Strength of Materials
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Lecture -15
Application of Stress by Strain
Thin-walled Pressure Vessels - I

Welcome to the first lesson of third module which is on thin-walled pressure vessels part one which is on the application of stress and strain.

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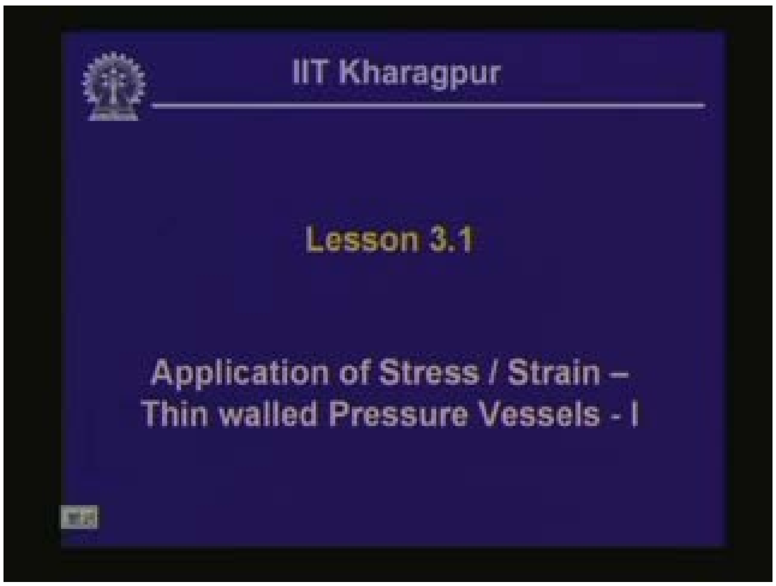

In the last two modules 1 and 2 we had looked into the aspects of analysis of stress and analysis of strain at a point in a stress body as to how we compute the stresses and strain?
Now we will look into the applications of this stress and strains at a point in thin-walled pressure vessels. The pressure vessel has wide industrial applications. In many industries we use this kind of vessel where the pressurized air or water is used. It is necessary to evaluate the strains and the stresses at any point on the surface of pressure vessels. We will also look into how to compute the strains and stresses on the body of such pressure vessels.

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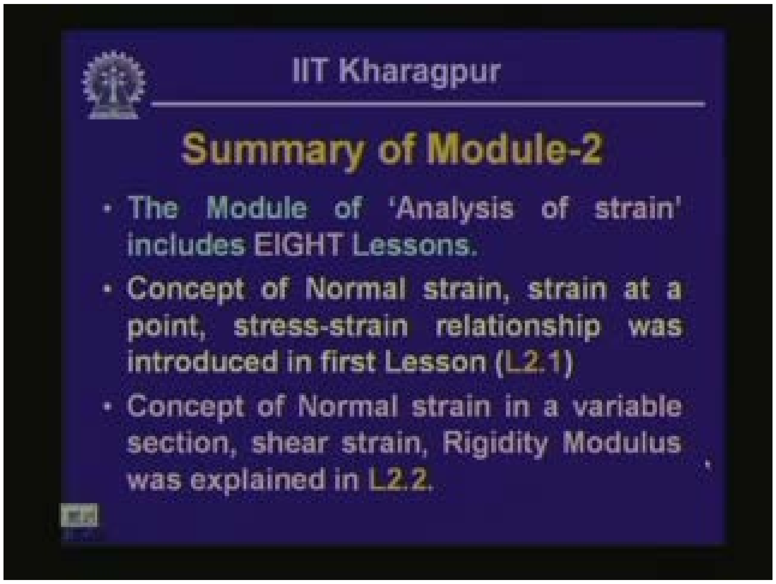

Before we go into the analysis of pressure vessels let us look into aspects which we discussed in the last module. Module-2 was devoted to on analysis of strain and we had 8 lessons, so let us quickly look into those lessons which we covered in Module-2.

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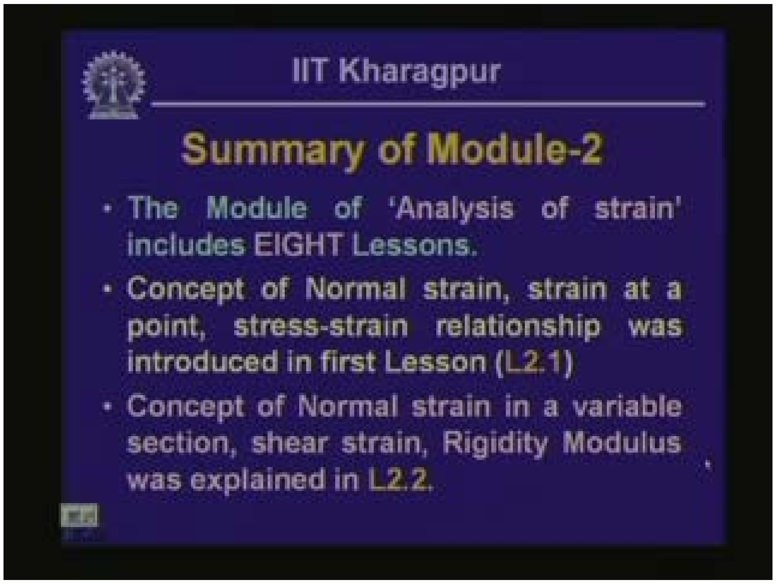

The module of analysis of strain includes 8 lessons. The 1st lesson was on the concept of normal strain, the strain at a point and stress strain relationship. The 2nd lesson was devoted to the aspect of normal strain in a variable section. In the first lesson we discussed the normal strain in a uniform body. If the body is having a variable section then what will be the strain in that and correspondingly the shearing strain in that. We also discussed the concept of rigidity modulus in the second lesson.

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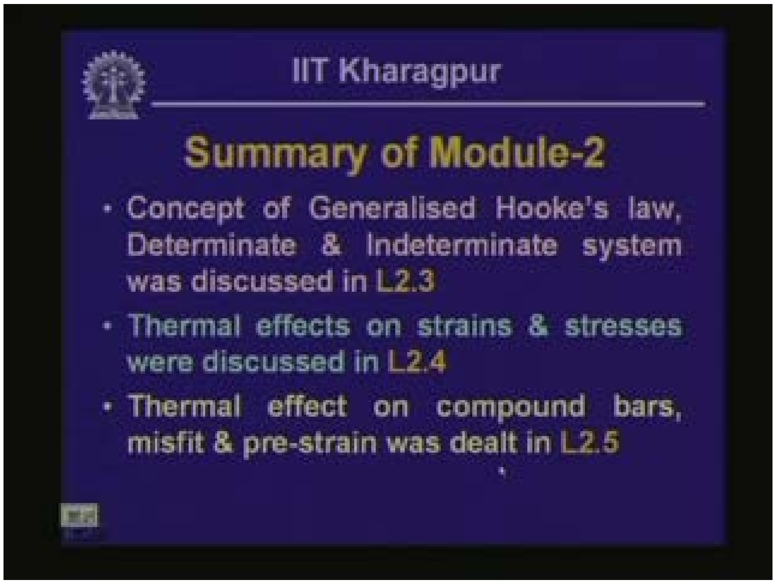

In the 3rd lesson we had introduced the concept of generalized Hooke's law and the determinate and indeterminate system were discussed. Thermal effects on strains and stresses were discussed in Module-2 and the fourth lesson. Thermal effect on compound bars and the misfit and correspondingly the pre-strain and pre-stresses was dealt with in the 5th lesson of Module-2.

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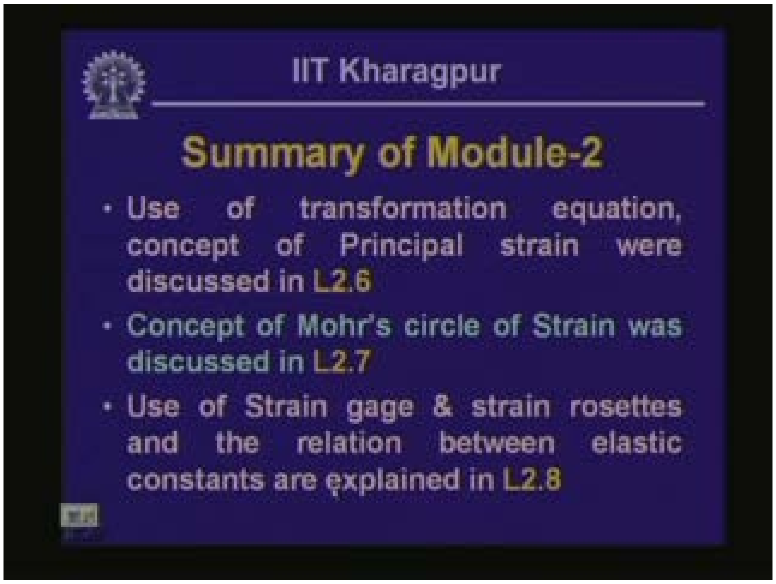

In the 6th lesson we discussed the transformation equation and we had discussed this aspect in the first module of stress as well wherein we say, at a particular point if you are interested to compute the values of stress and thereby the strains then we need to use the transformation equation if we orient the axes system from the rectangular axes system.

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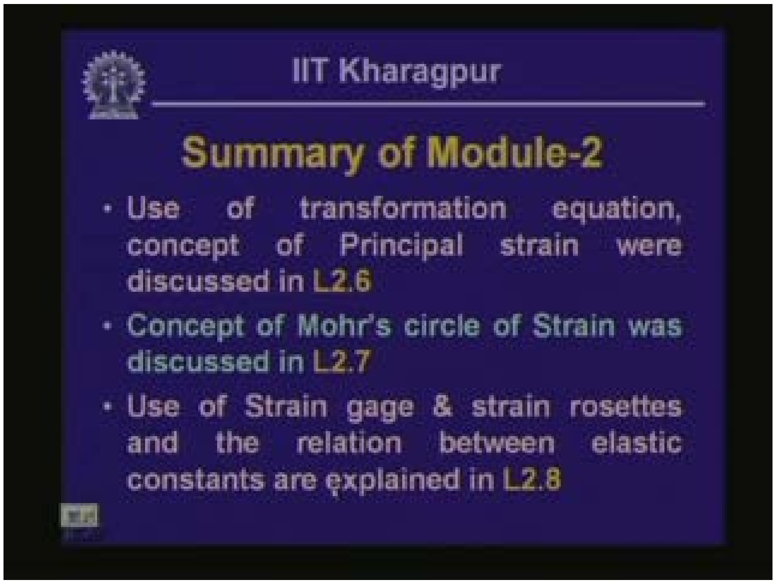

That is what was discussed in Module-2 lesson 6 in 2.6 where we had introduced the concept of transformation equation and thereby how to evaluate the principal strain. Then we saw the concept of Mohr's circle of strain, how to compute the strain at different orientations at that particular point where we know the rectangular strain components $\in_x$, $\in_y$, Υ_{xy} how to compute the values of principal strain and strain at that particular point at different orientations. If you like to compute we can do it through the use of Mohr's circle of strain and that is what was discussed in lesson 7.

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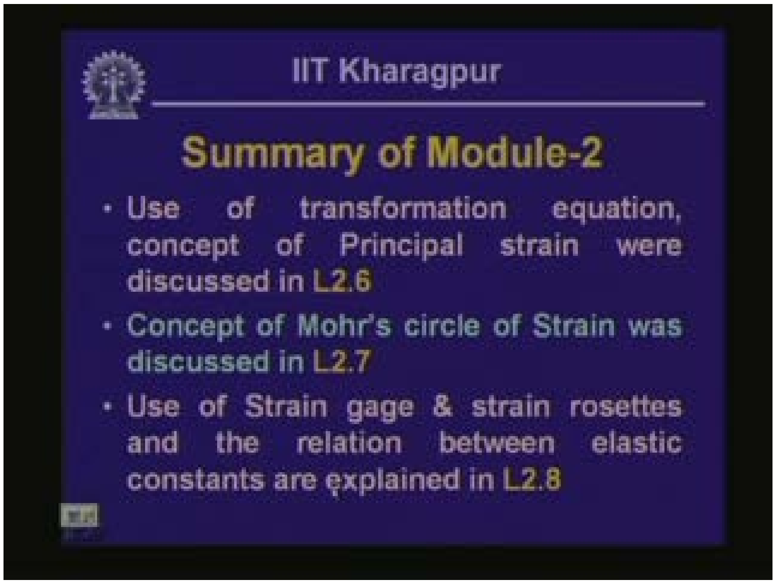

In the 8th lesson we introduced the concept of strain gage and strain rosettes. Thereby at any point it is easier to measure the strain and it is difficult to measure the stresses at any point when we talk about a stress body. We can measure the strains and thereby from those measures of strains we compute the stresses but as we have seen we measure the strains in the normal direction as the normal strain. Therefore out of three strain components $\in_x$, $\in_y$, Υ_{xy} it is very difficult to measure Υ_{xy} the shearing strain. Hence we take an indirect way to measure Υ_{xy} by using strain gages in three directions which we have called as strain rosettes. From the measured data of strain rosettes, we can compute the strains of $\in_x$, $\in_y$, Υ_{xy}. Using these strain components we can compute the principal strains and thereby the principal stresses.

