Ground Water Hydrology
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Module No # 2
Lecture No # 8
Determination of Permeability: Heterogeneity and Anisotropy

Welcome to this lecture number 8 on permeability and it is determination.

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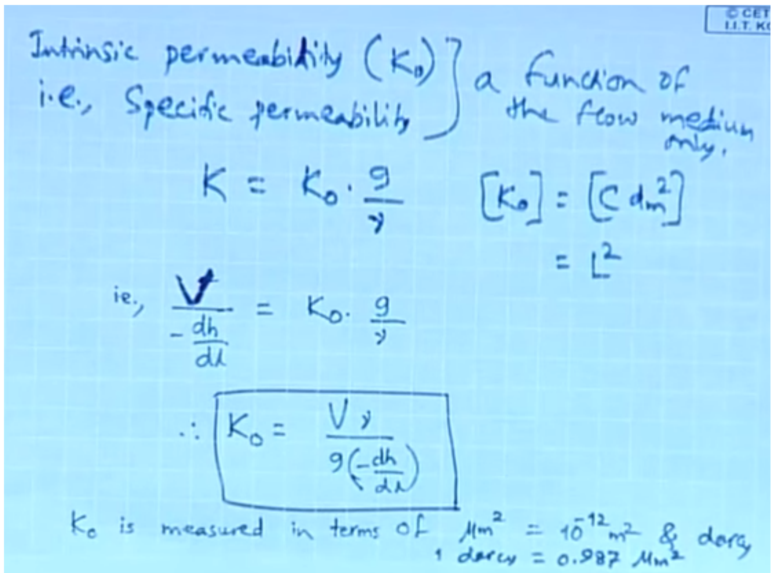

So in the previous lecture we were discussing about the intrinsic permeability which is essentially a function of flow medium only and it has the dimensions of square of length and it is expression it is this is the intrinsic permeability denoted by K0 and it is linked with the permeability or hydraulic conductivity K by the formula K = K0 into G / Mu where Mu is the kinematic viscosity.

And by this we also know that by Darcy's equation we have this hydraulic conductivity or permeability which can be expressed as velocity V of the ground water divided by the hydraulic gradient that is – DH / DL. So that will be the hydraulic conductivity or permeability and equating that K0 into G / Mu.

So finally after simplifying we get an expression for this intrinsic permeability also known as specific permeability has this V the product of V the ground water velocity multiplied by Mu the kinematic viscosity of ground water divided by the product of acceleration due to gravity G and the hydraulic conductivity I am sorry the hydraulic gradient that is – DH / DL. So this is the expression for the intrinsic permeability in terms of ground water flow velocity the ground water kinematic viscosity the acceleration due to gravity and the hydraulic gradient.

So this intrinsic permeability K0 is measured in terms of as we have already discussed that it has the units of square of length it is measured in terms of that is micrometer square of micrometer that is square of 10 to the power 6 meter which is 10 to the power – 12 meter square and in case of petroleum engineering. So another unit known as Darcy is also under use and this Darcy is related to this micrometer square by this expression 1 Darcy = 0.987 micrometer square.

So you can roughly say that 1 Darcy is almost = say 1 micrometer square so this is the intrinsic permeability which essentially is a function of flow medium only having the dimensions of square of the length.

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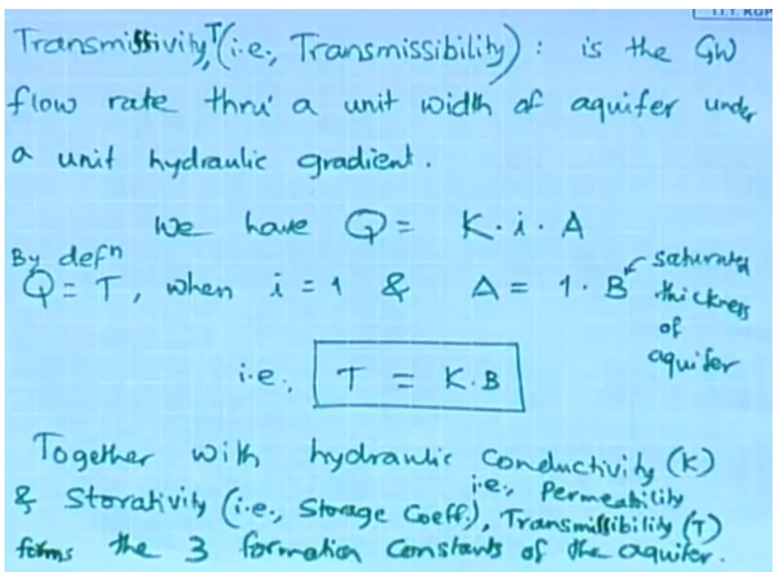

