Foundation Design
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Lecture - 5B
Bearing Capacity of Shallow Foundation- Part 2

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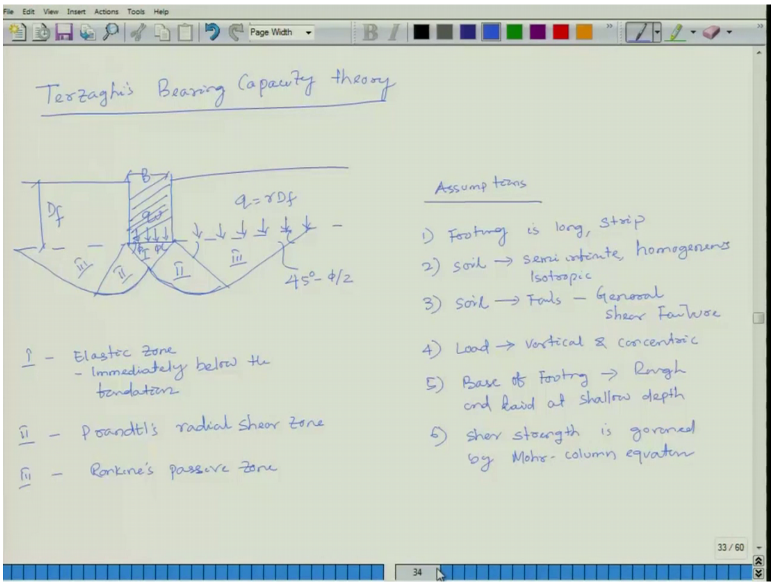

Now we will start Terzaghi's bearing capacity theory. Its first shallow foundations, let us draw a footing - this is your width B, depth of the foundation D f, pressure intensity q u, this is phi, this is phi, this has been called zone 1, this is your zone 2, this is your zone 3 and this will be q is equal to gamma D f and this angle this angle will be 45 degree minus phi by 2.

First one is called elastic zone which will be immediately below the foundation, second is zone 2 is called radial shear zone or Prandtl's radial shear zone and zone 3 - zone 3 is your Rankine's passive zones.

So, what are the assumptions? Assumption is your footing is long and strip soil is semi infinite homogeneous and isotropic, soil semi infinite homogeneous isotropic soil fails by means of general shear failure, load is vertical and concentric load is vertical and concentric, base of the footing, base of footing it is rough and laid at shallow depth and shear strength is governed by, shear strength is governed by Mohr column equations. Terzaghi's bearing capacity theory how it has come? It has come from a earlier from the mechanical by electronic microscopic with a strip they observed that these kind of failure surface or particularly plastic sheets they observed.

So, Terzaghi is the person who was taken these observations and taken a shallow foundation there is a footing here and it is zone 1 2 3, here also zone 1, zone 2 and zone 3 it is symmetry both the sides. So, he classified based on that observation from mechanical of a polythene sheets by means electronic microscopic whatever the failure surface observed it has been implemented or taken for soil mechanics. Zone 1 is your elastic zone which will be immediately below the foundations - if I say this is my foundation, so this elastic zone will be immediately below your foundation, second one is your radial shear zone this is your radial shear zone, third one is your Rankine's passive zone and later on it has been modified Terzaghi's assumption is a footing is a long and strip, soil is in all geotechnical engineering problems, generally assumption is for soil it is semi infinite homogeneous and isotropic; that means, the properties are not changing.

Soil fails by means of general shear failure that is his assumptions and in this case load is vertical and concentric, base of the footing is rough laid at shallow depth shear strength is governed by Mohr column equations these assumptions you can say that may be these are the limitations of Terzaghi proposed bearing capacity. The exact solution it is a long one, I will just Gibbs doing the free body diagram, so that you can have an idea how these exact solution has been done, but it is out of this scope.

Now, the derivation part is coming what we are going to get it from here, we are going to get it a bearing capacity equations.

