Water Resources Engineering

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Lecture No. # 5

In the last lecture we had looked at some of the limitations of the unit hydrograph theory. We have also looked at different ways to go about solving these problems. For example if the catchment area is very large we cannot apply the UH theory. Then we can subdivide it into smaller areas. Apply the unit hydrograph theory and route, the flow through the channel or if the rainfall is non uniform, we can derive an instantaneous unit hydrograph IUH and apply that to get the direct runoff because of any non uniform distribution of the rainfall. In the third problem which we faced, there was lack of data. So if the stream is not gauged at a particular point or if the storm events of that particular duration are not available then we cannot develop by unit hydrograph for that catchment for that purpose. We already described methods know as synthetic unit hydrographs.

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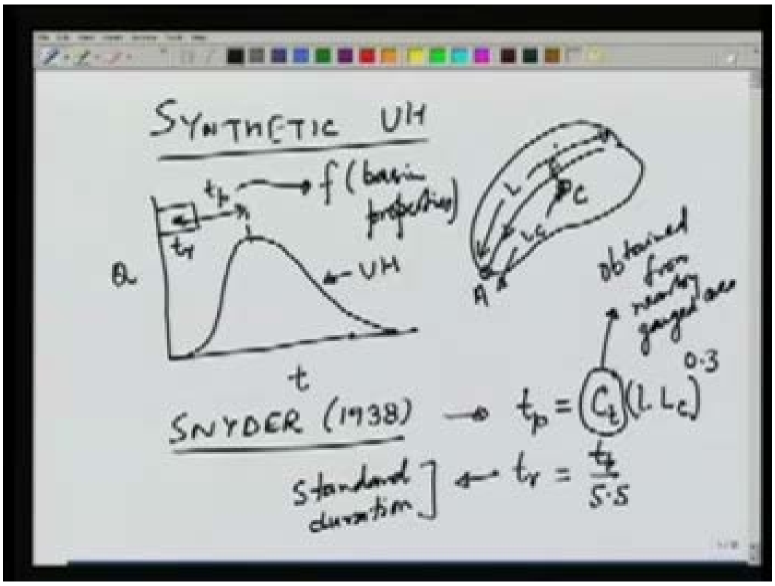

This correlates some of the hydrograph properties to some of the basin properties and since the basin properties can be easily obtained, we can obtain the synthetic unit hydrograph for that area using some relationships which are mostly empirical based on observations of existing gauged catchments. So let us look at a catchment where there is no gauging station.

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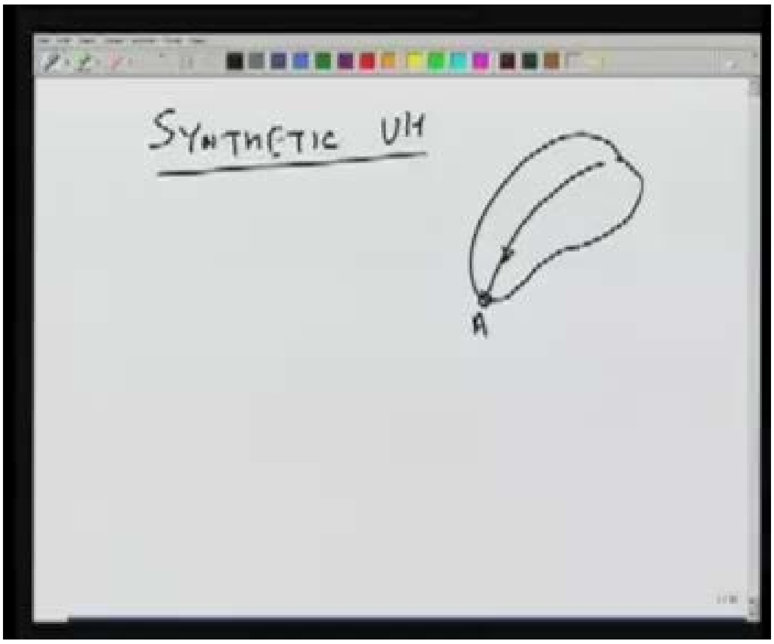

So the point A is not gauged and therefore we do not have any data evolved from the the runoff due to a corresponding storm event. In absence of this data we can try to correlate some of the features of the hydrograph.

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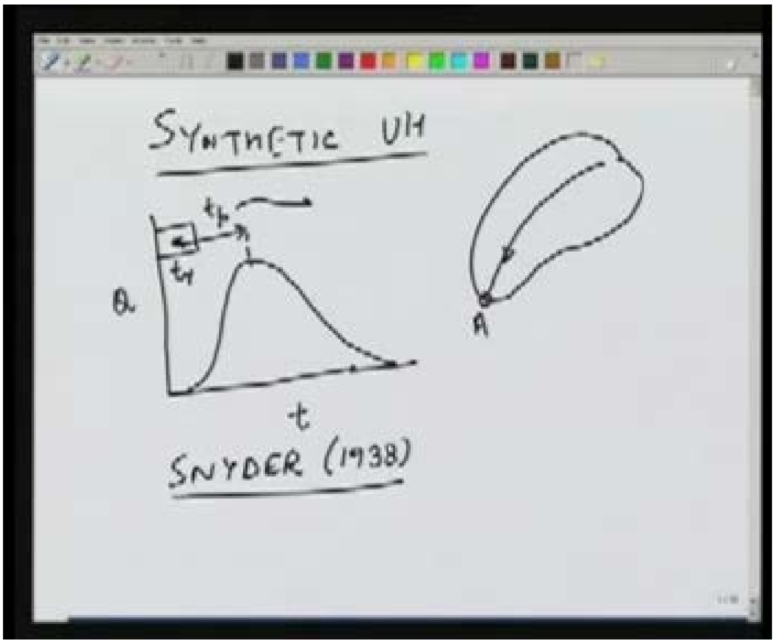

So let us say the duration of the rainfall is t_r and we can define a time to peak from the centroid of the rainfall as t_p. Based on a lot of data about existing gauged catchments in the Eastern United States, Snyder in 1938 came up with some relationship which relate the lag t_P, which is the time from centroid of the rainfall to the peak with catchment properties. Some basin properties can be used to obtain t_p. Snyder described the basin properties in terms of 2 main variables, length of the basin, which is from the point A, the outlet point to the water dividing alone the stream. So this length is not a straight line distance between 2 points. It is along the stream a curving length L. The other parameter which he chooses was based on the centroid of the area. If this point C is the centroid of the catchment area then there is a point directly opposite the centroid on the stream. Let us call this point C prime. The distance of C prime from A again along this was termed as length to the centroid and we would write it as Lc. So using these 2 parametres, L denotes the size of the basin because larger L value means a larger size of basin and Lc represents the shape of the basin. So if the basin is wider near the point A, Lc would be smaller. If the basin is wider away from point A then Lc would be larger.

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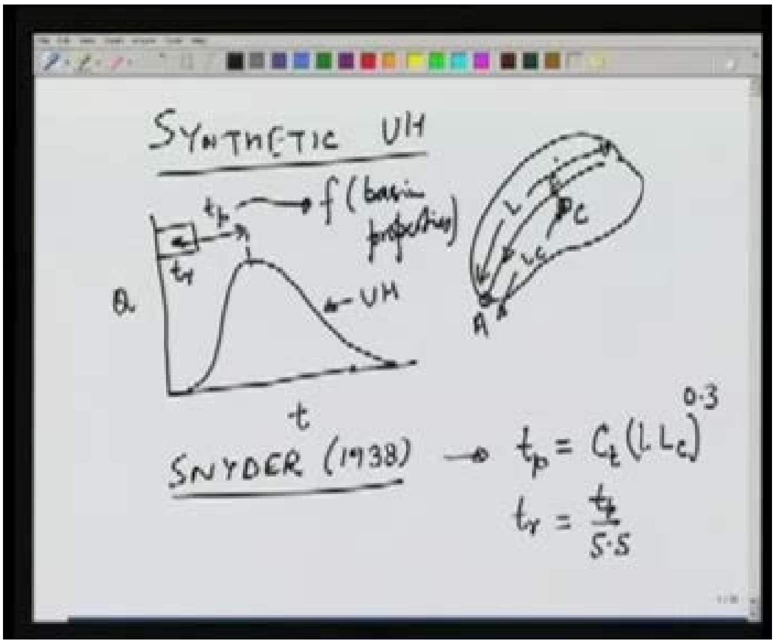

What Snyder found was that the time to peak from the centre of the rainfall was related to L, Lc to the power 0.3. So for any catchment if we know L and Lc, which we can measure from the topographic map of the area, then we can find out for a hydrograph of duration t_r, the unit hydrograph of duration t_r that was related with the time to peak as t_p over 5.5. So Snyder called standard duration of rainfall t_r and for this t_r, the time to peak will be given by this equation. In general we may not want to derive a unit hydrograph of duration t_r and we may have the value of C_t, typically obtained from a gauged catchment which is similar to the catchment under consideration. So C_t is obtained from nearby gauged catchment or gauged area. So in order to derive the synthetic unit hydrograph, we should have an area which is similar to the area under consideration. It may be a nearby basin where the properties are similar to the area under consideration and for that area we should have a gauging station from which we can derive a unit hydrograph for that particular area.

