Strength of Materials
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Lecture #12
Analysis of Strain – VI

Welcome to the 6th lesson on module 2 which is on analysis of strain.

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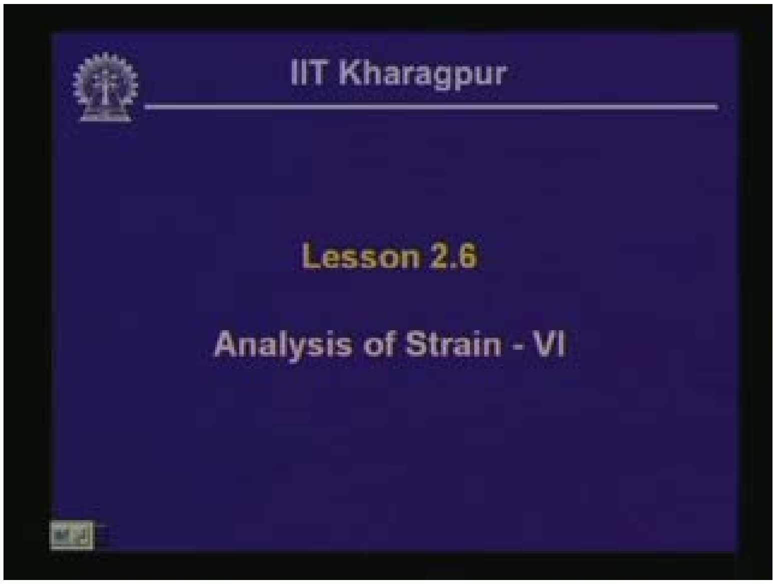

In the last lesson we have discussed some aspects of strain which occurs due to change in temperature material and we had looked into the aspects of lack of heat in a system and the stresses and strain which is generated thereby. And in this particular lesson we are going to discuss some more aspects of strain analysis.

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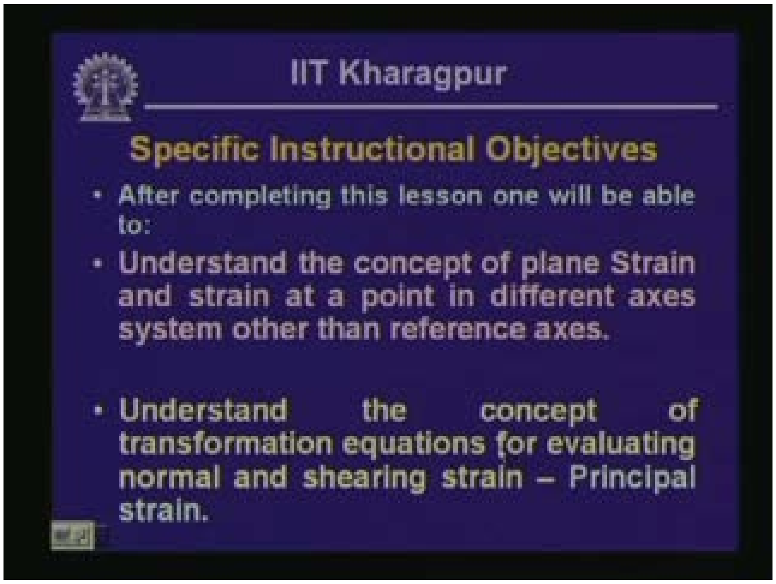

It is expected that once this particular lesson is completed one should be able to understand the concept of plane strain at a point in different axes system other than reference axes. In general when we refer to the strain, we refer in terms of epsilon_x, epsilon_y, gamma_{xy} in plane strain. If we orient the axes system other than the rectangular axes system which we consider then what will be the difference in strain value.

One should be able to understand the concept of transformation equations in fact for evaluating the strain at different orientation of axes, we need to evolve the equations which we term as transformation equations for evaluating normal and sharing strain. Thereby we will come across the maximum value of the normal strain which in analogous with the traces we will term them as principle strain. So we will look into how to evaluate the values of principle strain and at which orientation this principle strains act at that particular point.

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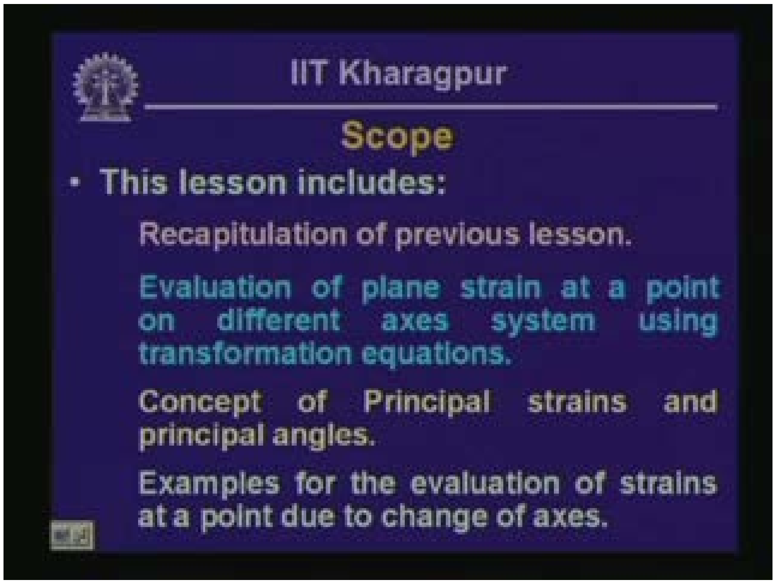

Hence this particular lesson includes recapitulation of previous lesson. In fact as we have done in the past, we will be discussing the questions and in the process we will recapitulate the aspects which we had looked into. Evaluation of plane strain at a point on different axes system using transformation equations and thereby the concept of principle strains and principle angles will generate from these equations of transformation and we look into some examples for the evaluation of strains at a point due to the different orientation of axes system from the rectangular axes system.

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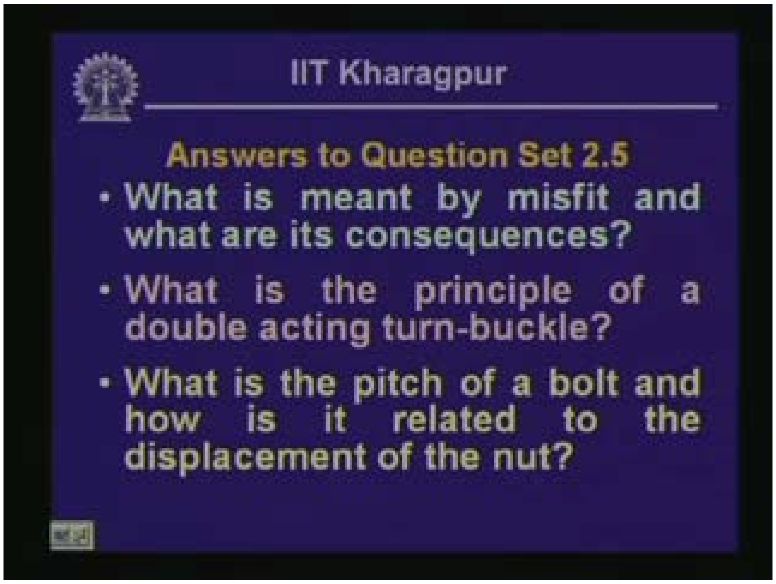

Let us look into the answers. The first question was what is meant by misfit and what are its consequences?

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In any system, if a particular member of a particular length is manufactured or fabricated in such a way that it varies from its original length then there is a problem of fitting that particular member into the whole structural system and that leads to some problems. If the system is a determinate one, the geometrical arrangement of the whole structural system will be in problem but there would not be any stresses or strains generated within that particular system. But if the structural system is an indeterminate one, then this length of the member which is not right which could be longer or shorter can induce strains and thereby stresses in such members and thereby if a member of a structure is fabricated different from its intended length then the member will introduce imperfection in the geometry of the structure.

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Now this is commonly referred as misfit or lack of fit now for a determinate system no strains or stresses are produced except that there will be change in geometry this aspect we have discussed last time.

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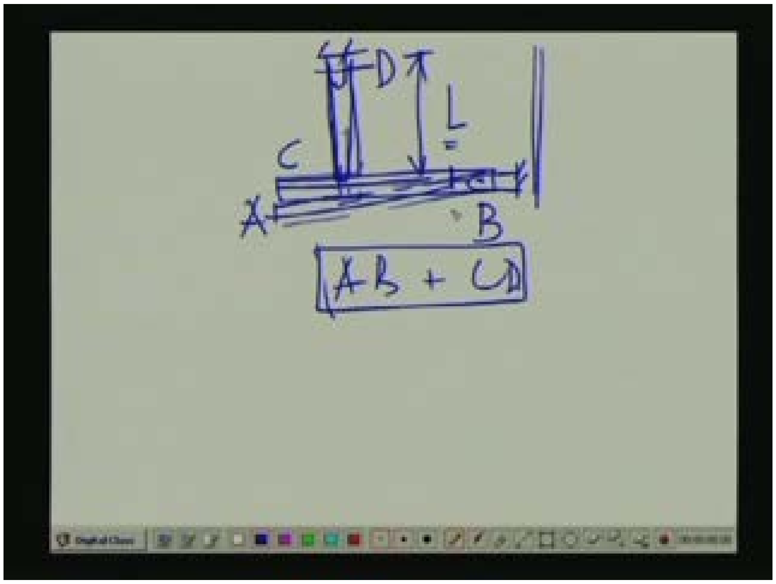

Let us say we have a bar which is supported at this particular point and we have one member here connected at this point and let us say this is member AB, this is member CD. If this particular member CD which is of length l, if this particular length differs; if it is shorter or longer than l then when we try to fit in this particular member CD into whole assembly there is a possibility that by changing the geometry of this particular system, for example if CD is longer than l then this particular member may undergo rotation over here to accommodate the longer length of CD. So, in the process the geometry of the whole system AB and CD which is consisting of these two members will be different than as expected because the geometry of this particular system looks like this bar will no longer be horizontal but it will be inclined. But in the process no strains or stresses will be introduced in this particular member.