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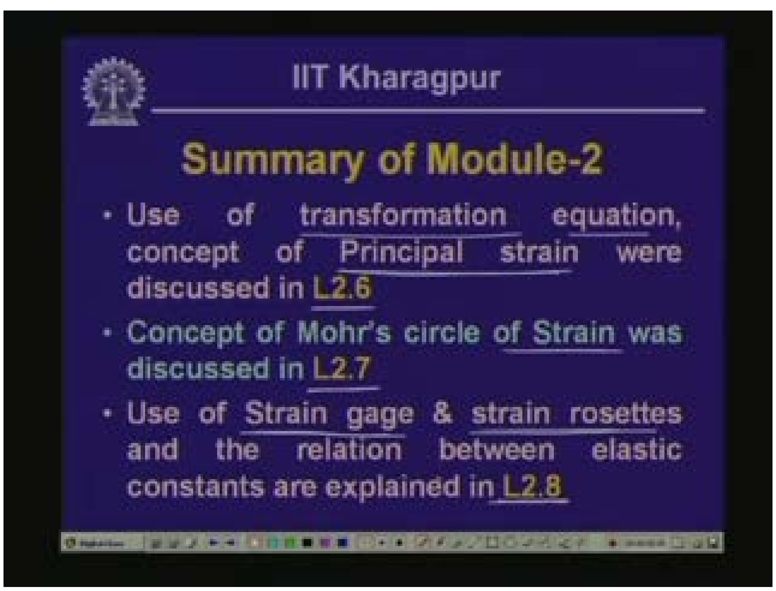

This is what was discussed in lesson 8 Module-2. Also, we looked into the relationship between the elastic constants which were E, G and mu.

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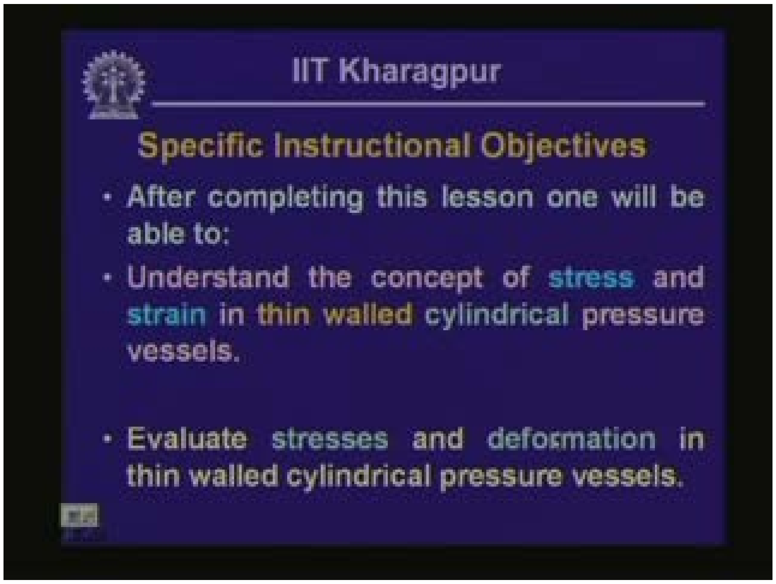

This is the summarization of the Module-2. It is expected that once this particular lesson is completed, one should be able to understand the concept of stress and strain in thin-walled cylindrical pressure vessels. Here we are concerned with two aspects, one is the vessel is subjected to internal pressure and this internal pressure is over and above the external pressure by overcoming the external pressure on the vessel the internal pressure is working and thereby there will be the stresses generated in the vessel and we are interested in evaluating those stresses.

Secondly, this is a thin-walled structure and this means that the thickness of the wall is very small so that the stress variation across the thickness is significantly small or we neglect that. Hence the thin-walled pressure vessel is of importance and we will look into how to compute the stresses in such vessels.

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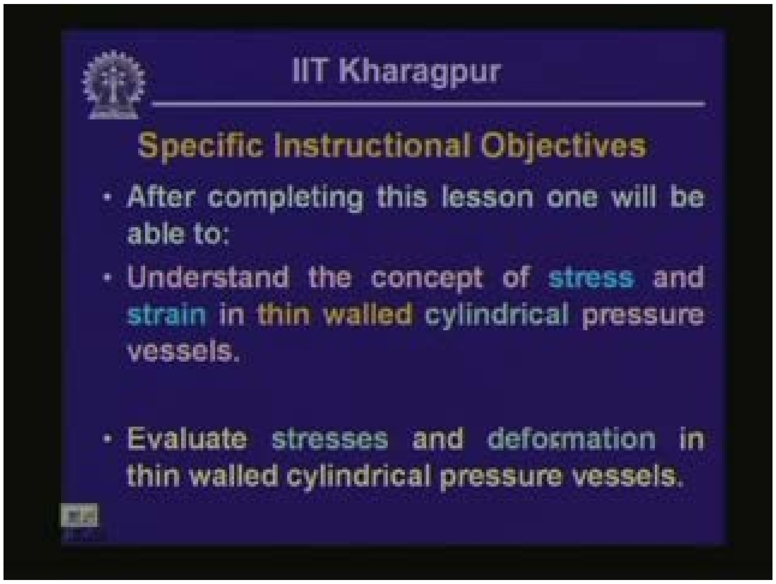

Thereby we will evaluate stresses and deformation in the thin-walled cylindrical pressure vessels subjected to internal pressure because of the content in the container.

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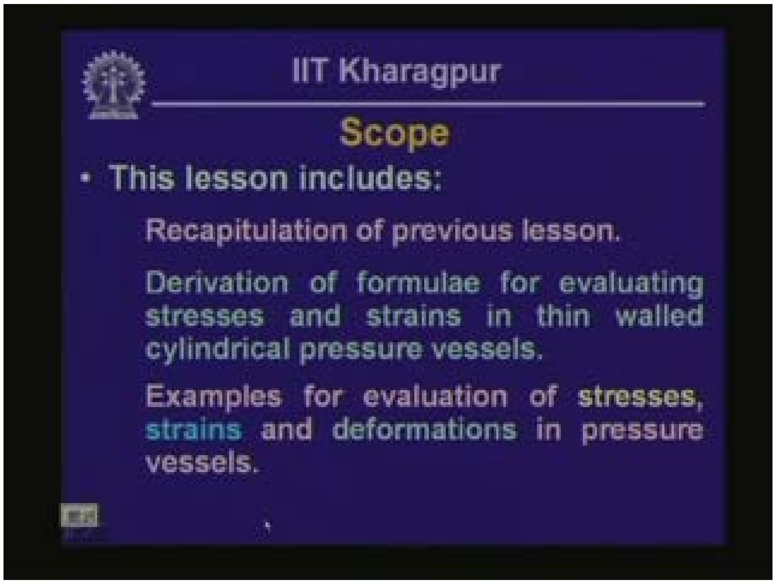

As we keep going through we will be looking into the aspects of previous lessons or recapitulation of previous lessons which we will be doing through the question answer session. We will look into the answers for those questions which I had posed for you in the last lesson and thereby will be scanning through the aspects which we had discussed in the previous lesson.

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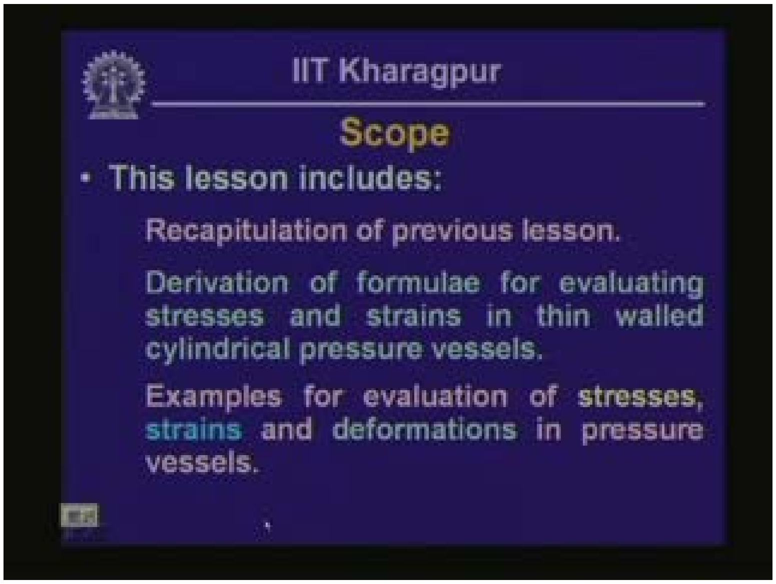

Also, we are going to derive the formulae for evaluating stresses and strains in thin-walled cylindrical pressure vessels. In this particular lesson we will be concentrating on the cylindrical type of pressure vessels and subsequently we look into different categories of pressure vessels. We will also look through a few examples for evaluation of stresses, strains and deformations in such pressure vessels. These are the two main aspects that we will be doing in this particular lesson.

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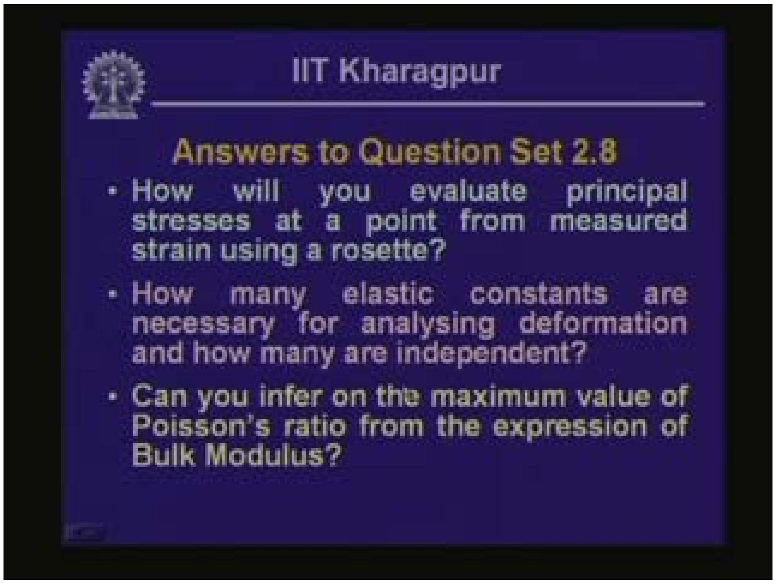

Let us look into the answers to this questions which I posed last time. The first question is how you will evaluate principal stresses at a point from measured strain using a rosette. When we try to measure stress at a point we cannot directly measure the stress. Hence we measure strain at that particular point and thereby we compute the values of stresses. The strain component as we have $\in_x$, $\in_y$, Υ_{xy} under a plane strain situation we cannot measure Υxy directly, instead we measure indirectly and from there we compute the values of principal strains. Now let us look into how that is done.

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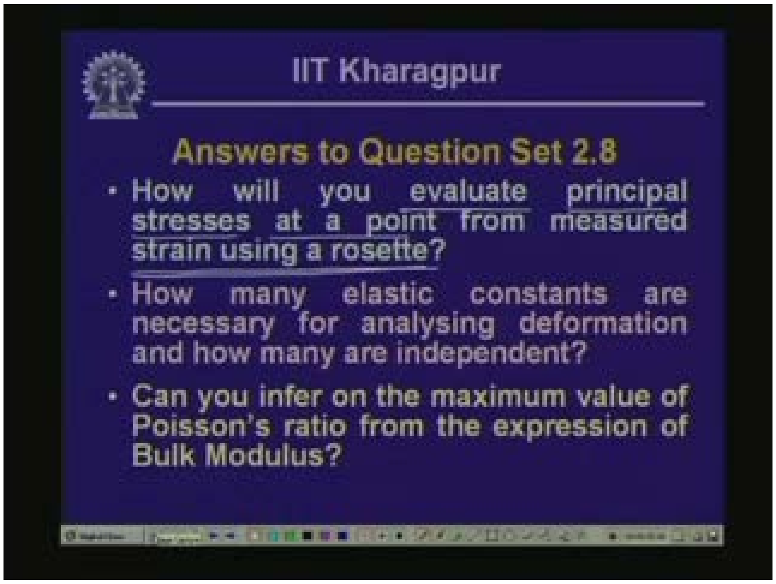

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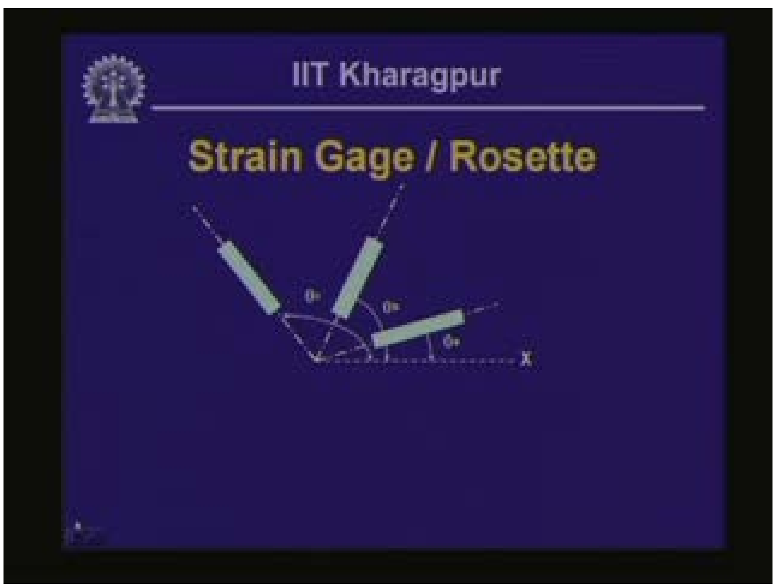

In the previous lesson we had discussed that this is the x direction and this is the y direction. We can use these kinds of strain gages which can measure the normal strain. If we place these types of gages in the x direction and y direction we can measure the strain $\in_x$, $\in_y$, but we need three quantities $\in_x$, $\in_y$, Υ_{xy} for evaluating the principal strains. Since we cannot measure Υ_{xy} directly what we do is that we place three strain gages in three different orientations. Let us call this direction as a, this as b and this as c. All three are oriented at three different angles; this particular angle is $theta_a$, this angle in the second gage which is along the b is at an angle of $theta_b$ and the third gage which is along c is oriented at an angle of $theta_c$. And thereby employing the transformation equation which is: $\in$x' is equal to $\in$ ($\in$x plus $\in$y) by 2plus ($\in$x' $\in$y) by 2 cos2theta plus ($\frac{\gamma xy}{2}$sin 2theta) is the transformation equation. We can employ this equation to evaluate strain along a, b and c which are $\in$a, $\in$b and $\in$c. Keeping in mind that $\in$a is acting along a which is at an orientation with reference to x axis at an angle of $theta_a$ hence in the place of theta we will substitute $theta_a$ and for $\in$b the orientation is along $theta_b$ and for $\in$c the orientation is along $theta_c$.