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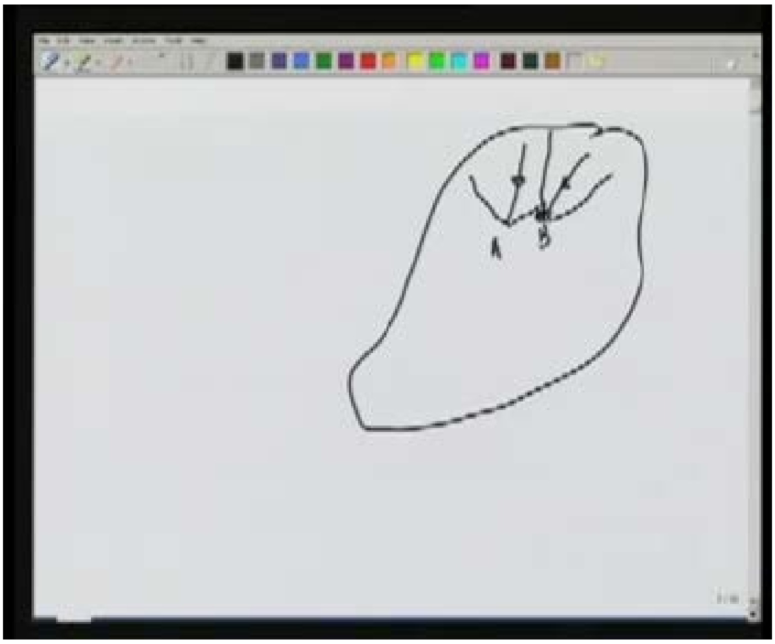

So let us look at the big basin and let us say that we have one stream here, another stream here. We know the catchment areas of these streams at some points. Let us call these points A and B. Suppose there is a gauging station at Point B, due to a rainfall event in the area B, we can measure the runoff at point B. But at A we do not have a gauging station. So our aim is to derive the UH at A. I will call it synthetic unit hydrograph SUH and this SUH may have a duration which is not equal to the standard duration $t_{r.}$ It may have some other duration and let us call it t capital R or the required duration. This capital R stands for required duration. Now the data is available at B. Again there is rainfall event for this catchment area may not be of a standard duration Tr. So let us say that at B we denote the properties by variables which have a star. So let the area of catchment B be called A star.

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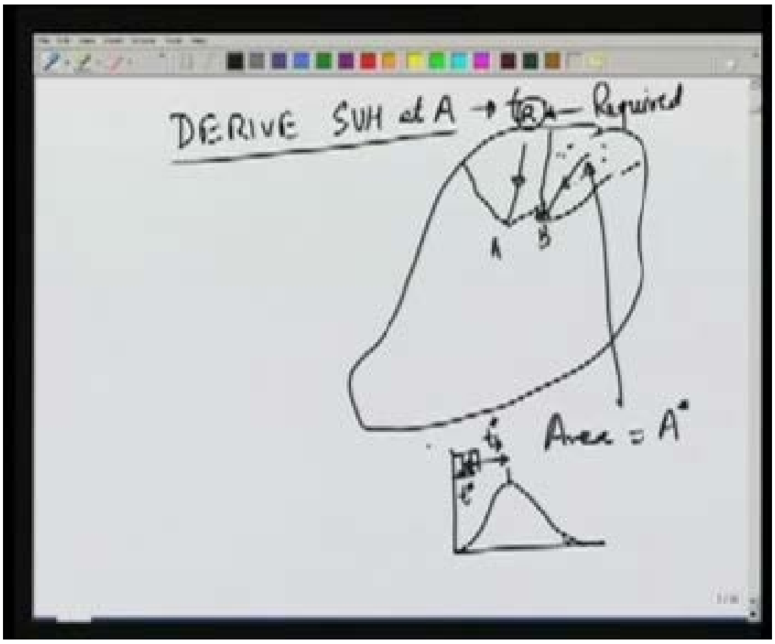

Also let us say that we have a unit hydrograph derived for this area. Since this is a gauged catchment area, we can derive the unit hydrograph for this area based on the storms occurring in that area and suppose we have derived the UH for a rainfall, duration t star and the time to peak for this UH is t_p star. Now this t_r star is the duration of rainfall for this unit hydrograph which has a time to peak of t_p star. Now based on these data what we now want is to obtain the coefficient C_t. What Snyder said was if there is any other duration, let us say t_r star which is not equal to t_r the standard duration.

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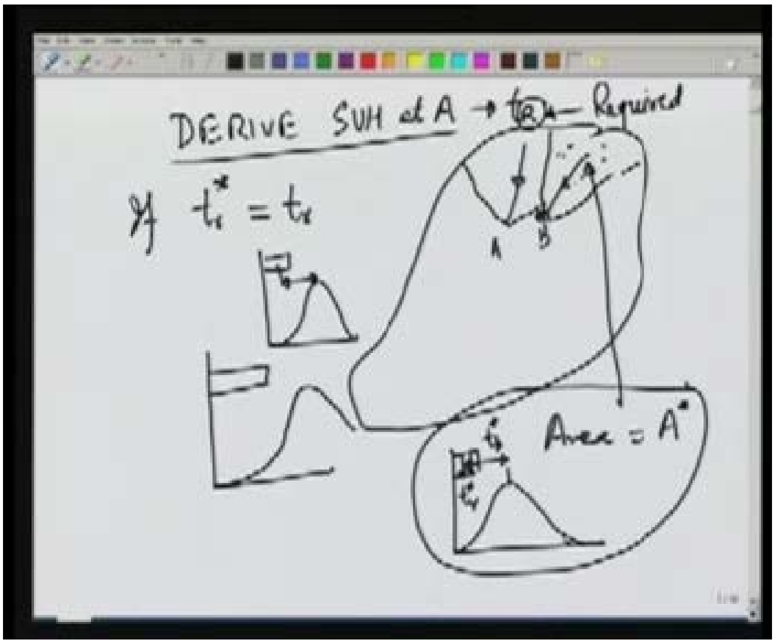

Then the time to peak will also have to be shifted. So if for a standard duration there is sometime to peak and if there is another rainfall which is not of a standard duration let us say, this is longer than a standard duration then that peak will shift to the right and the distance to peak t_p star. So if this is t_r and t_p, $t_p$star will not be same as t_p and there are relationships which is proposed by Snyder that t_p star should be taken as t_p+ t_r star – t_r over 4. So this accounts for the difference in duration of the rainfall. The standard duration is t_r and the actual occurrence on the catchment area of B is t_r star. This helps us in obtaining the lack from the catchment area B. Now using these values we can derive the coefficient C_t for the area B.

