Strength of Materials
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Lecture No #13
Analysis of Strain - VII

Welcome to the 7th lesson of module II on the course on Strength of Materials and this module and the lesson is on Analysis of Strain part VII.

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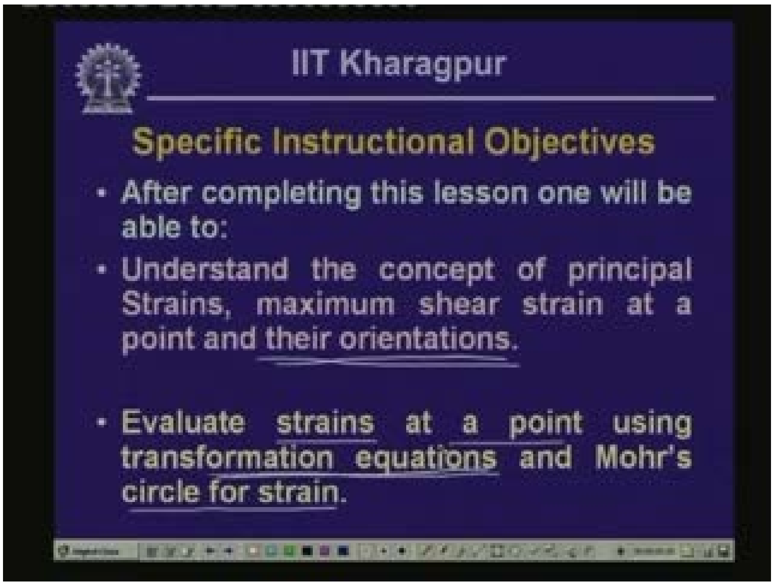

In the previous lesson on Analysis of Strain we had looked into some aspects of evaluating strain at a particular point at a different orientation. If we have axes system oriented

differently from the rectangular axes system then how do you compute the strains with reference to those axes system?
Once this particular lesson is completed one should be able to understand the concept of principal strain and maximum shear strain at a point and their orientations with respect to the rectangular axes system. One should be able to evaluate strains at a point using transformation equations and Mohr's circle of strain.

Let us look into the transformation equations today which we have already discussed. Also as we have looked into, in case of stresses we can evaluate stress at a point using Mohr's circle. We see that we can evaluate strain as well using Mohr's circle of strain.

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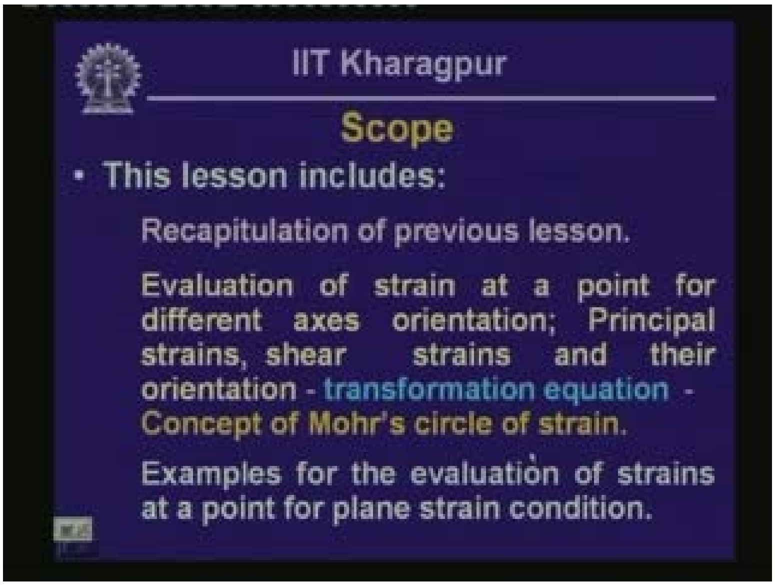

Hence the scope of this particular lesson includes the recapitulation of previous lesson and we will discuss the answers to the questions that I posed last time which will give you an overview of the previous lesson. Evaluation of strain at a point for different axes orientation; principal strains, shear strains and their orientation In fact with respect to the rectangular axes system as we looked into, the strain at a particular point varies if the axes system differs from the rectangular axes system which is x, y and z.

Also, we will be looking into the evaluation of principal strains, the evaluation of shear strains and their orientations with reference to the rectangular axes system through transformation equation which we have dealt with to a certain extent last time and also will introduce the concept of Mohr's circle of strain similar to the line of the Mohr's circle of stress. Then will look into some examples for the evaluation of strains at a point for plane strain condition. That means if we know epsilon_x epsilon_y gamma_{xy} then how to compute strain at a point if we have different axes orientation and how to compute the principal strains and how to compute the shear strains and their orientations with respect to rectangular axes system.

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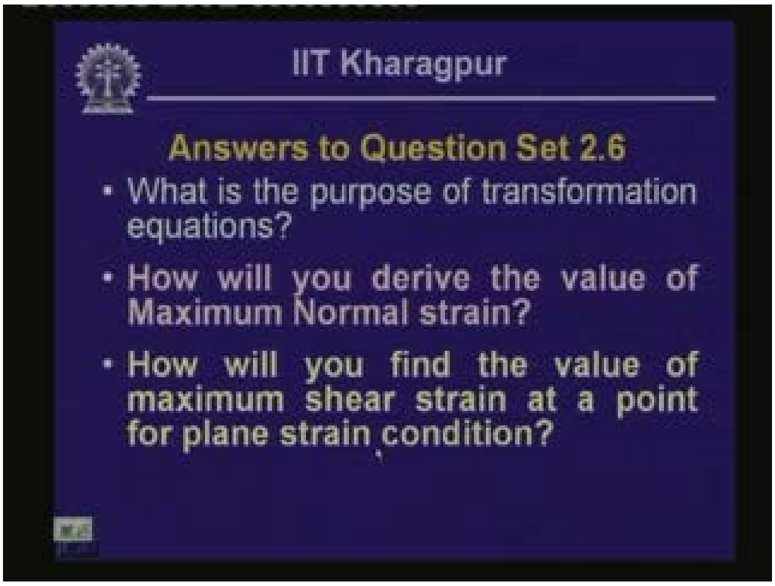

Let us look into the answers to the questions which I had posed last time. The first question which was given was what is the purpose of transformation equations?
Now that we had derived the transformation equations now we will look into what is the purpose of this particular equation.

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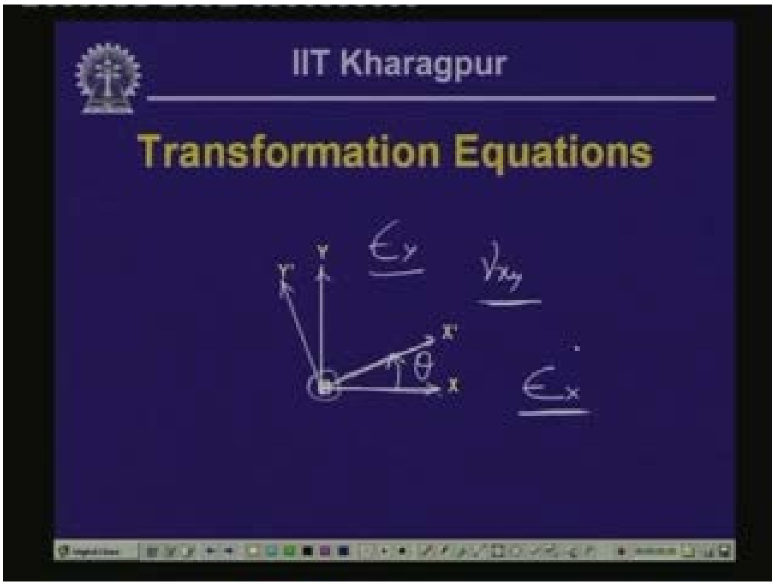

At a particular point if we are interested to evaluate the strains in terms of the strains which are known in the x direction, y direction and shearing strain i.e. $epsilon_x$ $epsilon_y$ and $gamma_{xy}$. Then in this particular point if we like to evaluate the strain with reference to another axis system which is oriented at an angle of theta with reference to x axis and let us say this is x prime and y prime axes then what will be the state of strain at this particular point? Will they still be $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ or will they be different?

Through transformation equation we tried to evaluate the strains at that particular point with reference to different axis system which are x prime y prime in this particular case. So the purpose of the transformation equations is to evaluate the strain at different orientations with reference to the rectangular axis system x and y.

