Ground Water Hydrology
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Module No # 02
Lecture No # 10
General Flow Equations through Porous Media (Contd.), Dupuit's Assumptions

Welcome to this lecture number 10.

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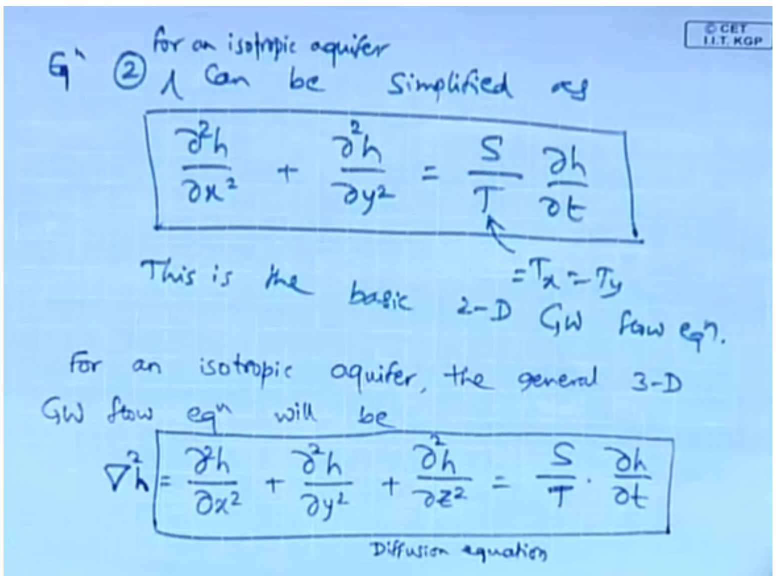

So we are continuing with the previous lecture so where in we went for we derived an expression for the two dimensional general ground water flow equation for an isotropic aquifer. So which is of the form D square H / DX square + D square H / Y square = S / T into DH / DT where here H represent the H, X and Y represents the flow direction S is the storativity T is transmissivity and since it is a isotropic aquifer the transmissivity in both the directions DX and DY are same = T.

And this DH represents the rate of change of head with time the partial derivative of the rate of change of head with time. Suppose it is a three dimensional flow and the flow in the aquifer is isotropic so in this case will be the third term also so therefore this the for an isotropic aquifer the general 3D groundwater flow equation will be D square H / DX square + P square H / DY square + D square H DZ square = S / T into DH / DT.

So this is the expression and obviously in this case the left hand is denoted by del square H so which is the which is the combined notation for these second order partial derivative terms in with respect to each of the directions and so this is when the flow is unsteady flow. So this is of the form this is known as the diffusion equation and when the flow is steady then the DH / DT will become 0 then the right hand side will become 0 and then this equation attains the form of Laplace equation okay.

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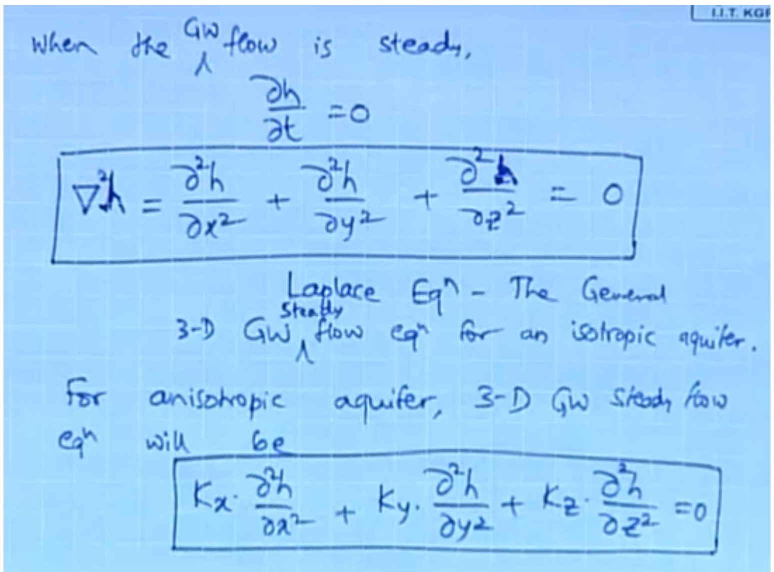

So when the flow ground water flow is steady in that case this DH / DT = 0 so therefore D square H / DX square + D square H / DY square + D square Z D I am sorry D square H / DZ square = 0 and obviously this is del square of H. So del square H which is = D square H / DX square + D square H / DY square + D square H / DZ square which is = 0.