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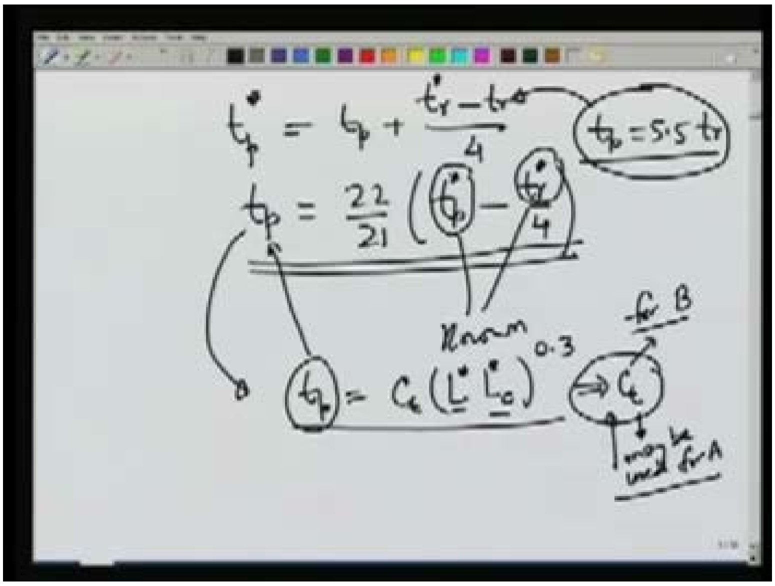

We have this equation t_p star and we also have the standard equation which says that t_p is equal to 5.55 t_r as proposed by Snyder. So using these equations we can correlate the standard time to peak with the observed values of time to peak and rainfall duration As t_p = 22/21 t_p star which is a rather straight forward operation by just substituting this value here and solving the resulting equation. So using this information t_p star and t_r star, these are known because the catchment is gauged and therefore we have unit hydrograph for duration t_r star and we also have the time to peak in terms of t_p star. So we can find out t_p and then since we know the relation of Snyder, t_p is = C_t and since this is on the catchment area B, I will put a star to the power 0.3. So using this relationship we can obtain C_t because once t_p is known from L star and Lc star from the basin. We already know this information because we have the data available for the gauged basin therefore we can get C_t from here. We can assume that this is the same C_t for B. This C_t value for catchment B but since we have assumed that both the catchments A and B are similar, we can say that the same value of C_t can be used for here too. So it may be used for A and so that takes care of one of the coefficients C_t.

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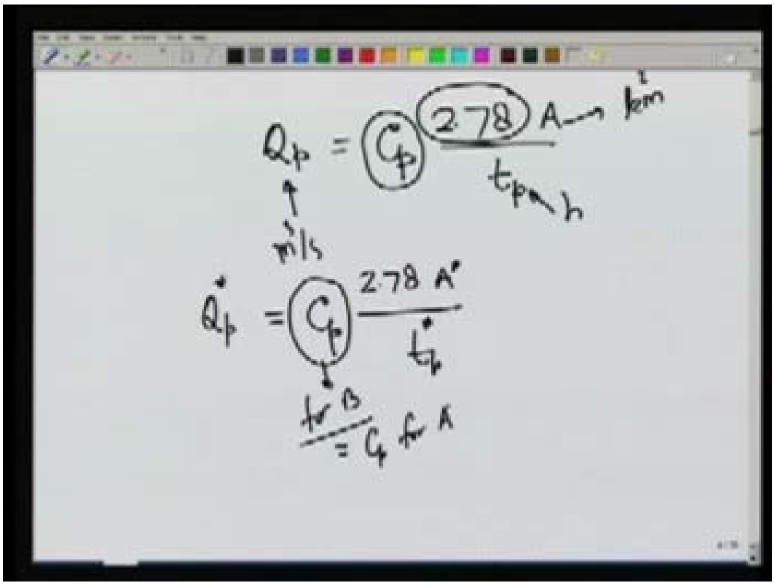

Now to find out the peak discharge, Snyder found another relationship and he says that Q_P will be given by some other constant C_P 2.78 A over t_p. Now this 2.78 factor comes by conversion of units of km square and hours to get Q_P in metre cube per second. So we take t_p in hours, A in kilometre square, Q_P in metre cube per second and this C_P again can be obtained from the data on the gauged catchment. So we know the value of Q_P star. We have the value of A star and we also have the value of t_p, we can write C_P and since everything else is known, we can obtain C_P for B and assume that the same C_P holds good at A also and same as C_P for A.

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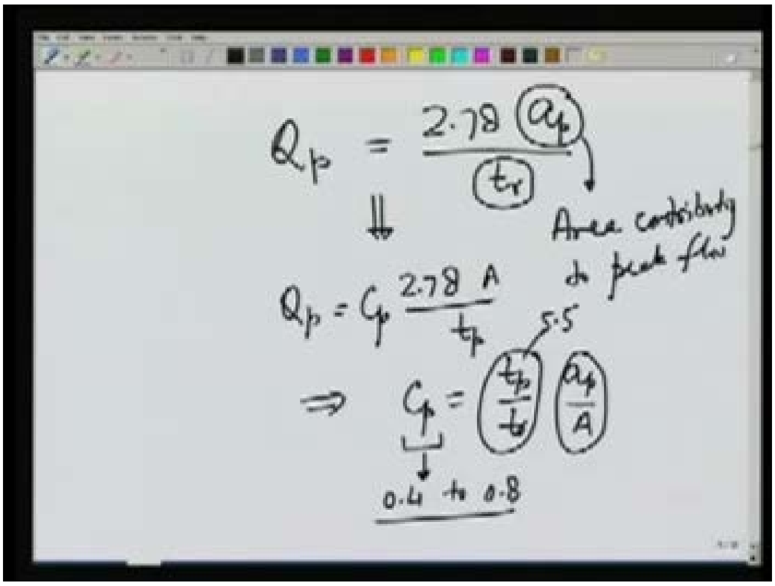

If we look at discharge, the peak discharge due to a rainfall of duration t_r, we will just say that should be one over t_r because we are talking about unit hydrograph and a_p is the area which is contributing to the peak flow. If we want to write this in terms of Snyder's equation which is Q_P equal to C_P, what we get is the value of C_P as t_p by t_r into a_p by A. t_p by t_r as per Snyder is equal to 5.5. a_p by a will generally vary from catchment to catchment and therefore the value of C_P also varies depending on the value of a_p over A. It has been found to vary from about 0.4 to 0.8. We have the value of C_P and C_t.

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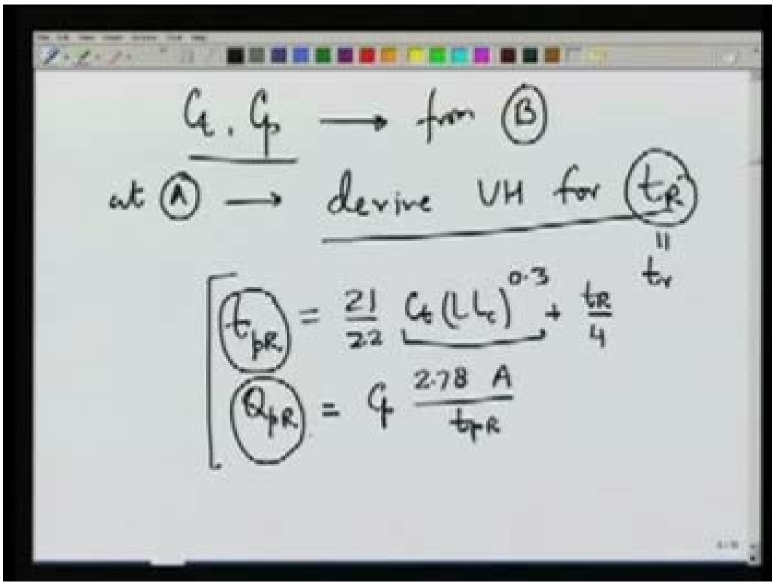

So we have obtained these values from B and now we can use these values at A to derive a unit hydrograph for any desired duration t_r. So at A we want to derive a UH for duration t_R, capital R. So now we have to see how to go about deriving the unit hydrograph at A for a duration t_r. If this t_r is equal to the standard duration t_r then we do not have to do anything but if it is not, then again we have to use the Snyder's proposal and what we do is write t_{pR}, where this term is nothing but t_p and then we can write Q_P for the required duration t_r as t_{pR}. So this will give us the time to peak for the required unit hydrograph and the discharge at peak Q_{pR} and draw the unit hydrograph.

