Strength of Materials

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Lecture No #16

Application of Stress/Strain

- Thin Walled Pressure Vessels - II

Welcome to the 2^{nd} lesson of 3^{rd} module which is on Thin Walled Cylindrical Pressure Vessels, in fact this is Thin Walled Pressure Vessels part 2.

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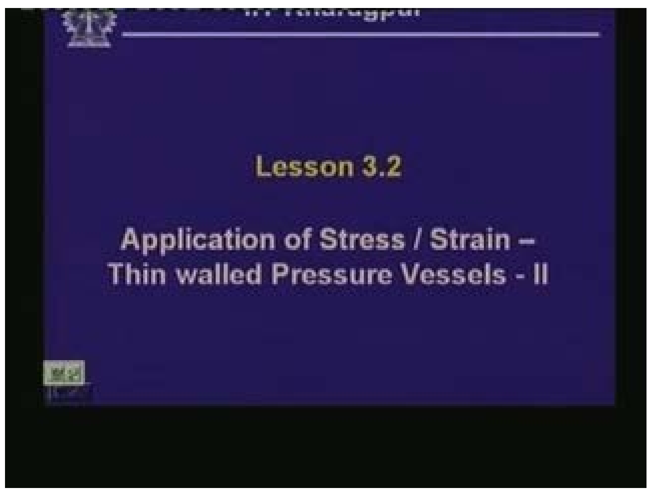

In the last lesson we discussed about the Cylindrical Pressure Vessels and in this lesson we are going to discuss some more aspects of pressure vessels. This is, thin walled pressure vessels part 2 which is the application of stress and strain.

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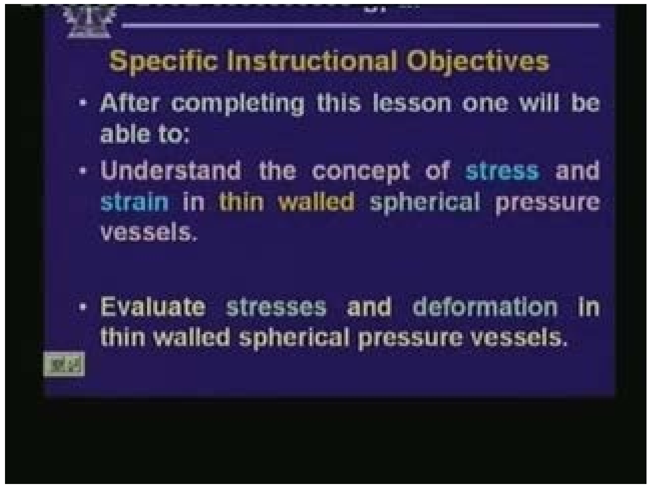

It is expected that once one goes through this particular lesson they will be clear on the concept of stress and strain. You will be able to understand the concept of stress and strain in a spherical pressure vessel. In fact in the last lesson we had discussed about cylindrical pressure vessels and here we will be dealing with the spherical pressure vessels. Also, one should be able to evaluate stresses and deformation in thin walled spherical pressure vessels.

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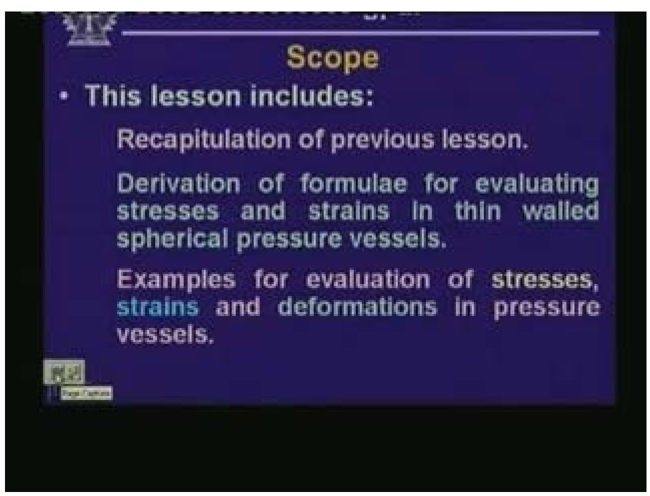

Hence the scope of this particular lesson includes the recapitulation of previous lessons or the previous one which we had discussed on thin walled cylindrical vessels.

Here we will be dealing with the spherical vessels and we will be deriving the formulae for evaluating stresses and strain in spherical vessels which is subjected to internal pressure. Then we will be solving few examples for evaluating stresses and strains and thereby deformations in pressure vessels. We will be looking into these examples both in terms of cylindrical pressure vessels as well as the spherical pressure vessels.

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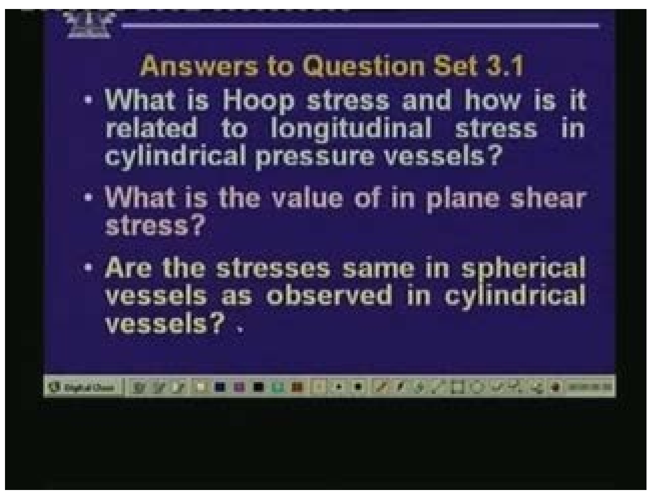

Let us recapitulate the previous lesson through these questions.
The first question which was posed is what is hoop stress and how is it related to longitudinal stress in cylindrical pressure vessels.

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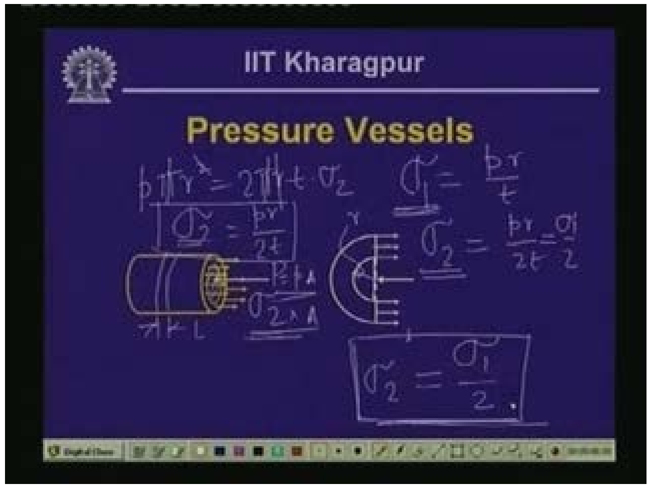

Now let us look into how we computed the two stresses which we had designating as hoop stress or the circumferential stress and the longitudinal stress.

Circumferential stress if you remember in the cylindrical pressure vessels along the circumferential direction, if we take the free body of half of the cylindrical vessel then the stress denoted over here, let us call this resulting force as p which is a function of the stress σ_1 multiplied by the area which is the thickness times the length which we are taking called as l.

So σ_1 times t the thickness times l is the force that is acting and the internal liquid which is exerting pressure on this particular surface if we say that the radius of this particular cylinder is r then if internal pressure is p so p × 2r × l is the total force that is being exerted by the content and if we equate these two p 2r × l = σ_1 × t × l and twice of that you have these two values which gives us $\sigma_1 = \frac{pr}{t}$ where these l and l gets cancelled.

