Ground Water Hydrology
Prof. Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology – Kharaghpur

Module No # 07
Lecture No # 32
Saline Water Intrusion in Aquifers: – BODON – Ghyben – Hergberg Principle (contd.) – Analytical Solution of Saline Water Intrusion in Coastal Aquifer

Welcome to this lecture number 32.

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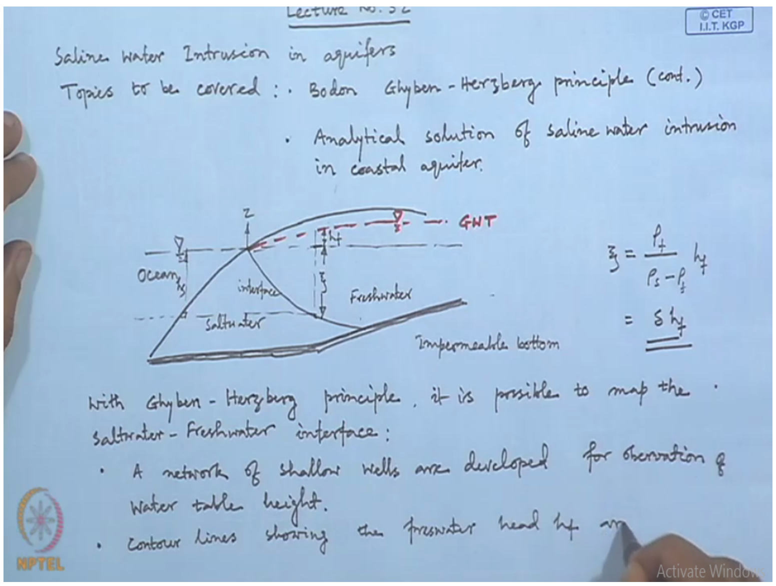

In this particular lecture we will continue our topic that is saline water intrusion in coastal aquifers and in this particular lecture we will cover our previous topic that is BODON GHYBEN HERGBERG principle. And basically we will continue from our previous lecture and second topic we will cover analytical solution of saline water intrusion in coastal aquifer.

So now at last lecture we have seen that if we have one phreatic aquifer this is basically impermeable bottom and this is the location of ocean with our Z axis pointing upward. And this is the location of interface that is interface between salt water and fresh water. And we have ground water table located at this position this is groundwater table.

So let us consider in arbitrary section in which this depth below ocean level to the position of interface we can denote it as Xi and head above ocean surface that we can denote it as HF and corresponding head in ocean we can denote it as ages. We have seen with our assumption that is GHYBEN HERGBERG principle we can express the Xi in terms of HF as HF Rho F / Rho S - Rho F into HF.

In this particular derivation we have assume that our salt water wedge that is basically stationary or stagnant. With this assumption we will continue our derivation for saline water intrusion in different coastal aquifer formations. That is first one will be confined then unconfined and for oceanic islands. So let us consider this ratio Rho FG / Rho S - Rho F as delta.

So with this assumption we can continue our derivation. So first of all let us see that how to use this BODON GHYBEN HERGBERG principle in the field to identify the salt water intrusion in coastal aquifers. So with this with GHYBEN HERGBERG principle it is possible to map the saltwater fresh water interface. So what are the steps for identification of this salt water fresh water interface?

So first point is that network of shallow wells are developed for observation of water table height. So first of all we need to install some good number of piezometers in the field to identify the location of ground water table locally. Next thing is that we can use contour plotting for this ground water table height distribution.

So contour lines showing the fresh water head HF are drawn using some interpolation method what are the interpolation method we can use one interpolation method by creasing or inverse weighting method we can utilize. So for example we can use creasing or IDW or any recent techniques for plotting our contour lines for the fresh water head HF.