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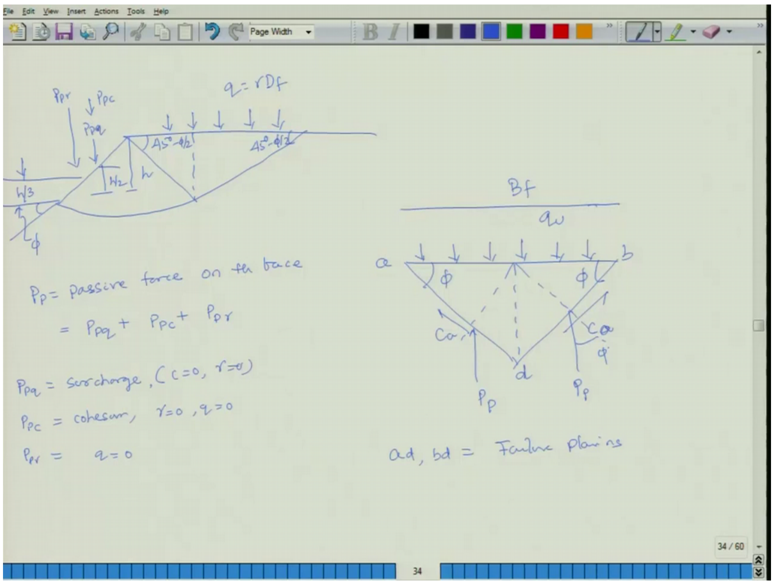

Now let me come back to 2 free body diagrams, 2 free body diagrams. So, let me take it half of this whatever I have shown it earlier half of this from here this surface to this surface I am taking it back. So, how it looks?

So, now I can name it or I can marked this forces this is your q is equal to gamma D f, this will be your 45 degree minus phi by 2 and this is also 45 degree minus phi by 2 and here this angle will be, this will be your phi, this will be your phi and there will be different terms 1 is your p p gamma other is your p pc and p p gamma if I put it here from here to here it will be h by 2 and here it is your h, and then sorry it is not p p gamma it will be p pq, then it is a p p gamma p p gamma this will be h by 3.

Now, particularly if I write it p p, if I write it p p, p p is your passive force p p is your passive force on the face, on the face then it will be p pq plus p pc plus p p gamma. If you look at the notations p pq passive force due to surcharge, p pc passive force due to cohesion, p p gamma passive force due to your unit weight.

So, generally how he has solved it may be at the end slightly I can say it is out of this scope. So, it has been super imposed. Considering p pq other 2 part has been neglected, considering p pc other the 2 part has been neglected free body diagram has been drawn, considering p p gamma also other 2 part has been neglected and free body diagram has been drawn then it has been super imposed then it has been super imposed. So, if I write in case of p pq. So, p pq what is that part? This is because of your surcharge, surcharge then in that case c is equal to 0 gamma is equal to 0 then p pc by soil cohesion by soil cohesion in that case gamma is equal to 0 q is equal to 0 then p p gamma in that case by weight of soil in shear zone. So, in that case q is equal to 0.

These are the cases. So, if I am taking it to a middle part of this free body diagram if I make it in this bigger one let it be a b d, let it be a b d. So, this will be your I am drawing the free body diagram, this will be your q u and this is phi and this is your phi, here is your ca here is your ca then I wrote it, it is completely p p, p p is your p pq, p pc and p p gamma. Here I wrote it then this side also it is p p it makes an angle phi.

This is my free body diagram and width of the foundation is your D f. I am taking the elastic zone part I am taking the elastic zone part. So, ad and bd of the elastic zone are the failure plains rising at an angle to the horizontal ad and db, ad and bd of the elastic base are failure plains, failure plains. So, if I take this free body diagram let me write it out downward force downward force that is your q u into B into 1, q u into B into 1 I am writing it only B width of the foundation only B.

