

**Foundation Design**  
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**Lecture - 6B**  
**Bearing Capacity of Shallow Foundation- Part 4**

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Example 1

Ultimate bearing capacity of a Strip Footing

→  $B = 1.5 \text{ m}$   $D_f = 1 \text{ m}$

Sand stratum

$\gamma_d = 17 \text{ kN/m}^3$

$\phi = 32^\circ$

$\phi$  lies between  $28^\circ$  and  $36^\circ$

$\phi = 32^\circ$ ,  $N_q = 25$  (General Shear Failure)

$\tan \alpha = \frac{2}{3} \tan 32^\circ$

$\alpha = 22.6^\circ \rightarrow N_q' = 10$  (Local Shear Failure)

Actual  $N_q = 10 + (N_q - N_q') \left[ \frac{32^\circ - 28^\circ}{36^\circ - 28^\circ} \right]$

$= 17.5$

For  $\phi = 32^\circ \rightarrow N_{\gamma} = 28$  (General)

$\rightarrow N_{\gamma} = 6$  (Local)

Actual  $N_{\gamma} = 6 + (28 - 6) \left[ \frac{32^\circ - 28^\circ}{36^\circ - 28^\circ} \right] = 17$

$N_q = 17.5$   $N_{\gamma} = 17$

$q_u = c N_c + \gamma D_f N_q + 0.5 \gamma B N_{\gamma}$

$= 17 \times 1 \times 17.5 + 0.5 \times 17 \times 17$

$= 514.25$

Earlier we have finished, I have solved 2 problems of the how to calculate bearing capacity of strip footing, given value for sand stratum, and phi is 32 degree it is in between local shear failure as well as general shear failure.

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**Example 2**

Sandy Soil

1.5m

1m

(i) at a depth 0.5m below Ground surface

(ii) at a depth 0.5m below base of footing

$\gamma = 17 \text{ kN/m}^3$     $\gamma_{sat} = 20 \text{ kN/m}^3$

$\phi = 38^\circ \rightarrow$  General shear Fail

$N_q = 60$     $N_r = 7.5$

$q_u = \underbrace{cN_c + \gamma D_f N_q}_{\text{Surcharge}} + \underbrace{0.5 \gamma B N_r}_{\text{Shear}}$

(i)  $z = z' \rightarrow r'$

$r \rightarrow$  changed

???

(ii)  $r_{modifi} \rightarrow$  shear zone

$r_{modifi} = r' + \left(\frac{D_f}{B}\right) (\gamma_c - \gamma')$

Second one I have also solved one example considering water table effect at a depth 0.5 meter below the ground surface and at a depth 0.5 meter below the base of the footing.

Now, besides the Terzaghi's bearing capacities theory, there are also other theories available.

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**Skempton - Bearing Capacity analysis for clay soils**

- Strip Footing  $N_c = 5 \left[ 1 + 0.2 \frac{D_f}{B} \right] \rightarrow$  Maximum 7.5
- Square and circular footing  $N_c = 6 \left[ 1 + 0.2 \frac{D_f}{B} \right] \rightarrow$  Maximum 9.0
- Rectangular Footing  $N_c = 5.0 \left[ 1 + 0.2 \frac{D_f}{B} \right] \left[ 1 + 0.2 \frac{B}{L} \right]$  for  $\frac{D_f}{B} \leq 2.5$   
 $= 7.5 \left[ 1 + 0.2 \frac{B}{L} \right]$  for  $\frac{D_f}{B} > 2.5$

So, Skempton has given bearing capacity analysis for clay soils applicable for saturated clay these are his empirical relations Skempton has given and for strip footing strip footing  $N_c$  is equal to 5 into 1 plus 0.2 D F by B with maximum 7.5. Then second is

your square in rectangular footing  $N_c$  is equal to  $6 \left( 1 + 0.2 \frac{D_f}{B} \right)$  with a maximum 9.0. For rectangular footing  $N_c$  is equal to  $5.0 \left( 1 + 0.2 \frac{D_f}{B} \right) \left( 1 + 0.2 \frac{B}{L} \right)$  then here it is  $7.5 \left( 1 + 0.2 \frac{B}{L} \right)$ , here it is for  $D_f$  by  $B$ , for  $D_f$  by  $B$  less than equal to 2.5 for  $D_f$  by  $B$  greater than 2.5.