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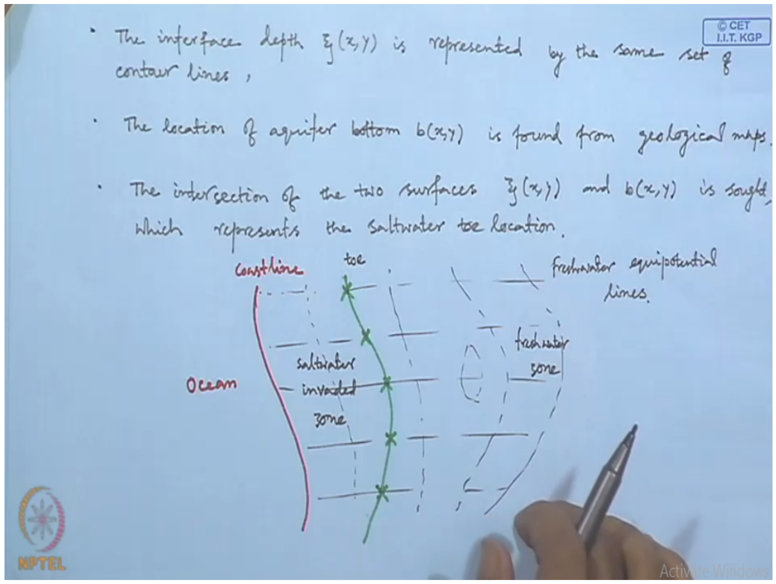

Then the interface depth the interface depth Xi which is a function of X and Y is represented by the same set of contour lines. And this is drawn within the plotting and fourth one the location of aquifer bottom aquifer bottom which is again function DXY is found from geological maps. And final one the intersection of these two surfaces that is the intersection of the two surfaces Xi XY and BXY is sought which represents the salt water toe location.

So with this method let us say we have our coastline like this which is coastline and this is our ocean and with dotted lines and representing the fresh water equipotential lines. These are basically fresh water zone these lines are fresh water fresh water lines and location of toe that is that is the intersection of two surfaces may be located as this. So this is basically location of toe so for this particular location salt water has entered into coastal aquifer.

So salt water invaded zone so we have salt water invaded zone then we our coast line like this which is coast line. And this is our ocean and with dotted lines and representing the freshwater equipotential lines. These are basically fresh water zone these lines are fresh water fresh water lines and location of toe that is that is the intersection of two surfaces may be located as this so this is basically location of toe.

So for this particular location salt water has entered into coastal aquifer so salt water invaded zone. So we have salt water invaded zone then we have fresh water zone. And these are our fresh water equipotential lines. So using GHYBEN HERGBERG principle we can easily identify toe location in the field however for complex geological formation and non stationary interface condition this is not valid. GHYBEN HERGBERG principle considers stationary salt water salt water wedge assumption for its derivation.

However in complete geological conditions we cannot directly use that assumption for practical modeling or practical use so next thing we will try to find out the analytical solutions for our coastal aquifers.

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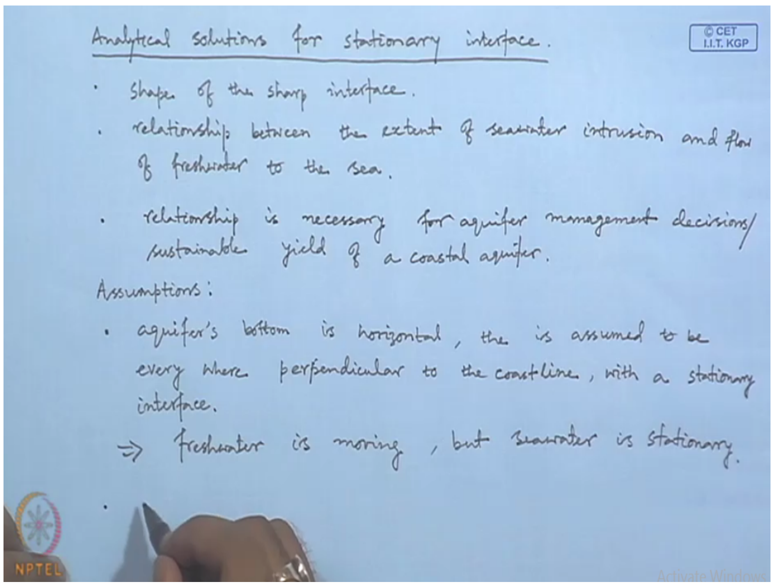

So for analytical solutions analytical solutions for stationary interface so basically for these solutions will utilize our GHYBEN HERGBERG principle. And we will try to see the simplified analytical solutions for simple geological condition. So first thing with this analytical solution what we can do we can identify the shape of the sharp interface. Shape of the sharp interface next we can find out the relationship between the extent of sea water intrusion and flow of fresh water to the sea.

