

Foundation Design
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Lecture – 10A
Stress Distribution in Soils-Part 2

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Stress distribution in soil

In-situ stress

Stress due to overburden are called in-situ or geostatic stresses

$$\sigma_v = \gamma z$$

$$\sigma_h = K \sigma_v = K \gamma z$$

$$\sigma_v' = \sigma_v - U_0 = \gamma z - \gamma_w h$$

$$\sigma_h' = \sigma_h - U_0 = K \gamma z - \gamma_w h = K_0 \sigma_v'$$

stress due to Foundation loading

$K_0 = \text{coefficient of earth pressure at rest}$

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So earlier covered stress distribution in soil last class; in situ stress due to foundation loading.

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Stress due to Foundation Loading

(a) Stress caused by a point load
 (Boussinesq, 1885)

homogeneous isotropic elastic

$$\Delta \sigma_z = \frac{P}{2\pi L^5} \left\{ \frac{3z^2z}{L^5} - (1-2\nu) \left[\frac{x^2+y^2}{L^3(L+z)} + \frac{y^2z}{L^3\sigma^2} \right] \right\}$$

$$\Delta \sigma_z = \frac{3Pz^3}{2\pi L^5}$$

$$= \frac{3P}{2\pi} \frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

$$\sigma = \sqrt{x^2+y^2}$$

$$L = \sqrt{x^2+y^2+z^2} = \sqrt{\sigma^2+z^2}$$

$\nu = \text{poisson's ratio}$

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So, in case of stress due to foundation loading it is primarily important for your design of a foundation; that means, increasing stress because of your external loading. So, stress due to foundation loadings stress caused by a point load we have started with boussinesq 1985.

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Vertical stress $\Delta\sigma_z$ is independent of poisson's ratio

$$\Delta\sigma_z = \frac{P}{z^2} \left(\frac{3}{2} \left[\frac{r^2}{z^2} + 1 \right]^{5/2} \right)$$

$$= \frac{P}{z^2} I$$

↑
influence factor or coefficient

$I \rightarrow \left(\frac{r}{z} \right)$

$r=0, I = 0.4775$

$$\sigma_z = 0.4775 \frac{0.05 P}{z^2}$$

$z \rightarrow 0 \sigma_z \rightarrow \infty \rightarrow \text{not true}$

Vertical stress distribution diagrams

- Vertical stress → isobar diagrams
- Vertical stress distribution on a horizontal plane below the ground surface
- Vertical stress distribution with depth at a distance or away from the line of action

Then vertical stress distribution diagram one is your isobar second is your vertical stress distribution on a horizontal plane below the ground surface. Vertical stress distributions with a depth at a distance are away from the line of action.

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$$\Delta\sigma_z = I \frac{Q}{z^2}$$

$$\sigma_z = 0.1 Q$$

$$\Rightarrow 0.1 Q = I \frac{Q}{z^2} \Rightarrow I = 0.1 z^2$$

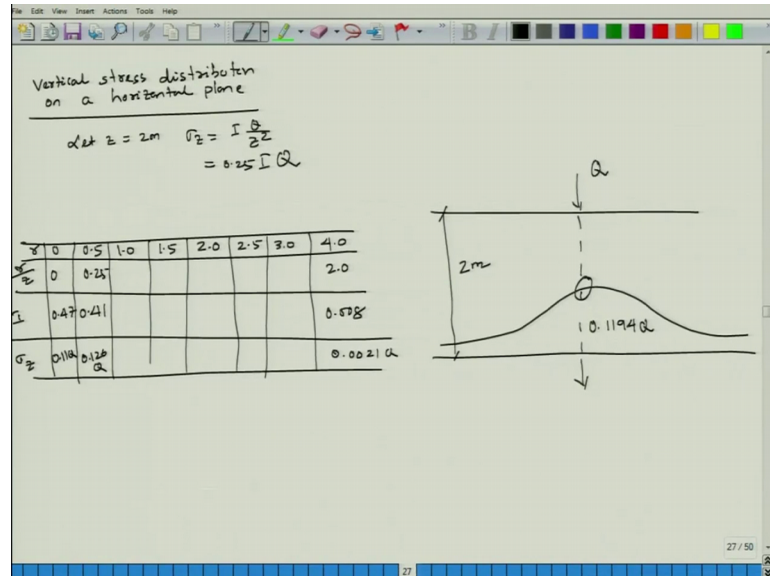
Depth	0.25	0.5	0.75	1.0	2.185
I	0.0625	0.025	0.0156	0.1	0.4775
r/z	2.17	1.5	1.16	0.93	0.000
σ	0.543	0.75	0.97	0.9	0.000