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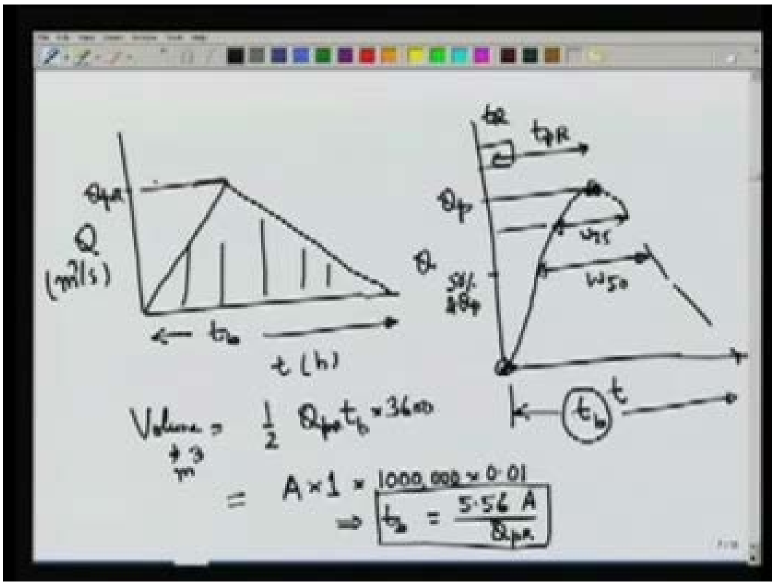

But it will not be very accurate. What we have is only 2 points. This is Q_P, this is t_R and t_{pR}, so we have one point here. We know it will start at 0, 0. The other information which we have is that the area under the curve should be equal to 1cm of depth over the catchment area. So we may have a shape like this or like this. There are various other options which will have the same area. Therefore some other points need to be given. One of the parameters which is normally given is the time base t_b. The time base of the UH and there are again equations provided for this time base in order to help us a sketch the unit hydrograph. There are some other parameters for example the width of the unit hydrograph at some location can be given. In this case suppose Q_P is the peak flow, we can find out what will be the width of the unit hydrograph at 50 percent of Q_P and call this W_{50}.

Similarly we can also provide some equation for W_{75} which is at 75 percent of Q_P. So once we know these 4 or 5 different parameters, it would be much easier for us to sketch the unit hydrograph. So let us first locate this t_b, the time base. What we do is we assume that the unit hydrograph can be approximated by a triangle which has a peak flow of Q_P and a time base of t_b. Once we assume it to be a triangle, it is very easy to find out the area under the curve knowing that Q is in metre cube per second and t is in hours and can be written as the volume can be written as half Q_P t_b. Now we have to take care of the units, so this Q_P is in metre cube per second and t_b is in hours, so we have to multiply it by 3600 to get the volume in metre cube. Now this should be equal to 1cm of depth over the entire catchment area. So this should be equal to area of the catchment into one and now to take care of the units A is in typically kilometre square so we have a million and this one is cms. So it could be 0. 01 in terms of metres and equating these 2 we can obtain an equation for t_b in terms of the peak flow and the area and the equation is given as 5.56 into A over Q_P. Now if the rainfall is not of a standard duration, for required duration, we will add subscript r here to show that this is the Q_P for the required duration of rainfall. Once we know t_b then we have 3 points fixed. One is the peak, one is the 0 0 and one is this t_b for W_{75} and W_{50}, there are some equations proposed again based on available data.

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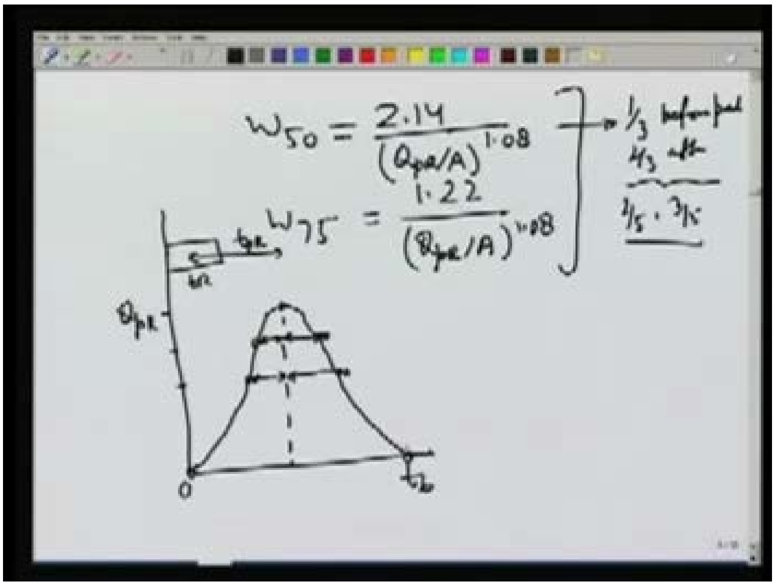

The equations which are proposed are for W_{50} which is 2.14; this is the peak discharge per unit catchment area. Q_{pR} over A to the power 1.08 and W_{75} would be smaller than this. So now we have these 5 parameters known so we have a point here, 0 point. Here t_b we have Q_{pR} for the required duration of rainfall and we also know t_{pR}. So the peak point is known. The base is known at 50 percent and 75 percent. We also know the width of the unit hydrograph. There are some suggestions to know the amount of width before the peak and the amount after the peak. For example some researchers have suggested 1/3 before peak and 2/3 after. If this is the peak line, W_{50} can be put, 1/3 here and 2/3 here. Some of the other researchers have suggested 2 over 5 and 3 over 5 in a ratio of 2:3. Some have suggested 1:2. Some have suggested 2:3. So once we know that we will have additional points. 2/3 is here, 1/3 here, so we have one point here, one point here, another point here, another point here, and the base here. So now we have 1, 2, 3, 4, 5, 6, and 7 points which can be joined by a smooth curve and obtain a synthetic unit hydrograph.

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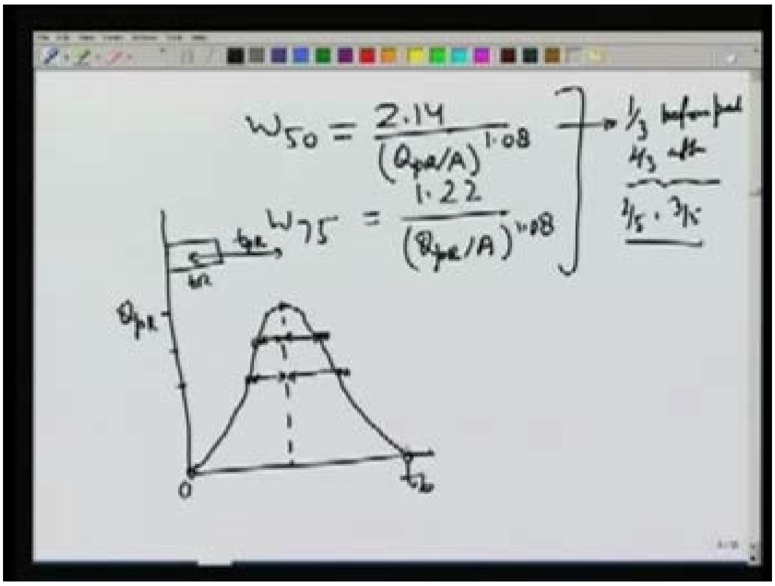

The check will be that the area under this curve should be equal to that of rainfall and if it is not, then we will have to adjust some coordinate. May be this t_b will have to be shifted this side or this side. A little bit of adjustment has to be made in order to maintain the area under the curve equal to 1 cm of depth. So in this way for any catchment, which is not gauged but a nearby catchment has a gauging station and this is similar to the basin under consideration. We can develop a hydrograph t_r hour unit hydrograph based on the synthetic unit graph.

