Ground Water Hydrology

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Module No # 08
Lecture No # 39
Modeling and Management of Ground Water Conjunctive Surface – Subsurface Modeling

Welcome to lecture number 39 of this ground water hydrology course.

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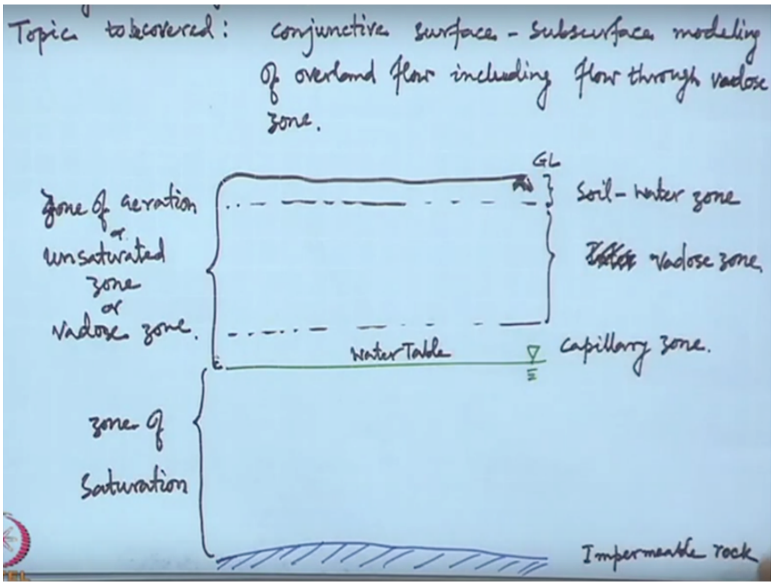

So in the last lecture we have seen that regional scale development of ground water. In that one, one important aspect is conjunctive use of surface water and ground water. In this particular lecture we will try to cover that aspect. So our main uh topic is modeling and management of ground water.

And under that topic to be covered is conjunctive surface subsurface modeling of overland flow including flow through vadose zone. So you already know that if this is our ground surface and this is our water table and this level is that, our lower portion or the lowermost portion of the triatic aquifer. Then this zone from starting from the water table to this bed level or if it is of rocky nature then we can say that this is impermeable rock.

And starting from this impermeable rock to this water table level we can say that this is our zone of saturation. And above that there is a thin region which is called as capillary zone. And near to surface there exists another region which is called as soil water zone. This is important for plants. And in between we have intermediate zone or intermediate or vadose zone. Sometimes this total region starting from water table to ground surface this is called as zone of aeration, zone of aeration or unsaturated zone or vadose zone.

So in this particular lecture we are concerned about the modeling of this zone of aeration or unsaturated zone or vadose zone. And the water above that ground level or ground surface. So what are the processes or what is the interaction mechanism between this overland flow and subsurface flow. That we will examine.

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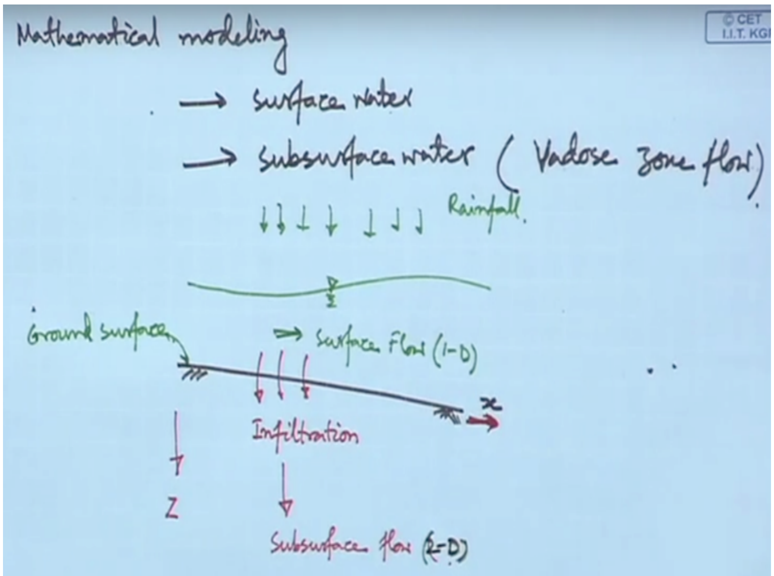

So to model this particular problem we need certain governing equations for flow representation. So for mathematical modeling, we need governing equations for surface water. Next we need governing equation for subsurface water or in this particular case this is for vadose zone flow. So the whole problem can be represented as this is let us say we have ground surface which is not always straight. It is always having some slope.