And why this relationship is necessary this relationship is necessary for taking aquifer management decisions and sustainable yield identification for coastal aquifers. So this relationship is necessary for aquifer management decisions or for identification of sustainable yield of a coastal aquifer. So we will use certain assumptions for our derivation. So what are these assumptions first of all we will consider that aquifers bottom is horizontal. So aquifer bottom is horizontal the flow is assumed to be everywhere perpendicular to coast line with a stationary interface.

That means our fresh water is moving but sea water is stationary and we will consider that sea water wedge length L and one important assumption is that dupuit assumption. Essentially horizontal flow is valid dupuit assumption of essentially horizontal flow is valid so let us consider a case of confined aquifer and for this condition we will try to find out the nature of interface.

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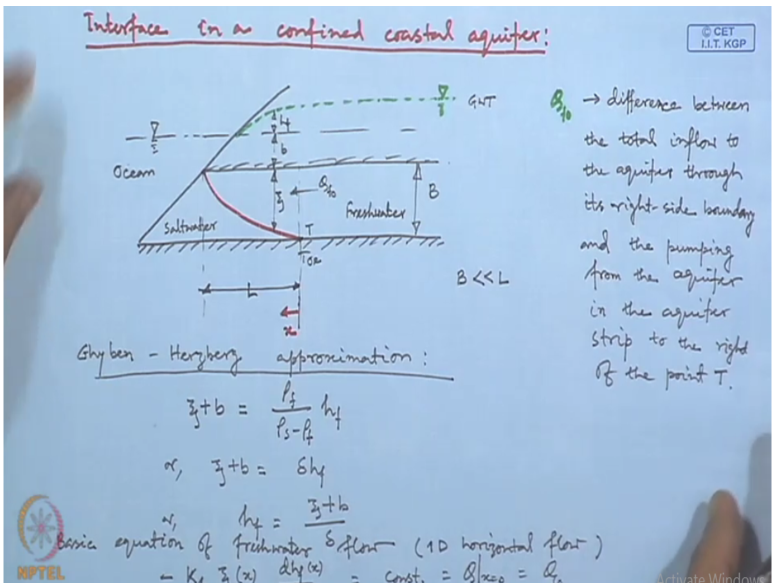

So interface in a confined coastal aquifer so interface in a confined coastal aquifer so let us draw a conceptual thing we have horizontal bottom which ensures the horizontal flow in the aquifer towards sea. So thickness of this aquifer is B again we have the interface this point T or capital T is called as toe location. Then length of this wedge that we have already assumed to be L then location of sea surface or ocean surface we have this.

And location of this ground water table is this green thing. So for a particular location we have three different distances. First one, we denote it with Xi next one this is constant depth between the ocean surface and the aquifer top that is B and this HF is the fresh water head above ocean surface up to ground water level. So we can consider this zone as fresh water zone and this one is basically salt water zone.

So with this configuration we can consider that there is horizontal flow which is occurring from aquifer towards the sea and the X axis starts from this toe location towards the sea. So with this fundamental assumptions and configuration we can start our derivation. What is this QF not in this position this is the difference between the total inflow to the aquifer through its right side boundary and the pumping from the aquifer in the aquifer strip to the right of the point T or location of toe.

So next thing in this particular case another assumption is there that is B is much much lesser than our wedge length and B is the thickness of aquifer. So using our GHYBEN HERGBERG approximation we have this HF is the fresh water head above ocean surface up to this ground water table. So we can express this Xi + B that are the depth below ocean level up to interface.

That we can denote as denote it as Xi + B = Rho F / Rho S - Rho F into HF or in simplified form Xi + B to delta HF or HF we can directly use it as B + Xi + B by delta. So for the small configuration we can say that our fresh water zone we have flow which is KF or we can write it as basic equation of fresh water flow.

And this is 1 D and only horizontal flow condition we have KF Xi X D H X / DX = constant. What is this constant this is Q or flow at location X = 0 which is = QF not. So what is this is basically an overflow or dartian flow is equals to the flow which is coming from right hand side boundary. And basically we have seen this is the difference between total inflow to the aquifer from the right hand side boundary and pumping from the aquifer.

