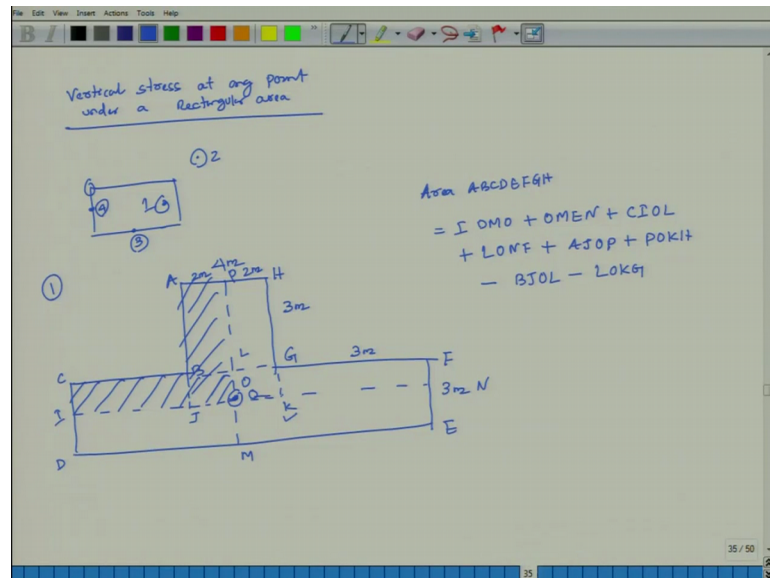


Foundation Design
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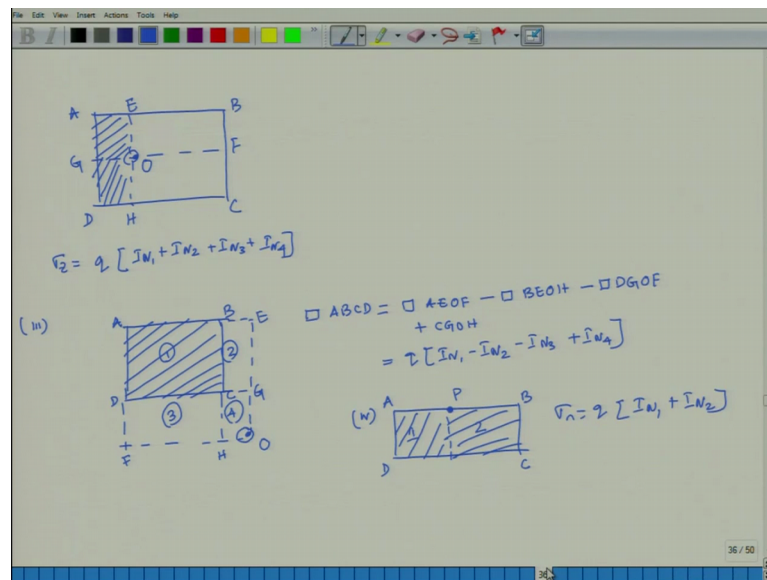
Lecture – 11B
Stress Distribution in Solis-Part 5

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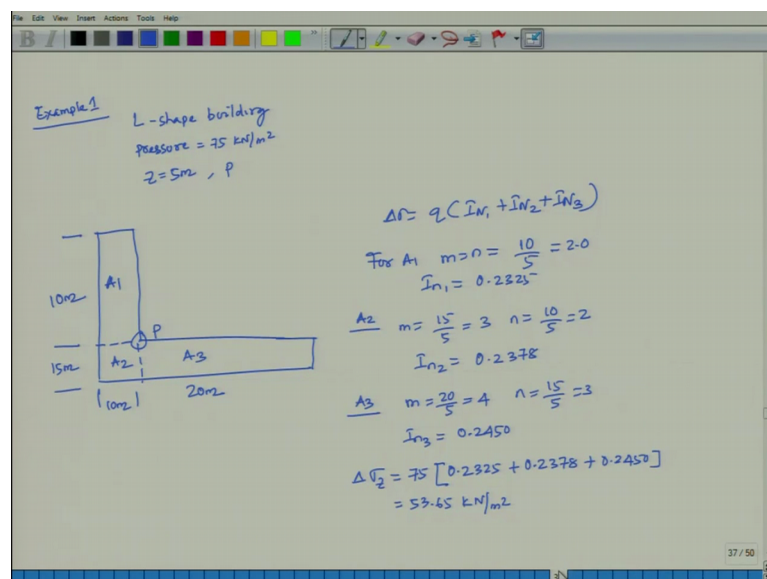
Last class we have discussed vertical stress at any point under rectangular loaded area by means of here, if it is either inside the rectangular area or outside the rectangular area are at one of the side of the rectangular area. So, by means of method of super positions.

