Strength of Materials
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Lecture # 14
Analysis of Strain - VIII

This is the 8 lesson of module 2 which is on Analysis of Strain. We have already discussed quite a few aspects of analysis of strain.

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Today we are going to discuss some more aspects of analysis of strain. It is expected that once this particular lesson is completed one should be able to understand the concept of strain gages and strain rosettes and their uses in evaluating strain and stresses at a point.

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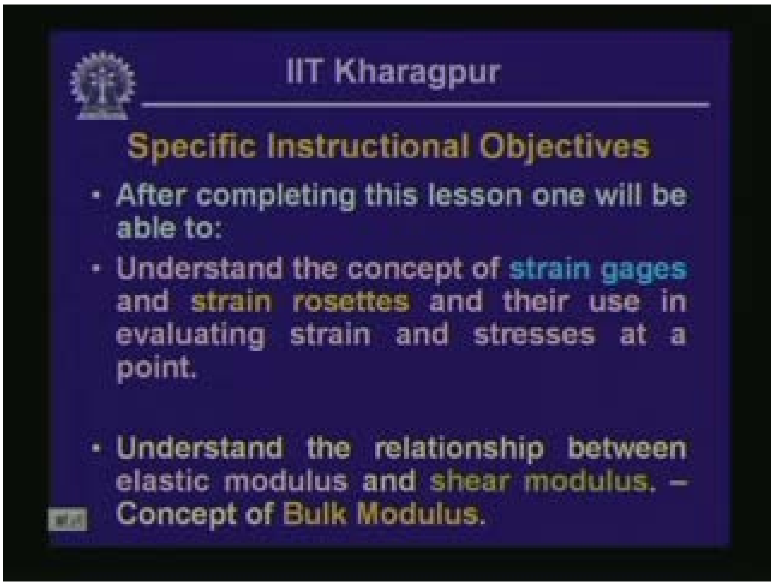

One should be able to understand the relationship between elastic modulus and shear modulus, how we relate the elastic modulus to the shear modulus and then consequently the concept of bulk modulus.

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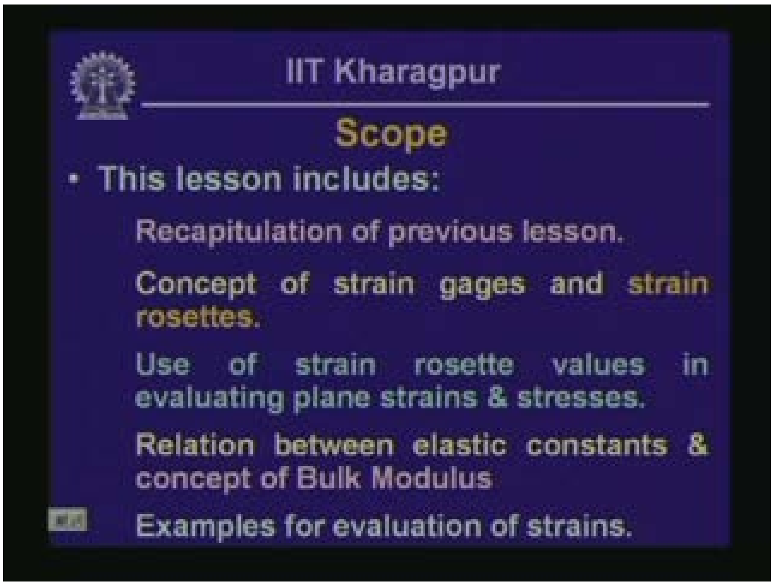

As the scope of this particular lesson includes the recapitulation of previous lesson we will be answering the questions posed last time. Then concept of strain gages and strain rosettes, use of strain rosette values in evaluating plane strains and stresses will be discussed. We will also discuss how the rosette dilutes the strains which we evaluate or measure through the use of strain rosette and how they are used to compute the values of strains and stresses at a point.
Then the relation between elastic constants and then the concept of bulk modulus will be dealt, and we will be solving few examples for evaluation of strains at a point from the given information.

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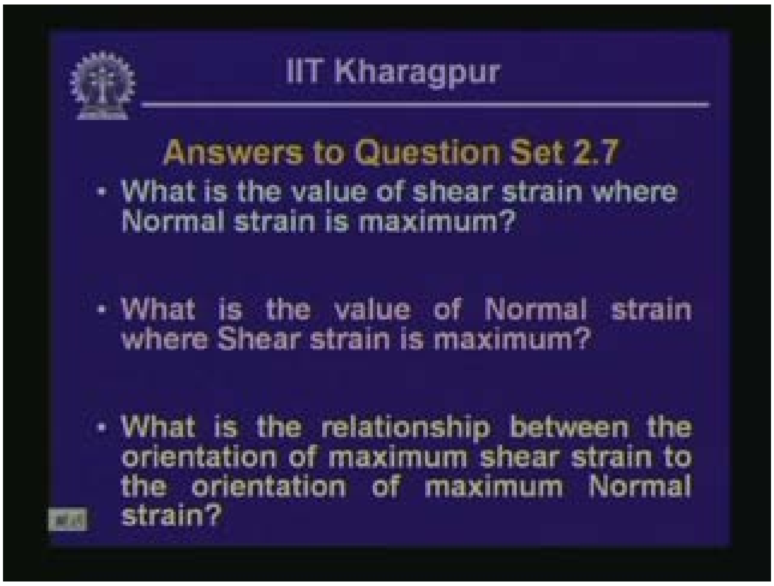

Let us look into the answers of the questions which we posed last time. The first question is what is the value of shear strain where normal strain is at maximum?
Probably the first and second questions can be discussed together.
What is the value of normal strain where shear strain is at maximum?

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Let us discuss these two questions through the Mohr circle of strain. If you remember last time we had discussed that we can plot the Mohr circle for evaluating strain at a point.

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Now if we have an element which is subjected to strain let us say this is reference x axis and this is y axis, the strain is epsilon$_x$, epsilon$_y$ and the shearing strain is gamma$_{xy}$, now we can plot them in the Mohr circle and this is the point this being the positive strain axis and this is the positive gamma by 2 axis, this particular point represent the positive epsilon$_x$ and gamma$_{xy}$ by 2 and this particular point which is diametrically opposite to this particular point represent epsilon$_y$ and minus gamma$_{xy}$ by 2.

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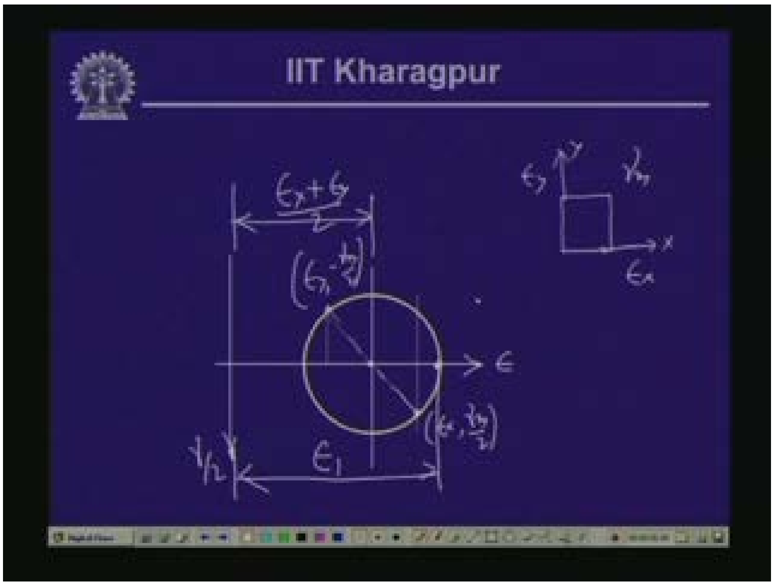

This is the center of the Mohr circle which is at a distance of epsilon$_x$ plus epsilon$_y$ by 2 and this particular point gives us the maximum normal strain which we normally denote as epsilon$_1$ the maximum principal strain. And as you can absorb from this particular point and from this diagram, at this point the value of shearing strain is 0. So along the axis where the principal strain or the normal strain is the maximum the value of the shear strain is 0.

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Consequently if you look into the Mohr circle the maximum value of the strains are at this particular point and at this particular point. This is the maximum positive shear strain and this is the minimum because magnitude-wise they are same but only it is negative. This is gamma$_{xy}$ maximum. The point where the shear strain is the maximum the corresponding normal strain is equal to this and this normal strain is equal to ϵ_x plus ϵ_y by 2. Therefore along the axis where the shear strain is maximum the corresponding normal strain is ϵ_x plus ϵ_y by 2. This answers the first two questions.

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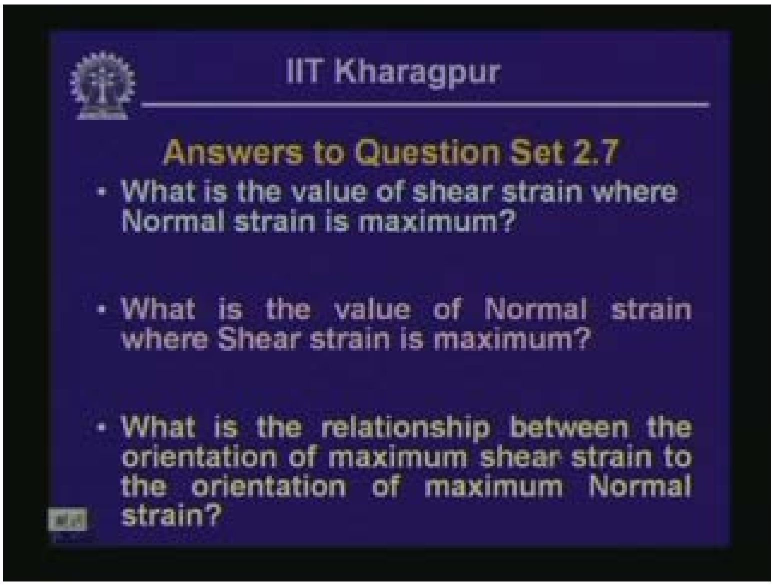

The third question is what is the relationship between the orientations of maximum shear strain to the orientation of maximum normal strain?

