Ground Water Hydrology
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Module No # 03
Lecture No # 11
1-D Unconfined Ground water Flows; Steady Flow into Wells

Welcome to this lecture number 11 and which will continue with the 1D ground water flow in an unconfined aquifer.

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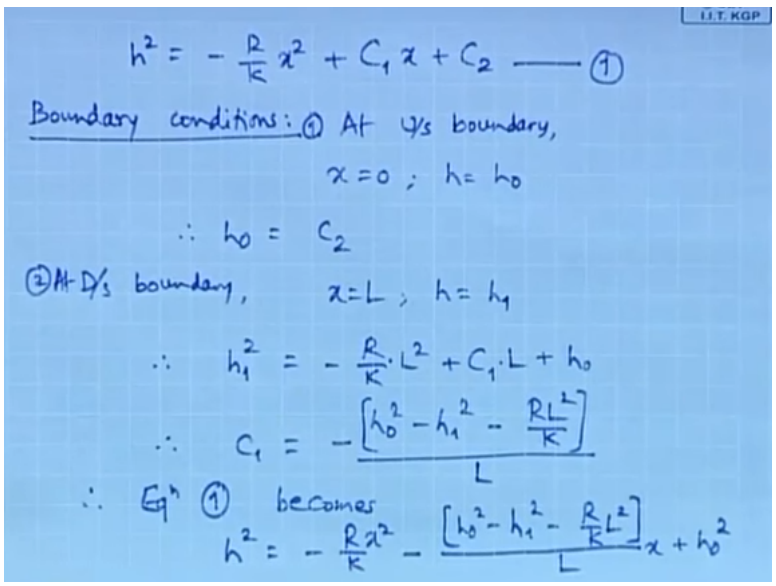

And firstly let us consider the case of 1D ground water and confined ground water flow between two water bodies having a discharge with the discharge in the vertically downward direction and here we know that when there is 1D ground water flow the obviously the flow is in the X direction and in this case the governing equation will be D square H / DX square and since the flow is in the X direction.

So therefore the second order partial derivative terms in the Y & Z direction they will not be there and this will be = - 2R / K where R is the rate of recharge and this is in the vertically downward direction and K is the aquifer permeability or the hydraulic conductivity. So this is the governing equation and so this equation if we I am sorry so they I made a small less 1. So this is a second partial derivative of H square so this is D square DX square of H square = - 2R / K.

So not let us integrate this governing equation price so we get X square = - R / KX square + C1 X + C2. So here the C1 and C2 are the constants of integration. Now let us apply the boundary conditions so the boundary conditions so at the upstream boundary upstream boundary so X = 0 H = H0 therefore so H0 = C2 so this is the first boundary condition the second boundary that is the downstream boundary at the downstream boundary we have X = L and at that location head H = H1 which is the head the ground water head in downstream water body.

So now so therefore here we get this H1 square will be = - R / KL Square + C1 into L and C2 = H0. So therefore here you can write down so this C1 = -X0 square – H1 square – RL square by K. So this whole thing divided by L So therefore if we call this equation one therefore equation one becomes H square = - RX square / 2 or R / KX square – of H0 square – H0 Square – R / K into L square / L this into X + H0 square.

So this is the expression for the square of the head the ground water head in an unconfined having 1D flow between two water bodies having a head of H0 in the round stream water body having a head of H1. And there is also a vertically downward recharge in the vertically downward direction with an intensity of R and the aquifer is a homogenous isotropy with a hydraulic conductivity of K the distance between upstream and downstream aquifer X is L now the upstream and downstream water bodies is L.