So this is the famous Laplace equation and so this is the general ground water flow equation general 3D 3 dimensional ground water flow equation here we can add one more this one 3D ground water steady flow equation for isotropic aquifer. So initially when the aquifer was an isotropic in that it will be different okay this the terms will be different. So for an isotropic aquifer the 3D ground water steady flow equation will be KX into D square H / DX square + KY into D square H / DY square + KZ into D square H / DZ square this is = 0.

So this is when the flow is steady and the aquifer is an isotropy and the flow is steady and 3 dimensional and so when the flow is an isotropic three dimensional through an anisotropic aquifer and flow is unsteady and in that case there will be right hand term the right hand side will not be 0.

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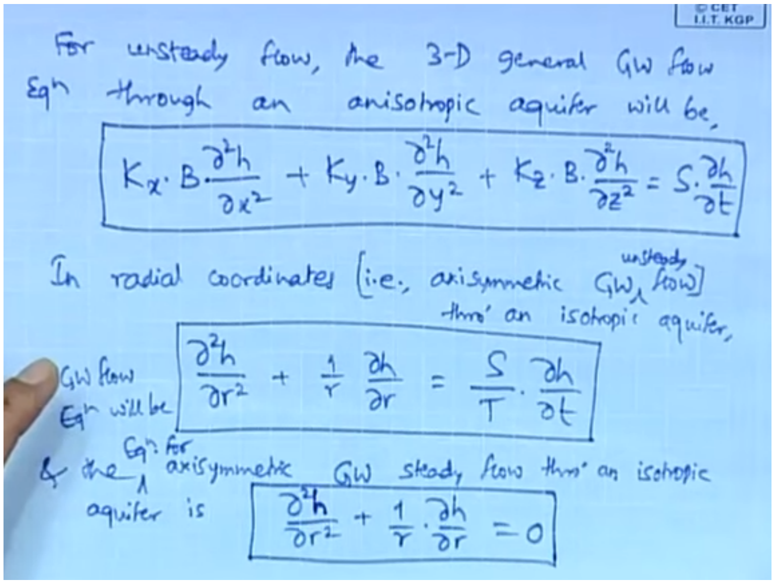

So here you can write down say for unsteady flow the 3D general ground water flow equation through an anisotropic aquifer will be this KX into B into D square H / DX square + KY into B into D square H / DY square + KZ into B D square H / DZ square. So this is = S into DH / DT so this is the expression here the flow is unsteady and three dimensional and the aquifer is anisotropy.

So in that case this is the general form and the flow becomes steady the right hand side terms become 0 and the aquifer is isotropic in that case KX = KY = KZ in that case it will be the Laplace equation. So therefore so this is how the general ground water flow equation it varies now the same expression for a say in radial coordinates that is axis-symmetric ground water flow.

So the axis-symmetric ground water flow through and isotropic aquifer the equation will be the ground water flow equation will be D square H / DR square + 1 by R DH / DR = S / T into DH / DT so this is the in the cylindrical coordinate this one and radial coordinates where R is a radial distance and H is the head and the radial coordinate and obviously this S is same a storativity or the storage coefficient T is the transmissivity so and the flow is unsteady.

So here the ground water unsteady flow so and the axis symmetric ground water steady flow the equation for axis symmetric ground water steady for through and isotropic aquifer is D square H / DR square + 1 by R DH/DR this = 0. In this case the flow is steady so therefore this DH / DT this term become 0 so the right hand side become 0 so this is the so this is the obviously the Laplace equation the equivalent in the cylindrical co-ordinate system or the radial coordinate system.

