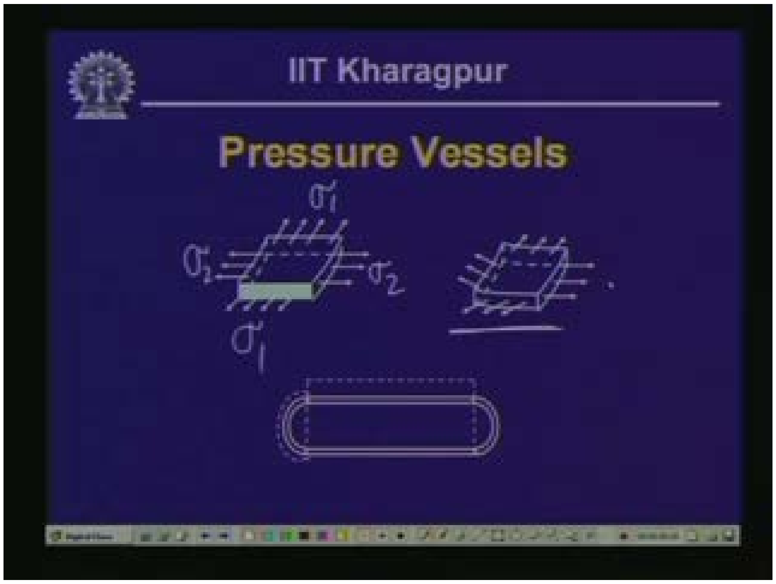

That means the stress or the pressure which is being exerted by the content of the spherical vessel is being resisted by the vessel in all directions. As we have seen in both the cases the cylindrical as well as the spherical vessel, the pressure which is exerted on the curved surface is normal to the surface. And since the wall is thin the variation of the stress across is negligibly small and we disregard that.

Hence we have a bi-axial state of stress which is in terms of $sigma_1$ and $sigma_2$. In the case of the cylindrical pressure vessel this longitudinal stress does not really contribute much to the equilibrium of the curved surface. Whereas in case of the spherical vessels the curved surface or the stresses from both directions comes from the resistance of the pressure exerted by the content. That is the reason of the stress.

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And in the cylindrical pressure vessel the stress is more when compared to the spherical vessel. In the case of spherical vessel we have uniform tensile stress over the entire surface which is normally called as the membrane stresses.

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If we have a vessel which is a combination of cylindrical vessel, at the two ends we have a spherical part, this is a combination of cylindrical and spherical vessel. The stress in cylindrical pressure vessel is $\frac{pr}{t}$ larger than the stress corresponding to the spherical vessel which is sigma is equal to $\frac{pr}{2t}$. If you compute the deformation, we will find that the deformation in case of cylindrical one which is given by delta, is larger than the deformation which is caused in the spherical $delta_1$. The problem arises at this particular junction where the container is a combination of the end of the cylindrical part and the starting of the spherical part. At this particular junction we have larger deformation from the cylindrical end and we have smaller deformation from the spherical end.

As a result, there is a mismatch in the deformation at that particular junction. Therefore as far as the whole assembly is concerned there should be continuity of the element. Thereby there will be some other kind of stresses generated apart from the stresses we are discussing over here. When we go for the design of such systems one should be careful in selecting the sizes as well as concentrating on the type of stresses that gets induced at that particular junction and accordingly the thickness has to be decided. These are some of the problems that can arise in case of such pressure vessels where you have the combination of cylindrical and the spherical pressure vessel.

For the time being we assume that geometrically there we will be smoothness and we will have a similar kind of pressure existing. Thus the corresponding deformation can be evaluated directly without considering the change in other kinds of stresses.

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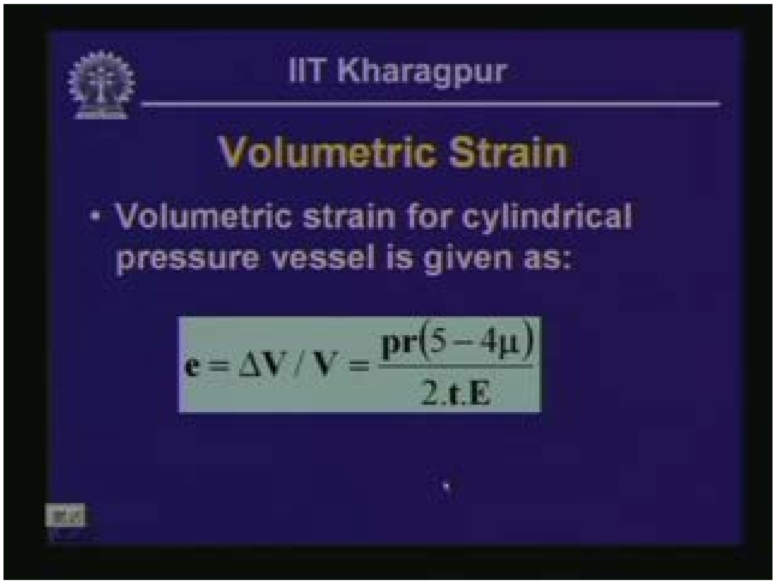

When you have a liquid inside a container whether it is a cylindrical or spherical vessel, it undergoes change in the diameter thus there is a change in the volume of the container. If a volume of a cylindrical or spherical vessel undergoes a change then we should be in a position to compute the changes in the volume.

As we have defined strain in the case of a linear element, the extension to the original length is the strength i.e. ratio of the extension or the deformation to the original length is the deformation. In case of volume change we define the change in volume to the original volume as the volumetric strength. Let us look into how you are going to evaluate the volumetric strain in cylindrical or spherical vessels. Let us look into the evaluation of volumetric strain for cylindrical vessels: The value of the volumetric strain e is defined as deltaV by V when we computed the value of the bulk modulus.

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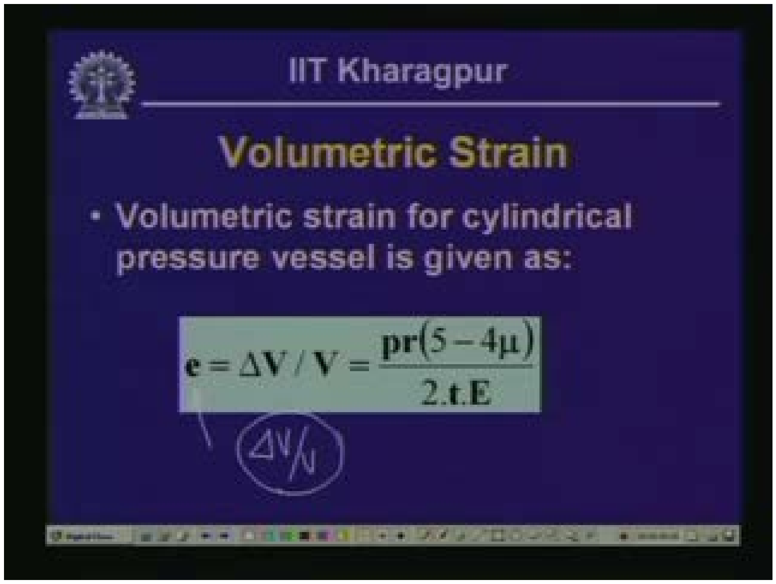

We can evaluate this volumetric strain as a function of the pressure, radius, thickness of the vessel and correspondingly the values of the material e and mu. Let us look into the evaluation of this volumetric strain. If we consider a cylinder of length L, radius r (Assume the internal radius is equal to r and length is equal to L).

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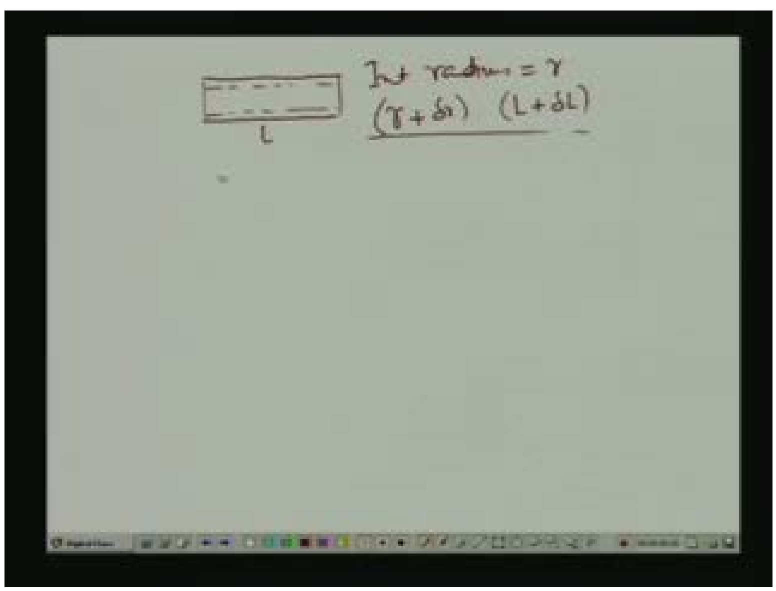

After deformation, the change is r plus deltar in the radius and consequently L plus deltaL is the change in the length after the content has exerted pressure and the dimensions have undergone some changes in terms of deformation. If these are the changes then the volume of the cylinder deltaV by V is equal to (pi r plus deltar) whole square into (L plus deltaL) the change volume minus pi r square L the original volume/pi r^2 L the final volume after the deformation has occurred. So it is; $\frac{(\pi r + \delta r)^2 \times (L + \delta L) - \pi r^2 L}{\pi r^2 L}$.