So let us say this is X direction. And this is X direction which is one dimentional thing. And which is in line with the or in the same direction of our bed slope and we have vertical direction is Z direction. This is for infiltration. Infiltration from our overland flow.

So infiltration, then we have subsurface flow, 2D model. And our surface flow is modeled as 1D. So this is our surface flow. This is 1D model. And this is our surface water level and we have rainfall above that.

So starting from rainfall, we have surface flow. And this is our ground surface. And below we have this infiltration. And due to this infiltration only there will be subsurface flow that is 2D or two dimensional in nature.

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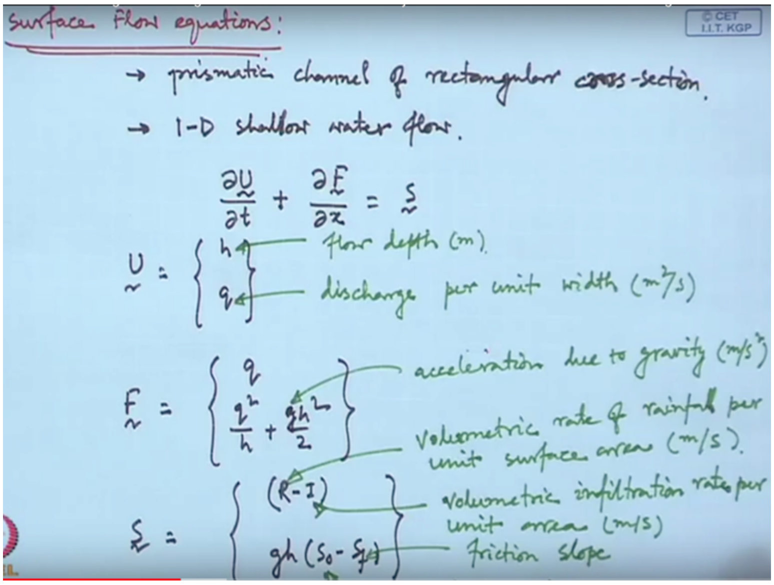

So considering prismatic channel we can write our governing equation. So we can write this surface flow equations. Surface flow equations so first assumption is that the surface flow occurs in prismatic channel of rectangular cross section. And flow second is flow is 1D shallow water flow. So in this case, we can write our governing equation for surface water flow as del U / del T.

Where U is vector + del F is vector by del X and this is another vector. Or we can say that these are column matrices. So in this case, U is basically H and Q. Then we have this F which is Q and Q square / H GH square / 2. And S this is R - I GH S naught - SF.

So term by term we need to define different variables. So H is flow depth in meters or any standard unit, Q is discharge per unit width meters squared per second, G is acceleration due to gravity. R is volumetric rate of rainfall per unit surface area. This is meter per second. Then I is volumetric infiltration rate per unit area, bottom slope, and SF is friction slope. So these are the key parameters.

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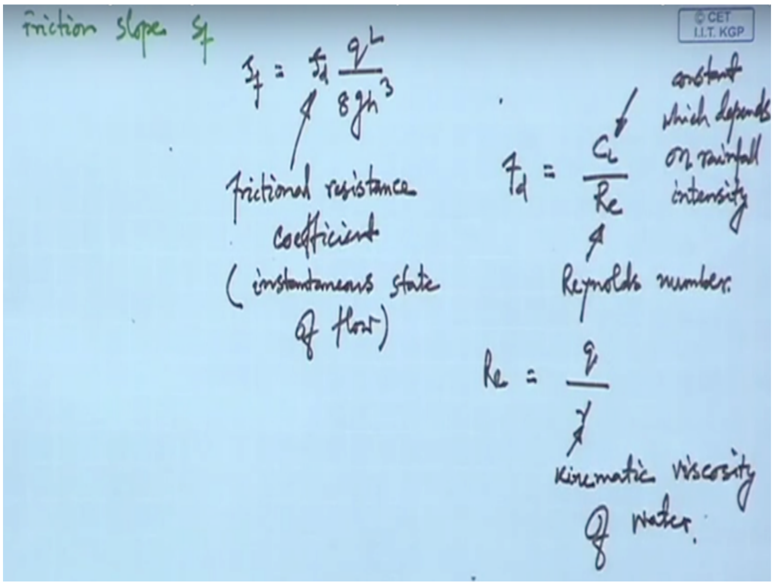