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So, there are different cases case one, is it is at any point inside the rectangular area. Case 2 it is outside the rectangular area, case 3, case 4, it will be one of the side of a rectangular area it has been discussed.

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Then 3 examples I have solved. Example one is your l shape building and pressure intensity is given depth is given 5 meter, find it out increasing stress.

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Example 2

Diagram: A rectangular area with width 2.5m and height 2m. Point P is located at a distance of 0.5m from the top edge and 1.0m from the right edge. The area is divided into four regions: 1 (bottom-left), 2 (top-left), 3 (top-right), and 4 (bottom-right).

Given: $q = 80 \text{ kN/m}^2$, $z = 2.5 \text{ m}$

Formulas and Calculations:

$$\Delta \sigma_z = q [I_{n1} - I_{n2} - I_{n3} + I_{n4}]$$

① \square A B P G

$$m = \frac{3.5}{2.5} = 1.4 \quad n = \frac{2.5}{2.5} = 1.0$$

$$I_{n1} = 0.1914$$

② \square B G F P

$$m = \frac{3.5}{2.5} = 1.4 \quad n = \frac{0.5}{2.5} = 0.2 \quad I_{n2} = 0.0589$$

③ $m = \frac{2.5}{2.5} = 1.0 \quad n = \frac{1}{2.5} = 0.4 \quad I_{n3} = 0.1013$

④ $m = \frac{1.0}{2.5} = 0.4 \quad n = \frac{0.5}{2.5} = 0.2 \quad I_{n4} = 0.0328$

Final Calculation:

$$\Delta \sigma = 80 [0.1914 + 0.0589 + 0.1013 + 0.0328] = 5.12 \text{ kN/m}^2$$

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Example 3

Diagram: A rectangular area with width 5m (2m + 3m) and height 4m (2m + 2m). Point P is located at the center of the rectangle.

Given: $q = 20 \text{ kN/m}^2$, $z = 2.5 \text{ m}$

Formulas and Calculations:

$$\sigma_z = q [I_{n1} + I_{n2} + I_{n3} + I_{n4}]$$

Second one is your outside the loaded area at point p, and find it out increasing in stress.
 Third I have said again it is in between in the loaded area, it will be very easy then you can find it out easily.

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Example-4

A Rectangular Foundation $3.0\text{m} \times 1.5\text{m}$ carries a uniform load of 40 kN/m^2 . Determine the vertical stress at 'p' which is 3m below the ground surface. Use equivalent point load Method.

Load on each area
 $= 40 \times (1.0 \times 0.5)$
 $= 20\text{ kN}$

For (1) & (4)
 $r = \sqrt{1.5^2 + 0.25^2} = 1.521$ $\frac{r}{z} = 0.527$

For (2) & (3), (5) & (6)
 $r = \sqrt{(0.5)^2 + 0.25^2} = 0.559$
 $\frac{r}{z} = 0.186$

(7) & (8)
 $r = \sqrt{0.75^2 + 0.5^2} = 0.901$ $\frac{r}{z} = 0.30$

(9)
 $r = \sqrt{1.5^2 + 0.75^2} = 1.672$
 $\frac{r}{z} = 0.559$

$$\sigma_z = \sum \frac{3Q}{2\pi z^2} \left[1 + \frac{3r^2}{z^2} \right]^{5/2}$$

So, let us start with one more example, equivalent point load method. Example 4, a rectangular foundation, 3.0 meter by 1.5 meter, carries a uniform load of 40 kilo newton per meter square. Determine the vertical stress, vertical stress at point p which is 3 meter below the ground surface. Use equivalent point load method. So, in this case what happened? This is what it is given. It is 3 meter by 1.5 meter. So, this will be 1 meter, 1 meter and 1 meter. This is 0.5, this is 0.5 meter, this is 0.5 meter, this is 0.5 meter.

Now, equivalent point load means, I divided into in such way that this is 3 meter. I make it in to equal. Here it is 1 meter, 1 meter and 1 meter. Here it is given 1.5 meter. I make it in equal 0.5 0.5, 0.5; that means, each size having 1 meter by 0.5 meter. So, it will be very easy to distribute. So, then load on each area will be 40 into 1.0 in to 0.5. 40 is your load intensity kilo newton per meter square in to area of each. That will be 20 kilo newton so; that means, each area having load intensity of point load, equivalent point load of 20 kilo newton. Let me set it has 1 2 3 then this is 4 5 6 7 8 and 9. I make it in to in terms of 1 2 3 4 5 6 7 8 9.