The second question was how will you derive the value of maximum normal strain? Please note here that we are going to find out the value of the maximum normal strain. We had computed the value of normal strain at a particular point when the axes system is oriented at an angle of theta with reference to x and y. Now what we need to do is that to first find out the value of the maximum normal strain and the location where that occurs.

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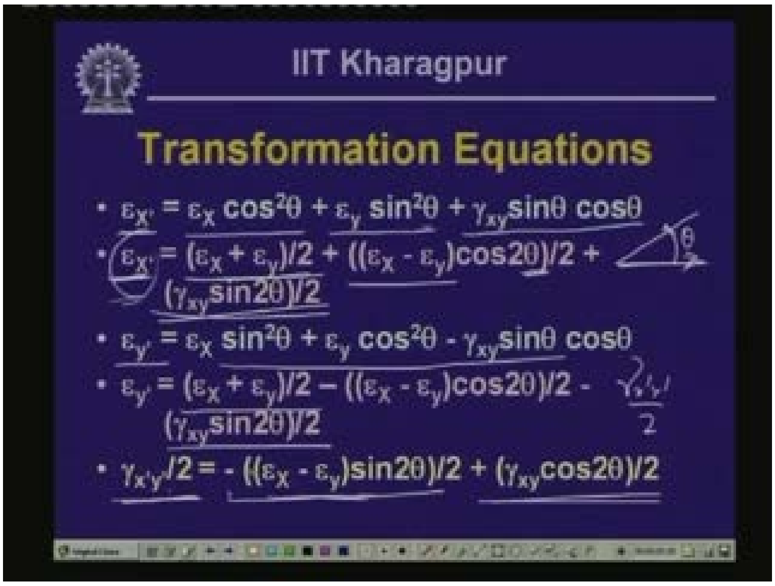

Now the transformation equations we had derived epsilon$_x$ the normal strain in the x direction is epsilon$_x$ cos square theta epsilon$_y$ sin square theta plus gamma$_{xy}$ sin theta cos theta where theta is the angle which is oriented in an anticlockwise form with reference to the x axis. And thereby epsilon$_{xprime}$ can be written as (epsilon$_x$ plus epsilon$_y$) by 2 plus (epsilon$_x$ minus epsilon$_y$) by 2 costheta plus gamma$_{xy}$ sin2theta by 2. And also if we substitute for theta as theta plus 90 then this gives us the value of epsilon$_y$ which is epsilon$_x$ plus epsilon$_y$ by 2 minus epsilon$_x$ minus epsilon$_y$ by 2 cos 2theta minus gamma$_{xy}$ by 2 sin 2theta.

And the shearing strain with reference to x prime y prime axis which is gamma x prime y prime we have written in terms of gamma x prime y prime by 2 equal to minus epsilon$_x$ minus epsilon$_y$ by 2 sin 2theta plus gamma$_{xy}$ by 2 cos 2theta. Now let us look into, that if we have to find out the maximum normal strain, now this is the normal strain epsilon$_x$ prime which is acting in the x direction. What we need to do now is to find out the value of the maximum normal strain. To do that if we take the derivative of this strain with respect to theta we can get the value of the angle at which the maximum normal strain will be oriented. Let us look into that aspect first.

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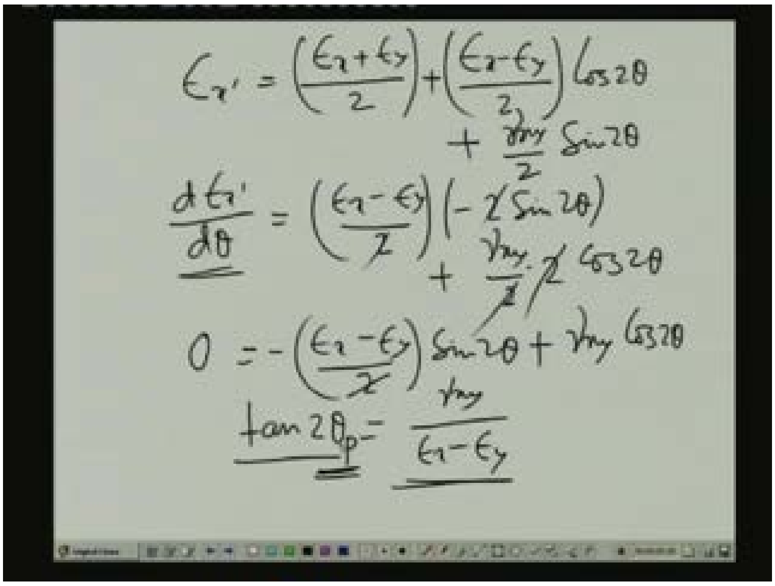

Now epsilon$_x$ prime as we have noted down is equal to epsilon$_x$ plus epsilon$_y$ by 2 plus epsilon$_x$ minus epsilon$_y$ by 2cos 2theta plus gamma$_{xy}$ by 2 sin 2theta. Now if we take the derivative of this with respect to theta this is equal to epsilon$_x$ minus epsilon$_y$ by 2 minus 2 sin 2theta plus gamma$_{xy}$ by 2 cos 2theta these two gets cancelled. So this is now maximized using the normal strain this is equal to 0 so 0 equal to theta x minus epsilon$_y$ by 2 sin 2theta with minus sign here plus gamma$_{xy}$ cos 2theta. Now this gives us the value of tan 2theta equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$ so these two gets cancelled, so it is epsilon$_x$ minus epsilon$_y$ sin 2theta so tan 2theta equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$.

Now let us call this angle with the suffix p or the maximum normal strain we generally designate as principal strain. As we had done in case of stresses the maximum normal stress and the minimum normal stress are designated as principal stresses but here the strain values the maximum normal strains we call as principal strains and their orientation angles are denoted with theta$_p$. Now this particular equation has two solutions, one is 2theta$_p$ and tan 180 plus 2theta$_p$ which is also the same value. So we will have two angles; one is theta$_p$ and the other one is 90 plus theta$_p$ in the physical plane.

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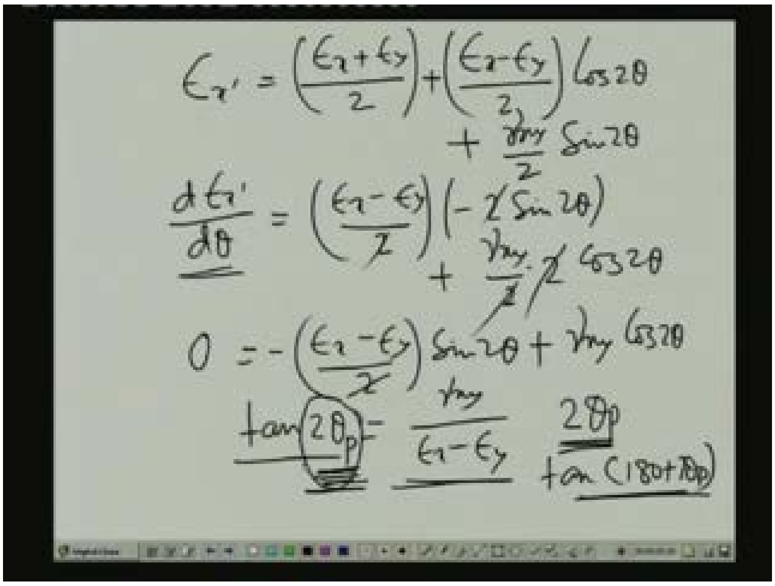

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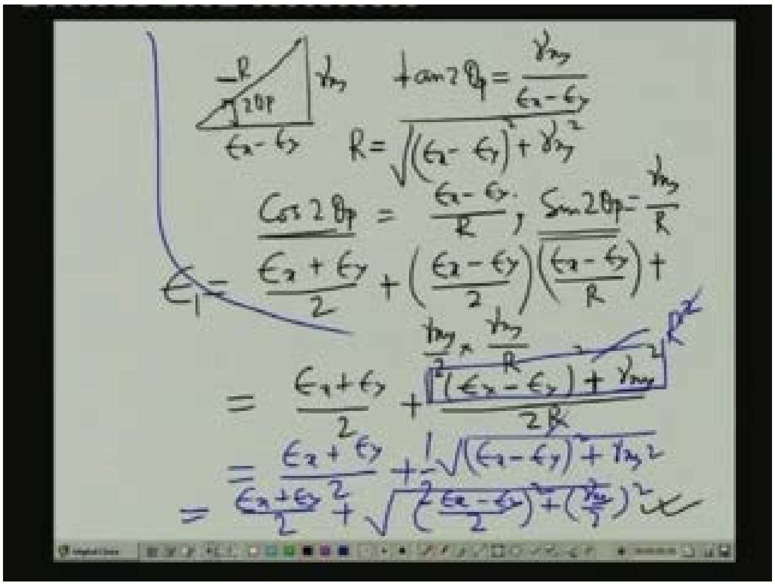