We had seen this last time called as circumferential stress or hoop stress.

Also, in the longitudinal direction if we call the stress as σ_2 now on this surface the content is exerting pressure which is p the pressure times area and on this periphery (on the wall) that is stress σ_2 multiplied by the area will give the force. Now if we take the equilibrium of this which is p × a which is pressure p × πr^2 = $2\pi r$ t ×σ_2.

Hence from here we get r^2 cancelled π and π gets cancelled, $\sigma_2 = \frac{pr}{t}$ and this is called as the longitudinal stress or the axial stress. Hence from these two expressions as you can see we have σ_1 which is hoop stress, $\sigma_1 = \frac{pr}{t}$ and $\sigma_2 = \frac{pr}{2t}$.

Now σ_1 is called as the circumferential stress or the hoop stress and σ_2 is known as the Longitudinal Stress or the Axial Stress. Hence $\sigma_2 = \sigma_1/2$.

So the relationship between the hoop stress and the longitudinal stress we can call as, σ_2 the longitudinal stress = $\sigma_1/2$ the hoop stress. So this is the relationship between σ_1 and σ_2.

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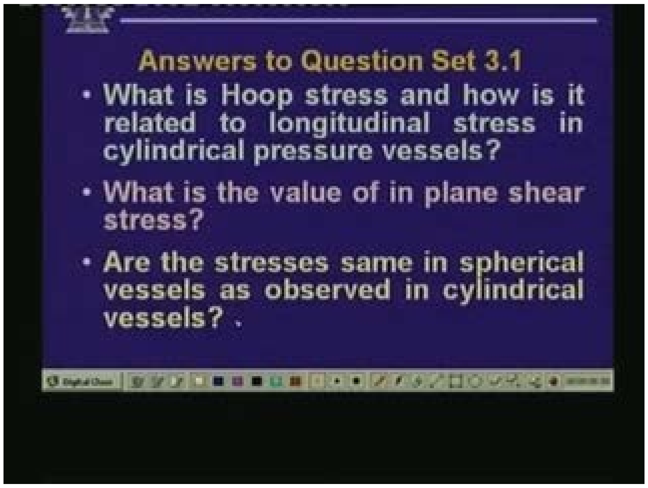

Now the second question posed was what is the value of in Plane Shear Stress? This particular term is important the in plane sheer stress in cylindrical pressure vessels.

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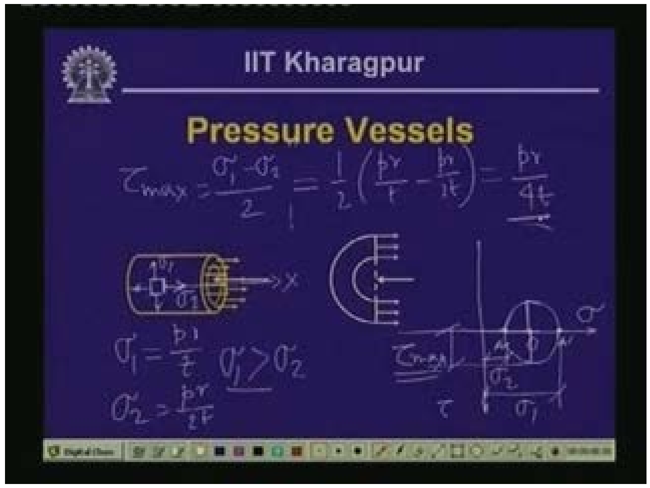

Now let us look into with reference to the expression which we are computing here. We have σ_1 which is the circumferential stress equal to $\frac{pr}{t}$ and we have longitudinal stress σ_2 = $\frac{pr}{2t}$. Now on this surface if we take a small element on which we plot this σ_2 and σ_1, this is σ_1 which is in the circumferential direction and this is σ_2 which is in the longitudinal direction where we have computed the values. Now let us say σ_2 is in the x direction, so let us call σ_x as σ_2 and σ_y as σ_1. If we plot these stresses in Mohr circle this is our σ axis, this is the τaxis and here please note that as we have obtained the value of σ_1 and σ_2, σ_1 is $\frac{pr}{2}$ and σ_2 is $\frac{pr}{2t}$.

Hence $\sigma_1 \rangle \sigma_2$, so if we plot these values of σ and since there are no sheering stresses in this so the point of σ_x and τ, the σ_x is here which is σ_2 and $\sigma_y \rangle \sigma_x$ and this is σ_1 and τ being 0 so both the points are on this σ axis only and the center of these two will give the center line of the Mohr circle. If are we taking this as center and taking radius as oa and oα' and considering oa or oα' as radius if we plot the circle this will give the Mohr circle of stress.

From this the value of the shear stress, this is the positive shear which is maximum τ we call this as τ_{max} and the value of τ_{max} = σ_1 -σ_2/2.

So τ_{max} the maximum shear = σ_1 -σ_2/2.

Now σ_1 -σ_2/2, σ_1= $\frac{pr}{2}$ so ½ ($\frac{pr}{2} - \frac{pr}{2t}$) = $\frac{pr}{4t}$ so this is the value of in plain sheering stress.

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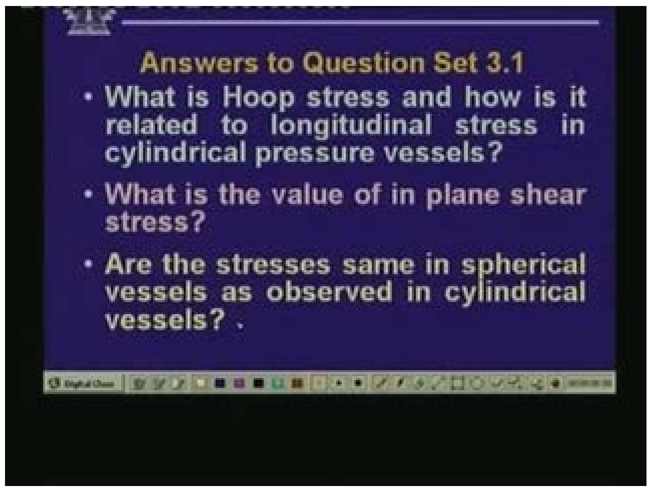

The third question posed was; are the stresses same in spherical vessels as observed in cylindrical vessels, whether they are the same. The question is will the stresses in the cylindrical pressure vessels be the same as in the spherical pressure vessels.

This particular question will be answered through the lesson we are going to discuss today. In fact we will see how to compute the stresses in a spherical pressure vessel.

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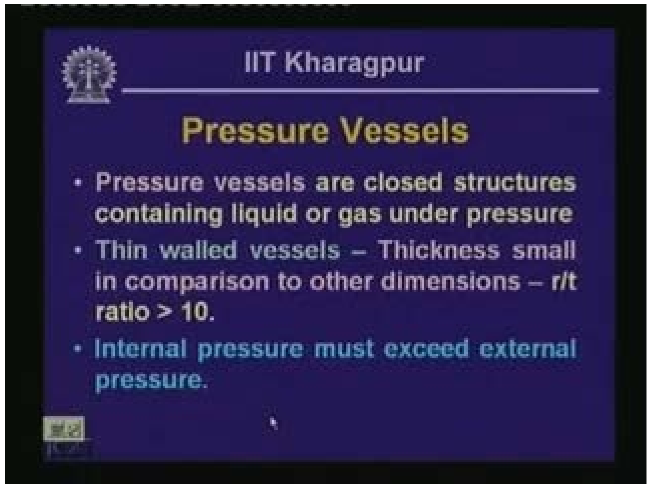

Here are some aspects to look into before we go into the discussion on Spherical Pressure Vessels.

