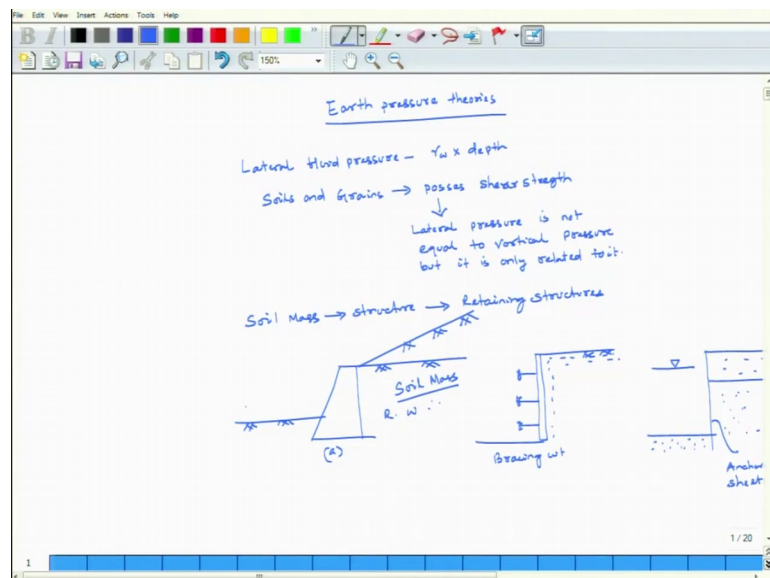


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**Lecture - 16B**  
**Earth Pressure Theories- Part 2**

So, last class I have started with this earth pressure theories.

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Where it is required particularly to return this soil mass retaining structures.

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Effect of Wall Movement on earth pressure

1) when wall is rigid and un-yielding, the soil mass is in a state of rest  
 → No deformation and displacement  
 → Earth pressure at rest (A)

B Wall rotates about toe  
 → Moving away from soil  
 → Soil mass will expand  
 → decrease in earth pressure  
 → Mobilization of shearing resistance in a direction opposing the movement of earth mass and to move outward and downward relative to the wall  
 → A resultant decrease in earth pressure which continues until, at a certain amount of displacement failure will occur  
 → Forces acting on the wall does not decrease

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And effect of wall moment on earth pressure, I consider one is your earth pressure at rest, where it there will not be any deformation or displacement. One is wall rotate about the tough, and then wall movie is moving away from the soil mass, that is called your active stage. Other is your wall is moving towards the soil mass, that is called your passive state.

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C Wall pushed towards backfill  
 → Soil is compressed  
 → shearing resistance build up in directions towards wall  
 → earth pressure gradually increases  
 → reaches limiting value  
 → Failure occur  
 → Passive earth pressure

Earth pressure at rest

$P_0 = K_0 \sigma_z$  ← effective vertical stress at depth  $z$   
 Lateral pressure      coefficient of earth pressure at rest

Let  $\epsilon_x =$  strain in horizontal direction  
 Plain strain condition  
 $\epsilon_x = \frac{1}{E} (\sigma_x - \mu(\sigma_z + \sigma_y))$   
 earth pressure at rest  $\epsilon_x = 0$

$\sigma_x = \frac{\mu}{1-\mu} \sigma_z$   
 $P_0 = \frac{\mu}{1-\mu} \sigma_z$   
 $K_0 = \frac{\mu}{1-\mu}$        $P_0 = K_0 \sigma_z = K_0 \sigma_z$

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Then earth pressure at wall we have derived  $p_0$  is equal to  $\mu$  by  $1 - \mu$  into  $\sigma_z$ .



is in a state of plastic equilibrium, soil is in a state of plastic equilibrium. Both in case of active and passive earth pressure conditions. Then rupture surface is a planar surface then is your rupture surface is a planar surface. Backfill surface is horizontal, backfill surface is horizontal then back of wall is vertical, then back of wall is smooth.

