

Ground Water Hydrology
Prof. Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology – Kharagpur

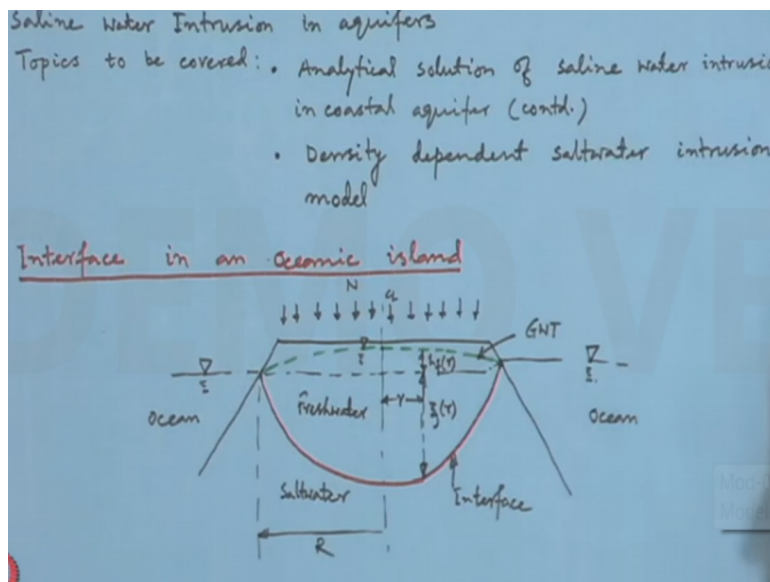
Module No # 07

Lecture No # 33

Saline Water Intrusion in Aquifers: -Analytical Solution of Saline Water Intrusion in Coastal Aquifer (contd.) -Density Dependent Saltwater Intrusion Model

Welcome to this lecture number 33 and in this particular lecture we will talk about sea Water intrusion in aquifers and topics to be covered is analytical solution of salt water intrusion coastal aquifer and we will continue from previous lecture class and we will also talk about density dependent salt water intrusion models.

(Refer Slide Time: 00:58)



So this is pure lecture number 33 and in this one we will talk about Saline water intrusion in aquifers and topics to be covered are first one as analytical solution of Saline water intrusion in coastal aquifer we will continue from previous lecture class next one is density dependent salt water intrusion model we have already talked about salt water intrusion in confined and unconfined aquifers.

Now let us derive the relationship that exists in oceanic island so interface in an oceanic island so

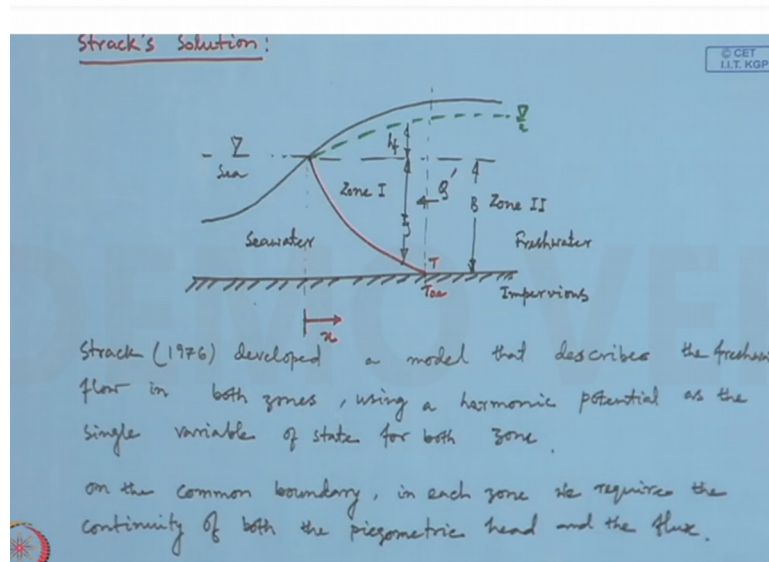
in this one let us say we have an oceanic Island which is surrounded by ocean from all the sides and the interface is like this we have assumption we have for this particular thing is that if R to capital R then we have Z value = 0.