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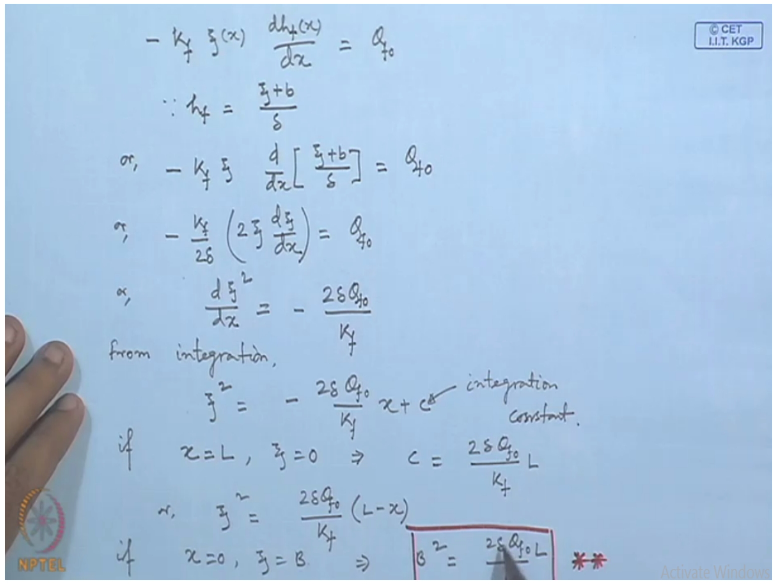

So using this thing we can write again we will just write KF Xi DHF X DX = QF not for this particular situation now we can use our GHYBEN HERGBERG principle. So we can replace this HF with Xi + P / delta or KF Xi + D / DX B / delta = F or this is KF / 2 2 Xi D Xi / DX = QF not or this thing we can directly write it as Xi square DX and the right hand side will have delta QF not by KF.

With this we can use integration so from integration we have Xi square = delta QF not / KF 2X + C where C is the integration constant. Now if we have X = L then this X value is 0 which implies that C = 2 delta QF / KF into L so final equation we can write it as this Xi square = 2 delta QF / KF L – X.

So important thing is that we have considered our coordinate system from two location towards left. So if we have X = 0 that is the location of toe we have Y = B which is again B is the depth of the confined aquifer. So this implies that we can write it as B square 2 delta QF not / KF into L because X = 0.

So basically this particular relation relates with our depth of the confined aquifer this is Del is the density ratio which is used for GHYBEN HERGBERG principle. KF is the fresh water hydraulic conductivity QF as we have defined in our assumptions and L is the sea water wedge length so this is somewhat parabolic in nature. So we can say that salt water interface in confined aquifer is parabolic in nature.

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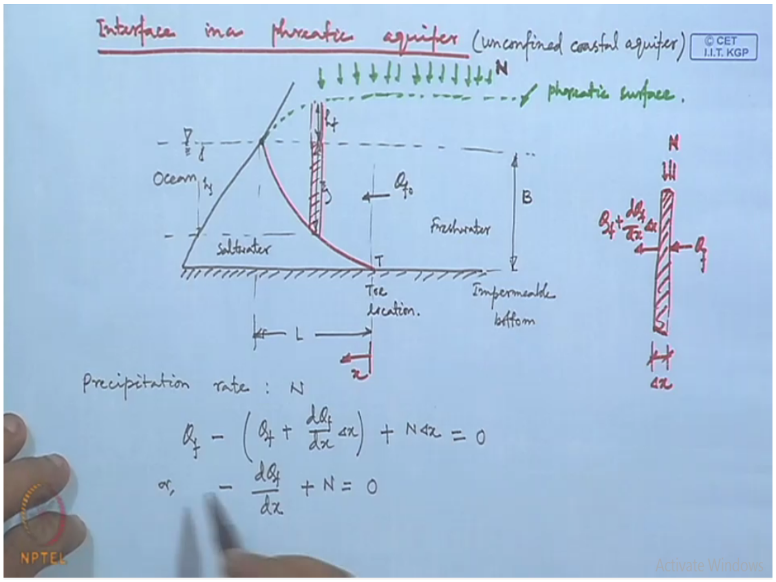

So next thing we will start our interface for interface in a phreatic aquifer. So in this phreatic aquifer this is also our unconfined aquifer unconfined coastal aquifer. Again we can draw our basic configuration for the problem we have horizontal impermeable bottom this is the location of ocean and location of interface can be denoted with this red line. And this is T or toe location with salt water and fresh water thing.