What is the basic equation of here boussinesqs theory? σ_z is equal to summation of $\frac{3Q}{2\pi z^2} \left[1 + \frac{3r^2}{z^2} \right]^{5/2}$. This is what your equation. Now in this equation each load intensity having Q, it is known, z is given 3 meter which is it is known. Now r by z, what is your radial distance you have to find it out. Now for 1 and 4, 1 and 4, this area and this area. So, r is equal to 1.5 whole

square plus 0.25 whole square in to root over which is equal to 1.521. 1.5 whole square and 0.25 whole square for 1 and 4, then it will be 1 root over, r is equal to how you get the r. If you look at how the how do you get the r or these case r is your radial distance. This radial distance is your 1.5 plus 0.25 and for 2 3 5 6 for 2 3 5 6, 2 3 5 6. If you look at here 2 3 5 6.

So, it will be r is equal to 0.5, 0.5 whole square. So, here it will be 0.5 whole square plus 0.25 whole square root over this distance is your 0.5. This distance half of this this will be 0.25. So, in this case it will be it will be coming out to be 0.559. Then here first one, r by z is equal to 0.507. Here r by z is equal to 0.186. Similarly, for 8 and 9, look at here 8 and 9, 8 and 9 your r will be, now this distance will be your 8 and 9. 8 and 9 will be if I am starting with this it will be 0.75 and 0.5, r is equal to 0.75 whole square plus 0.5 whole square. Then it is root over which is equal to your 0.901. I forgot to tell, this is the point p. This is the point p with respect to the stress increase has to be found out.

Now, if you look at here load on each area; that means, if you look at here, it is 20 kilo newton for 1 and 4, I am joining here. And I am joining here for 1 and 4, this distance is one and this will be your 0.5. Hence 1.5 whole square and this distance is your 0.25 whole square root over this will be 1.5 2 r by z is equal to 0.507. If this is my point p from here to here to here these are all same these are too, for 8 and 9, if I put it 8 and 9 this is my point, distance from here to here what is the radial distance. Now if I take it this will be 0.5 whole square, and this will be 0.5 plus and 0.25, this will be 0.75 whole square this comes out to be 1.677.

So, r by z is equal to 0.30. Now come to 0.7, similarly 0.7, because what happened? 1 4 are identical then 2 3 5 6 are identical equal distance from point p. And 8 9 are equal distance from point p. Similarly, 0.7 draw a line here. For 0.7 r is equal to 1.5 whole square, r is equal to 1.5 whole square; that means, if I draw it here, this is your 0.5. This is your 0.25, 0.75 this is one this is 0.5. So, 1.5 whole square plus 0.75 whole square root over. It comes out to the 1.677. Then r by z is coming over to be 0.559. This is how it has to be calculated.

If you look at here once again I am explaining there is an area 3 meter by 1.5 meter is there. Load intensity in that area is a 40 kilo newton per meter square. And there is a point p has been given you can do it, earlier method the way I have said you can take that

point and make it small sur small rectangle and at the edge can find it out, but here it has been asked by means of equivalent point load then we convert we have to think how you are going to derived in to equal number of smalls. So, here 3 meter equally divided 1 meter, 1 meter, 1 meter. Here it is 1.5 meter equally divided 0.5 meter, 0.5 meter and 0.5 meter.

So, load on each area will be 40 load intensity in to 1.0 in to 0.5. This have to be 20 kilo newton. Now as per boussinesqs equations sigma z is equal to 3 Q by 2 p I z square 1 by 1 plus r by z whole square in to 5 by 2. Now what are the things Q is known, z known 3 meter only r or r by z you have to find it out now. Here if you look at 0.14 from point p it is equal distance. So, for one 4 it has been calculated 2 3 5 6, even it is equal distance calculated 8 9 again it is equal distance calculated 7 it has been calculated.

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$$\sigma_z = \sum \frac{3Q}{2\pi z^2} \left[\frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \right]$$

$$= \frac{3 \times 20}{2\pi (3)^2} \left[\frac{1}{\left[1 + (0.507)^2\right]^{5/2}} + \frac{4}{\left[1 + (0.186)^2\right]^{5/2}} + \frac{2}{\left[1 + (0.3)^2\right]^{5/2}} + \frac{1}{\left[1 + (0.559)^2\right]^{5/2}} \right]$$

$$= 1.061 [1.129 + 3.474 + 1.612 + 0.507]$$

$$= 7.31 \text{ kN/m}^2$$

Now, is a sum it of, the summation will be sigma z equal to summation of 3 Q by 2 p I z square in to 1 by 1 plus r by z whole square to the power your 5 by 2. And this comes out to be if I write it 3 in to Q is equal to 22 p I in to z square is equal to 3 whole square in to here it is 1 by 1 plus 0.507 whole square to the power 5 by 2.