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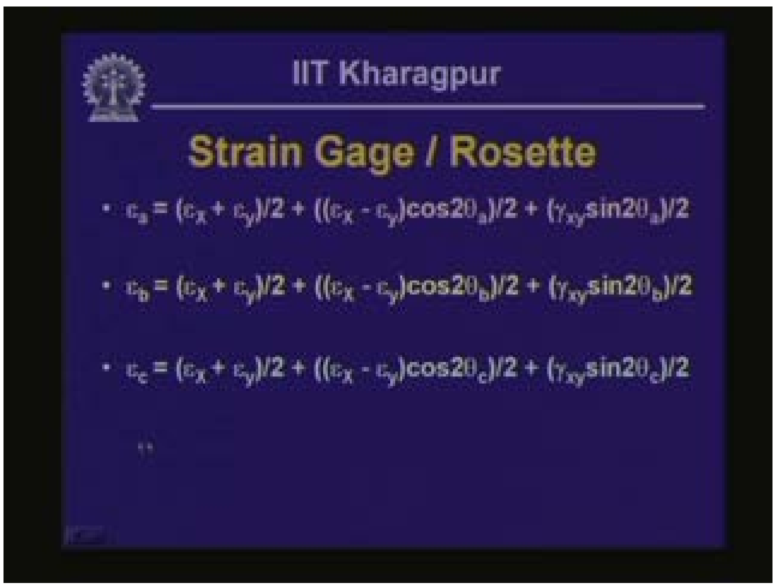

So, if we substitute that we can get the equations in this particular form where we have three equations $\in$a, $\in$b and $\in$c and they are oriented at an angle of $theta_a$, $theta_b$ and $theta_c$. In these three equations if we look we know $\in$a, $\in$b, we know $\in$c. Also, we know $theta_a$, $theta_b$, $theta_c$ as these are predetermined orientations that gages along which it will be placed $theta_a$, $theta_b$, $theta_c$ are predetermined hence $theta_a$, $theta_b$, $theta_c$ are known. And since we are measuring strain along a, b and c so $\in$a, $\in$b and $\in$c are known.

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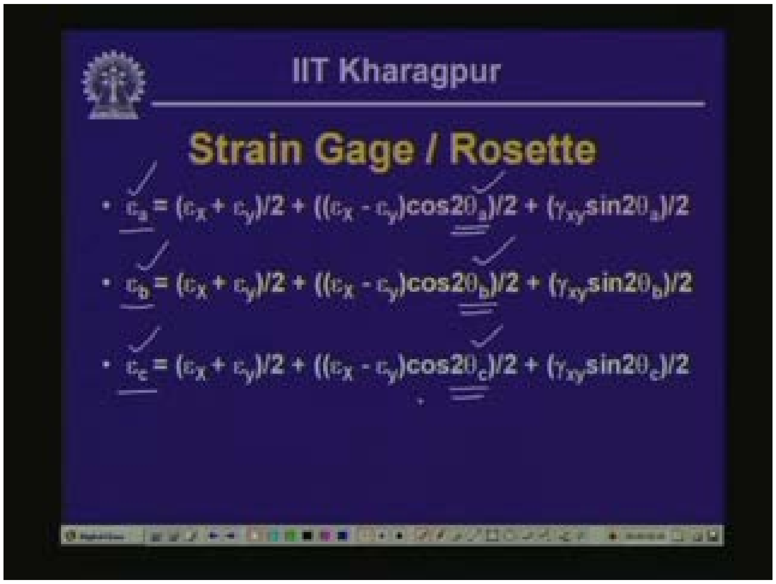

So in these three equations six quantities are known so the only unknown quantities are $\in_x$, $\in_y$, Υ_{xy} which we can evaluate from these three equations. Once we know $\in_x$, $\in_y$, Υ_{xy} now we can compute the values of $\in_1$ and $\in_{2.}$ We can compute as a function of $\in_x$, $\in_y$, Υ_{xy} which is ($\in_x$, $\in_y$) whole square plus $\sqrt{\ }$ ($\in_x$,$\in_y$ by 2) plus (xy by 2) whole square. So this is the value of maximum principal strength and same quantity with minus will give the quantity of $\in_{2.}$ So we can evaluate the value of $\in_1$ and $\in_2$ from $\in_x$, $\in_y$, Υ_{xy}. We computed the values of strain in terms of stresses and there E_1 is equal to sigma_1 by E - musigma 2 by E where sigma_1 and sigma_2 are the principal stresses and $\in_1$ and $\in_2$ are the principal strain and sigma_1 and sigma_2 are the principal stresses. And as we know the direction of principal stresses and direction of principal strain coincides.

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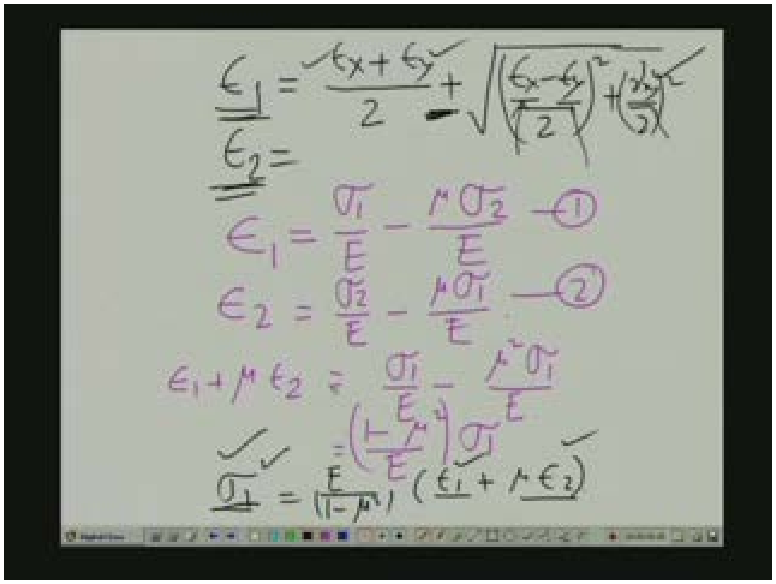

So we can compute the values of principal strain from the principal stresses in this form and likewise $\in_2$ the strain in the minimum strain direction is equal to sigma$_2$ by E minus mu sigma$_1$ by E. If we multiply the second part of this equation with mu and add of with the first one we get $\in_1$ plus mu star $\in_2$ is equal to sigma$_1$ by E.

Now this minus mu sigma by E and this is sigma$_2$ by E and when it is multiplied with mu this gets cancelled so we have minus mu square sigma$_1$ star e or this is equal to 1 minus mu square star e star sigma$_1$. From this we get the value of sigma$_1$is equal to E by (1- mu) whole square ($\in_1$ plus mu$\in_2$). So this is the value of sigma$_1$ in terms of $\in_1$ and $\in_2$ and $\in_1$, $\in_2$ we have obtained in terms of $\in_x$, $\in_y$, Υ_{xy}. So once we measure strains along three directions we can compute $\in_x$, $\in_y$, Υ_{xy} and from those measured values of $\in_x$ or evaluated values of $\in_x$, $\in_y$, Υ_{xy} we can compute $\in_1$ and $\in_2$ and once we know $\in_1$ $\in_2$ .We can find out stress sigma$_1$ likewise stress sigma$_2$ stress is equal to E by (1- mu) whole square ($\in_2$ plus mu$\in_1$). So these are the values of the principal stresses sigma$_1$ and sigma$_2$.

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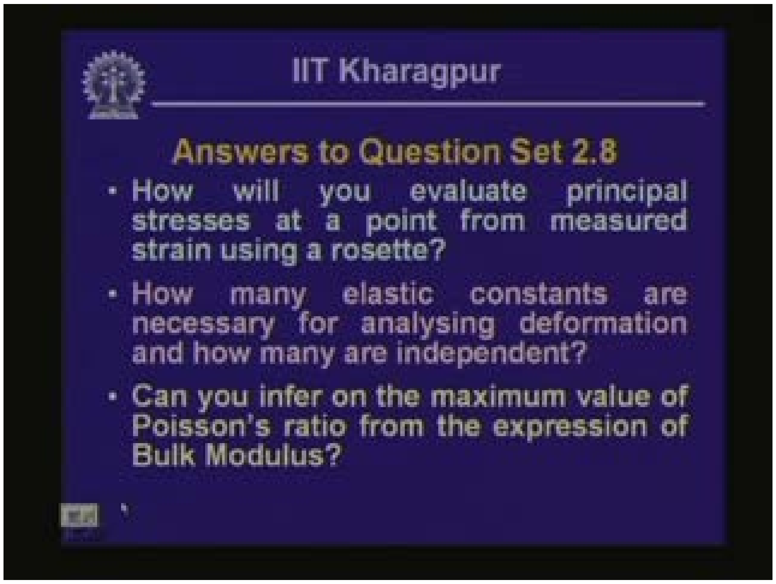

How will you evaluate principal stresses at a point from measured strain using a rosette?
This is how from the measured strain data we can compute the values of principal stresses which is $sigma_1$ and $sigma_2$. Let us look into the second question. How many elastic constants are necessary for analyzing deformation and how many are independent?

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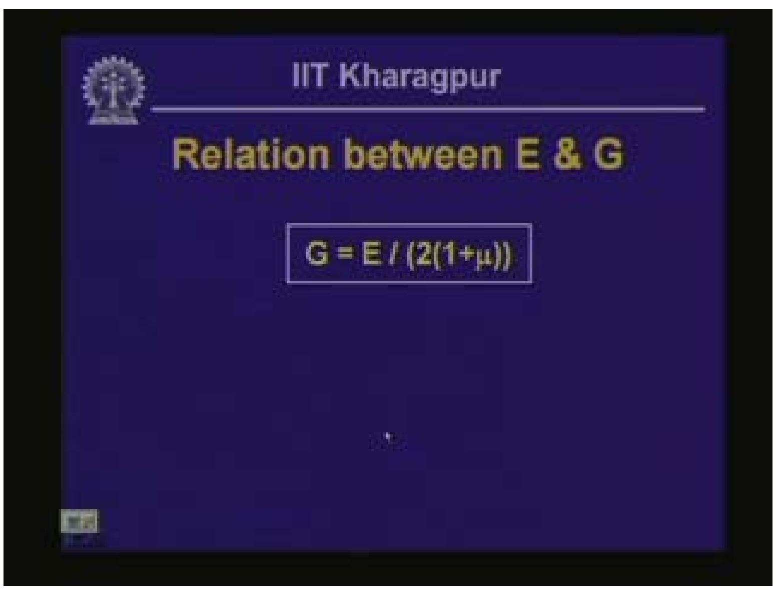

Now we have seen that when we are trying to compute the stresses and the strains we are requiring three constants which are E, G and mu. The modulus of elasticity is E and G is the shear modulus and mu is the Poisson's ratio. As we have seen last time when we were relating to shear modulus with elastic modulus out of these three constants; E, G and mu which are necessary for analyzing deformation in a stress body two are independent and in fact G can be evaluated once we know the value of e and mu. So, out of these three elastic constants two are independent and one is dependent on the other two. This is what is demonstrated here that G is equal to E by 2(1 plus mu) as we derived last time. Once we know the value of E and mu we can get the value of G. Though we need these three constants the two are independent.