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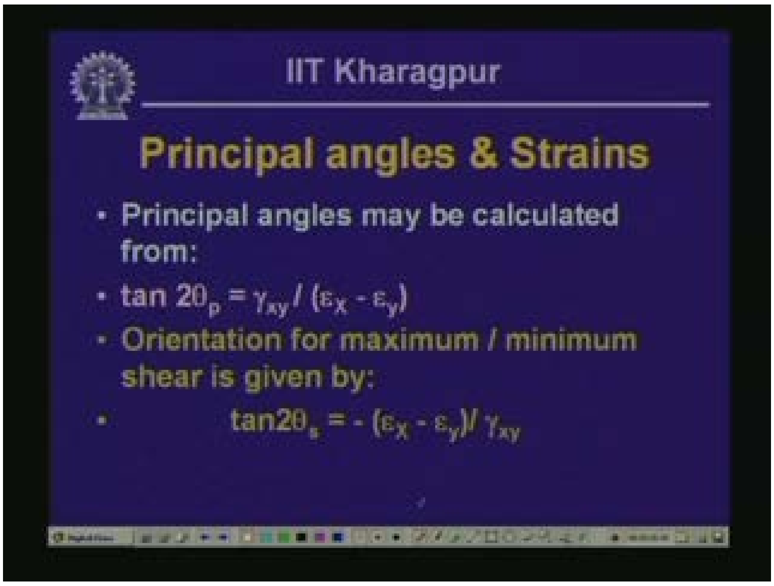

Last time we discussed this aspect. The principal angle $2\theta_p$ for the maximum principal strain can be located using the expression γ_{xy} by (ϵ_x minus ϵ_y).

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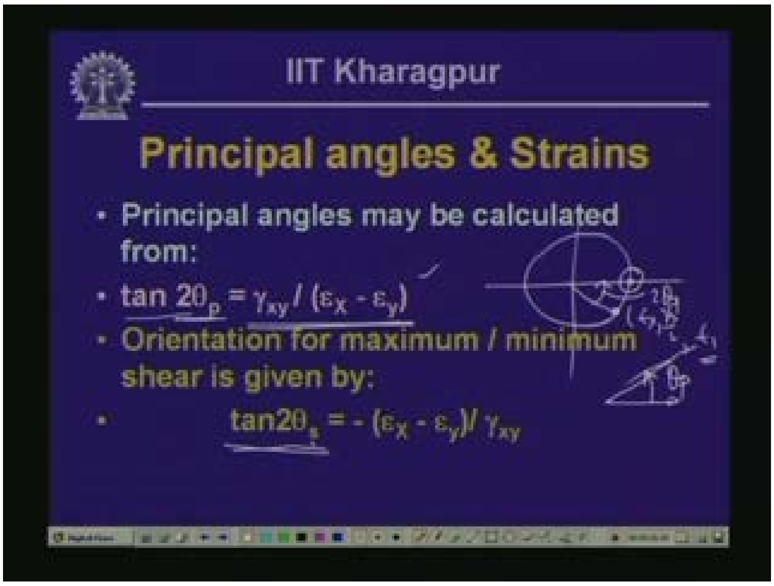

In the Mohr circle if we try to locate the principal plane, this is the position of the maximum normal strain and if this is the reference plane or reference point where strain is epsilon$_x$ and gamma$_{xy}$ by 2 from this particular orientation which is along the x direction if we move anticlockwise by 2theta$_p$ we get the point of maximum strain. In the physical space if we move in anticlockwise form by an angle of theta$_p$ this gives the direction of the principal strain epsilon$_1$ and the expression for measuring this angle theta$_p$ is tan 2theta$_p$ is equal to gamma$_{xy}$ by (epsilon$_x$ minus epsilon$_y$). Consequently we had seen that the expression for evaluating the angle for maximum shear strain which is tan 2theta$_s$ is equal to epsilon$_x$ minus epsilon$_y$ by gamma$_{xy}$.

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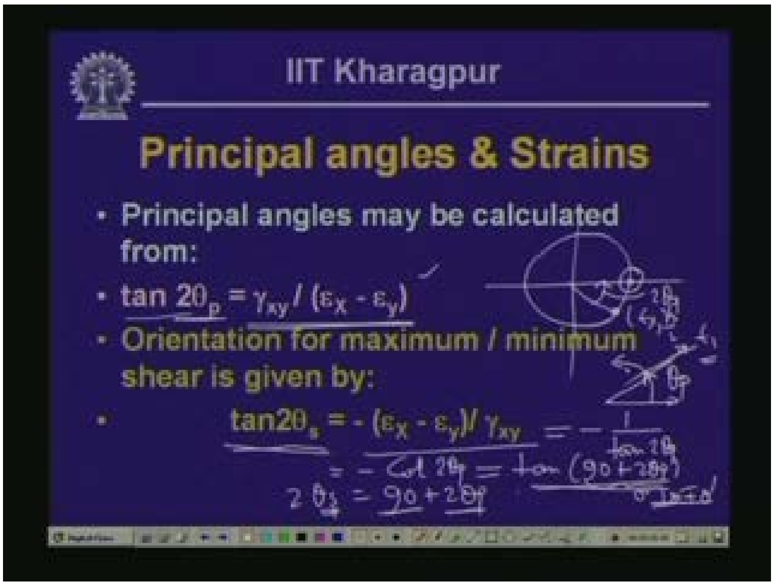

Now this particular value can be written as minus1 by tan 2theta$_p$ or this is nothing but equals to minus cot 2theta$_p$. We can write this as tan (90 degrees plus 2theta$_p$) hence 2theta is equal to 90 plus 2theta$_p$ and consequently the solution for this can be 90 plus theta angle and 180 degrees plus theta will be another angle. So the relationship between the orientation of shear strain which is $theta_s$ is equal to 45 degrees plus $theta_p$. That means if we know the direction of the principal strain from that if we orient by 45 degrees we can get the direction for the maximum shear strain. This is the relationship between the orientations of shear strain to the orientation of principal strain direction. These two directions are interrelated and they are at 45 degrees apart in the physical plane which is at 90 degrees apart in the Mohr circle of strain.

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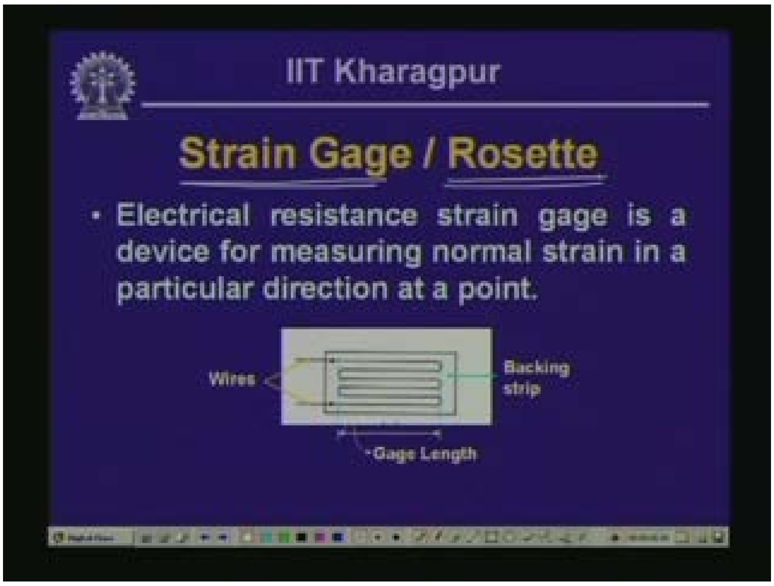

Having looked into these questions now let us look into the aspects which we are going to discuss today: the concept of the strain gage and consequently the term which is called as Strain Rosette. Many a times what we need to do is that experimentally we need to measure the strain at a point in a stressed body and that can be done using a device which we call as Strain Gage.

Electrical resistance strain gage is one such device using which we can evaluate strain at a point in a stressed body. The mechanism of this particular strain gage works with a wire which is pasted on a paper base and this can be fixed onto the stressed body as the body undergoes strain or deformation the gage also along with the body undergoes deformation and thereby there is a change in the resistance of that particular wire which is evaluated in terms of the strain and that is the basic principal. The strain gages can be used for measuring strain at a point or specifically the normal strain at a point in a particular direction.

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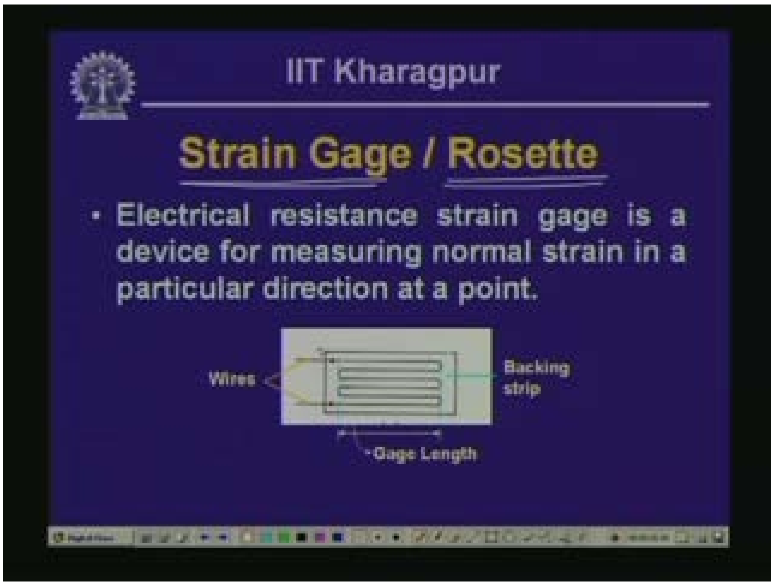

This is the configuration of the strain gage as shown here where this particular area is the backing strip over which the wire is placed and this is the gage length of the strain gage. Various sizes of the strain gages are available, they range between 2 mm to even 40 mm to 50 mm size.

Depending on the usage depending on the type of strain we are looking for or the accuracy in the strain we are looking for we use such gages to measure the strain at a particular point. From this particular strain values we need the normal strain epsilon$_x$, epsilon$_y$, gamma$_{xy}$ and if we have those information then we can compute the value of principal strains epsilon$_1$, epsilon$_2$; we can evaluate the stresses from those strains. So if measurement of strain at a point using such strain gages is a useful aspect in Strength of Materials.

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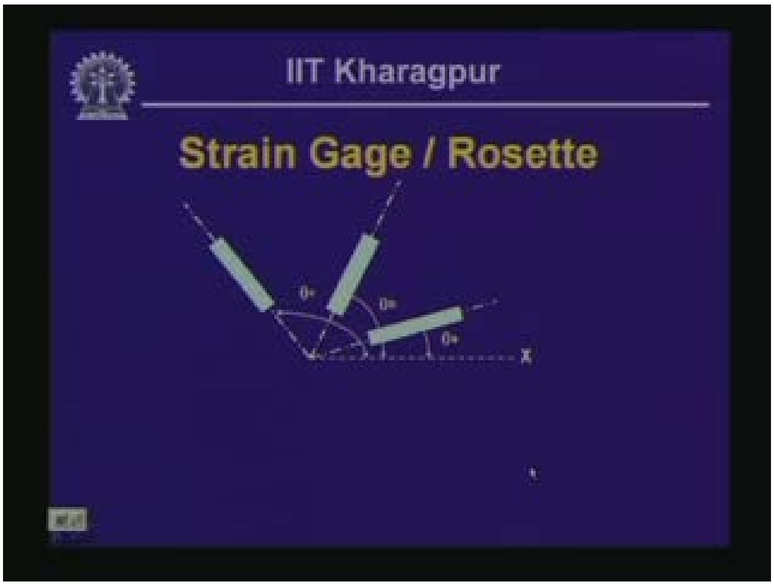

In a plane strain we need to have three quantities. They are: epsilon$_x$, epsilon$_y$ and gamma$_{xy.}$ And if we can have these quantities then we can compute the strain values which are epsilon$_1$, epsilon$_{2,}$ the principal strains, they are directions, the maximum shearing strain which is gamma$_{xymax}$, and once we have the values of the principal strains we can compute the values of principal stresses as well which are related between the strains and the strain. Therefore we can evaluate the stress at that particular point. But the point is the strain gage which we use can be used for measuring the normal strain. If we place the strain gage along the x-direction we can get the value of epsilon$_x$ and if we place them in the y-direction we can get the value of epsilon$_y$ but the problem is with the gamma$_{xy}$.