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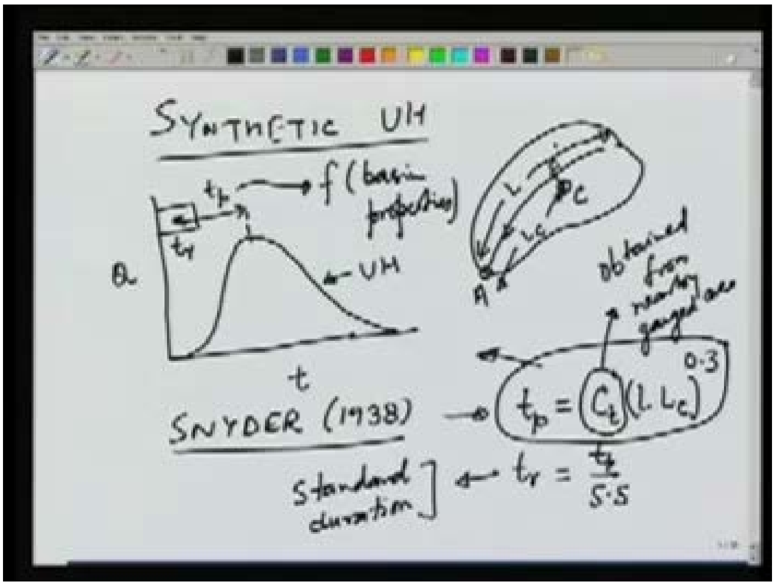

There are some modifications with this synthetic hydrograph which have been proposed, for example the equation C_t parameter depends on the slope of the basin. Some of the investigators have put the slope of the basin too inside this variable, and then they have suggested the usage of some formula which involves,

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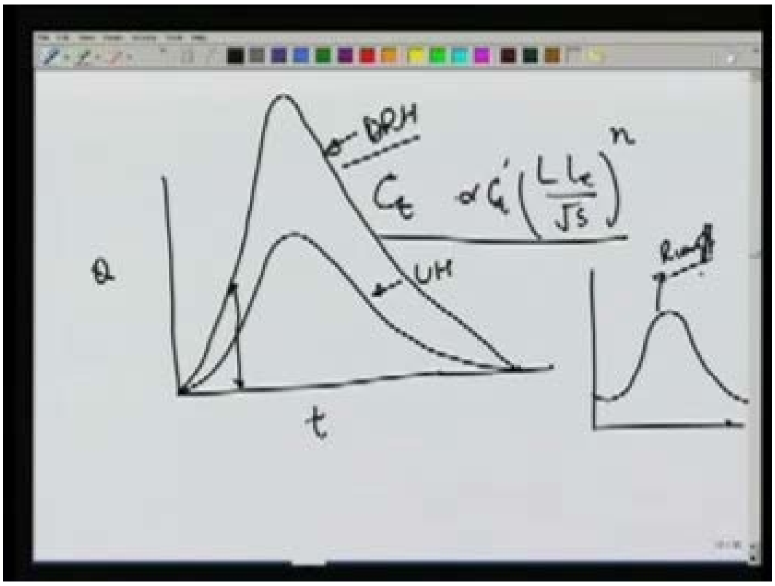

the variable L Lc over a square root of S and to the power n, and some factor C_t prime. So these suggestions have been there in the lecture available to us but we will not discuss those advanced techniques right now. The description of synthetic unit hydrograph should be sufficient for this purpose. Synthetic unit hydrographs or instantaneous unit hydrograph or a regular unit hydrographs give us how the discharge varies with time. Based on these UH we can derive the DRH. For example if we know the intensity of rain is 1 and a ½, the depth of rain is 1 and a ½ cms, we can derive a DRH which will have every where ordinates equal to 1 and ½ times by the UH ordinate. Once we get the DRH, we can add the base flow and get the total runoff hydrograph which would look like this. Our aim of developing this total runoff hydrograph is to be able to answer questions like what is the maximum flow in the stream, what is the minimum flow in the stream, what will be the total amount of water which passes the stream for a fixed period of time and there could be other questions like what is the reliability of certain discharge? For example let us say 85 percent of the times, we determine the amount of discharge from which we can assure that it will be available to us 85 percent of the times. So to answer all these questions, we would have to look at alternative ways of presenting the same information.

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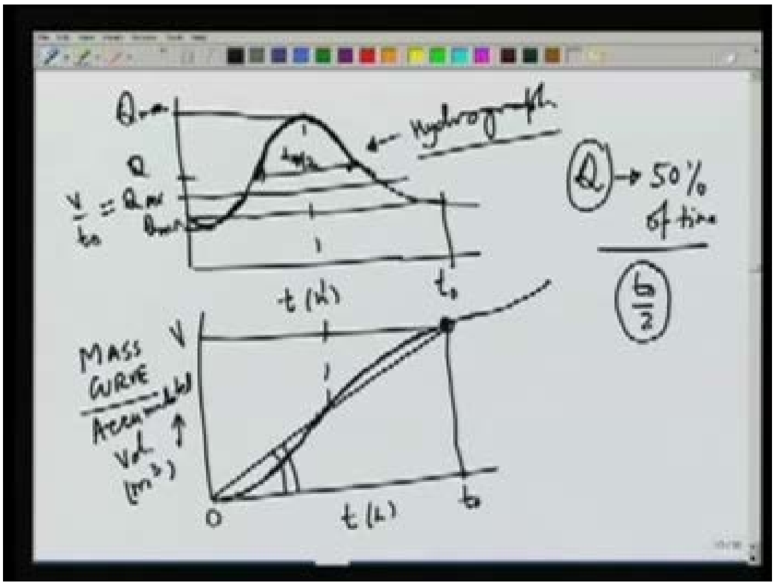

So let us choose a time period. It could be 2 or 3 days, it may be a week, it may be a month but let us say that the hydrograph during that time period is given by this total runoff hydrograph. If there are some questions which are very easy to answer, for example if somebody asks what the minimum flow in the stream is, we can say that this is the minimum flow. Then if you are designing for a flood control structure, you may have to answer a question that goes, what is the maximum flow in the stream? That is also very easy to answer. Q max. Now somebody might ask what the average flow over this time period is. Let us call it $t_{0.}$ Over this time period of t_0 what is the average flow in the stream? What is the total volume of water which has passed the stream at that location A in this time t_0? For this purpose, there is a curve which we can generate from this hydrograph which is known as the mass curve. Mass curve shows the cumulative volume of water passing through that point in a given time. So we have t and let us say we are concerned about time period from 0 routine hours only. So if you look at the accumulated volume in metre cube, time may be in hours. It may be in days or months but let us give the single storm hydrograph and see how we can develop these curves. So at the time t = 0, the cumulative volume will be 0 and as water starts coming in the river the cumulative volume will be increasing.

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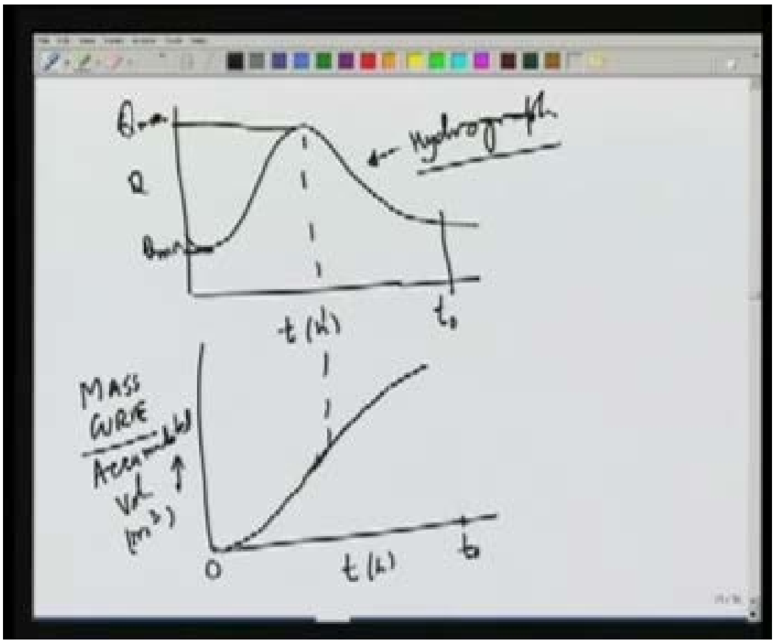

It will be increasing at a fast rate, till the peak and then the rate of increase will become slower as the discharge decreases. So we get a curve which is concave then convex then again, it may become concave here, which tells us the average rate of flow, by taking the slope of this line. So the slope of this line will give us total volume coming in time t_0 and slope of this line will tell us what the average flow rate in the stream for period 0 to t_0 is. We cannot satisfy this flow rate. Suppose we demand this flow rate, we will not be able to satisfy it because some of the times our discharge is less. Let us show the average flow rate somewhere here and call it Q average. Q average is nothing but the total volume V divided by t_0. This is the average flow rate if we consider the entire period 0 to t 0 but as you can see here in this portion the supply in the river is less than Q average the rate at which water is coming in is less than Q average. In this portion it is more and then again it would be a smaller than Q average.