If you take out pi, and expand this (r plus delta square) is equal to (r square plus delta r square plus 2r into deltar) into (L plus deltaL minus r square L divided by pi r square L) (pi gets cancelled) is equal to $\frac{1}{r^2 L}$ is equal to [r square L plus Ldeltar plus 2rLdeltar plus r square deltaL plus deltaLdeltar plus 2(rdeltardeltaL) minus r square L]. This r square L and r square L gets cancelled, and deltar being small if we neglect the multiplication of deltar into deltaL) is equal to $\frac{1}{r^2 L}$[2rLdeltar plus r square deltaL and we neglect this quantity deltaL deltar, this is Ldeltar square so we neglect this term also and deltar being small the square of that will be very small, the product of deltar and deltaL also we neglect so we are now left with only these two terms. If we divide (r square L) then it is $2\frac{\delta r}{r}$ plus $\frac{\delta L}{L}$.

Now as you can realize that $\frac{\delta r}{r}$ is the strain in the radial direction which is known as $\in 1$ or sigma$_1$ is the stress in the circumferential direction for cylindrical vessel and $\frac{\delta L}{L}$ is in the longitudinal direction which makes the change in the original length.

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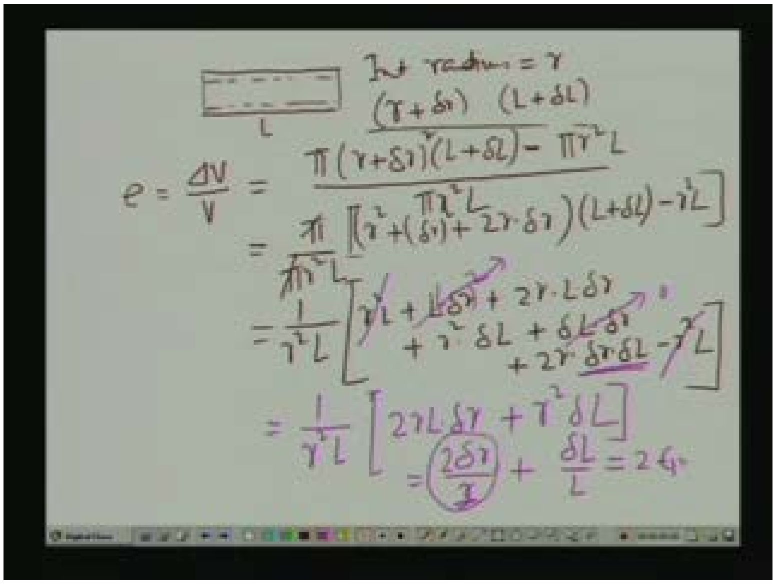

Now this is equal to $2\in 1+\in 2$ so notice that $\in 1$ is the strain in the circumferential direction and $\in 2$ is the strain in the longitudinal direction.

e is equal to $\frac{\Delta V}{V}$ is equal to $\frac{\pi(r+\delta \mathrm{r})^2\,(L+\delta \mathrm{L})-\pi \mathrm{r}^{-2}\mathrm{L}}{\pi \mathrm{r}^2\mathrm{L}}$ is equal to $\frac{\pi}{\pi \mathrm{r}^2\mathrm{L}}[(\mathrm{r}^2+(\delta \mathrm{r})+2r\,\delta \mathrm{r})\,(\mathrm{L}+\delta \mathrm{L})-\mathrm{r}^2\mathrm{L}]$ so the volumetric strain e is equal to $2\in 1+\in 2$.

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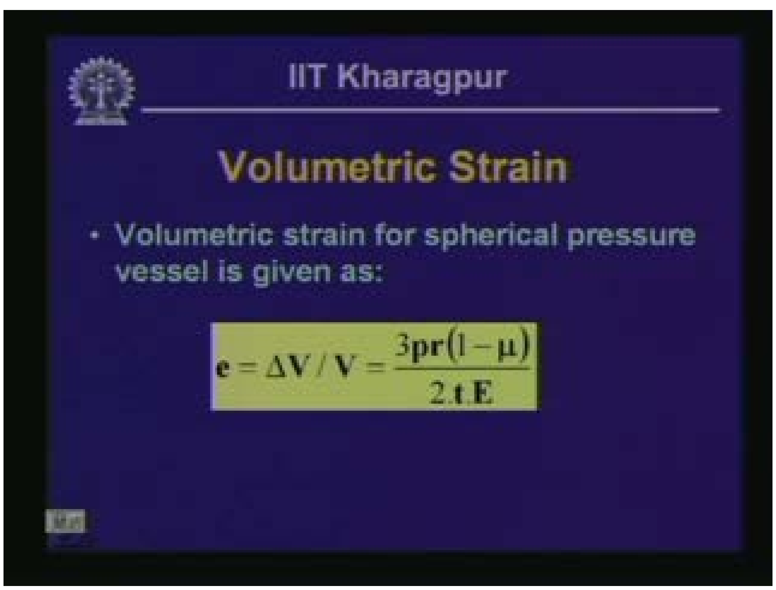

If we write down these in terms of the stresses we have obtained e is equal to $2\in 1+\in 2$ and

$\in 1$ if we write down in terms of stresses as $\frac{\sigma 1}{e}$ minus $\frac{\mu\sigma 2}{e}$ plus $\in 2$ is equal to $\frac{\sigma 2}{e} - \frac{\mu\sigma\ 1}{e}$ and as we know sigma$_1$ is equal to $\frac{pr}{t}$ and sigma$_2$ is equal to $\frac{pr}{2t}$ so we write this as $\frac{pr}{t.E}$ minus mu $\frac{pr}{2.t.E}$] plus $\frac{pr}{2.t.E}$ minus mu $\frac{pr}{t.E}$. So if you take out $\frac{2pr}{t.E}$ is equal to 1 minus $\frac{\mu}{2}$ plus $\frac{pr}{t.E}$ we have ½ minus mu so from here if you take out $\frac{pr}{t.E}$ out for the whole we have 2 minus mu2 plus $\frac{1-2}{2}$ is equal to $\frac{pr}{t.E}$ 2 4 plus 1 5 so $\frac{5-2\ 4\mu}{2}$ and this is what is the expression which we have obtained as $\frac{pr}{t.E}$ $\frac{5\ -\ 4\mu}{2}$ or $\frac{pr}{t.E}$ 5 minus 4mu and this is the expression which we have obtained.

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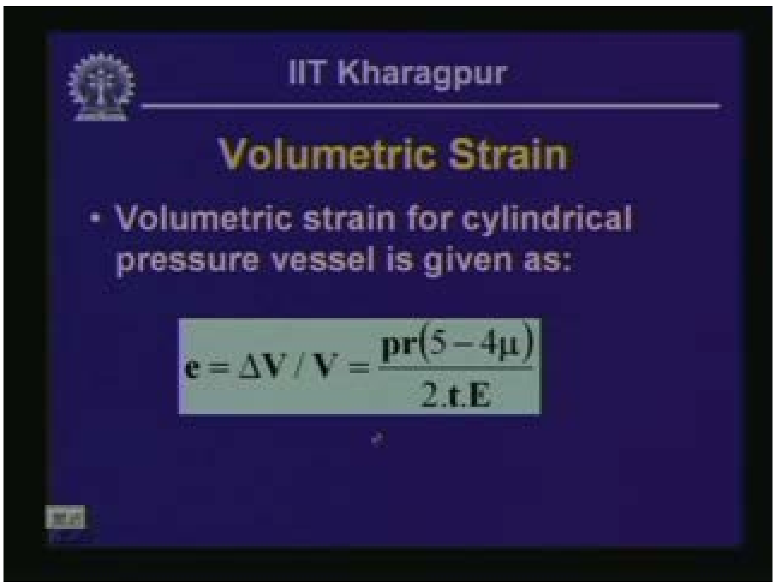

The $\frac{\Delta V}{V}$ is equal to $\frac{pr(5\ -\ 4\mu)}{2.t.E}$ this is the volumetric strength in case of cylindrical pressure vessel now.