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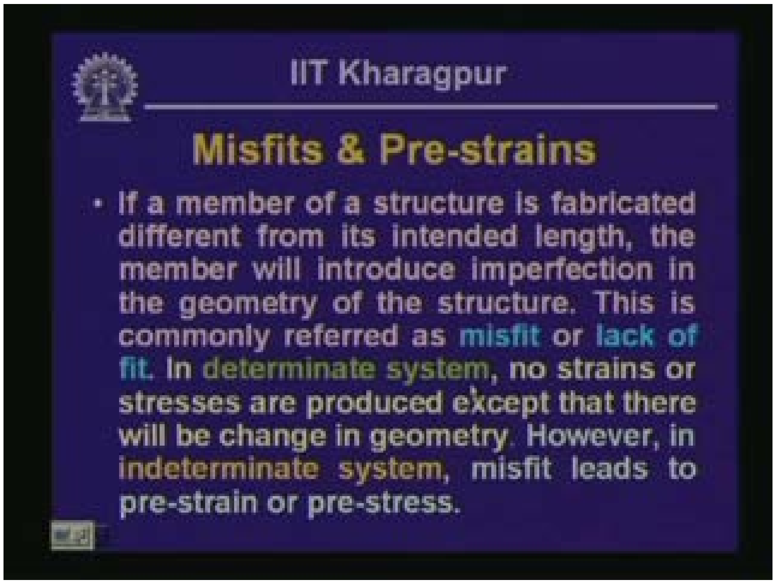

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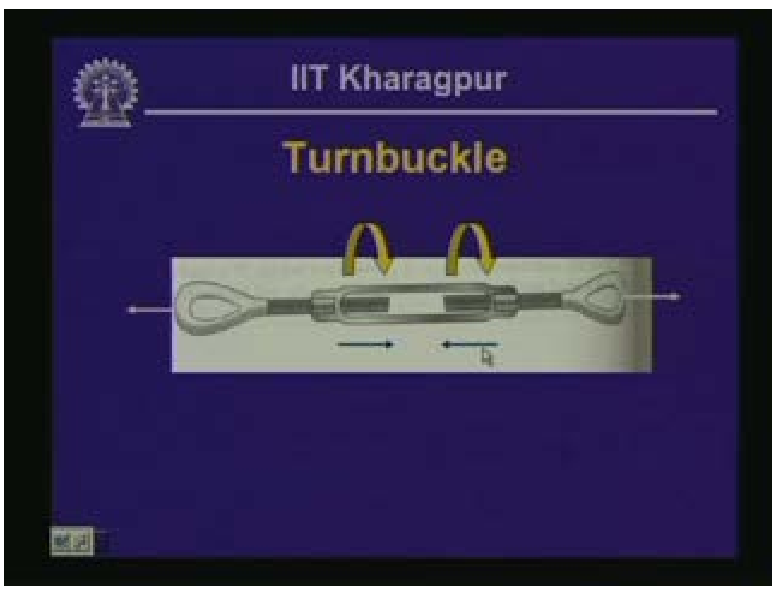

But if we have a system which is indeterminate, then this particular misfit will lead to pre-strain or pre-stress. The next question posed last time was, what is the principle of a double acting turnbuckle?

Now if we remember that we introduce the lack of heat by introducing or turning this kind of system which we call as turn-buckle. Here we have the threaded part in this particular zone and this particular part which is equivalent to or not in the whole system and if this whole assembly is rotated, we have one threaded part in this particular zone and another threaded part in this particular zone. On one side if the whole assembly is rotated on right hand side, we have a thread on the left hand side also we a have a thread. It is like the whole thing is rotated and there will be movement of the screw towards inside or if we are rotating on the other side the screw will move outside. Thereby there will be change in the length of the whole screw assembly system either inside or outside.

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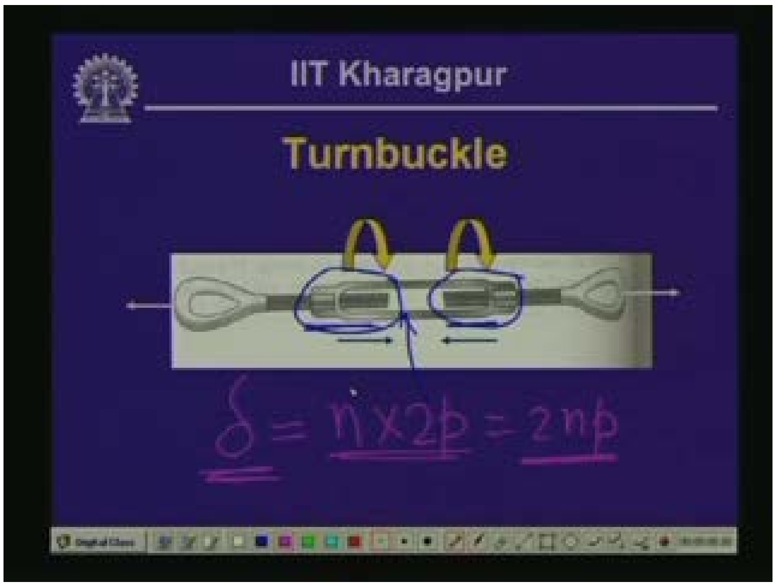

Since in one turn both the screws moving and as we had defined last time that by one turn of a knot or by one turn of this turn buckle, the movement of the knot for one 360 degree revolution, the movement we call as pitch which we had defined as p, now if we give a turn buckle by one full revolution then both the screws are moving by one pitch. So thereby there is a total movement of the whole turn buckle by an amount which is 2p.

If we have n number of turns given n into 2p will be the total movement of this turn buckle. So that deformation delta or the deformation which we are introducing by turning this turnbuckle will be equal to 2np. This is the system or the principle on which it works which can be introduced where we need to introduce the tensile or the axial pull or we like to reduce the length by turning the screws of the turnbuckle.

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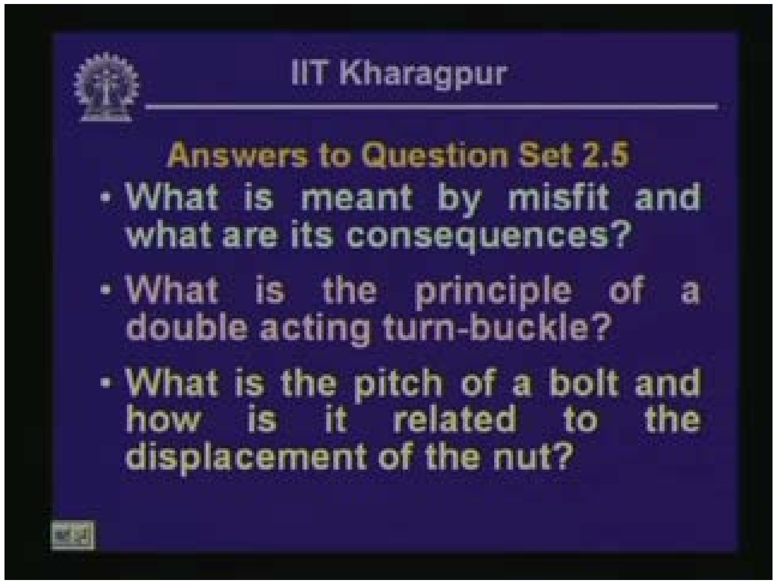

Now the third question similar to the second one is what the pitch of a bolt is and how is it related to the displacement of the nut?

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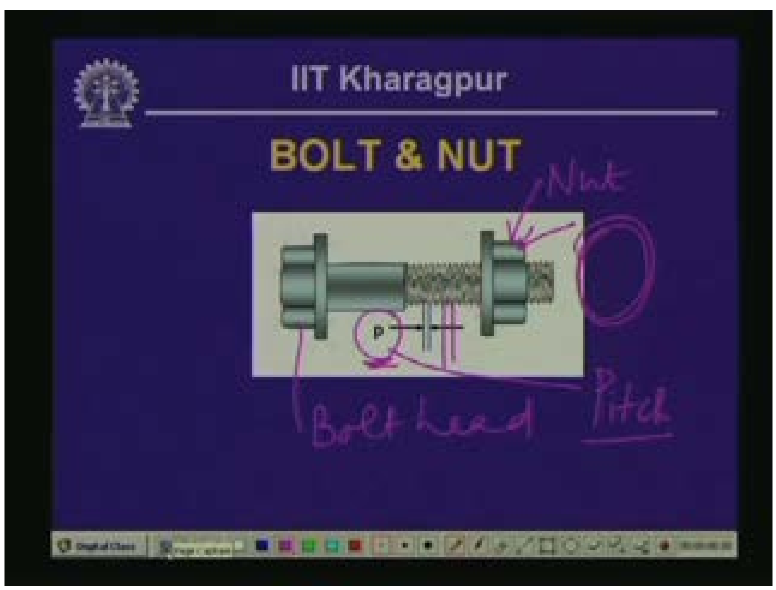

Let us look in to this nut bolt assembly. Here this is the nut and this is the bolt assembly, this is the bolt head. When we turn this nut, this nut is turned on for one full revolution of 360 degrees. The nut moves along the length of the threaded part by these two peaks and these consecutive two peaks in fact is denoted by letter p which we call as pitch. So one movement or one turn of the nut over the full circle moves the nut by a distance p and this is what is the displacement of the nut by giving a full circle turn along the threaded part of the bolt.

Now let us look into the strain that we are looking for at a particular point. We are defining the strain with reference to the rectangular axes system as normal strain in the x direction as epsilon$_x$, normal strain in the y direction as epsilon$_y$ and the normal strain in the z direction as epsilon$_z$.

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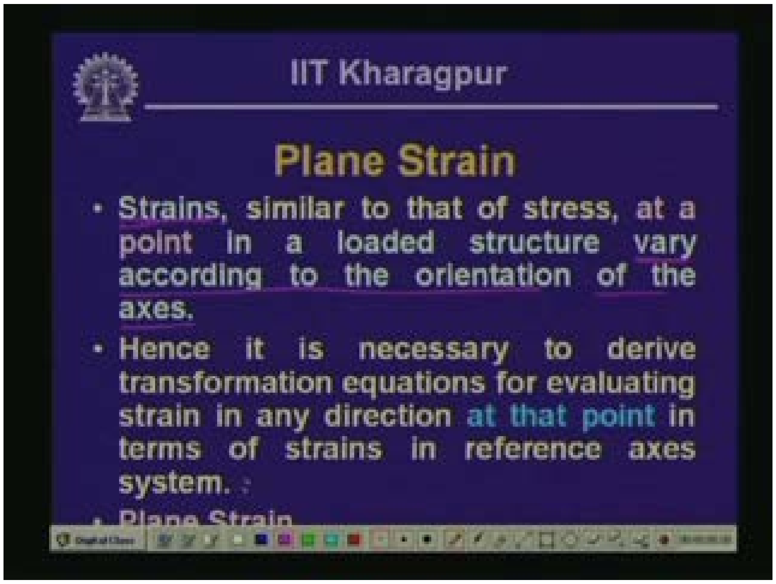

Now it is similar to the situation as we had in case of stresses. The strain at a particular point in a loaded structure varies according to the orientation of the axes. If we have a rectangular axes system then what stress we are expecting?

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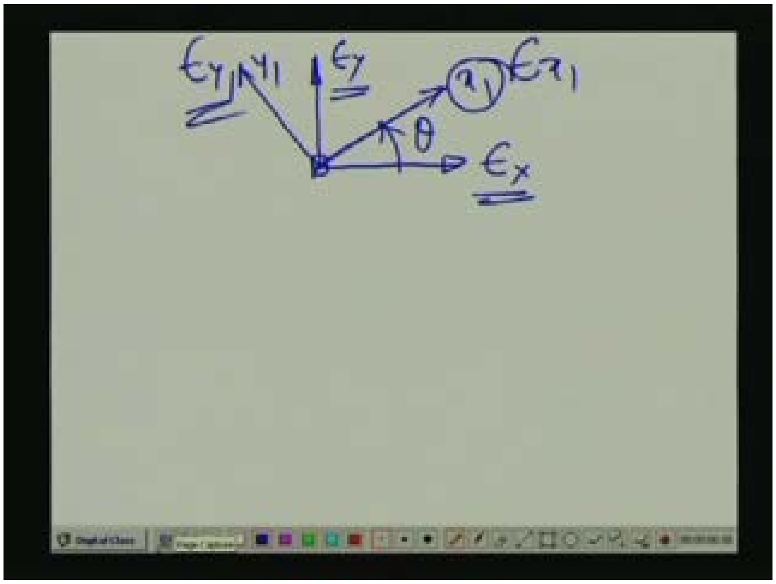

For example, we are interested to evaluate the stress strain at this particular point. Now at this particular point the strain which we are referring with reference to the rectangular axes system epsilon_x and epsilon_y, if we try to find out the strain at this point at a different orientation of the axes; let us call this as axes x_1 y_1 now the strain corresponding to this as epsilon_{x1} and epsilon_{y1} will be different from epsilon_x and epsilon_y. Our objective is to find out the strain along this orientation or in the orientation of the axes which could be in the general form as theta at any orientation with reference to x and y plane or x and y reference axes so that we can compute the strain at that particular point at different orientation.