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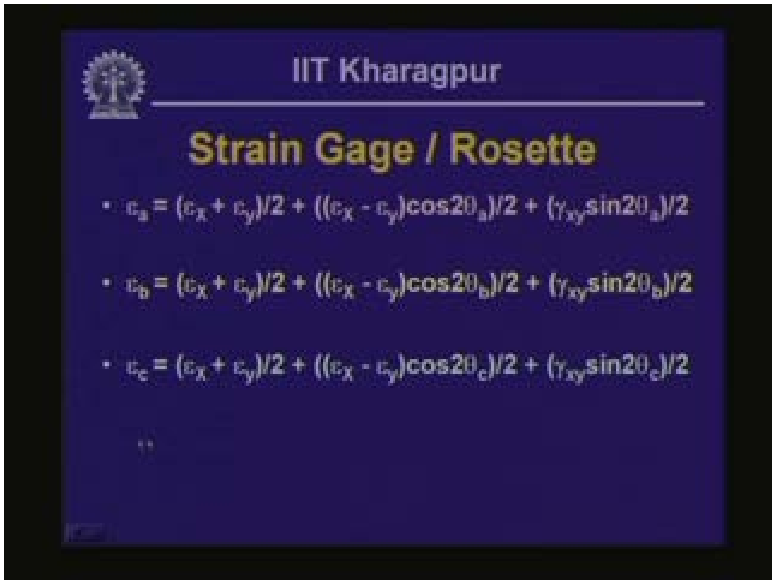

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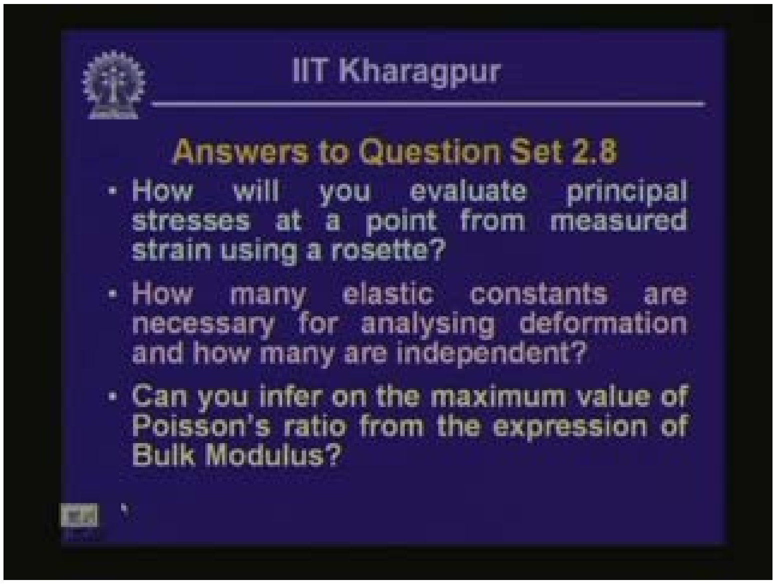

The third question is can you infer on the maximum value of Poisson's ratio from the expression of bulk modulus?

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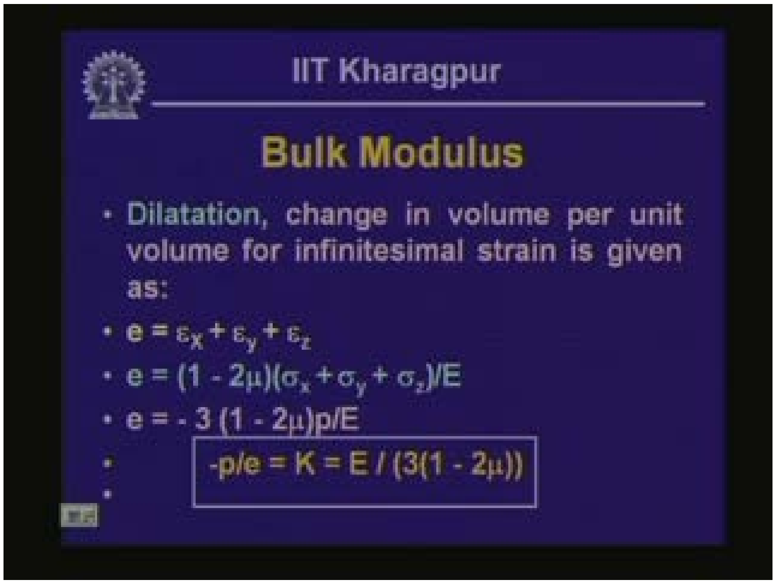

Let us look into the bulk modulus. Dilatation which we have denoted as e is defined as Δ_v by v where Δ_v is the change in the volume and V is the original volume and that is how we say that the dilatation is the change in volume per unit volume. If this is equal to 1 then E is equal to Δv. Now in the expression for the change in volume or the dilatation we have the Poisson's ratio mu. There is another term here which is p, and e you are already acquainted with which is elastic modulus and p is the term which is the hydrostatic pressure acting at that particular point in the small element. Now p is the stress quantity which is acting in all directions in place of σ_x σ_y and σ_z. These are the values of p which are called as the state of hydrostatic pressure. That means that element is subjected to a compressive stress from all sides and it is under the state of a compressive force.

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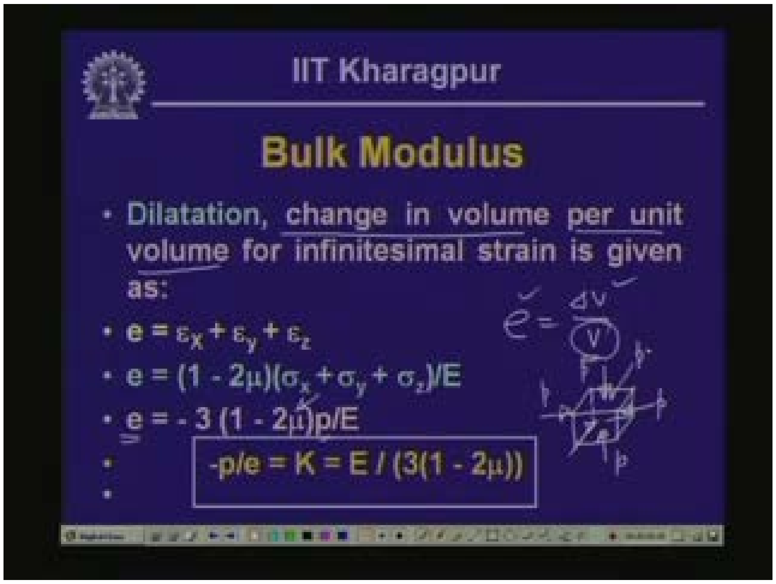

So if you look into this particular expression where e is equal to minus 3(1 minus 2mu), the minus indicates that it is under compression. Now if we have the value of mu ≥ 0.5 then 1 minus 2mu this particular quantity becomes negative. And once this becomes negative means the minus and minus becomes positive which indicates that Δ_v is increasing that means the volume is increasing. That means if the value of mu is greater than ½ then it states that there is an increase in the volume which is contradictory to the physical strain.

In the physical strain it has to see to that particular element is subjected to hydrostatic pressure from all directions. Thereby σ_x σ_y and σ_z are all minus p and now this is trying to compress the body. If we have the higher value of mu ≥ 0.5 then it shows that it is expanding which is contradictory to the physical strain. So the maximum value of mu that you can have is 1 by 2. So as we had stated earlier also the maximum value of mu is equal to 0.5. These are the three questions that we had. And in fact these are the aspects which we had discussed in the previous lesson and if you go through the previous lesson you should be in a position to answer these questions.

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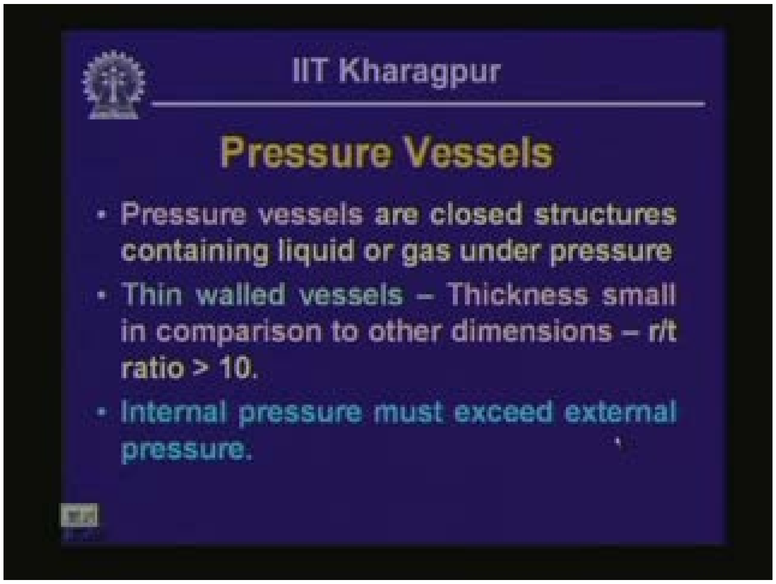

Pressure vessels are the containers in which the gas or liquid is put under pressurized condition and thereby it exerts pressure in the internal surface of the container and if that internal pressure is higher than the external pressure the body will be experiencing stresses and basically there will be tensile stress in the body. Our objective here is to evaluate those stresses in such pressurized containers. We use compressed air in most applications and another example of this type of pressure vessel is the water flowing through a thin-walled pipe.

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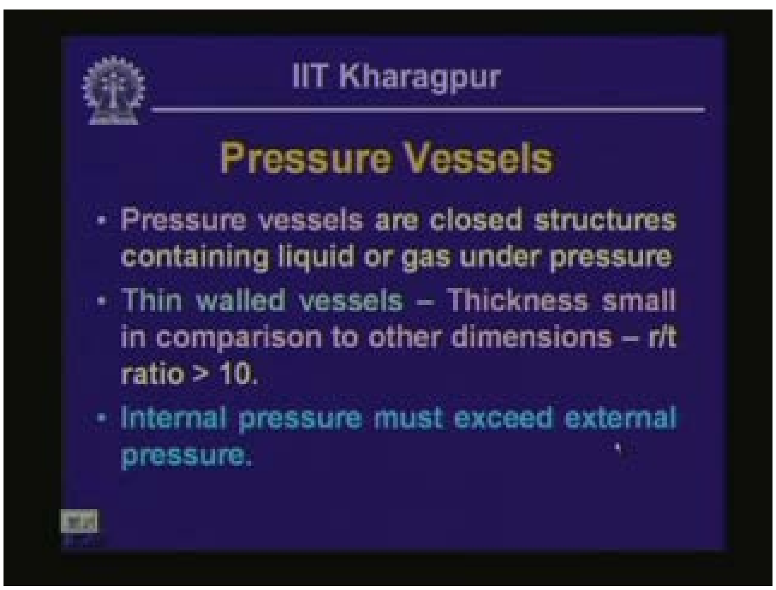

What is stated here is that the pressure vessels are closed structures containing liquid or gas under pressure. And we have qualified the terms pressure vessels with thin-walled because the thickness of the wall or the thickness of the container wall is sufficiently small. It is so small that the variation of the stress across the thickness is insignificant and thereby we consider that there is a state of stress which is on the surface in two directions and the third direction stress is 0.

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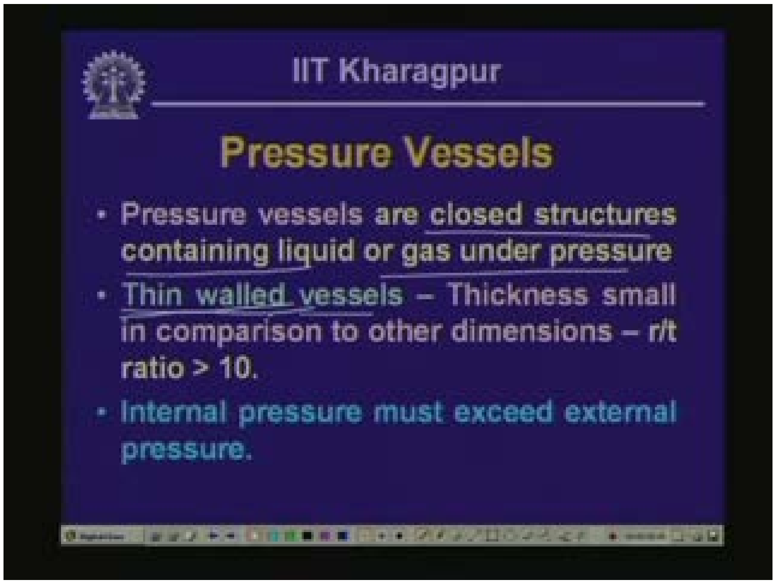

For those vessels for which the thickness is so small that in comparison to the other dimension that the ratio of r by t where r is the radius of the cylindrical tank and t is the thickness so if r by t ratio is ≥ 10 then we call those kinds of vessels as thin-walled vessel. The term ten or number 10 here actually is considered that it has been observed if r by t ratio exceeds value of 10 then the error in the stress level is the minimum and thus it is the limiting value considered for thin-walled vessels.

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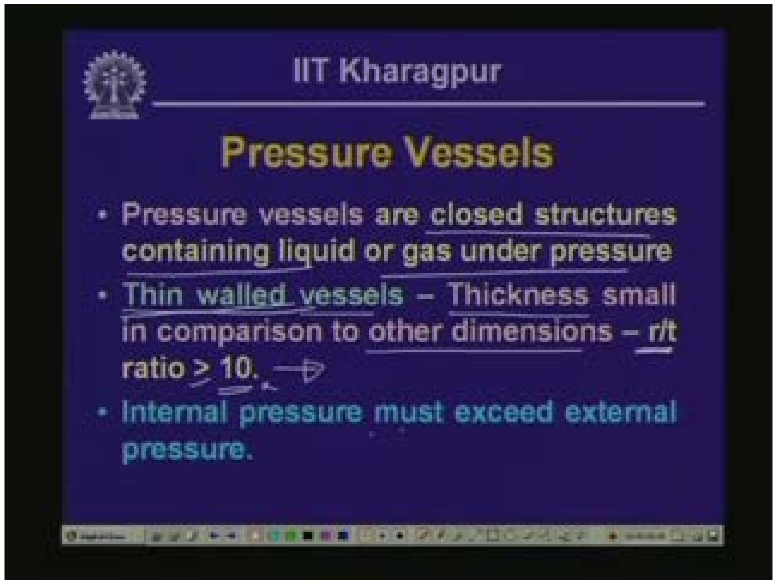

Actually the internal pressure must exceed the external pressure otherwise there will be other kinds of problems.