So at the end of the oceanic island we have the sign value 0 with this let us say write as 33.1 and next one we can just write it as $0 - N \Delta^2 K F 1 + \Delta$ this is capital R square $C 2$ and we can write it as 33.2 and if we subtract this equation 33.2 from 33.1 then we can directly write it as $\sin^2 2 - N \Delta^2 K F$.

$1 \Delta R$ square + this is $N \Delta^2$ to $K F 1 + \Delta R$ square or finally we can write it as $N \Delta^2 2 K F 1 + \Delta R$ square - R square so this shows the relationship between the location of interface the recharge rate and the recharge rate and different radiuses this is radius of island this is hydraulic conductivity and interestingly in this case if we have N which is pure recharge rate.

If $N = 0$ then the \sin value is always 0 that means if there is no recharge in oceanic islands so it means an intrusion from the bottom so next thing we will discuss another important solution in case of our salt water intrusion problem that is again sharp interface approximation of the problem.

(Refer Slide Time: 18:01)



So in this case this is popular only known as stacks solution and stracks solution we consider two different zones for modeling our salt water intrusion problem in our three consecutive derivations that is for confined unconfined aquifers and our oceanic islands we have talked about single zone modeling but in case of stracks solution.

We consider two zone model for problem solving in salt water intrusion modeling so we have location of sea surface or ocean surface we have interface located as to T and we have ground water table so in this one the problem is divided based on the location of two so on the left hand side we denote this particular zone as zone 1 and right hand side we denote it as zone 2.

So in this zone 1 and zone 2 we have two different situations zone 2 consists of only fresh water aquifer and in zone 1 we have both salt water and fresh water thing that exists with a sharp interface so strap what I did it define certain potential for defining different piezometric heads so we can simplify or we can write it directly according to our previous conventions.

This is our HF this is sigh and Q prime that is entering from zone one to zone two and we have thickness of this phreatic aquifer below sea level, ocean level is B and this is our sea water position ad this is our fresh water location and this is impervious bottom with instructs solution the coordinate system is taken from left to write and it starts at the beginning of the interface so in this case we can say that the stack that is it derive the particular formation in 1976.

Developed a model that describes the fresh water flow in both zones using a harmonic potential as the single variable of state for both zones or both zone so what is this stack solution in common boundary it requires that so on the common boundary in each zone we require the continuity of both the piezometric head and the flux so the interface level we require that both piezometric head and flux.

(Refer Slide Time: 25:41)

© CET
I.I.T. KGP

Zone I (phreatic flow above interface)
 Stock's potential, ϕ , is defined as:

$$\phi = \frac{1}{2} (h_f + \bar{z}) h_f = \frac{1}{2} (B h_f + h_f) h_f = \frac{1}{2} (1+B) h_f^2$$

$$\bar{z} = \delta h_f$$

$$Q' = -K (h_f + \bar{z}) \nabla h_f = -K \nabla \phi$$

fresh water discharge per unit aquifer width.

Zone II (freshwater only)

$$\phi = \frac{1}{2} [(h_f + B) h_f - \frac{1+B}{\delta} B^2]$$

$$Q' = -K (h_f + B) \nabla h_f = -K \nabla \phi$$

at location T $\bar{z} = \delta h_f$

$$\phi = \phi_{\text{ave}} = \frac{(1+B) B^2}{2\delta}$$

So using this thing two things can be defined for both zone 1 and zone 2 so for zone 1, we have phreatic flow above interface so in this case stacks potential pie is defined as equals to HF + I into HF if we utilize our things so we can see that this is basically our half this is again we have this for HERZBERG principle.

For phreatic aquifer we have $Z = \delta$ into H so in this case for in case of HF we can replace it with $\delta HF + \delta HF + HF$ into HF so we have half $1 + \delta HF$ square and for flux Continuity we have Q' . So which is in factored form $K HF + Z$ this is HF and K tells Z and Delphi and this thing is basically fresh water discharge fresh water discharge for unit aquifer width in case of zone 2 in case of zone 2 which is fresh water only.

We can define our stacks potential as $5 HF + B$ square $1 - \delta$ by δB square and again we can define our flux thing as $Q' = -K$ this is $HF + B$ prime $HF - K 5$. So at location T at location T or at two location we have B which is = HF into δ as per our Continuity of zone 1 and zone 2 and by using HERZBERG approximation.