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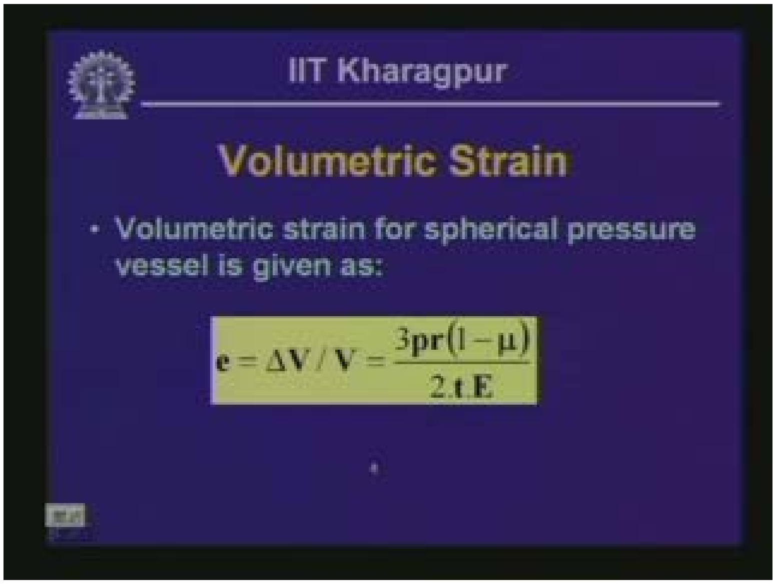

Likewise, I we can compute the volumetric strain for spherical vessel given by e is equal to $\frac{\Delta V}{V}$ is equal to $\frac{3pr(1-\mu)}{2.t.E}$ and we can compute the value exactly in the same way as we just now did. Supposing if we have spherical vessel for which internal radius is again as r and because of internal pressure there is a change in the radius which is (r plus deltar) this is the change radius hence, the value of volumetric strength which is e is equal to $\frac{\Delta V}{V}$ is equal to (deltaV is the final volume minus the original volume) is equal to 4/3pi (r plus deltar) whole cube minus 4/3pi r cube [by the original volume, which is 4/3pi r cube, if you take 4/3 pi out] is equal to [(r plusdeltar cube) − r cube]/4/3(pi r cube); (1 / r cube)[r cube plus 3r square deltar plus 3r(deltar square) plus(deltar cube minus r cube)] so this gets cancelled.

Again with the same logic that Δr being small we neglect the value of deltar square and deltar cube so we are left with 3r square deltar is equal to 3r square deltar divided by r cube and this gives us 3($\frac{\delta r}{r}$) and again ($\frac{\delta r}{r}$) is the radial strain that means deltar is elongation in the radial direction and that divided by the original radius will give you the strain. So this is nothing but equal to three times epsilon. So e the volumetric strain is three times equal to the normal strain.

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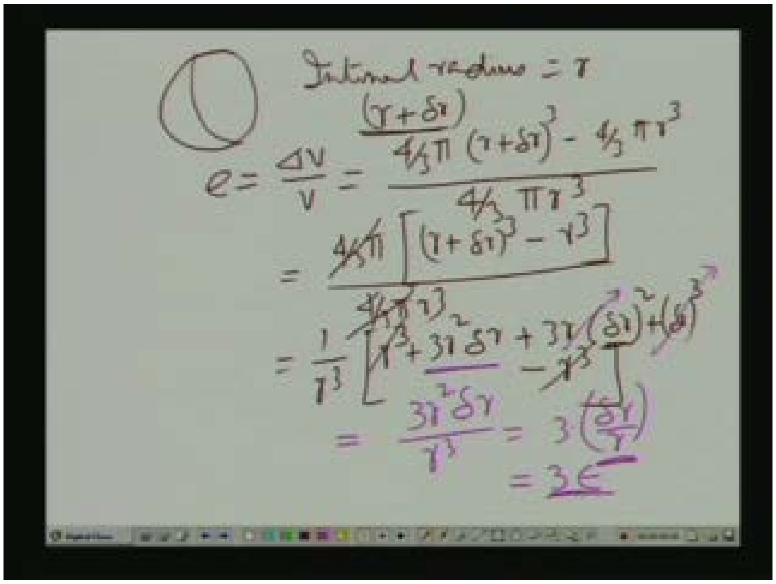

Now we can compute the value of e in terms of stresses and it comes as e is equal to $3\in$ and $\in$ we can write in terms of sigma is equal to $3\frac{(1-\mu)}{E}\sigma$. And in case of spherical vessels sigma is equal to $\frac{pr}{2t}$ is equal to $\frac{3(1-\mu)}{E}\frac{pr}{2t}$ is equal to $\frac{3(1-\mu)}{E}$ into $\frac{pr}{2t}$ is equal to $\frac{3pr}{2tE}(1-\mu)$, this is the value of the volumetric string in terms of the internal pressure, the radius and the thickness modulus of elasticity and the Poisson's ratio.

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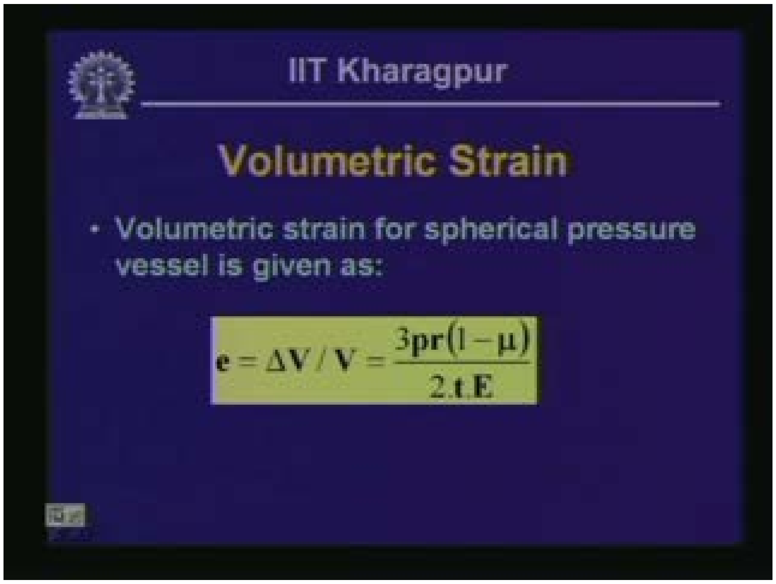

This is what is represented here that the value of the volumetric string e is equal to $\frac{3pr(1-\mu)}{2.t.E}$.

Now we know the value of volumetric string for a cylindrical vessel and for the spherical vessel. We looked into the difficulties we encounter when we go for a combination of a spherical and cylindrical vessel at the junction where there is a mismatch in the deformation and to make the geometrical compatibility or if we have a smooth deformation over there then there will be additional stresses which has to be taken into account when you go for the design of such vessels.

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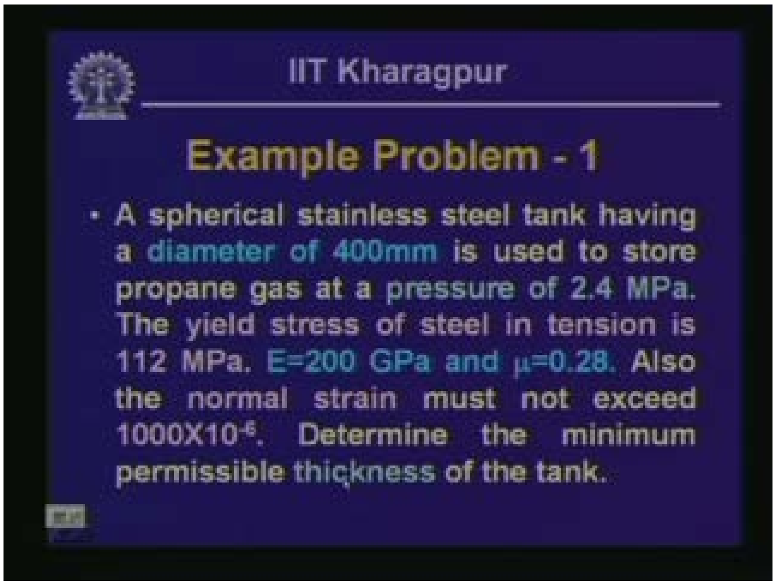

Let us look into the examples related to the cylindrical and spherical vessels. Now, the first example is that a spherical stainless steel tank having a diameter of 400 mm is used store propane gas at a pressure of 2.4 MPa. Now the yield stress of steel in tension is 112 MPa, the value of e is given as 200 GPa and the Poisson's ratio as 0.028. Also, it is stated that the normal strain in the vessel should not exceed 1000×10 power 6 so under these constraints we have to determine the minimum permissible thickness of the tank. So a spherical tank is used for storing gas.

Now certain parameters are given, the diameter is given but the thickness is not given so we will have to find out the thickness of the tank if the tank has to contain this gas that the pressure is subjected to maximum stress of the limit given and the maximum strain limit given. So from these two criteria we have to evaluate the value of thickness.

Let us see how to compute the value of the thickness. The diameter of the tank given is 400 mm, the pressure inside is equal to 2.4 MPa, the stress limitation the maximum tensile stress that can be applied on the vessel is equal to 112 MPa, the value of e is equal to 200 GPa (Gigapascal), mu is equal to 0.28 and also it is stated that the strain limitation is 1000(10 to the power minus 6). If we compute the stress, since there is a limitation on the stress as well as on the strain we have to compute the thickness of the vessel from two considerations.

First we have to compute the maximum stress that can be generated because of the pressure exerted by the content and from there we will get one thickness, also when the vessels is subjected to internal pressure there will be strain on the surface and there is a limitation on the strain value as well and corresponding to that strain we will get another thickness. Now we have to go for the thickness which you will satisfy both the criteria.