Isobar - Line joining all points of equal vertical stress below the ground surface.

For loading system → Many isobar can be drawn

So, we have covered vertical stress distribution; that means isobar then let us start with vertical stress distribution on a horizontal plane.

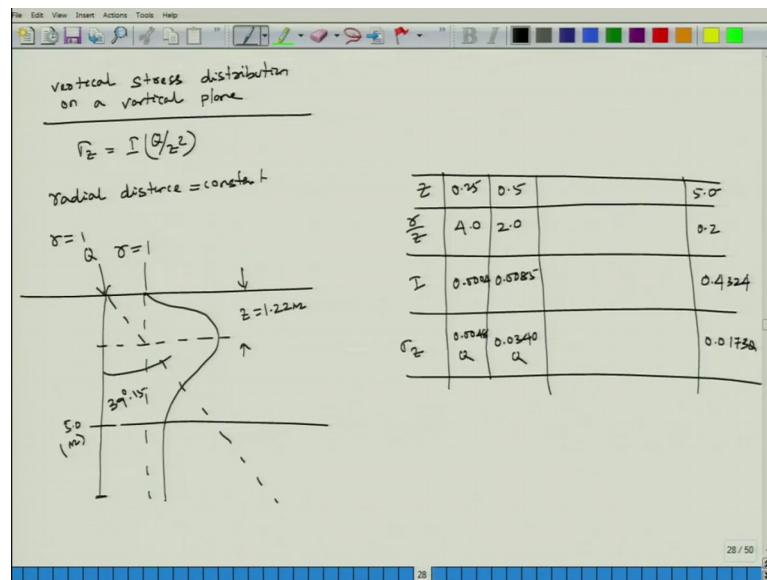
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So, let us assume let take an example z is equal to 2 meter. σ_z is equal to $I Q$ by z square. So, then it will be $0.25, I$ into p . Now take the values this is your r , this is 0 this is 0.5, this is 1, 1.5, 2, 2.5, 3 and 4. Let us see how it looks this is your r this is the 0 this is the 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0. This will be your r by z . This will be your I , this will be your σ_z . Suppose 0 r by z is equal to 0 sorry it is not I it will be I and this was σ_z this will be a 0.47, this will be 0.11 Q . This will be 0.11 Q . And this will be 0.25 this will be 0.41, this will be 0.126, Q 4.0 this will be 2.0 0.08. And this will be 0.0021 Q .

Now, if I draw it other values you can find it out if I draw it, how it looks this is my load intensity, say suppose Q . And there is a depth this is at 2 meter. This is your vertical stress distribution on a horizontal plane below 2 meter horizontal plane. If I draw it and this is my center line. So, this will be 0.1194 Q . This is my how your distribution is there vertical stress distribution on a horizontal plane.