Now, let us consider a retaining wall. This is my wall; this is your ground surface. Then I have marked it here both the sides this is your passive this is your passive, and this case is your active. And let us consider at this distance  $Z$  a soil mass is there, in the soil mass let us say this is my  $\sigma_x$  and this will be the sorry, this is my  $\sigma_z$  and this will be the  $\sigma_x$ . Now let me draw it in such a way that it will be more convenient for you to understand, original soil mass this is a dotted line.

Then another one is I can write it this is my active, and this will be my passive. If I can write it a soil mass I can show it in a bigger way this is my soil mass. So, this is the case of your active, then there is a soil mass here is the soil mass, and this is the case of passive, active and passive. Now if I say there is a soil mass here this is a soil mass and this is a soil mass in active case and here in soil mass in passive case. Now what will happen once wall moves away from the backfill that is the case your fear active state, then what will happen in case of active first part is your the soil element expand, soil expand and what will happen,  $\sigma_x$  decreases.  $\sigma_x$  is your horizontal stress  $\sigma_z$  is your vertical stress,  $\sigma_x$  decreases to a minimum value. So, that it reaches plastic equilibrium.

Now, this is the case of active, now what will happen case of the passive? In case in this case if this is my original soil mass try to understand. And hear it will be  $\sigma_z$  and here it will be  $\sigma_x$ . Once wall moves away from the field to  $\sigma_x$  decreases and it reaches a minimum value decreases to a minimum value where is your plastic equilibrium observe. Now let us plot in terms of Mohr circle if I plot in terms of Mohr circle.



So, what does it mean?  $\sigma_3$  is equal to  $\gamma Z$  into  $k_a$  or I can write it  $P_a$  is equal to  $\gamma Z$  into  $k_a$ . Now  $k_a$  can be written as  $\frac{1 - \sin \phi}{1 + \sin \phi}$ , which is equal to  $\tan^2 45^\circ - \phi/2$ . Look at the physics for active state. In active state what will happen? There is a soil mass retaining and the wall is moving now away from the soil mass.

Once wall is moving away from the soil mass the  $\sigma_x$  decreases to a minimum value, where is your plastic equilibrium reached. Then what will happen if I take a Mohr circle it will touch your failure and envelope, then in that case what will happen,  $\sigma_x$  decreases and it becomes minor principal stress, that is your  $\sigma_3$  as compared to your  $\sigma_z$ . Then vertical stress  $\sigma_z$  becomes your major principal stress this is your  $\sigma_1$ .

If I take it this is my  $\phi$  value  $\phi_{plan}$  value if I take it,  $\sin \phi \sin \phi$  is equal to this divided by this then this will be your  $\frac{\sigma_1 - \sigma_3}{2}$  divided by  $\frac{\sigma_1 + \sigma_3}{2}$  which is equal to  $\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$ . Then  $\sigma_3$  comes out to be  $\sigma_1 \frac{1 - \sin \phi}{1 + \sin \phi}$  then  $\sigma_1$  is equal to  $\sigma_z$ , which is equal to  $\gamma Z$  then  $\sigma_3$  is equal to  $\gamma Z$  into  $k_a$ . So,  $k_a$  is equal to  $\frac{1 - \sin \phi}{1 + \sin \phi}$  which is equal to  $\tan^2 45^\circ - \phi/2$ . Then  $P_a$  is equal to  $\gamma Z$  into  $k_a$ .  $P_a$  is your active earth pressure which is equal to  $\gamma Z$  into  $k_a$ .

$k_a$  value is your  $\frac{1 - \sin \phi}{1 + \sin \phi}$ . Now similarly if I draw it for passive state; if I draw it for passive state what will happen? What will happen in passive state? So, then this will be your  $\theta$  of course, this will be your  $2\theta$ , then this will be your  $\theta$ . So, in these case it will be your  $\sigma_x$  maximum. If I say this is my passive state, what will happen?