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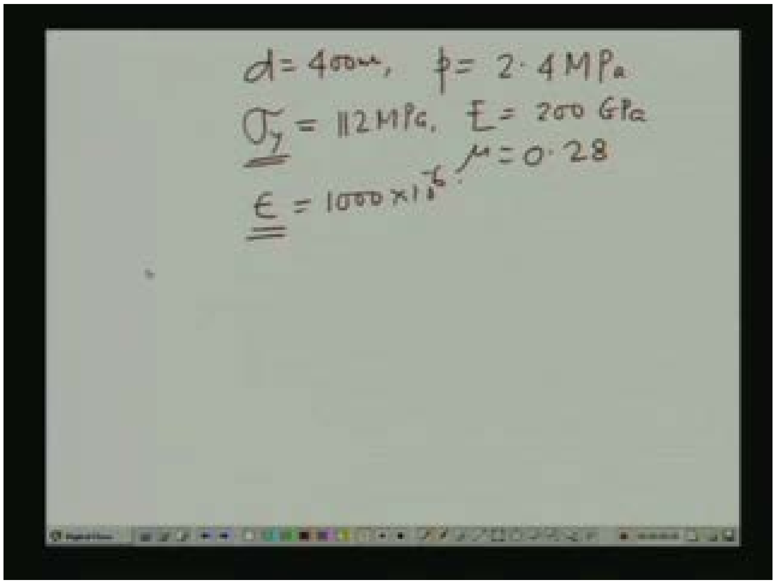

First let us look into with reference to stress. So maximum stress that can be generated in a spherical vessel sigma is equal to $\frac{pr}{2t}$ and here p is equal to 2.4 MPa so 2.4 times diameter given is 400 so radius is 200 divided by (2 into t) $\frac{2.4\times 200}{2\times t}$ so t is the thickness which we have to evaluate. And this particular stress is limited 212 MPa so this is equal to 112 MPa. Sigma is equal to $\frac{pr}{2t}$ is equal to $\frac{2.4\times 200}{2\times t}$ is equal to 112 MPa.

Now you can compute the value of t from this equation which comes as 2.143 mm. The value of t as required satisfying the stress requirement of the maximum stress that should not exceed 112. Therefore to satisfy that criteria the thickness which we need is this. For the strange requirement as we know $\in$ is equal to $\in$ x $\frac{\sigma_x}{e}$ minus $\frac{\mu\sigma_y}{e}$ is equal to $(\frac{1-\mu}{E})\sigma$. Now in this particular case of spherical vessel σ_x and σ_y are same and their sigma so this is equal to $(\frac{1-\mu}{E})\sigma$. This is the value of strain $\in$. Here the value of Poisson's ratio μ is equal to $\frac{1-0.28}{200\times 10^3}\times\frac{2.4\times 200}{2\times t}$. And this strain cannot exceed the value of 1000(10^ minus 6) is equal to (10^ minus 3). So (10^ minus 3) is the limiting value of the strain and that is written as a function of t through this expression is equal to 0.72 and this 200 and this 200 gets cancelled times 2.4 divided by 2 into 10^3(t) is equal to 10^ minus 3. So it is: 10 to the power minus 3 is equal to $\frac{0.72 \times 2.4}{2 \times 10^3 \times t}$. From this, if we compute the value of t is equal to 0.864 mm.

Now we have two values of thicknesses, one is corresponding to the verification of the maximum strain value and the other one is corresponding to the satisfaction of the stress criteria. So if we have to satisfy both, if we go for the lower thicknesses there is a possibility that the stress level here will be higher and as a result it will not satisfy this criteria. But if you go for higher thickness then it will satisfy the strain criteria. So the thickness of the vessel to be adopted is t is equal to 2.143 mm so that it satisfies both the criteria of stress as well as the strain. The maximum value of the stress requirement is 112 MPa and the maximum value of the strain requirement is 10^ minus 3. These two can be satisfied if we use the higher thickness between the two which is 2.143 mm.

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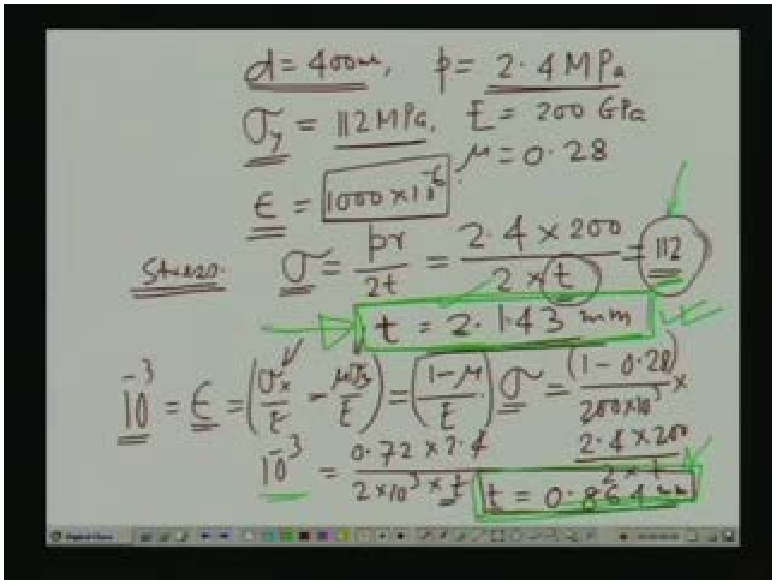

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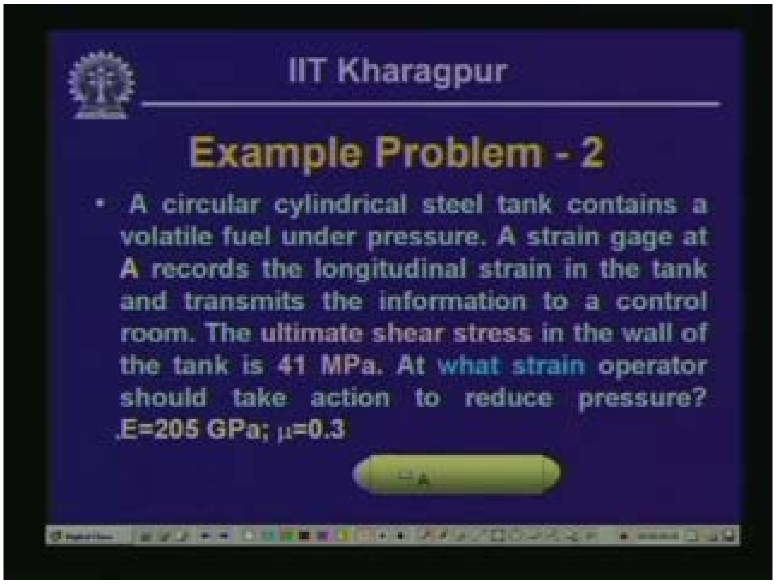

Now, let us look into another example problem wherein we have a circular cylindrical steel tank which contains a volatile fuel under pressure. Now a strain gage A is fixed at this particular location. Now that gage records the longitudinal strain in the tank. Here we have a cylindrical tank, this part is the cylindrical part and on this there are hemispherical ends attached to the tank and this contains of volatile fuel inside it which exerts pressure. On this cylindrical surface there is a strain gage mounted. This strain gage transmits the information to a control room about the strain on such vessels. Now it is indicated that the ultimate shear stress in the wall of the tank is 41 MPa.

Since the operator is recording at the control room he is getting a constant record of the strain. Now what you need to find out is at what strain the operator should reduce the pressure so that the stresses in the vessel does not go beyond this limiting capacity. So that is what is being controlled. In the vessel we had fixed a strain gage and the strain gage data is being transmitted to the control room.

Now the content is a giving a pressure inside and because of that the strain is changing thereby the stresses are changing. Now there is a limiting value beyond which there is a possibility where the whole thing may burst. So the operator who is controlling that has to keep an eye so that the strain value goes beyond the particular value it is under rest and the pressure has to be reduced inside the container. So we have to compute that value of the strain for which the operator should reduce the pressure and the value of e is given and the value of the Poisson’s ratio is given here.

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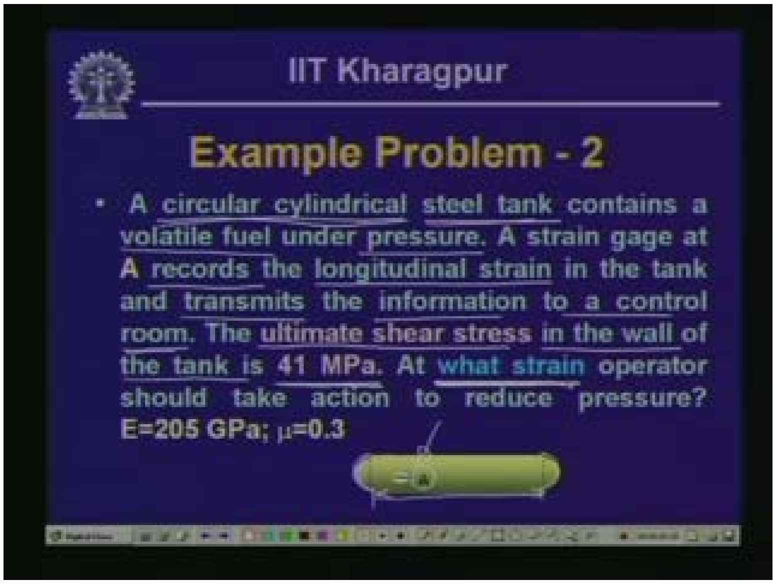

Now let us look at how we compute the value of the strain at which the operator should send the signal that is to be stopped or the pressure is to be reduced. The value of e is equal to 205 GPa(Giga Pascal) and the value of mu is equal to.3 Poisson's ratio. As we know in case of spherical vessel or cylindrical vessel the value of $sigma_1$ and $sigma_2$ can be computed in terms of the pressure and if you take a small element we have $sigma_2$ in the longitudinal direction and $sigma_1$ in the circumferential direction and the in-plane shear stress τ is equal to $\frac{\sigma 1 - \sigma 2}{2}$.