Now let us come to the we have already seen and the previous class we have seen the storativity or storage coefficient which is the volume of ground water which is the aquifer and release or absorb per unit surface per unit change in the head and so this is the denoted by the letter S and we have also seen the hydraulic conductivity which is given by the Darcy's expression as the ground water velocity = hydraulic K into the hydraulic gradient that is – DH / DL.

So now these two storativity as well as permeability also known as hydraulic conductivity and storativity is also known as storage co efficient along with that there is a third parameter which forms what is known as the what are known as the three formation constants of the aquifer and that is this transmissivity also known as transmissibility and it is defined as the ground water flow rate through a unit hydraulic gradient.

So here we are considering a unit width of aquifer and a unit the flow is taking place under the unit hydraulic gradient. So we know that we have an expression based on the Dary's law that the ground water flow rate Q = the ground water velocity which is given by K into I and the area of flow. So therefore Q = K into I into A and b this definition of transmissivity or transmissibility so these Q will be = T when this hydraulic gradient I = 1 as well as the area of ground water flow is = the unit width multiplied by the saturated thickness of aquifer denoted by B.

So therefore when I = 1 and A = B the saturated aquifer thickness so then Q = T so therefore this transmissivity or transmissibility T is given by the product of hydraulic conductivity K multiplied by the saturated thickness of aquifer that is B. So in case of unconfined aquifer so this B will be varying because the unconfined aquifer is not confined at the top and therefore this B will be varying as per the water table shape of the water table shape obviously the slope also.

Whereas in case of confined aquifer so the saturated aquifer thickness is constant at the particular location and so therefore so there the transmissibility or transmissivity T is more constant as compare to that of an unconfined aquifer. So the hydraulic conductivity K the transmissivity also known as permeability the transmissivity T also known as transmissibility and the storage coefficient is S also known storativity.

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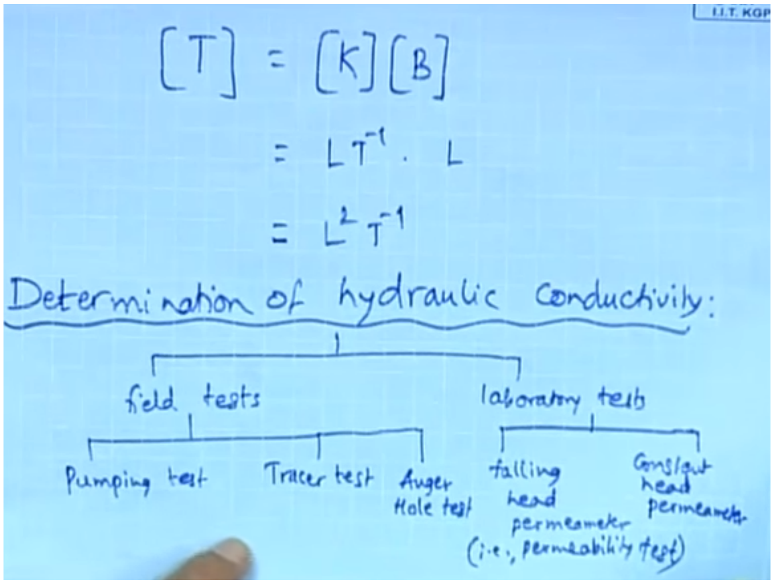

So these three forms what are known as formation constants of the aquifer and here this transmissivity T it has the expression given by K times B. K is hydraulic conductivity B is the saturated aquifer thickness so therefore this transmissivity T this dimensions are L T to the power – 1 into L.

So this is L square to the power – 1 so this transmissivity is expressed in terms of meter square per second or meter square per day or it can be meter square per hour depending upon the time unit in which the ground water flow is described. Now let us come to the determination of hydraulic conductivity or permeability so this hydraulic conductivity or permeability can be determined by either field test, laboratory test.

And in the field test we have pumping hole test that is pumping test of course we also have say tracer test and we also have this auger hole test. So these for what are known as a field test in which the hydraulic conductivity or permeability K is determined in the field.