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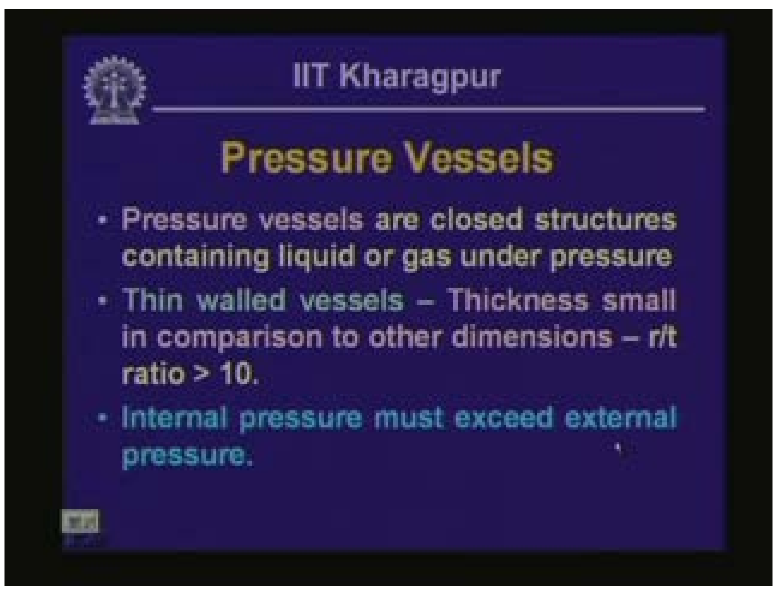

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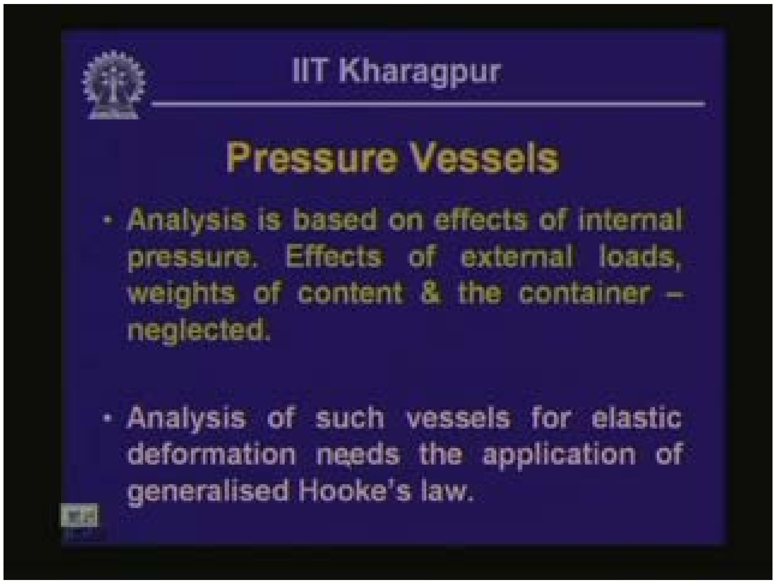

Basically in pressure vessels when we try to analyze we try to go for a simplified analysis and this analysis is based on the effect that the internal pressure exerts pressure or the forces on the body and thereby the body is subjected to stress. That is why we try to analyze how much stress the body undergoes when the container is subjected to pressure but we neglect other effects such as effects of the external loads, the weights of content and the weight of the container.

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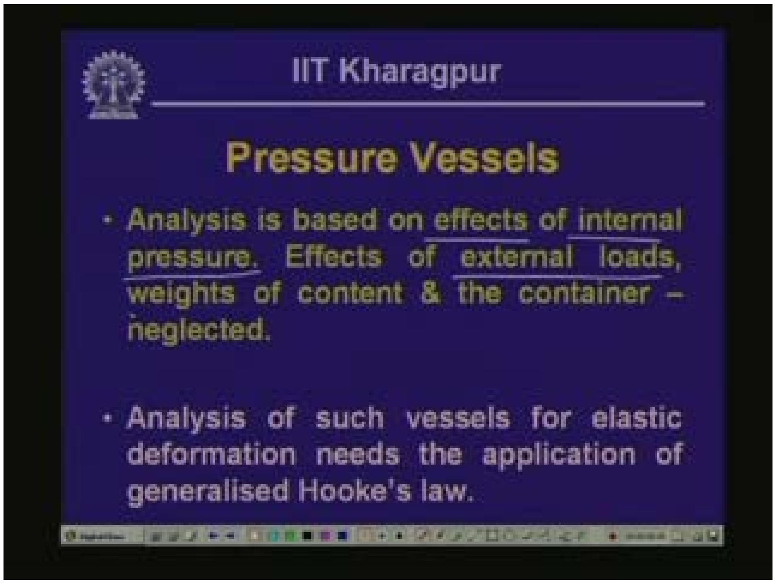

These aspects are neglected in this analysis. So when we try to analyze these kinds of pressure vessels we analyze the container for only stresses because of the pressure which is being exerted by the content from inside. We do not take the weight of the content, we neglect the weight of the container and we do not take the effect of any external loads that are acting on that particular container.

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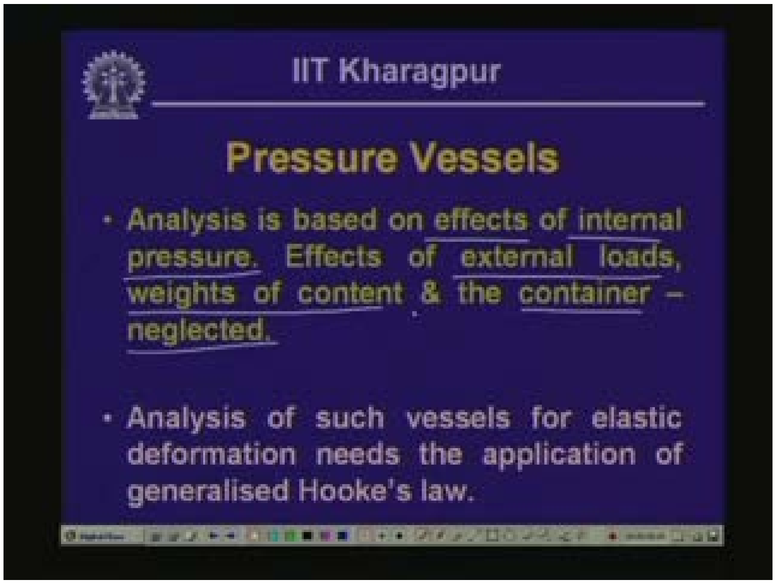

Also the analysis of such vessels for elastic deformation, when we try to compute the elastic deformation of such a container this needs the application of generalized Hooke's law. We have seen this where we compute the strain in terms of stresses in a generalized Hooke's law form where epsilon$_x$ is equal to sigma$_x$ by e - mu star sigma$_y$ by e - mu star sigma$_z$ by e and so on.

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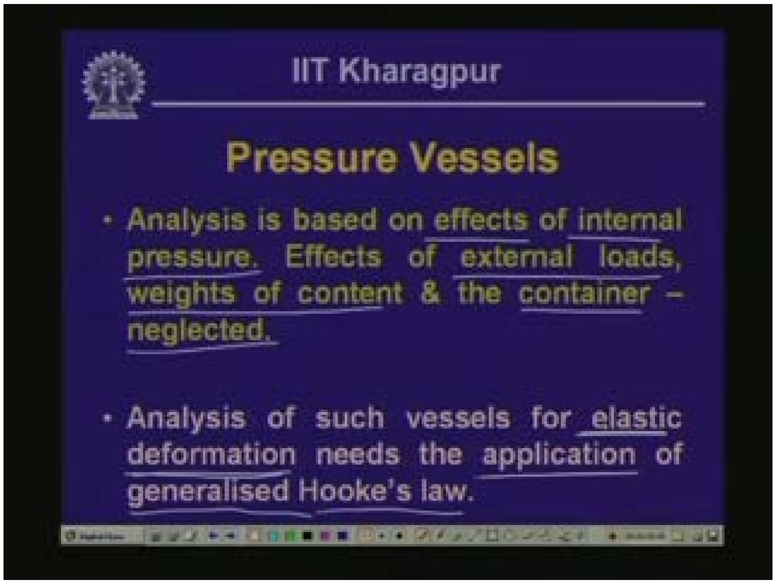

These are the generalized Hooke's law application and we take the applications of these for analyzing strains and thereby the deformation in the pressure vessels.

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We can have different forms of pressure vessels but in this particular lesson we are concentrating on pressure vessels which are of cylindrical form. If we have a cylindrical pressure vessel which is exerted by pressure inside which is more than the external pressure then what will be the state of stress that will be acting on such containers?

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Let us look into one such configuration that this is the cylinder, if we cut this cylinder and take a free body let us say this is a cut AA and BB these are the two planes by which we are cutting this cylinder and also we are cutting this cylinder along this center half then we get a configuration which is something like this. Here this is the thickness and this thickness is sufficiently small and this thickness is r by t where r is the radius of this cylindrical structure, r by t is equal to ≥ 10 and we call this kind of cylindrical form as thin-walled cylinder.

If we cut there are forces which will be acting and there will be internal pressure which will be exerted by the content. If we like to write down the equilibrium now this configuration can be written down in this form that this is the half cross section of the cylindrical container whereas this is the cylindrical part and this is the thickness of the cylinder and this is the center and this is the radius of the cylinder. This is internal radius and this is external radius so we can call this as r internal and this as r outer. And the stresses that will be acting, if we take an element on this cylindrical surface we have stresses acting in this form and stresses acting in the circumferential direction.

Let us call this as sigma_1 and this one acting in the longitudinal direction as sigma_2. So, in the circumferential direction the stress acting is sigma_1 and the resulting force which is acting here is p, this also is p and the content is exerting pressure on this side. If we try to evaluate the value of stresses, here this p is equal to stress time area. The sigma_1 is the stress which is acting on the circumferential direction so sigma_1 into t, now if we take the distance between these two cuts as l which is this length of the segment and if we are computing stress for this particular length so sigma_1 into t into l where t into l is the area, sigma_1 is the stress so p is the force that is acting over here.

We can compute the values of sigma_1 in two ways. One is, supposing if we take this thin wall part and we say that this is subjected to the pressure from inside and here we have p, here we have p is equal to sigma_1 into t into l. Now if we consider a small element which is at an angle of theta making an angle dtheta here so this is theta and this is dtheta. Then the force which will be exerted here is equal to the internal pressure p star r star dtheta that is p(r(dtheta)) and this is over the length l. If we take the component of this force in the horizontal direction which is theta then this is this times star cos theta so the force which is acting is p r l dtheta costheta and this is acting over the whole of this segment so this side we have p and this side also we have p so 2p is equal to integral.

Now, if we integrate from 0 to 90 degrees for the two halves so 2 of 0 2pi by 2 into p r l costheta dtheta. This gives us the value of costheta dtheta is equal to sintheta is equal to 1 so this is 2p r l and p is equal to sigma 1 t l. So in place of we can substitute these as 2sigma 1 into t into l is equal to 2 p r l, if we equate this 2, this 2 this 2 gets cancelled so we have sigma_1 is equal to p into r by t now this is called as sigma_1 which is called as a circumferential stress as it is acting along the circumference of the cylinder or sometimes we call this as Hoops stress. So, circumferential stress or the Hoop stress is equal to pr by 2t Hoop stress. So this stress sigma_1 since it is acting around the circumferential direction we call that as a circumferential stress or Hoop stress, which is given as p is the internal pressure, r is the radius of the of the cylindrical vessel and t is the thickness of the wall.

We can compute this directly as well. We can compute the values of circumferential stress or Hoop stress from this particular figure. Now here again p is equal to (the stress into the thickness of the wall) into segment of length over which we are concentrating, and this is equal to p. And since the content is applying the pressure along with the liquid inside so this is the projected length over which this force is acting which is 2r into l, this is 2r into l into the internal pressure p is equal to 2sigma 1 t into l. If we equate these two then from this we get the value of sigma$_1$ is equal to p r by t. So we can see that the value of the circumferential stress in a pressurized cylindrical vessel is equal to pr by t where sigma$_1$ is called the circumferential stress or hoop stress.

The point to be noted here is that I am using the value here as r though initially I stated two values of r where one is r internal and another one is r outer or the external. Since the thickness of the vessel is very small we consider that there is little variation between the internal radius and the external radius and we consider the radius as internal radius and compute stresses for all as we have assumed in the beginning that since the thickness is small the variation of stress across that thickness is negligibly small and hence we consider only one r which is the internal radius r. That is why here this is always written as r.