And this is width B which is depth below ocean surface level up to impermeable bottom. Again we have this ground water table which is phreatic surface and we can assume some constant recharge rate N. So with this height ground water fresh ground water table above ocean level is HF and below this ocean surface level till the interface we have distance Xi and salt water wedge length this is again L.

And our coordinate system we assuming from the toe location towards left and we have again this QF not which is the difference between the flow which is coming from right side boundary and this is the difference between flow and pumping. So with this situation this is HS and this precipitation rate or recharge rate this precipitation rate we can consider it as N.

So within this our interface if we consider one small elemental portion then we will see that for this elemental portion we have this is N which is recharge from the top and QF is a flow coming to this strip. And QF + DQF / DX into Del X. This is the distance the flow which is coming from left boundary and which is going out from which is coming from the right boundary and going out from the left boundary.

So if we do mass balance for this particular configuration then we will see that QF - QF + DQF by DX into delta X + we have N into delta X = 0 or we have situation where DQ F / DX. This is N or 0 so if we use our dartian flow conditions then we can easily see that this QF is basically - KF into HF + Xi into DH / DX. So this QF at the right hand side boundary we have this value QF is + QF not.

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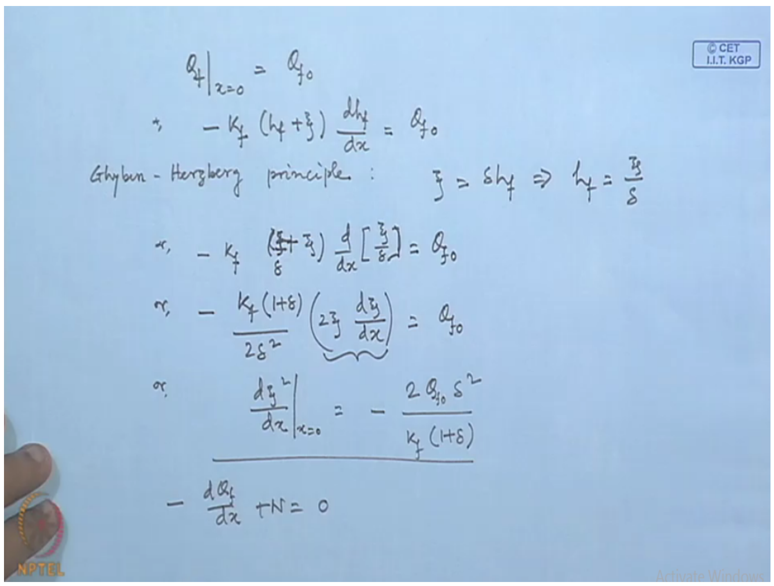

So at this right hand side boundary QF at X + 0 + QF not or we can say that - KF HF + Xi DH / DX = KF not. Now if we apply our GHYBEN HERGBERG principle we have Xi = delta + HF for unconfined aquifer or HF is = Xi / delta. Now using this GHYBEN HERGBERG principle relationship we can simplify our basic equation as this - KF this is Xi / delta + Xi and this is D by DX Xi / delta or we can say that KF 1 + delta / delta square into 2 Xi D Xi / DX.

This is + QF not or we can simplify this particular term as D Xi square DX and X not = 0 sorry 0 - 2 QS not F delta square divided by KS 1 + delta KF 1 + delta. Again from our fundamental relationship that is we have derived that is KQ F DX N = 0 and in this one if we substitute here QF then we will get D / DX which is - KF HF Xi DHF DX + N = 0. This is our relation number one or 32.1.

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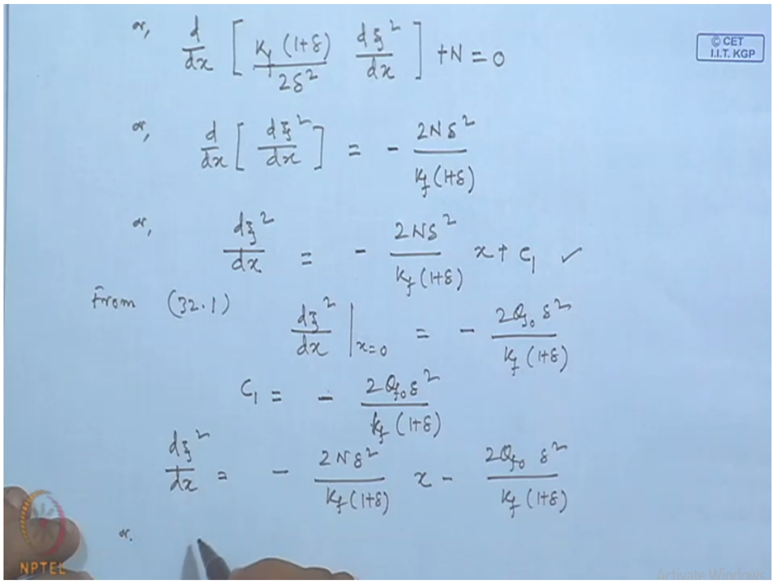