We will not be able to supply water at this rate at the outlet point. If somebody wants to guarantee certain rate of supply that would be Q minimum. We cannot supply anything more than that because, if you supply a little more than that during some period of time you will not be able to satisfy the demands. The way to supply this demand is to create a storage facility. So we will look at that in a little while. The other question which we were asking is about what is the percentage of time that is required to satisfy certain requirements. For example we may say what is the flow rate, what is the Q which can be guaranteed. For instance 50 percent of the times, if we look at what the Q, which can be guaranteed at 50 percent of a time, means for a period of $t_0/2$. The Q should be more than that guaranteed Q and for that, we must draw a line somewhere here which will have a width of $t_0/ 2$. In this $t_0/2$ time, our flow will always be more than Q. Let us call it Q_{50}. One question which needs to be answered by water resource engineer is Q_{50} or Q_{85}. What is the flow rate which we can satisfy 85 percent of the time, so that we will look at curve which is known as a flow duration curve. So let us see that from a given hydrograph, how we can obtain these quantities which are important for a water resource engineer to know.

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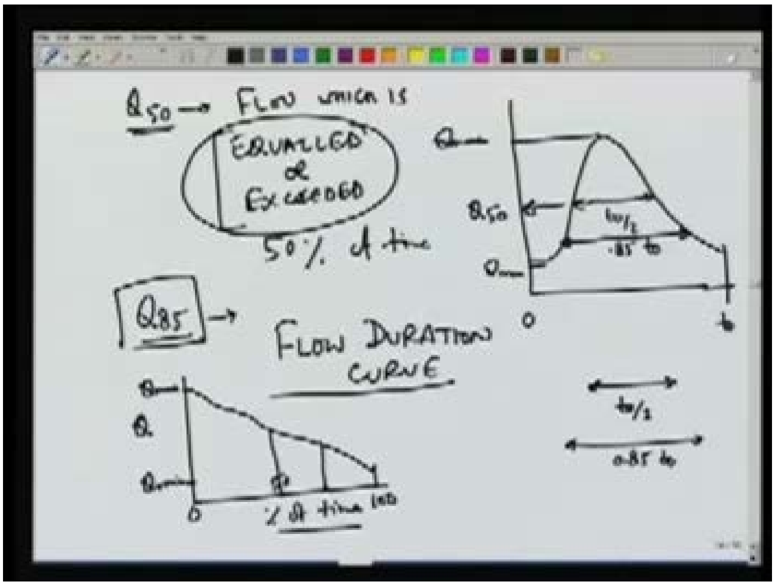

We have a minimum flow Q min, maximum flow Q max, and let us say we are analysing this period 0 to t_0 and we want to say, what Q_{50} means, what the flow which can be guaranteed at least 50 percent of the time is. So this is the flow which is used. The term equalled or exceeded. So at least this amount of flow Q_{50} would be available to us 50 percent of the times. So in order to find out this Q_{50} what we will do is we will take a line which is of width $t_0/2$, 50 percent and then we will have to shift this line up and down till this line fits exactly between the 2 curves and this will give us Q_{50}. Similarly if you want to find out Q. Then we must see the amount of flow which is equalled or exceed 85 percent of the times. So that means only 50 percent of the time the flow would be below that.

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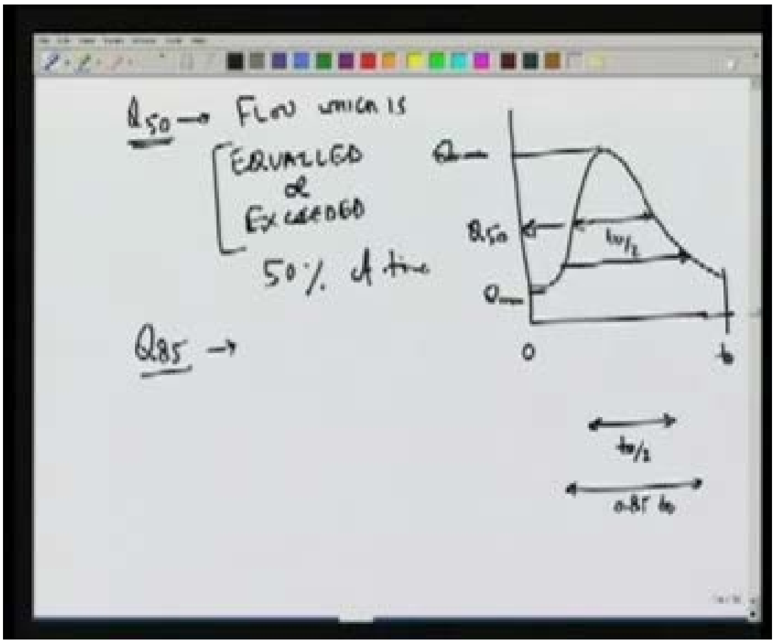

Let us draw this line here again. This will be 0.85 t_0 and again that line which will be 0.85 t_0. So this flow would be equalled or exceeded 85 percents of the time. This is an important parameter. Some times when we are creating a facility for power generation, we take this 85 percent dependable discharge as our designed discharge. That means at least 85 percent of times that much flow will be available to us.

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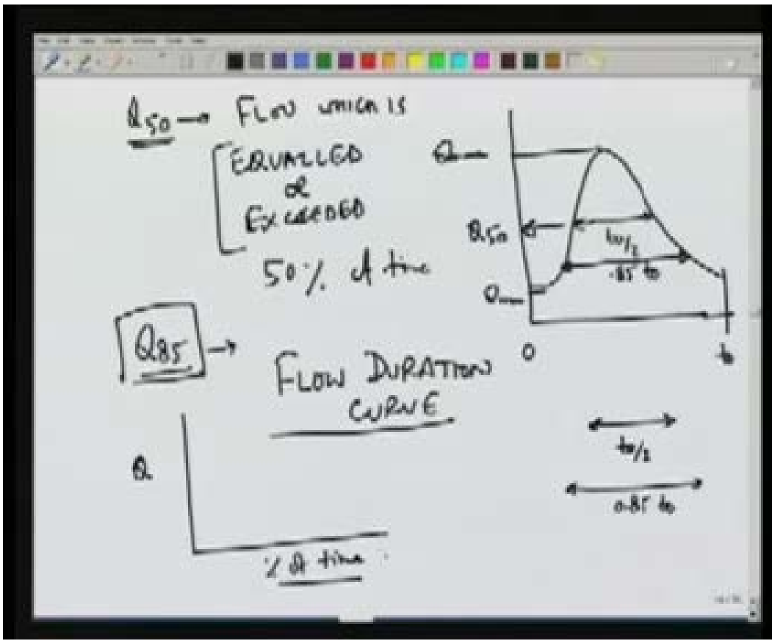

To answer these questions, we use what is known as the flow duration curve which plots this percent against Q. This percent of time when we write here equalled or exceeded and the curve would look like this (Refer Slide Time: 49:59).There is some minimum flow which would have equalled or exceeded 100 percent of the times, Q min. There is some maximum flow which will not be exceeded, so it will have a 0 probability at 0 percent of the time equalled or exceeded. Then from this curve we can answer all the questions like what is 50 percent Q_{50} what is Q_{85} and so on. Now the flow duration curve and the mass curve are both very critical in water sources or maybe power generation design. We would first look at the mass curve to find out how much storage we would require in order to satisfy a given demand.