The pressure vessels are the closed structures which contain the liquid or gas under pressure. We called these vessels as the thin walled vessels where the thickness of the wall in comparison to the other dimension is much less where $\frac{r}{t}$ ratio r is the radius of the vessel to the thickness of the wall and if it is more than 10 then we call those kind of vessels as thin walled vessels because the stress distribution across the thickness is negligibly small and we assume that same stress exists or the stress distribution in the vertical direction is 0.

Also, for stresses to exist in these kind of vessels the internal pressure must be greater than the external pressure otherwise we will experience different kinds of problems in such vessels. So the internal pressure must be more than the external pressure.

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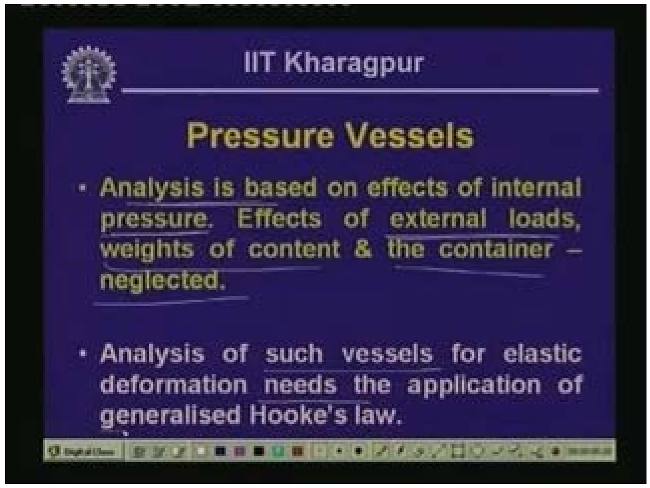

We had also looked into these aspects that analysis is based on the effect of internal pressure only and we are neglecting the other effects such as the effect of the external loads and the weights of the content and the container weight. So we are neglecting these aspects and basically we are simplifying our analysis on the presumption that the stress that is being induced in the vessel is primarily because of the pressure that is being exerted by the content of the container. Lastly the analysis of such vessels for elastic deformation needs the application of generalized Hooke's Law.

When the containers are subjected to pressure from inside by the liquid content or the gaseous content the vessels are subjected to tensile stresses. So we are computing stresses as a function of the internal pressure and once we get these stresses then we can compute strain utilizing or using the generalized Hooke's Law from which we can compute the deformation in the vessels.

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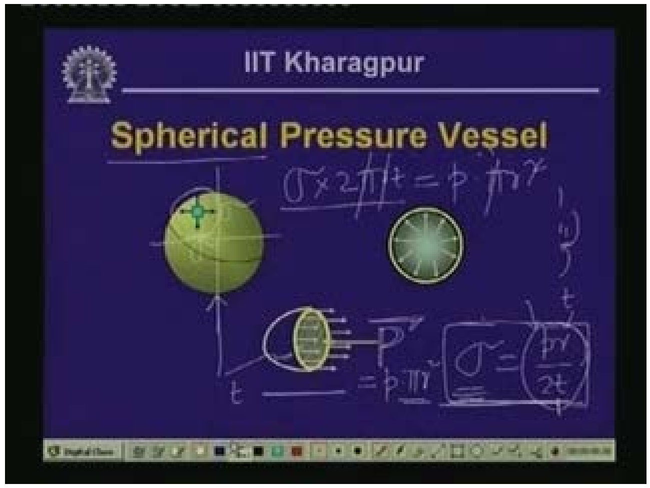

Let us look into the aspects of how you compute the stresses in a spherical vessel. Now this is the one which shows a spherical vessel which is different from the cylindrical vessels. In the previous lesson we had discussed a cylindrical vessel which is a one dimensional substance long in comparison to its cross sectional dimension and thereby we had stresses in the circumferential direction as well as in the longitudinal direction.

In this particular case, the spherical vessel is uniform everywhere having the same radius with respect to the centre so if we cut across the sphere through the centre, at any orientation if we pass the plane we will get a similar type of situation for spherical vessels.

In that sense the spherical vessel is a little different from the cylindrical one. Now let us look into how to compute the stress from the internal pressure. This is the cross-section; if we cut across the vessel through the centre then this is the plan of this spherical vessel. The pressure which is exerted by the content inside is p and let us say with respect to the centre the internal radius is r_i and the external radius is r outer so r outer and r inner are the radii to radii and p is the internal pressure being exerted by the content.

The thickness of this container is so small that practically we take r outer equivalent to r inner and we define by one radius which is r which is the internal radius of the vessel. The same aspect or the same concept holds good in this particular case also that the thickness of the cylinder or thickness of the sphere being small the internal radius of the spherical vessel is equivalent to the outer radius and we deal with only one radius which is the internal radius. Henceforth for all calculations we will deal with the internal radius only.

If we take equilibrium of the forces, if we cut across vertical section through the sphere and if we look into this force distribution, this is the force p which is being exerted by the content, the liquid or the gases so this half is the container along with the content.

Therefore the force exerted by the content p = p pressure × πr^2 the cross sectional area and r is the internal radius of this particular sphere.

The thickness of this sphere is t; hence if we say the stress which is acting on the periphery of the sphere is σ then the force which is being registered P = σ(area) and the area here is 2πr the peripheral distance multiplied by the thickness t.

This area the ring area we calculated in terms of the internal and external radii and if this is r outer and if this is r inner then the area which is shaded is equal to πr $outer^2$ – r $inner^2$ and this we can write as r outer + r inner × r outer – r inner and r outer – r inner is t, and since r we are taking equivalent to r outer or r inner so this is 2r and basically this is 2rt.

Hence this 2πrt is the area multiplied by σ is the force.

Now if we equate these two forces then we have the stress σ × 2πrt the area so this is the resistive force = p × πr^2.

Hence from this we get, the π and π gets cancelled the r^2 gets cancelled so σ = pr/2t.

In this particular case we have only this stress which is acting everywhere. So, in a sphere wherever we take a section through the centre of the sphere we get identical situation which is a little different from the cylindrical form. So whichever section we take in that we have the stress σ; hence we do not distinguish between two stresses σ_1 and σ_2 so both in x and y direction if we take a small element on the surface of the spherical body then we have σ_x and σ_y as the same, they are σ.

In fact if we look into this particular small element which is acting on the surface of the spherical body, now in the x direction we have σ, in the y direction also we have σ so everywhere the same state of stress exist in the case of spherical pressure vessel and the stress is equal to $\frac{pr}{2t}$ where p is the internal pressure, r is the internal radius of the spherical vessel and t is the thickness of the wall.

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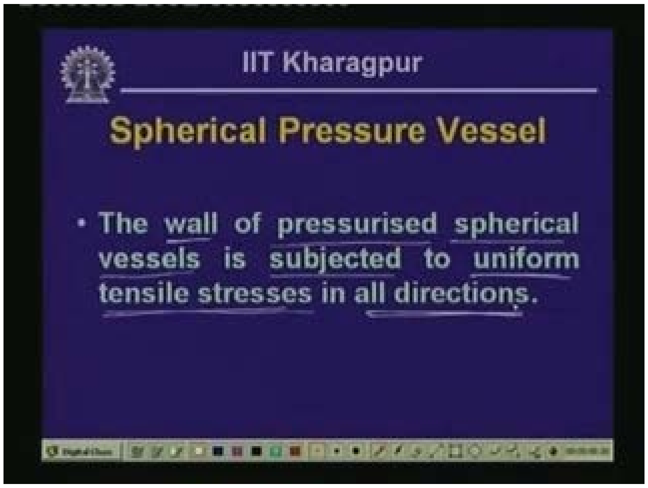

Therefore the wall of pressurized spherical vessels is subjected to uniform tensile stresses in all directions. As we have seen wherever we cut across in a spherical vessel we get the same kind of distribution and as we have equated the resistive force $\sigma \times 2\pi rt$ is equal to the pressure exerted by the content multiplied by the area on which the surface is exerting which is πr^2 and if we equate these two we get $\sigma = \frac{pr}{2t}$.