And in the laboratory it can also be determined by what is known as the falling head permeameter or falling head permeability test also permeability test and the other one is constant head permeability test or permeameter. So the instrument which is used the experimental setup which is used for determining the hydraulic conductivity or permeability in the laboratory is known as permeameter and it is specific purpose is the determination of K or the hydraulic conductivity value.

So therefore this is both is whether it is a constant head whether it is operating at constant head in that case it is known as constant head permeameter and whether it is operating at falling head that means the head is falling continuously as the ground water flow takes so that case it is known as falling head permeameter. So these two forms the laboratory test whereas the other this one that is the pumping test or tracer test and auger hole test they form what is known as the field test.

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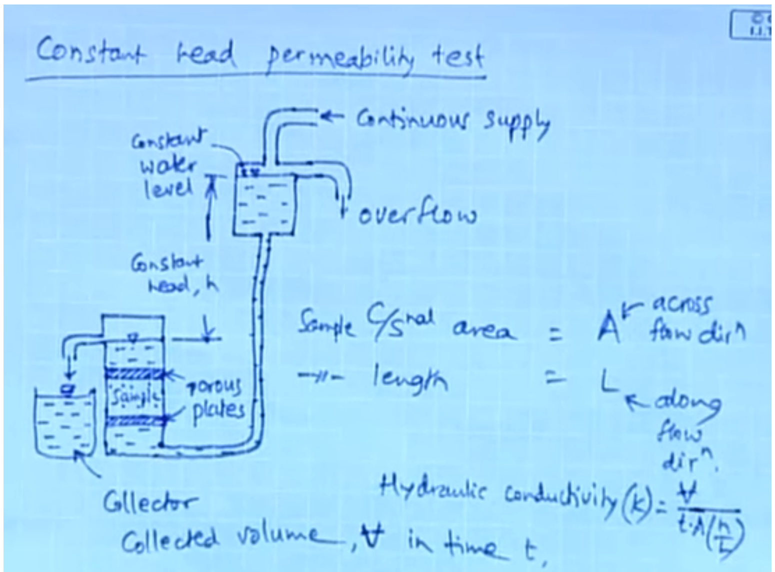

Now let us come to firstly the constant head permeameter or say permeability test so here we have in the laboratory a setup in which the ground water is made to flow through a soil or a rock sample under constant head. So let me draw this schematic diagram here so there will be supply of water so this is the supply this is the continuous supply so this is the overflow and here this is the constant water level.

And here through the ground water or this water is made to flow through a setup in which this is the sample here soil or rock sample and here to retain this soil sample. So there are porous plates one at the top one at the bottom of soil sample and then there is this so there will be a collecting tank. So this is the collector in which the water after flowing through this sample which is enclosed between two porous plates one at the top and one at the bottom.

So this sample so this has a so that is a sample cross sectional area is a so where is perpendicular to the flow direction in this case the flow is from bottom to top and this is known as constant head H. So this is the constant head H which is essentially the level difference between the constant level in the apparatus after the flow takes place through the soil or rock sample as well as a ground water constant water level in the supply side.

So this is the sample cross section area A and in this case so collector suppose let us say the collected volume in the collector as V. Let us show this by HV okay. And this sample cross section area this sample length. So this is L so this sample cross sectional area so this is a across flow direction and this L is along flow direction. So in this case and the collected volume is V in say time T okay.

So in that case the hydraulic conductivity K is given by V the volume multiplied by so divided by Time T so this will give the discharge divided by area that will give the velocity and divided by the hydraulic conductivity.

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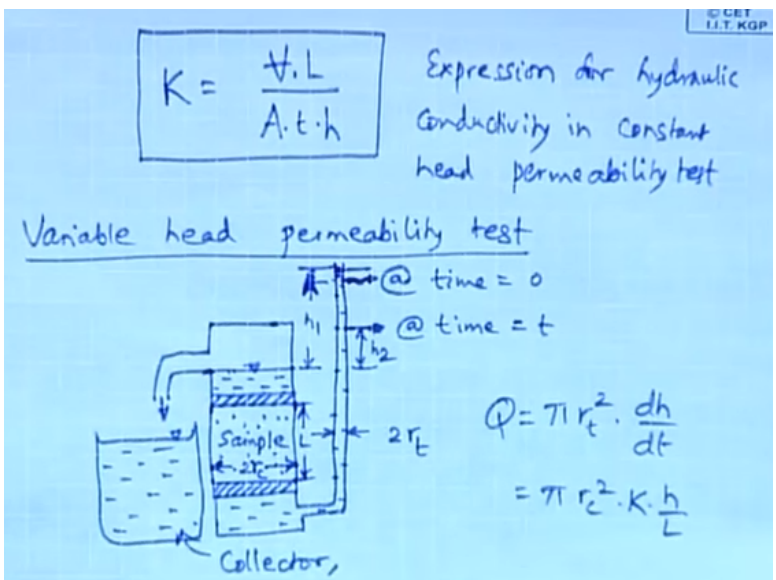