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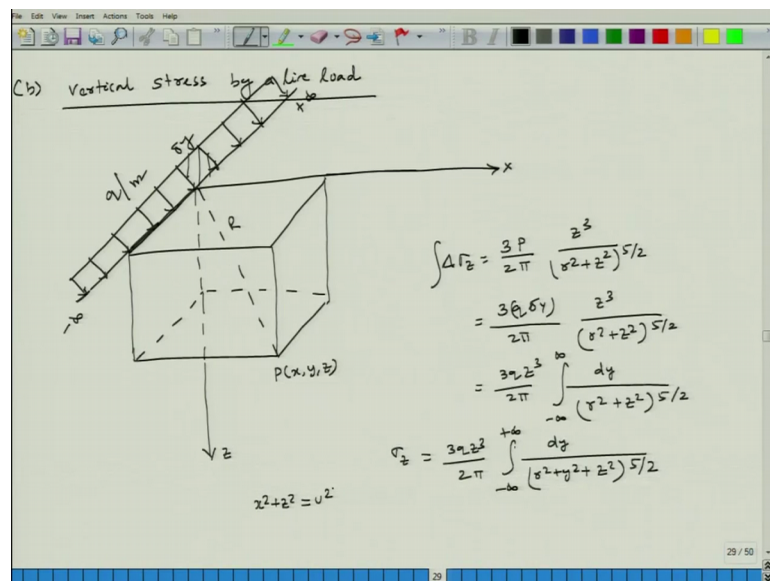
Similarly, we can find it out vertical stress distribution on a vertical plane.

So, σ_z is equal to let me write it equation, I into Q by z square on a vertical plane means; that means, your radial distance radial distance is your constant. And say r is equal to 1, say r is equal to 1. So, r is equal to 1; that means, this is your z , and this is your 0.25. This is 0.5, then somewhere else it is coming 0.5 intermediate values we can calculate, r by z I and σ_z for 0.5 it is 4.0, it is 0.004. And it will be 0.0048 Q . And 0.5 it will be 2.0. It will be 0.085. It will be 0.0340 Q . And it is not 0.5 it will be 5.0 in case of 5.0, it will be 0.2, 0.4324 0.0173 Q .

Now, if I draw it, if I plot it in this way. Suppose say r is equal to 1 this is your Q . So, in this way if I plot it, this is how it comes out. So, this maximum suppose per r is equal to 1 maximum intensity, I am getting at a depth z is equal to 1.22 meter. And this is coming 39 degree 39 degree 0.15. And it will look at why have to taken of to 5, beyond 5 means here is suppose to be your 5 beyond 5 there is no changing stress distribution on a vertical plane and maximum stress distribution observe for r is equal to radial distance r is equal to 1 at a depth z is equal to 1.22 meter. This is how first one is a isobar means it is suppose to be below the base of the 14 and base of the load intensity at the just below the load intensity it is supposed to be 0.4775. Then second one is your vertical stress distribution on a horizontal plane have taken a for example, z is equal to 2 meter here intensity maximum intensity located below your point load. This will be 0.1194 Q .

Similarly, if you go to now vertical stress distribution, vertical stress distribution on a vertical plane where sigma z is equal to I into Q by z square, any radial distance I have taken r is equal to 1, r is equal to 1 any radial distance. After 5.0 meter, there is no change in there is no variation in vertical stress distribution, in a vertical plane that is why for a example for r is equal to 1, r drawn up to 5.0 depth. And for r is equal to 1 maximum vertical stress distribution object to be at z is equal to 1.22 meter. And angle made is your 39 degree point 39 degree.

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Now, let us come back to this is about your point load. So, vertical stress by a line loads. Second one is your vertical stress by a line load. So, basically it started this derivation is basically from the extended version from your vertical stress below a point load. Let me draw in such a way it will be more you can understand more easily. This will be a z, and this will be a p into x y and z. And this is your x, and now I am taking in these directions.

So, let us consider small part here. This will be your delta y, and pressure intensity is Q pi meter. Here it will be minus infinity here it will be plus infinity. And this distance will be this is your radial distance. So, how it can be happened? I can take what will be there initially power point load if you come back to here. Point load what is that equation 3 by 2 pi p by z square or may be Q by z square into 1 by r by z hole square plus 1 to the power 5 by 2. If this equation can be return in this form, consider a point load delta

σ_z is equal to considering it will be your $3 p$ by $2 \pi z q$ by r square plus z square to the power 5 by 2 .