So, sorry it is not square and rectangular rather it is square and circular footing. So, he has given for 3 conditions particularly clay soils strip footing  $N_c$  is equal to  $5 \left( 1 + 0.2 \frac{D_f}{B} \right)$  maximum is your 7.5. Second is your square and circular footing  $N_c$  is equal to  $6 \left( 1 + 0.2 \frac{D_f}{B} \right)$  maximum is your 9.0, here it is a 7.5 rectangular footing  $N_c$  is equal to  $5.0 \left( 1 + 0.2 \frac{D_f}{B} \right) \left( 1 + 0.2 \frac{B}{L} \right)$  for  $D_f$  by  $B$  less than equal to 2.5 and with  $c$  is equal to again for rectangular footing  $D_f$  by  $B$  greater than 2.5 it is  $7.5 \left( 1 + 0.2 \frac{B}{L} \right)$ . So, this is what this Skempton has given for your saturated clay initially it has been given by Terzaghi's.

Now, it has been modified again Vesic, it is called Vesic bearing capacity theory or bearing capacity factors.

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Inclined Surface

$a_d$  and  $b_d$

$(45^\circ + \phi/2) \rightarrow$  (instead of  $\phi$ )

$$N_c = (N_q - 1) \cot \phi$$

$$N_q = \left( \frac{e^{\pi \tan \phi}}{\pi} \right) \tan^2(45^\circ + \phi/2)$$

$$N_c = 2(N_q + 1) \tan \phi$$

$\phi$	0	5	10	15	...	...	...	...	$45^\circ$
$N$	0	0.4	1.2	2.4					

If you look at what is the difference in Terzaghi and Vesic, if I draw it Terzaghi and Vesic, then Vesic free body diagram of the Terzaghi then we will be understanding what is the difference of these in Terzaghi what Terzaghi has given. Generally I allow these charts in examinations no need to remember or memorize this chart. So, particularly

Terzaghi if you look at here this is your  $\gamma D F$  and this is  $B$  and this is  $a b d$  this is zone 1, zone 1, this is your zone 2, and this is your zone 3.

So, Terzaghi bearing capacity assumption of the zone 1 and zone 2 and zone 3, zone 1 is your elastic zone, zone 2 is your radial shear zone, zone 3 is your Rankine's passive zone and this will be  $45^\circ - \phi/2$ . If I come to the zone 1 in Terzaghi's. So, in zone 1 if you look at here this is your width  $B$  and this is your  $q_u$  and this is your  $\phi$ , this is your  $\phi$  then here it is  $c_a$  then there is a  $p$   $p$  both the sides that is what I have explained earlier Terzaghi's bearing capacity theory.

Now, you come back to Vesic, this is your Terzaghi these 2 is your Terzaghi. Now come to the Vesic what Vesic assumption is. So, inclination in inclined surface particularly, if I look at inclined surface what is your inclined surface -  $ad$  and  $bd$  makes horizontal makes horizontal with your  $\phi$  angle here their (Refer Time: 07:49) saying that this angle suppose to be  $45^\circ + \phi/2$  this is the only change as compare to Terzaghi's. Vesic what happened, in zone 1 is your elastic zone, zone 2 is your radial shear zone, zone 3 is your Rankine's passive zone this is as per your Terzaghi's.

So, in zone 1 as per the Terzaghi's the angle of this particularly inclined surface  $ad$  and  $bd$  they makes an angle  $\phi$ , but Vesic said it makes an angle with  $45^\circ + \phi/2$  instead of  $\phi$  instead of  $\phi$ . Now based on that he modified  $N_c$  is comes out to be  $N_q$  minus 1 into  $\cot \phi$  then  $N_q$  is equal to  $e$  to the power  $\pi \tan \phi$  into  $\tan^2 45^\circ + \phi/2$  and  $N_\gamma$  is equal to  $2$  into  $N_q$  plus 1 into  $\tan \phi$ . So, if you look at here they have also given charts in terms of value  $\phi$  and  $N_\gamma$  0, 5 degree, 10, 15, so on up to your 50 degree. So, 0, here it is 0.4, here it is 1.2, here it is your 2.6.

So, instead of making  $\phi$  with your inclined failure surface it makes an angle  $45^\circ + \phi/2$  in case of your Terzaghi's. After Terzaghi's what happened, after Terzaghi's the value has been given by Meyerhof what are the implementation of your Meyerhof's.