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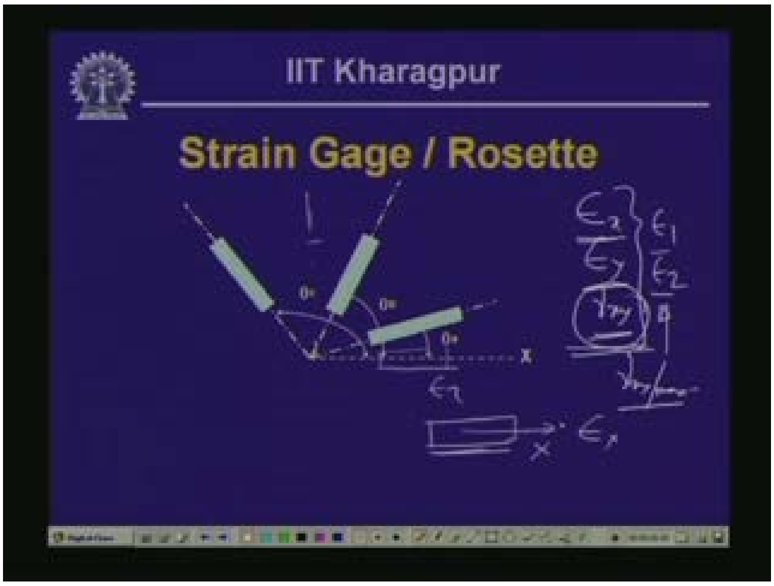

We cannot measure the shearing strain gamma$_{xy}$ directly as we can measure the normal strain epsilon$_x$ and epsilon$_y$ using the strain gages oriented in that particular direction because strain gage gives us the normal strain at that particular point so we can evaluate epsilon$_x$ and epsilon$_y$ but measurement of gamma$_{xy}$ is difficult. Therefore we take an indirect path to evaluate gamma$_{xy}$ so that we can utilize this information for evaluating principal strains and thereby the stresses. The indirect path is like this, what we do is that if we install strain gages along three directions, let us call this is direction a, this is direction b, and this is direction c.

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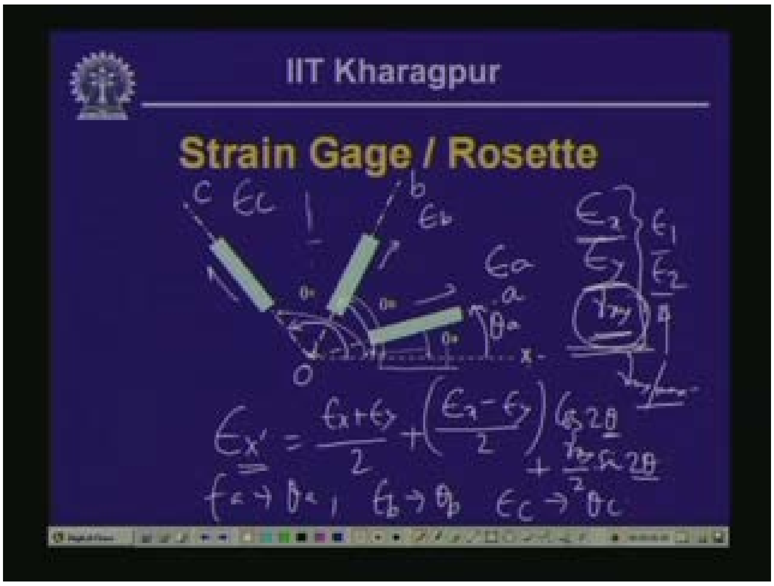

Now if we place strain gage along Oa, Ob, and Oc then we can get the normal strains epsilon_a, epsilon_b and epsilon_c in three directions. Let us assume that the direction Oa is oriented at an angle of theta_a with reference x-axis and likewise Ob is oriented at an angle of theta_b with reference x-axis, and Oc is oriented at an angle of theta_c with reference x -axis. If we have these information, if you remember, we calculated the transformation equations as $\text{epsilon}_{x\ \text{prime}}$ is equal to (epsilon_x plus epsilon_y) by 2 plus (epsilon_x minus epsilon_y) by 2 cos2theta plus gamma_{xy} by 2 sin2theta. In this particular case x prime direction with theta the epsilon_a is corresponding to thetaa, epsilon_b is corresponding to theta_b and epsilon_c is corresponding to theta_c. If we substitute for theta_s, theta_a, theta_b, theta_c correspondingly we can get the values of epsilona, epsilon_b and epsilonc and this is what is done over here.

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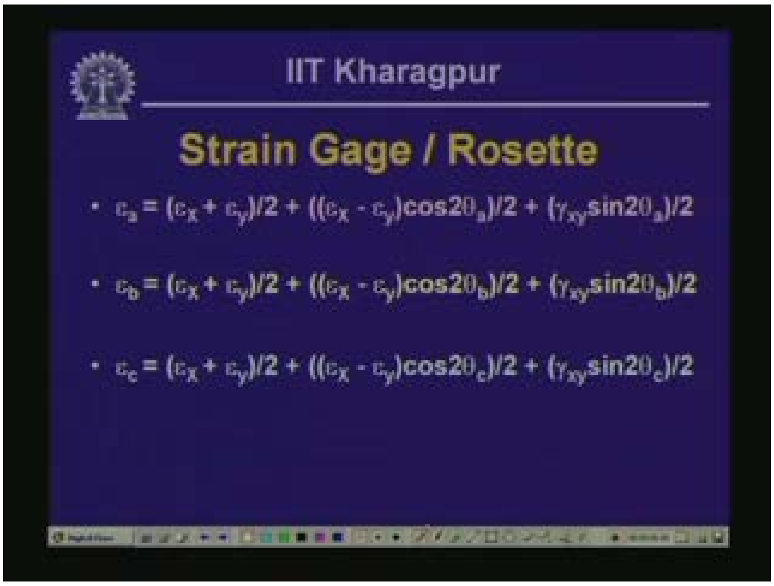

The epsilon_a is equal to (epsilon_x plus epsilon_y) by 2 plus (epsilon_x minus epsilon_y)cos2theta_a by 2 plus (gamma_{xy} sin2theta_a) by 2 likewise $\text{epsilon}_b$is equal to (epsilon_x plus epsilon_y) by

2plus(epsilon$_x$ minus epsilon$_y$)cos2theta$_b$ by 2plus (gamma$_{xy}$ sin 2theta$_b$) by 2 and epsilon$_c$ is equal to (epsilon$_x$ plus epsilon$_y$) by 2 plus (epsilon$_x$ minus epsilon$_y$)cos 2theta$_c$) by 2 plus (gamma$_{xy}$sin2theta$_c$) by 2. We have three equations: equation 1, equation 2, and equation 3 and out of these three equations we know the value of epsilon$_a$, we know the value of epsilon$_b$, and we know the value of epsilon$_c$.

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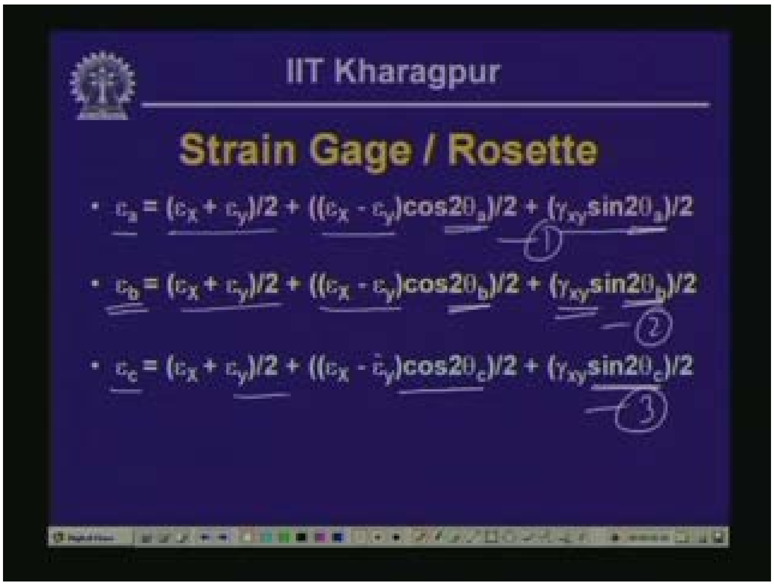

Also, we know the orientation angles theta$_a$, theta$_b$ and theta$_c$. Since these three parameters or six parameters are known epsilon$_a$, epsilon$_b$ and epsilonc are known; consequently thetaa, theta$_b$ theta$_c$ are known. The unknown parameters epsilon$_x$, epsilon$_y$ and gamma$_{xy}$ can be evaluated from these three equations. Therefore we can compute epsilon$_x$ we can compute epsilony, and we can compute gamma$_{xy}$ from these equations. Once we know epsilon$_x$, epsilon$_y$ and gamma$_{xy}$ then using these three quantities we can evaluate strain at any orientation as we desire as we have seen either using transformation equations or using Mohr circle or we can evaluate even the principal strains and consequently evaluate principal stresses from the material properties.

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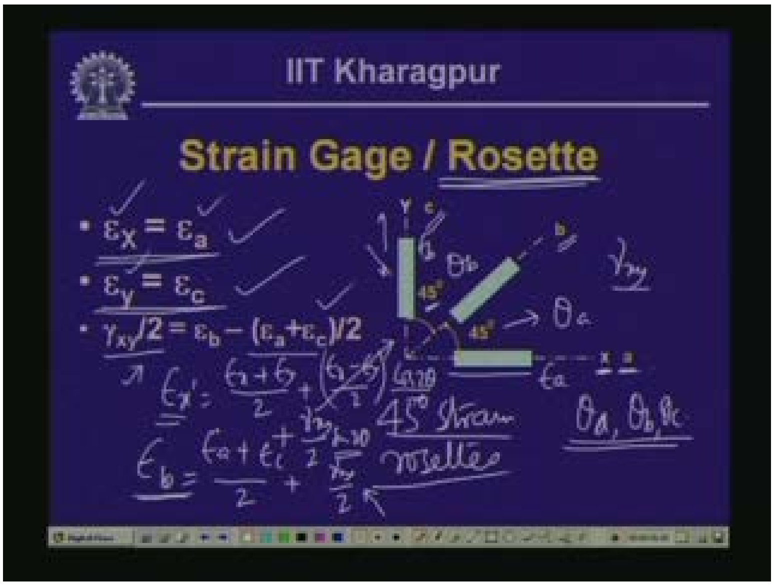

When we use the strain gages together to measure the strain at a particular point we call that assembly as the strain rosette; the formation of three gages together is termed as strain rosette. These angles shown as theta$_a$, theta$_b$ and theta$_c$ can be chosen according to our own convenience and normally the angles used are the 45 degrees or 60 degrees and now this is called as a 45 degrees strain rosette. Here two angles used are; theta$_a$ is 45 degrees and theta$_b$ is 45 degrees. Now along the x-direction we have oriented a, along 45 degrees we have b, along 90 degrees we have c.