Then in this case I can take it p is equal to there is a infinity small. That is your Q into Δy , p is your load intensity. Here Q is equal to load intensity per meter length I have consider is small infinitely small element Δy , if I write it change it into it will be 3 small Q into Δy by 2π . And it will be $z q$ by r square plus z square to the power 5 by 2 . And which is equal to $3 Q z q$ by 2π . Because it is varying from minus infinity to plus infinity let us minus infinity to plus infinity, it will be dy by r square plus z square to the power it will be a 5 by 2 to the power 5 by 2 .

Now, if I write it will be integrated, if I write it σ_z is equal to $3 Q z q$ by 2π minus infinity to plus infinity dy by r square plus y square plus z square to the power 5 by 2 . So, what will happen let us say x square plus z square is equal to u square.

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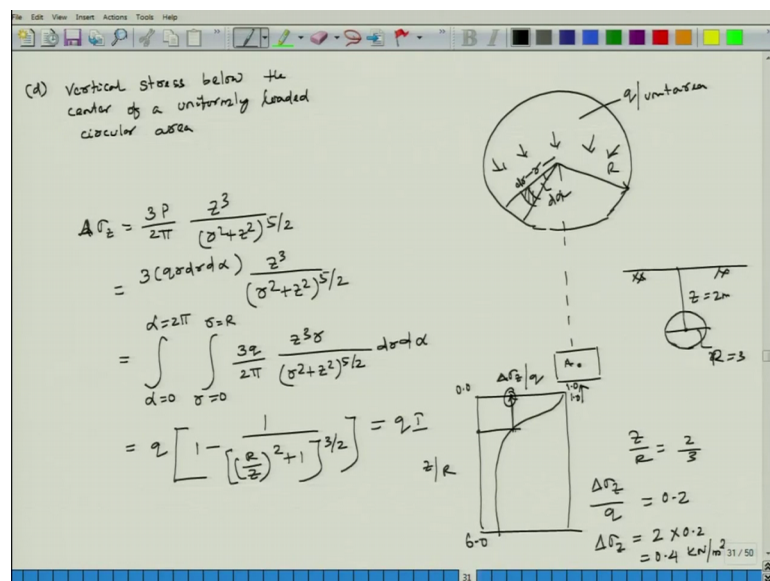
- Equations: $y = u \tan \theta$ and $dy = u \sec^2 \theta d\theta$
- Equation: $\sigma_z = \frac{2q}{\pi z} \left[\frac{1}{1 + \left(\frac{x}{z}\right)^2} \right]^2$
- Text: (c) Vertical stress distribution by a strip load
- Diagram: A horizontal strip of width B is shown above a vertical z -axis. A differential element dr is indicated at a distance r from the center of the strip.
- Equation: $\Delta \sigma_z = \int_{-B/2}^{+B/2} dz = \int_{-B/2}^{+B/2} \left\{ \frac{2q}{\pi} \frac{z^3}{(x^2 + z^2 + r^2)^2} \right\} dr$

Now, So, if I put it y is equal to $u \tan \theta$ and dy is equal to $u \sec^2 \theta d\theta$, then how it comes out to be in the form of if I solve it is coming out to be σ_z is equal to $2 Q$ by πz into 1 by 1 plus x by z hole square here it is a hole square. After solving this equations after solving this equation $2 Q$ by πz 1 by 1 plus x by z hole square to the power hole square.