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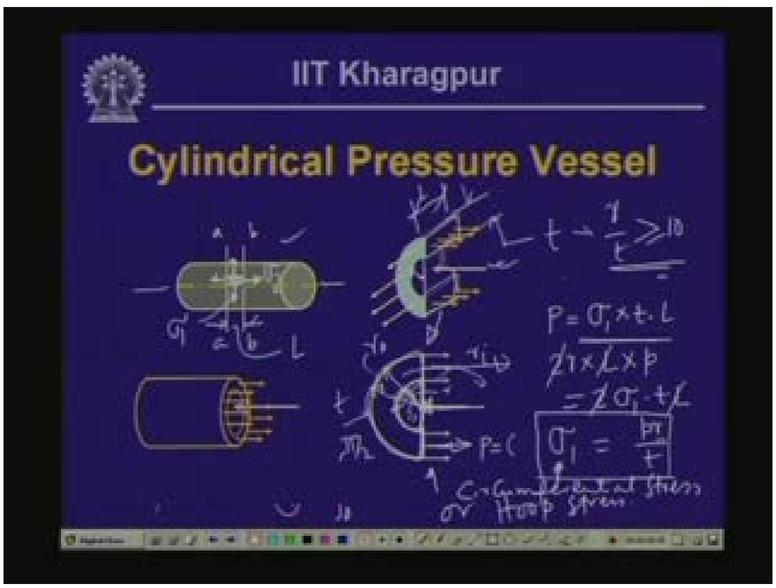

Now if we would like to compute the value of the other stress which is sigma$_2$ acting in the longitudinal direction then let us take the section here shown in this particular figure. Here you see that we have internal pressure which is acting on this particular surface and the thin wall which is subjected to the forces in this form resisting this internal one. Let us write in terms of internal and external radius over here, this is internal r and this is external radius. The pressure which is exerted by the content inside the pressure vessel is equal to the area which is pir internal square multiplied by the intensity of the pressure.

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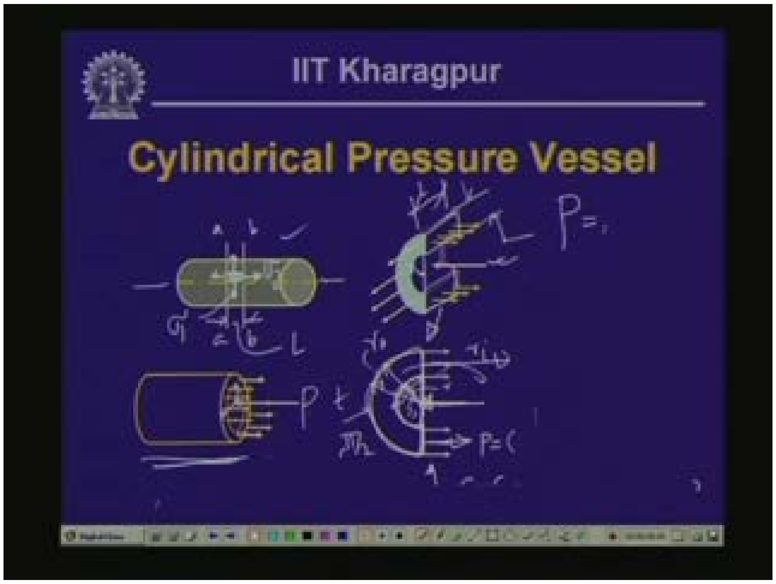

We can write this as pressure p into pir internal square. Now on the outer periphery on the thin wall which is between r internal and r external this is subjected to a force which is resisting this pressure from within.

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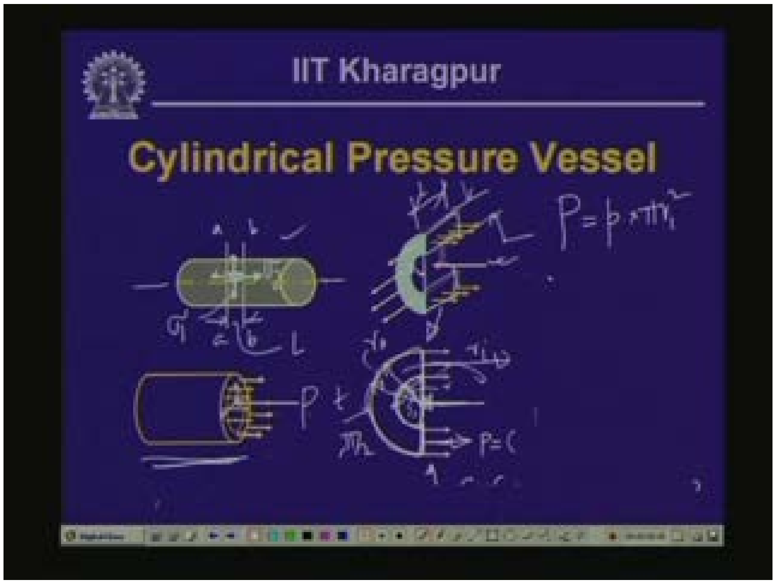

If we write down that as p prime is equal to sigma_2 the stress so now sigma_2 multiplied by the area will give us the force and the area is p**i**r outer square minus r pir internal square. So, from the whole of the cylindrical surface with external or outer radius r if we subtract the internal part with the r internal then the force given by the walls is equal to pir outer square minus pir internal square multiplied by the stress. Thus the force that is going to resist this internal pressure is exerted by the content.

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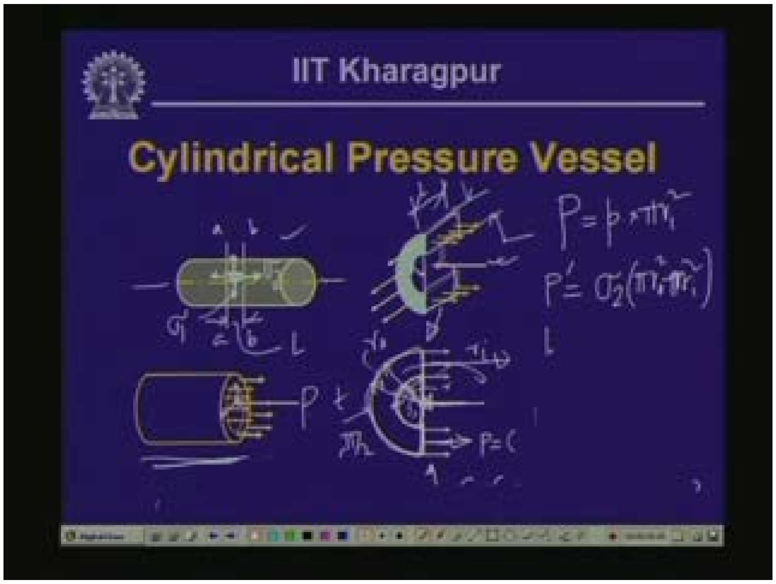

If we equate these two we can write p pir internal square is equal to $sigma_2$ into pi**.** Now r outer square minus r internal square we can write this as r0 minus r i or r. It is r outer minus r inner into r outer plus r inner now r outer minus r inner basically gives the thing; r outer is this and r inner is this. Now the difference between these two is nothing but the thickness. So these we can replace as t, so this is equal to $sigma_2$ pi into t, here you see we have two quantities r outer and r inner. Since the thickness is very small and as I am telling you constantly that there is little variation between r, r internal and r outer because the thickness is significantly small hence we consider r outer is equal to r inner is equal to r.

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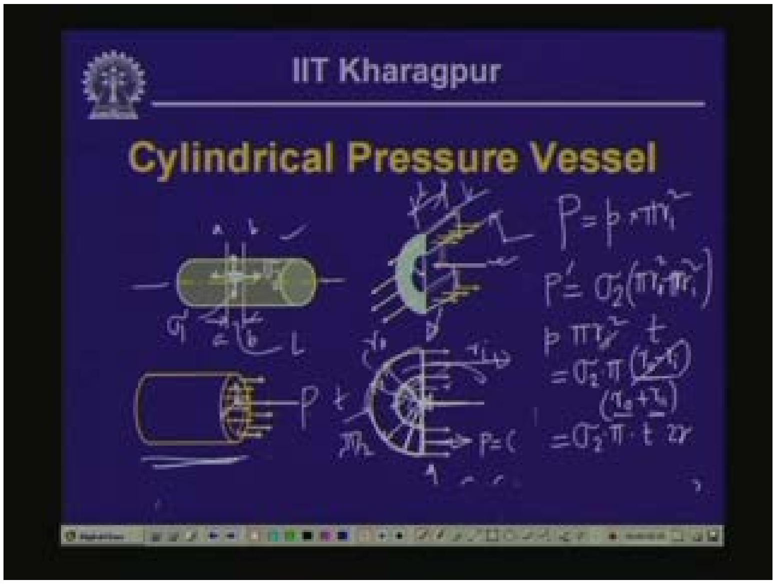

If you do that then this is basically 2r and we have on the left hand side r square which is also r so we have now from this pi gets cancelled r square and r gets cancelled so we have $sigma_2$ is equal to pr by 2t. Now here we have obtain the expression for the longitudinal stress which is $sigma_2$ which is equal to pr by 2t. Earlier we have seen that we have circumferential stress or the hoop stress $sigma_1$ as is equal to pr by t and now we have obtained the longitudinal stress which is $sigma_2$ which is in fact that half of this circumferential stress which pr by 2t. So if we know the internal pressure, if we know the radius of the cylindrical vessel, if we know the thickness of the wall, we can compute the stresses; the circumferential stress and the longitudinal stress and both the stresses $sigma_1$ and $sigma_2$. The circumferential stress or the Hoop stress and the longitudinal stress are basically tensile in nature.

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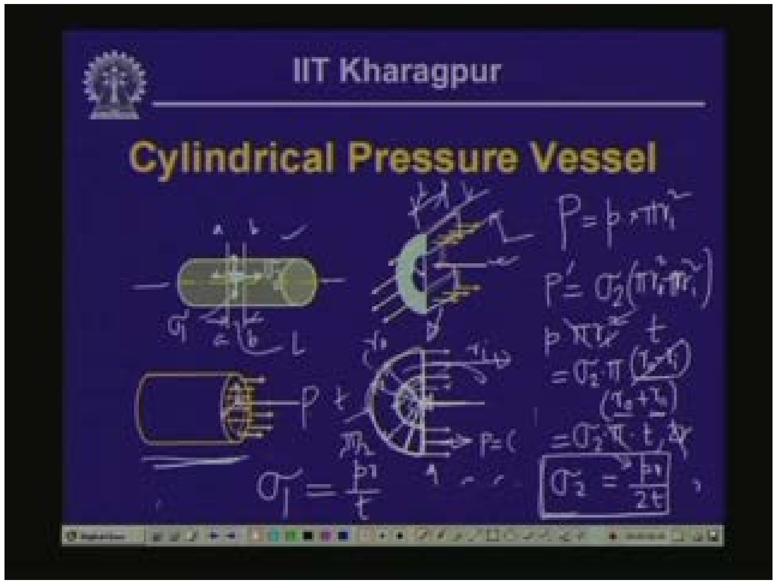

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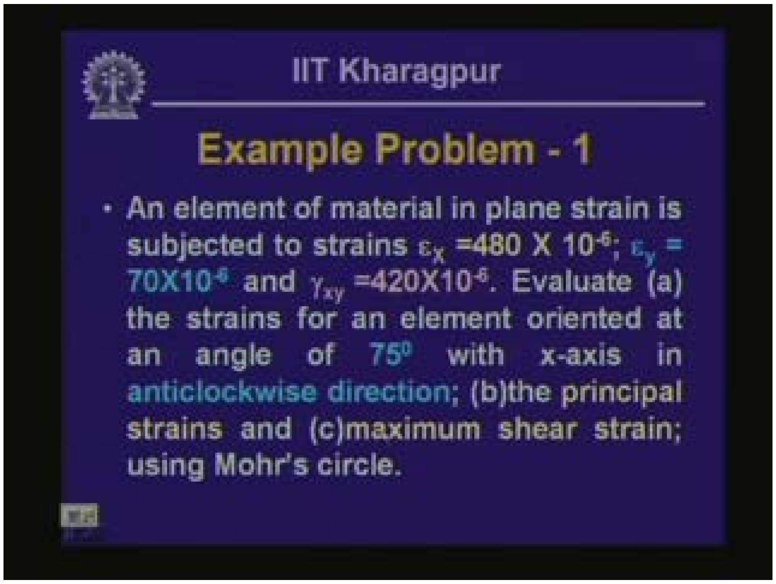

Having looked into the concept of the stresses in the cylindrical vessels, let us look into some of the problems. Here are some problems with reference to the stress system or with reference to the Mohr circle of strain and the transformation equation. Let us also look into the examples of the pressure vessels.