Since epsilon$_a$ are the strain along a is coinciding with the x axis so epsilon$_x$ is epsilon$_a$ directly. Likewise since this particular gage c is oriented in the direction of y the epsilon$_y$ is equal to epsilon$_{c;}$ so directly we can get the values of epsilon$_x$ and epsilon$_y$ from the measured strain value of epsilon$_a$ and epsilon$_{b.}$ What we need to do is evaluation of gamma$_{xy}$. Again if we go back to the transformation equation which is epsilon$_{x\ prime}$ is equal to (epsilon$_x$ plus epsilon$_y$) by 2 plus (epsilon$_x$ minus epsilon$_y$) by 2 (cosine2theta plus gamma$_{xy}$) by 2 sin2theta. Now theta here is 45 degrees so sine 90 is 1, cosine 90 is equal to 0, so this term goes off and this is gamma$_{xy}$ by 2 and this is in direction of 45 degrees which is epsilon$_b$ so epsilon$_b$ is equal to epsilon$_x$ is epsilon$_a$ plus epsilon$_y$ is equal to epsilon$_c$ so epsilon$_a$ plus epsilon$_c$ by 2 plus gamma$_{xy}$ by 2. From this it gives gamma$_{xy}$ by 2 is equal to epsilon$_b$ minus epsilon$_a$ plus epsilon$_c$ by 2 so we get the value of epsilon$_x$, epsilon$_y$ and gamma$_{xy}$ from the measurement of this particular type of strain rosette which can be utilized for evaluating the strain or the principal strain at that particular point and those principal strains can be utilized for evaluating stresses at that particular point.

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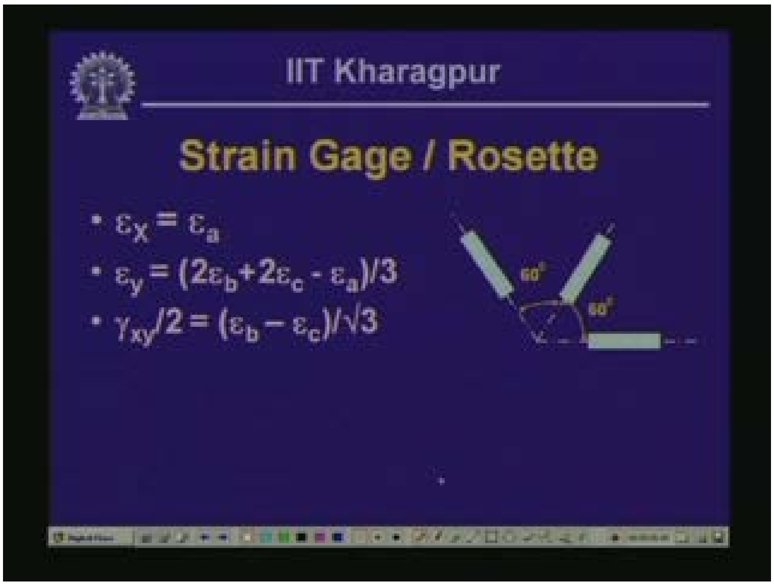

Another kind of strain rosette is oriented at an angle of 60 degrees, again the whole assembly the three strain gages fixed in this particular form can be fixed on the surface where we like to measure the strain, and at that particular point we are measuring strains in three different directions which are oriented at an angle of 60 degrees with reference to the different x axis. This is reference x-axis and now the y-axis here is in this direction; so let us call this as direction a, this as b, and this as c. So as usual since the strain in the gage a is in the x-direction, so epsilon$_x$ will directly give us epsilon$_a$. Now here neither the gage is oriented in the direction y so epsilon$_y$ we cannot get directly and we cannot evaluate the value of gamma$_{xy}$ directly as we have seen earlier so we have to find the values of epsilon$_y$ and gamma$_{xy}$ from the measure strain in the direction of b and c.

To evaluate the strain value at epsilony and gamma$_{xy}$ we make use of the transformation equation which is again epsilonx' is equal to epsilon$_x$ plus epsilon$_y$ by 2 plus epsilon$_x$ minus epsilon$_y$ by 2 cosine 2theta plus gamma$_{xy}$ by 2 sine 2theta. When we substitute for b that is epsilon$_b$ is equal to epsilon$_x$ plus epsilon$_y$, now epsilon$_x$ is already epsilon$_a$ by 2plus epsilon$_a$ minus epsilon$_y$ by 2 cosine 2theta where theta being 60 degrees this is cosine 120 degrees plus gamma$_{xy}$ by 2 sine 120 degrees.

Likewise we evaluate the strain in the c direction epsilon$_c$ is equal to epsilon$_x$ plus epsilon$_y$ by 2 plus epsilon$_x$ minus epsilon$_y$ by 2 now theta here is 120 degrees so cosine 240 degrees and sine 240 degrees and once we substitute the values of that then we get two equations in epsilon$_b$ and epsilon$_c$ and we have two unknown parameters epsilon$_y$ and gamma$_{xy}$ and these two can be solved and if we solve that we will get the values of epsilon$_y$ and gamma$_{xy}$ in terms of epsilon$_b$ and epsilon$_c$.

We have again three quantities epsilon$_x$, epsilon$_y$ and gamma$_{xy}$ in terms of epsilon$_{a,}$ epsilon$_b$ and epsilon$_c$ and that was our objective as to find the values of epsilon$_{x\ prime}$ epsilon$_y$ and gamma$_{xy}$ in terms of epsilon$_{a,}$ epsilon$_b$ and epsilon$_c$ and that was what our objective: to find out the values of epsilon$_x$ epsilon$_y$ and tau$_{xy}$ in terms of epsilon a epsilon b and epsilon c and these are the three normal strains which we have measured at a point based on which we can compute other quantities as we desire.

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This is another important aspect wherein we need to relate the quantity which is E which we call as modulus of elasticity to the quantity G which we call as the modulus of the rigidity. We have already seen the definition of E and G earlier and now we like to relate these two quantities E and G.. Let us look into state of stress at a point where the stresses are all shearing, stresses acting at that particular point in this element. Here the normal stresses $sigma_x$ and $sigma_y$ is equal to 0 and the shearing stresses are tau_{xy}. If you remember, this kind of stress distribution is called as the state of pure shear. This is a state of pure shear.

Now if we try to plot this stress in the Mohr circle, and as we have seen, this is our sigma-axis and this is our tau-axis and now on this particular plane sigma the normal stress is equal to 0 when we have only positive tau_{xy} which is causing anticlockwise moment and correspondingly if you place sigma this is sigma axis, $sigma_x$ is 0 and we have positive tau so this is a point which can be represented by this particular stress value and in the perpendicular direction again $sigma_y$ is 0 so we have normal stress 0 and negative of tau which is this particular point. So, considering this as the center let us call this o as center and oA or oA prime as the radius and now if we draw the circle this gives us the Mohr circular stress corresponding to the pure state of shear at a particular point.

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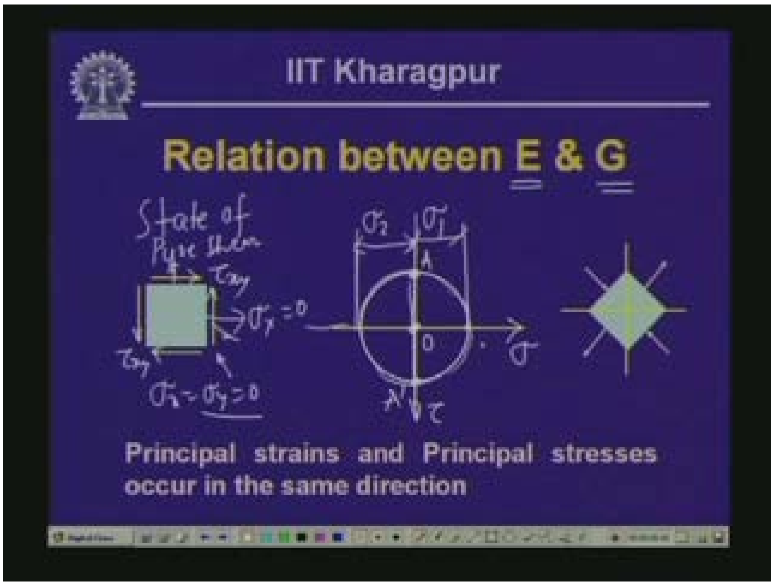

In this diagram this is the maximum normal stress which we call as $sigma_1$ and this is the minimum normal stress which we call as $sigma_{2.}$ Please observe that the maximum stress point oo prime or the minimum stress point oo double prime are of the same magnitude which is equal the radius of the circle. The radius of the circle is tau_{xy} so $sigma_1$ is equal to tau_{xy} and $sigma_2$ is equal to minus tau_{xy} and this is the element where the principal stresses $sigma_1$ and $sigma_2$ are acting. Now this aspect must be noted, we have seen consequently when we were evaluating the direction for the principal strain. We have seen that the direction of the principal stress at a point and direction of the principal strain match. In fact they are oriented at an angle of 45 degrees with reference to the reference x-axis.

Hence the principal strains and principal stresses occur in the same direction. This is quite important. We know that in this particular point if we would like to find out the strain $epsilon_1$ we can compute that $epsilon_1$ is equal to $sigma_1$ by (E) minus mu [$sigma_2$ by E]. This is the plane state of stress that is acting in this $sigma_1$ and $sigma_{2.}$ And if we like to evaluate the strain $sigma_1$ by E is the direct because of the Poisson effect so $epsilon_1$ is equal to $sigma_1$ by (E) minus mu $sigma_2$ by (E). Now if we substitute for the value of $sigma_1$ and $sigma_{2,}$ we have epsilon $_1$ is equal to $sigma_1$ by E minus mu $sigma_2$ by E and as we have seen that $sigma_1$ is equal to tau_{xy} and $sigma_2$ is equal to minus tau_{xy} hence epsilon $_1$ is equal to tau_{xy} by E minus mu (minus tau_{xy}) by E is equal to 1 plus mu tau_{xy} by E. Let us call this as equation 1.