So these give us the orientation of the direction along which the normal strain will be maximum. Now if this is the orientation along which normal strain is the maximum then what is the value of the corresponding normal strain which is the maximum normal strain. We have obtained that tan $2theta_p$ equal to $gamma_{xy}$ by $epsilon_x$ minus $epsilon_y$. If this is the angle $2theta_p$ then this is $gamma_{xy}$ and this is $epsilon_x$ minus $epsilon_y$ so let us call this as hypotenuse R so R equal to square root of $epsilon_x$ minus $epsilon_y$ square plus $gamma_{xy}$ square. So cos $2theta_p$ equal to $epsilon_x$ minus $epsilon_y$ by r and sin $2theta_p$ equal to $gamma_{xy}$ by r. If we substitute the value of cos $2theta_p$ and sin $2theta_p$ in the expression for $epsilon_x$ prime that will give us the maximum value of the normal strain so let us call that as $epsilon_1$ equal to $epsilon_x$ plus $epsilon_y$ by 2 plus $epsilon_x$ minus $epsilon_y$ by 2, in place of cos $2theta_p$ we write $epsilon_x$ minus $epsilon_y$ by r plus $gamma_{xy}$ by 2 and in place of sin $2theta_p$ we write $gamma_{xy}$ by R equal to $epsilon_x$ plus $epsilon_y$ by 2 plus $epsilon_x$ minus epsilon$_y$ square plus gamma$_{xy}$ square by 2R and the numerator epsilon$_x$ minus epsilon$_y$ square plus gamma$_{xy}$ square equal to R square.

This we write as R square so this R square and this R square cancelled so it becomes R by 2 equal to epsilon$_x$ plus epsilon$_y$ by 2 plus this is R which is ½ square root of epsilon$_x$ minus epsilon$_y$ square plus gamma$_{xy}$ square. If we take these two on the inside then this can be written as epsilon$_x$ plus epsilon$_y$ by 2 plus square root of epsilon$_x$ minus epsilon$_y$ by 2 square plus gamma$_{xy}$ by 2 square. This is the value of the maximum normal strain acting at that particular point.

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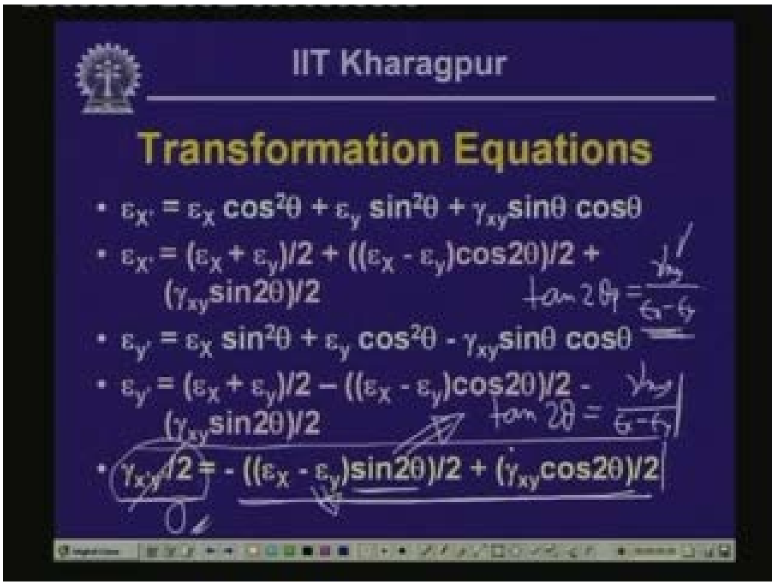

Here if you look into the shearing strain gamma x prime y prime is given as this, now we have already looked into the angle at which the maximum normal strain occurs is tan(2theta$_p$) equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$.

In this particular expression for shearing strain if we substitute for gamma x prime y prime equal to 0 this expression leads us to the value of tan 2theta equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$. This expression indicates that if this gamma x prime y prime is substituted as 0 then tan 2theta equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$. This indicates that the angle at which the maximum normal strain occur the shearing strain is 0 at that place.

In case of stresses the plane along which the maximum normal stress acts at that particular plane the shearing stress is 0. In similar line here where the orientation at which we get the maximum normal strain which we are calling as principal strain then in that element the shearing strain is 0 where maximum normal strain occurs.

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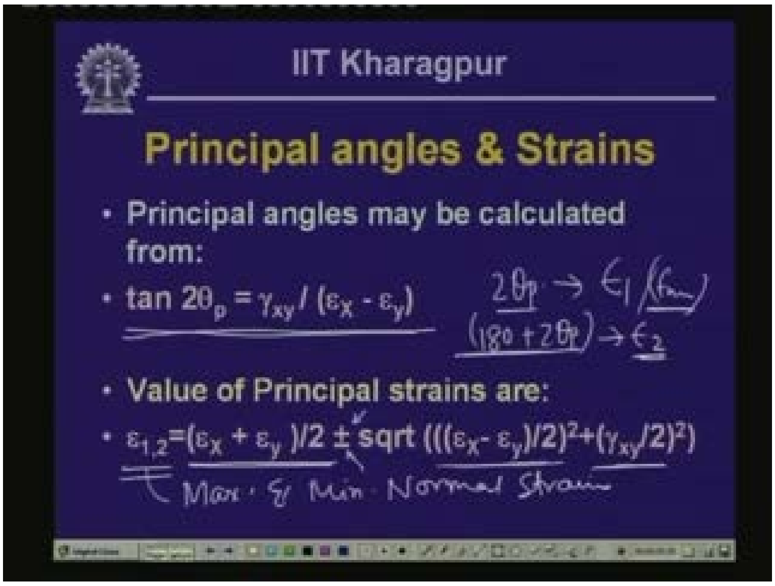

Now we have seen that the tan 2theta$_p$ equal to gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$ and corresponding to the angle of 2theta$_p$ we get the value of epsilon$_1$ with odd epsilon$_{max}$ we call this as epsilon$_{max}$. Now if we substitute as 180 degrees plus 2theta$_p$ as the other angle and substitute for cos and sin we can get the value of epsilon2. So the epsilon$_1$ or epsilon 2 the maximum and the minimum normal strain are given as epsilon$_x$ plus epsilon$_y$ by 2 plus minus, plus is for epsilon$_1$ and minus is for epsilon$_2$ as square root of epsilon$_x$ minus epsilon$_y$ by 2 square plus gamma$_{xy}$ by 2 square as we have derived just now.

The third question was how you will find the value of maximum shear strain at a point for plane strain condition. Now let us look into this particular answer that how we are going to compute the value of maximum shear strain.

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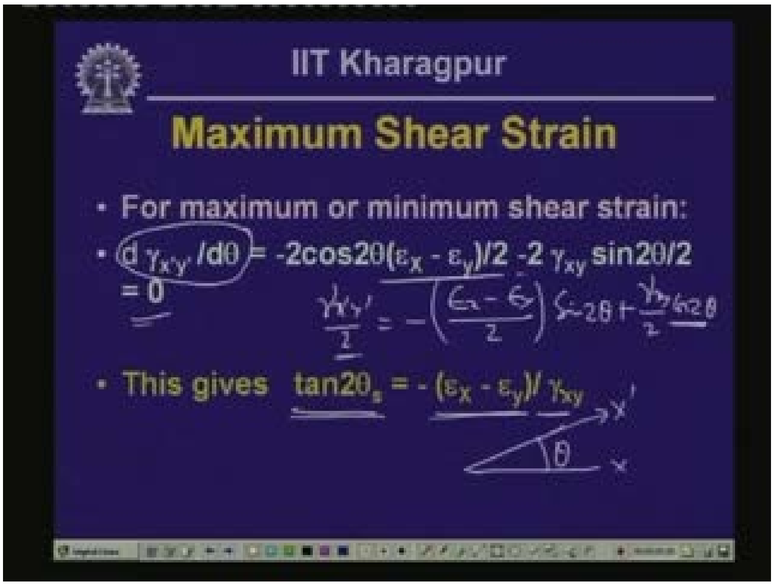

As we have seen the shearing strain at any orientation which is oriented at an angle of theta with reference to x axis which is our x prime gamma x prime y prime corresponding to that particular direction is epsilon$_x$ minus epsilon$_y$ by 2 cos 2theta plus gamma$_{xy}$ by 2 sin 2theta plus gamma$_{xy}$ by 2 cos 2theta.