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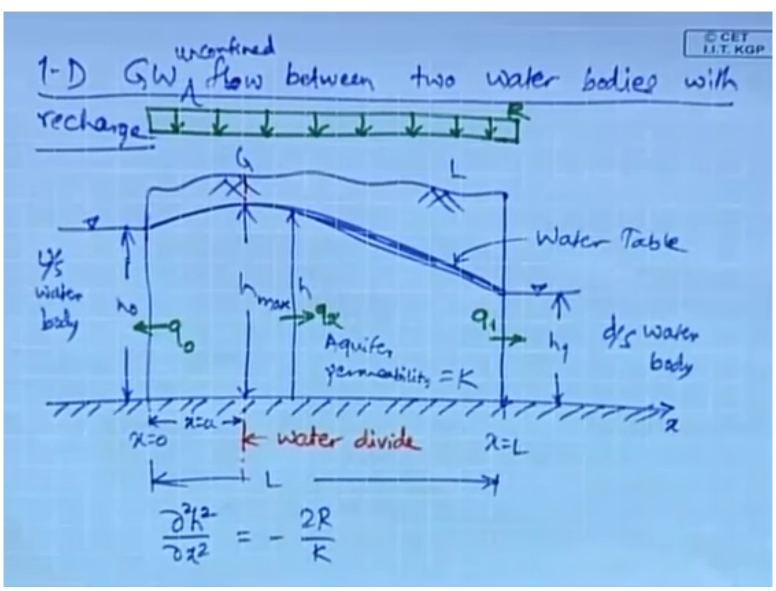

So here as you can see from the from the figure due to this concept or uniform rate of recharge over in the vertically downward direction the water shows the hump somewhere in the middle where the this H values maximum = H Max. And so from there he water table dips slightly towards the upstream water body to the left and then more towards the water body to the right of this and this this section is known as the water divide.

So this is known as the water divide wherein the H is maximum the ground water head is maximum and the to the left of that the water will be flowing towards the upstream water body into the right of that the groundwater will be flowing towards the downstream water body and now let us and this the water divide so let us denote the distance of this water divide at X = A from the upstream water body.

So now let us find out the expression for this this the distance A of the water divide from the upstream water body and we know that the expression is so this is the expression and if we take the derivative of H with respect to X and equate that derivative to 0. So that will give the condition for maximum that is H = H Max so we will get the expression for this one.

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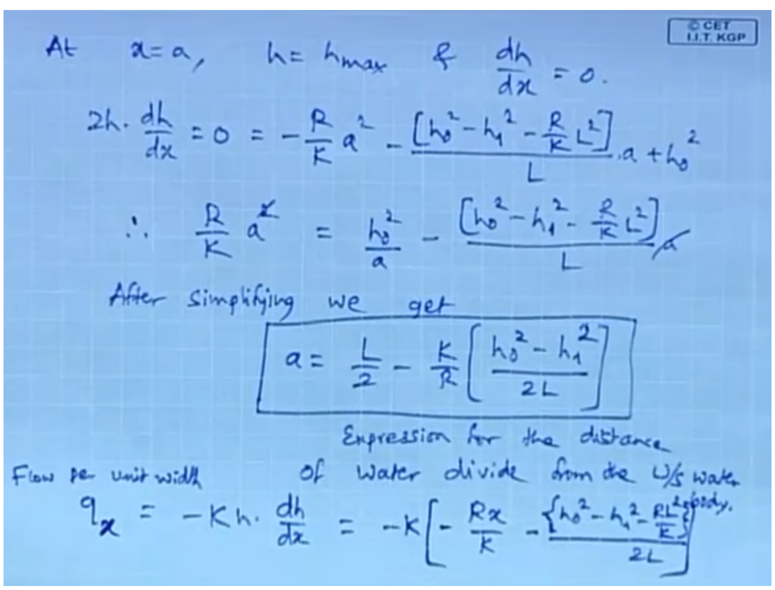

So we know that at X = A H = H max DH / DX = 0 so this is the condition for maximum. So therefore let us differentiate this expression so that is 2H into DH / DX this is equal to so this and let us substitute because this is DH / DX which is 0. So therefore we are equating it to 0 and then the right hand side and that in place of X we have to substitute the value of A.

So therefore here on the right hand side we get – R / K into A square this is H0 Square – H1 square – R / K into L square this divided by L into A + H0 square. So therefore we get so this is R / K A square which = – H0 square – H1 square – R / K L square into divided by L into A. So if you substitute this if we simplify this after simplifying this here we will get and obviously here we can cancel out 1A here and so when we cancelling out this A.

So obviously here there will be a here so after simplifying we get this is = L / 2 – K / R into H0 square – H1 square divided by 2L. So this is the expression for the distance of this water divided from the upstream water body okay. So now we will get the what will do is let us find the expression for this the flow from the water divide from the left of the water divide as well as to the right of the water divide and obviously at the water divide there is no flow.