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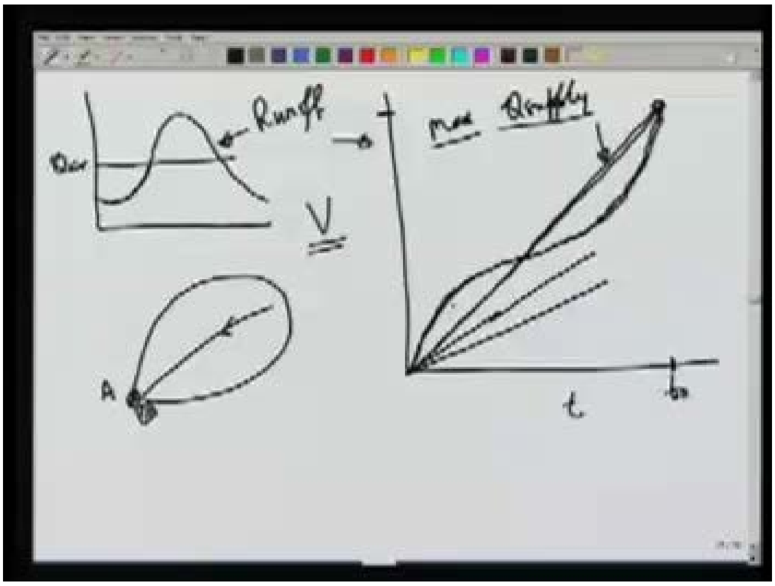

So if we draw the mass curve which would look like this. During periods of high discharge the slope will be larger, and then as the discharge reduces, the slope becomes flatter and then if another storm comes it will go up. Now we are discussing this on a very short time bases, may be a few weeks or few days but generally, this analysis is done for the annual flows. But to understand the concept let us look at this curve first, which let us say is in response to at a storm with the DR with the runoff given by this.

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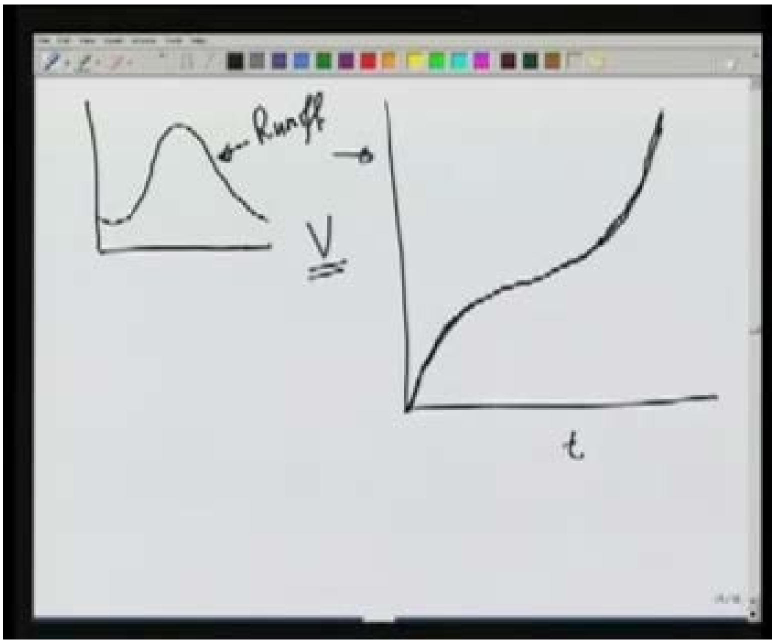

So this runoff will produce this mass curve where V is the cumulative volume. As we have seen, if this is the time t_0 there is an average flow which can be thought to occur during this time t_0 and we cannot supply this average flow as we have discussed because some of the times, the flow in the river is not sufficient and it will not be able to meet the demands.

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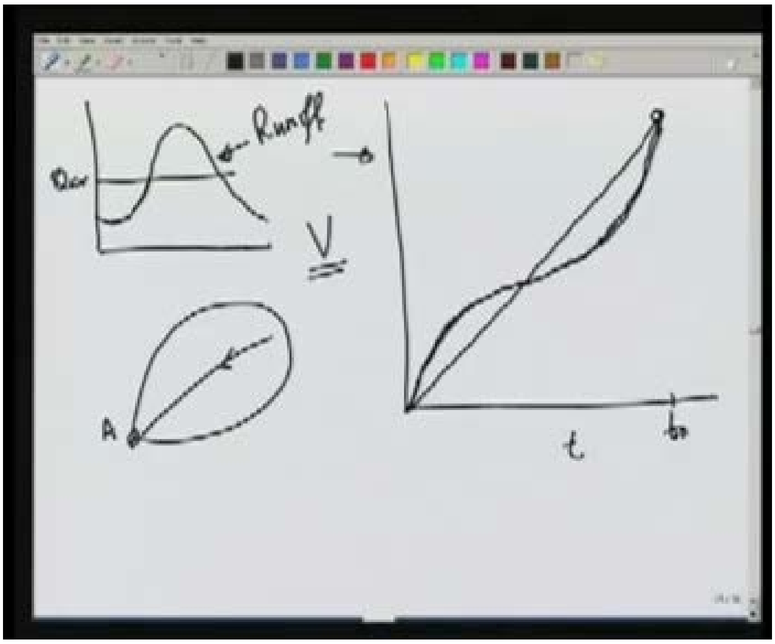

If we want to meet this demand, suppose at point A, we can create some facility for storing water. It is a dam built on the river to store but we can assume that there is some structure here which stores water. Now during periods of low flows when the flow is less than the demand, we can get water from the storage to supply the demand or to supply the requirement which exists. We cannot supply water more than this rate; our Q supply is the maximum rate at which we can supply water because this is the total amount of water available to us. V is the total amount of water available to us in time t_0. We cannot supply more than this water. So this is our maximum rate of supply, less than that of course, we can always supply. So we can always supply anything at a smaller slope but more than that we will not, unless we take water from somewhere else. So let us say that we want to supply this maximum rate by creating some storage facility here. The question which is to be answered is what should be the capacity of that storage structure so from this figure. We can see if we draw the mass curve again like this.

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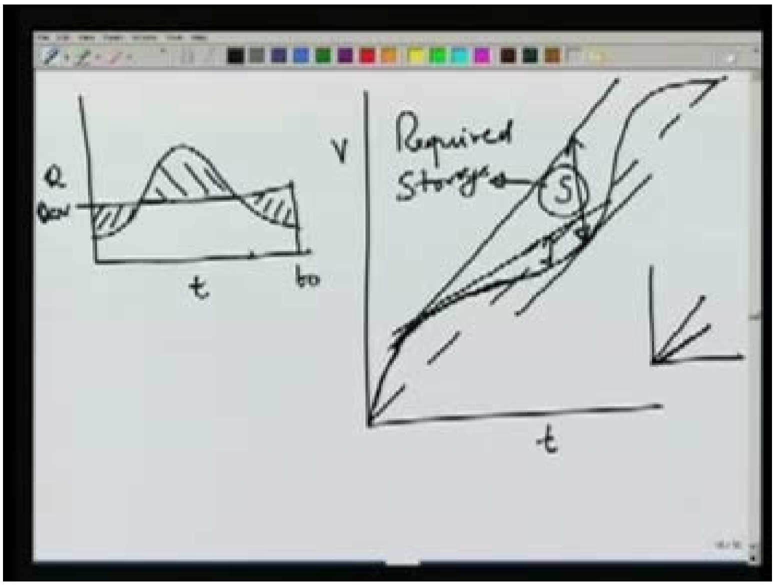

The idea is that during period of low flows we will take water from the storage and during period of high flows we will divert the additional water from the stream to the storage. If you look at the figure of runoff, during this time, our supply is less than the demand and therefore we must take water from the storage.

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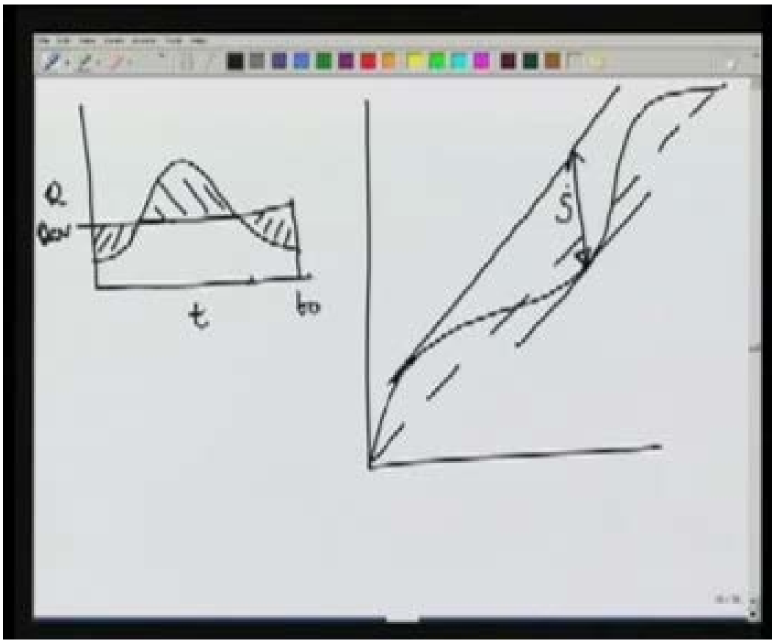

During this period water will be going from the stream to the storage after that satisfying the demand and again during this period up to t_0 water will be taken from the storage to satisfy the demands. In order to find out this capacity, this is the average flow rate. What we do is we draw lines parallel to this and then the difference between these 2 lines is the required storage. These lines are drawn at the bridge and the valley so the highest portion and the lowest portion of the mass curve is the volume verses time. The difference will be the required storage. This S is the required storage. To satisfy a continuous demand at a constant rate which is equal to the average flow rate, this is the maximum sustainable demand at that location.