So that is H / L so therefore so the hydraulic conductivity K is given by volume V multiplied by the sample length L divided by the sample cross sectional area A multiplied by the time of collection of volume that is T multiplied by constant head H. So this is the expression for hydraulic conductivity in constant head permeability test. So simply this hydraulic conductivity is expressed in terms of say 4 linear parameter rather 2 linear parameters, 1 area parameter, 1 volume parameter and 1 time parameter.

So here this is the collected volume as V the cross sectional area is A and then the two linear parameters are the sample length along flow and the constant head between the supply the constant level supply tank and the level of constant ground water level after it as flown through the soil or rock sample and then T is the time of collection. So this is the constant head permeameter or which is known as constant head permeability test.

Now let us go to another laboratory method which is the variable head permeability test here unlike the constant head permeability or concentrate permeability test the head in the supply side will be varying. So obviously due to gravity flow so these head in the supply side this continuously decreasing so the sample and that part is almost same. So there is a soil sample or soil rock sample is enclosed between two porous plates.

So this is the soil or rock sample through which permeability variable head permeability test is conducted and so here this is the collector and here we have this water and the sample it is assuming this to have a circular cross section let us consider the diameter of the sample as RC and here so this is the supply side the circular tube with the diameter = RT or rather 2RT 2RC is so the sample the diameter of the circular sample and 2RT is the diameter of the this pipe which supplies water for flow through the soil or rock sample and the initial head.

So this is let this be the head H1 and this head is measured with respect to say this one so this H1 is the initial head and then say let us say this is H2. So this is the H1 is time = 0 and H2 so this is the water level so this is at time = T. So here in this case let us say this is the H1 so the initially head is at this level which is above the reference level which is the constant water level maintained after flow through the soil or rock sample.

So this is the datum which with the heads are measured so that is H1 and then as the water flows through the sample. So this head decreases and at arbitrary time T so this head decreases from H1 to H2. H1 is at time = 0, H2 is at time = T so now and the collector now with the collected volume. So now we can say here that the ground water flow rate or the flow rate through the soil or rock sample is given by the cross sectional area it is Pie into RT square multiplied by DH / DT which is the rate of change of head with time.

So that is the represents the velocity in the vertical direction and this Q is also = the Pie into RC square where RC is the radius of the circular sample through which the water flows circular soil or rock sample multiplied by hydraulic conductivity K into H / L and this L is the sample length along flow.

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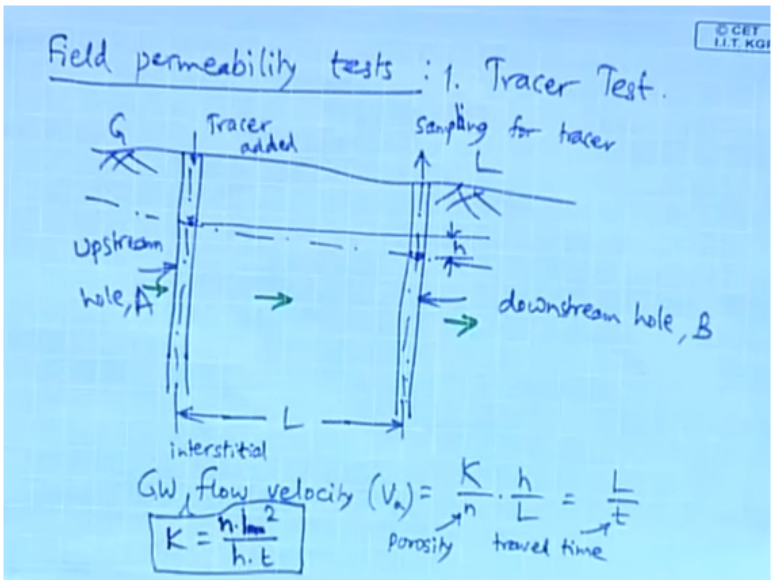

So now if we quit these two expressions and integrate so here this is Pie into RT Square DH / DT = Pie into RC square into K into H / L. So let us simplify this one so therefore here we can write this as a DH / H with limits say from initially initial limit of H1 to unlimit of H2 = so DH / H and of course if you want you can keep this RC square / RT square which is constant you can take it outside into K / L.