Let us denote it and from this one if we substitute our GHYBEN HERGBERG principle relationship so KF again this is 1 + delta divided by delta square DX + N = 0 or we can have a relationship where this D / DX Xi DX = - N 2 and delta square divided by KF 1 + delta. So from this one we can derive this spare X = - 2 N delta square KF 1 + delta X + C 1.

So from our relationship from 32.1 we have seen that D of Xi square DX at X 0 or X = 0 X = to 0 this value is equals to 2 QF delta square by KF 1 + delta. So if we utilize this thing for our case then we have C 1 is = - 2 QF not delta square by KF 1 + delta. So finally this particular equation can be written as 2 Q not F delta square KF 1 + delta. So again integrating this one we will get Xi square = - 2 so 2 2 will cancel this X square so we have N delta square KF 1 + delta X square - 2 QF delta square KF 1 + delta X + C 2.

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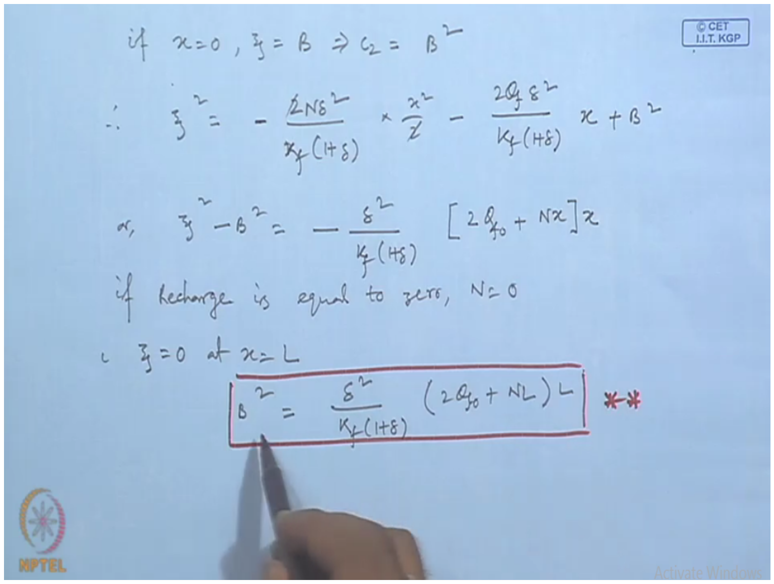

This is again one integration constant so if we have X = 0 we will get Xi = B. So with this one we can get that C 2 = B square. So finally we can write our equation as Xi square = - 2 N delta square / KF 1 + delta into X square / 2. So 2 2 cancels - QF delta square / KF 1 + delta X + B square or Xi square - B square this is - delta / KF 1 + delta this is 2 QF not + NX into X.

If we have recharge is equal to 0 then N = 0. And we can get simplified relationship with N = 0 and if we have Xi = 0 at X = l. We can get this is B square = delta square KF 2 Q F not + 10 L into L. So again we have found out one relationship between depths of this phreatic aquifer from ocean surface to impermeable bottom delta is the GHYBEN HERGBERG density relationship.

KF is freshwater hydraulic conductivity QF as we have defined in our assumptions and N is the recharge L is the length of the wedge. So with this relationship we can easily find out the shape of the wedge and this is the final relationship between different parameters in confined aquifer. So we can utilize these things for our calculations so we can conclude our lecture with this particular confined aquifer unconfined aquifer condition unconfined aquifer condition that we have derived here.

And in the next lecture we will start with case where we will try to find out the ocean island case in which we will try to utilize the condition for a symmetric flow symmetric radial flow conditions thank you.