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Meyerhof (1951)

$$q_u = cN_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$F_{cs}, F_{qs}, F_{\gamma s}$  = Shape Factor  
 $F_{cd}, F_{qd}, F_{\gamma d}$  = depth Factor  
 $F_{ci}, F_{qi}, F_{\gamma i}$  = Load inclination Factor  
 $N_c, N_q, N_\gamma$  = Meyerhof's Bearing Capacity factors

Terzaghi

$$q_u = cN_c + qN_q + 0.5\gamma B N_\gamma$$

$\alpha = \phi$   
 $\alpha = 45^\circ + \phi/2$   
 $\alpha \rightarrow$  closer to  $45^\circ + \phi/2$  than  $\phi$   
 $N_q = \tan^2(45^\circ + \phi/2) e^{\pi \tan \phi}$   
 $N_c = (N_q - 1) \cot \phi$      $N_\gamma = 2(N_q + 1) \tan \phi$

So, it is called Meyerhof 1951, Meyerhof 1951 get comprehensive analysis of bearing capacity of strip footing at any depth, Meyerhof 1951 he has modified particularly Terzaghi's case. What he has given the general bearing capacity theory if you look at here  $q_u$  is equal to  $C N_c, F_{cs}, F_{cd}$  plus  $F_{ci}$  plus  $q N_q, F_{qs}, F_{qd}, F_{qi}$  plus  $0.5 \gamma B N_\gamma, F_{\gamma s}, F_{\gamma d}, F_{\gamma i}$ . If you look at this terms in terms of Terzaghi what is this  $q_u$  here it is only  $C N_c$  plus  $q N_q$  plus  $0.5 \gamma B N_\gamma$ .

So, what he has introduced if you look at here  $F_{cs}, F_{qs}, F_{\gamma s}$  it is your shape factor, then  $F_{cd}, F_{qd}, F_{\gamma d}$  this is your depth factor, then  $F_{ci}, F_{qi}, F_{\gamma i}$  it is your load inclination factor and  $N_c, N_q, N_\gamma$  are your Meyerhof's bearing capacity factors.

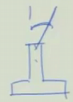
So, if you will go back to Terzaghi's what they have to given only 3 steps initial it is your strip it has been modified to square then rectangular or circular, then depth is it has been made up to a shallow depth. In this case at any depth there is a bearing capacity depth factor then load inclination, in case of Terzaghi it is only vertical no inclined load. So, he has taken all the assumptions whatever all the limitations in the Terzaghi's bearing capacity theory it has been taken care means it has been taken into considerations and put it as a Meyerhof's bearing capacity theory.

Then if I draw it Earlier's Terzaghi's bearing capacity theory with if there is a footing here footing here and this is your base of the footing this is what your this, this, this, this, then there is a wedge then if you look at here, here it is your  $45^\circ - \frac{\phi}{2}$ , here it is  $45^\circ - \frac{\phi}{2}$  then your  $\alpha$  this angle is your  $\alpha$ ,  $\alpha$  is equal to for Terzaghi's bearing capacity theory it is  $\phi$  or Vesic bearing capacity theory it is  $45^\circ + \frac{\phi}{2}$ .

Now, if you look at here particularly in case of Terzaghi what he says  $\alpha$  is closer to  $45^\circ + \frac{\phi}{2}$  than  $\phi$ , it is in between Terzaghi  $\alpha$  is equal to  $\phi$ , Vesic  $\alpha$  is equal to  $45^\circ + \frac{\phi}{2}$  and in case of Meyerhof's general bearing capacity theory he has included 3 factors any shape of the foundation any depth factor and load inclinations. It should not be vertical it should be any load inclination and  $\alpha$  particularly this angle in case of your elastic zone if it is zone 1, zone 2 and it is zone 3 it is closer to  $45^\circ + \frac{\phi}{2}$  than your  $\phi$  it is neither  $\phi$  nor  $45^\circ + \frac{\phi}{2}$  rather it is closer  $45^\circ + \frac{\phi}{2}$ . Then your  $N_q$  is coming about to be  $\tan^2(45^\circ + \frac{\phi}{2})$  and  $e$  to the power  $\pi \tan \phi$ ,  $N_c$  is equal to  $N_q \sin \phi$  into  $\cot \phi$  and  $N_\gamma$   $N_\gamma$  is equal to  $2$  into  $N_q \cos \phi$  into  $\tan \phi$

He has solve it with free body diagram and solutions it will be kind of in advanced foundation engineering not foundation design particularly in B.tech laboratory if you know the Vesic's and how it has been come up and what are the basic difference between your Terzaghi's Vesic and Meyerhof's theories. Details are there, detail derivation is there if time permits I will show you the free body diagrams and how the forces are coming and how it has been solved, so that you can have an idea about what is the difference between Terzaghi, Vesic, as well as in case of Meyerhof's.