Now let us write down so this is the so flow per unit width that is QX is given by K into H into TH / DX and in this case so this is – K and H into DH / DX is given by that is – RX / K – so here we get this is H0 square – H1 square – RL square / K this whole thing divided by 2L. So this is the expression for QX and here in this case we are taking this one the substituting the value of H into DH / DX and multiplying it with – K.

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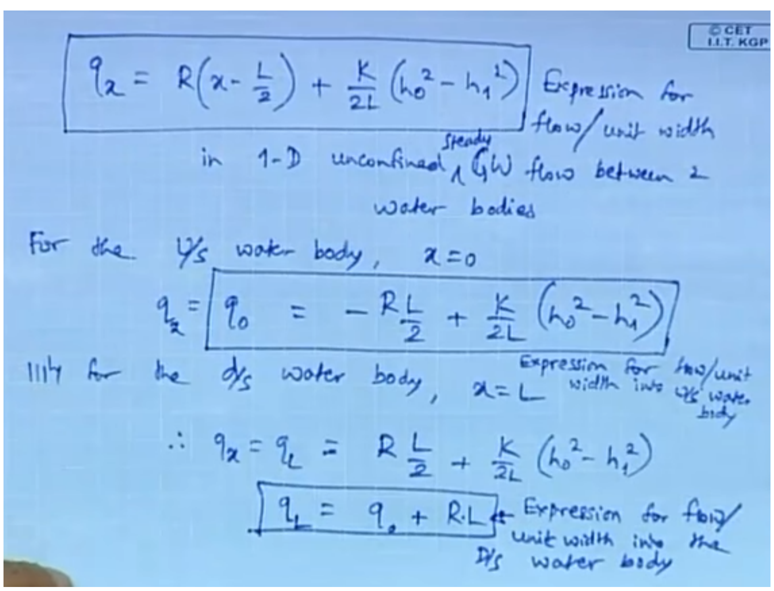

So we get this expression so now if we simplify this so we get this QX will be = R into X – L / 2 + K / 2L into H0 square – H1 square. So this is the expression for QX now to get the upstream and the downstream the flow per unit width into the upstream water body and the downstream water body we need to substitute X = 0 to get the flow pet unit width into the upstream water body and we need to get X = L substitute X = L to get the expression for flow per unit width into the downstream water body.

So here you can write down this this is the expression for flow per unit width in 1D unconfined flow and confine study flow steady ground water flow between two water bodies. So now for the upstream water body Q = Q0 and which is simply given by substituting X = 0 in this expression so therefore this Q0 is given by- RL / 2 + K / 2L into H0 square – H1 square. Similarly for the downstream water body so for the upstream water body X = 0 and for the downstream water body X = L.

So therefore this QX in this case QX = QL which is = this is R into L / 2 here X in place of X we are substituting LR into L by 2 + K / 2L into H0 square – H1 square. So therefore this can be expressed as this QL = Q0 + RL that means the flow into the downstream water body.

Obviously it has to be more because there is more gradient between the water divide and the downstream water body so that is obtained by adding this the terms the product R into L the recharge intensity as well as the length the distance between the two water bodies upstream and downstream water bodies to the flow per unit width into upstream water body.

So therefore so here we get this is the so this Q0 so this is the expression for flow per unit width into upstream water body. And obviously this is for say 1D confined steady ground water flow and this is the expression for flow per unit width into the downstream water body and obviously this is also for 1D confined steady ground water flow. So this is the expression for this one so we have a discussed the 1D ground water flow without a with recharge.

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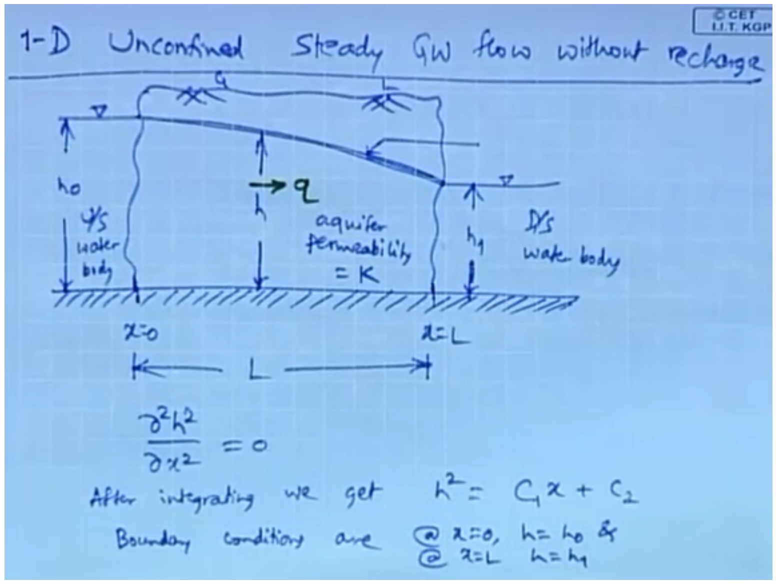