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Passive state  
Wall Moves towards the backfill  
 $\sigma_x > \sigma_z$   
 $\sigma_x =$  Major principal stress  
 $= \sigma_1$   
 $\sigma_z =$  Minor principal stress  
 $= \sigma_3$   
 $P_p = \gamma_z \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right)$   
 $= k_p \gamma_z$   
 $k_p = \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) = \tan^2(45^\circ + \phi/2)$   
coefficient of passive earth pressure  
 $\theta = 45^\circ + \phi/2$   
 $\theta = 45^\circ - \phi/2$   
Two sets of failure planes will develop  
Active  
Passive

In passive state wall moves towards the backfield, then you what will happen once walls moves towards the backfield look at the diagram. Once wall moves towards the backfield this is your passive state, then this will be your sigma Z, and this will be your sigma x.

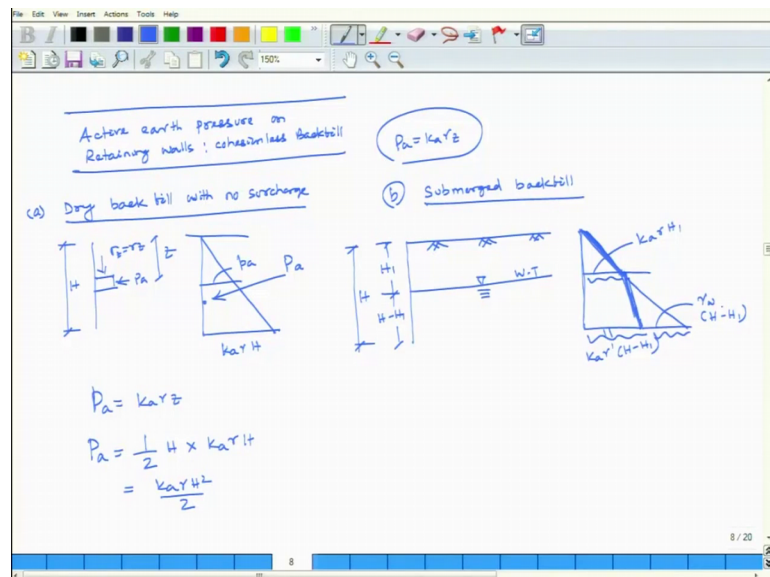
So, sigma will be more, sigma Z will be it will be less, sigma x will be more. So, in that case sigma x is greater than your sigma z. So, then sigma x is equal to your major principal stress, which is equal to your sigma 1. Sigma Z in passive state it is your minor principal stress, minor principal stress which is equal to sigma 3. Then if I reverse it, what will happen? Pp I can get it Pp is equal to passive force gamma Z into 1 plus sin phi by 1 minus sin phi. You can get this derivation from this diagram also directly. Then which is equal to k p gamma z. So, k p is equal to 1 plus sin phi by 1 minus sin phi which is equal to 10 square 45 degree plus phi by 2.

This is your coefficient of k p is coefficient of passive earth pressure. So, then in these case theta is equal to 45 degree minus phi by 2. So, I am expecting we are expecting 2 sets of curves we are supposed to get it, solve particularly active and passive case if I write it if I draw this diagram, this is my active. So, angle will be for active case this will be this will be your theta, and theta is equal to 45 degree plus phi by 2. Similarly if you are passive, passive I am supposed to get it. So, this angle as well as this angle will be theta is equal to 45 degree minus phi by 2. This is case of active, this is the case of passive. So, in this case 2 sets of failure plain will develop, 2 sets of failure plain will

develop for active case as well as for passive case also, 2 states of failure plan also will develop.