This is identical; whichever section you orient and in whichever angle it is as long as it passes through the centre of the sphere you are going to get same stress. Everywhere the wall experiences the same state of stress in case of spherical vessels. So, the wall of the pressurized spherical vessel is subjected to uniform tensile stresses in all directions.

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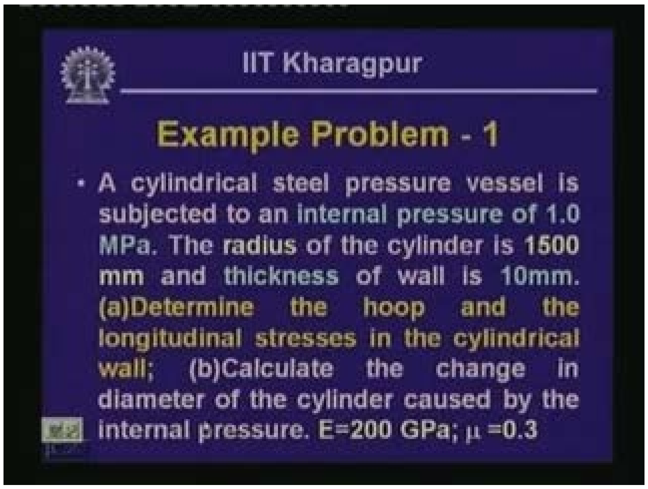

This is the difference between the cylindrical pressure vessels and the spherical pressure vessels.

This was the question posed last time:

Will you get the same stresses in case of spherical vessels as we get in case of cylindrical pressure vessels?

Now the answer to this is that, it is not really the same as we get in case of cylindrical pressure vessels.

Cylindrical pressure vessels are being different in form in comparison to the spherical pressure vessels. In case of cylindrical pressure vessels you get the axial stress, the longitudinal stress as well as the circumferential stress.

Now in this particular case, the sections, since it is identical everywhere the same state of stress exists and thereby the same amount of tensile stress exists in case of spherical vessels.

Let us look into some of the examples of cylindrical and spherical vessels.

This is the example we have seen already but let us look only into the last part which says, what is the change in the diameter of the cylinder caused by the internal pressure?

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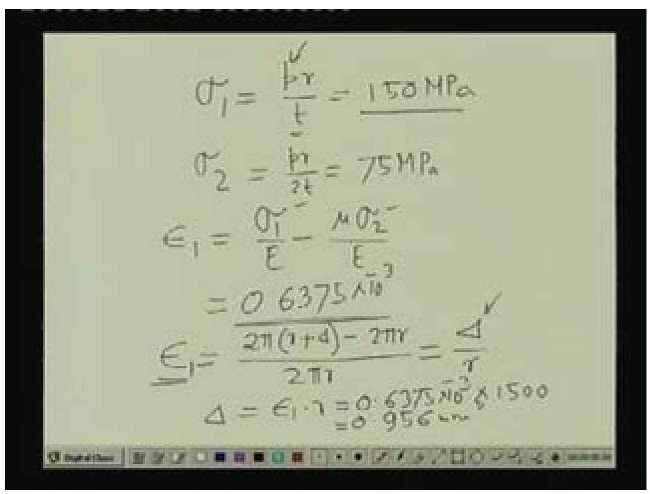

if you remember, the values which we had calculated last time was the value of $\sigma_1 = \dfrac{pr}{t}$ and this we had computed as 150 MPa where p was 1 MPa, r was 1500 and t has a thickness of 10, we had the value of σ_1 as 150 MPa and the value of σ_2 which we had obtained was half of this which was $\dfrac{pr}{2t}$ = 75 MPa.

Based on these we had computed the strain ϵ_1 and we need to use the generalized Hooke's Law to compute the strains from the stresses. So we have computed σ_1 and σ_2 where σ_1 is the circumferential stress or the hoop stress and σ_2 is the longitudinal stress.

The σ_2 is acting in the x direction so let us call σ_x as σ_2 and σ_y as σ_1. Hence from these if we like to compute the strain using generalized Hooke's law then strain $\epsilon_1 = \sigma_1/E - \mu \times \sigma_2/E$.

Hence $\sigma\epsilon_1$ we can write as $\sigma_1/E - \mu \times \sigma_2/E$ so it is the strain in terms of stresses. So the values which we had obtained corresponding to these if we substitute the value of σ_1, σ_2, μ and E this comes out to be .63775(10^{-3}) which is the value of strain.

Now what happens is that when this is being pressurized the peripheral size of the cylinder expands; so there is a change in the radius or the diameter. Now if we assume that there is an extension of Δ in the radial direction then we can write down the strain and epsilon which is in the peripheral direction as $\in$ =

$$\frac{2\pi(r+\Delta)-2\pi r}{2\pi r} = \frac{\Delta}{r}.$$

Now 2πr + Δ is the extended periphery – 2πr which is the distance or the length of the original periphery of the cylindrical vessel divided by 2πr which is the original one equal to Δ/r.

So this strain which is also in the circumferential direction is called as ϵ_1 so ϵ_1 =

$\frac{\Delta}{r}$ where Δ is the extension in the radius.

We can evaluate the value of $\Delta = \in \times r$ and $\in = 0.6375(10^{-3}) \times r \times 1500$ and this gives us a value of the Δ = 0.956 mm. Hence this is the extension in the radius. Now the question was that how much elongation the diameter undergoes, so it will be twice of this Δ which is the extension in the diameter of the cylinder. So the extension in the diameter can be written as extension or the elongation in the diameter of the cylinder equal to 2Δ = 1.912 mm. This is the answer of the question which was discussed last time.

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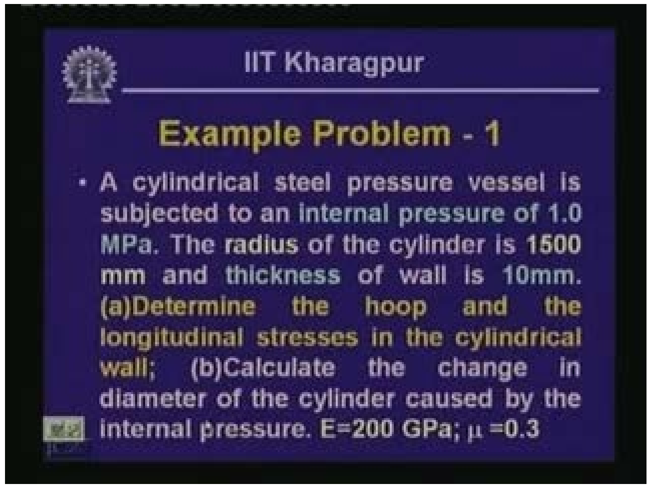

This was a cylindrical steel pressure vessel subjected to an internal pressure of 1 MPa, radius of the cylinder is 1500 mm and the thickness of wall is 10 mm. Last time we had computed the values of hoop stress which is the circumferential stress and the longitudinal stresses in the cylinder. Now we have calculated the change in the diameter of the cylinder which is being caused by this internal pressure.