Now energy slope or friction slope be calculated using Darcy Weisbach formula. So friction slope SF can be calculated using the formula SF = FD Q square 8 GHQ, where FD is the frictional resistance coefficient. And this depends on instantaneous status of flow. So flow is laminar by assumption so we can write this FD = CL / RE where this RE is Reynolds number.

Cl is the constant which depends on rainfall intensity; FD depends on rainfall intensity and Reynolds number. And this Reynolds number can be calculated as Q / Mu. Mu is kinematic viscosity of water. So this is all about the equations related to surface flow.

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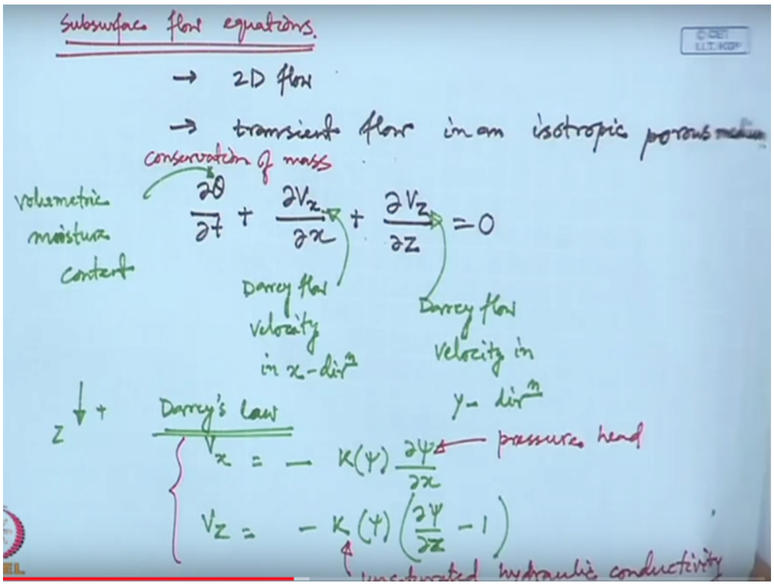

Now subsurface flow equations are subsurface flow equations are one is related to the flow. That is first assumption is 2D flow. Second assumption is transient flow in an isotropic porous medium. So with the conservation principles we can write this equation without any source or sink as del theta / del T del VX del X. And this VZ Z = 0, where this theta is volumetric moisture content.

VX and VZ these are Darcy flow velocities. Darcy flow velocity in X direction. And this is Darcy flow velocity in Y direction. So in this case X and Z are distances along two coordinate directions. And Z is taken as positive downwards. So for Darcy's law the velocity components can be calculated as, this is for Darcy's law.

This VX can be calculated as - K PSI del PSI del X and VZ can be calculated as - K PSI del PSI / del Z - 1. PSI is pressure head and K PSI this is unsaturated hydraulic connectivity. So by substituting these two expressions in our original equation for conservation of mass.

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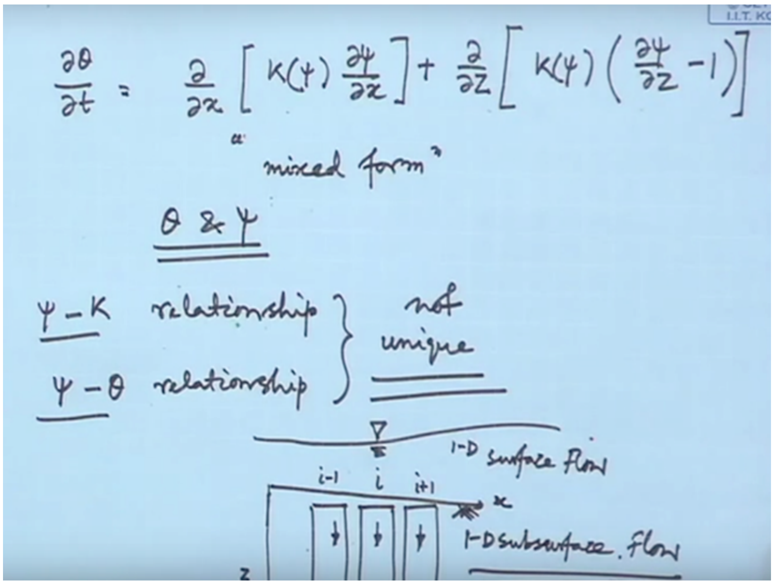