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Factor	Relationship	Authors
Shape	$F_{cs} = 1 + B/L \frac{N_c}{N_c}$ $F_{qs} = 1 + B/L \tan \phi$ $F_{\gamma s} = 1 - 0.4 B/L$	Hansen (1970)
Depth	$D/B < 1$ $F_{cd} = 1 + 0.4 \frac{D}{B}$ $F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \frac{D}{B}$ $F_{\gamma d} = 1$ $\frac{D}{B} > 1$ $F_{cd} = 1 + 0.4 \tan^{-1} \left( \frac{D}{B} \right)$ $F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \tan^{-1} \frac{D}{B}$ $F_{\gamma d} = 1$	Hansen (1970)
Inclination	$F_{ci} = F_{qi} = \left( 1 - \frac{\beta}{\phi} \right)^2$ $F_{\gamma i} = \left( 1 - \frac{\beta}{\phi} \right)^2$ <p><math>\beta =</math> Inclination of load</p> 	Meyerhof (1963)

So, if you look at here there are different relationship based on your Meyerhof's theory, different relationship has been given this is your factor, this is your relations relationship and who are the authors? You can collect this information from the books if I write bit by bit will be time consuming, now if you look at here for example, this is shape this is depth, first one is your shape, this is your depth and this is your inclination.

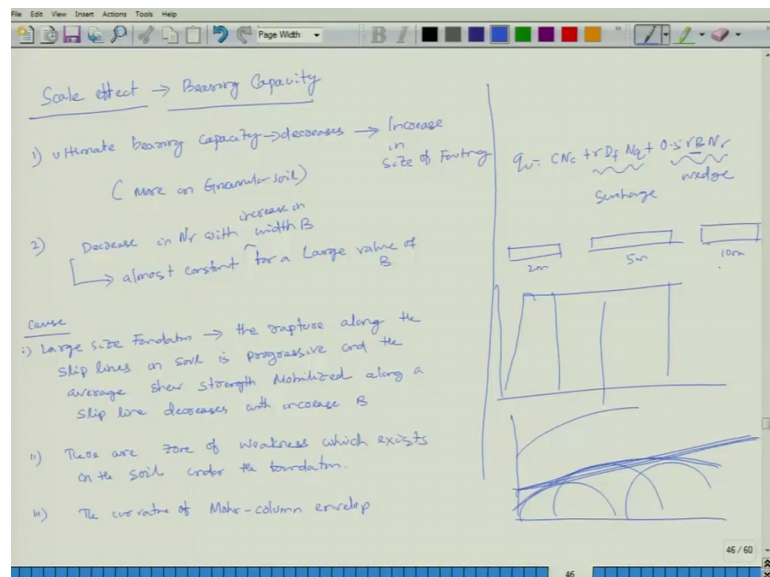
Now, if you look at this shape  $F_{cs}$  is equal to 1 plus  $B$  by  $L$   $N_c$  by  $N_c$   $F_{qs}$  is equal to 1 plus  $B$  by  $L$   $\tan \phi$   $F_{\gamma s}$  is equal to 1 minus  $0.4 B$  by  $L$  and this has been given by Hansen 1970. Then depth factor depth factor again it has been given by Hansen 1970. So, there are 2 conditions in one condition if  $D/B$  is less than 1. So, then  $F_{cd}$  is equal to 1 plus  $0.4 D/B$   $F_{qd}$  is equal to 1 plus  $2 \tan \phi (1 - \sin \phi)^2 D/B$  and  $F_{\gamma d}$  is equal to 1. Condition 2 if  $D/B$  is greater than 1 in that case  $F_{cd}$  is equal to 1 plus  $0.4 \tan^{-1} D/B$  then  $F_{qd}$  is equal to 1 plus  $2 \tan \phi (1 - \sin \phi)^2 \tan^{-1} D/B$   $F_{\gamma d}$  is equal to 1.

Then for inclinations it has been given by Meyerhof 1963. So, what they have given?  $F_{ci}$  inclination is equal to  $F_{qi}$  which is equal to  $1 - \beta/\phi$  whole square,  $\beta$  is your inclination of the load on the foundation, inclination of load suppose this is my load this suppose this is my footing, then this is the case and this here I can put it inclinations.