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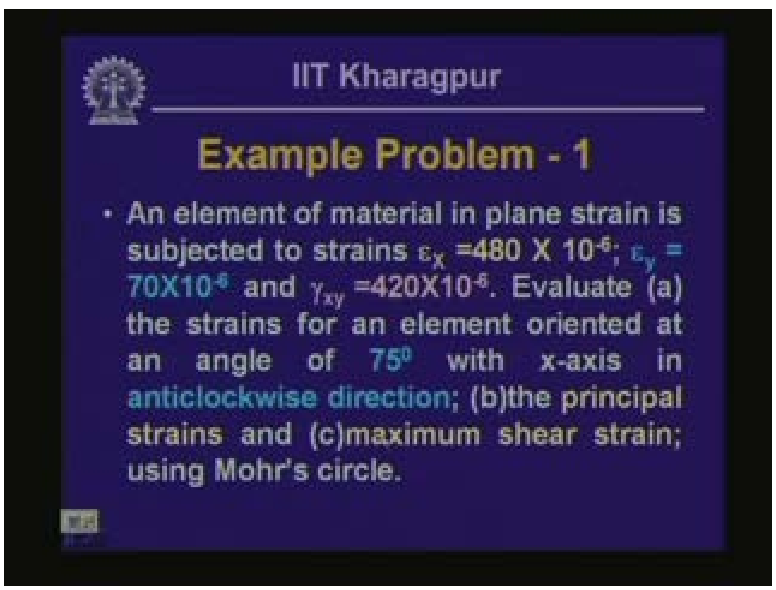

Here is the first example. What is stated here is that an element of material in a plane strain condition is subjected to $\in x$ so much $\in y$ is equal to 70[10 to the power minus 6 Υ xy is equal to 420[10 to the power minus 6. What you will have to do is to evaluate the strains for an element which is oriented at an angle of 75 degrees x -axis in anticlockwise direction and secondly the principal strains and thirdly a maximum shear strain using Mohr circle. Now let's us look into how you compute the values of the principal strains such as the maximum shear strains and the strains which are at orientation which is oriented at an angle of 75 degrees with reference to the x axis.

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Now, let me draw the Mohr circle corresponding to the state of stress which is given $\in$x is equal to 480[10 to the power minus 6] epsilon$_y$ is equal to 70[10 to the power minus 6] Υ xy is equal to 420[10 to the power minus 6]. If we try to plot the Mohr's circle based on these values of strains then we have, let us say this is epsilon axis and this is positive Υ xy by [2] or Υ [2]axis. Now here we have the point which is the positive epsilon$_x$ and Υ xy by [2] and correspondingly we have epsilon$_y$ and Υ xy by [2]. If we join these two points together the line which crosses the epsilon point we get as the centre of Mohr's circle and this if we call as O, OA prime or OA. If we take this as the radius then we can compute or we can draw the Mohr circle in this form.

Here this particular point indicates the value of maximum strain which is the principal strain equal to epsilon$_1$. This strain is the minimum normal strain which we call as epsilon$_2$ and as you have seen the distance of the center from the origin is equal to (epsilon$_x$ plus epsilon$_y$) by [2] and the distance between these two points AA prime is (epsilon$_x$ minus epsilon$_y$) by [2].

If we designate them, let us say this is B prime and this as O prime and this as X and this as Y. So the distance OO prime is equal to epsilon$_x$ plus epsilon$_y$ by [2] is equal to 480 plus 70 is equal to 550 by 2 is equal to 275[10 to the power minus 6] epsilon$_x$ minus epsilon$_y$ is equal to 410[2] is equal to 205[10 to the power minus 6]. Now this distance is epsilon$_x$ - epsilon$_y$ and from here to here is epsilon$_x$ minus epsilon$_y$ by [2] and this distance is Υ xy by [2] is equal to 210[10 to the power minus 6]. Thereby from this particular triangular configuration, this is 210 and this is 205 and this is the radius of the circle which is OA where OA is equal to R is equal to $\sqrt{210^2+205^2}$ is equal to 293.5[10 to the power minus 6]. Hence let me first compute the principal strains epsilon_1 where epsilon_1 is equal to OO prime or O prime O plus r so O prime O plus r and O primeo is equal to (epsilon_x plus epsilon_y) by [2] is equal to 275. So 275 plus 293.5[10 to the power minus 6] is the strain and this gives us the value of 568.5[10 to the power minus 6].

Now epsilon_2 which is the minimum principal strain or minimum normal strain is given by this distance and that is equal to r or O prime O minus r and eventually that will become negative. Now O prime O is equal to 275 and minus 293.5[10 to the power minus 6] gives us a negative value and that is why the end of he circle has gone below the OO point and this is equal to minus 18.5(10 to the power minus 6). So it is epsilon_2 is equal to (275 minus 293.5)10 to the power minus 6 is equal to minus 18.5(10 to the power minus 6). So this is the maximum principal strain and this is the minimum normal strain or the principal strain. The maximum shearing strain is the maximum value of the r is equal to Υ by [2] so r is equal to 293.5 so Υmax is equal to 2r is equal to 587[10 to the power minus 6]. This is the maximum value of the shearing strain. These are the three quantities; maximum value of the principal strain epsilon_1 epsilon_2 and Υmax is this.

Now let us look into the angle. Since this is the point of the maximum principal strain the angle from the reference point is this which is 2theta_p, now this angle 2theta_p. So let us see the value of 2theta_p. Now tan of 2theta_p is equal to 210 by 205 and thereby theta_p or 2theta_p is equal to 45.7 degrees and theta_p thereby comes as 22.85 degrees. This is the orientation along which the principal strain acts. Now we will have to compute the value of strain with reference to axis. We have the reference axis x and y so now we have to find out the state of strain at an orientation of the axis which is 75 degrees in an anticlockwise form with reference to x. We will have to find out the strain along x_1 and y_1, now when we take 75 degrees in the reference plane in the Mohr plane then that becomes 2theta is equal to 150 degrees. From this particular reference point if we go to 150 degrees somewhere here so this is the point which will give us the state of stress which is oriented at an angle of 75^0 with reference to x axis.

From here to here this is 150 and 2theta_p we have obtained 45.7, this is 90 so this angle if we call this as alpha then alpha- is equal to 150 minus 45.7 minus 90. So this gives us an angle of 14.3 degrees so alpha is equal to 14.3. Now from here if we draw a perpendicular then this distance is going to give us the value of the normal strain which we call as epsilon is equal to 75 and if we take diametrically opposite this point we will get epsilon_y and this value will give us the value of $\Upsilon\gamma$ by [2]. If we know that this angle is 14.3 degrees and this is r then we can compute these distances and let us call this distance a and the vertical distance which is b. So cos14.3 is equal to b or sin14.3 is equal to a.

Now let us calculate the value of a and b. So a is equal to r(sin14.3) is equal to 293.5[10 to the power minus 6] [sin14.3 degrees] is equal to 72.5[10 to the power minus 6] and the vertical distance b is equal to 293.5[10 to the power minus 6] [cos14.5 degrees] is equal to 284.4[10 to the power minus 6]. Once we know a and b then the epsilon_x or epsilon_{x1} is equal to OO prime minus a the distance. Now distance OO prime is equal to 275 minus 72.5 which will give us the value of epsilon_{x1}. And epsilon_{y1} will be likewise 275 plus 72.5[10 to the power minus 6] and Υxy by 2 rather x_1 y_1 by [2] is equal to 284.4 the distance b and Υx_1y_1 is equal to 568.8[10 to the power minus 6]. These are the values of strain epsilon_{x1}, epsilon_{x1}, epsilon_{y1} and Υx_1y_1 with reference to the axis which is oriented at an angle of 75 degrees with reference to the reference x and y plane. So those are the values of the strains and the principal strains and the maximum shearing strength we have seen how to compute it.

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Now this is the orientation, the final form of the reference plane. If this is the x direction and this is the y direction now when we have the orientation for the principal plane which is theta_p is equal to 14.3 degrees. So this is the direction along which h the principal strain acts, so this is epsilon_1 and in this direction we have epsilon_2. Now epsilon_2 was negative and that is why it is compressed and this is elongated. Therefore this is the form of the principal strain.

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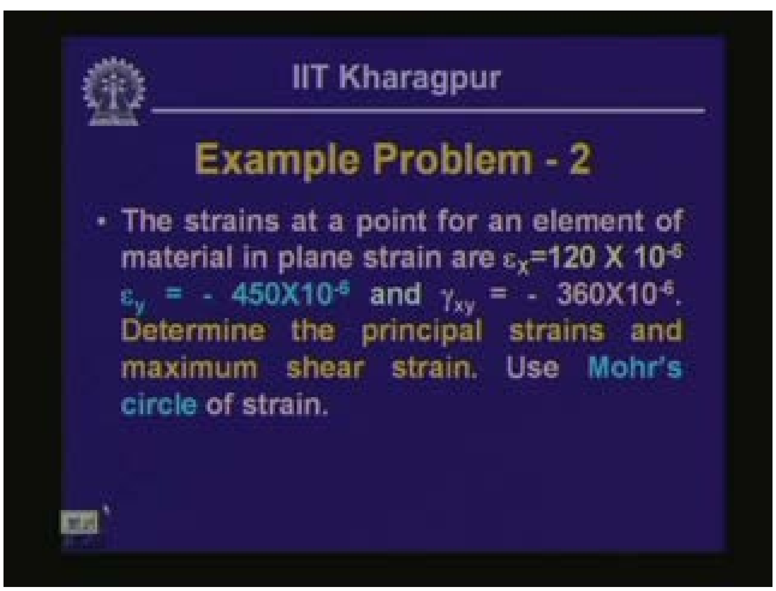

Now let us look into the second problem which was stated as that the strain epsilon$_x$ is equal to 120 [10 to the power minus 6] epsilon$_y$ minus 450[10 to the power minus 6] ϒxy is also negative 360[10 to the power minus 6] you will have to find the principal strains and the maximum shear strain using Mohr's circle of strain.

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Now if we plot the values of the Mohr's circle or if we plot the Mohr's circle based on the values given here, note that here we have the values of epsilon$_x$ is equal to 121[10 to the power minus 6] epsilon$_y$ is equal to minus 450[10 to the power minus 6] xy is equal to minus 360[10 to the power minus 6]. Now if we plot this that epsilon$_x$ is equal to 120 and ϒxy is minus 180. Again this is positive epsilon$_x$ axis, this is equal to ve ϒ by [2] axis. So we have this as 120 and this as 180 so this is 180 because ϒ by [2] and this distance is equal to 120 which is epsilon$_x$ and likewise we have this point which is epsilon$_y$ is equal to 450 is equal to 180. With these if we join them and plot the circle then this is going to give us epsilon$_1$ and this is epsilon$_2$. So we can compute the value of principal strain because r is equal to root of this square and this square and the principal strain is equal to the distance from centre to this plus r is equal to epsilon$_1$ and epsilon$_2$. And epsilon$_1$ comes out to be 172[10 to the power minus 6] and epsilon$_2$ which is negative comes out to be minus 502[10 to the power minus 6] and ϒmax the shearing strain comes as 674[10 to the power minus 6].

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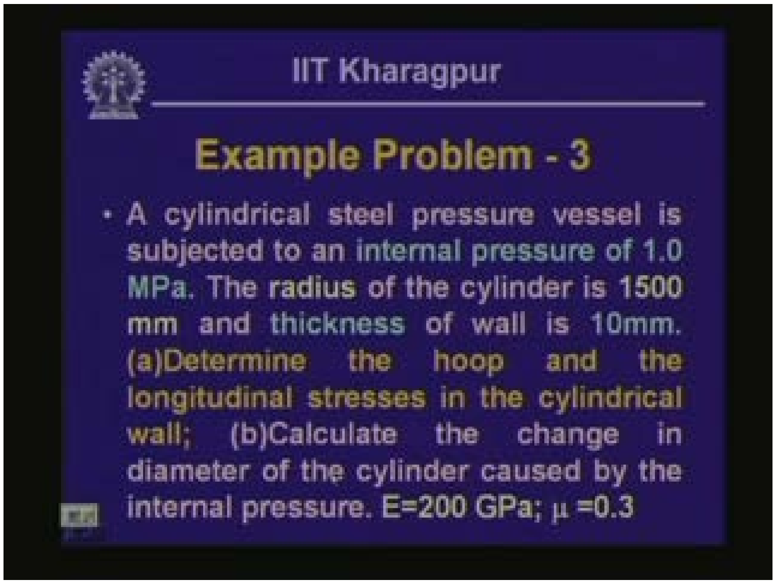

Here is the third problem which is related to the example as discussed today. Now here the cylindrical steel pressure vessel is subjected to an internal pressure of 1 MPa and the radius of the cylinder is 1500 mm and thickness of wall is 10 mm. We will have to determine the Hoop stress which is the circumferential stress and the longitudinal stresses in the cylindrical wall. Also, we will have to calculate the change in diameter of the cylinder which is caused by this internal pressure. Let us look into the values of the stresses as we compute.

As we have seen the circumferential stress sigma_1 is equal to pr[t] and the longitudinal stress sigma_2 is equal to pr[2t]. Now here p is given as 1 MPa, r is given as 1500 mm and t is equal to 10 mm. Hence sigma_1 the circumferential stress or the hoop stress is equal to 1[1500 by 10] is equal to 150 MPa and thereby sigma_2 the longitudinal stress the longitudinal stress is equal to pr[2t] is equal to sigma_1[2] is equal to 750 MPa. The sigma_2 acts in the longitudinal direction of the vessel which is longitudinal stress and sigma_1 acts in the circumferential direction. Now if we like to find out the increase in the diameter then it means you must know how much strain it is undergoing in the circumferential direction. So we are interested to evaluate the strain in the circumferential direction which we call as epsilon_1.