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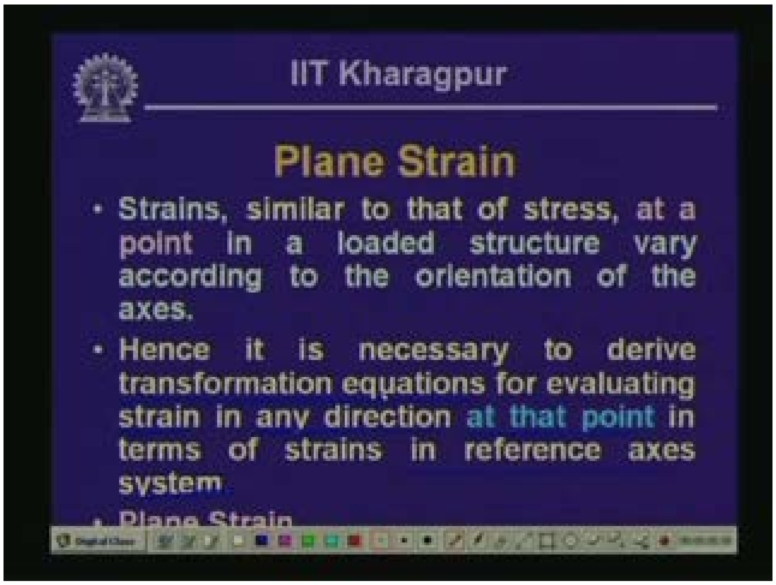

Hence it is necessary to derive the transformation equations for evaluating strain in any direction at that particular point in terms of strains in reference axes system. Before we go in to the evaluation of transformation equations let us look into what we really meant by the plane strain condition.

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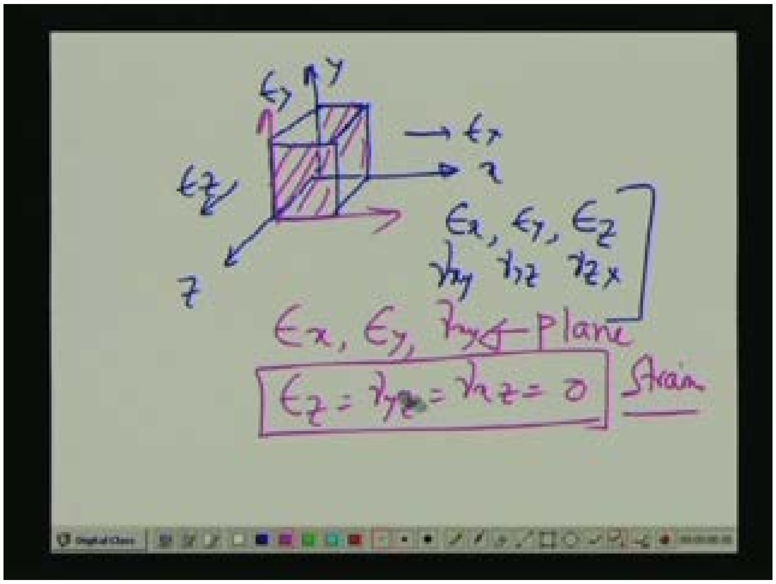

At a particular point the strains are represented in this particular form. This is the small part in a material and these are the reference axes x, y and z. The strains acting on this in the x direction is epsilon$_x$, epsilon$_y$ as the normal strain and epsilon$_z$ in the z direction. We have the strain components which are epsilon$_x$ epsilon$_y$ and epsilon$_z$ the normal strains and the sharing strains are gamma$_{xy}$, gamma$_{yz}$ and gamma$_{zx}$. So these are the six strain components which will be acting at this particular point with reference to this rectangular axes system.

Now if we like to represent the strain on a plane which is the z plane having x and y axes the strain that will be existing will be epsilon$_x$ epsilon$_y$ and gamma$_{xy}$ and rest of the strains like epsilon$_z$, gamma$_{yz}$ and gamma$_{xz}$ are 0. So this particular condition of strain we term as plane strain condition. That means these strains will be acting in this particular plane either in the front part or at the backside which is represented by this x and y axes system which is basically the z plane.

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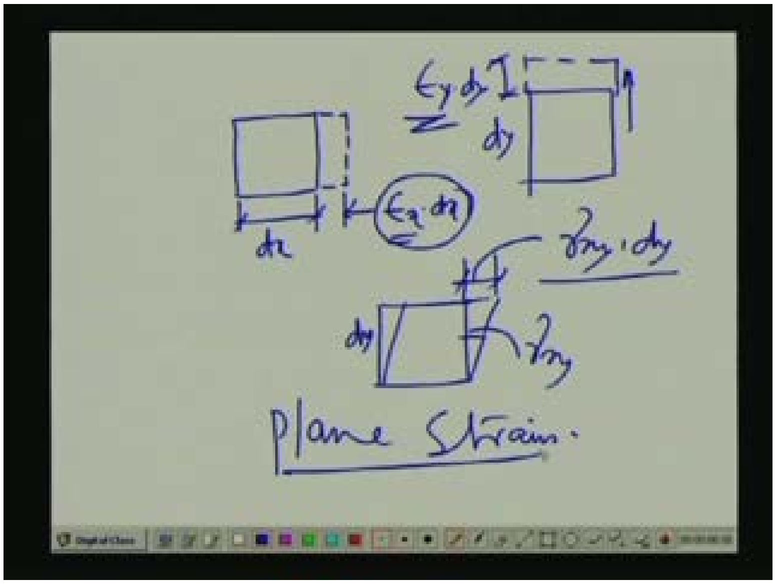

Now in this plane if we like to find out the strains corresponding to three different directions, if we consider the material in this particular plane, in the x direction when it is stretching it is undergoing deformation in this particular direction and if we say this length as dx, then it will undergo stretching in this particular direction which is $epsilon_x$ dx where $epsilon_x$ being the strain and dx being the original length then the elongation is $epsilon_x$ dx or it can have stretching in the y direction this length is dy so this stretching is $epsilon_y$ dy where $epsilon_y$ is the strain in the y direction. Or if we have the shearing strain which is $gamma_{xy}$ then the extension is equal to $gamma_{xy}$(dy) if we call this depth as dy. So these are the extensional aspects when we talk in terms of the strain in the plane, these are related to the plane strain conditions.

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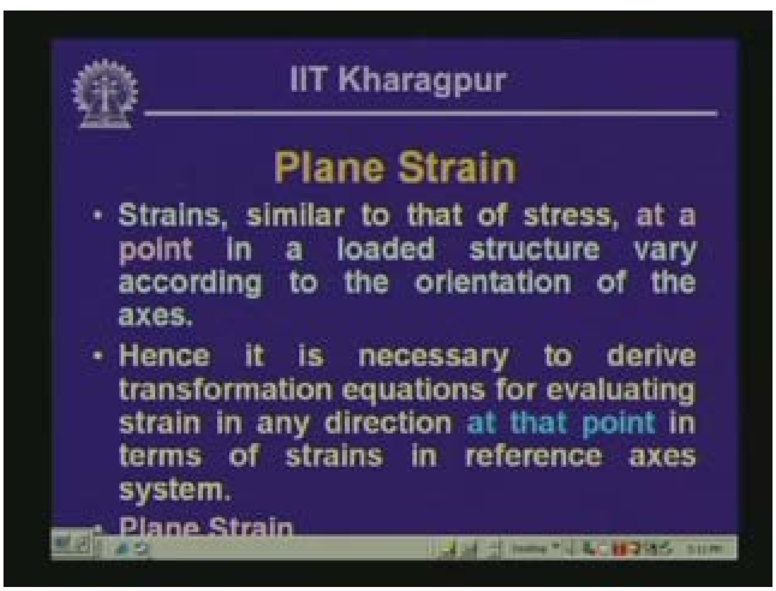

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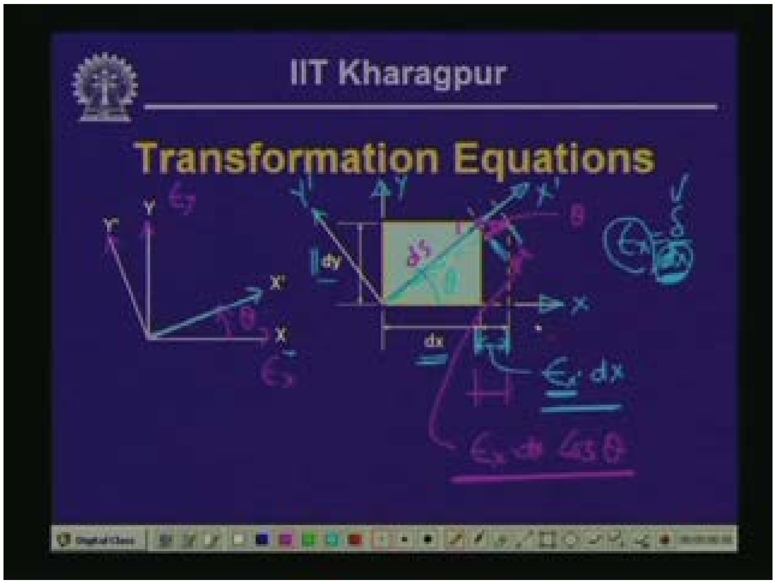

Now let us look into the transformations. We are interested to evaluate what will be the strain values at that particular point if we orient the axes with reference to our rectangular axes system. If it is oriented at an angle of theta with reference to x axis then if we are interested to find out the strains we should look into how to compute it and what are the corresponding equations which we term generally as transformation equation. Now x and y are the reference rectangular axis and the strain corresponding to these directions are epsilon_x and epsilon_y as we are defining. We are interested to evaluate the strain at this point with reference to the axes system x dashed and y dashed which is at an angle of theta with reference to x axes.

Let us consider different cases which are corresponding to the plane strain situation in which this is the length dx, this is length dy. Now we consider that the axis x prime lies along the diagonal of this particular element which we have considered of length dx and dy. Thereby this diagonal direction is our x prime axis and perpendicular to that is our y prime axis, this is the rectangular x and y direction. When this particular element is getting stretched in the x direction the extension in the x direction is given by this particular length which is equal to $\text{epsilon}_x(d_x)$ and epsilon_x is the strain in the x direction.