Now if we take the derivative of that gamma x prime y prime by 2 equal to minus epsilon$_x$ minus epsilon$_y$ by 2 sin 2theta plus gamma$_{xy}$ by 2 cos 2theta. Now we take the derivative of this with respect to theta and eventually this gives as epsilon$_x$ minus epsilon$_y$ by 2 star minus 2 cos 2theta and minus sin 2theta minus 2gamma$_{xy}$ sin 2theta by 2. And if we say this as 0 dgamma x prime y prime by dtheta then we get the value of tan 2theta again from this expression which is equal to epsilon$_x$ minus epsilon$_y$ by gamma$_{xy}$.

In the previous case we evaluated the maximum normal strain and we had the value of angle two theta$_p$ as gamma$_{xy}$ by epsilon$_x$ minus epsilon$_y$ and in this particular case when we are taking the derivative of the shearing strain with respect to theta we are getting the value of the angle which we are calling as the angle for the shear tan 2theta$_s$ equal to epsilon$_x$ minus epsilon$_y$ by gamma$_{xy}$.

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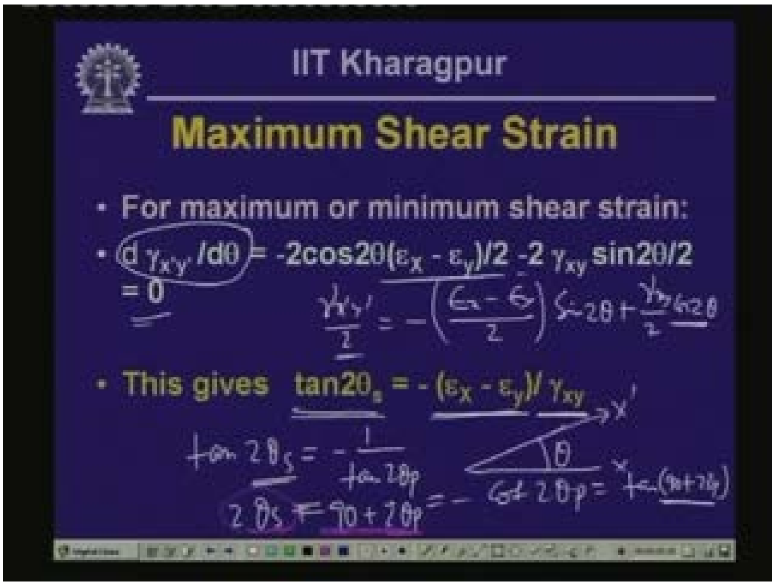

In general we can write tan 2theta equal to minus 1 by tan $2theta_p$ equal to minus $cos(2theta_p)$ and this we can write as tan 90 plus $2theta_p$. So 2theta equal to 90 plus $2theta_p$. Hence $theta_s$ equal to 45 degrees plus $theta_p$. This shows that the angle at which the shearing strain is maximum that orientation is 45 degree with respect to the orientation with reference to the principal axes strain. So the direction along which maximum normal strain occurs where we get epsilon 1 which is the maximum $epsilon_x$ if we orient by another 45 degrees the values of maximum and minimum shear strain we get will be corresponding to the axes which we will get.

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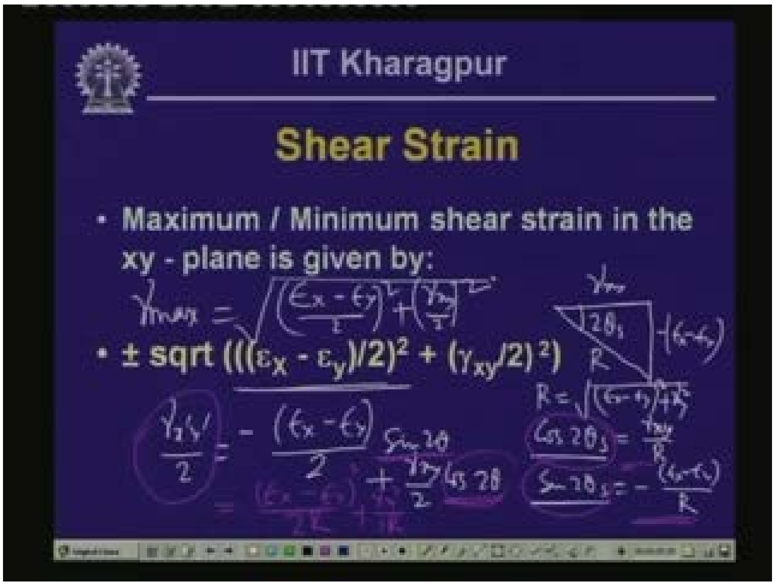

If we take the values of tan $2theta_s$ as we have obtained, now this is the angle $2theta_s$, this is the $gamma_{xy}$ and this is minus $epsilon_x$ minus $epsilon_y$. So this is at tan $2theta_s$ equal to minus $epsilon_x$ minus $epsilon_y$ by $gamma_{xy}$ and the value of R equal to square root of $epsilon_x$ minus $epsilon_y$ square plus $gamma_{xy}$ square. Hence the value of cos $2theta_s$ equal to

$gamma_{xy}$ by R and sin $2theta_s$ equal to minus $epsilon_x$ minus $epsilon_y$ by R. Now if we substitute the values of cos 2theta s and $sin2theta_s$ in the expression for gamma which is gamma x prime y prime by 2 equal to minus $epsilon_x$ minus $epsilon_y$ by 2 sin 2theta plus $gamma_{xy}$ by 2 cos 2theta.

Now in this particular expression if we substitute for cos 2theta and sin 2theta in terms of $gamma_{xy}$ by R and $epsilon_x$ minus $epsilon_y$ by R we get these as $epsilon_x$ minus $epsilon_y$ square by 2R plus $gamma_{xy}$ square by 2R and eventually again $epsilon_x$ minus $epsilon_y$ square plus $gamma_{xy}$ square is R square equal to R by 2 and hence the value of gammamax gamma x prime y prime when we are substituting in terms of cos $2theta_s$ and sin $2theta_s$ then the maximum value of $gamma_{max}$ equal to square root of $epsilon_x$ minus $epsilon_y$ by 2 square plus $gamma_{xy}$ by 2 square.

Similarly, in the case of the maximum normal strain we had two angles 180 degrees plus $2theta_p$. Here also we have two angles $2theta_s$ plus 180 plus $2theta_s$ and if we substitute corresponding values we will get the value of minimum shearing strain and the magnitude of the shearing strain maximum and minimum value is equal to $epsilon_x$ minus $epsilon_y$ by 2 square plus $gamma_{xy}$ by 2 square and maximum is plus of this and minimum is minus of this so this is plusminus square root of $epsilon_x$ minus $epsilon_y$ by 2 square plus $gamma_{xy}$ by 2 square. This is the value of the maximum and the minimum shearing strain corresponding to the plane strain condition.

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Now, as we have evaluated the strain at a point, if the reference axis is oriented at an angle of theta with reference to the rectangular axis system x and y we have seen how to compute the value of the maximum normal strain which we have called as the principal strain, we have see how to compute the value of the maximum shearing strain and the orientations of the principal strain and shearing strain through the angles $theta_p$ and $theta_s$.

As we did in case of stress evaluation at a point we had calculated the values of the maximum normal stress which we called as principal stress, we calculated the maximum shearing stresses and their orientations with reference to the rectangular axis system.

Now as we did in case of stress evaluations we had evaluated at any orientation through transformation equations, also we had evaluated stresses at a point at any orientation through Mohr's circle of stress. In case of strain as so long we have calculated the values at a point at different orientations the normal strains and shearing strains the maximum value of the normal strain the principal strain and the maximum shearing strain through the use of transformation equations. We can find out the strain at any orientation at a particular point using Mohr's circle of strain.

Now we will look into the concept of Mohr's circle of strain. Before we get into that, let us look into the sign conventions of different strain components. As we have seen that positive normal strain is basically the elongation per unit length. When we said that epsilon$_x$ is the strain in the x direction we referred to the extension or the elongation in the x direction. Likewise when we are talking about the positive strain in the y direction it is its elongation in the y direction.

When we are talking about the negative strain both in the x and y direction basically they are under compression. So, positive normal strain is the elongation per unit length and compressive normal strains are considered as negative. In case of shear strains, this is little different than the normal strain. They are positive when the angle decreases and negative when angle increases. Based on this particular definition of positive and negative shears we can say that positive shear stress cause positive shear strain and the vice versa.