Now let us come to this another case that is the 1D unconfined steady ground water flow without recharge. So here we know that so in this case so let me draw the figure here in this case this is the bottom confining layer for the unconfined aquifer and then there is upstream water and then this is the general ground here and then there is a downstream water body.

So this upstream water body has a head of H0 so this is the upstream water body has a head of H1 so this is the downstream water body and here this is the ground level. And obviously since there is no recharge so the water table will simply will assume a curve curved shape and here X = 0 and at the downstream at X = L. So this is the distance L between the upstream and downstream water bodies and here so at any general point so this is H and obviously let us say at this the at this general section X.

The discharge per unit width Q and this aquifer permeability so permeability aquifer permeability = K. So now here in this case the expression that is the in this case the governing equation will be simply that is a D square H square / DX square will be = 0 and therefore if you differentiate this one twice or twice if you integrate this twice. So after integrating we get X square = C1X + C2 and again the condition of there the boundary conditions are at X= 0, H = H0 and at X = L, H = H1.

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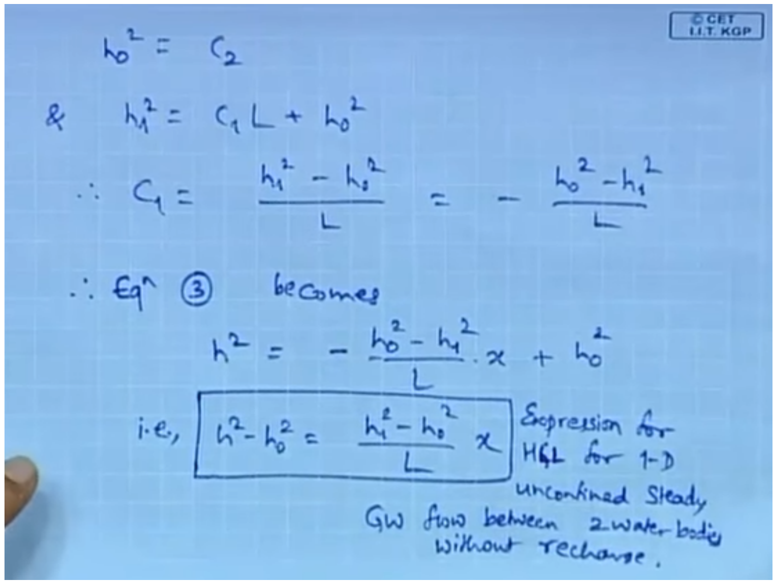

So here so therefore so this is H0 square = C2 and H1 square = C1L + C2, C2 in this case is H0 square therefore C1 = H1 square – H0 square / L which you can write this as – H0 square – H1 square / L. So therefore so this equation if you call this as if I call this as equation say three so this equation three becomes so H square = -H0 – H1 square / L into X + H0 square.

Or in other words H square – H0 square = H1 square – H0 square / L into X so this is the so this is the expression for hydraulic grade line for 1D confined steady ground water flow between two water bodies without recharge. So it gets simplified so therefore we can also write if you differentiate this so the we get this 2H into DH / DX and so that can be half of that term can be used can be multiplied with K so we will get the discharge per unit length unit width.