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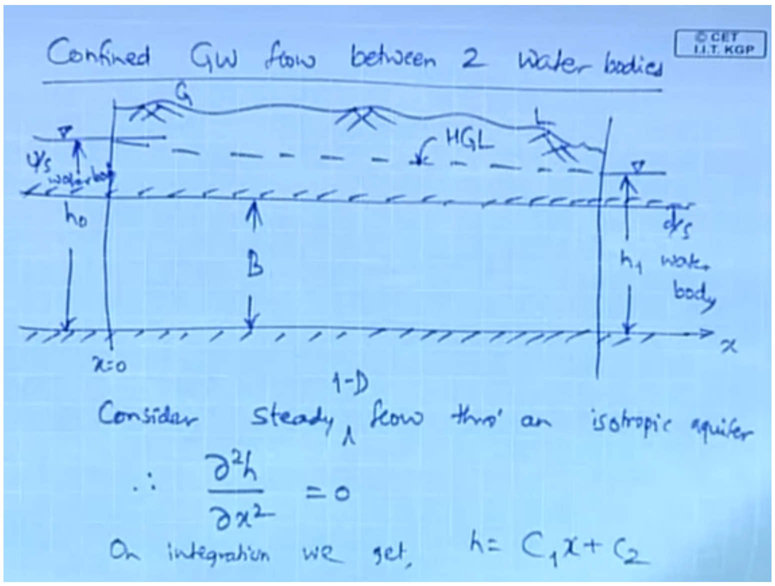

So now let us consider say this is the confined ground water flow between two water bodies. So here let us say this is the upstream water body so let this be X and this X = 0 and here we have the so this is the water level at the upstream water body where the head is H0 and of course here we have the top and bottom confining layers that is the impervious layers of this confined layers having a thickness B and so here this is the downstream water body where the head is H1.

So this is the so here this is the upstream water body so this is the downstream water body and the flow is taking place between this one and here let us say this is the general the ground between these two this one and obviously so this is how the ground water flow takes and so this is the hydraulic grade line and this is the downstream water body and this is upstream water body and so in this case obviously the consider steady follow through and isotropic aquifer.

And in this case the flow is e dimensional okay therefore the governing equation will be D square H / DX square = 0 because the flow is only X direction so therefore the this D square H /

DY square so they will be 0. So only we get this is D square H / DX square = 0 so this is the governing equation so on equation we get so this is H which is = C1 X + C2 okay and now need to substitute the boundary conditions to evaluate this C1 the constants of integration C1 and C2.

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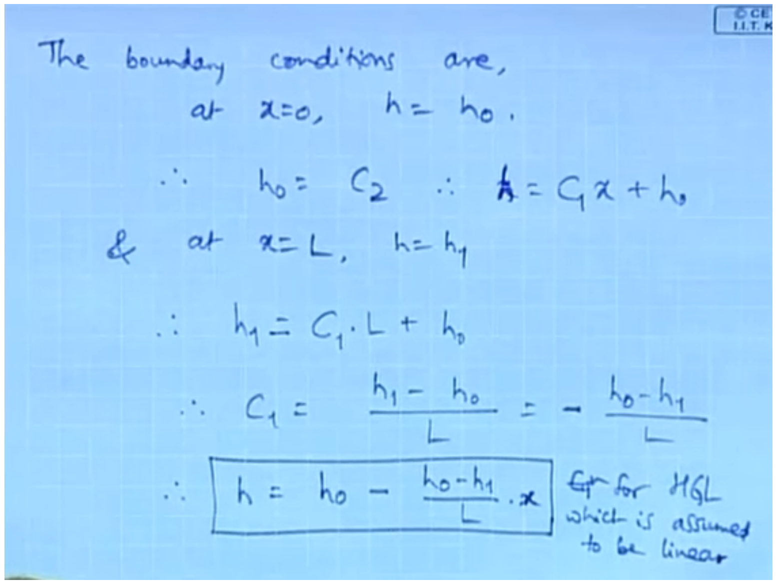

So the boundary conditions are at X = 0, H = H0 so therefore H0 = C2 and the second boundary condition is at X = L and sorry here they just forgot to show this 1. So this is = L so this is the distance between the upstream water body and the downstream water body so here at X = L then H = H1. So then C2 = H0 therefore get the expression H = C1 X + H0.

Now let us substitute the second boundary condition so this therefore H1 will be = C1 into L + H0 therefore C1 = H1 – H0 / L which we can also write this as – H0 – H1 / L since H0 is bigger the head in the peizometric surface in the upstream water body. So it is having this so having this one so can write that if we write that so therefore we get the expression after evaluation both the constant we get this H = – H0 – H1 / L into X.

