

Foundation Design
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Lecture - 17B
Earth Pressure Theories- Part 4

So, last class I have started active earth pressure on rigid retaining wall for cohesive soils.

(Refer Slide Time: 00:17)

Active earth pressure on Retaining wall for cohesive soils

C-φ soil

Relationship between Major principal stress σ_1 and Minor principal stress σ_3 at plastic equilibrium

$$\sigma_1 = \sigma_3 \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2c \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$$

$\sigma_1 = \sigma_3 = \gamma z$ $\sigma_3 = P_A$

$$\gamma z = P_A \left[\frac{1 + \sin \phi}{1 - \sin \phi} \right] + 2c \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$$

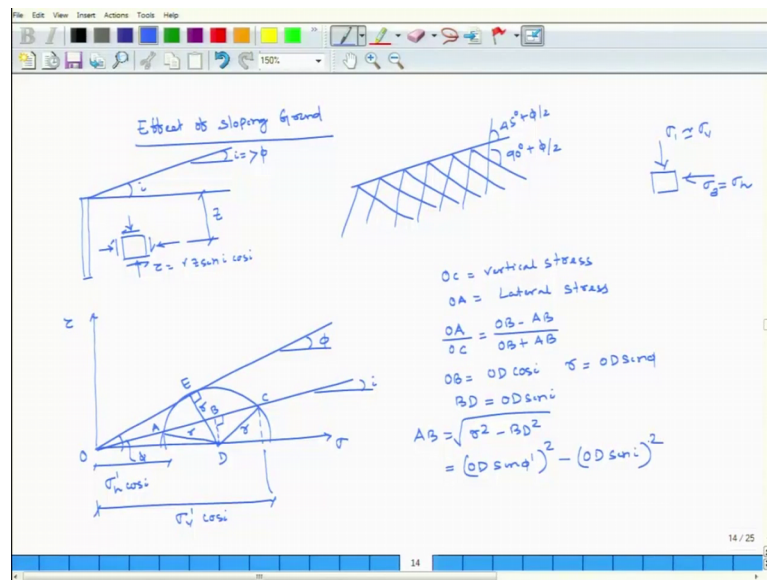
$$P_A = \gamma z \left[\frac{1 - \sin \phi}{1 + \sin \phi} \right] - 2c \sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}}$$

$$P_A = \gamma z k_a - 2c \sqrt{k_a}$$

Diagram showing stress distribution with $z=0$, $P_A = -2c \sqrt{k_a}$, $P_A = 0$ at $z = z_0 = \frac{2c}{\gamma \sqrt{k_a}}$, and $P_A = \frac{1}{2} k_a \gamma H^2 - 2c H \sqrt{k_a} + \frac{2c^2}{\gamma}$. $H_c = 2z_0 = \frac{4c}{\gamma \sqrt{k_a}}$ is the critical depth of vertical cut.

And best on relationship between major principal stress and minor principal stress at plastic equilibrium. So, active earth pressure has been derived for passive conditions sorry, for cohesive soils P_a is equal to $\gamma z k_a$ minus $2 C$ root over of k_a . Then we bring it back to critical depth of vertical cut h_c . Up to what depth we can go for a cohesive soil? Vertical cut. Now come to next this is effect of sloping ground.

(Refer Slide Time: 00:59)



Take an example; let us this is my wall. And there is a soil in between. Then here it is, then this is at a distance z , and this is your angle ϕ . And τ I can write it $\gamma z \sin i \cos i$ sorry this is not ϕ , let us say this is i which is greater than ϕ .

So, τ is equal to $\gamma z \sin i \cos i$ this angle is your i , and if I draw it one more term, in this way this is 45° plus ϕ by 2. This is your 90° plus ϕ by 2. Then let us draw the Mohr circle, for sloping ground surface. This is τ , this is origin, and this is σ scale. So, this is the Mohr circle. This angle is this is a failure envelope touching your Mohr circle. This is your ϕ . Then this is e , 90° . And this is your σ , and these value your D . Then let us draw another line at an angle i , angle i this is your A this is your C . And this part is your AD , this is your CD . And this is your radius r . Let draw perpendicular here and this will be 90° . This is BE . And then draw it a perpendicular here, with this a vertical intercept. With this will be your $\sigma_h \cos i$ and this will be your $\sigma_{B'} \cos i$.