So, this is what our summary. In active state  $P_a$  active earth pressure is equal to  $\gamma Z$  into  $k_a$ .  $k_a$  is equal to  $\frac{1 - \sin \phi}{1 + \sin \phi}$ . In passive state  $P_p$  is equal to  $\gamma Z$  into  $\frac{1 + \sin \phi}{1 - \sin \phi}$ . And  $P_p$  is equal to  $k_p$  into  $\gamma Z$ ,  $k_p$  is equal to coefficient of passive earth pressure. So,  $k_p$  is equal to  $\frac{1 + \sin \phi}{1 - \sin \phi}$ . Let us start with this few examples, active earth pressure on retaining walls Cohesion less backfield.

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So, case by case let us we will go. Dry backfield with no surcharges case a, so here  $P_a$  is equal to you can write it  $P_a$  is equal to  $k_a$  into  $\gamma Z$ , this is our equation.

So, dry back field with no surcharges. So, this is my wall. In this wall there is a soil mass and this is your  $\sigma_z$  is equal to  $\gamma Z$ . And this is your active earth pressure  $P_a$ . And his distance is your  $Z$ , and this is your height  $H$ . Now what should be it? It is will be like, this  $k$  is constant  $\gamma$  is constant  $Z$  it is wearing with this. So, then it will be  $k_a \gamma H$ . And this will be your small  $P_a$ . And this will be acted as a capital  $P_a$  active earth pressures. So, here it will be  $P_a$  is equal to  $k_a \gamma z$ . So, then if you want to find out what is your active earth pressure take this is the distance  $H$ , take the triangle. So, then it will be what actual  $P_a$  this is a small  $P_a$   $P_a$  is equal to half  $H$  into  $k_a \gamma H$



$H$   $k$  a into  $\gamma H$ . And from there it will be  $k$  a  $\gamma H$  square by 2, and this will be act as a  $c$  g here 2 third from here it will be one third.

So, this is summation of total force, then submerged backfield. Let us consider your submerged backfield, consider this wall. This is water table, this is your  $H_1$ , and this is my total height  $H$ . What will happen if there is there is a dry; dry backfield, with no surcharges. Here in this case it is a submerged backfield what will happen? How your earth pressure distribution will be there? Look at carefully. So, up to this part is your dry up to  $H_1$ , simple triangle in this case it will be  $k$  a  $\gamma H_1$ . Then there will be a water table, because of water table it will be a submerged conditions. So, because of submerged condition what will happen? Here in this case it will be  $k$  a  $\gamma' H$  minus  $H_1$ . This part, because  $\gamma$  is your unit weight is your submerged unit weight.

So, that is why it is a  $\gamma'$ . Then what will happen there is a water pressure will come into picture. That water pressure will be taken into considerations. This water pressure will be your  $\gamma_w H$  minus  $H_1$ . So, look at there are 2 cases I put it drive backfield with no surcharges. Another one is your submerged backfield. In drive backfield there is a very simple way it is a very triangle,  $k$  a  $\gamma H$  are the below and  $P_a$  we will act in the triangle at the  $c$  g. Total force you can find it out the area half  $H$  into  $k$  a  $\gamma H$   $k$  a  $\gamma H$  square by 2. Then it will look at the submerged backfield suppose part of this there is no water table up to  $H_1$ . Water table lies below the ground surface at a distance of  $H_1$  total height is  $H$ .

So, for  $H_1$  it is  $k$  a  $\gamma H_1$ , here to here. Then from  $H$  minus  $H_1$  this will be  $H$  minus  $H_1$ . In this case the  $\gamma$  will be a submerged because the water table is there. So, this will be  $k$  a  $\gamma$  submerged into height. This is your  $H$  minus  $H_1$  then what additionally we will be come into picture because water is there. Water pressure we will come into picture. Here it will come  $\gamma_w H$  minus  $H_1$ . So, this entire diagram looks like here, here and this will be your this, this part will be this, this part will be this and this will be because of your water pressure. I will stop it here.

Next class I will proceed with more like surcharge, effective of surcharge, then effect of 5 layer soils. Next class I will discuss.

Thank you.