As we know strain is equal to δ by dx delta by the original length so original length multiplied by epsilon_x epsilon_x will give you the extension delta. If we are interested to know how much extension this diagonal has undergone with reference to this epsilon_x, this particular part is the diagonal after it has undergone extension.

Now this particular angle is theta as we have defined that x prime axis is lying along the diagonal. Hence x prime axis is making an angle of theta with x so this particular angle is theta. If this is the horizontal distance epsilon_x dx which is equal to this, and since this angle is theta, this is theta and this particular one also we are calling as theta then the extension of the diagonal is horizontal distance times cos theta which is equal to; so this particular extension is given as epsilon_x dx cos theta. This is the extension of the diagonal. Let us call that this particular diagonal is of length ds so originally it had a length of ds and it is undergoing an extension which is in terms of $epsilon_x$ which is $epsilon_x$dx cos theta because of the extension in the x-direction.

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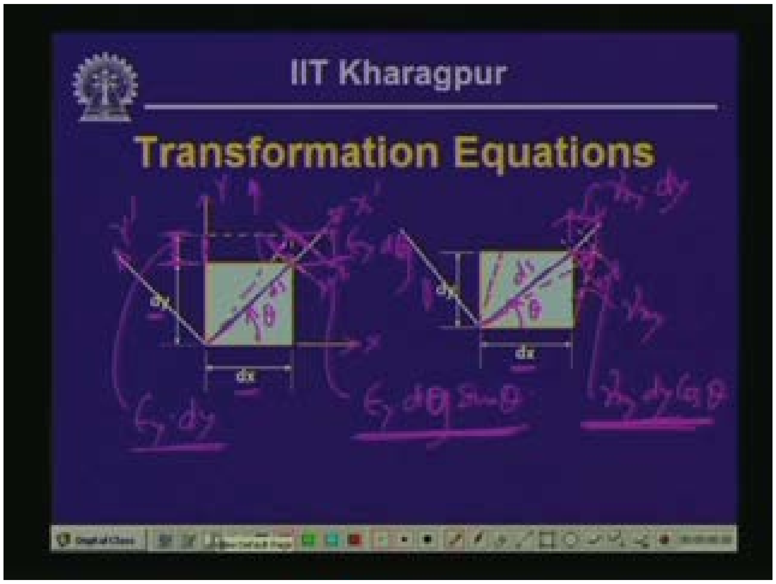

Now let us look into what happens if it undergoes strain in the y direction and thereby the shearing strain. When this particular element undergoes the strain having length dx and dy and it is undergoing strain in the y direction so this is x axis. As we had defined this is y axis and as for our definition we said that the diagonal of this particular element is representing the x prime axis and perpendicular to that is the y prime axis. This is the angle theta and here in the y direction it is undergoing extension thereby the extension which is given by this particular length from its original form length is equals to $epsilon_y d_y$ so $epsilon_y d_y$ $epsilon_y$ is the extension of this element and because of this stretching the diagonal which was originally of length ds now, is getting changed. This is the change length of the diagonal and extension is given by this particular part of the diagonal which is equals to now this angle theta, thereby this particular angle also is theta so this is theta. Now this horizontal extension is given by $epsilon_{yd}$ $epsilon_y$ theta hence the extension which is happening is equals to $epsilon_{yd}$ $epsilon_y$ theta sin component of this which is equal to this particular extension and this extension is given by $epsilon_{y\ dy}$ sin theta. $Epsilon_y$theta thetaepsilon$_y$ theta $epsilon_y d_y$ $epsilon_y$ is the vertical extension and the component along the diagonal direction is $epsilon_y d_y epsilon_y$ sin theta.

Similarly, the element which is of size dx dy undergoing a sharing strain and because of sharing strain it is having the sharing strain angle which is $gamma_{xy}$. Thereby the diagonal which was originally of length ds is undergoing extension which is of this particular form and thereby the extension of this, is given by this particular length and again this angle is theta as we have defined theta hence this is theta. So the extension which is this stage is given by now this particular extension because of shearing strain is equals to $gamma_{xy}.d_y$ and the extension in the diagonal is equals to is given by $gamma_{xy}$ d_y cosine theta. So we have three extensions of the diagonals obtained now in the previous case where it was getting stretched in the x direction. We have the extension as $epsilon_x d_x$ $epsilon_x$ cos theta. In case of stretching in the y direction or strain in the y direction we have extension of the diagonal as epsilon$_y$d$_y$ epsilon$_y$ sin theta and the shearing strain the stretching of the diagonal is equal to gamma x y d y cos theta. Hence the total stretching of the diagonal if we look in to the total stretching of the diagonal.

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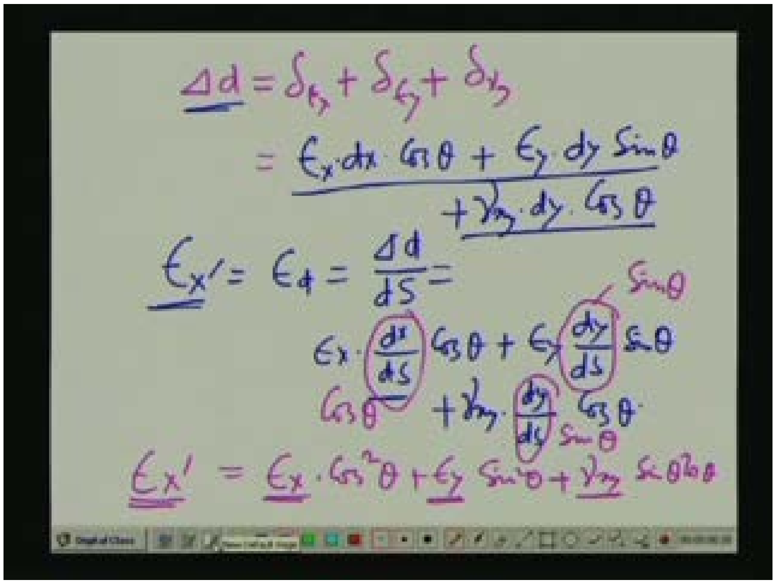

Let us call that as delta d, this is given by the extension due to epsilon$_x$ plus epsilon$_x$ extension in the diagonal due to epsilon$_y$ plus epsilon$_y$ extension due to gamma xy. These are given as epsilon$_x$d$_x$ epsilon$_x$ cos theta plus epsilon$_y$d$_y$ epsilon$_y$ sin theta is equal to gamma$_{xy}$ dy cos theta. So these are the three extensions we have obtained now if we propose that all three cases because of the plane strain condition, then this is the extension delta d we get for the diagonal one hence the strain in the diagonal direction which is that of epsilon$_x$ prime.

Now epsilon$_x$ prime are the strain in the x prime direction is nothing but the strain in the diagonal is equals to the extension of the diagonal divided by its original length and this is nothing but equals to d s divided by whole this expression which is equals to epsilon$_x$.d$_x$ by d$_s$.epsilon$_x$ cos theta epsilon$_y$ plus epsilon$_y$d$_y$ by d$_s$ sin theta plus gamma x$_y$d$_y$ by d$_s$ cos theta.

Now if I look into the diagram from which we have evaluated this extension dx by ds is again cos theta dy by ds is nothing but sin theta. Hence the whole thing gives us; this is equal to epsilon$_x$ cos square theta epsilon$_y$ plus epsilon$_y$.sin square theta plus gamma$_{xy}$ sin theta cos theta. So epsilon$_x$ prime is the strain in the x direction. In terms of the strain epsilon$_x$ epsilon$_y$ and gamma$_{xy}$ is given as epsilon$_x$ cos square theta epsilon$_y$ plus epsilon$_y$ sin square theta plus gamma sin theta cos theta. Now this expression called epsilon prime further can be simplified now writing cos square theta as in terms of cos 2 theta.

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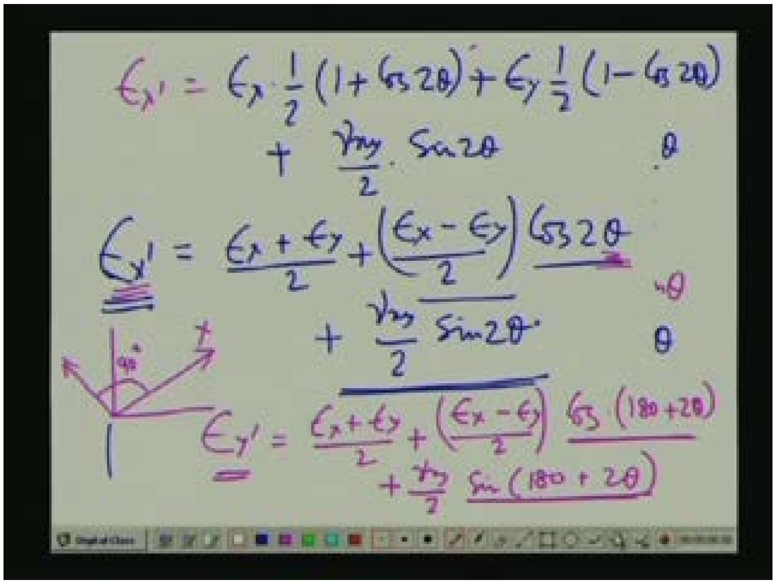

Let us write $epsilon_x$ prime as equal to $epsilon_x$ in place of cos square theta write this as ½ into 1 plus cos 2theta plus $epsilon_y$(1 by 2) into 1 minus cos 2theta plus $gamma_{xy}$ by 2. Now twice sin theta cos theta gives sin 2theta so this is equal to $epsilon_x$ plus $epsilon_y$ by 2 plus $epsilon_x$ minus $epsilon_y$ by 2(cos 2theta plus gamma xy by 2 sin 2theta so this is the expression for $epsilon_x$ prime or the strain in the x prime direction in terms of the strain $epsilon_x$ $epsilon_y$ and gamma xy or $epsilon_x$ plus $epsilon_y$ by 2 plus $epsilon_x$ minus $epsilon_y$ by 2 cos 2theta plus gamma xy by 2 sin 2theta.

Similarly, we can compute the value of $epsilon_y$ prime from the expression of $epsilon_x$ now since the y prime axis is at 90 degrees with reference to x prime this is being 90 degrees. If we place theta as theta plus 90 we can get the strain in the one y prime direction so it is $epsilon_y$ prime. We can write as $epsilon_x$ plus $epsilon_y$ by 2 plus $epsilon_x$ minus $epsilon_y$ by 2 cos of 2theta, theta is theta plus 90 which is 180 plus 2theta plus $gamma_{xy}$ by 2sin of 180 plus 2theta. Now this will give us the value of strain with the y direction and cos 180 plus 2theta is equal to minus cos theta and sin 180 plus 2theta is equal to minus sin 2theta.