So all this will be outside the integral sign and here so there will be definite integral when H = H1 time = 0 when H = H2 the time = T. So in here so this is DT so after simplifying this after integrating and simplifying we get an expression for the hydraulic conductivity through this variable permeability test as RT square / RC square into L / T. L is a sample length T is the time of collection into natural log of H1 / H2.

Where H1 is initial head H2 is the final head after the flow which results in the continuous decreasing of head through this variable head permeable. So this is the expression for K in variable head impermeability test for variable head permeameter. While in the constant head permeability test so the expression for the hydraulic conductivity is simply given by the product of collected volume multiplied by sample length divided by product of the sample cross sectional area.

And time of collection and the constant head whereas in this case in case of variable head permeability test. So this hydraulic conductivity K is given by RT square / RC square of course you can also write this K as the area of the tube divided by area of the column into L / T. So this is the AT is the tube area and AC is the sample column area into L / T okay into natural log of H1 / H2.

So therefore so this is the expression of the hydraulic conductivity and if both the tube as well as a sample which are sample which are in circular shape generally that is the case. In that case simply this K = RT square / RC square into L / T into natural log of H1 / H2 where H1 is the initial head at time = 0 and H2 is the final head after the flow of water through the soil or rock sample at time = T.

So these are the two laboratory test which used to determine the hydraulic conductivity or permeability and in this case the laboratory test many times they do not give the proper results because in most of these cases the sample through soil or rock which the hydraulic conductivity the laboratory test is conducted okay. It will be a disturbed sample so therefore there may not give the actual value.

So that we go for the field test firstly let us consider the field permeability determination test a permeability test and in this firstly let us say the tracer test. So in this tracer test what is done is suppose there is a ground water or a soil or rock sample so here two holes are drilled one at the upstream one at the downstream. So this is the upstream hole which is A and then this is the downstream hole which is B and so in the upstream hole so this is tracer is added and so this is sampling for tracer is done at the downstream hole.

And here in this case obviously there will be hydraulic gradient so this is the and the level difference between the water level and the upstream and downstream is H and say suppose this so this is the flow direction and this is the length between these two holes L. So in that case the velocity of flow is given by the flow velocity the ground water flow velocity ground water it is the interstitial flow velocity.

So this is a VA so this is given by the hydraulic conductivity K / the porosity N multiplied by H / L which represents hydraulic gradient and this is also given by L / T so this is the T is the travel time N is the porosity and H is the difference in the water level between the upstream hole and the downstream hole upstream hole where in the tracer is added and downstream hole where the sampling of the tracer is done that means the time measurement as well as it is concentration.

And so here by equating these two equation expression so from this expression we get this K = H I am sorry L square N into L square / H into T. That is the hydraulic conductivity K is given by the porosity of the soil or rock sample N multiplied by the square of the distance between the two holes the upstream one at the upstream and downstream.

So let me show the center line here for these two so this is obviously L is the center to center distance and H is the level difference of ground water in the upstream hole A and the downstream hole B and T is the time of travel. So sample is added is here so they initially and then they the stop watch is started and as soon as the sample reaches this downstream hole B this stop watch is closed so that it will give the travel time T.

So therefore so this hydraulic conductivity is given by the porosity multiplied by this square of the length divided by the H which is the difference in the water the ground water level between upstream hole and downstream hole and T the travel time. So this is the first of the field permeability test for determining the permeability or the hydraulic conductivity of course there is also another technique known as part deletion method anyway let us not go into that and so there is also.