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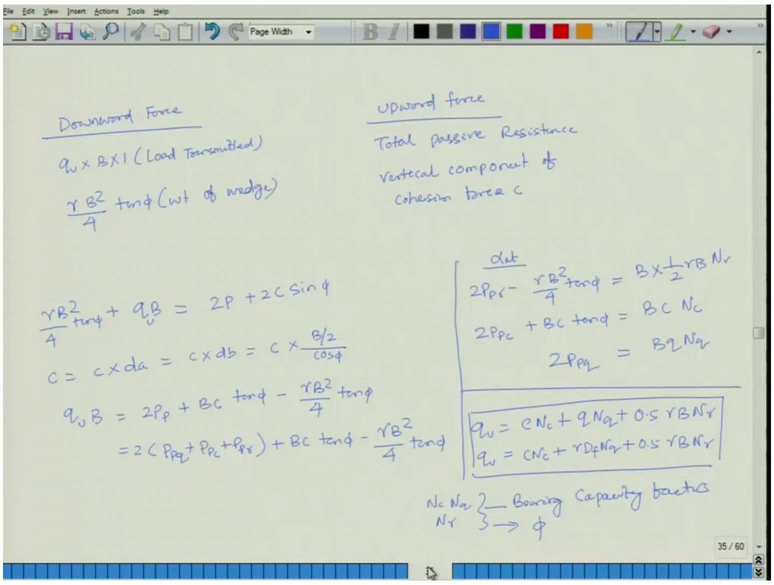

So, downward force q u load intensity major width into 1, this is your load transmitted. So, then weight of the soil wedge gamma B square by 4 tan phi this is your weight of wedge, this is your downward force downward force and upward force total passive resistance up upward force is your total passive resistance. Then vertical component of cohesion c vertical component of cohesion force, force c; if I write it qB which is equal to 2 p plus 2 c sin phi here I am writing plus gammas B square by 4 tan phi.

Look at here once again downward force this is the free body diagram of the elastic zone downward force - q u into B into 1 because it has been assumed is a long, it is a 2 dimensional problem and plain strain if it is a plain strain per meter length, q u into B per meter length is a 1, this is a vertical force acting downward. Then within this wedge what is the weight of the wedge? Weight of the wedge is your gamma B square by 4 into tan phi then look at the upward force total p p, it may be p p is p pq, p pc and p p gamma is that upward force and this c, ca or c has 2 component - one is your vertical component, one is your lateral component.

So, lateral component will equal and opposite and it will cancel out. So, vertical component will are your upward force, now here it is your 2 p p plus 2 c sin phi, now I can write it capital C is equal to small c unit cohesion into da which is equal to c into dB. Capital C is equal to ca or c unit cohesion small c da over the length this da over the length this dB which is equal to again which is equal to c into B by 2 by cos phi, c over the length here which will be B by 2 cos phi. So, if I am writing it here I am putting it q u, q u into B which is equal to 2 p p plus Bc, Bc tan phi minus gamma B square by 4 tan phi. As I said the p p has 3 component it is your total passive. So, p p gamma, p pq and p pc, if I write it so this will be again 2 p pq - q is your surcharge, p pc this is your cohesion plus p p gamma unit weight plus Bc tan phi minus gamma B square by 4 tan phi.

Now, let writing in this way let 2 p p gamma minus gamma B square by 4 tan phi which is equal to B into half gamma B N gamma, then 2 p pc with this is because of your unit weight, this is because of your cohesion plus Bc tan phi which is equal to B c Nc and 2 p pq which is equal to Bq Nq. Why it has been rearranged like this? We have to bring out in a dimensionless form Nc, Nq and N gamma are called bearing capacity factors and if you look at here if I am saying p p gamma because of your unit weight, unit weight which part is coming? This is a weight of the wedge I am taking it here this minus this footing into B into half gamma B by 2 into N gamma.

If I am talking about the unit cohesions, this will be your 2 p pc unit cohesions plus Bc tan phi around the periphery how your cohesion force this is your Bc Nc, Nc is your bearing capacity factor considering cohesions, p pq surcharge Bq or Bqu, Bq or Bqu Nq. If I am rewriting it, if I am rewriting it q u is equal to all the terms if I am rewriting it, it will be C Nc plus q Nq plus 0.5 gamma B N gamma or C Nc plus gamma D f Nq plus 0.5 gamma B N gamma. This is your Terzaghi's bearing capacity theory, ultimate bearing capacity q u is equal to C Nc, look at here definitions q u is your ultimate bearing capacity C Nc plus gamma D f Nq plus 0.5 gamma B N gamma and Nc, Nq, N gamma these are all your bearing capacity factors, bearing capacity factors it depends on phi.

So, this is all total in detail details you have to take a free body diagram individually then calculate it and Nc, Nq, N gamma are the bearing capacity factors.