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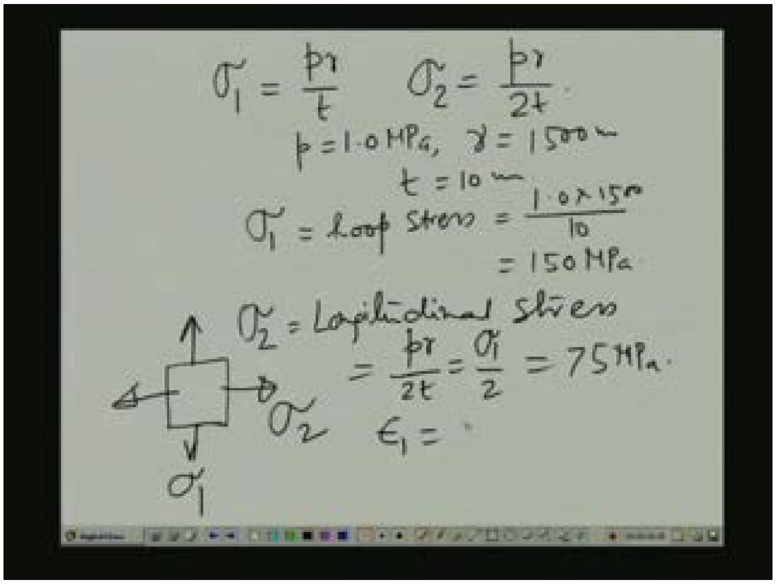

Now epsilon$_1$ is equal to sigma$_1$ by E minus mu [sigma$_2$ by E]. Now if we substitute the values of sigma$_1$ and sigma$_2$ we will find that the value of strain which we are getting from here is equal to 0.6375[10 to the power minus 6] whole power cube. So this is the value of strain it is undergoing. Now we can write this strain in the circumferential direction as is equal to 2pi(r plus delta) minus 2pir by 2pir. This is the extension by the original length which is equal to delta by r. So the extension in the radius is equal to epsilon[r] is equal to 0.6375 mm. Hence the increase in the diameter 2delta is equal to 2(.6375) mm. This r is equal to 0.6375[1500] and this is 0.6375[1500]. So this comes out as 1.912 mm.

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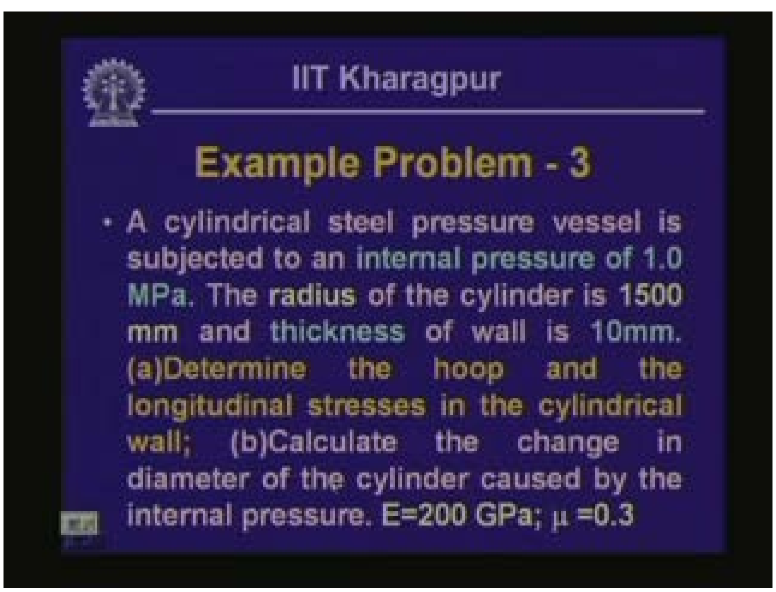

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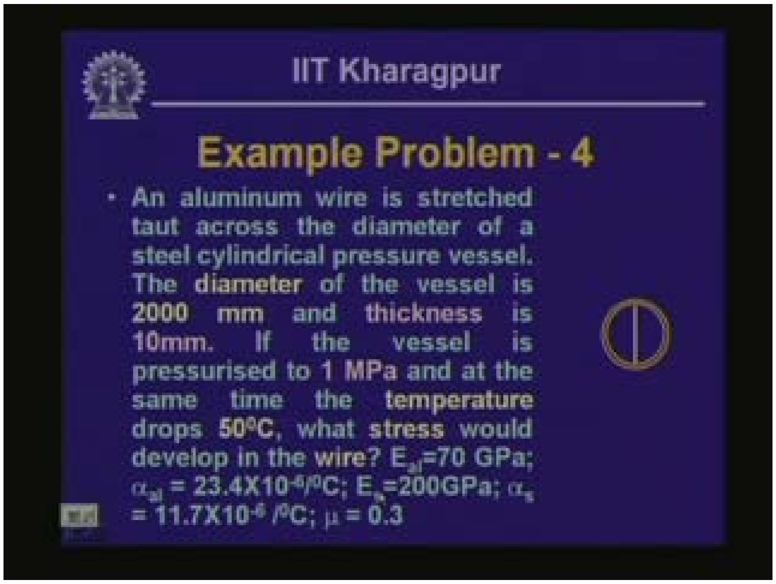

Here is another problem set for you which is an aluminum wire stretched out across the diameter of a steel cylindrical pressure vessel. The diameter of the vessel is 2000 mm and thickness is 10 mm. If the vessel is pressurized to 1 MPa and at the same time the temperature drops to 50 degree C. Therefore what stress do you expect in the thin wall pressure vessel?

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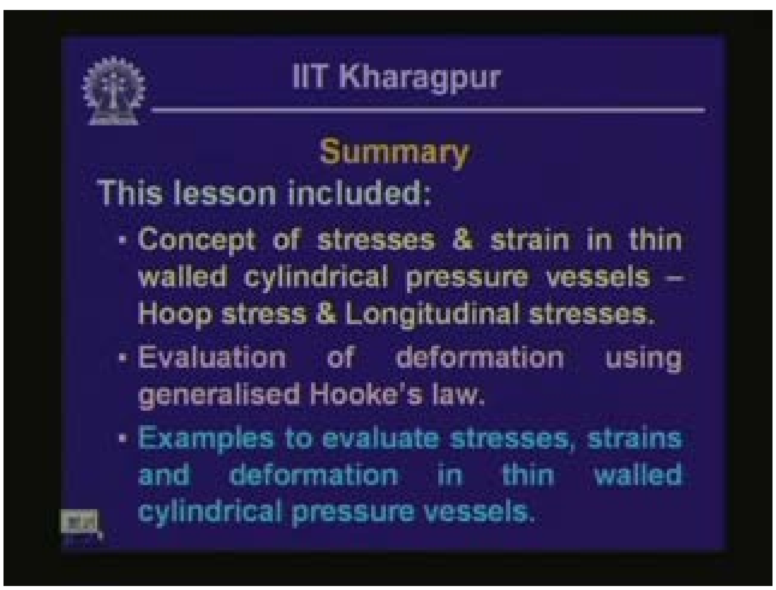

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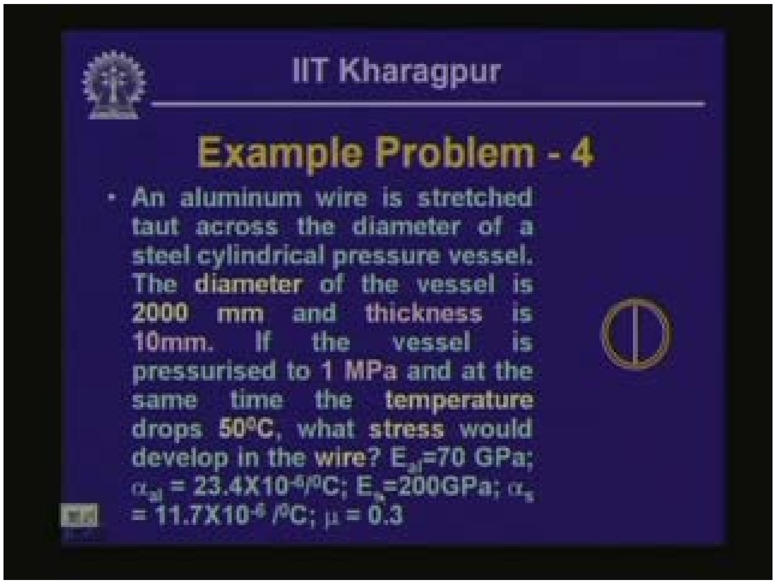

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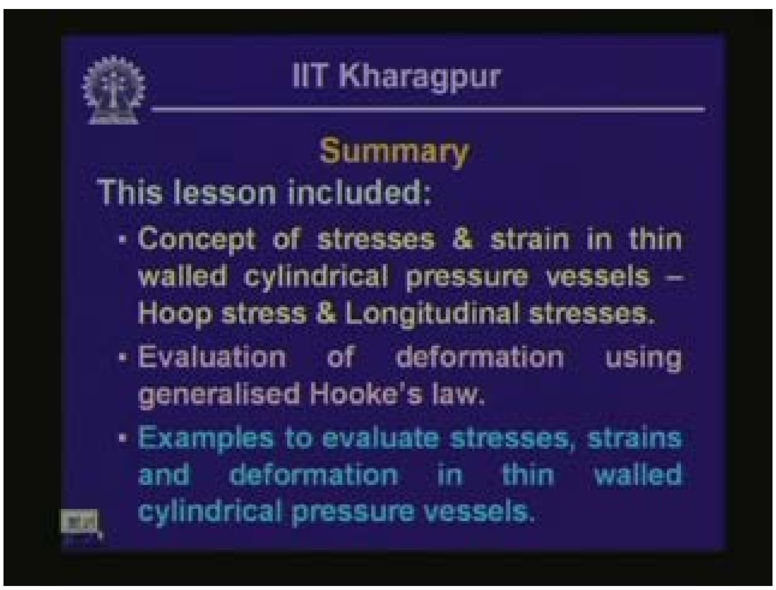

Summary:
We have discussed the concept of stresses and strain in thin-walled cylindrical pressure vessels.

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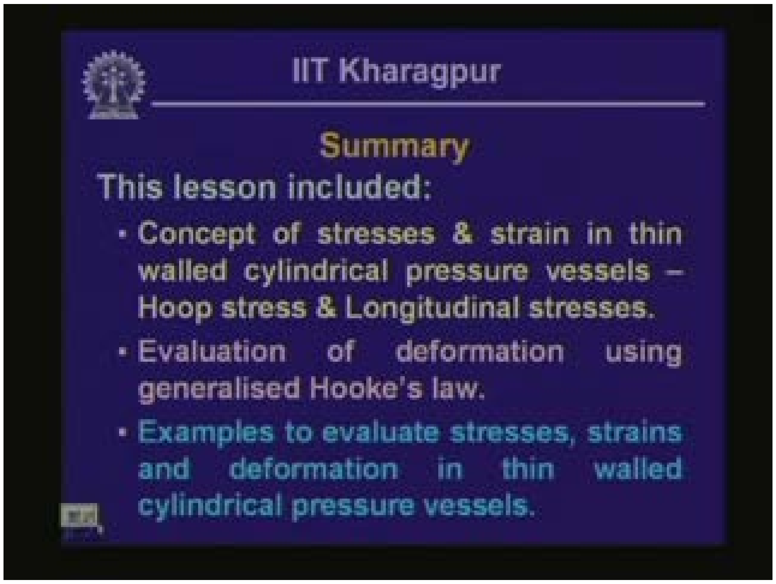

We have also discussed Hoop stress and longitudinal stresses, evaluation of deformation using generalized Hooke's law and examples to evaluate stresses, strains and deformation in thin-walled cylindrical pressure vessels.

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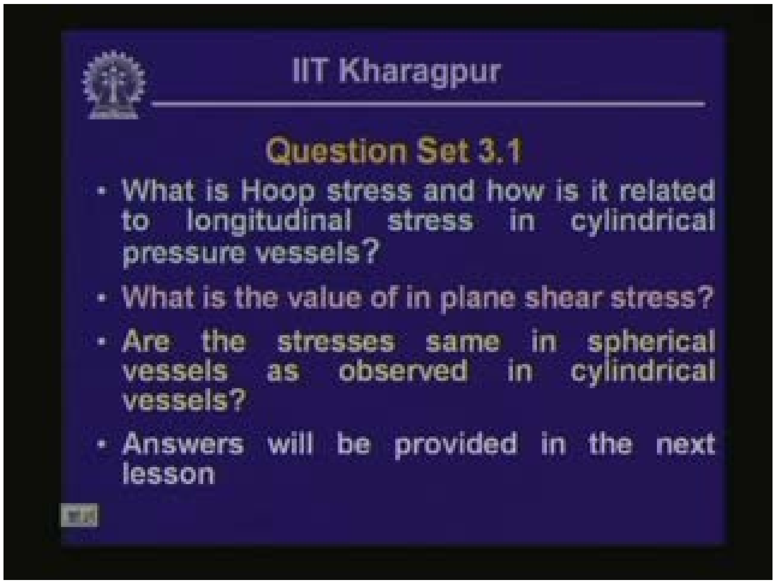

The question set for you is:
What Hoop stress is and how is it related to longitudinal stress in cylindrical pressure vessels?
What is the value of in plane shear stress and are the stresses same in spherical vessels as well as we seen in cylindrical vessels?