Now the maximum shear stress is equal to 41 MPa so the value of this stress differential $sigma_1$ minus is equal to 82 MPa now, we know that in case of cylindrical vessel $sigma_1$ in terms of pressure can be computed as $\frac{pr}{t}$ where p is the internal pressure, r is the radius internal radius and t is the thicknesses and incase of longitudinal stress $sigma_2$ is equal to $\frac{pr}{2t}$ so this longitudinal pressure $sigma_2$ is equal to sigma ½. Now if we adopt this value we can write this as ($sigma_1$ minus sigma (1/2)) is equal to 82 MPa so $sigma_1$ is equal to 164 MPa. So this is the value of the circumferential stress and thereby the longitudinal stress is half of that which is 82 MPa. On the cylindrical vessel we have a strain gage fixed in the longitudinal direction which will record the strain in the normal longitudinal direction. So we have to compute the value of the strain in terms of the stresses.

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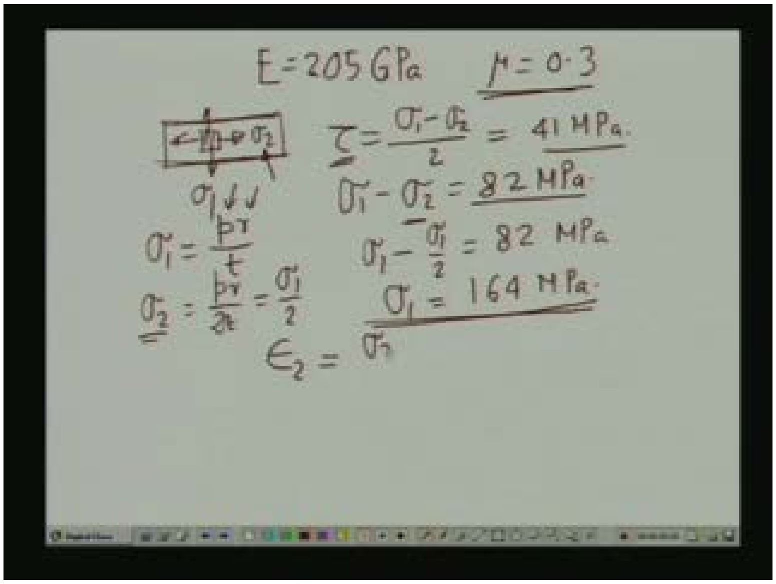

So the strain in the longitudinal direction which we are calling as $\in 2$ is in line with the stress sigma$_2$ the $\in 2$ is equal to $\frac{\sigma\ 2}{e}-\frac{\mu\sigma 1}{e}$. Now sigma$_2$ is equal to $\frac{\sigma 1}{2}$ since we are writing every thing in terms of $\frac{\sigma 1}{2}$ so this is $\frac{\sigma 1}{2E}$ minus $\frac{\mu\sigma 1}{E}$. This is equal to $\frac{\sigma 1}{E}=(\frac{1}{2}-\mu)$. Now mu is equal to 0.3 so this is 0.5 minus 0.3 and sigma$_1$ is equal to 164 so this is 164/205(10^3) so this many Mega Pascal, this was 205 GPa so 205(10^3) so much of Mega Pascal times 0.5 minus 0.3; $\frac{\sigma 1}{E}=(\frac{1}{2}-\mu)$ is equal to $\frac{164}{205\times 10^{3}}$ so this gives you the value of the strain which comes as is equal to 0.16(10^ minus 3) is equal to 160(10^ minus 6) so this is the limiting strain. So, as soon as the operator records the strain going closer to 160(10^ minus 6) the operator should reduce the pressure from the cylindrical tank so that the stress level does not go to the limiting value of that particular material with which the vessel has been fabricated. So this particular information is necessary that is to be kept with the operator that if the strain value as it is recorded under the pressure that particular strain as it goes to (16 minus 10 to the power 6) or it is little below that the pressure inside the vessel has to be reduced so that the stress level does not go to criticality.

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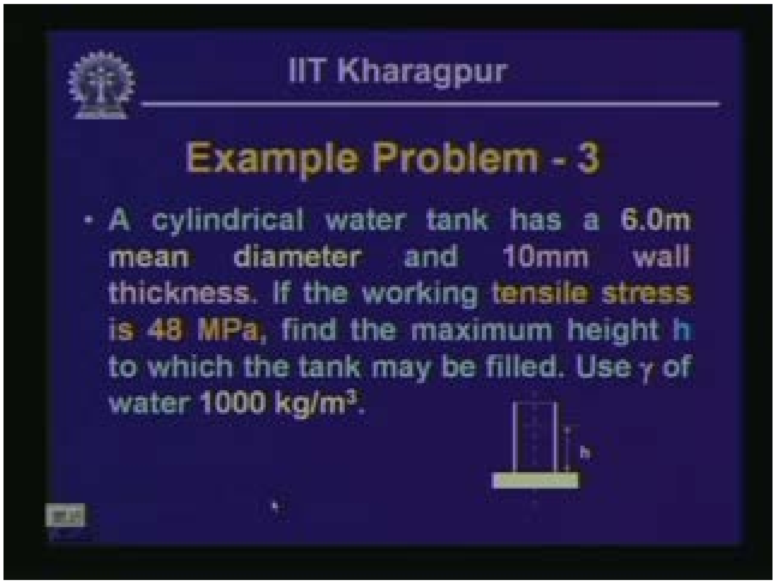

Let us look into the third example; a cylindrical water tank has a 6m diameter. Let us take this as our internal diameter only which is 6m as 6000 mm and the thickness of the tank wall is 10 mm. Now if the working tensile stress is 48 MPa the limiting stress on the vessel is 48 MPa, find the maximum height h to which the tank may be filled. Use Γ of water unit weight of water as 1000 kg/m cube.

Now this particular tank is filled up with water. This height h is to be determined up to which if we fill it up the maximum stress that can be allowed to go in the tank is up to 48 MPa. So this height has to be evaluated from this particular criterion of the stress. Let us look into how to get the value of h if we limit our stress to 48 MPa. The values which are given here are; the diameter of the tank is 6m which is 6000 mm. Now the value of t the thickness of the wall is 10 mm, the limiting stress, the working stress of the wall or the vessel is equal to 48 MPa. So this is the limiting value sigma$_w$ the working stress that is limited in the tank is 48 MPa, the unit wet of water Γ is 1000 kg per m3.

In that particular tank as the water level is going up there will be change in the pressure which will be exerted by the water. This is the tank and as the water level is going up to level h the maximum pressure which will be exerted by the water is equal to γ w(h). So we got to compute the value of the stress with reference to this maximum pressure so that the maximum stress can be evaluated either time and corresponding to that we can find out the height because corresponding to the maximum pressure the maximum stress will be generated and of course as the pressure is reduced the stresses will be different.

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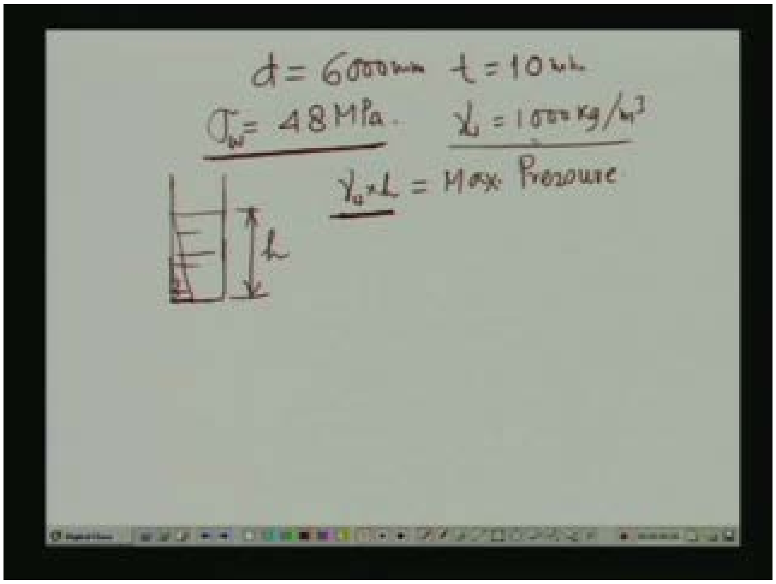

Here γ is equal to 1000 kg per m cube so 1000(h) is so much of kg per m square is the pressure that is being exerted by the water on the tank. Now if we write in terms of N approximately taking 10 this is 10000 hN by m square. So this is equal to the pressure p in this particular example.