This is from conservation of mass we can get our final form of the equation that is del theta by del T = del X K PSI del PSI X. And Z this is K PSI del PSI / del Z - 1. So this particular equation is called as mixed form. Mixed form equation because it includes both theta and psi in single relation equation, in single equation. So it is important to have the expression for K PSI which is unsaturated hydraulic connectivity.

And this PSI K relationship dictates the flow regime. And this moisture content and theta, there is a relationship and interestingly this K PSI or PSI K relationship and PSI theta relationships. These are not unique, not unique. So it is always problem dependent and it is related to soil type and it varies with uh problem to problem. To solve these two equations we can start discretizing the governing equations or for our surface flow and subsurface flow.

To have this first we need to have some kind of conceptual definition of surface flow and subsurface flow. So we can have situation like this where this is X direction, this is Z direction. Although these are not having 90 degree but approximately these are considered to be 90 degree situations. Then we have regions like this where this is cell, this is I - 1, this is I + 1. And from here we have 1D flow, 1D subsurface flow.

Surface flow and above that we have water level and 1D, this is surface flow. And this is our ground surface. So with this picture in our mind we can start discretizing our governing equations. So first is surface flow.

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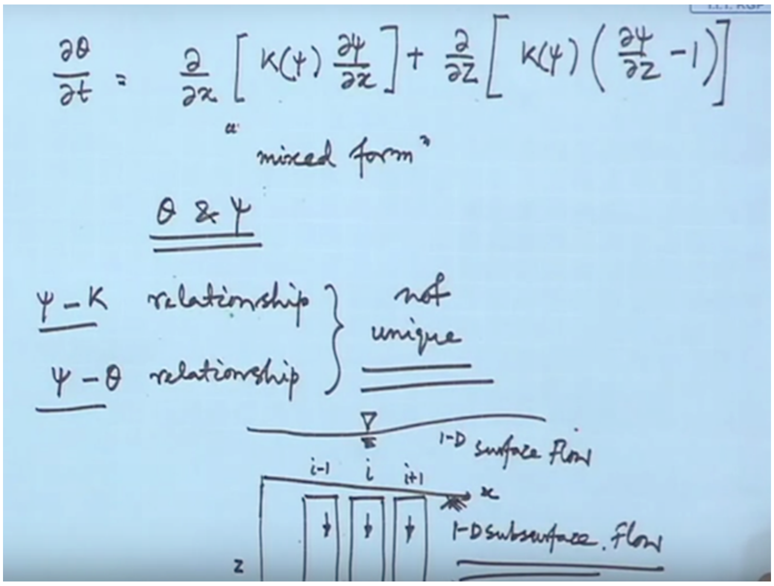

So in surface flow this is can be solved using some kind of predictor corrector method for surface flow. So our governing equation is del U del T + del F del X =S. So for that the predictor step. Predictor step can be written using n and n and star level; N is known time level. This is predictor level.

The star is basically predictor level. So with these two levels. We can write this UI star = UI N -del T / del X and this is FI + half nth level. And this is FI - half, this is again at nth level. And this is del T SI this is also calculated at nth level.

So this represents, this F at I + half represents numerical flux through the cell face between nodes I + 1 and I. And del X is basically grid spacing. All terms in the right hand side of this particular equation are at know time level. Therefore U star can be computed directly or explicitly. So we can write it as explicit equation.

So if I + half can be calculated as, if I + half = half FR + FL - UR - UL. So what is this? This is basically, alpha is positive coefficient. And FR is the flux computed using information from the right hand said of the cell face. So FR is basically flux computed with the information from right hand side and this is with the information from left hand side of the cell face.