So this is the expression for head at any general section which is at a distance of X from the upstream water body and so this is the here you can this is the equation for so this is the equation for hydraulic grade line at GL which is assumed to be linear okay. And let us also write down expression for the discharge per unit width of the through the confined aquifer between two water bodies.

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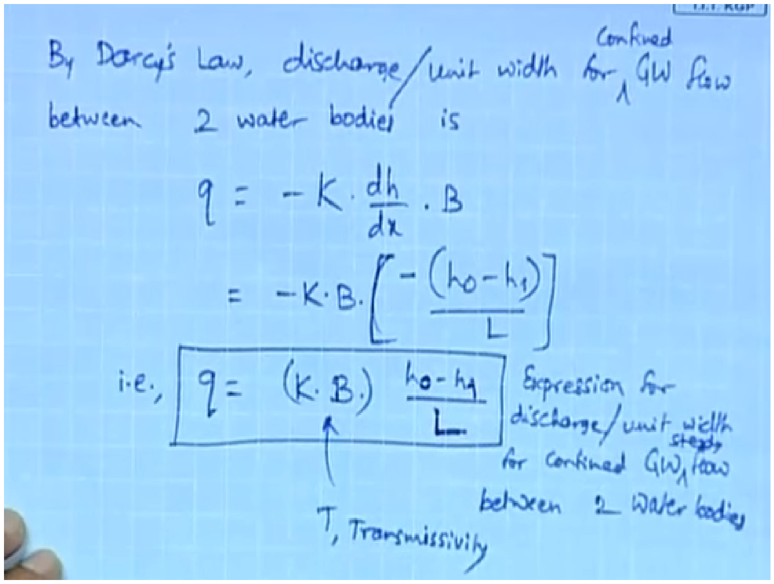

$$q = -K \cdot \frac{dh}{dx} \cdot B = -K \cdot B \cdot \left[-\frac{(h_0 - h_1)}{L} \right]$$

$$q = (K \cdot B) \frac{h_0 - h_1}{L}$$

So the by Darcy's Law discharge per unit width for ground water flow between for here you can say confined ground water flow between two water bodies is given by this Q = - hydraulic conductivity K into DH / DX where with this – DH / DX is the hydraulic gradient into the area. Since this is unit width so this will be 1 into B or simply B okay so in this case so this is a – K into B into the hydraulic gradient into – H0 - H1 / L that is Q = K into B into H0 – H1 / L.

So this is the expression for discharge per unit width for confined ground water flow between two water bodies. So the discharge per unit width for confined ground water flow between two water bodies and obviously here this we all start with this steady flow confined ground water steady flow between two water bodies. So in this the discharge per unit width is given by K into B which we can also represent by transmissivity T the trasmissivity or transmissibility.

And H0 and H1 are the heads in the upstream and downstream water body and L is the distance along the flow direction between the two water bodies, So if we know the transmissivity or the hydraulic conductivity and the thickness of the confined aquifer through which the flow takes place between the two water bodies and upstream water bodies having a head of H0 and the downstream water bodies having a head of H1 with the distance of L separating them.

So then this Q is given by K into B H0 – H 1 divided by l okay and now let us come to the unconfined flow that means flow through a unconfined aquifer where in the Dupit's assumption.

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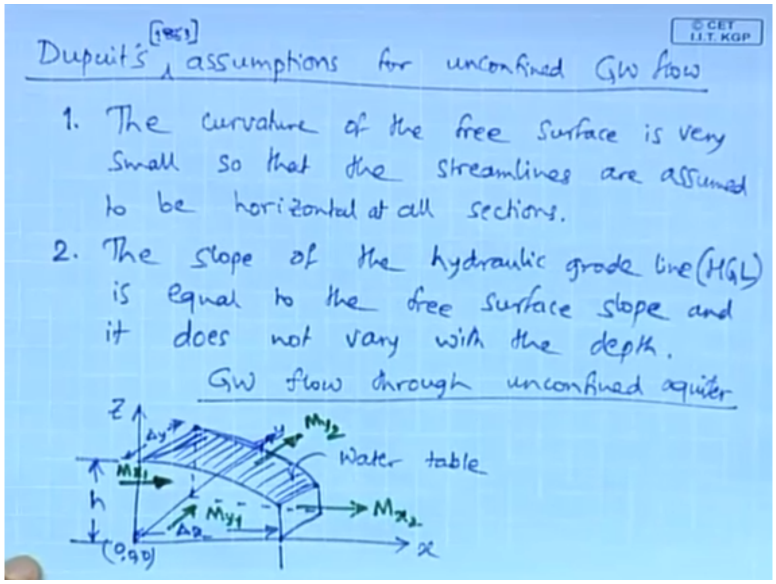