So, I draw a Mohr circle considering active conditions. At the same time I draw sloping ground that i line I draw it. Then let us start with this OC . OC is equal to, it is my vertical stress, this is my vertical stress in i . Then OA , this is lateral stress. In active conditions this is my σ_1 which is equal to σ_3 . And this is my σ_3 , which is equal to σ_h . So, OA is your lateral stress. So, let us say OA by OC which is equal to OB minus AB divided by OB plus AB . Then OB is equal to $OD \cos i$. Then r is equal to, r is

your radius OD sin phi. This r will be OD or this is my r, this is my r, this is also r, this is also r this will be OD if this angle is phi then it will be OD sin phi. Then BD is equal to OD sin i.

Now, let us consider AB: AB is equal to, A and B is equal to r square minus BD square root over which is equal to OD sin phi prime whole square minus OD sin i whole square into root over. So, this comes out to be OC is equal to r z cos i. Then if I write in terms of OA by OC which is equal to OD cos i minus OD sin square phi minus sin square i root over divided by OD cos i plus OD sin square phi minus sin square i root over.

(Refer Slide Time: 08:32)

The image shows a whiteboard with handwritten mathematical derivations. The derivations are as follows:

$$\frac{OA}{OC} = \frac{OD \cos i - \sqrt{OD^2 \sin^2 \phi - \sin^2 i}}{OD \cos i + \sqrt{OD^2 \sin^2 \phi - \sin^2 i}}$$

$$\sin^2 \phi = 1 - \cos^2 \phi \quad \sin^2 i = 1 - \cos^2 i$$

$$\frac{OA}{OC} = \frac{\cos i - \sqrt{1 - \cos^2 \phi - 1 + \cos^2 i}}{\cos i + \sqrt{1 - \cos^2 \phi - 1 + \cos^2 i}}$$

$$\Rightarrow \frac{OA}{OC} = \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}$$

$$\sigma_{ha} = \gamma z \cos i \left[\frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right]$$

$$P_a = K_a \gamma z$$

$$K_a = \cos i \left[\frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right]$$

$$K_p = \cos i \left[\frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}} \right]$$

Then sin square phi is equal to sin square phi is equal to 1 minus cos square phi, then sin square i is equal to 1 minus cos square i. Then OA by OC is equal to which is equal to cos i minus root over of 1 minus cos square phi minus 1 plus cos square i divided by cos i plus root over of 1 minus cos square phi minus 1 plus cos square i. That implies OA by OC is equal to cos i minus root over of cos square i minus cos square phi divided by cos i plus root over of cos square i minus cos square phi.

So, then sigma prime h a is equal to gamma z cos i. Then which is your cos i minus root over of cos square i minus cos square phi divided by cos i plus root over of cos square i minus cos square phi. And Pa is equal to; Pa is equal to k a gamma z. Now comes out to be k a is equal to cos i into cos i minus cos square i minus cos square phi root over divided by cos i plus cos square i minus cos square phi root over. And similarly k p

considering this k_p is equal to $\cos i$ then, into $\cos i$ plus $\cos^2 i$ minus $\cos^2 \phi$ root over divided by $\cos i$ minus root over $\cos^2 i$ minus $\cos^2 \phi$.

So, this is this derivation for sloping ground. What I have done? There is a soil element below this. So, τ is equal to $\gamma z \sin i \cos i$. And then this sloping ground is making an angle of i , then I have drawn the Mohr circle for active case. In the same Mohr circle this sloping ground has been plotted i . So, each co ordinate has been calculated, OC is your vertical stress; obviously, OC is your vertical stress if it is a sloping ground. OA represent your lateral stress or horizontal stress. Then this coordinates has been calculated then ratio I make it. Then in this ratio it is it comes out to be OC by OC is equal to $\cos i$, minus root over of $\cos^2 i$, minus $\cos^2 \phi$ by $\cos i$ plus root over of $\cos^2 i$, minus $\cos^2 \phi$. Then if I put it OA is equal to OC here OC is what OA is your lateral stress OC is your vertical stress.