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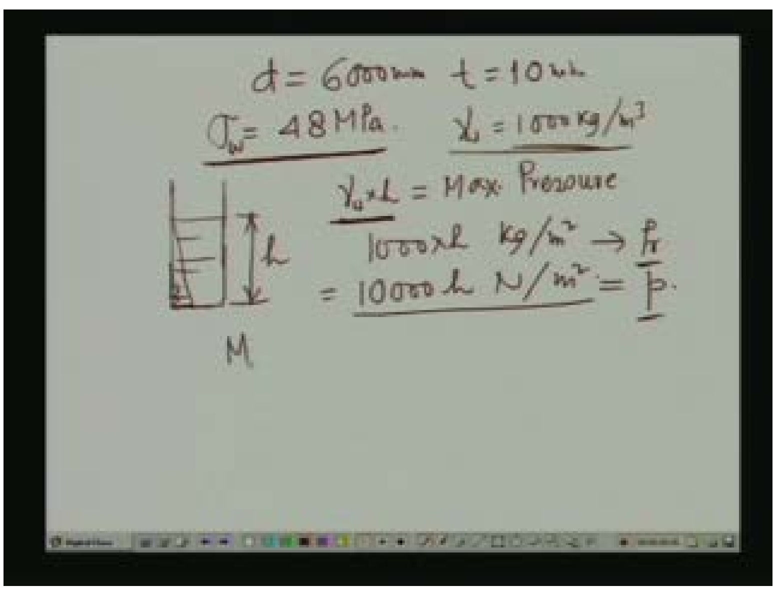

This particular vessel is a cylindrical type so the maximum normal stress for this particular vessel is equal to sigma$_1$ which is the circumferential stress is equal to $\frac{pr}{t}$. And in this particular case 10000 into h is the value of p and r diameter of the tank is given as 6000 so times 3000 is the radius divided by thicknesses t is equal to 10. Now this is the magnitude of the stress and the maximum normal stress is limited to 48 MPa because it cannot go beyond that then the member will fail or the tank will fail. Here this is N^{m^2} , so meter and this is mm this is also mm, so if we convert that m square to mm square so this is 10 power 6. So this is 10 to the power 4, 10 to the power 7. So this is equal to $\frac{3.h\times10^7}{10^7}$ so this gets cancelled, h gives us $\frac{48}{3}$ is equal to 16m because h was in meter as we assumed over here. So if the height of the water goes up at the level of 16m then it can push the pressure level up to 48 MPa. If we like to keep the pressure in the tank wall up to 48 MPa then the maximum height up to which we can pour water is 16m.

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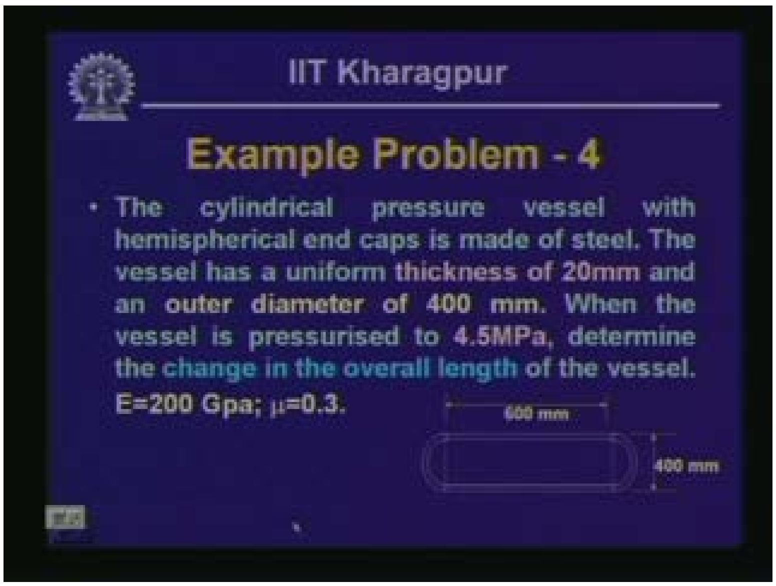

Let us look into another problem. Here we have a pressure vessel which is basically a cylindrical pressure vessel, this is the cylindrical part having 600 mm in length and is connected with a hemispherical end. The two ends are hemispherical having a diameter of 400 mm and the diameter of this particular hemispherical end is the external diameter. The cylindrical pressure vessel with hemispherical end caps is made of steel.

The vessel has uniform thicknesses of 20 mm and an outer diameter of 400 mm. Now when the vessel is pressurized to 4.5 MPa, now determine the change in the overall length of the vessel. This is the length of the vessel originally. Now we will have to compute that because of this pressure 4.5 MPa which is exerted on the vessel how much increase in the length occurs because of this internal pressure?

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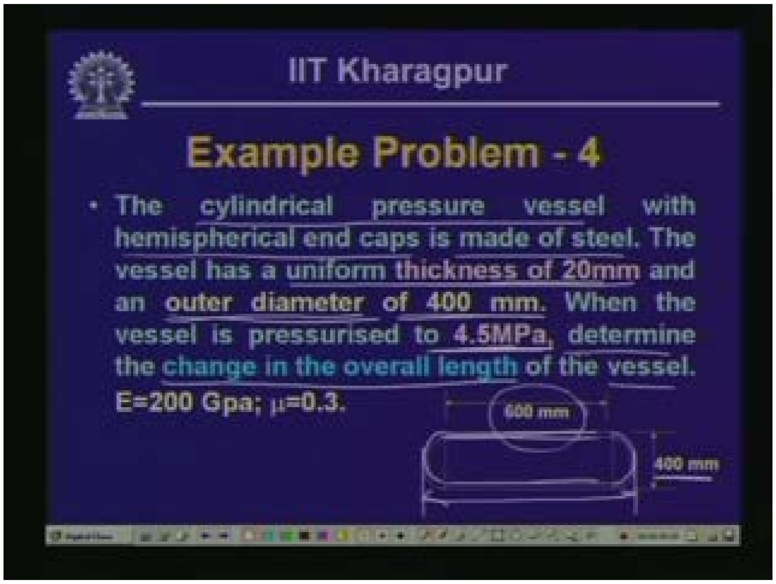

So this is what is to be computed to determine the change in the overall length of the vessel. The value of e is equal to 200 GPa and mu is equal to 0.3. Now let us look into the evaluation of these particular values that how much is the elongation or change in the length. The outer diameter of the vessel d outer is equal to 400 mm and thickness is 20 mm. P is given here as 4.5 MPa and the value of e is equal to 200 GPa, mu is equal to 0.3. The outer diameter is given as 400 mm so the radius r outer is equal to 200 mm and since we are dealing with the radius which are inner radius, so r inner is equal to 180 mm which is this minus the thicknesses.

First let us look into the cylindrical part and then subsequently we will go to the spherical part. For the cylindrical part the change in the length will be the elongation in the longitudinal direction. So we got to compute the strain in the longitudinal direction and then correspondingly find out the strength and then look into the hemispherical end and know what change it is in the radius.

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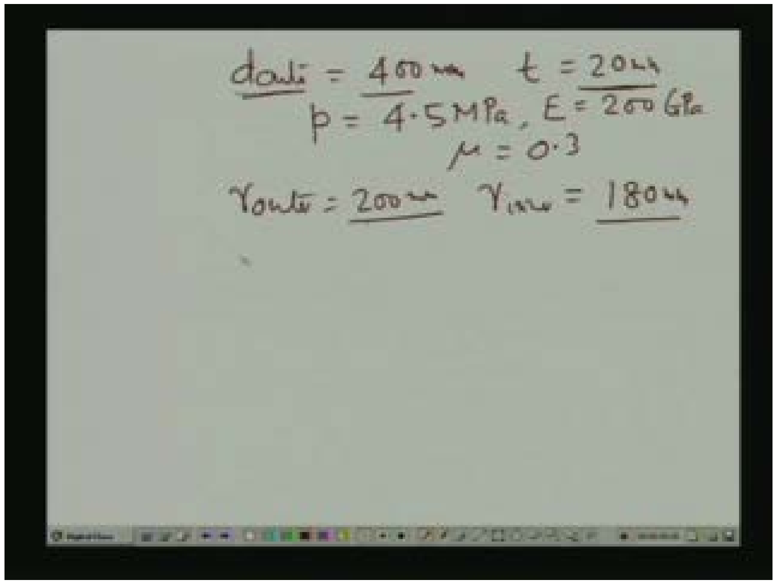

For the cylindrical one if you look into; the value of sigma$_1$ is equal to $\frac{pr}{t}$ is equal to $\frac{4.5\times180}{20}$ is equal to 40.5 MPa. The value of sigma$_2$ which is half of sigma is equal to 20.25 MPa and corresponding the strain in the longitudinal direction which is $\in 2$ is equal to $\frac{\sigma 2}{E}$ minus $\frac{\mu\sigma 1}{E}$ and this gives us a value of, if you substitute the values of sigma$_2$ and sigma$_1$, e and mu this comes as a value of 0.0405 into 10 to the power minus 3. So this is the value of the strain in the longitudinal direction. Now the strain in the longitudinal direction in terms of the deformation, if deltal is the deformation in the length to the original length is the strain $\in$ is equal to 0.0405 into 10 to the power minus 3. So change in the length deltal is equal to the original length times this which is 600 multiplied by this and this comes as 0.0243. So this if it is multiplied with the 600 gives the value of 0243 mm. This is the change in the length in the longitudinal direction for the cylindrical part. Now let us look into what is the change in the two hemispherical ends or what is the change in the radius and that will give us the total change.