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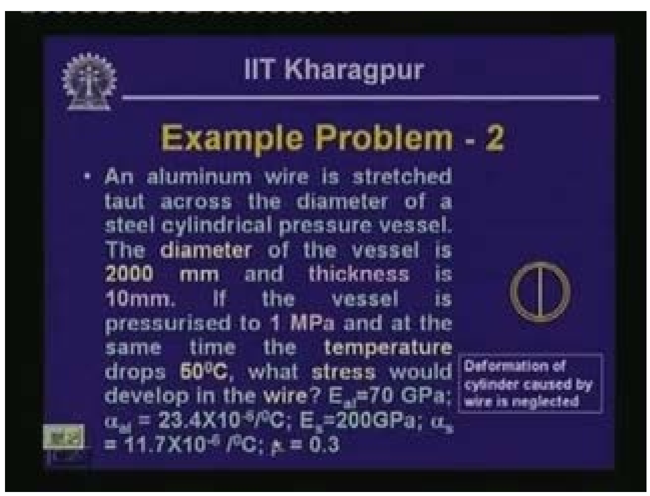

Now let us look into an interesting problem.

An aluminium wire is stretched taut across the diameter of a steel cylindrical pressure vessel. We have a cylindrical pressure vessel and here we have one aluminum wire which is taut across the diameter.

So this is the aluminum wire and this is taut across the diameter of this cylindrical vessel. Now the diameter of the vessel is 2000 mm and the thickness of the vessel is 10 mm.

Now if the vessel is pressurized to 1 MPa and at the same time the temperature drops 50°C then what stress would develop in the wire?

We have pressure from inside which is being exerted at a pressure of 1 MPa and also the whole assembly is undergoing a change in the temperature which drops at 50°C. Also please note that the deformation of the cylinder is caused by the wire because of the pull by the wire, the aluminum wire is connected to the cylinder and if the wire is under compression or tension it is going to deform the cylinder. The deformation of the cylinder is because of the change in this wire tension or compression that is neglected. also one point to be noted is that both these elements the aluminum wire and the steel pressure vessels are undergoing change in temperature simultaneously.

So if this happens then what will be the stress in the wire?

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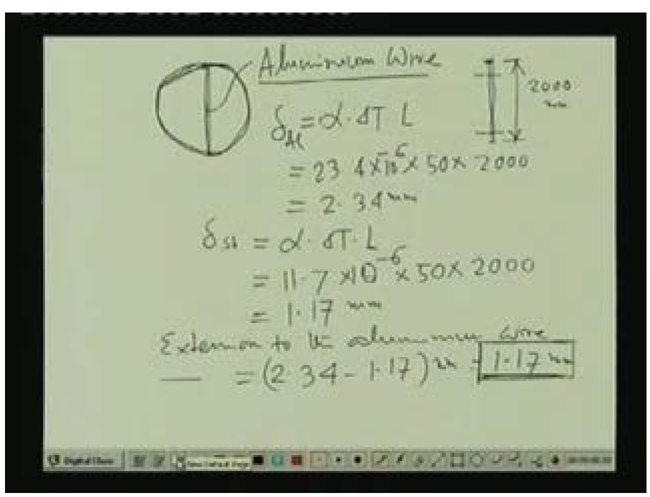

Let us look into this particular example.

First let us compute the value of what happens in the wire because of the change in temperature. Now mind that both the aluminum wire and the vessel are undergoing change in the temperature and at the same time because of the content inside the vessel it is exerting pressure on the wall. Because of that, the aluminum wire will be subjected to a tensile pull.

Let us see what happens if there is a drop in the temperature. We have seen this already in the analysis of strain; what happens in a structural system because of the change in the temperature.

Let us say that this is the aluminum wire which is taut in the steel vessel. Now, to evaluate the effect of the change in the temperature what we can do is that we can release this wire from the vessel. That means if we delink the wire from the vessel and allow the wire to undergo the changes and since the temperature drops here there will be contraction of the aluminum wire.

This is the original length of the aluminum wire which is across the diameter and that is equal to 2000 mm. Let us assume that because of the change in the temperature it undergoes deformation and this is the decrease. Now the decrease in the length of the aluminum wire δ will be equal to α the thermal expansion × δT × the length. Now the coefficient of thermal expansion of aluminum is given as $23.4(10^6)$, so $23.4(10^6)$ × the temperature drop to 50°C so there will be contraction in fact times length which is 2000 so this gives us a value of 2.34 mm.

Since both aluminum and steel are undergoing change in the temperature the steel cylinder also will undergo deformation because of the change in the temperature. And change in the temperature in the steel cylinder is equal to α δT × L.

Here one aspect is to be noted that when the cylindrical vessel is undergoing change because of the temperature there is a change in the periphery of the cylinder and because of that there will be change in the diameter.

If there is a strain ϵ_1 in the peripheral direction then the change in the length in the radius = $\in$ × r.

Likewise the change in the diameter of the cylindrical vessel will be the d × ϵ_1. So we can compute the change in terms of the change in the diameter as well.

so in the steel vessel in the diameter dimension the change will be α δTL = 11.7 α in case of steel, and δT is (10^{-6}) × 50 × 2000 that is L.

The equation is; = 11.7 × 10^{-6} × 50 × 2000.

This is the deformation in the steel cylinder diameter or the diameter of the steel cylinder undergoes deformation of this much which is equal to 1.17 mm.

Now having discussed this particular problem exclusively from the evaluation of strain and thereby the stresses from thermal point of view, if you remember that we had taken down the compatibility where the compatibility is this that the aluminum wire is undergoing contraction by 2.3/mm, steel is undergoing a contraction by 1.171 mm so there is a gap of 1.17 mm between these two so we try to pull the aluminum and we try to compress the steel so that they come to a common place and thereby we evaluate the compatibility and then the corresponding equilibrium equation.

But in this particular problem it has been stated that because of the change in the aluminium wire because of the induction of the stresses that is because of these thermal changes the cylinder is not going to deform so only the aluminum wire has to be stressed and put back in its original position.

As we have seen from this evaluation the aluminum contracts by 2.3/mm whereas the steel contracts by 1.17 mm so this balance 2.34 – 1.17 = 1.17 this aluminum wire has to be pulled and then get connected with the cylindrical vessel.

In the process the aluminum wire will be subjected to a tensile pull to the tune of the extension of 1.17 mm. So the extension that has to be applied to the aluminum wire is 2.34 – 1.17 mm = 1.17 mm.

This is the first part of it, this is because of the temperature, let us keep this in mind that the aluminum wire is subjected to a tensile pull wherein we will have to apply an extension of 1.17 mm.

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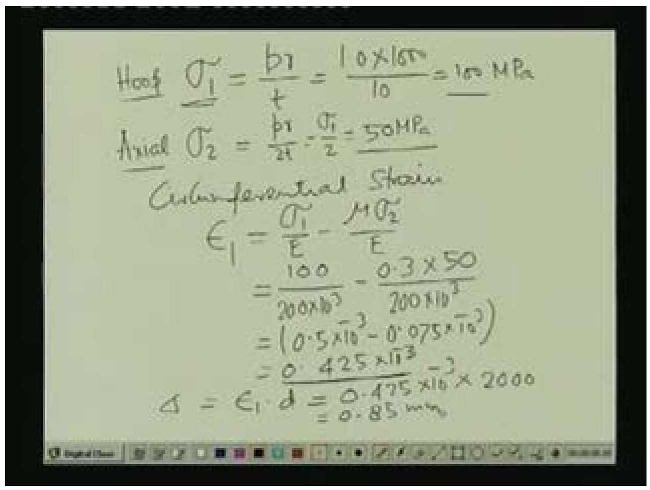

If we try to compute, what is the change that is necessary for pressure, as the cylinder is undergoing internal pressure it will be exerting pressure on the wall the wall is subjected to tensile stress and because of the stresses the circumferential hoop stress and the longitudinal stress there will be a change in the diameter. Now let us look into how much change it undergoes because of these pressures.