So, once OC is your vertical stress this is your $\gamma z \cos i$ into this term. And if I equate this equation in terms of P_a active earth pressure which is equal to $k_a \gamma z$. So, k_a is equal to $\cos i$ into $\cos i$ minus $\cos^2 i$ minus $\cos^2 \phi$ root over by $\cos i$ plus $\cos^2 i$ minus $\cos^2 \phi$ root over. This is my active earth pressure coefficient. K is your coefficient of active earth pressure, considering sloping ground. Similarly if I take the passive earth pressure coefficient, k_p is equal to $\cos i$ into $\cos i$ plus here talked on, then here bottom will be here minus. This is what is your derivations.

Now, we can solve one problem, let us say one or 2 problems let us say, how this is varying with this sloping ground and other example we will solve it. Let us consider one example.

(Refer Slide Time: 14:55)

Example

Layer	γ	C	ϕ
1	20	0	35°
2	18	20	25°
3	16	35	0

Layer 1 ($C=0$) $K_a = \frac{1 - \sin 35^\circ}{1 + \sin 35^\circ} = 0.27$
 $z=0$ $P_v = 0$
 $P_A = K_a \gamma z = 20 \times 5 \times 0.27 = 27 \text{ kN/m}^2$

Layer 2
 $z=5\text{m}$ $P_v = 20 \times 5 = 100 \text{ kN/m}^2$
 $P_A = K_a \gamma z - 2C \sqrt{K_a} = 0.41 \times 100 - 2 \times 20 \sqrt{0.41} = 15.4 \text{ kN/m}^2$
 $z=10\text{m}$ $P_v = 100 + 18 \times 5 = 190 \text{ kN/m}^2$
 $P_A = 0.41 \times 190 - 2 \times 20 \sqrt{0.41} = 52.3 \text{ kN/m}^2$

Layer 3
 $\phi=0 \Rightarrow K_a = 1$
 $z=10\text{m}$ $P_v = 190 \text{ kN/m}^2$
 $P_A = K_a \gamma z - 2C \sqrt{K_a} = \gamma z - 2C = 190 - 2 \times 35 = 120 \text{ kN/m}^2$
 $z=15\text{m}$ $P_v = 190 + 16 \times 5 = 270 \text{ kN/m}^2$
 $P_A = 270 - 2 \times 35 = 200 \text{ kN/m}^2$

Diagram shows a trapezoidal active earth pressure distribution with values 27, 15.4, and 52.3 at depths 0, 5, and 10m respectively. A vertical line at the bottom right indicates a total height of 20m.

This is the case has been given. 5 meter, 5 meter, 5 meter, 3 layers are there. So, let us put it this is layer 1 this is layer 2 this is layer 3. And the properties has been given gamma C and phi. This is your kilo Newton per meter cube; this is your kilo Newton per meter square, phi in degree. For layer 1 it is 20, C is equal to 0, phi is equal to 35; that means, it is a cohesion less soil, for layer 2 it is 18, C is given 20 phi is equal to 25.

For layer 3 it is 16, 35 and phi is equal to 0. There are 3 layers typical case I put it. Layer 1 having C is equal to 0 purely cohesion less soil. Layer 2 is a C phi soil, both C both C and phi are there. Layer 3 is purely cohesive soil phi is equal to 0. Let us start with for layer 1. Layer 1 C is equal to 0 then k a is equal to 1 minus sin 35 degree by 1 plus sin 35 degree which is equal to 0.27. A z is equal to 0 P_v is equal to 0. So, let us put it P_a is equal to k a gamma z, which is equal to 20 into 5 into 0.27 which is equal to 27 kilo Newton per meter square.

Similarly, for layer 2, to let us consider layer 2: layer 2 is a C phi soil. Layer 2 is a C phi soil, what has it mean? Z is equal to 5 meter, P_v is equal to 20 into 5, what is equal to 100 kilo Newton per meter square, then P_a is equal to k a gamma z minus 2 C root over of k a, which is equal to 0.41 into 100 minus 2 into 20 root over of 0.41, 15.4 kilo Newton per meter square.

This is what we have per C phi soil active condition this is P_a is equal to k a gamma z minus 2 C root over of k a straight forward. Then a z is equal to 5 meter means at here.