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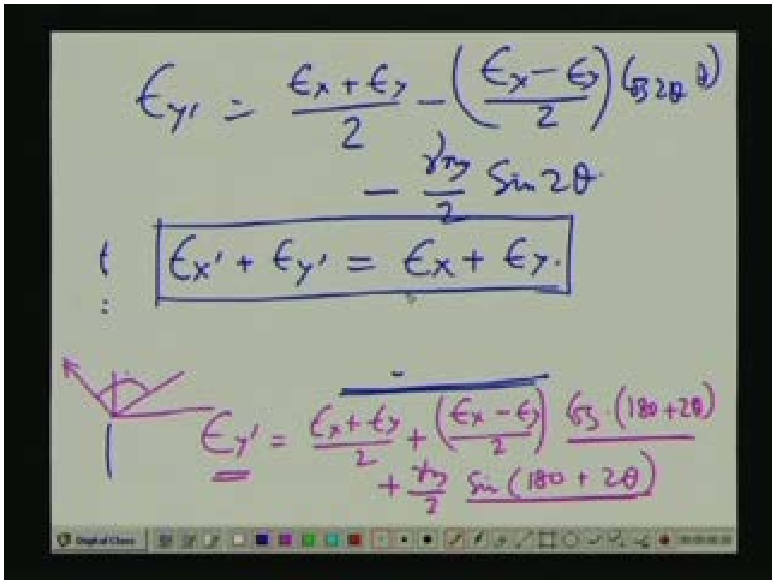

So the expression called epsilon_y prime is equal to then epsilon_x plus epsilon_y by 2 minus epsilon_x minus epsilon_y by 2cos 2theta minus gamma xy by 2sin 2theta and interestingly, if we note here that if we add this epsilon_x prime plus epsilon_y prime this gives us the value of the equation as epsilon_x plus epsilon_y now because other terms gets cancel hence epsilon_x prime is equal to epsilon_y prime.

If you remember, when we had evaluated the stresses at a point with reference to different orientation of axes we had seen that the total stress values remain constant. The values of sigma x plus sigma y is equal to sigma x prime plus sigma y prime if it is oriented at a different angle and the same thing holds good for the strain as well as the strain with reference to rectangular axes system epsilon_x plus epsilon_y is equal to epsilon_x prime plus epsilon_y prime with reference to x prime and y prime axes.

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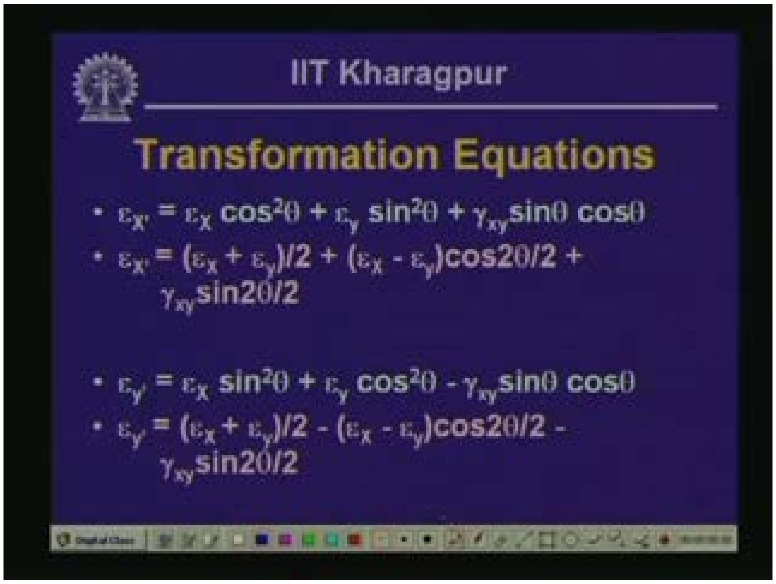

These are the values which we have just now seen that epsilon$_x$ prime is equal to epsilon$_x$ cos square theta epsilon$_y$ sin square theta plus and gamma$_{xy}$ sin theta cos theta and epsilon$_x$ prime in terms of cos 2theta and sin 2theta given as epsilon$_x$ plus epsilon$_y$ by 2 plus epsilon$_x$ minus epsilon$_y$ cos 2theta by 2 and gamma$_{xy}$ sin 2theta by 2 and thereby epsilon$_y$ prime is equal to epsilon$_x$ sin square theta epsilon$_y$ cos square theta minus gamma$_{xy}$ sin theta cos theta which eventually comes as epsilon$_x$ plus epsilon$_y$ by 2 minus epsilon$_x$ minus epsilon$_y$(cos 2theta) by 2 minus gamma$_{xy}$ sin 2theta by 2 so these are the values of the strain with reference to x prime and the y prime axis.

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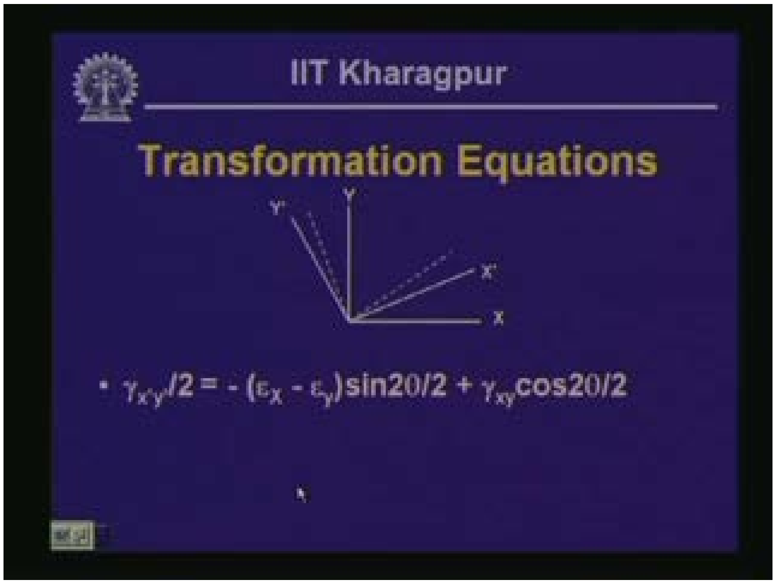

Now what we need to do is, since we said that, under plane strain condition we have three strain components which are epsilon$_x$ epsilon$_y$ and gamma$_{xy}$ so under the changed or the oriented axes system with reference to x prime and y prime we will have the strains which are equivalent to the plane strain situation which are epsilon$_x$ prime epsilon$_y$ prime and gamma x prime y prime and gamma x prime y prime we call as the shearing strain with reference to x prime y prime axis. Now, let us evaluate the value of the shearing strain with reference to x prime y prime axis.

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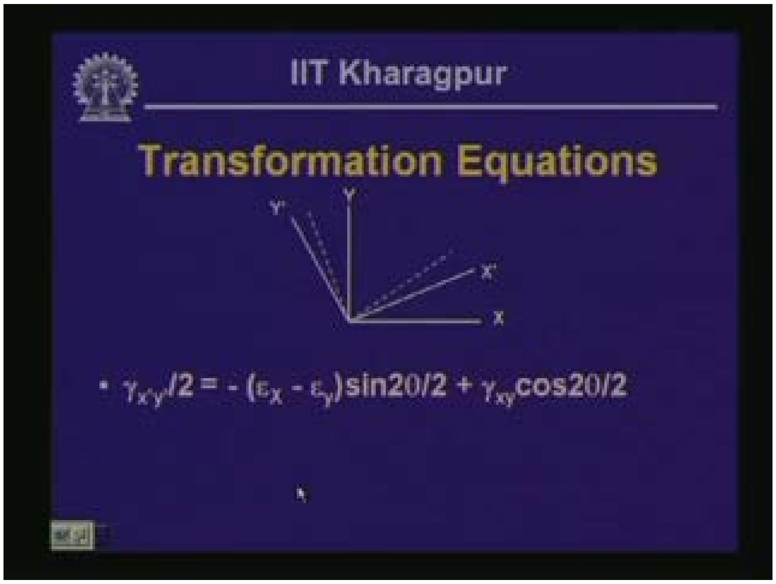

As we know that sharing strain is the angle which is changed from its original 90 degree position. If we have two reference axes which is at 90 degrees with each other the shearing strain causes the change in this 90 degrees angle.

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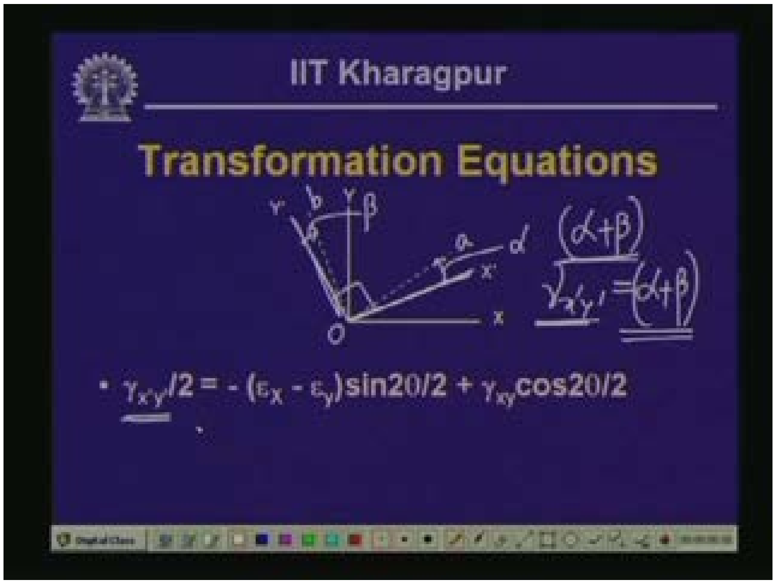

So here in this particular case when we try to change, evaluate the shearing strain gamma x prime y prime let us assume that we had a line oa which was originally lying along this o x prime now and a line ob originally, which was lying on the line oy now because of the shearing strain. They have undergone a change and let us call this changed angle as alpha and this changed angle as beta, thereby the total change is alpha plus beta from the original 90 degrees of ox prime oy prime and this change is the shearing strain gamma x prime y prime so gamma x prime y prime is equal to alpha plus beta.

Now what we need to do is corresponding to the three changes that we have seen in case of plane strain corresponding to stretching in the x direction stretching in the y direction and because of the shearing strain that the diagonal which is along the x dashed axis had undergone changes. The diagonal head changes in the angle and if we join those angles together we will get the final position of the diagonal. Similarly, the line which is along o y which is originally we are designated as ob now under its changed consideration after it has undergone deformation. It has come to position o b, so there is a change in the angle from ox prime oy prime to oa and ob. This changed angle will give us the value of the shearing strain.