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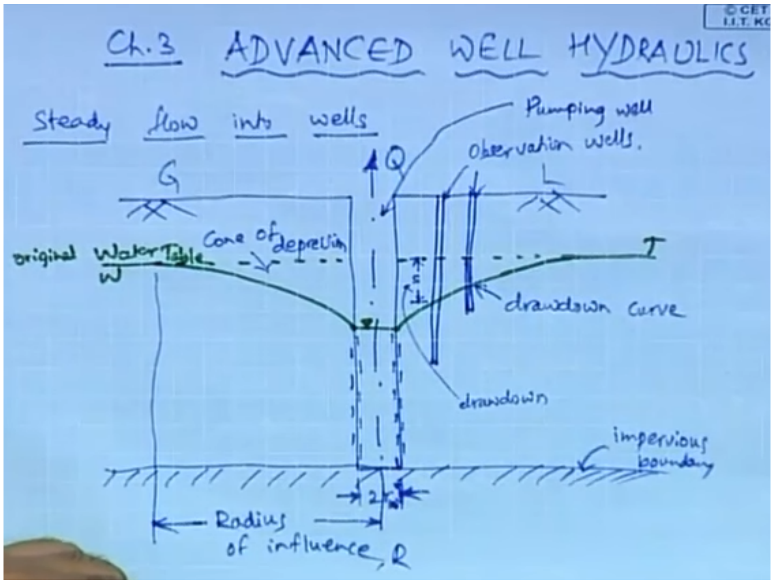

So the discharge per unit width so that is Q so this is = -KH DH / DX so this is = - K and X into DH / DX is simply given by it is H1 square – H0 square divided by 2L into so that is the H into DH / DX. So therefore the discharge per unit width so this is the expression for so this is the expression for the discharge per unit width. So for 1D unconfined steady ground water flow between two water bodies without recharge okay.

So therefore so thus we have come to the end of the second chapter that is an occurrence in the moment of ground water now we will go to the third chapter that is the advanced well hydraulics and in this case specifically we will discuss with the initially we will start with the flow through the wells and that too initially the steady flow through the wells.

So we are entering the third chapter that is the well once well hydraulics so this is chapter three of this NPTEL video course on ground water hydrology. So this chapter three is on advanced well hydraulics and specifically we start with the steady flow through the wells steady flow into the wells. So firstly we will consider the confined aquifer wells in the confined aquifer and then we go for the wells in the unconfined aquifer.

And firstly let me start with the basic figure of the diagrammatic representation of a well which is used for extracting of groundwater. So this is the well and let us consider this well to the fully penetrating aquifer. So this aquifer may be confined aquifer or an unconfined aquifer and in this case this is the ground level and here what happens is so this is the water table the original water table.

So this is the original water which is a horizontal in this case and then because of this pumping so this water table will show a depression like this and this is known as cone of depression. So this is the cone of depression so this is the steady the rate of pumping from this well and obviously the flow is radially there is a radially symmetrically flow into this well and the diameter of this well let us consider the diameter to be 2RW and here so this is the water surface and this this curve is known as the drawdown curve.

Obviously when there is no pumping so the original or the static water table is a horizontal surface with no drawdown and no cone of depression and in this case suppose we have a an observation well let us say this is one observation well and this is another observation well which we are so in this case the draw down in this observation well that is basically the head difference between original water table and the depressed water table.

So this S is known as the drawdown and the maximum radius up to which the drawdown is felt. So this is from the axis of the well is known as the radius of influence and in this case here say we may obviously these are the strainers basically well casing with perforations so through which the ground water enters radially in a radially inwards flow and obviously so this is axisymmetric flow.

So it is symmetrical in all the directions because we are assuming the aquifer to be homogenous and isotropic and in this case of course I have considered this is an unconfined aquifer and same thing can also be shown as a confined aquifer. So this is the original and this is the water table that is WT and these are terminology here RW the well radius and this radius of influence.

So this is generally denoted by R and this is the impervious boundary in this case the bottom impervious boundary of the aquifer and then this is draw down curve and then this is the cone of depression and this is the pumping well with a steady pumping rate of Q. And these two are the observation wells so these are the observation wells and this is the pumping well and it is also known as the dischargeable.

So this is the typical sketch indicating all the terminology and let us consider first the steady flow into a well fully penetrating a confined aquifer.