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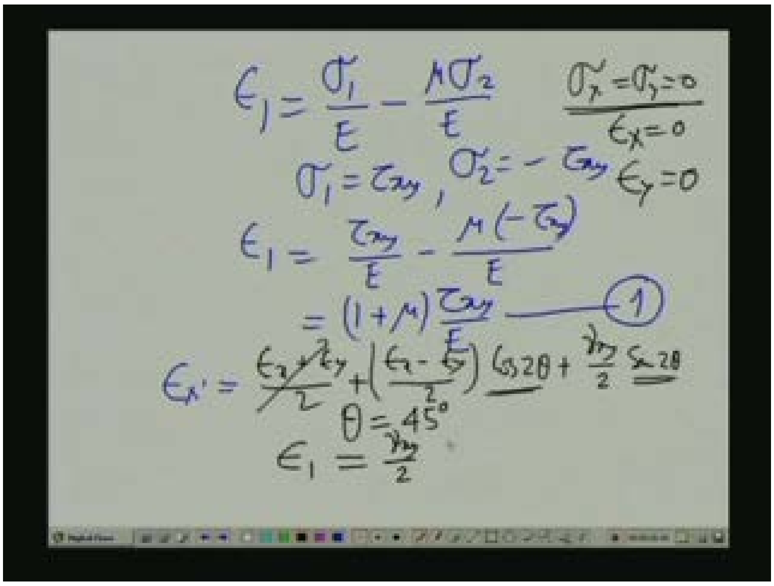

We have seen that the strain at any direction epsilon$_{x\ prime}$ is equal to epsilon$_x$ plus epsilon$_y$ by 2 plus (epsilon$_x$ minus epsilon$_y$ by 2) cos2theta plus gamma$_{xy}$ by 2 sin2theta. Since the direction of principal stress coincides with the direction of the principal strain then also that we have noted that sigma$_x$ is equal to sigma$_y$ is equal to 0 so since normal stresses are 0 and the corresponding normal strains are also 0; epsilon$_x$ is equal to 0 and epsilon$_y$ is equal to 0 hence the first term is 0.

Now since the direction of the principal strain is also theta is equal to 45 degrees then cosine 90 is 0 and thereby sine 90 is 1 so epsilon$_1$ the strain in the principal direction is equal to gamma$_{xy}$ by 2. Now the relation between the shearing stress and the shearing strain can be related through the shear modulus using the Hooke's law. So tau$_{xy}$ is equal to G(gamma$_{xy}$) where tau$_{xy}$ is the shearing stress and G is the shear modulus. Now gamma$_{xy}$ you can substitute in terms of tau$_{xy}$ is equal to tau$_{xy}$ by 2G so this is the second equation.

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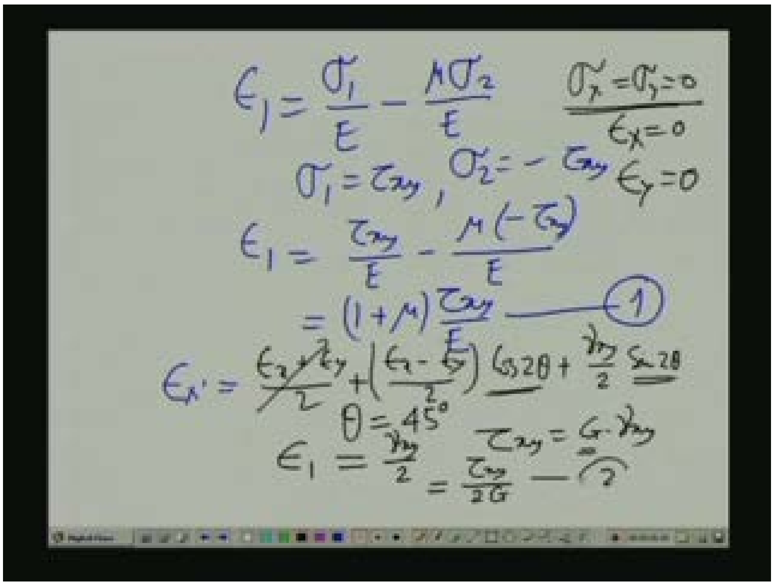

So, in the first equation we have is $\epsilon_1$is equal to 1 plus mu τ_{xy} by E and the second equation which we have is ϵ_1 is equal to τ_{xy} by 2G. If we equate these two, in the first place we have 1 plus mu(τ_{xy}) by E is equal to τ_{xy} by 2G. Hence the τ_{xy} and τ_{xy} gets cancelled so G is equal to E by 2 into 1 plus mu.

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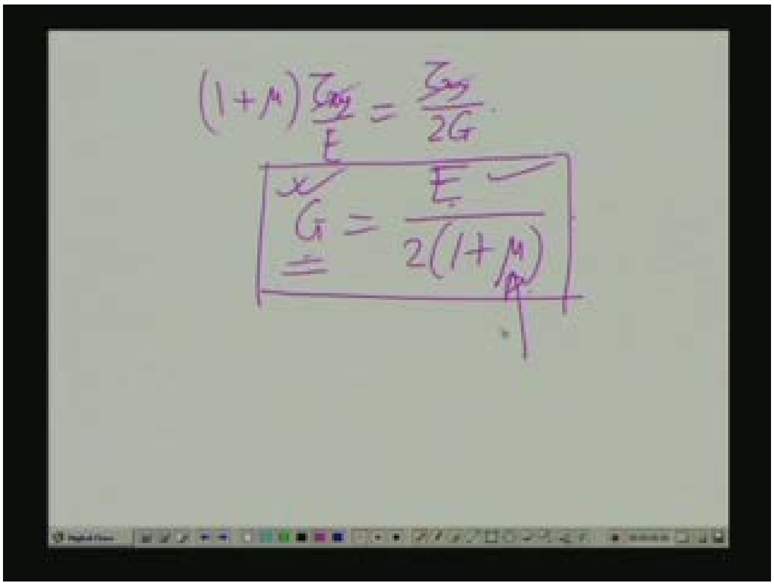

This is the relationship between the shear modulus G and the elastic modulus E through the term Poisson's ratio. This is the relationship between E, G, and tau. Now please note that these are the three quantities E, G, and tau these three elastic constants are necessary for evaluating deformation at a particular point but out of these three quantities two are independent and one can be evaluated in terms of the other two.

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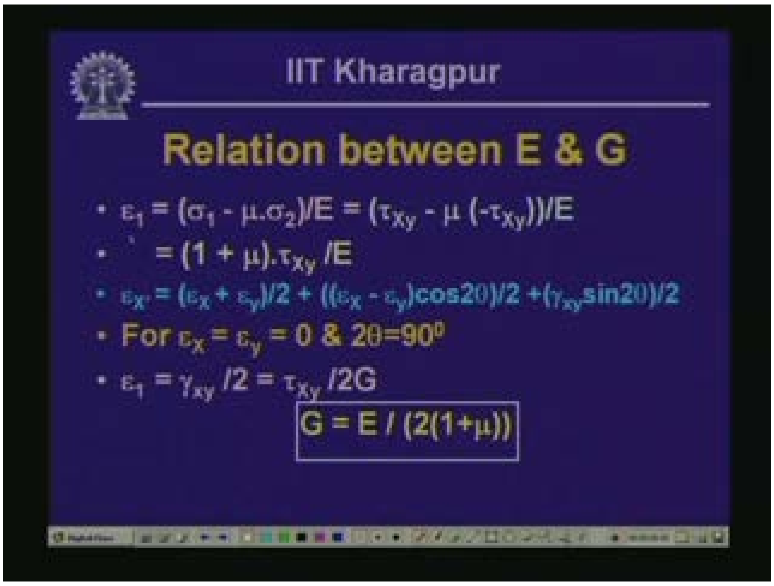

These are the expressions which we have just derived. It is written over here epsilon_1 is equal to 1 plus mu tau_{xy} by E and then epsilon_{xprime} is equal to epsilon_x plus epsilon_y by 2 plus epsilon_x minus epsilon_y cos 2theta by 2 and for epsilon_{xprime} epsilon_y is equal to 0 and 2theta is equal to 90 degrees if we relate these two we have G is equal to E by (2(1 plus mu)). This is the relationship between E and G through the term Poisson's ratio mu is the Poisson's ratio. Let us look into another quantity which we define as bulk modulus.

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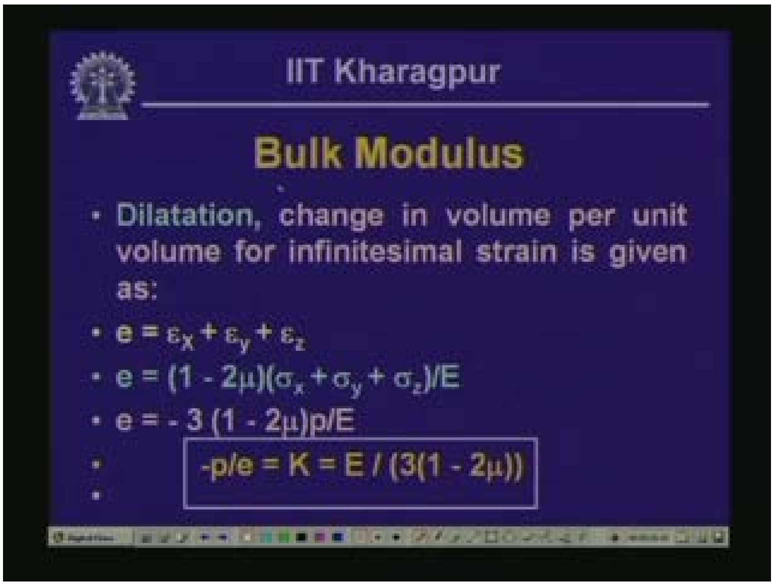

Many a times we need to know the volume change at a particular stressed body and if we try to find out we generally come up with another constant k which we call as bulk modulus which is related to the change in the volume in a stressed body. Now let us look at a point where a small body which is having a dimension dx, dy, and dz.