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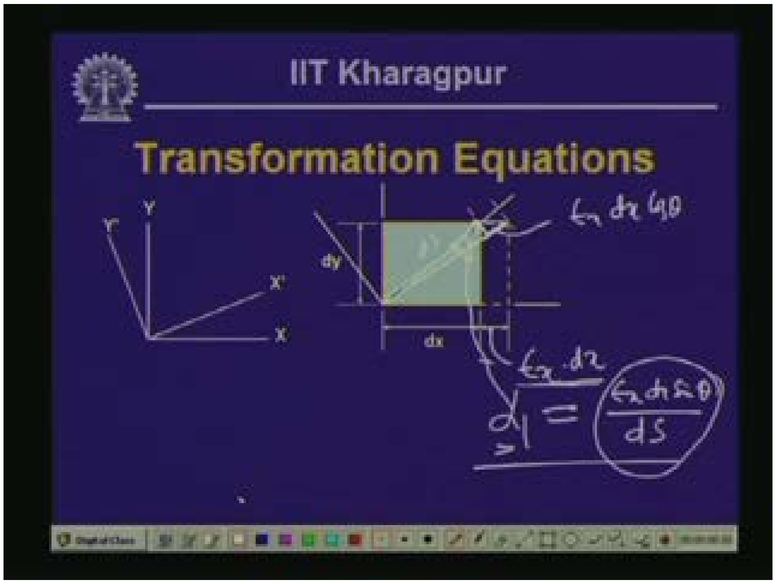

Now to compute those angles let us go back. In this particular case where we computed it the deformation of the diagonal which was originally ds then finally it came to this particular position. Now here we have already seen that this particular extension is epsilon$_x$ dx and this particular stretching of the diagonal was epsilon$_x$ dx cos theta. This is the angle that has been changed because of this stretching so let us call this angle as alpha$_1$ and then the alpha$_1$ can be given as this divided by this distance. Now this particular length is nothing but the extension which is epsilon$_x$ dx(sin theta). So alpha$_1$ is equal to epsilon$_x$ dx sin theta by ds. So epsilon$_x$ dx sin theta by ds is the angle, alpha$_1$ is equal to this. Now please note that that this particular angle is undergoing a change in the clockwise direction. Let us look in to the deformation that we are getting corresponding to the other two cases.

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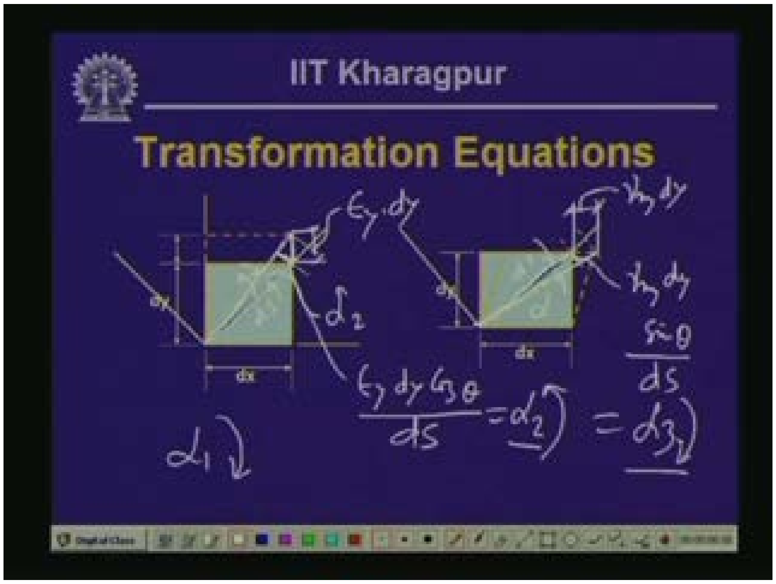

Now in this particular case the deformation which we have the diagonal this is the original length d s and because of the stretching in the y direction this has taken this particular form. Thereby this is the extension in this particular direction so let us call this angle change as $alpha_2$ and this angle change is in an anticlockwise form. Whereas in the previous case it was in the clockwise direction and the horizontal stretching of this particular element is $epsilon_y$ dy as we have seen and this particular angle is theta. So this stretching is equal to $epsilon_y$ dy cos theta and that divided by the original length ds will give the value of angle $alpha_2$.

Similarly, when we have the sharing strain the diagonal was original length of ds finally it has taken the form which is here and the extension of this particular length in terms of the shearing strain is $gamma_{xy}$ dy. Again, since this angle is theta this particular value is $gamma_{xy}$ dy sin theta and that divided by ds will give us the angle $alpha_3$ which is again in the clockwise direction so this is equal to $alpha_3$. Therefore we have three angles $alpha_1$ $alpha_2$ $alpha_3$ and please note that angle $alpha_1$ is in the clockwise direction angle, $alpha_2$ is in anticlockwise direction and angle $alpha_3$ is in clockwise direction.

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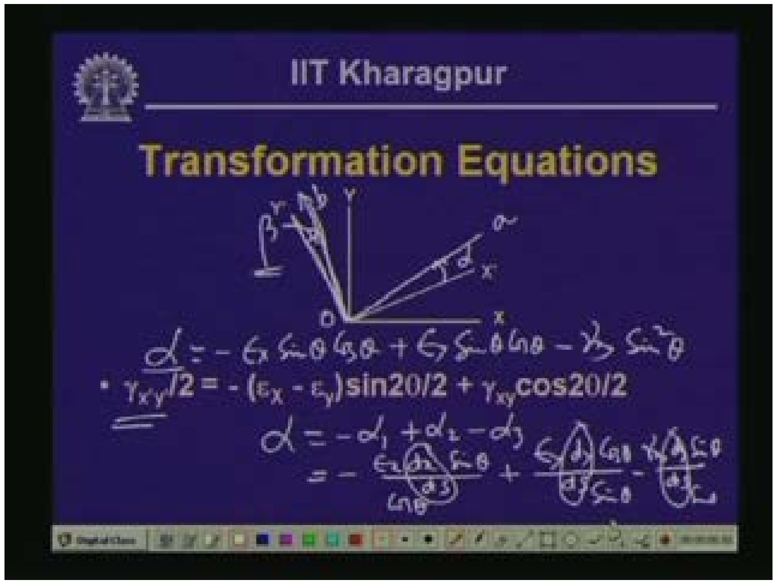

When we are computing the strain here which is we have designated as a and this particular angle is alpha which is in an anticlockwise direction so alpha is equal to minus $alpha_1$ which is in a clockwise direction plus $alpha_2$ which is anticlockwise direction minus $alpha_3$ which is again in a clockwise direction so this state value gives in the anticlockwise direction and if we sum up this values of alpha we get the values as alpha one we got as $epsilon_x$ dx sin theta so this is equal to minus $epsilon_x$ dx sin theta by ds plus $alpha_2$ is equal to $epsilon_y$ dy cos theta by ds minus $alpha_3$ is equal to $gamma_{xy}$ dy sin theta gamma xy dy sin theta by ds.

Now again dx by ds as we have seen is cos theta dy by ds dx by ds is equal to cos theta dy by ds is equal to sin theta and dy by ds again is sin theta. So this expression turns out to be alpha is equal to minus $epsilon_x$ sin theta cos theta plus $epsilon_y$ sin theta cos theta minus $gamma_{xy}$ sin square theta. Now note that this is the value of the angle alpha. Similarly, we will have to compute the value of beta which is taken by the length ob from it is original position. Therefore oy prime theta has come to this particular position by changing this angle beta, so alpha plus beta sum will give me the angle gamma x prime y prime now beta can be computed from the expression of alpha itself by placing theta as theta plus 90. Also, noting the fact that alpha is computed in an anticlockwise from whereas beta is in a clockwise direction so if we substitute for theta as theta plus 90.

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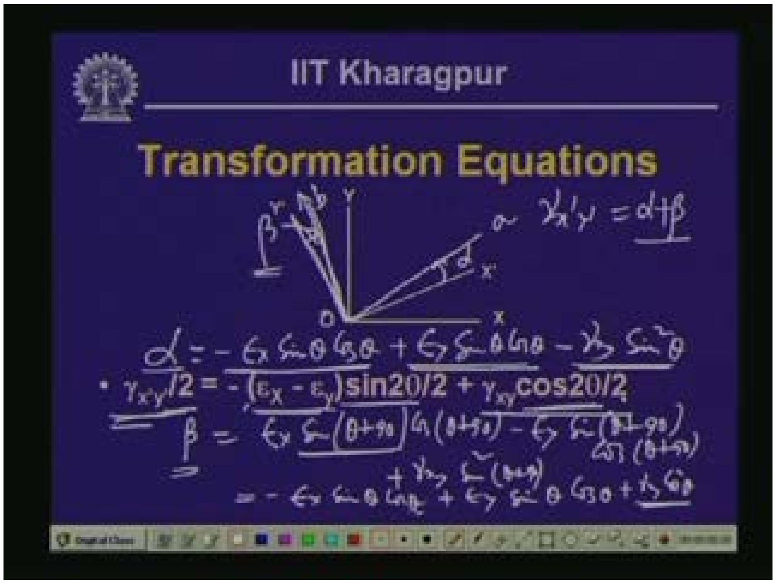

Now writing down for beta which is the negative of this, because it is going in a clockwise direction it is epsilon$_x$ sin of theta plus 90 cos of theta plus 90 minus epsilon$_y$ sin of theta plus 90 cos of theta plus 90 plus gamma$_{xy}$ sin of theta plus 90 square. The signs have been changed because beta is in the clockwise form and alpha was in anticlockwise form. Now this gives us the values as; again cos theta and sin theta which is negative so this is minus epsilon$_x$ sin theta cos theta, this is again plus epsilon$_y$ sin theta cos theta and this is gamma$_{xy}$ cos square theta. So if we add this alpha and beta we get the value of gamma x prime and y prime.

The gamma x prime y prime is equal to alpha plus beta. If we add them up we get epsilon minus 2 epsilon$_x$ sin theta cos theta 2 epsilon$_y$ sin theta cos theta and gamma xy cos square theta minus sin square theta, gamma xy square theta minus sin square theta will give you cos 2theta and 2 sin theta cos theta will give you sin 2theta. If we write down gamma x prime y prime by two forms it gives us gamma x prime y prime is equal to minus (epsilon$_x$ minus epsilon$_y$ sin2 theta by 2 plus gamma xy cos 2theta by 2 so this is the value of the shearing strain gamma x prime y prime.

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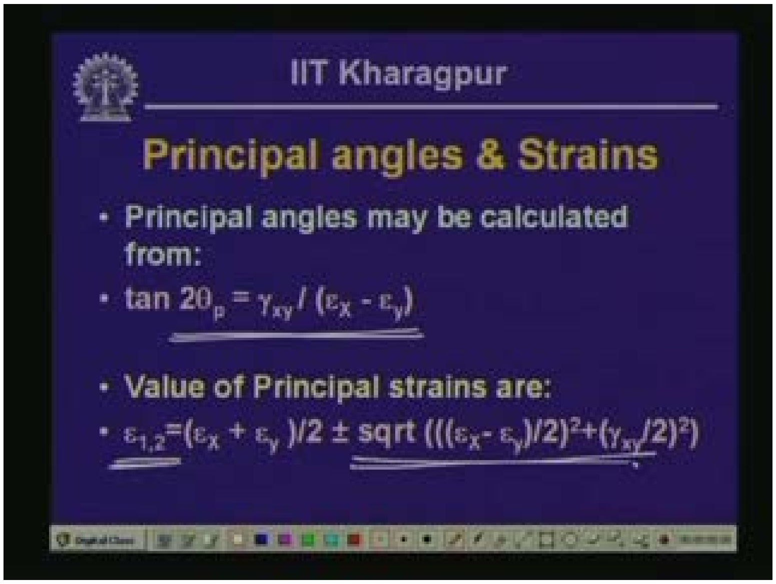

Now from this we can compute the values of the principle strains and the angle of principle planes which is given by twice 2theta p and the principle strain as epsilon_1 epsilon_2. This is similar to the one which we computed for the principle stresses and the principle angles. If you remember when we had computed the stresses we evaluated them in the same form, and this gives us the value of principle strains and the principle angles.