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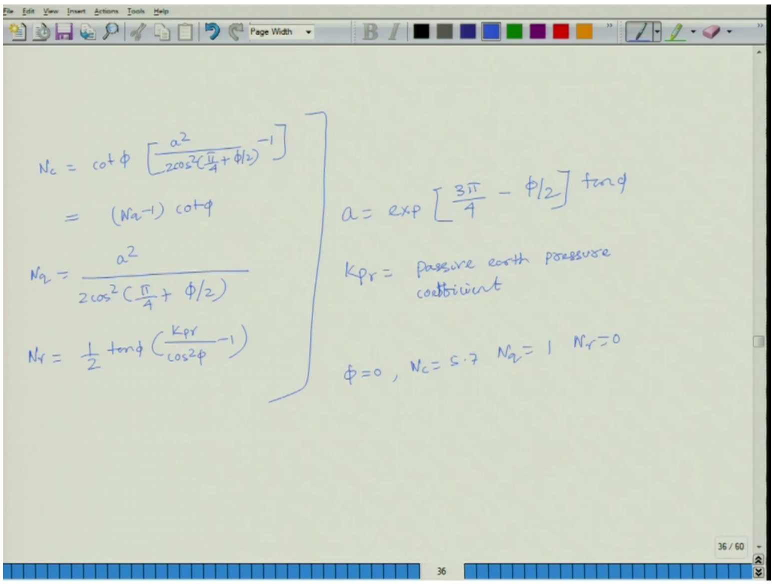

If I write it Nc, Nq, N gamma after the detail solutions bearing capacity factor for cohesions it will be cot phi a square by 2 cos square pi by 4 plus phi by 2 minus 1 which is equal to you can write it Nq minus 1 into cot phi. Nq is equal to bearing capacity factor for your surcharge a square by 2 cos square pi by 4 plus phi by 2 N gamma is equal to half tan phi k p gamma by cos square phi minus 1. So, this is a after solving all free body diagram individually taking into considerations integrating if I write it Nc, Nq and N gamma we will come into in this form. So, basically it depends on your phi depends on your phi. So, a, what is a? a is exponential 3 pi by 4 minus phi by 2 into tan phi, k pr is equal to passive earth pressure coefficient.

So, this is your bearing capacity factors and ultimate bearing capacity as per Terzaghi's bearing capacity theory you have to remember this, this will be required throughout your foundation design. C Nc gamma D f Nq plus 0.5 gamma B N gamma or C N cq Nq plus 0.5 gamma B N gamma. Once you know the phi value you can find it out bearing capacity factors Nc Nq and N gamma when phi is equal to 0, phi is equal to 0 Nc is equal to 5.7, Nq is equal to 1, N gamma is equal to 0.

So, Terzaghi has given a bearing capacity factor chart that you can keep it for examination point of view. For different value of phi Nc Nq has been given. So, in between phi it has been given 0 - 10 degree, 20 degree, 30 degree, 40 degree, 50 degree like this given in between you have to interpolate, you have to interpolate this value of your Nc, Nq and N gamma. So, this is about your Terzaghi's bearing capacity ultimate bearing capacity of your foundations resisting of your soil to find it out considering a general shear failure.

Later on Terzaghi has modified if there is a local shear failure local shear failure if you go back to as I have said which cases it will be your local shear failure.

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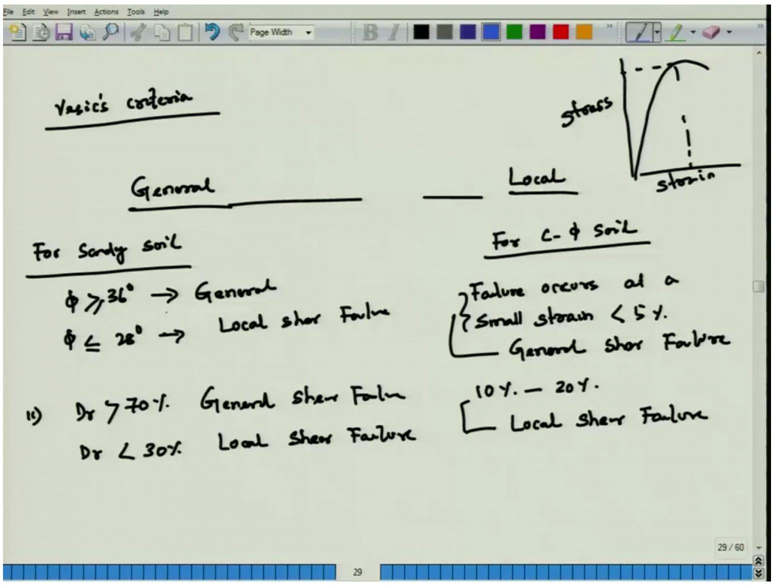

Phi is greater than or equal to 36 degree general shear failure it has been assumed for sandy soils. Local shear failure if it is phi is less than equal to 28 degree then it is a local shear failure.