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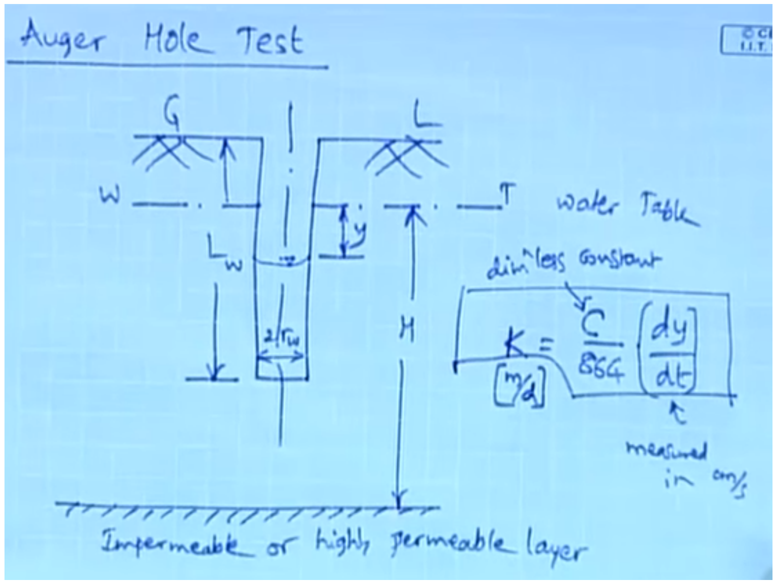

Now let us come to another field test which is known as auger hole test here what is done is an auger hole is drilled in the ground consisting of soil or rock where we need the permeability or the hydraulic conductivity value and so this is LW which is the length of the well or the auger this one. And this auger hole or well it has diameter of 2 into RW and in this case.

So let this be the so this is the impermeable or highly permeable layer and with respect to this so this is the water table denoted by WT. And the draw down in this auger hole let this be Y the level difference between the water table and the water level in the S auger hole and the height of the water table above the datum of impermeable or highly permeable layer let this be H. So here so it can be shown that the hydraulic conductivity K is given by that is C a coefficient.

So this is a dimensionless constant C / 864 and this K so this is directly give the hydraulic conductivity value in meters per day and 864 comes from this is 24 into 3600 which is the number of seconds in a day that is 86,400 into DY / DT which is the rate of change of the Y with time. So this DY / DT so this is a measured in centimeter per second.

So initially this auger hole is drilled and then so some water hole is a remote that is water comes out from this auger hole. So therefore there will be a drawdown of Y created and then so because the water table is above the when there is a drawdown so what happens so this Y goes on decreasing so therefore this DY / DT is the represents the rise of the way the rate of change of this draw down with time. So this K is directly given by this expression C is constant.

So this is the dimensionless constant okay so this is the expression for hydraulic conductivity through this auger hole test. Now let us come to the last this one anyhow of course we will go into this and that is the pumping test.

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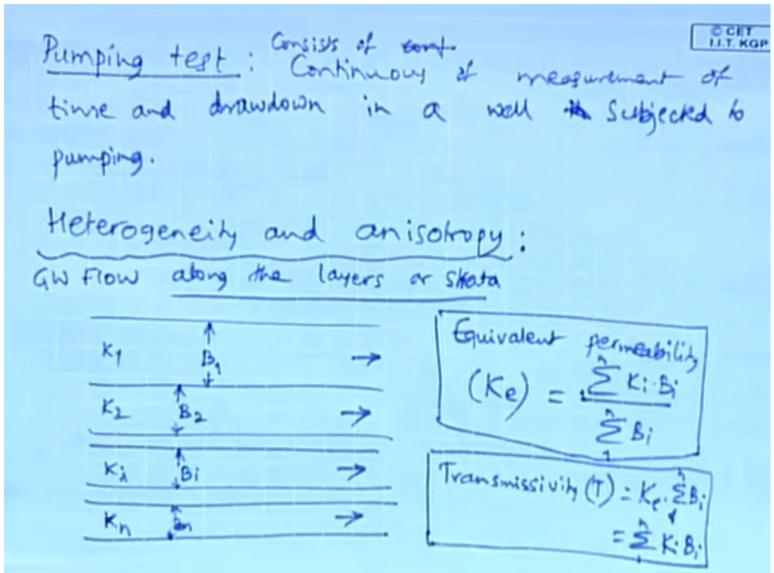

And in this pumping test and in this pumping test through a well field so the time versus drawdown that means the time and is continuous measurement is done and then this consists of continuous measurement of time and drawdown in a well subjected to pumping and by using the appropriate expressions.

So the hydraulic conductivity K value is determined so these are some of the methods for the determination of the hydraulic conductivity initially we discussed about the the laboratory test wherein we started with the constant head permeameter or concentrated permeability test and we also discussed the variable head permeability test or the variable head permeameter then we moved on to the field test which will give a better result that simple reason that the sample is undisturbed in case of field test as compared to the laboratory sample of soil or rock which is generally disturbed.