And UL and UR are obtained using a particular, this UL and UR these are calculated based on a particular expression that is called as uh MUSCL approach. MUSCL is monotone upwind scheme for conservation laws. So under this this UL is calculated as UI + del UI and half. And UR is calculated as UI + 1 + half uh this is - half del UI + 1. There are several ways of calculating these two variables.

We can calculate it is using min mod approach. That is called min mod approach. In that 1 this del UI is calculated as min mod of UI + 1 - UI and this is UI - U - 1. What is this min mod? Min mod is defined as A, B where A if modulus of a is less than modulus of B and AB is greater than 0; B if modulus of B is less than modulus of A and AB greater than 0. And 0 if AB is less than equals 0 or negative.

This positive coefficient alpha in our previous expression is determined as alpha should be greater than maximum of modulus of this QI divided by HI + root over GHI. And this is for all I, this is for all I. That means considering all i that is i uh starting from 1 to N, 1 to N, N is the number of grid points. We can calculate the minimum value of alpha. So alpha should be greater than this level. So next step is corrector step.

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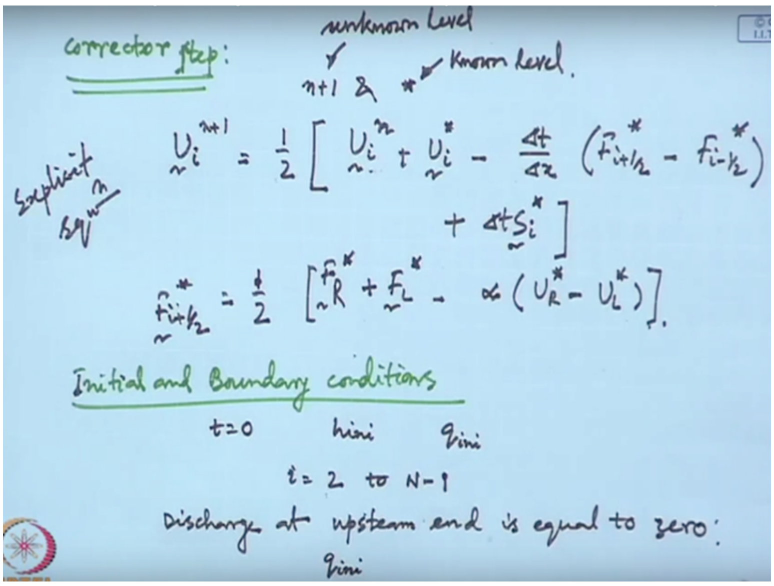

So in this corrector step, the star values are predicted values are utilized to get the corrected value. So in this 1 it involves N + 1 level and star level, star level is already known level from our predictor step. And n level is unknown level. So in this 1 this UI N + 1 this can be written as UI nth - UI star - del T del X. This is FI + half FI - half star + del T into SI.

This is at star level where this FI + half is calculated as FR star + FL star - alpha UR star - UL star. So UL and UR these can be calculated using that min mod approach we have utilized in our predictor step. And we can solve this particular equation. Again it is explicit equation. So we can directly solve this.

So in this particular equation, only the source term is evaluated using predicted value of H and Q. So these two steps we can calculate using explicit equations. And finally we can get N + one level value for this 1. So what are the initial conditions? At T = 0, initial boundary conditions.

So at T = 0, we need to consider 1 H initial depth and Q initial. Although it will have impact on the final solution but we need to specify to remove the discontinuity in the initial condition. So although it will have impact on the final results this can eliminate the numerical singularity of the solutions. So at this level we have I2 to N - 2 N - 1 we have internal nodes. And for the first node and the last node, we can specify the boundary condition.

So values of the variables at upstream and downstream ends domain are determined by appropriate values. So discharge at the upstream end is = 0. So discharge at upstream end is = 0. However one initial or discharge is specified to remove the singularity. But it should be small, sufficiently small compared to the other values.