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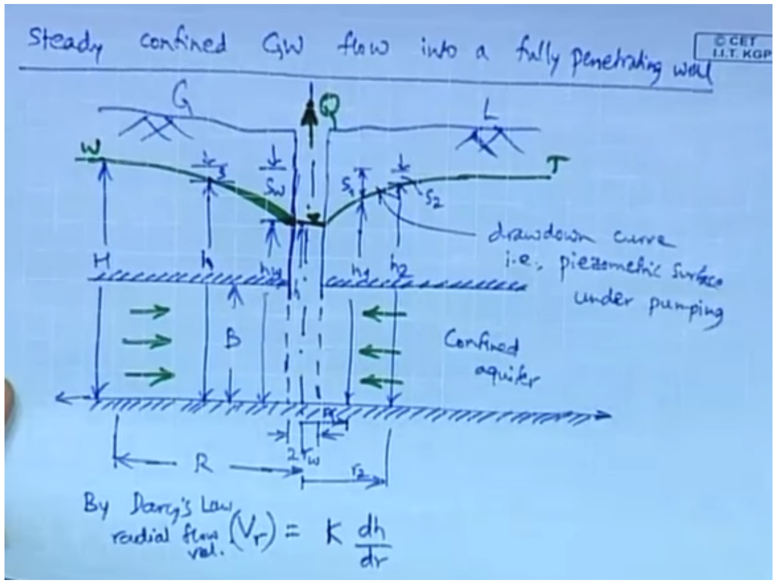

Let us consider steady confined ground water flow into a fully penetrating well in this case the well is penetrating for the entire depth of the confined aquifer. So let us draw the sketch here so this is the bottom impervious layer of the confined aquifer and here so this is the well. In this case let us say this is the top confining layer of this confined aquifer and obviously so here the strainers basically well casing with perforations and so this is the confined aquifer.

And here the total let us consider this is the ground level and here let us consider the so this is the water table. So this is the water table with the draw down and the total height of the water table with respect to the bottom confining layer of this aquifer is H and here the flow is radially inward flow and by Dupuit's assumptions. We are assuming the streamlines to be horizontal and this well which is penetrated in the confined aquifer has a diameter or 2RW and this is the radius of the influence is R and at any general points.

So the drawdowns are S1 and this is H1 and then say for the second one so this is a S2 and this is H2. And obviously at the well so this is a HW and this drawdown here is a SW and at any general point the drawdown is S and the variable head is denoted by small H and this is the pumping well with the discharge it is Q. So this is a fully penetrating well through a confined aquifer and then there is a steady ground water flow into this confined aquifer.

And obviously here this S1 so this is at a radius of R1 and similarly this S2 the draw down S2 is observed at observation well which is at a radial distance of R2 and the let the thickness of confined aquifer be B. And obviously this one that is the radial direction is the axis this one is radially the there is a radially involved flow through out and so this are the drawdown curve represents the piezometric surface.

So here this is the draw down curve that is piezometric surface under pumping so now for such a well which is having which is receiving a steady flow which is radially inward flow and then the flow is axisymmetric. So the flow is symmetrical about the vertical axis of this fully penetrating well. So now let us write down the expression for radial velocity VR and this is simply given by the Darcy's expression by Darcy's law radial flow velocity that is VR is simply given by K into DH / DR.

K is the aquifer permeability and then DH / DR is the hydraulic and so this is the expression for velocity and if you multiply this by area of flow.

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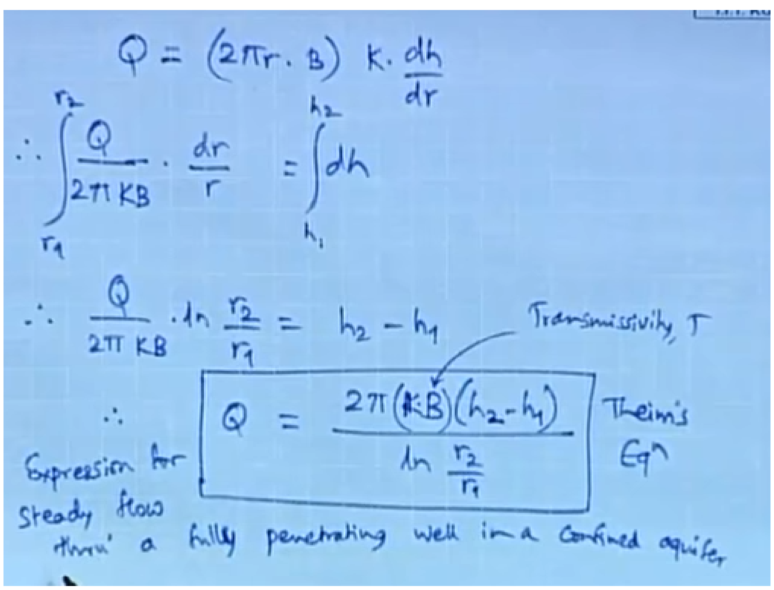

So in this we will get the expression for the steady rate of pumping so this Q is simply given by 2PieR into B so this is the area of flow for the radially inward flow into K into DH / DR. So this is the area multiplied by velocity that will give the expression for the steady flow rate into this well. So therefore let us rewrite this terms so this Q / 2 PIE KB into DR / R this = DH.