The pressure σ_1 which is circumferential pressure which is = $\frac{pr}{t}$ which is evaluated in this particular problem as = 1 is the pressure, r is 1000 because the diameter is 2000 mm/t is10 = 100 MPa. That is; σ_1 = $\frac{pr}{t}$ = $\frac{1.0 \times 1000}{10} = 100 MPa$.

And σ_2 the longitudinal stress is = pr/2t = σ_1/2 = 50 MPa. So; σ_2 = $\frac{pr}{2t}$ = σ_1/2 = $50MPa$.

Therefore these are the values of σ_1 and σ_2 the hoop stress and the axial stress. The circumferential strain because of these stresses we have called this as ϵ_1 in terms ofσ_1 = σ_1/E – $\mu\sigma_2$/E. This is the circumferential strain and E is given as 200 GPa so this is = σ_1 = 100 MPa so $\frac{100}{200}$(10^3) MPa – μ is given as = 0.3 this multiplied by σ_2 which is 50 MPa/E which is 200(10^3). So; $\frac{100}{200}$(10^3) = $\frac{0.3\times 50}{200\times 10^3}$.

Now if we evaluate these values this comes as = .5(10^{-3}) – 0.075(10^{-3}) = 0.425(10^{-3}). This is the strain that it is undergoing the circumferential direction. As we have seen in the previous example that due to the circumferential strain there will be a change in the diameter which is equal to d times this strain.

So the change in the diameter Δ = ϵ_1 × d = .425(10^{-3}) × 2000 = 0.85 mm.

So, if you look into this we have two kinds of extension that we have obtained now in the wire.

In the first case we had the extension that is occurring in the aluminum wire. Because of the drop in the temperature the extension is being exerted on the wire and because the aluminum wire is contracting more in comparison to the steel hence it is subjected to a pull and that extension is coming as 1.17 mm because of the change in the temperature.

Now subsequently as we have looked into, because the cylindrical vessel is subjected to internal pressure it is undergoing another extension in the diameter direction = .85 mm.

So the total extension that the aluminum wire will be subjected to is 1.17 + 0.85 mm.

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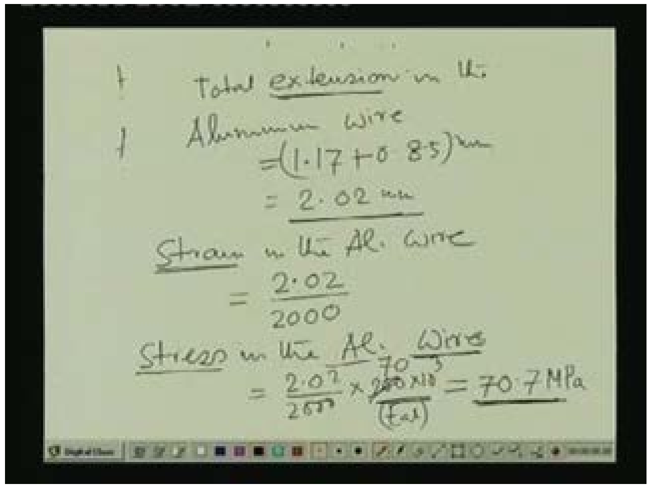

Hence the total extension that the aluminum wire will be undergoing is equal to 1.17 + 0.85 mm = 2.02 mm.

This is the extension that it is undergoing and the strain in the aluminum wire is equal to strain in the aluminum wire is equal to $\frac{2.02}{2000}$ the initial length so the extension by initial length is the strain in the wire and the stress thereby in the aluminum wire, this is equal to the strain multiplied by the value of aluminium that is E 70 Giga Pascal × 10^3 so this gives us the value of 70.7 MPa. Therefore it is: $\frac{2.02}{2000}$ = × 70 × 10^3 So this is E of aluminum which is 70 × 10^3. So this is the stress that the aluminum wire will be undergoing because of the change in the temperature as well as because of the internal pressure that the content is exerting on the cylindrical vessel and in the process the aluminum wire is also getting extended and is subjected to the tensile stress.

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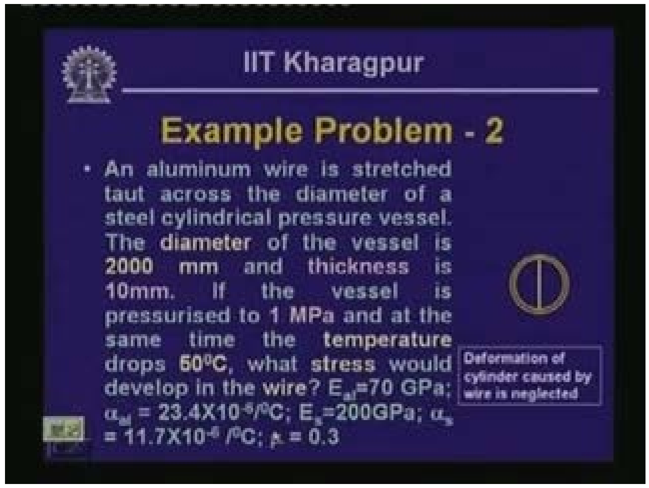

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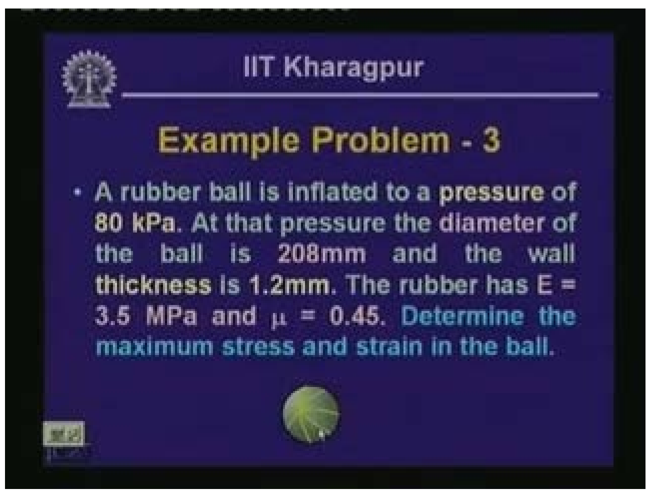

This is the example that we have seen and these two were in respect to the cylindrical vessel. Let us look into the problem which is for spherical vessel which we have discussed today. A rubber ball is inflated to a pressure of 80 kPa and at that pressure the diameter of the ball is 208 mm and the wall thickness of this ball is given as 1.2 mm. The rubber has the modulus of elasticity as 3.5 Mpa and μ = 0.45, so we will have to evaluate the maximum stress and strain in the ball. This is a rubber ball which is inflated to a pressure of 80 kPa and the diameter in that inflated position is 208 mm. Now we got to compute the value of maximum stress and the strain in the ball. This is the problem of a spherical vessel.