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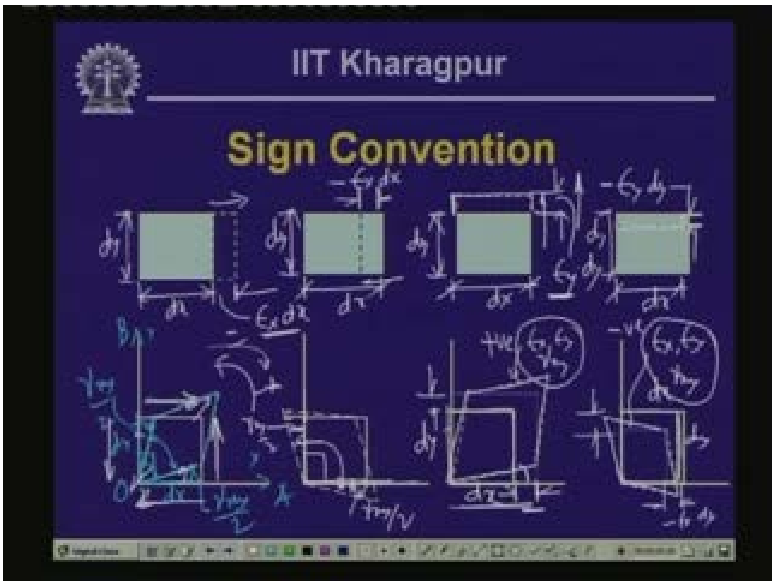

Now this will be probably clearer when you look into the figures. Now in this particular figure if we look into let us say that this is a small element of length dx at a point where the member is undergoing strain. Now this is the extension and if the strain in the direction is epsilon$_x$ we write this as epsilon$_x$dx the extension and this is the elongation and we call this is as positive strain.

Again in this particular element this is dx and this is dy. In this particular case the compression of this element is denoted by this dotted line and this is again epsilon$_x$dx but it is gone in the opposite direction or it has compressed. So here in the particular case epsilon$_x$ is negative.

In this particular case here this is again dx and this is dy. Now in the y direction the element is elongated and it has taken this particular position. So this extension is equals to epsilon$_y$dy and here since it is elongation we call this epsilon$_y$ as positive. In this particular figure again this element's length is dx and this is dy. Now here this part has come to this particular position wherein again the compression of this element is equal to epsilon$_y$dy but since it has undergone compression so epsilon$_y$ here is negative. That is how we define the positive and negative normal strain.

In case of shearing strain as we say that the angle decreases then this was the position x and y and from this x and y position this was the element dx and dy, here this is oriented in this particular form thereby the angle which was 90 degrees before is getting reduced. Hence this particular shearing strain is called as the positive shear strength and here in an anticlockwise direction it is moving in this form and here in a clockwise direction it is moving in this position and this we call as gamma$_{xy}$ by 2 and gamma$_{xy}$ by 2.

Now if you remember last time when we were computing the values of gamma x prime y prime which was the summation of two quantities alpha and beta and alpha was the orientation of x axis in the position OA and beta was the orientation of y axis along OB and the sum of alpha and beta gave us the value of the shearing strain gamma x prime y prime. Here the position from OA and OB is moving to this position and thereby the angle is decreasing and we call this shearing strain total gamma$_{xy}$ by 2 plus gamma$_{xy}$ by 2 which is gamma$_{xy}$ as the positive shearing strain and this is the position that this particular element is going to take because of the shearing strain.

Now if you remember, we discussed about positive shear. Now if we draw the shearing stress in this particular element, the direction of positive shear stress is this on this phase and on this phase it is downwards, on this phase it is in this direction and on this phase it is in this direction. Now this positive shear as we had taken in case of the construction of the Mohr's circle this upward shear along with this complementary shear causes an anticlockwise rotation and this shear along with the complementary on the other phase causes clockwise rotation. This anticlockwise rotation we had taken as positive and the clockwise one we had taken as negative.

This positive direction of shear tries to deform the element in this direction, this direction of positive direction tries to deform the element in this direction and that is why this is positive strain. In contrast if we look into the strain which is the negative one here the angle between the two axes OX and OY was in 90 degrees and now because of the strain it has increased. This increase is due to shearing strain and this we define as the negative shear strain so this is again gamma$_{xy}$ by 2 and this is gamma$_{xy}$ by 2.

Please note that in the previous case this moment was in the anticlockwise direction but in this particular figure it is in the clockwise direction and here it was in a clockwise direction and here it is in an anticlockwise direction. So anticlockwise movement in Mohr's circle we will consider as positive as before and clockwise direction we will consider as negative as before. This is the configuration which shows that this particular element which was of length dx and dy in the initial stage undergoes deformation in the x direction as given here, and it undergoes deformation in the y direction as given here and also it has shearing strain. So the final position after undergoing the $epsilon_x$ epsilony and $gamma_{xy}$ this is the position that it is going to take. So this particular configuration corresponds to $epsilon_x$ $epsilon_y$ $gamma_{xy}$ where all are positive.

In this particular figure again if we look into this particular element which is originally again of length dx and dy it undergoes strain but in the opposite direction or it gets compressed. So this is the negative $epsilon_x$ dx and again here in this particular direction this is the deformation it gets compressed which is $epsilon_y$dy and it has a negative shearing strain and because of that the angle increases and when we combine all three strains it takes the shape in this particular form. So this particular configuration is corresponding to $epsilon_x$ $epsilon_y$ $gamma_{xy}$ where all three quantities are negative. This is the sign convention which we follow and we will be following this convention while plotting the Mohr's circle for the strain. Exactly in the similar line we did in case of stress evaluation we will be computing the strain identically using the Mohr's circle of strain.

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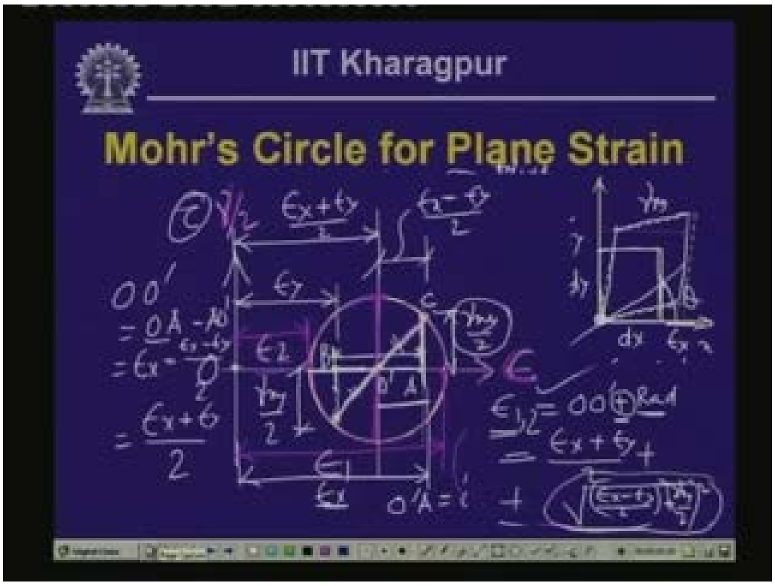

Now here let us say we have strain at a point or this is a small element where the lanes are dx and dy and it has undergone strain $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ and from this configuration you can make out that all quantities are positive; $epsilon_x$ is positive $epsilon_y$ is positive and $gamma_{xy}$ is positive. So if we take these quantities we can compare our relationship with the stresses and in case of stresses at any orientation x prime, it was written as a function of sigma x sigma y and tau_{xy}.

Now here in contrast to this the normal strain $epsilon_{xprime}$ we are writing in terms of $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ by 2. If you look into the expression for $epsilon_{xprime}$ and $sigma_{xprime}$ we will find $sigma_{xprime}$ equal to $sigma_x$ plus $sigma_y$ by 2 plus $sigma_x$ minus $sigma_y$ by 2 cos 2theta plus tau_{xy} sin 2theta. In case of $theta_{xprime}$ we have $theta_{xprime}$ equal to $theta_x$ plus $theta_y$ by 2 plus $epsilon_x$ minus $epsilon_y$ by 2 cos 2theta plus $gamma_{xy}$ by 2 sin 2theta. So in place of tau_{xy} we have $gamma_{xy}$ by 2, in place of $sigma_x$ we have $epsilon_x$ and in place of $sigma_y$ we have $epsilon_y$, otherwise the expression is identical.