And this we need to integrate between the appropriate limits so in this case the limits are between say R1 and R2 the lower limit is R1 the upper limit is R2 and then similarly for the right hand side that lower limit is H1 and upper limit is H2. So this case we get so this is Q divided by 2 PIE KD into natural log of R2 / R1. So this = H2 – H1 so if we rewrite this one so this the expression for the steady flow rate which is given by 2PIE KB into H2 – H1 divided by natural log of R2 / R1.

So this is the expression for a steady flow through a fully penetrating steady fully penetrating well in a confined aquifer and this is generally known as the themes equation after the hydraulic hydraulicien theme who initially proposed this equation. So and obviously so this is K into this B so this can be replaced by this tranmissivity T. So if we know the transmissivity and then the depths of the water table above the unconfined the above the lower confining layer in two observation wells and also the radial distance from the center of the vertical well axis.

So then we can determine the expression flow through the fully penetrating well this also be equal to the steady discharge which is given out by this which this well is giving out so in that equilibrium condition the amount of for the inflow which is through the radially inward direction that is equal to the pump age the rate of pumping through the this well.

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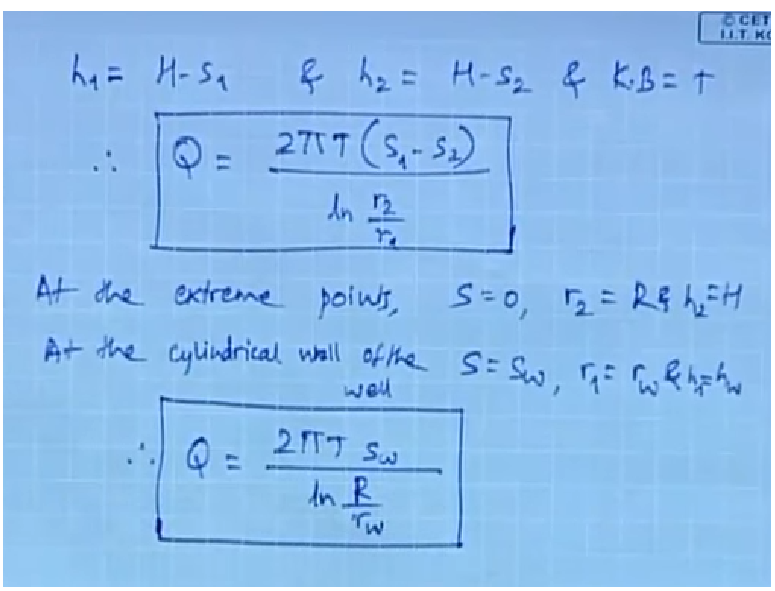

And so in this case so this H1 they head at the observation well the peizometric head at the observation well is simply given by H – S1 and H2 = H – H2. So therefore this K into B = T so therefore we can write down this Q = 2 PIE the tranmissivity or transmissibility multiplied by so this is S1 – S2 divided by natural log of R2 / R1. So this is another expression for the steady flow into a fully penetrating well through a confined aquifer.

And suppose we consider the extreme points that means we consider the downstream extreme points which is the radial surface the cylindrical curved cylindrical curved cylindrical surface of the well and the upstream extreme points which represents the point on the circle of influence which is at the radial distance of R the radius of influence.

So in that so we get at the edge at the extreme points this the drawdown S = 0 and R2 = R which is the radius of the influence and H2 = H at the cylindrical wall of the well this S = SW, R1 = RW and H1 = HW. So therefore we can write down that Q = 2PIE into T into SW divided by natural log of R divided by RW.

So this is another expression wherein if we know the drawdown in the well the radius of influence as well as well radius. So then we can and of course the transmissibility we can get the expression for the discharge through the steady flow rate through this fully penetrated well in a confined aquifer which is the same as the rate at which the water is pumped out through this well.

Because the flow is steady obviously inflow is equal to outflow so in the next class we will continue with the steady flow through an unconfined through a fully penetrating well in an unconfined aquifer thank you.