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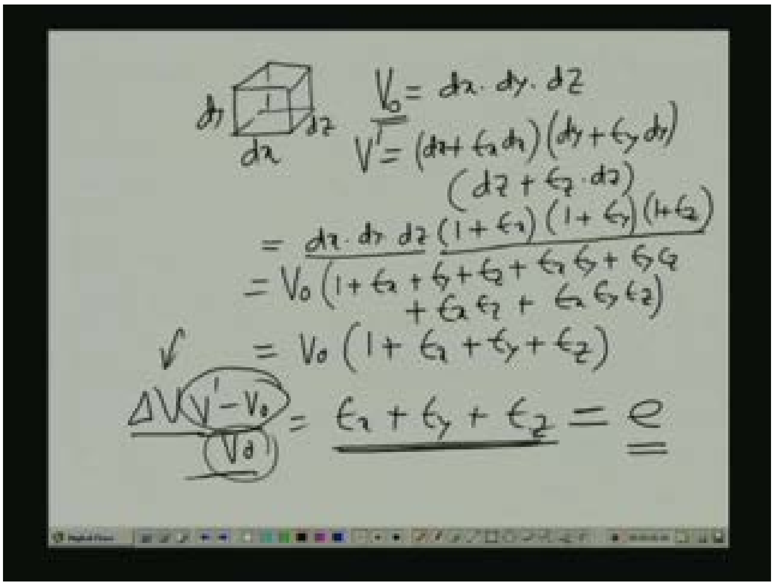

This is dx, dy, and dz the elemental length so the V_o is equal to dx(dy(dz)). Now after this particular element has undergone strain there will be extension in all three directions and thereby there will be change in the volume. Let us say that the strains in the x, y, and z directions are epsilon$_x$ epsilon$_y$ and epsilon$_z$ and thereby the extensions in the x direction will be epsilon$_x$(dx); and the extension in the y direction will be epsilon$_y$(dy); and the extension in the z direction will be epsilon$_z$(dz).

The changed volume accordingly will be equal to let us call this V prime as dx plus epsilon$_x$dx is the changed length in the x direction; dy plus epsilon$_y$dy is the changed length in the y direction (dz plus epsilon$_z$dz). Or this we can write as dx dy dz, if we take out dx, dy, dz, we have 1 plus epsilon$_x$(1 plus epsilon$_y$) (1 plus epsilon$_z$). If we expand this, now dx dy dz as we have seen is the initial volume which is V_0 times if we expand this then it is 1 plus epsilon$_x$ plus epsilon$_y$ plus epsilon$_z$ plus epsilon$_x$ epsilon$_y$ plus epsilon$_y$ epsilon$_z$ plus epsilon$_x$ epsilon$_z$ plus epsilon$_x$ epsilon$_y$ epsilon$_{z.}$ This is the expression which we are going to get if we multiply these three quantities.

Since the strain is small epsilon$_x$ epsilon$_y$ epsilon$_z$ they are small quantities so the product of these quantities epsilon$_x$ epsilon$_y$ and epsilon$_x$ epsilon$_z$ and epsilon$_y$ epsilon$_z$ and epsilon$_x$ epsilon$_y$ epsilon$_z$ we consider them as 0 or insignificant. Hence this is equal to V_0 (1 plus epsilon$_x$ plus epsilon$_y$ plus epsilon$_z$) and this V prime minus V_0 by V_0 is equal to epsilon$_x$ plus epsilon$_y$ plus epsilon$_{z.}$

Now V prime minus V_0 is the change in the volume deltaV. Now deltaV by V_0 is equal to epsilon$_x$ plus epsilon$_y$ plus epsilon$_z$ and this we denote by quantity e. And in fact this is known as dilatation that the change in volume per unit V_0 is unit then change in volume deltaV gives e. So, dilatation is the quantity which is called as the change in volume per unit volume represented in terms of strain as epsilon$_x$ plus epsilon$_y$ plus epsilon$_{z.}$ Now we know the relationship between the strain and stress which is epsilon$_x$ is equal to sigma$_x$ by E minus mu(sigma$_y$ plus sigma$_z$) by E.

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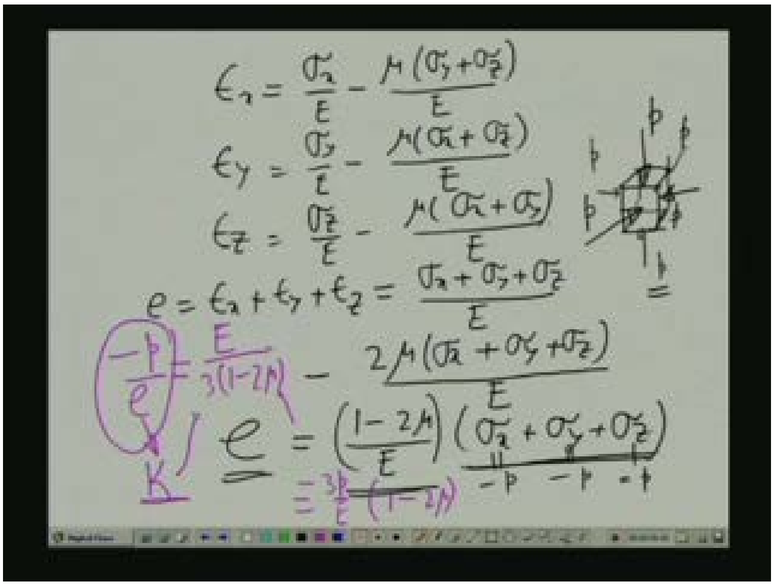

Likewise epsilon$_y$ is equal to sigma$_y$ by E minus mu(sigma$_x$ plus sigma$_z$) by E. This we have seen while discussing the generalized Hooke's law and epsilon$_z$ is equal to sigma$_z$ by E minus mu(sigma$_x$ plus sigma$_y$) by E. Now if we add together e is equal to epsilon$_x$ plus epsilon$_y$ plus epsilon$_z$ as we have seen and this equal to in terms of stresses as sigma$_x$ plus sigma$_y$ plus sigma$_z$ by E minus 2mu(sigma$_x$ plus sigma$_y$ plus sigma$_z$) by E is equal to 1 minus 2mu by E(sigma$_x$ plus sigma$_y$ plus sigma$_z$). So e is related to the stress as 1 minus 2mu by E(sigma$_x$ plus sigma$_y$ plus sigma$_z$).

Now, if we take that small body from which we evaluated this quantity, and if we say that the stresses acting on this particular body sigma$_x$ sigma$_y$ and sigma$_z$ are all of magnitude p acting in the compressive direction on all sides then it is called as the state of hydrostatic pressure. That means a body is subjected to pressure from all directions. Thereby the sigma$_x$ sigma$_y$ and sigma$_z$ are of equal value and they are of magnitude p and this particular state of stress in a body we call as state of hydrostatic pressure.

If we replace the value of sigma$_x$ sigma$_y$ and sigma$_z$ in terms of p then since it is acting in the compressive direction they are minus so sigma$_x$ is equal to minus p; sigma$_y$ is equal to minus p and sigma$_z$ is equal to minus p and minus e is equal to minus 3p by E(1 minus 2mu) and this is the quantity minus p by e is equal to E by 3(1 minus 2mu) and this particular quantity minus p by e is normally designated as the bulk modulus k. So k is related to this elastic modulus E and Poisson's ratio mu as k is equal to e (3 by 1 minus 2mu) and this is what has been discussed here.

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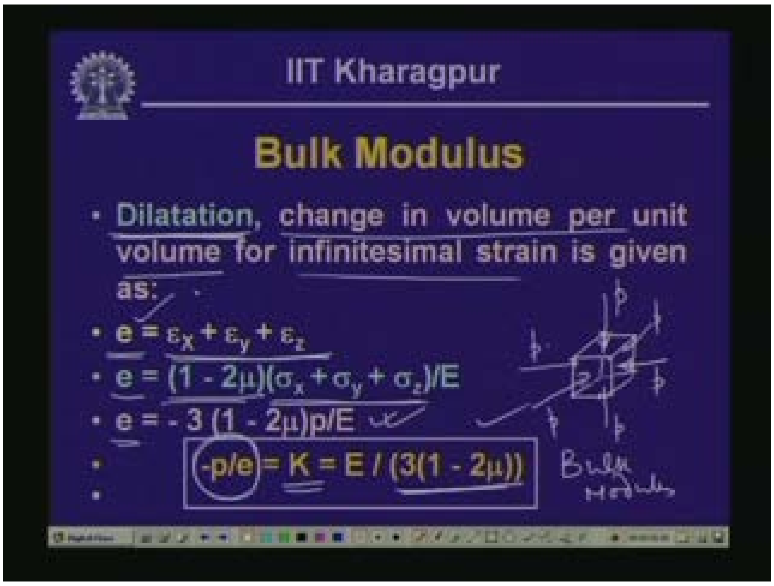

These are the expressions as we have derived right now that epsilon is equal to epsilon$_x$ plus epsilon$_y$ plus epsilon$_z$ and e is equal to 1 - 2 mu (sigma$_x$ plus sigma$_y$ plus sigma$_z$ by E and e is equal to minus 3(1 minus 2mu) p by E when the element is subjected to the state of hydrostatic pressure that in all directions it is subjected to p and because of this the state of stress sigma$_x$ plus sigma$_y$ plus sigma$_z$ is minus 3p this gives us the expression e equal to this and thereby the quantity minus p by e we call as the bulk modulus and bulk modulus value is equal to E by 3(1 minus 2mu). This term is defined as dilatation which is the change in volume per unit volume for a small infinitesimal strain which is given by this expression e.

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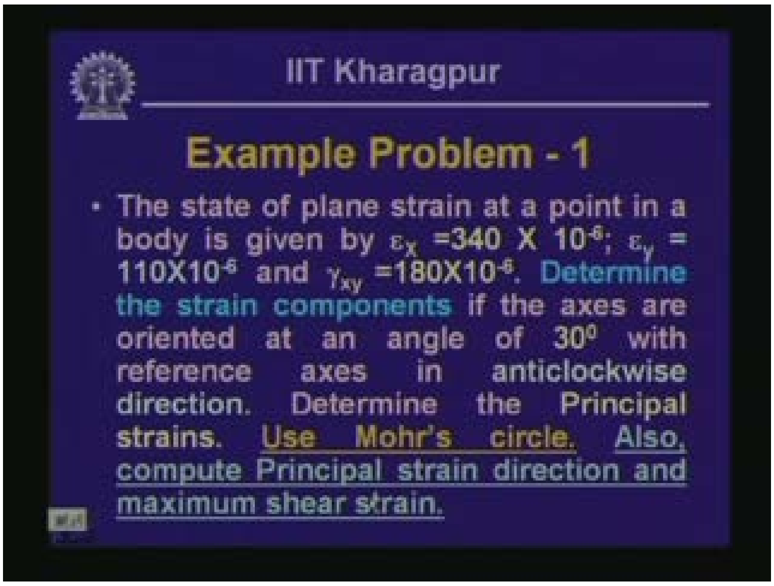

Example problem:
After looking into the aspects of strains as discussed through transformation equations, through Mohr circle, and also we have seen usage of strain gages and the strain rosettes from

which we can find out the values of normal strains in different directions which can be utilized for the evaluation of the principal strain at a particular point from which we can compute the values of principal stresses. Last time I solved one example using the Mohr circle of strain. Now the values of epsilon$_x$ is given as this, epsilon$_y$ as this and gamma$_{xy}$ as this. What you need to do is to compute the values of the principal strains and their directions and also the maximum shearing strain based on these data using Mohr circle of strain and we will have to find out the strains at a direction which is oriented at 30 degrees in an anticlockwise direction with reference to the x-axis.