Because there are cases. So, then it will be 1 plus 1, it will be 2 here there are 4 cases here it will be 4 by 1 plus 0.186 whole square to the power 5 by 2. Here it will be 2 by 1 plus 0.3 whole square to the power 5 by 2. Here it will be 1 by because this is 7th this is your one 4 this case is your 1 and 4, 1 and 4 this is your 2 3 5 6. This is your 2 number 3

number 5 and 6. This is your 8 and 9 and this is your 7. So, it will be 1 by 1 plus 0.559 whole square to the power 5 by 2. So, then it comes out to be 1.061 into 1.129 plus 3.674 plus 1.612 plus 0.507 this comes out to be 7.34 kilo newton per meter square.

So, this is how you can solve same problem, suppose this is the problem as I have said explained here by equivalent point load. Now this is the point p located at a distance 1 meter and here at a distance 0.5 meter is this is your 2 meter, and this is your 1 meter. So, I can do it by making into in this way also this point p. So, it is summation, what happened? This is your rectangular area here it is at the corner again this is your rectangular area here it is at the corner, again this is your rectangular area here it is at the corner again this is your rectangular area here it is at the corner. So, you can solve also this way. Equivalent point load, what I did? I am made it in to 3 parts here it is 1 meter 1 meter I distribute in to here then. So, point p 1 2 this is my radial distance I calculate for each cases at the central calculating radial distance and from there increase in stress has been calculated.

So, this comes out to be 7.34 kilo meter per meter square. I completed most of the examples. So, let me start with this new marks influence chart or influence sheet that will be interesting, new marks influence chart.

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Newmark's Influence chart

Vertical stress under a uniformly loaded area of other shapes

→ based on the vertical stresses below the center of circular area

R_1 → Radius of first circle
→ 20 equal sectors

$$\Delta \sigma_z = \frac{1}{20} q \left[1 - \frac{1}{\left(1 + \left(\frac{R_1}{Z}\right)^2\right)^{3/2}} \right]$$

→ $0.005q$

$$0.005q = \frac{1}{20} q \left[1 - \frac{1}{\left(1 + \left(\frac{R_1}{Z}\right)^2\right)^{3/2}} \right]$$

$$\frac{R_1}{Z} = 0.270$$

$$R_1 = 0.270Z$$

Diagram: A circular area divided into 20 equal sectors. A point p is located at a radial distance R_1 from the center. A larger circle with radius R_2 is also shown.

$$2 \times 0.005q = \frac{1}{20} q \left[1 - \frac{1}{\left(1 + \left(\frac{R_2}{Z}\right)^2\right)^{3/2}} \right]$$

$$\frac{R_2}{Z} = 0.42$$

42/50

So, if you look at here new mark influence chart, it is vertically new mark has been proposed. What happened earlier, there are cases where increase in stress below a point

load. Increase in stress below it is strip load, increase in stress below a line load, increase in stress at the corner of the rectangular loaded area. Increase in stress at the center of the circular loaded area, these are the stresses we have discussed; however, for new marks chart if you look at for new marks influence chart. New marks has been given to find it out vertical stress, under a uniformly loaded area under a uniformly loaded area, of other shapes. What do it mean other shapes? It is not a strip footing it is not a circular loaded area it is not a rectangular loaded area not a line load.