Dupuit's assumption for unconfined ground water flow so in 863 so these Dupuit's assumptions Dupuit's is in hydraulic engineer and in the year 863 the proposed two assumption for the unconfined groundwater flow the first assumption is the curvature of the free surface is very small so that this stream lines are assume to be horizontal at all sections.

So this is the first assumption and as per this assumption even though there is a slight inclination for the streamlines so that inclination is neglected in the streamlines are assumed to be horizontal at all sections. In this second assumption is the slope of the hydraulic grade line that is HGL = free surface slope and it does not vary with the depth. So this is a second Dupuit's assumption the first assumption in which the streamlines are assumed to be horizontal in all directions.

So that whatever little inclination the streamline is there it is neglected in the second assumption the slope of the hydraulic grade line is equal to the free surface slope and it does not vary with the depth so these are the two Dupuit's assumptions and which are applicable for ground water flow through unconfined aquifers and now using this Dupuit's assumptions let us determine the ground water flow through unconfined aquifer.

So here let us consider the ground water flow through unconfined aquifer so let us consider let us consider this as the Z direction this are the X direction and this is the Y direction and here let us consider. So this is the water table and so this dimension along the Y direction so this is delta Y and this is the origin and this dimension so this is the delta X and this head is H and here let us consider the ground water inflow.

Let us consider the flow to be in the positive direction of X and Y so let this be MX1 and the ground water outflow the mass outflow let us take the this to be MX2 in the X and Y direction . Similarly the ground water inflow in the Y direction let us take to be MY1 and the ground water outflow through the other phase in the Y direction let us take this to be MY2 and this is the water table.

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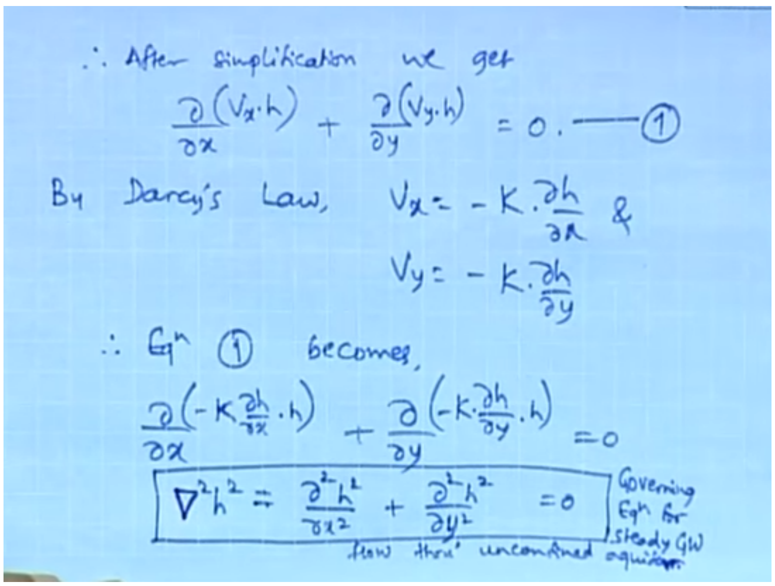

Now let us take down the expression for the mass flux entering the element so this is the given by MX 1 so this Rho which is the velocity groundwater flow velocity in the X direction into H into delta Y is the cross section area. So this VX into cross section area of flow that will be discharge into Rho that will be the mass flux or mass rate of flow, mass rate of inflow.

Similarly mass flex leaving the element so this is MX 2 so this is given by Rho VX H delta by + let us add the mass rate of change term in the X direction that is D / DX of Rho VX H delta Y into this delta X okay. So therefore now let us write down there is the net mass F flux in X direction so this is MX1 – MX 2. So this will be given by – D / DX of Rho VX H delta Y Delta X.