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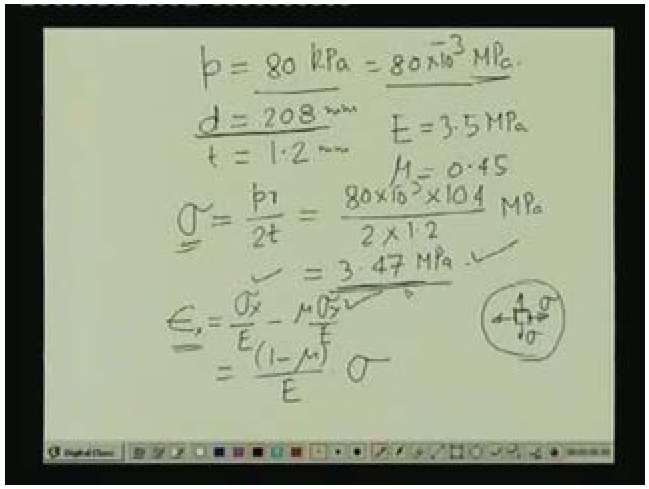

Let us compute the values of the stresses in this it is given that the spherical ball is subjected to a pressure which is equal to 80 kPa which is 80(10^{-3}) MPa. If we write down everything in terms of MPa which is Newton and millimeter t will be easier because we write all dimensions in terms of Newton and millimeter.

The internal pressure given is p = 80(10^{-3}) MPa and at that pressure the ball has a diameter of 208 mm and the thickness of the ball is 1.2 mm the value of the E the modulus of elasticity of the material is given as 3.5 MPa and the value of the Poisson's ratio μ= 0.45. We will have to compute the value of maximum stress and thereby the maximum strain.

In the case of spherical vessel the stress $\sigma = \frac{pr}{2t} = \frac{\left(80 \times 10^{-3} \times 104\right)}{\left(2 \times 1.2\right) \text{ MPa}} = 3.47 \text{ Mpa}$.

As we have seen when we try to compute the strain, in the spherical form on the surface, if we take a small element and if we write down the stresses then we have σ_x and σ_y and in this particular case both σ_x and σ_y = σ.

So in terms of the generalized Hooke's law if we try to write down the value of strain then strain = ϵ_x = (σ_x/E) – (μ × σ_y/E). Now in this particular case both σ_x and σ_y is the valueσ.

And this is equal to (1- μ)/E × σ is the strain. Hence we have already computed the stress so the value of the strain which we get, this is the maximum stress that the wall will be subjected to. So, in terms of that maximum stress if we compute the strain that the wall will be subjected to is as a function of that particular maximum stress which is given by this expression ϵ = (1 – μ)/E σ = μ is 0.45 so this is (1 – 0.45)/E is 3.5 × 3.47, this is the value of the strain and this comes as = 0.545. Hence it is: $\epsilon = \frac{(1\ -\ \mu)}{\mathrm{E}}\ \sigma = \frac{1-0.45}{3.5} \times 3.47$.

So value of stress σ = 3.47 MPa and the value of maximum strain ϵ = 0.545 so these are the values for that particular ball which is pressurized by 80 Kpa and coming to the diameter 208.

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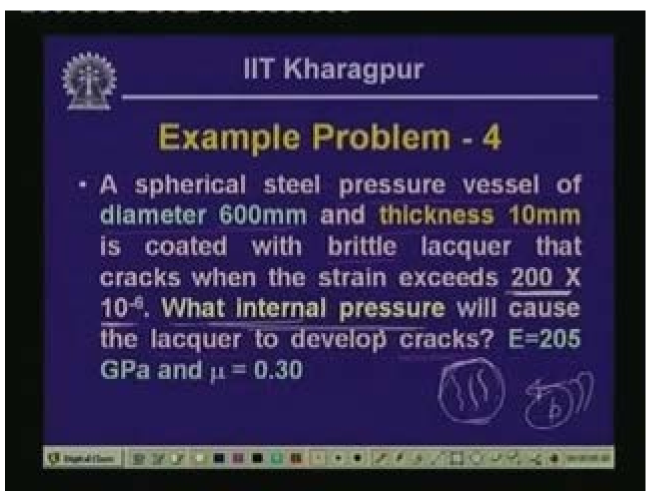

Now let us look into another example of a spherical ball or a spherical vessel where we have a spherical steel pressure vessel of diameter 600 mm and thickness 10 mm coated with a brittle lacquer and that cracks when the strain exceeds $200(10^{-6})$. What internal pressure will cause the lacquer to develop cracks?

If we have a spherical vessel or a container, in case of measuring the strain on a particular stressed body we fix up the strain gauge so that we can acquire the strain data from the surface. Many a times what is done in vessels like cylindrical vessels or spherical vessels are on the surface of these vessels we put some coating called as a brittle coating.

When internal pressure is exerted on such vessels the surface undergoes strains and thereby the stresses and because of such strains or expansions the coating which is put on the surface of the vessels undergo a crack because of being too brittle. These cracks indicate that the pressure vessel has gone to a limit of a particular strain if we know the strain value at which this coating cracks. This is one way of carrying out some experimentation onto what extent a pressure vessel has been strained.

This is one such example where the coating has been applied onto this spherical vessel and because of this pressure some kind of cracks are generated on this lacquer.

Now it has been checked that at a strain of around a limiting strain of $200(10^{-6})$ the lacquer cracks. If we apply this lacquer onto a spherical surface then what is the maximum internal pressure that we can apply on this pressure vessel so that the initiation of the cracking in the lacquer tells us that this is the maximum value of the pressure you can apply onto this particular vessel.

We can carry out the experiment and keep on applying the pressure. As soon as we see that there is generation of the crack we know that it is the maximum value of pressure that can be applied and this can be computed numerically through this example.

Here the vessel has a diameter of 600 mm and a thickness of 10 mm and the value of E = 205 Gpa and μ = 0.3.

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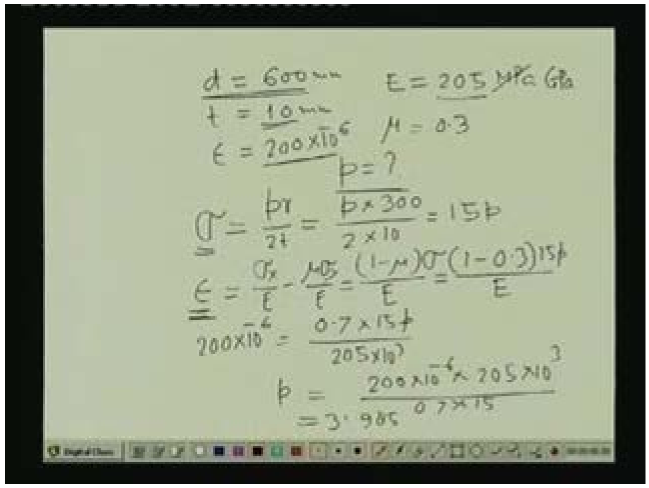

Let us look into the numerical exercise of this.

What we have is the diameter of the vessel which is equal to 600 mm and then we have the thickness of this vessel as 10 mm and it is given that the limiting strain ϵ = 200(10^{-6}**)** and value of E = 205 Gpa and value of μ = 0.3.

These are the values as indicated here. Now we have to find out the value of pressure at which the lacquer starts cracking. In the case of spherical vessel the stress $\sigma = \frac{pr}{2t}$. Now the value of p is unknown, the diameter is given as 600 mm so this is 300/2 × 10 = 15p. So it is; $\sigma = \frac{pr}{2t} = \frac{p\times 300}{2\times 10}$ = 15p.

Here the strain = $\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$ and here σ_x and σ_y is the same which is equal to $\frac{(1-\mu)\sigma}{E}$ so this is the value of ϵ the strain and this is equals to 1 – μ = 0.3 and σ is 15p/E.

Therefore the equation is as follows:

$$\epsilon = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} = \frac{(1-\mu)\sigma}{E} = \frac{(1.03)15p}{E}.$$

As it is said the lacquer cracks at a strain of 200(10^{-6}**).** This the limiting value of the strain. The lacquer can take a strain of 200(10^{-6}**)** beyond which it cracks. And if we calculate the value of pressure corresponding to that and limit our pressure below that then it will not be strained to the desired extent.