So, this is what your Meyerhof's has been given, Hansen has been given at this shape depth inclinations factor it can be utilized in case of Meyerhof's theory and then you can put it and the bearing capacity can get it any shape any depth any load inclination of your foundations.

Now, there is one term called scale effect what are the scale effect on bearing capacity before I further proceeds there is also IS code provisions, Bureau of Indian standard provisions also from the field test like SPT we can find it out what are the bearing capacities.

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So, scale effect next come to your scale effect. Scale effect particularly on your bearing capacity ultimate bearing capacity decreases number one, first one ultimate bearing capacity decreases with increase in size of the footing. It is more in case of more in granular soil; that means, the decrease is more in granular soil, then second part is your decrease in  $N_{\gamma}$  with foundation width B and it remains almost constant for a large value of B.

Now, what are the cause, why it is happening? If you look at here before I am writing this cause if you look at here ultimate bearing capacity  $q_u$  is equal to  $c N_c$  plus  $\gamma D_f N_q$  plus  $0.5 \gamma B N_{\gamma}$ ,  $N_{\gamma}$  this is your surcharge, this is your wedge at the base of your footing; that means, what is it mean B every everything is constant same soil if you look at this equation everything is constant same soil, as I increase the B



width of the foundation  $B$  what will happen; that means, my bearing capacity is increasing it is not true look at here - decrease in  $N \gamma$  with width  $B$  that means, with width  $B$  means with increase in width  $B$ .

Student: (Refer Time: 25:20).

It does not mean that suppose there is a 2 meter size of the footing, here it is a 2 meter, here it is a 5 meter, here it is a 10 meter it does not mean that if I increase the size of the foundations 2 5 and 10 or 20 meter the my bearing capacity will increase. It will be kind of for a large value of  $B$  it is kind of it will increase decrease or you can say that it remains constant. For large value of  $B$  large value of the  $B$  whatever you increase beyond  $B$ 's width of the foundations the bearing capacity remains constant.

So, why this is happening this is called your scale effect. So, what is the cause? The cause first one is for large size foundation the rupture along the slip lines, slip lines in soil is progressive and the average look at here average shear strength mobilized mobilized along a slip line decreases with increase  $B$ . So, second part is your there are zone of I can say weakness which exists in the soil under the foundation, the curvature of Mohr column envelop, the curvature of Mohr column envelop if I take a Mohr column here Mohr circle the failure envelop should be it should be a curve.

But in shear strength what happened? It has been assumed and approximately it has been put as a straight line straight line. So, because of this curvature of your Mohr column failure envelop that may be one of the reason, large size foundation look at this 3 are very vital you should understand large size foundation the rupture along this slip lines in soil. What is your slip line for Terzaghi? Inclined phase is progressive and the average shear strength mobilized along a slip line, average look at here the average shear strength mobilized along a slip line decreases with increase in  $B$ , it is not necessarily that average shear strength will increased as we are going to increase the  $B$ .

There are zone of weakness also below the foundations, but most of important part the curvature of your Mohr column, the Mohr column is a curvature because of it is a curved. So, this kind of, so it is certain part it will increase then it is not going to be increase rather it will be decreasing. So, this is what is your scale effect; that means, as I go on increasing size of the footing it does not mean that my bearing capacity will increased the bearing capacity of the foundations up to a certain value of  $B$  will increase

then there after either will decrease or it remains constant even if I increase the bearing capacity, even if I increase the size of my footing size suppose I increase from size of the footing from 2 meter to 5 meter expected bearing capacity will increase from 5 meter to 10 meter I expect also bearing capacity to increase, but it does not mean that if I increase from 10 meter to 15 or 20 meter the size will be going to increase. So, it will be remain constant.

The size after a certain size the width B after a certain size it has no impact on the bearing capacity factor this is called your scale effect. Why this has been explained a later phase as soon as this bearing capacity theory and settlement is over then we will go slowly slowly one by one design, design of the strip footing in sand as well as in clay, design of your mat foundations there you have to play with your size of the footings. Beyond certain size it has no impact on your bearing capacity of your footing.

So, I will stop it. So, next class I am going to start few of them are remaining as for your Bureau of Indian Standard what is your bearing capacity and from the fields state study particularly SPT and CPT whether you can get directly your bearing capacity of a soil.

Thank you.