Similarly net mass efflux in Y direction so this is given by MY1 – MY2 so this will be given by – D / DY of Rho VY into HDX H delta X into delta Y therefore since there is a neither inflow nor outflow in Z direction. So the continuity equation gives so here this MX1 – MX2 is denoted by this is delta MX similarly MY1 – MY2 is denoted by delta MY. So therefore this delta MX + delta MY + delta MY = 0.

Therefore we get so after simplification we get D / DX of VX into H + D / DY of VY into H = 0 and obviously so here this delta X delta Y as well as Rho okay. So that can be cancelled ok so this is the expression we get and we know that say by Darcy's law we get this VX = - K into DH / DX and this VY = - K into DH / DY therefore if we solve this equation one. So this equation 1 becomes D / DX of – K DH / DX into H + D / DY of – K into DH / DY into H this = 0.

And here we can take out this K outside the one we can cancel out so therefore we are left with this is a D square / DX square of H square + D square / DY square of H square = 0. And this we can denote this as del square of H square so this is the governing equation for study ground water flow through aquifer unconfined. So this unconfined aquifer so study flow this its satisfy Laplace equation in H square where is the confined aquifer will satisfy Laplace equation H well in case of unconfined aquifer it satisfies the Laplace equation in H square.

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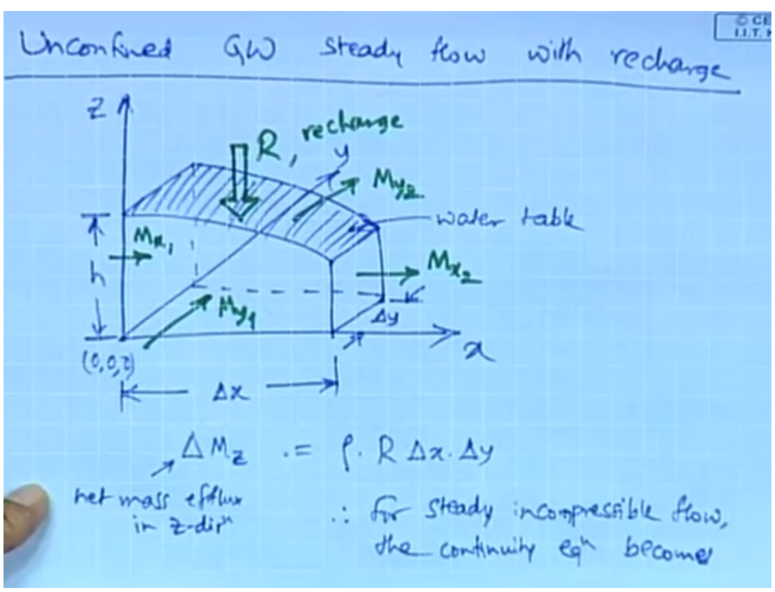

So now let us consider the unconfined aquifer that is the confined ground water steady flow with recharge suppose there is some recharge in the Z direction say then there is a due to rainfall or precipitation. So this the ground water getting recharged so in that case what will be the expression let us see? So here let us draw the basic figure with X Y and Z direction and so this is the water table and it is getting a recharge at the rate of R and here so this is the origin and this is the dimension in the X direction and this is the dimension of this element.

In the Y direction that is delta Y and the variable head H and in this case so let this the mass inflow through X direction let it be MX 1 the mass inflow let this be MX 2 similarly the mass inflow in the Y direction let it be MY1 and the mass inflow in the Y direction mass outflow along the Y direction be MY2 and obviously this is the water table and this R is the recharge Z direction.

And now in this case it is all similar to the previous except that in this case there will be additional term that is mass efflux in Z direction that is the net mass efflux in Z direction. That is delta MZ is given by Rho into R into rate of recharge into the area of flow perpendicular to Z direction that will be delta X into delta Y.