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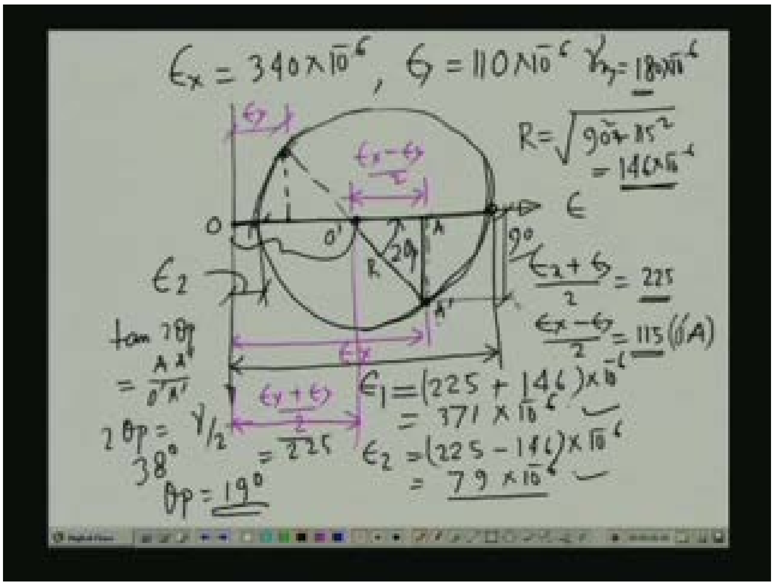

Plotting the Mohr circle:

Now given values are epsilon$_x$ is equal to 340 into 10 to the power minus 6, epsilon$_y$ is equal to 110 into 10 to the power minus 6 and gamma$_{xy}$ is equal to 180 into 10 to the power minus 6· If we plot this in the Mohr circle this is the gamma by 2 direction and this is epsilon direction. Here we have epsilon$_x$ is equal to 340 and gamma by 2 is equal to 90 so this is the point which we choose here; we have another point epsilon$_y$ which is 110 and gamma by 2 is minus 90 so we place the point here and as per the norm of the Mohr circle we join these two points and thereby this gives us the center of the Mohr circle. And if we plot the circle considering this as center and this as radius we get the circle. This particular point gives us the point of maximum normal strain which we define as epsilon$_1$ and this is the minimum normal strain which we define as epsilon$_2$.

In this figure this is epsilon$_x$ and epsilon$_y$ hence this is epsilon$_x$ minus epsilon$_y$ by 2 and thereby this distance is epsilon$_x$ plus epsilon$_y$ by 2. The values of this if we calculate epsilon$_x$ plus epsilon$_y$ by 2 is equal to 340 plus 110 is equal to 450 hence 225 into 10 to the power minus 6 then epsilon$_x$ minus epsilon$_y$ by 2 is equal to 230 by 2 is equal to 115. Now the normal strain epsilon$_1$ will be the distance from o to o prime plus the radius. The radius can be computed from this triangular configuration where let us call this as a and this as A prime now AA prime is equal to tau by 2 is equal to 90(10 to the power minus 6) and this distance is epsilon$_x$ minus epsilon$_y$ by 2 is equal to 115 so the radius R is equal to square root of 90(10 to the power 6) square plus 115 square and this gives us a value of 146(10 to the power minus 6). Now this quantity is with 10 to the power minus 6 which I am keeping silent here, so R is equal to 146(10 to the power minus 6). So the value of maximum normal strain $epsilon_1$ is equal to oo prime plus o prime plus R and oo prime is equal to $epsilon_x$ plus $epsilon_y$ by 2 which is equal to 225. So 225 plus 146, this gives us the value of the normal strain maximum which is 371; 371 (10 to the power minus 6) is the maximum normal strain.

The minimum normal strain $epsilon_2$ is equal to this particular distance again is the distance from o to o prime minus the radius so oo prime is again 225 minus radius is 146 (10 to the power minus 6) this gives us the value of 79 (10 to the power minus 6). So these are the values of the maximum normal and the minimum normal, the principal strains. We need to know the orientation of the principal strains and this is the reference x axis where the normal $epsilon_x$ and $\frac{\gamma_{xy}}{2}$ two positive occurs now from here. If you rotate by angle $2theta_p$, this gives us the position of the maximum normal strain and from this particular triangle o prime AA prime we can get tan $2theta_p$ tan $2theta_p$ is equal to AA by o prime A prime and that gives us the value of which is AA prime is 90 and o prime A is equal to 115. So 19 by 115 tan inverse gives us the value of 38; $2theta_p$ is equal to 38 degrees and thereby $theta_p$ is equal to 19 degrees. So this is the orientation along which the maximum principal strain occurs. Now what we need to do is to evaluate the strains at a direction which is oriented at an angle of 30 degrees with reference to the reference x-axis. So if it is oriented at an angle of 30 degrees in the Mohr's plane, it is twice the theta so it is 60 degrees.

Now with reference to the positive $epsilon_x$ and $\frac{\gamma_{xy}}{2}$ if we orient by 60 degrees so this is the 60 degrees point and if we join the line from center to this we get this particular point which is the direction where we need the strain and this will give us the value of this is the normal strain, and this is the value of the shearing strain $gamma_{x\ prime\ y\ prime}$ by 2. Now this particular angle is 38 degrees and this total angle is 60 degrees; so 60 minus 38 this is if we call this as alpha is equal to 22 degrees. Now if we can compute the value of a, and this distance b, then we can find out the normal stress which is oo prime plus b and shearing strain is $\frac{\gamma_{xy}}{2}$; x prime y prime by 2 is equal to a or which is minus a that is negative in the Mohr's plane. So if we compute those two values now this is the radius R cos alpha will give us this particular distance and our sin alpha will gives us value a. So once we know the value of a and b we can compute $epsilon_x$ and $gamma_{x\ prime\ y\ prime}$. Now let us compute the values of a and b.

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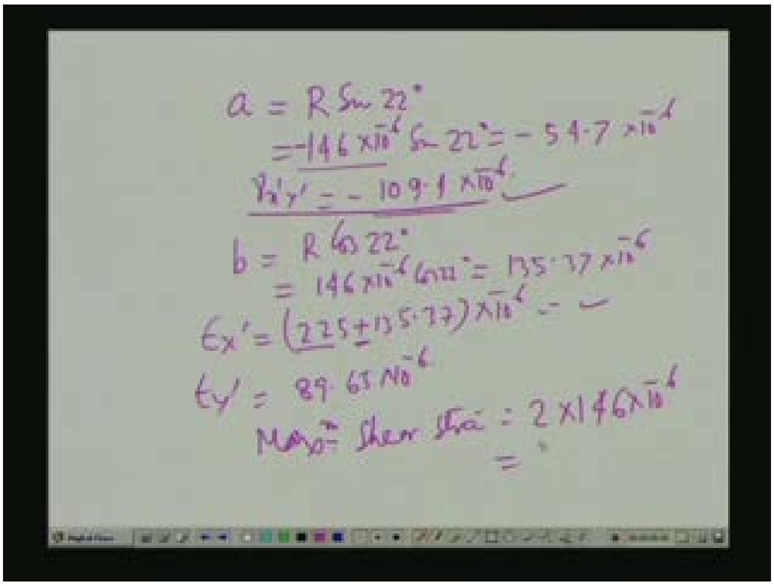

Now a is equal to R sin 22 degrees is equal to 146 (10 to the power minus 6) (sin22 degrees) is equal to minus 54.7 10 to the power minus 6 and writing this as a minus because it is in the negative direction. So gamma$_{x\ prime\ y\ prime}$ in fact the shearing strain is twice of this so minus 190.4(10 to the power minus 6) is the shearing strain. Now the distance b is equal to R cos 22 degrees is equal to 146 (10 to the power minus 6) cos 22 degrees is equal to 135.37(10 to the power minus 6). So epsilon$_{x\ prime}$ is equal to oo prime distance is equal to 225 plus 135.37(10 to the power minus 6) the normal strain and epsilon$_{y\ prime}$ consequently will be the minus of this which is diametrically opposite is equal to 89.65 (10 to the power minus 6) and gamma$_{xy}$ is equal to this. So these are the values of the strain and maximum shear strain is equal to the radius of the Mohr circle is equal to 2(146); $\frac{\gamma}{2}$ is the radius so 2(146)(10 to the power minus 6) is equal to 292 (10 to the power minus 6). These are the values of the strain quantities.

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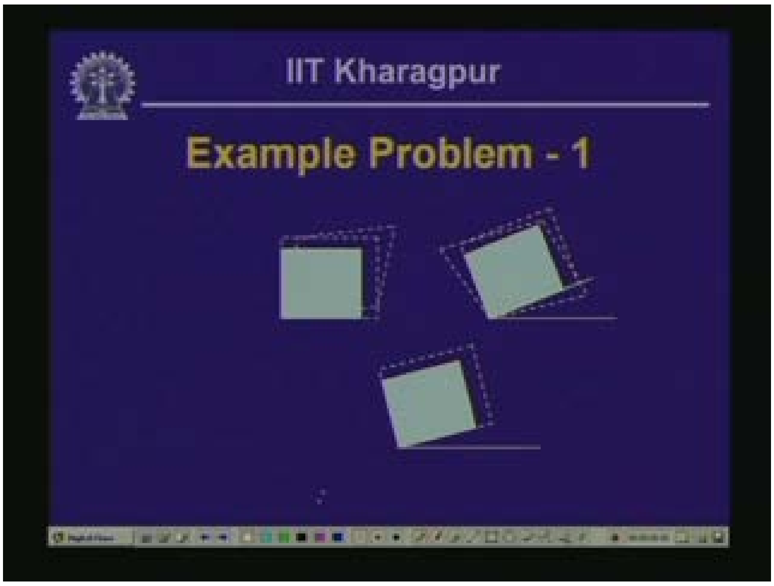

Once this particular element undergoes strains now this is the configuration which we expect. In the initial stages we have epsilon$_x$ as 340 and epsilon$_y$ as 110 and we had the positive shear which is denoted by this as 180 and we had the orientation of the element as the initial strain. Finally when it is rotated at an angle of 30 degrees, it has epsilon$_{x\ prime}$ epsilon$_{y\ prime}$ and gamma$_{x\ prime\ y\ prime}$ is negative so the angle here increases and this is the form which it takes. This is the final shape of the body after it has undergone the strain. And when it undergoes the principal strain, the principal strain direction is 19 degrees. This is the direction of epsilon$_1$ and this is the direction of epsilon$_2$ and both are positive and here there is no shearing strain.