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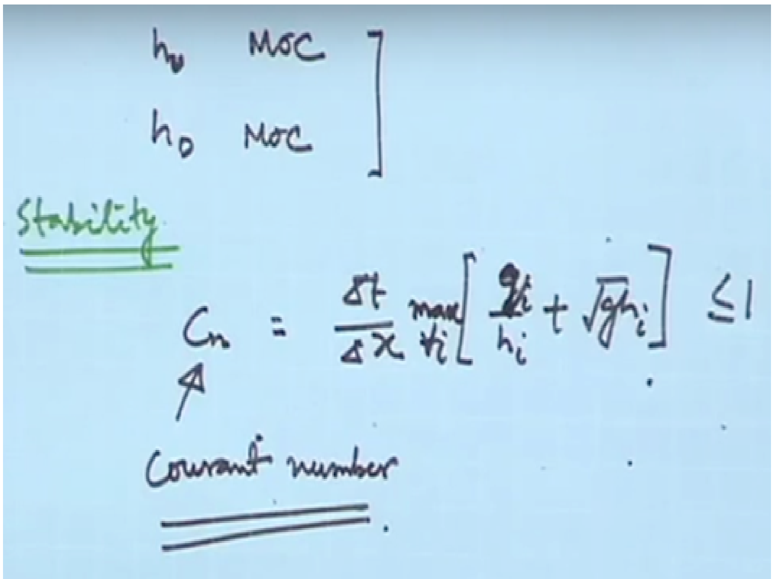

The flow depth at upstream. We can say that flow depth at upstream level can be determined using negative characteristics equations or MOC, or method of characteristics equation. And downstream also this MOC can be utilized to get the levels for H. For stability the courant number condition or CFL condition, this G / H + root over GH.

This should be less than one. The CN is called courant number. And in this particular problem delta T and delta X should be such that this G and H values, this should be less than 1. We can also say that this is actually maximum value for all I. This is GI or this is actually Q, QI, Hi + root over G into HI less than = 1.

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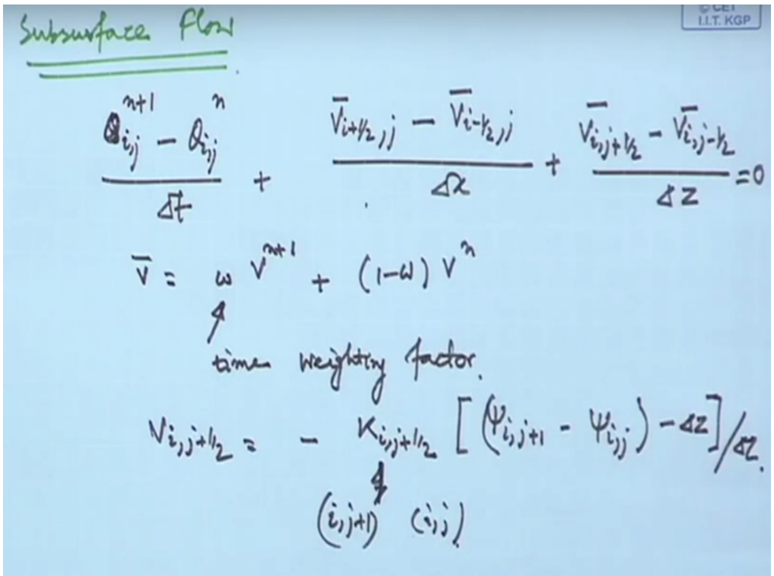

For subsurface flow again we need to discretize those equations. Subsurface flow so in this one way is that we can discretize our original equation that is theta IJ because it is a two-dimensional thing. So theta IJ at nth level. This is basically VI + half J bar represents time averaged values. And VI- half J this is there.

Plus we have IJ + half VIJ - half bar divided by del Z = 0. So this V bar is basically W into VN + 1, 1 - W VN. And this is time weighting factor. And VIJ + half this is calculated using our expression of Darcy's law this is J and PSI is IJ + half PSI IJ.

And this is - del Z and divided by del Z. So K IJ + half is unsaturated hydraulic connectivity. And it should be evaluated at the inter block phases between IJ + half and IJ. So this is basically calculated based on IJ + half and IJ.