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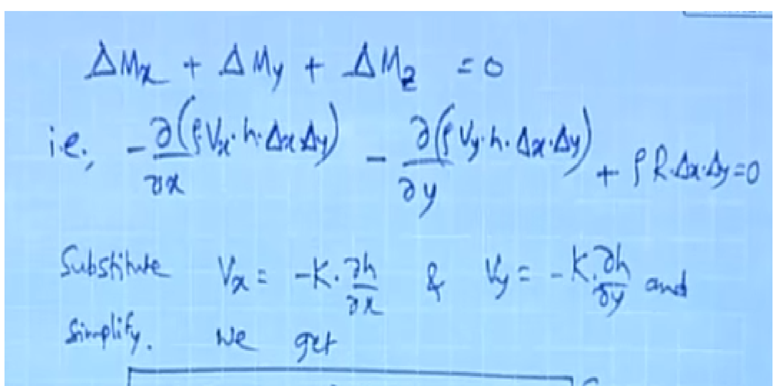

So therefore for steady incompressible flow the continuity equation becomes delta MX + MY + MZ = 0. So that is we get = D by DX f Rho VX H delta X delta Y – D / DY of Rho VY H delta X delta Y and the this recharge that is the net efflux in Z direction that is given by Rho R delta X delta Y this becomes 0. So again let us substituting the substitute VX = - K into DH / DX VY = - K into DH / DY and simplify.

So here we get this is a of course this is a Rho delta X delta Y that can be taken out and in this case where let with that is D square / DX square of H square + D square / DY square of H square this is equal to – 2R / K. So here this term R is their so therefore it will be H square / 2 so it will be – 2R / K so this is the general ground water steady flow equation through unconfined aquifer with recharge okay.

So the general ground water so here this is the governing equation so incase steady flow the unconfined aquifer the it satisfies Laplace equation in H square whereas when there is a recharge through the this one the unconfined aquifer at the rate of R along the Z direction the general governing equation for steady flow through confined aquifer will be given by this D square / DX square of H square / DY square of H square.

So this is = - 2R / K okay so this is how the ground water governing flow equation it changes its form in case of unconfined aquifer when there is no charge as well as there is recharge. And now let us consider the one dimensional Dupuit's flow with recharge between say two water bodies.

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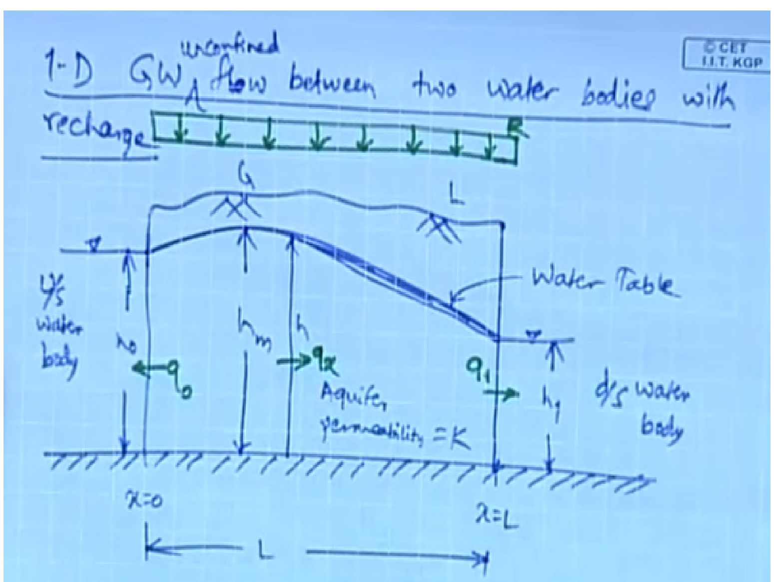

So let us say 1D ground water flow between two water bodies one day ground water unconfined flow with recharge. So let us consider unconfined ground water flow and let this be the upstream water body and let this be the ground surface and let this be so this is the upstream water body the head is H0 and downstream water body where the Head is H1 and there is a constant recharge and here this is the constant recharge of rate R.

And in this case the water will assume a shape like this there will be a peak somewhere in between and wherein the H is HM or h max and this is the water table. And so the aquifer permeability is a or hydraulic conductivity as K and this case there is obviously flow into the upstream water body as well as flow to a downstream water body this is Q0 this is Q1.

And in the next lecture and obviously at this general section H say here this is the general section H so this is a X = 0, X = L and so obviously this is L and a general section 8. So this case this is QX so in the next lecture we will discuss this thank you.