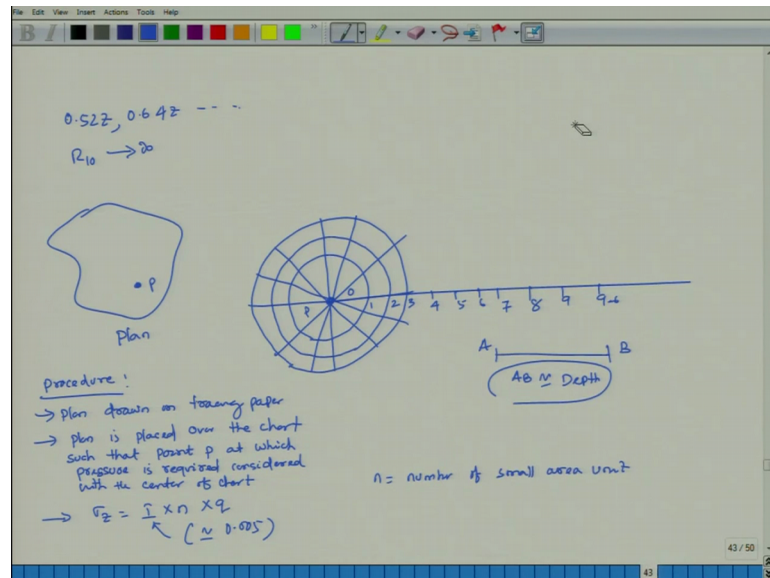
If I say this is my things find it out increasing stress it is known of the shapes. So, new marks has been given, and in this case the concept has been taken in to carried forward like this if I take it a circle right. Then here derived in to number of equal parts, let us say this is my R_1 , this is my R_1 . And this part will be let us say this is my R_1 . So, it is based on your vertical stress below circular loaded area, it is the concept is based on the vertical stress, below the center of circular area, below the center of circular area. If R_1 radius of the first circle let us say this is the first circle, R_1 is radius of first circle, then it derived in to 20 equal parts sectors. So, vertical stress at a point p at a depth z , below the center of the loaded area due to load on one sector, any one of the sector if I say this is my only one sector. Then it should be $\Delta \sigma_z$ should be $\frac{1}{20}$ of Q in to $1 - \frac{1}{1 + \frac{r_1}{z}}$ whole square, 3 by 2 .

If you apply if take a concentric circle divided into 20 equal parts. Then because it is a circular loaded area, at the center here let us see at the center here, O below the ground surface this is my point p then for circular loaded area this a increase in stress Q in to this. If I divided in to 20 parts if I taken only one sector it should be Q by 20. Let us say vertical stress is given by arbitrary fixed value of shape. So, arbitrary fixed values let us say it is $0.005 Q$, here and from there if I put it $\Delta \sigma_z$ is equal to $0.005 Q$ and here $\frac{1}{20} Q$ in to $1 - \frac{1}{1 + \frac{r_1}{z}}$ whole square to the power 3 by 2 . Then it comes out to be r_1 by z is equal to 0.270 .

So, every $\frac{1}{10}$ center of the circle with radius R_1 what does it mean? Every one-tenth one-twentieth; that means, because you have divided in to 20 parts, every parts will be equal to your R_1 will be $0.270 z$. R_1 will be $0.270 z$. Now it is small area exist 0.5 5 p . So, similarly consider another circle R_2 . So, this will be add to from here to here. Then what will happen if I take the same concept same concept. So, this and this 2 part will be 2 in to $0.005 Q$, which is equal to one twentieth of Q in to $1 - \frac{1}{1 + \frac{r_1}{z}}$

whole square to the power 3 by 2 right. Then what will happen it will be R 2 by z which is equal to 0.4 z. So; that means, now if I increase then if I go for third fourth like this. So, what will happen.

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Suppose here it is circle here it is, here it is, here it is. So, this is point O, and this is the value of the p.

Now, if you do it what will happened. So, if I continue like this. So, this is first second third fourth fifth 6th 7th 8th 9th and 9.6. So, what will happened it is increasing like third to 9 circle, it is increasing 0.52 z 0.64 z like this, and r 10 will be tends to infinity; that means, once tenth circle I am drawing, it is tending towards infinity. It is tending towards the infinity. So, the procedure, I will elaborate how the procedure has been drawn. So, consider here it is like this take a scale this, is your AB, A and B, AB is equal to your depth. So, what is your procedure will be plan of the loaded area is drawn on a trussing paper. Whatever this shape if this is the plan of the loaded area, plan of the area loaded drawn on trussing paper.

So, this scale has to be drawn in such way that length AB, length AB is equal to depth. Depth means I thought depth you are suppose to get it. Then plan is placed over new marks chart. So, you prepare the new marks chart up to 9.6 circle, not 10 circle 10 circle the value is going above to be infinity. So, plan is placed over the chart. Chart means new marks chart such that point p have to which pressure is required, such that point p at

which pressure is required consider with the center of the chart. So, which here is the point p, you prepare in the trussing paper then plan is placed here in such a way that the point p will be your center of the chart. It is merged at point O. Then count this plan area how many number of circles it is covering, count then from there you can find it out σ_z is equal to I in to n in to Q . Because pressure intensity every point you can know. So, only n , n is equal to number of small area unit, n is equal to number of small area unit.

So, in this case I is your influence coefficient or influence factor influence coefficient is your in this case 0.005. So, like this you can find it out any shape by new marks chart other than we have discussed about the point load circular loaded, line load, then rectangular loaded, all cases we have covered. Other than this regular shapes if any other shape is there you can find it out what is the stress at a point z below the ground surface. I will stop it here.

Thank you.