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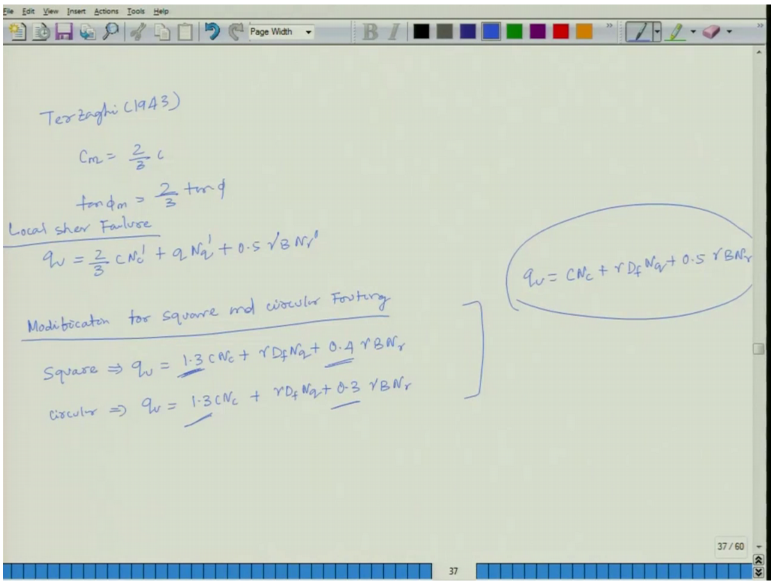

So, in this case for local shear failure Terzaghi has given empirical relations for adjustment, Terzaghi which has proposed 1943. So, you have to consider mobilized cohesion equal to two-third c, tan phi m is equal to 2 third tan phi. Now ultimate bearing capacity for your considering local shear failure q u is equal to two-third C Nc prime effective in terms of if it is a phi effective, there will be a total stress as well as there will be effective stress plus q Nq prime plus 0.5 gamma B N gamma prime this is what he has given the modifications for these case local shear failures.

So, now again he has modified for your square and circular footings we will solve also problems in this case how it will going. First one Terzaghi has given for it considering it is a cohesion less soil considering c phi soils considering it is general shear failures q u is equal to C Nc plus q Nq plus 0.5 gamma B N gamma. Then later in 1943 as modified for your considering it is as local shear failure q u is equal to two-third, C Nc prime q Nq prime plus 0.5 gamma B N gamma prime.

Now, again he has modified because initially assumption it has been considered for a strip footing. Again he has been modified for square as well as circular footing. Now for square footing q u is equal to 1.3 c Nc plus gamma D f Nq plus 0.4 gamma B N gamma and for circular footing circular footing q u is equal to 1.3 C Nc plus gamma D f Nq plus 0.3 gamma B N gamma. Look at initial bearing capacity considering general shear failure for a strip formulations q u is equal to C Nc plus gamma D f Nq plus 0.5 gamma

B N gamma, this is what initially he has given for a strip footing considering general shear failure.

Now, it has been modified for your considering local shear failure, again it has been modified for square as well as circular, square what are the terms? 1.3 C Nc and 0.4, instead of 0.5 it is 0.4, here it is 1.3 C Nc here it is 0.3. This modification based on the studies, based on some few experiments studies he has been modified.

Then, I think I will stop it here. Next class I will start the effect of the water table, after finishing this effect of water table I will solve few problems I will solve few problems how to calculate your bearing capacity ultimate bearing capacity before I go for other bearing capacity theories basics and Meyerhof's because these are all preliminary part, basic part how to calculate your bearing capacity of foundation resisting on soils, then once it is over we will go for settlement analysis then once that is over then we will go for different foundation designs.

Thank you.