Now let us find out how you compute the strain at any orientation with reference to this x and y at a particular point. If we know the strain at a particular point which is given by $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ we are interested to find out strain at that point which is at any orientation at an angle of theta in an anticlockwise form with respect to x so we need to find out epsilontheta and gamma also in that orientation in theta.

Last time in the x axes for stresses we had written down in sigma but here we write this strain for epsilon and the y axis instead of τ we write this as gamma by 2 axis and this is the origin. From here if we plot the value of $epsilon_x$ and $gamma_{xy}$ by 2 then we get one point in this plane which we call as Mohr's plane, also $epsilon_y$ and $gamma_{xy}$ by 2.

If you remember that $gamma_{xy}$ by 2 when we were looking into the sign convention from x it is oriented at an angle of $gamma_{xy}$ by 2 in the anticlockwise form which we are calling as positive and in the y direction it is oriented again by $gamma_{xy}$ by 2 in the clockwise direction which is the opposite to this. If we have plotted $gamma_{xy}$ in this particular position, now here we take $epsilon_y$ and $gamma_{xy}$. So this is the distance which is $epsilon_y$ and this is $gamma_{xy}$ by 2 and this point is at a distance of $epsilon_x$ from origin and this is $gamma_{xy}$ by 2. Now if we join these two points and when they cross the epsilon axis then it denotes the centre of the circle and this line is the diameter of the Mohr's circle.

If we look into this particular triangle and this particular triangle now this is equals to this, this is equals to this and this is equals to this so the distance from here to here this particular distance is divided equally by this particular point. From O to A this distance is $epsilon_x$, from O to B the distance is $epsilon_y$ so the distance AB equal to $epsilon_x$ minus $epsilon_y$ and the central point from O you call this as O prime so O primeA or O prime B, O prime A equal to O prime B equal to $epsilon_x$ minus $epsilon_y$ by 2.

Hence the distance of the centre from the origin, this particular distance is equal to OA minus AO prime the distance OO prime equal to OA minus AO prime. Now OA equal to $epsilon_x$, so this is equal to $epsilon_x$ minus AO prime equal to $epsilon_x$ minus $epsilon_y$ by 2 so this minus $epsilon_x$ minus $epsilon_y$ by 2 gives the value of $epsilon_x$ plus $epsilon_y$ by 2. So the distance from the origin to the centre is $epsilon_x$ plus $epsilon_y$ by 2.

Now if we look into this particular circle, if we take this O prime as the centre and OC as the radius and if we plot a circle eventually we get the circle which we call as the Mohr's circle for strain wherein the axis in the x direction corresponds epsilon and the axis in the y direction corresponds to gamma by 2. This is the point which represents the value of the maximum normal strain and this we designate as $epsilon_1$. This is the value of the minimum normal strain and this we designate as $epsilon_2$, also this is the value which gives us the maximum value of the shearing strain which is equal to the radius of this particular circle.

So, if we know $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ then we can compute the value of $epsilon_1$ then compute the value of the $epsilon_2$ and the shearing strain. Now from this you can make out the value of $epsilon_1$ which will be equal to the distance OO prime plus the radius. From this particular diagram O prime A and C from this particular triangle CA prime equal to $gamma_{xy}$ by 2 O prime A equal to $epsilon_x$ minus $epsilon_y$ by 2 this distance is $epsilon_x$ minus $epsilon_y$ by 2 so the distance CO prime which is the radius is equal to square root of $epsilon_x$ minus $epsilon_y$ by 2 square plus $gamma_{xy}$ by 2 square. So, the value $epsilon_1$ equal to OO prime which is $epsilon_x$ plus $epsilon_y$ by 2 plus square root of $epsilon_x$ minus $epsilon_y$ by 2 square plus $gamma_{xy}$ by 2 square. So this gives us the value of $epsilon_1$ which we have seen already through transformation equations and epsilon_2 will be OO prime minus the value of the radius. So once we substitute this as minus we will get the value of epsilon_2 and this is the radius which is equal to the plus and minus that gives us the value of the shearing strain. So using Mohr's circle of strain again we can compute the values of the principal strains epsilon_1, epsilon_2 and the maximum value of the shearing strain.

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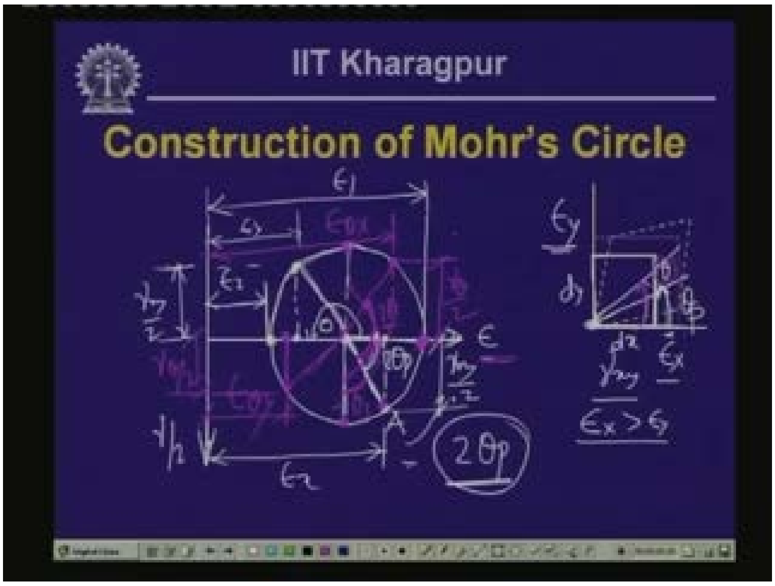

Let us look into how to plot the Mohr's circle of strain if we know the normal strain components epsilon_x epsilon_y and gamma_{xy}. For this particular element at a point we have the values of this element as dx and dy and the strains in the x direction as epsilon_x, in the y direction as epsilon_y and the shearing strain as gamma_{xy}. Now if we know these components our job will be evaluate the strains at this particular point the maximum normal strains, the maximum shearing strains and their orientations.

For example, if we have a set of rectangular axis which is oriented at an angle of theta with reference to x and y axes then what are the values of the strain corresponding to those orientations can also be evaluated using the Mohr's circle. Now let us look into that if we know the value of epsilon_x epsilon_y and gamma_{xy} at a particular point in a plane strain condition then how we can extract the other information using Mohr's circle of strain.

As we discussed in case of stresses regarding the direction of the axis, remember that you said this is the positive x direction in which we call this as epsilon. Now in the gamma by 2 we put the y in the downward direction as positive, the reason behind this was that if we are interested to evaluate the strains at a particular direction which is oriented at an angle of theta in the anticlockwise direction then we can keep the same orientation of angle anticlockwise in the Mohr's plane and to make this compatibility we consider the direction of gamma in the downward direction.

If we consider the gamma positive in the upward direction then this orientation of anticlockwise in the physical plane will be clockwise in case of the Mohr's plane. Now to keep the clarity between the physical one and the Mohr's plane we keep the axis direction as positive for epsilon on the right hand side and gamma by 2 in the downward y direction. Now assuming that the normal strain $epsilon_x$ is greater than the normal strain $epsilon_y$ we know $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ so we choose a point say $epsilon_x$ and gamma by 2 and this is a point on the Mohr's plane.

Another point we get corresponding to a value of $epsilon_y$ and correspondingly the gamma by 2 so this distance is $epsilon_x$ and this is $gamma_{xy}$ by 2, this is $epsilon_y$ and this is $gamma_{xy}$ by 2. If we join these two points it crosses the epsilon axis at this particular point and this is the diameter of the Mohr's circle and this is the centre of the Mohr's circle. Now with this if we plot the circle we get the Mohr's circle of strain. Now corresponding to this particular circle the value of the maximum normal strain is this which we have denoted as $epsilon_1$, the minimum normal strain is this which we have denoted as $epsilon_2$ and the value of the maximum shearing strain positive shear strain is this and value of negative shear strain maximum or the minimum shear strain is this.

Now this is the point which denotes the maximum normal strain $epsilon_1$ and this particular point is oriented at an angle of $2theta_p$ with reference to this, now this is our reference line where $epsilon_x$ $gamma_{xy}$ occurs so from this particular reference line let us call this as OA so if we orient by an angle of $2theta_p$ then we get the position for the maximum normal strain. Then if we orient from the position of the maximum normal strain by 180 degrees then we get the position for minimum normal strain. These are the values of $epsilon_1$ and $epsilon_2$ and this is the angle which gives us the value of maximum normal strain which is $2theta_p$ in the Mohr's plane and in the physical plane it will be $theta_p$.