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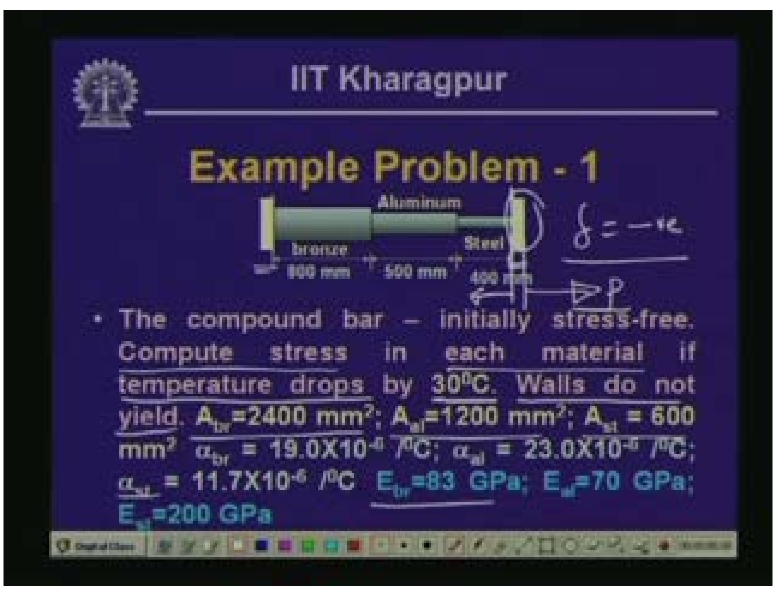

Here is a problem to look at. The compound part which is made out of bronze, aluminum and steel are supported between these two supports and the stress in each of these material is to be computed because of the drop in the temperature by 30 degree C and here what is to be noted is that the walls do not yield and the properties of the cross-sectional area of the bronze part, cross-sectional area of aluminum part, cross-sectional area of steel part are given and corresponding thermal expansion coefficients and the modulus elasticity values are given.

Here the point to be noted is that, if we remove this support then if we allow these bars to undergo changes because of the change in the temperature then there will be shortening because that is a reduction in the temperature so there will be deformation in delta which will be negative. Now we will we will apply a force p to pull this bar and bring it to this particular position and thereby there will be stresses.

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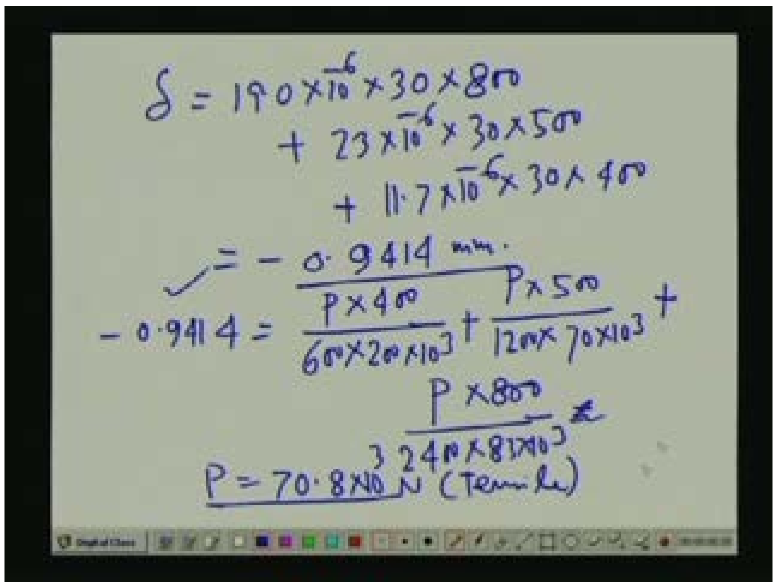

So if we compute the values of deformation delta this is equal to 19.0(10 caret minus 6)(30) is the temperature times the length alpha_l alpha_t plus 23(10 caret minus 6) is the alpha hence 3 into 1 plus 11.7 (10 caret 6(30(400). This gives us a shortening of 0.09414 mm and when we pull this by load p then the this elongation 0.9414 is to be compensated by the force which is p into l by a which is 600(200)(10 caret 3 plus p into 500 into 1200(70)(10 caret 3 plus p(800) by 2400(83)(10 caret 3 equal to the values of extension.

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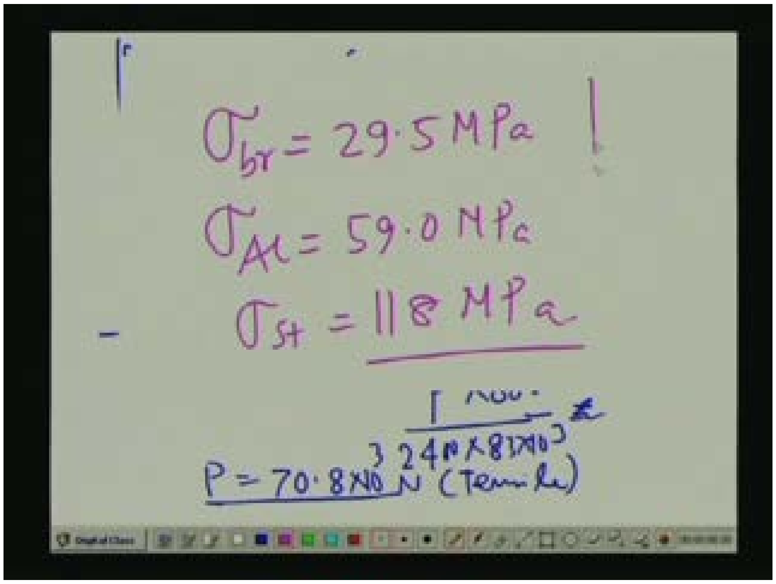

From this we get the value of p is equal to 70.8(10 caret 3 so much of Newton which is tensile and hence the stresses which we get and the stresses which we get from this are in the bronze part sigma bronze equal to 29.5 mp a sigma aluminum is equals to 59 Mpa and sigma steel is equal to 118 Mpa which will be p divided by respective cross-sectional area.

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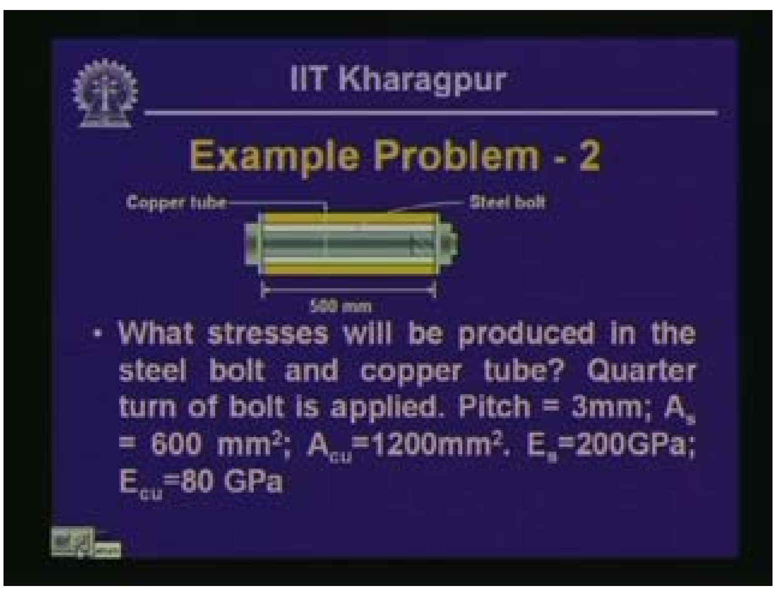

Now let us look into the second problem which will be the stresses that will be produced into this steel. When there is a quarter turn of the bolt there is a nut here and this nut is given a quarter turn then and the pitch of this bolt is 3 mm. This is the threaded part, now area of cross section is given of the copper also is given this the steel bolt and these are the copper tube. Now this is given a turn and then naturally the copper is under compression, the bolt is tensed so we will have to find out that what the stresses are which will be induced in this assembly because of the turning of the nut.

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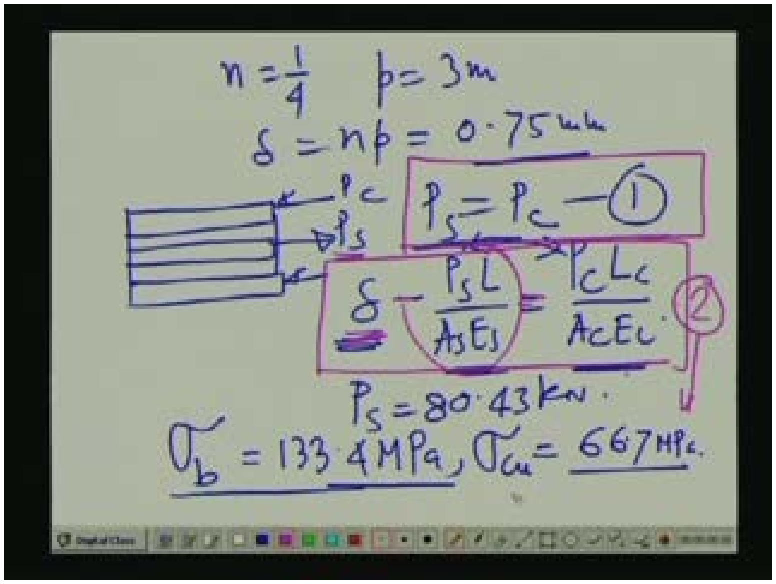

Since the turn is 1 by 4th so n is equal to 1 by 4th, pitch is equal to 3 mm so the delta is equal to n into p is equal to 0.75 mm and this is the copper sleeve and this is the steel bolt, the compatibility gives that the compression in the copper sleeve is equal to the tensile pull in the steel bolt. So P_s is equal to P_c this is the equilibrium criteria and the compatibility gives that the deformation is same. The deformation delta minus the deformation of the steel which is P_sL by A_sE_s is equal to P_cL_c by A_cE_c, this is the deformation that is been exerted by the nut minus the extension of the steel should be equals to the compression of the copper tube.

From this if we substitute the value of delta which we have already seen here we get another relation between pa sp as and pc and also we know that P_s is equal to P_c. From this P_s comes as 80.43 kN and the P_s is equal to P_c. So stress in the bolt sigma bolt is equals to P_s divided by the cross-sectional area of the bolt which is 600 mm square is equal to 133.4 Mpa and the sigma in the copper sleeve is equal to this force divided by the cross-sectional area which comes to 66.7 Mpa. Therefore these are the stresses that will be induced into the bolt and the sleeve. So here we have the equation of equilibrium which is P_s is equal to P_c and the compatibility equation is that deformation that has been created by the nut minus the strain because of extension of the bolt equal to the strain that is being induced in the sleeve. So this criteria along with this gives us the solution for P_s and P_c.