This is how your pressure intensity for a line loads. Similarly, vertical stress distribution by a strip load. This will be, this will be your x and this will be your b, this will be b, and this will be your vertical distance z. This will be your r, dr this, your r dr. So, then this is the distance x. So, what will happen vertical stress distribution by a strip load? So, it will be vertical stress distribution by a strip load, it will be $\Delta\sigma_z$ is equal to integration minus by 2, 2 plus by 2 dz which is equal to minus b by 2, 2 plus by 2 2 Q by pi into zq by x square minus r square plus z square into hole square into dr. There are charts available this instead of either you can solve it by whatever the formula as available or you can integrated, it is very easy it is started with a point load and it as been extended for line load.

Now it has been extended for a strip load all charts are allowed for a examinations.

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Now, come to next part that is your vertical stress it is your d, vertical stress below the center of a uniformly loaded circular area. Now look at here. Why where solving it many times we encounter a circular footing. So, this kind of load intensity may come into picture. This is your r and if I am taking a small part, this is your dr and this will be r, and this will be your stress distribution at here. And this will be your load intensity Q bar unit area load intensity per unity area because this is an load distributed of your this circular loaded area.

So, in this case same thing can be extended as per the point load you can start with d or may be $\Delta \sigma_z$ is equal to $3p$ by 2π . Then it will be zq by r^2 plus z^2 to the power $5/2$. Then here $3p$ will be 3 into qr dr $d\alpha$. So, this is your dr and this angle is your $d\alpha$. So, what happen small loading intensity, Q small Q loading intensity over the d area dr and over then angle $d\alpha$ it has to be integrated? So, then it will be zq by r^2 plus z^2 , here to 2 the power $5/2$. Then this will be α is equal to 0 2α is equal to 2π , r is equal to 0 $2r$ is equal to r , here it will be $3Q$ by 2π zq by r^2 plus z^2 into $5/2$ dr $d\alpha$.

So, basically what I am doing. This is for a as if it is a point load acting at this center. Then point load what is the course here either it is a p or Q . This p and Q will be converted here taking into a small element. This is your small element dr and this small element dr then in this case what is there load intensity Q per unit area. So, Q into r distance this is your r Q into r , dr into $d\alpha$ then it will be zq by r^2 plus z^2 to the power $5/2$. And it is varying because, there are 2 unknowns one is α on is your r . So, r is varying from here it is 0 to r , r to r ; that means, r is equal to 0 to r is equal to r ; that means, entire radius then α is varying from here 0 and it is going towards here it will be a 2π completely this is your 2π . So, α is equal to 0 to 2π integrated over this then will get the value the value is your Q , into 1 minus 1 by r by z hole square plus 1 into $3/2$ which is equal to Q into I .

So, graphical form they have given graphical form, this is your z by r . And this is your $\Delta \sigma_z$ by Q . So, it is varying from 0.02 1.0 and this comes out to be in this form. So, either you use this chart you know the load intensity Q you know the z by r z is your distance r is your radius. For example, suppose there is a circular loaded this is your ground surface. This is your ground surface; below there is a circular footing here. At a distance the footing is suppose at a distance z is equal to 2 meter right. Suppose there radius is equal to r is equal to 3 meter. So, you can get it z by r is equal to 2 by 3 . So, this you mark it here. It is varying from 0.02 , 6.0 and $\Delta \sigma_z$ by Q is 0.02 1.0 right.

So, then you mark it here, what the value where getting. You extend it is intersecting this point curve of the chart you extend it. Once you extend it then you are getting the value of $\Delta \sigma_z$ by Q ; suppose this value is equal to 0.2 . Now increasing stress is equal to $\Delta \sigma_z$, below z is equal to 2 meter. So, in this case which is equal to Q , what is

your load intensity, for example, load intensity is your say 2 kilo newton per meter square. So, it will be a 2 into 0.2. So, it will be your 0.4 kilo newton per meter square.

So, this is how you are going to find it out from the charts and other part, I will start with a long one; what it that? That is your; similarly, we can find it out vertical stress by a rectangular loaded area as well as approximate methods and few more examples. I will stop it here.

So, next class I will start with this vertical stress by a rectangular loaded area as well as approximate methods.

Thank you.