Now if we are interested to compute the strain at any orientation which is at an angle of theta with reference to x then we plot an angle 2theta from this particular reference line and if we go in an anticlockwise direction, if this is the line which represents 2theta and this particular point gives us the value of a corresponding normal and the shearing strain. So this is value of epsilon the normal strain at theta and this is the corresponding value of gammatheta by 2. And if we go in the opposite direction diametrically opposite point this will give the value of $epsilon_y$ epsilontheta or $epsilontheta_x$ and $epsilontheta_y$ and correspondingly this value is gammatheta by 2.

In this particular plane this theta is oriented at an anticlockwise fashion with theta and here also in the Mohr's plane we go in the anticlockwise direction by 2theta to locate that particular orientation in the Mohr's plane and that is why we consider the direction of epsilon axis and gamma axis in this form so that this compatibility between the orientation is maintained. Also, if you note that this is the orientation along which the maximum and the minimum shear strains are occurring so they are with reference to this particular reference plane which will be at an angle, this is minus $theta_s$ and from here again if we orient by 180 degrees we will get this.

Now from $theta_p$ this is the position of the maximum normal strain and this is the position of the maximum shearing strain and these two make an angle of 90 degrees in the Mohr's plane and hence these two directions make an angle of 45 degrees in the physical plane and that we have seen through the transformation equations. So what you observe here is that the state of transformation equations as we have derived for evaluating strain components at any orientation with reference to the rectangular axis system xy corresponding to the plane strain condition $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ we can achieve similar results through this Mohr's circle of strain as we have observed in case of the Mohr's circle of stresses before.

Having looked into this let us look into the problem example. If we know the state of strain at a particular point then how do you compute the values of the other strains at a particular angle or the principal strain?
Now the state of plane strain at a point in a body is given by epsilon$_x$ as positive, epsilony as this which is positive and gamma$_{xy}$ also as positive. Now determine the strain components if the axes are oriented at an angle of 30 degrees with reference axis in anticlockwise direction and also determine the principal strains.

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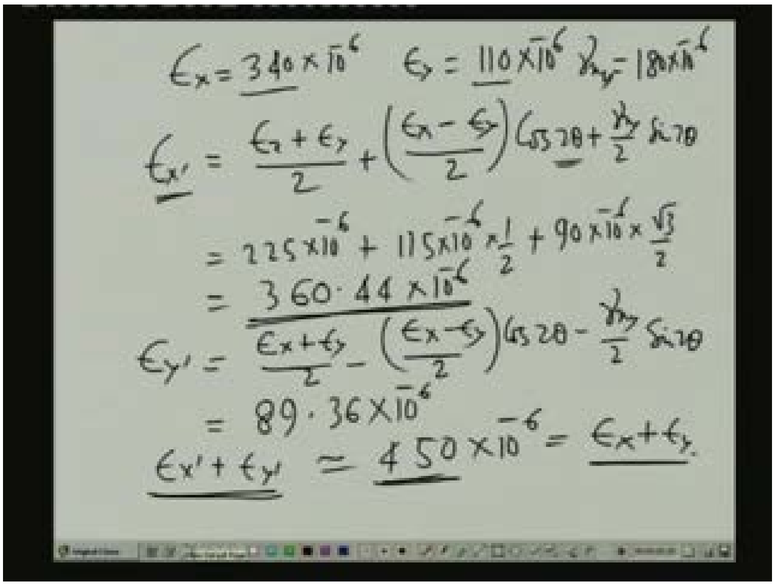

If we like to compute the values corresponding to the given values we have epsilon$_x$ equal to 340 into 10 to the power minus 6 we have epsilon$_y$ equal to 110 into 10 to the power minus 6 and we have gamma$_{xy}$ equal to 180 into 10 to the power minus 6. Now from the transformation equations we know epsilon$_x$ prime which is 30 degrees in this particular case is equal to epsilon$_x$ plus epsilon$_y$ by 2 plus (epsilon$_x$ plus epsilon$_y$ by 2) cos 2theta plus gamma$_{xy}$ by 2 sin 2theta.

Now theta equal to 30 degrees so 2theta equal to 60 degrees so if we substitute these values epsilon$_x$ equal to 340, epsilon$_y$ equal to 110 so this gives us a value of 225 into 10 to the power minus 6 plus 115 into 10 to the power minus 6 and cos 60 is ½ plus gamma$_{xy}$ by 2 equal to 90 into 10 to the power minus 6 and sin 60 is square root of 3 by 2 and this if you compute it comes as 360.44 into 10 to the power 6. So this is the value of epsilon$_x$ which is oriented at 30 degrees with reference to the x direction.

Corresponding epsilon$_y$ prime equal to epsilon$_x$ plus epsilon$_y$ by 2 minus epsilon$_x$ minus epsilon$_y$ by 2 cos 2theta minus gamma$_{xy}$ by 2 sin 2theta and if we substitute these values it will be 89.36 into 10 to the power minus 6. So, if we add the values of epsilon$_{xprime}$ plus epsilon$_{yprime}$ this will eventually be approximately equal to 450, it should ideally be 450 into 10 to the power minus 6 equal to epsilon$_x$ plus epsilon$_y$.

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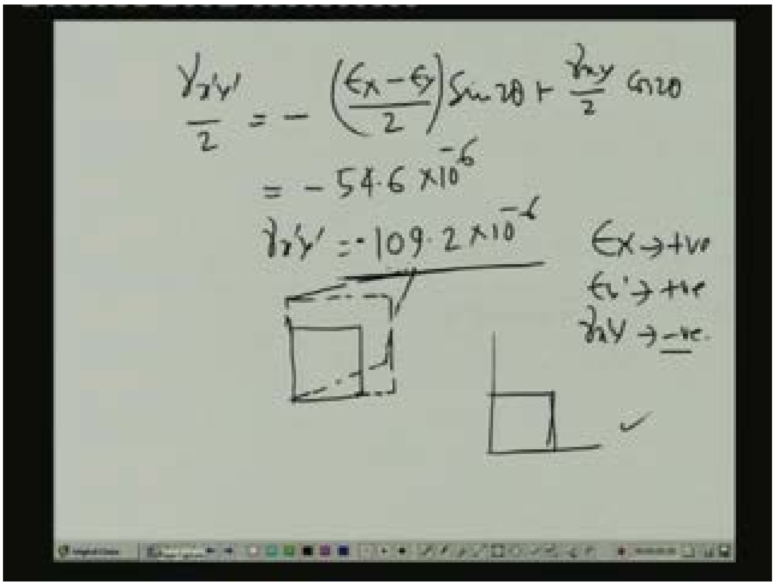

Now the value of gamma x prime y prime by 2 equal to minus epsilon$_x$ minus epsilon$_y$ by 2 sin 2theta plus gamma$_{xy}$ by 2 cos 2theta equal to minus 54.6 into 10 to the power minus 6 if we substitute the values which gives us the value of gamma x prime y prime as twice of this equal to 109.2 into 10 to the power minus 6 and this is negative. So initially we had the element which is in this form and we had a strain which is positive in the x direction, we had a strain which is positive in the y direction so this is the positive strain and then epsilongamma$_{xy}$ also is positive which means the angle is reducing which is in this form. This is the position of the element and in this particular case when we are computing the strain at 30 degrees we have epsilon$_{xprime}$ as positive we have epsilon$_{yprime}$ as positive we have gamma x prime y prime as negative and hence if we plot that you can get the configuration corresponding to that particular element.