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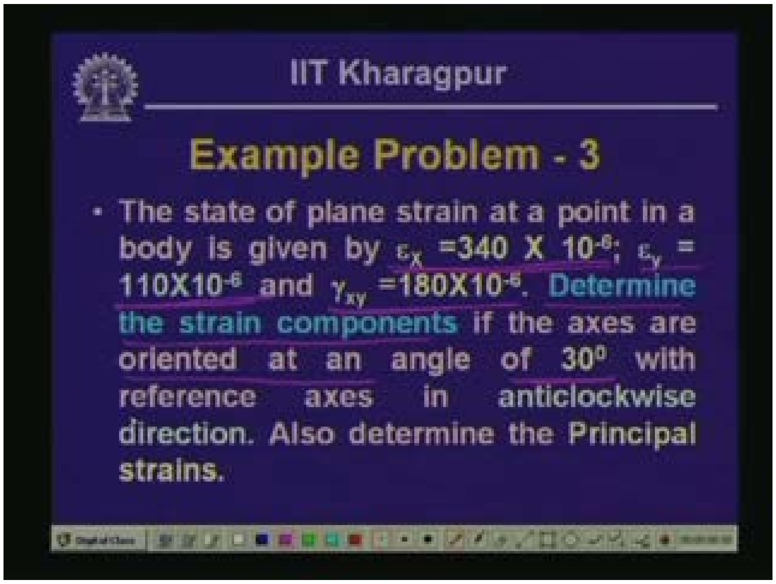

We have another example problem. Later on let us discuss bout the state of plane strain at a point in a body given by epsilon_x which is 340 into 10 to the power minus 6 epsilon_y is equal to 110 into 10 to the power minus 6 gamma_{xy} is 180(10 caret minus 6). What you need to do is that, determine the strain components if the axes are oriented at an angle of 30 degrees with reference axes in anticlockwise direction and also determine the principle strain. We need to compute it. We have reference axes strain epsilon_x epsilon_y and with this reference axes the site of axes is oriented at an angle of 30 degrees in an anticlockwise form. So you will have to find out the strain at that particular point which is the state of plane strain. You got to evaluate the strain epsilon_x prime epsilon_y prime and gamma x prime y prime and also we will have to find out principle strains epsilon_1 and epsilon_2.

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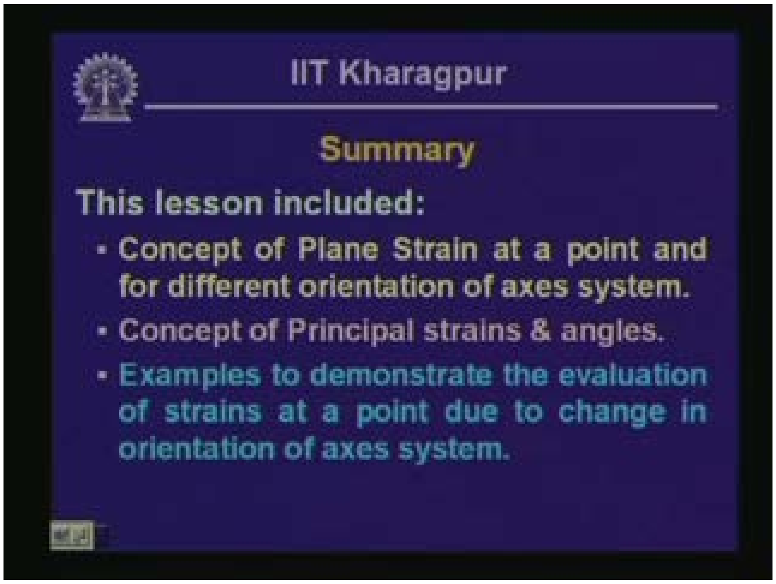

With reference to rectangular axes system we have to plot plane strain the three different strains as $epsilon_x$ $epsilon_y$ and $gamma_{xy}$. Now at that particular point for different position of any rectangular axes system which is oriented at an angle of theta with reference to x and y we can compute the values of strain in terms of those $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ and those strains we have termed as plane strain.

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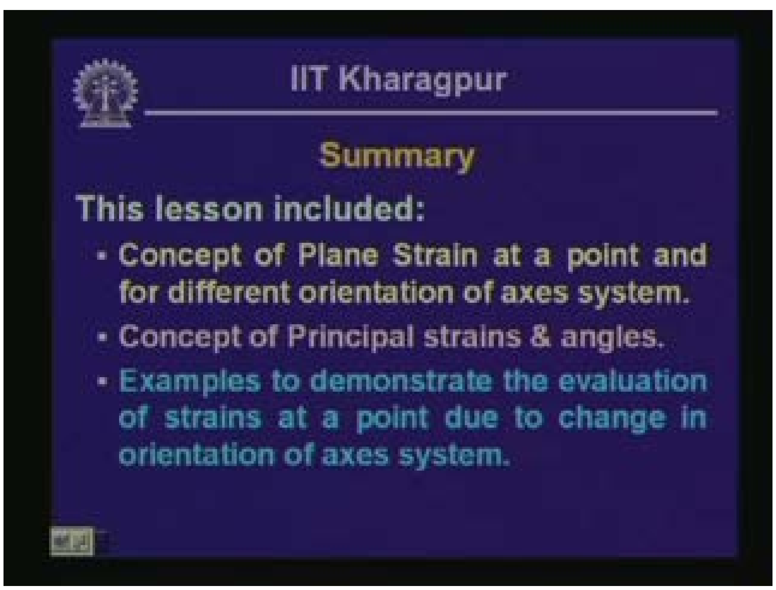

These strains which we evaluate in other reference axes in terms of $epsilon_x$ $epsilon_y$ and $gamma_{xy}$ we call them as transformation equation.

Concept of principle strains and principle angles:
As we did in case of stresses, from the stress we can compute the values of principle strains and principle angles. Also we have solved some examples. I have solved one problem in which we got to compute the value of the strain with reference to the rectangular axes system in axes which is oriented at an angle of 30 degrees with x in anticlockwise form.

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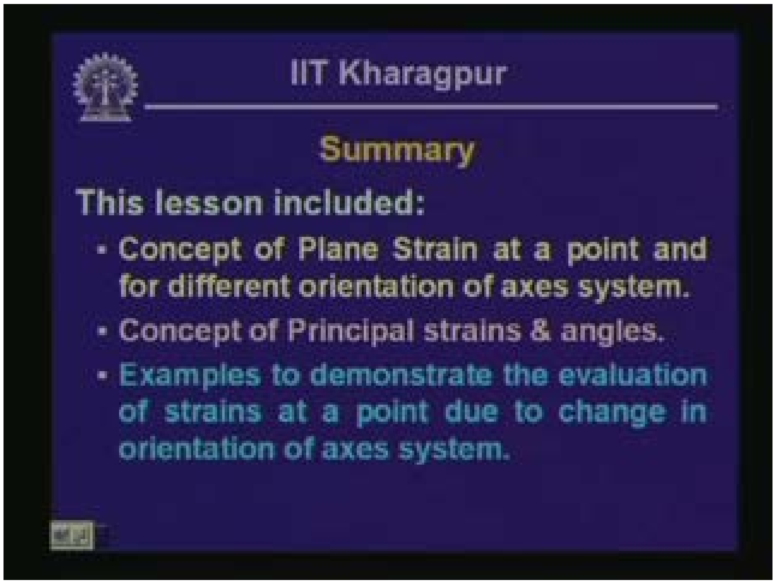

Now we know how to compute the stresses with reference to that change of orientation of the axes system.

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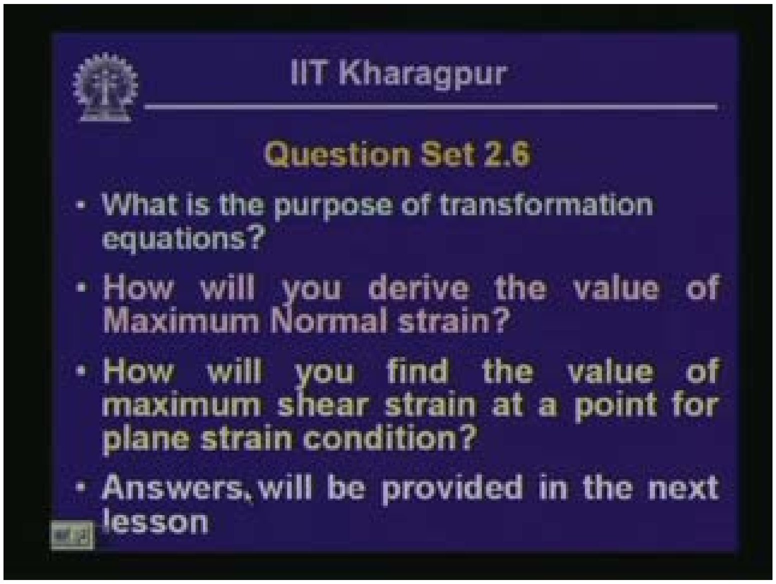

These are the questions:
What is the purpose of transformation equations?
From this discussion you will be able to understand why we really need the transformation equation to be derived, what is the need for that etc which you should be able to answer.

How you will derive the value of maximum normal strain?
You should be familiar with this particular part maximum normal strain with reference to the stresses.

How will you find the value of maximum shear strain at a point called plane strain condition?
This is also in a similar line with the stresses. We have computed the values of maximum and the minimum shear when in a loaded structure at a particular point. We evaluated the stresses and thereby we had computed the principle stresses, we had computed the value of the shearing stress and we had computed maximum and minimum sharing stress exactly in the similar line as we have done here. We have computed the transformation equations, we have computed the principle strain, and we have computed the principle angle. Now we can compute the maximum shearing strain and the angles for the maximum shearing strain in which the maximum and minimum shearing strain will be acting.

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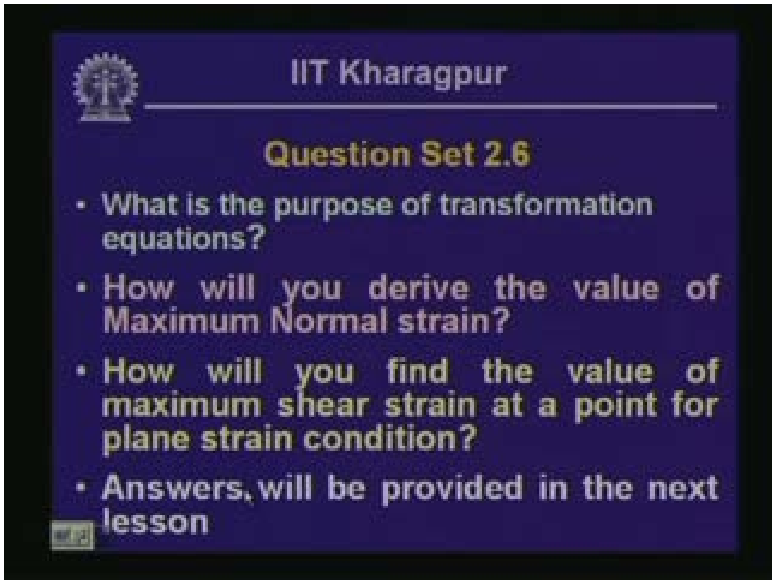

Question:
How will we find out the maximum shearing strain at a point for plane strain condition corresponding to the strains epsilon$_x$ epsilon$_y$ and gamma$_{xy}$?