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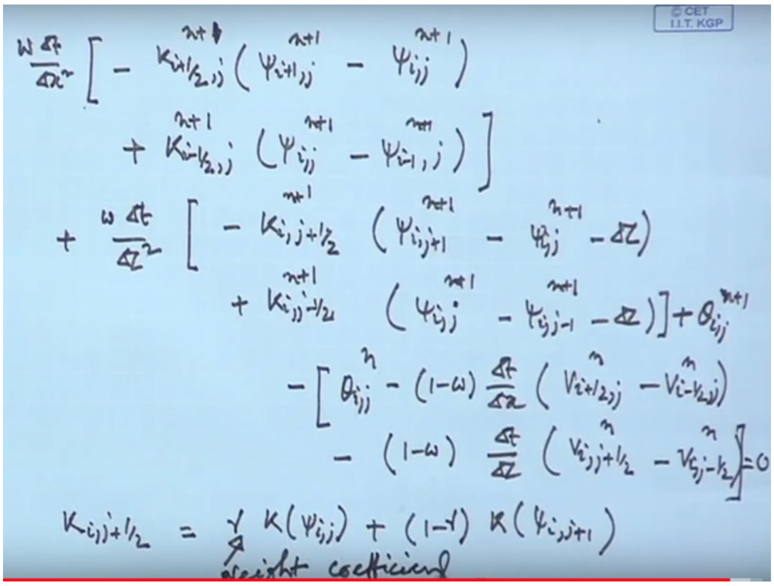

So if you discretize our original equation in terms of this one that our VIJ + half; and if we utilize this in this equation. And we can replace it here then we will get the final equation as del T del X square - KI + half N + 1 PSI I + half I + 1 N + 1 PSI IJ N + 1 + KI – J+ N PSI IJ N + 1 PSI I – 1 N+ 1J. And next one is related to this Z which is Z squared - KIJ + half. This is I + half j, I - half J. This is IJ + half.

And this is at PSI J + half N + 1 - PSI IJ at N + one level del Z + KIJ - half, both are at N + one level. PSI IJ N + half - PSI Ij - half N + 1 - del Z. So with this another term will be there that is theta IJ N + 1 - theta IJ N + 1 + W is del T / del X.

This is VI + half J N VI – half J N – 1 - W. This is del Z VJI + half N - VIJ - half N = 0. So in this one the inter block hydraulic connectivity calculation is important. So that is calculated as gamma K PSI IJ + 1 - gamma K PSI IH + 1. So gamma is weight coefficient.

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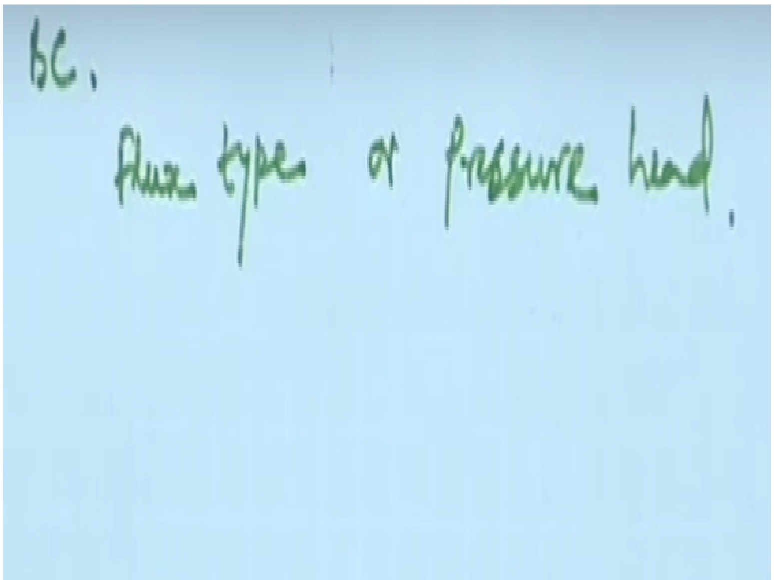

So now we can have boundary conditions. So we can have flux type or pressure head type boundary condition. So with these two boundary condition we can model that. And there will be interaction with our concept that is, for each cell there will be interaction between the surface and ground water. We need to solve our surface water flow subsurface flow at time level N.

Then surface flow calculation and we need to repeat this process to get the convergence and we need to proceed for future time levels. This subsurface flow equation can be solved using Newton absent technique. So this is the method for solving our surface water ground water interaction. But we have reached up to the zone of aeration or unsaturated zone. So we need to consider the saturated interaction with the saturated zone. So maybe in the next class or next lecture we will discuss that aspect. Thank you.