So in this case we have 5 which is = 52 and we can directly write it as B square / 2δ Square. So if you utilize this equation or this particular equation we will get same result because this is at the transition of zone 1 and zone 2 so using this thing or continuity of the flow is considered and for the steady state flow.

(Refer Slide Time: 31:36)

Steady state flow, $\nabla \cdot \mathbf{q}' = 0$ in each zone

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\phi = \frac{Q_0'}{K} x + \frac{Q_{w1}}{4\pi K} \ln \frac{(x-x_w)^2 + y^2}{(x+x_w)^2 + y^2}$$

$$\phi_{\text{sea}} = \frac{(1+\delta) B^2}{2\delta^2} = \frac{Q_0' x}{K} + \frac{Q_{w1}}{4\pi K} \ln \frac{(x-x_w)^2 + y^2}{(x+x_w)^2 + y^2}$$

Steady state flow condition we have $\nabla \cdot \mathbf{Q}' = 0$ in each zone so from this one we can directly write the Laplace equation for this particular case so this means that ϕ is harmonic function in the respective zone so it satisfies our Laplace equation.

So utilized this particular approximation and theory for derivation of a coastal aquifer modeling with single pumping well and he showed that that ϕ can be constructed as $\phi = \frac{Q_0'}{K} x + \frac{Q_{w1}}{4\pi K} \ln \frac{(x-x_w)^2 + y^2}{(x+x_w)^2 + y^2}$ which is location of x_w is the location of pumping well and this is can be utilized for Modeling of pumping wells.

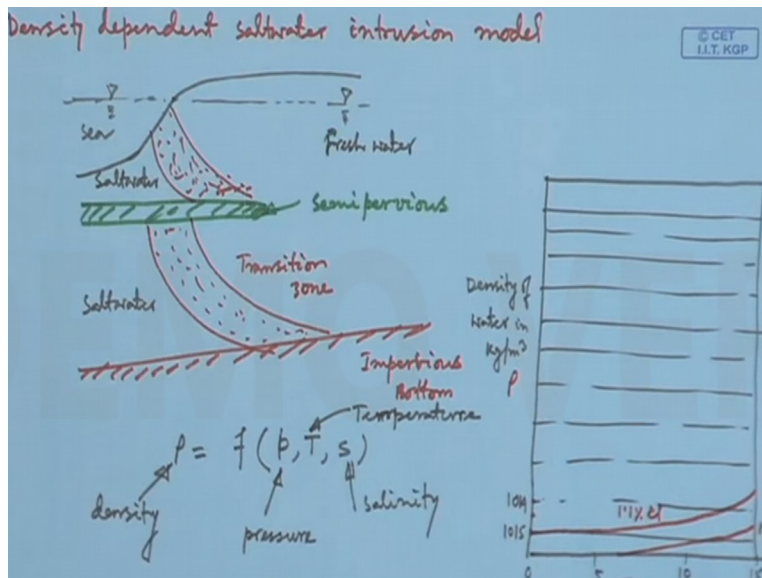
In coastal aquifers with simple approximation again this ϕ can be replaced with ϕ^2 and which is $= \frac{1}{2} \Delta P^2 \Delta^2 = \frac{Q^2}{2K} \Delta^2$ where Q we have is = pumping rate from the aquifer from with the pumping well $\frac{Q_0'}{K} x$ and this is a the flow that is occurring horizontal flow that is occurring from coastal aquifer phrase zone towards sea.

So if we write this thing we have one top view this is basically top view in tip view we can see that this is Y axis this is X axis and this is the location of the pumping well so with this location of the pumping well this is x_w for different pumping values we can have different location for this is different location for to this is ocean and this side we have this flow horizontal flow that is coming from fresh water aquifer towards sea ocean $\frac{Q_0'}{K} x$.

This is prime and this is this thing is basically K / X so as it is in pure pervious thing so initially if we have this $Q_w = 0$ the location may be like this however if we have if we have higher

pumping values so the location of interface that will change and that will be symmetric with this X axis and finally it has been observed that or specific value it shows a critical pumping location which will signify the end of no flow zone this is the influence zone of pumping and at his point shows the 0 flux situation and it is called as QC or critical pumping.