In the hemispherical end we have sigma is equal to $\frac{pr}{2t}$ which we have computed as 20.25 MPa. And correspondingly $\in$ is equal to $(\frac{1-\mu}{E})\sigma$ is equal to 0.0708 into 10 to the power minus 3 so this is the strength. Therefore strength $\in$ is equal to $\frac{\delta r}{r}$ is equal to 0.0708 into 10 to the power minus 3 change in the radius by original radius and this gives us a value of 0.0708 into 10 to the power minus 3. So δr is equal to 0.0255 mm delta r the change in radius is equal to this times the original radius which is 180mm and since we have two hemispherical end so it is twice of this value and this comes as is equal to 0.0255 mm].

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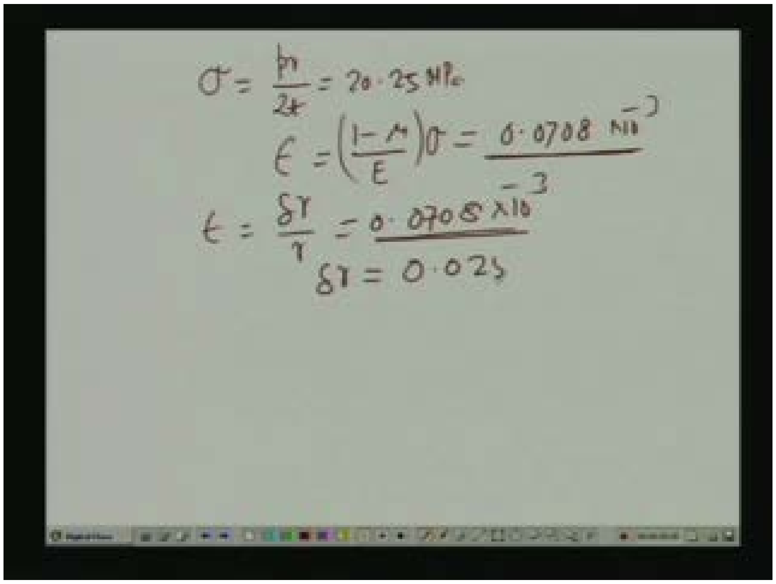

Hence the total change in length is equal to deltal plus deltar is equal to 0.0498 mm. So this is the total length in the change as we get in the in this particular vessel.

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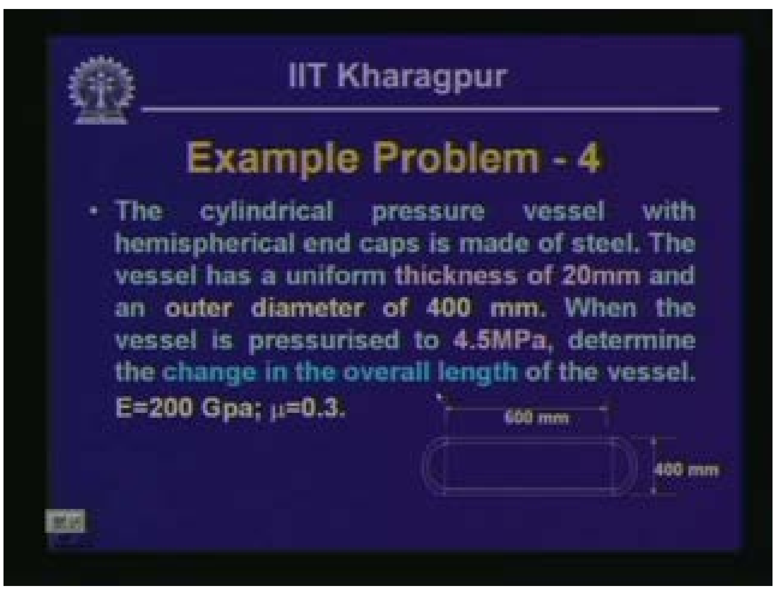

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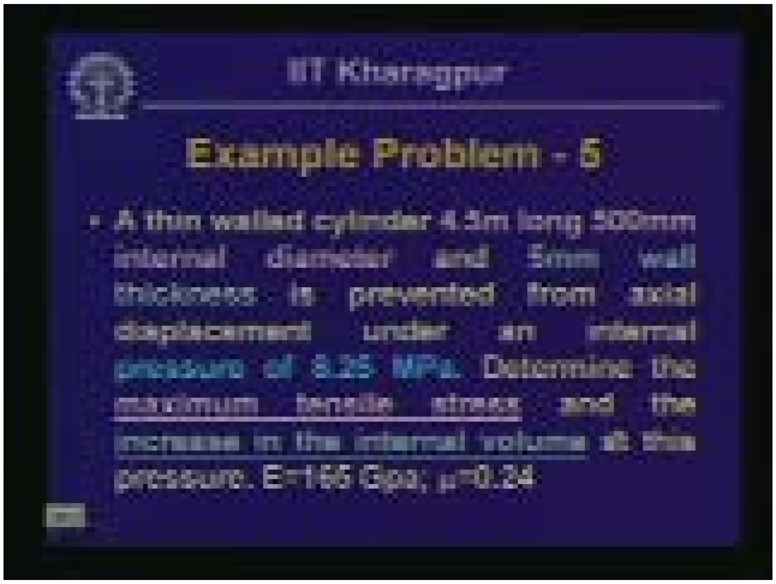

We have another example which is a thin-walled cylinder 4.5m long, 500 mm internal diameter and 5 mm wall thicknesses is prevented from axial displacement. Here the cylinder is not allowed to move in the axial direction so it is restrained and thereby there is a change in volume and this is corresponding to the volumetric strain.

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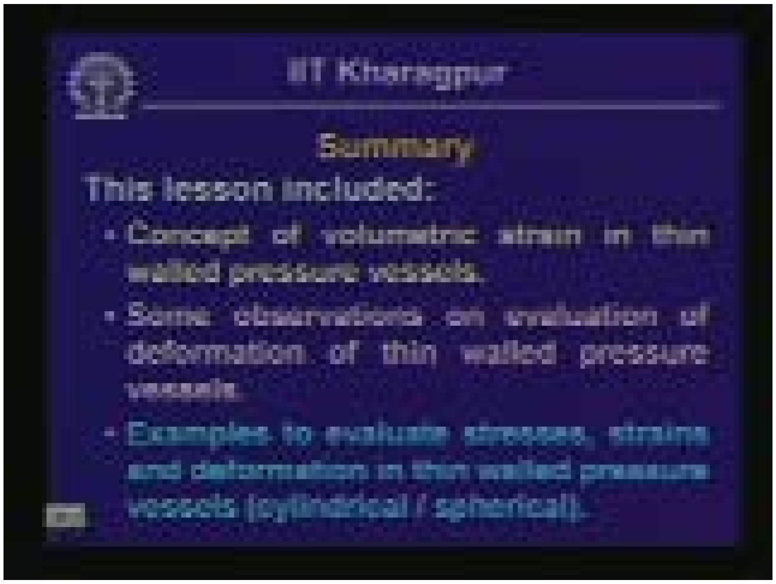

So determine the maximum tensile stress and the increase in the internal volume. You go to compute the volumetric strain and thereby the change in the internal volume for this particular vessel.

Now to summarize; what we had discussed in this particular lesson is that we have looked into the concept of volumetric strain in thin-walled pressure vessels. So in this particular lesson we have looked into, for cylindrical or spherical vessels how the volumetric strain occurs or change in volume occurs.

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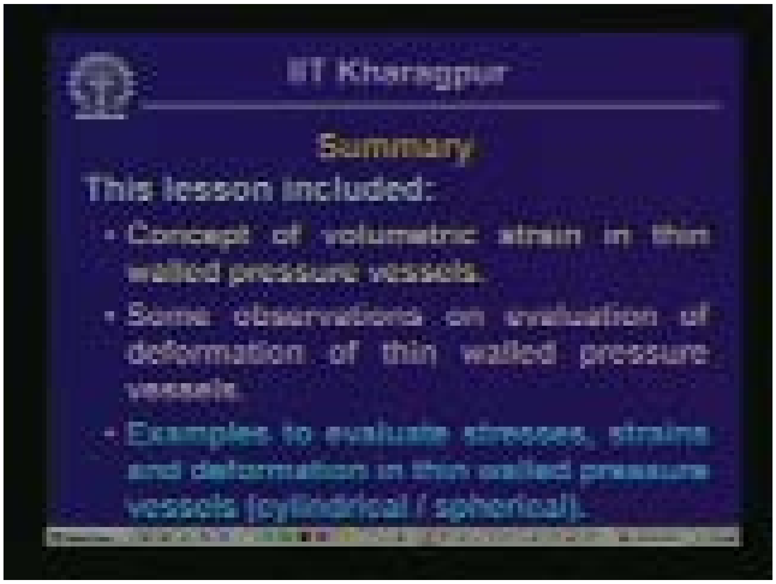

Then we have observed on some aspects on the deformation when specifically the cylindrical and spherical vessels are combined together what are the problems we encounter. Then we have looked into some examples where we could compute the values of strains and thereby the stresses and the deformations in the vessels. And also of course we have assigned one problem which is related to the change in the volumetric strain.