So which consisted of three types of person one is the tracer test and auger hole test and lastly the pumping test and of course the actual pumping the this one we see the subsequent chapters. Now let us come to the heterogeneity and anisotropy so here in all this we made an assumption that the soil or rock layer through which the ground water flows so is a homogenous and isotropy that means it is a hydraulic conductivity.

It is same in all directions in all the three direction that is KX, KY, KZ whereas in actuality it is not the case. Now let us consider the sum of the an isotropic aquifers where in this let us consider say separate layers where in the let us say this is the where is the flow is taking place firstly let us consider that is the ground water flow along the layers or strata okay.

And then this case let us consider then this is the so this are the three layers the first layer let us consider the permeability value as K1 and let us consider the width as the thickness of aquifer as B1 and let us consider the second as K2 the second layer having hydraulic conductivity K2 and thickness B2 and then let us also consider an aquifer having a permeable KI having a thickness of BI.

And this be the last and lower most aquifer having a Nth aquifer having K and permeability having K and permeability or hydraulic conductivity KN and thickness BN okay and in all this let the flow be along the layer or the along the strata. So in this case so this is a case of heterogeneous aquifer of course here in this case we are not considering the anisotropy and we are considering only flow in one direction so in this case the equivalent permeability that is KI.

So this is expressed as summation from 1 to N KI into BI / summation BI from 1 to N. So and this is the expression for equivalent this permeability and expression for transmissibility or transmissivity is simply given by KE into summation BI that is 1 to N which is simply summation 1 to N that is KI into BI okay. This is the expression for transmissivity and this is the expression for the equivalent permeability okay.

So the ground water is along the layers or strata sometimes happen that in a heterogeneous aquifer the ground water flow is taking place across the stratum or layers Okay.

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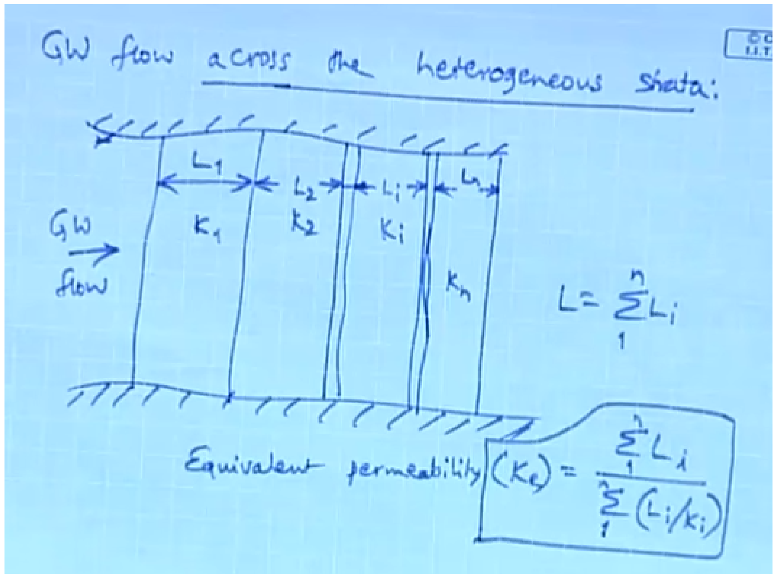

So this is the ground water flow across the heterogeneous strata so here let us consider say this is the impervious strata and this is the impervious strata may be top or bottom and say. And then suppose we say this is the ground water flow and let us consider the this strata the flow are perpendicular to the flow direction each having the length of say L1 and hydraulic conductivity K1, L2 hydraulic conductivity K2 and so on.

So suppose here say let us say this is a say this is LI the hydraulic conductivity KI and lastly let us say this is LN with the hydraulic conductivity KN. So in this case the ground water flow is perpendicular or across the perpendicular layers. So in this case so the total flow length is given by this L = summation LI, 1 to N and the equivalent permeability that is KE is given by summation LI 10 N / summation 1 to N of LI / KN.

So basically this is length and LI / KN represents the travel time okay so this is the expression for the equivalent permeability in case of a heterogeneous aquifer whereas the ground water flow is across the heterogeneous flow strata. So we will stop here and we will continue in the next class in the next lecture on the move on to this further article in this thank you.