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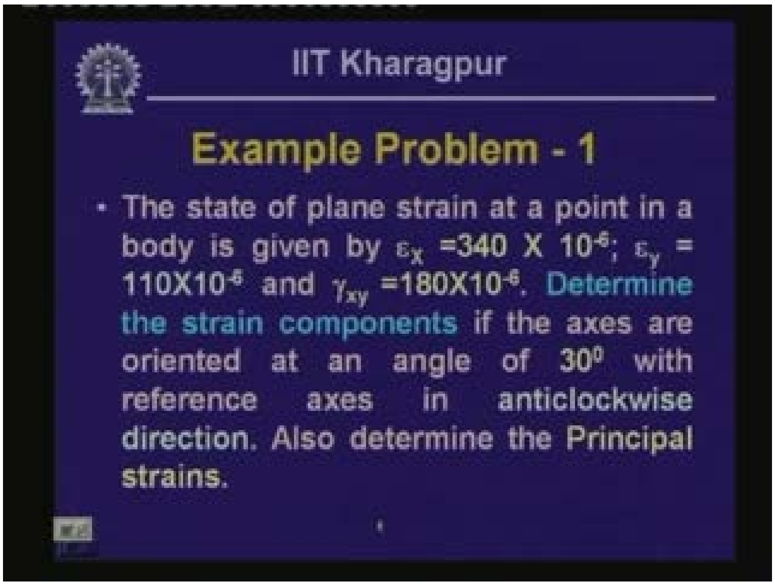

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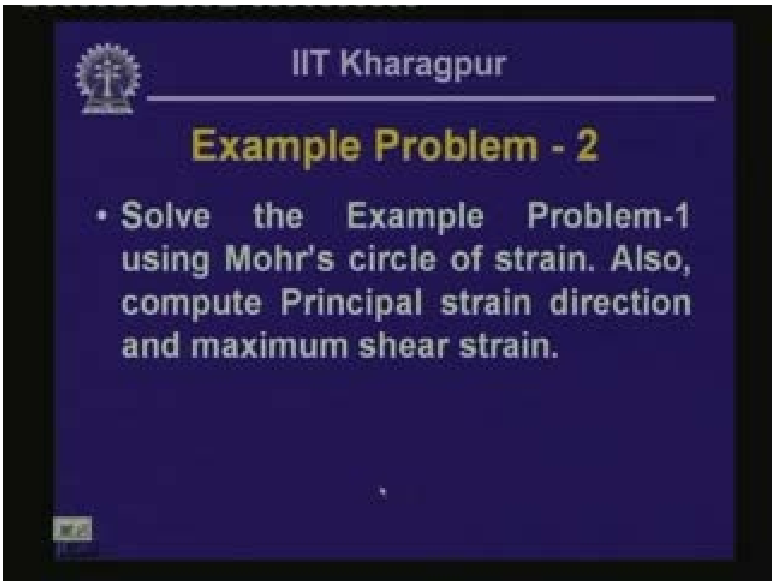

The next problem is that solve the same problem as we have given in one. But using Mohr's circle of strain compute the principal strain direction and maximum shear strain.

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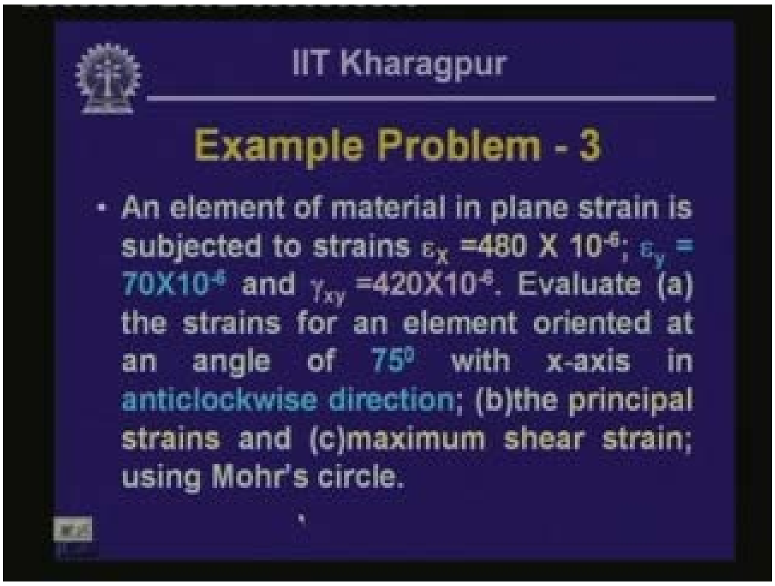

The next example is that an element of material in plane strain is subjected to strains of these. We got to evaluate the principal strain and the shear strains and also strain at an angle of 75 degrees.

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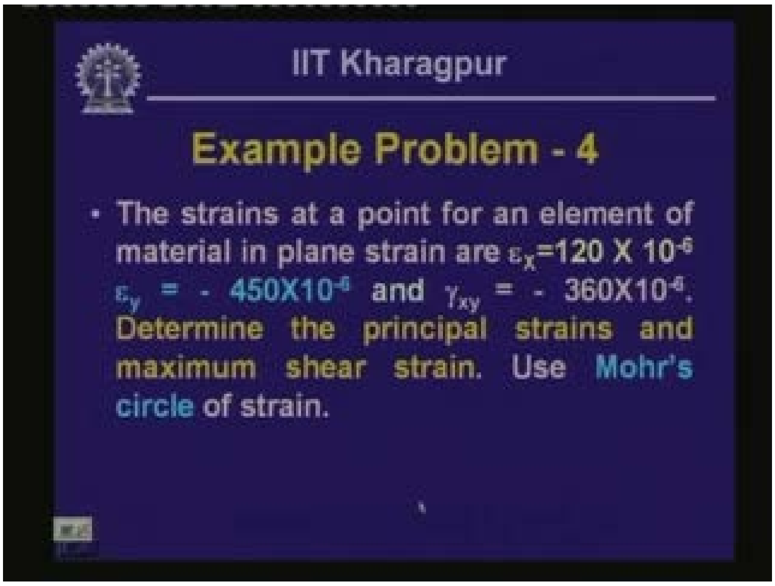

The fourth problem is that the strain at a point for an element is given by these values. We will have to determine the principal strain and maximum shear strain using Mohr's circle of strain.

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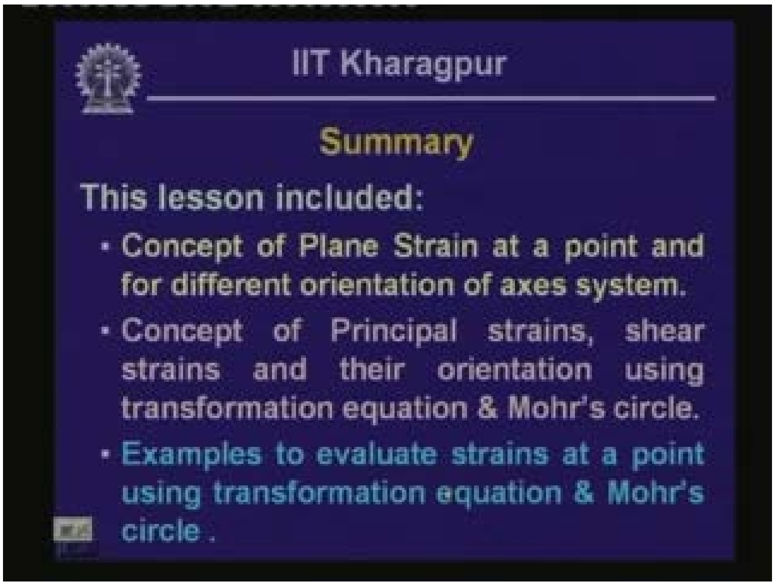

This particular lesson included the concept of Plane Strain at a point and for different orientation of axis system, concept of principal strains, shear strains and their orientations using transformation equation and Mohr's circle and examples to evaluate strains at a point using transformation equations and Mohr's circle.

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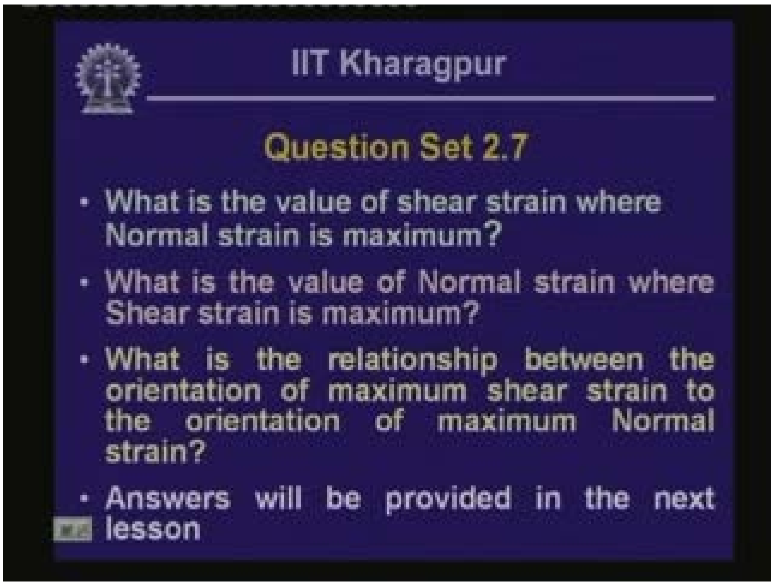

Questions:
What is the value of the shear strain where normal strain is at maximum?
What is the value of normal strain where shear is at maximum?
What is the relationship between the orientation maximum shear strain to the orientation of maximum normal strain.