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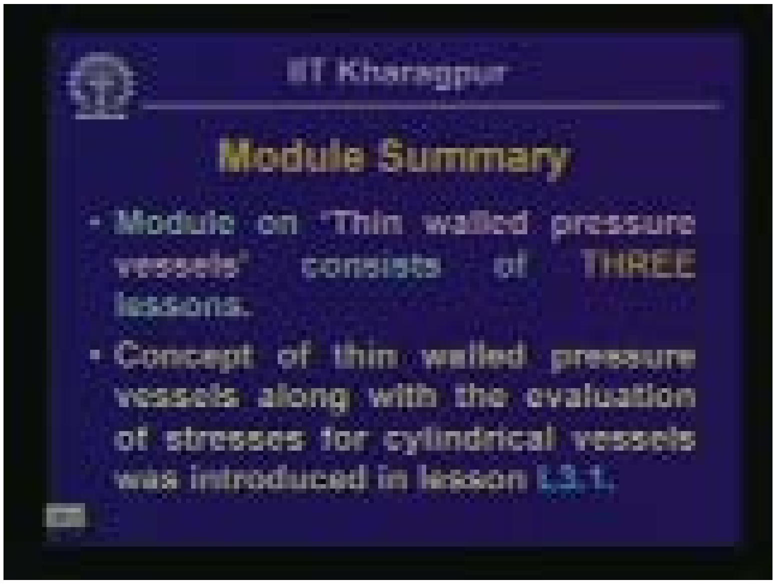

With this particular lesson in fact we come to the conclusion of this particular module. In this particular module we had three lessons. Now this particular module is on thin-walled pressure vessels and it consisted of three lessons.

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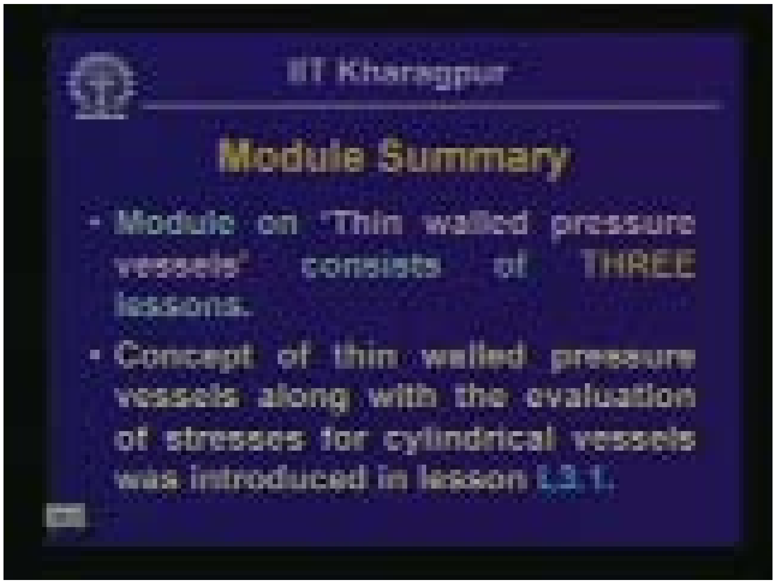

In the first lesson we had introduced the concept of what is meant by thin-walled pressure vessels, what are the kind of stresses it gets induced to and we had discussed that aspect with reference to the cylindrical pressure vessels.

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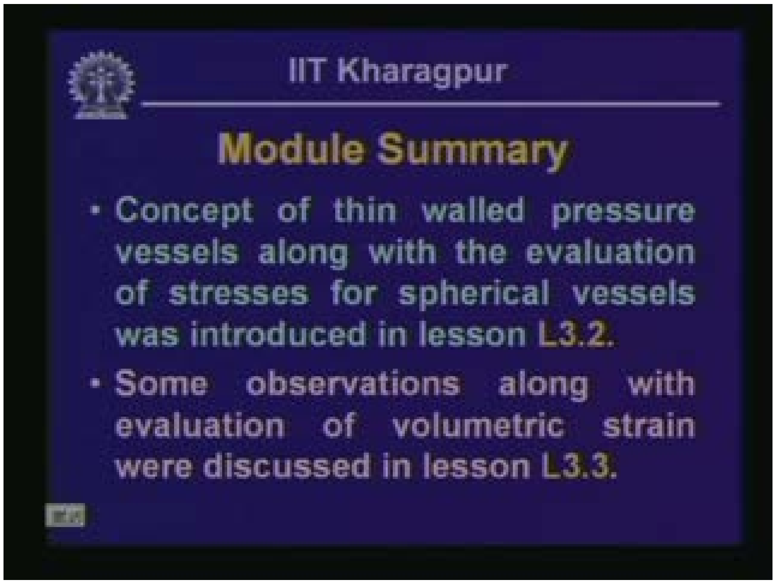

Consequently, in the second lesson we discussed again with reference to thin-walled pressure vessels but with a specific reference to the spherical vessel that what are the kinds of stresses and strain occurs and thereby what are the deformations in spherical pressure vessels.

Finally in today's lesson we discussed the volumetric strain with reference to the cylindrical and spherical vessels and thereby some critical observations on such kinds of pressure vessels and how to take care of that and that we have discussed through several examples related to both the cylindrical and spherical vessels and the combinations of that.

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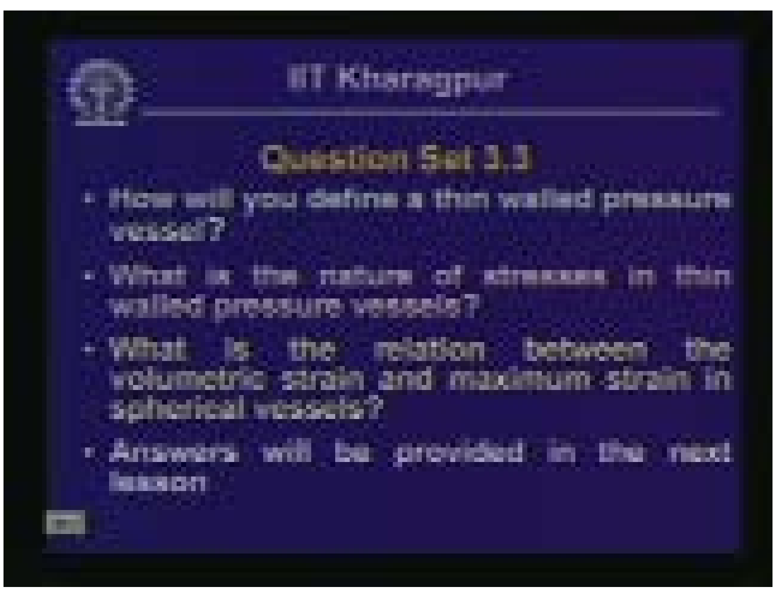

Now some questions are set for you. The first question is how you will define a thin-walled pressure vessel which is quite clear. Now from this lesson you can find out the definitions for a thin-walled pressure vessel.

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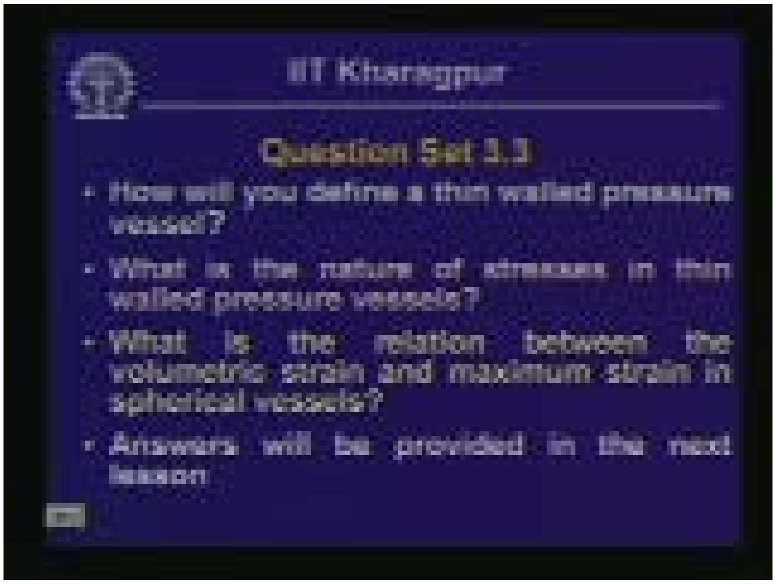

What is the nature of stresses in thin-walled pressure vessels?
What is the relation between the volumetric strain and maximum strain in spherical vessels?