Here it is starting 0 this is 5 then 5 to 5 this is 10. Then you can put it z is equal to 10 meter. So, then what is the value of P_v ? Vertical stress, vertical stress will be P_v will be 100 plus 18 into 5, 18 into 5 100 means 20 into 5 is your 100 vertical stress which is equal 190 kilo Newton per meter square. Now z is equal to 10 meter P_a is equal to 0.41 into 190 minus 2 into 20 root over of 0.41, which is equal to 52.3 kilo Newton per meter square.

Now, come to layer 3, come to layer 3. Layer 2 is a C phi soil. Layer 3 is purely cohesive soils; that means, once phi is equal to 0 what does it mean? K_a is equal to 1 k_a is equal to 1. Say z is equal to 10 meter, then P_v is equal to vertical stress is equal to 190 kilo Newton per meter square. So, P_a is equal to $k_a \gamma z$ minus 2 C root over of k_a which is equal to γz minus 2 C because k_a is equal to 1, which is equal to 190 minus 2 into 2 into 35 which is equal 120 kilo Newton per meter square.

Say z is equal to 15 meter, 15 meter then P_v is equal to how much; 190 plus 16 into 5 which is equal to 270 kilo Newton per meter square. Then P_a is equal to 270 minus 2 into 35 which is equal to 200 kilo Newton per meter square. So, let me summarize layer 1 which is a C is equal to 0. In this case P_a is will be calculated active bar pressure considering for cohesion less soil. P_a is equal to $k_a \gamma z$. Layer 2 layer 2 it is C phi soil. Once it is a C phi soil P_a is equal to $k_a \gamma z$ minus 2 C root over of k_a . This has been calculated considering z is equal to phi; that means, here considering z is equal to 10 meter total 10. Now for layer 3 phi is equal to 0, phi is equal to 0 means k is equal to 1 minus sin phi by 1 plus sin phi. So, that is equal to 1. So, in that case P_a will be γz minus 2 C, considering z is equal to 10 meter; that means, here and considering z is equal to 15 meter at this point has been calculate.

So, let me plot it how it looks like, this is your 27. Then this part is your, this part is your 15.4, 4. Then this part is your 52.3. Then it will go here this will your 120. This is your 120. And these will your 200. Why it is varying like this? Let us look at here this will be more appropriate; 0 2 5 meter. So, this is a C is equal to 0 purely cohesion less soil. So, this that is why it is 27, then at the 5 meter if I am considering, because 5 meter is a junction' 5 is there one case 5 is there C is equal to 0 other case C and 5 has there; that means, if I consider this one, then it comes out to be 15.4. Here I mark 15.4, because total is 27.

Then next is your 10 meter. At 10 meter how much it is considering? Second layer both C and phi it is your 52.3. Now after 52.3 if this is my junction at this junction what is the other soil property, because what happened? Look at here there is a junction here. Here it is a C and phi. Are the junctions other side there is a value of C and phi is equal to 0. At the interface both these values are there. Then phi is equal to 0 if I take it then what is the value at z is equal to 10 meter what is the value it is your 120. So, here it is 52.3 then I extend up to 120 this is the beginning of this layer.

Now, at 15 meter it is a straight forward this is at the end this is your 200. Generally we write it at the end the value this is your 120, this is your 52.3. This is your 15.4, this is your 27. And this depth is your 5 meter, this is your 5 meter, this is your 5 meter. Now find it out based on this result and earth pressure P_a and it acted at a distance z from vertical. At a distance z from the vertical we can find it out this distance is no resultant force, and at a distance z you can be easily calculating it.

So, while doing the problems, same problem same problem if I interchange it, this is for your home assignment, I just interchange it. Nothing else, I just interchange it this is my 5 meter, this is 5 meter, this is 5 meter, then if I write it gamma C and phi. I put it from the beginning gamma is equal to 16. C is equal to 35. Phi is equal to 0 then I put it gamma is equal to 20. C is equal to 0. Phi is equal to 35. Then I put it gamma is equal to 18. C is equal to 25, is equal to 25. The question is find it out earth pressure active earth pressure distributions and find it out the resultant P_a and acted at what distance from the bottom with this (Refer Time: 27:34) in this case just I just reverse it. Bottom part I put it. C phi middle C is equal to 0 only phi, top phi is equal to 0 only c.

So, I will stop it here next class I will solve few more problems. And I will start your coulombs theory Rankine's theory is over. Now I will start your coulombs theory.

Thank you.