(Refer Slide Time: 39:42)



So next thing we will discuss is this density dependent flow models what is this density dependent density dependent salt water intrusion model so in this model a transition zone is considered in state of a sharp interface that is used for our sharp interface modeling using Herzberg principle let us say we have some semi pervious semi pervious location and we have another.

This is again impervious bed or impervious bottom this is the location of transition zone so in transition zone that is consider that the density is in-between freshwater and saltwater and during this kind of modeling we need to consider the composition of sea water because the composition of sea water is important for exact modeling of using exact modeling of salt water intrusion using density dependent flow models.

So in this case this density is considered to be the function of pressure temperature and salinity so this density is a function of pressure temperature and salinity so if we see he variation of density with this is density of water in kg per meter cube this is temperature is 0, 5, 10, 15 and

degree Celsius so this is with difference of one kg per meter cube this is 1014 1015 like that we have.

So with this thing it has been observed that this density varies like this red kind with we have a chloride concentration of is considered to be 1.2 % of chloride next if we increase this we decrease this chloride percentage 1.1 % of chloride and again this is 15 say in-between also there will be different curves so we can see that there is some kind of non-linear relationship between density this is temperature and this chloride which signifies the salinity for salt water.

(Refer Slide Time: 47:45)

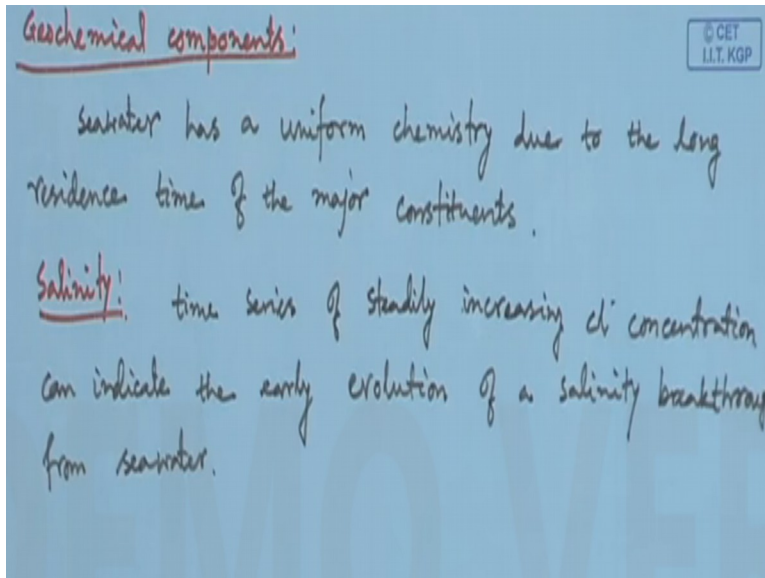
Classification of groundwater on the basis of Cl concentration

Type of GW	mg Cl/l
Oligohaline	0-5
Oligohaline-fresh	5-30
Fresh	30-150
Fresh-brackish	150-300
Brackish	300-1000
Brackish-saline	1000-10,000
Saline	10,000-20,000
Hypersaline or brine	> 20,000

So depending on the chloride concentration depending on this chloride concentration type of ground water we can define so classification of ground water on the basis of chloride concentration so type of groundwater and milligram chloride per liter first category is a oligohaline we have the range 0 to 5 oligohaline fresh we have 5 to 30 then fresh we have 3 to 150 30 to 150 fresh brackish that is 150 to 300.

Then we have brackish which is 300 to 1000 then brackish saline we have 1000 to 10,000 then saline we have this range 10000 to 20,000 and the final range is hyper saline or brine this is greater than 20,000. So this is the classification of groundwater on the basis of chloride concentration,

(Refer Slide Time: 50:58)



So different geochemical composition is required for modeling of detailed modeling of saltwater geochemical components sea water sea water has a uniform chemistry due to the long residence time of the major constituents so for this one to know something about salinity chloride is important thing.

So time series of steadily increasing chloride concentration can indicate the early evaluation of salinity breakthrough from seawater can indicate the early evolution of salinity break through from seawater so there are other components or other indicators exists that can be used for identification and evaluation of salinity level so we will discuss those topics in the next lecture class thank you.