Keeping ϵ = 200(10^{-6}) = $\frac{0.7\times 15p}{205\times 10^3}$ so this gives us the value of p and from this if we compute p = $\frac{200\times 10^{-6}\times 10^3}{0.7\times 15} = 3.905MPa$.

If we allow this much of pressure within that particular vessel then the strain will be up to a limit of 200(10^{-6}).

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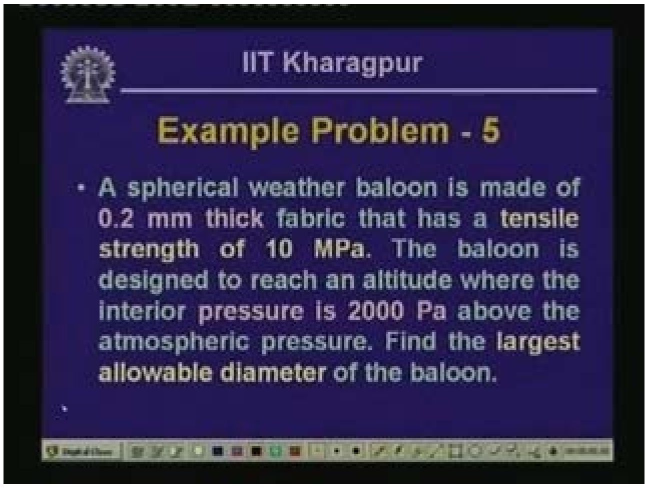

Let us look into another example of this particular category where we have the spherical vessels; a spherical weather balloon is made of 0.2 mm thickness fabric and that has a tensile strength of 10 MPa and the balloon is designed to reach an altitude where the interior pressure is 2000 Pa above the atmospheric pressure. We will have to find out the largest allowable diameter of this particular balloon. The thickness of the balloon is 0.2 ml and the tensile strength of the fabric has 10 MPa.

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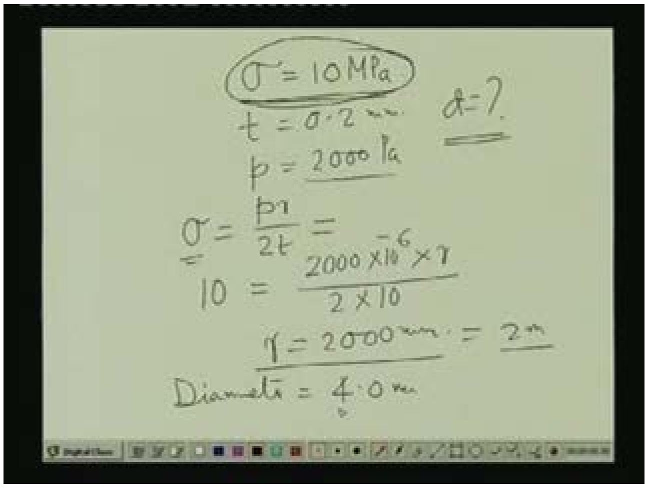

If we look into the stress in this particular balloon, σ is limited to 10 MPa which means the fabric can withstand a tensile strength of 10 MPa. The thickness of the fabric is 0.2 mm and the pressure to which this is applied is 2000 Pa.
We will have to find out largest diameter of this particular balloon that we can adopt.

As we know; $\sigma = \frac{pr}{2t}$, the permissible stress = 10 MPa and σ can be allowed up to an extent of 10 MPa = $\frac{2000 \times 10^{-6} \times r}{2 \times 10}$ and this gives us a value of r = 2000 mm = 4 mm which will be the diameter of the balloon.

This is 2 meters, so the diameter of the balloon that we can adopt is 4 m. As we can see here the limiting stress which is given for the fabric with which the balloon is composed of is given as 10 MPa. That is the maximum amount of tensile stress that can be applied onto the fabric. The fabric has a thickness of

0.2 mm. So if we apply the internal pressure in 2000 Pa and then we try to apply the pressure then we try to find out up to what extent we can go for what diameter of the balloon. As we see, we can go up to a diameter of 4m for such a problem.

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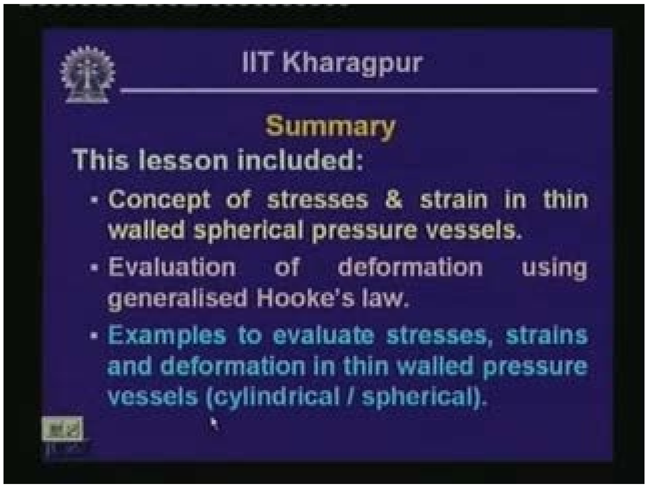

Let us summarize what we have learnt in this particular lesson. We have learnt the concept of stresses and strain in thin walled spherical pressure vessels. In fact in the previous lesson we had introduced the concept of cylindrical pressure induced in a cylindrical pressure vessel. Here we have seen the stresses induced in spherical pressure vessels.

Now you are in a position to compare how the stresses get induced because of the internal pressure of the content, either the liquid or the gas for which the vessel is subjected to the stresses.

We also looked into the evaluation of deformation using generalized Hooke's law, we are computing the value of the stresses on the pressure vessel surface which are σ_1 and σ_2 in case of cylindrical pressure vessel and σ which is uniform everywhere in case of spherical pressure vessel.

Once we compute the values of these stresses then we can compute the value of strain based on these stresses and these strains can be computed using the generalized Hooke's law which is $\sigma/E - \mu \times \sigma_y/E$. Based on that you can compute the value of the strain at any point based on the stresses.

We have also seen some examples to evaluate stresses strains and deformation in thin walled pressure vessels both in terms of cylindrical and spherical.

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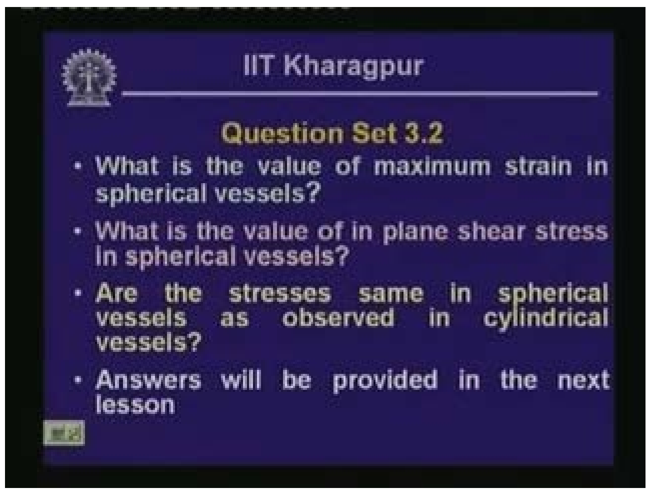

Here are some of the questions:

What is the value of maximum strain in spherical vessels, what is the value of in plane shear stress in spherical vessels?

As we have seen in case of cylindrical pressure vessels, what is that value in spherical pressure vessels?

Are the stresses same in spherical vessels as observed in cylindrical vessels?