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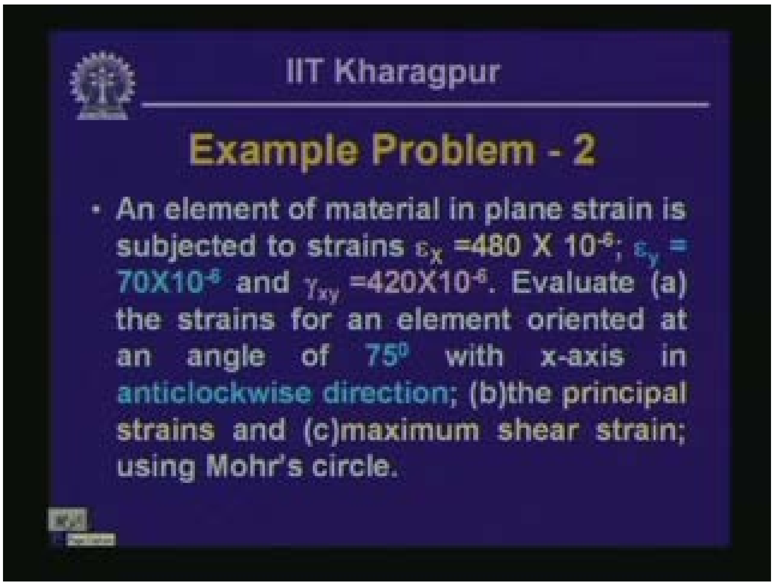

Hence this would be the shape. We have another example which is similar type that any element a material in plane strain is subjected to strains epsilon$_x$ is 48 (10 to the power minus 6); epsilon$_y$ is 70 (10 to the power minus 6); gamma$_{xy}$ is 420 (10 to the power minus 6). Let us evaluate the strain at an element which is oriented at an angle of 75 degrees with x -axis in anticlockwise direction and the principal strains and the maximum shear strain using Mohr's circle.

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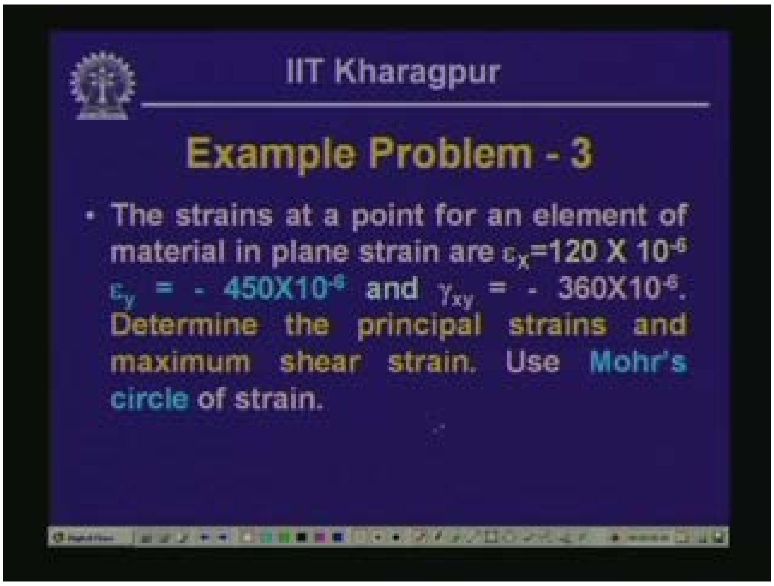

Here is another example wherein this is also identical. We have epsilon$_x$, epsilon$_y$ and gamma$_{xy}$. The strains at a point for an element of material in plane strain are of this form where epsilon$_x$ is 120 (10 to the power minus 6) which is positive epsilon$_y$ is negative is minus 45 0(10 to the power minus 6) and the shearing strain also negative minus 36 (10 to the power minus 6). We will have to determine the principal strains and the maximum shear strain using Mohr circular strain. This is also being similar to the previous problem. Now let us look into this particular problem.

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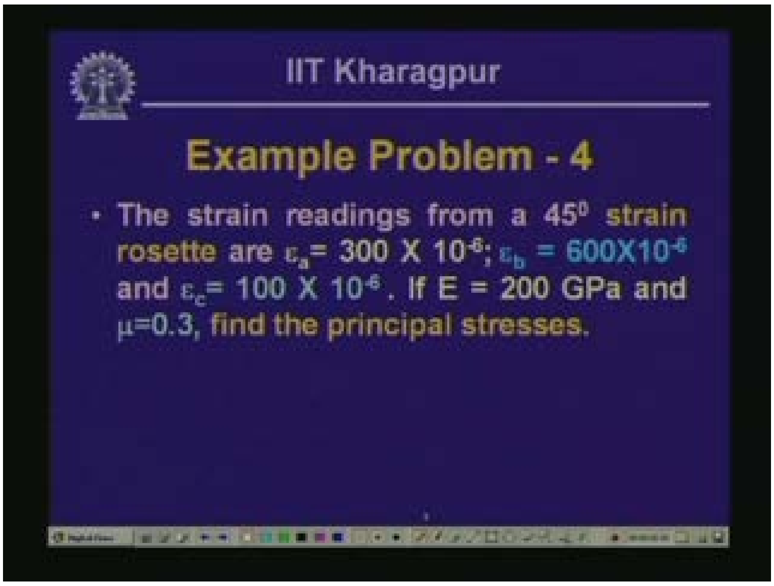

It is based on the aspects we have discussed where the strain readings from a 45 degrees strain rosette are epsilon$_a$ is this much, epsilon$_b$ is this much, and epsilon$_c$ is this much, so if E is equal to 200 GPa and mu is equal to 0.3 what will be the values of the principal stresses.

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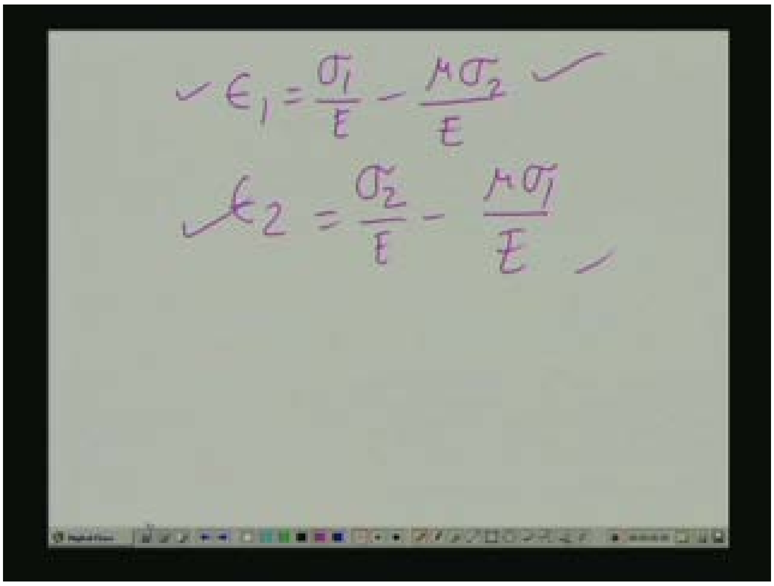

Now as we have seen that we have rosettes which are oriented in three different directions this is epsilon$_a$, this is epsilon$_b$, and this is epsilon$_c$; and as you have seen that epsilon$_x$ is equal to epsilon$_a$ and epsilon$_y$ is equal to epsilon$_c$ and $\frac{\gamma_{xy}}{2}$ is equal to epsilon$_b$ minus epsilon$_a$ plus epsilon$_c$ by 2. Now from this we can get the values of epsilon$_x$, epsilon$_y$, and gamma$_{xy}$ and we can compute the values of the stresses.

The values of epsilon$_a$ is equal to 300 (10 to the power minus 6), epsilon$_b$ is equal to 600 (10 to the power minus 6), and epsilon$_c$ is 100 (10 to the power minus 6) $\frac{\gamma_{xy}}{2}$ is equal to 600 minus 300 plus 100 by 2 (10 to the power minus 6) is equal to 400 (10 to the power minus 6). If we plot these in a Mohr circle with the values of epsilon$_a$ is epsilon$_x$ and epsilon$_c$ is epsilon$_y$ this is epsilon axis and this is the gamma$_{xy}$ axis.

We have epsilon$_x$ as 300 and $\frac{\gamma_{xy}}{2}$ is equal to 400. We have epsilon$_y$ as 100 and gamma by 2 is 400 and if we join this together we get the center and if we plot the circle we get the Mohr circle of strain and this is the value which is epsilon$_1$ and this is the value which is epsilon$_2$. Now this distance is equal to epsilon$_x$ plus epsilon$_y$ by 2 300 plus 100 is equal to 400 by 2 is equal to 200 and this distance epsilon$_x$ minus epsilon$_y$ by 2 which is 300 minus 100 is equal to 200 by 2 is equal to 100 and this distance is 400. So R is equal to $\sqrt{(100^2) + 400^2\left(10^{-6}\right)}$. We have and this is equal to 412.3 (10 to the power minus 6). Hence the value of epsilon$_1$ is equal to this distance plus the radius and this distance is equal to 200 so 200 plus 412 is equal to 612.

Consequently epsilon$_2$ is this distance minus the radius so this is negative which is 200 minus 412 (10 to the power minus 6) is equal to minus 212 (10 to the power minus 6) so that is the value of epsilon$_2$. Hence once we have the value of epsilon$_1$ and epsilon$_2$, as you know the value of epsilon$_1$ is equal to sigma$_1$ by E minus mu sigma$_2$ by E and consequently epsilon$_2$ is equal to sigma$_2$ by E minus musigma$_1$ by E. Now from this if we substitute the value of epsilon$_1$ and epsilon$_2$ we can get the value of sigma$_1$ and sigma$_2$.

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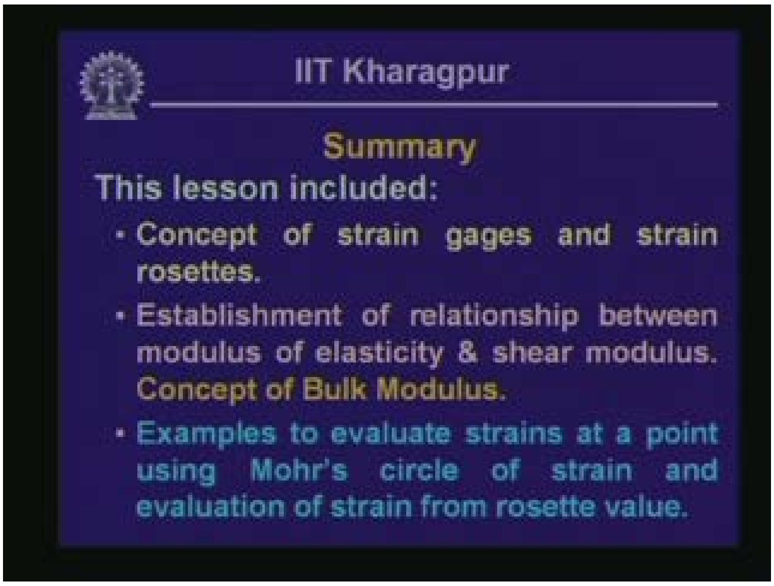

Summary:
In this particular lesson we have included the concept of strain gages and strain rosettes, and some of the relationships between the elastic modulae.

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