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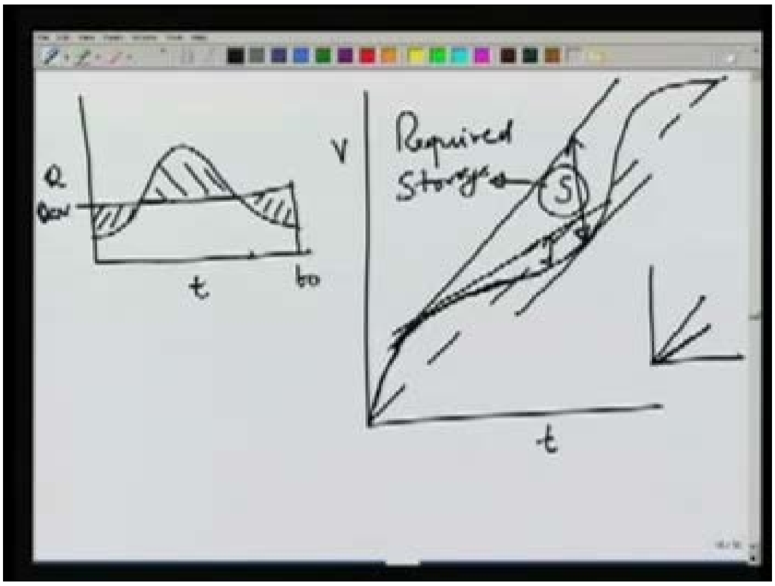

If you look at this figure it is clear that when the mass curve is flatter, that means we are getting water at smaller rates and therefore water has to come from the storage into the stream to satisfy the demand. There are various other options or various other variations in demand. For example the demand may not be constant. The demand may not be as high as Q average. The demand may be smaller, in which case our storage requirement will be a smaller. For example the demand curves in this case have taken parallel to this Q average line. If it is flatter then we will have to draw a line which is parallel to this demand line and this will result in a smaller storage for satisfying that particular demand. The demand itself may be varying.

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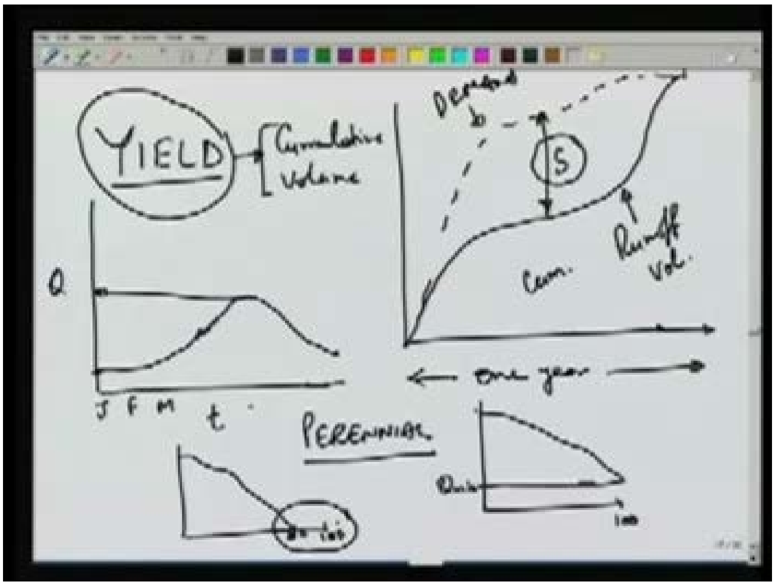

Your mass curve may look like this and the demand itself may look like this. Instead of a continuous constant demand, we say that the demand may be varying with time. If you are looking at a large term let us say annual time period, one year which is what we do in water resource project because the rain, runoff all are cyclic based on an annual cycle. We look at a 1 year period and see sometimes the demand may be very high. May be in summer months the demand may be high depending on the locality whether the demand is for irrigation or residential requirements. The demand may be high during summer; it may be low during winter and so on. The demand itself may be changing. Water supply of course has no changes. So in that case what we do is we find out the maximum difference between the demand and the available water which is runoff, cumulative runoff volume, so the maximum difference between these 2 ordinates will give us the required storage capacity. We will now discuss one more aspect which is yield related to the mass curve and we will talk about annual flow in the stream. As we have said looking at short term, we got S. Also possible but generally when we say yield, it would be annual yield of the stream. So if you look at the hydrograph of the stream at this time, it will now the January February March and so on.

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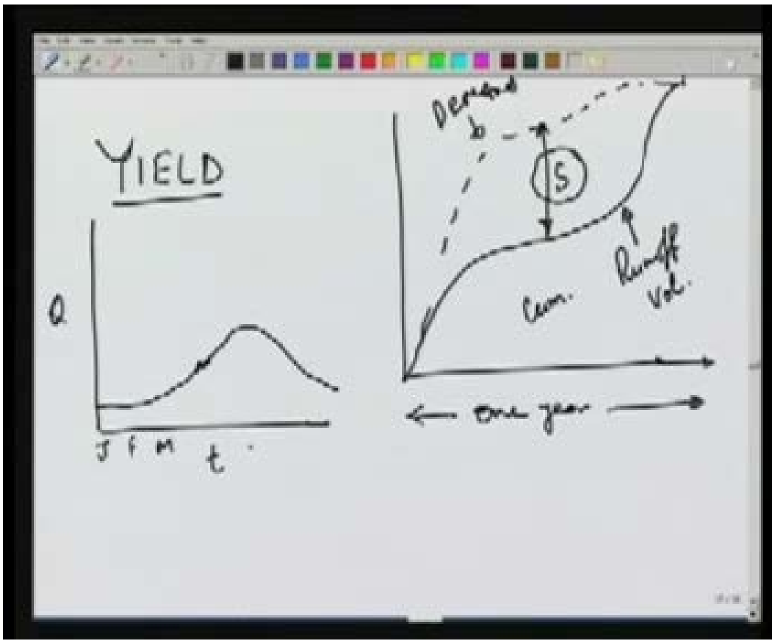

The hydrograph may look like this, small discharge here, then month to month it will go up and then again in winter it may come down. There may be smaller peaks here, values in response to individual storms but in general, the hydrograph will look like this. Yield would mean the amount of water available in the stream over the period of a year. This would be a cumulative volume typically over the year so yield is an important concept because it tells us the amount of water is available to us over a period of the year. Hydrograph will give us the maximum flood in the river, so we can use that to design flood control works. We also have this minimum flow here and as we have discussed earlier, there are 3 different kinds of streams.

We have perennial and if you draw this hydrograph for perennial, you will see that throughout the year there will be some water available in the stream and therefore if you draw the flow duration curve, the 100 percent will correspond to some finite non zero value of Q. But if you have an intermittent or ephemeral stream then the flow duration curve would look like this, where 100 percent may be here and this may be 70 or 80 percent. This means that 20 percent of the times there have been no flow in the stream. This is also an important aspect which has to be taken into consideration when we are designing a project that 20 percent of the time, there has been no flow in the stream. If we are generating power, there has been no power available during this time. We will finish here and in summary, in today's lecture what we have discussed is how to develop hydrograph for ungauged basins using synthetic unit hydrographs, how to develop mass curves and flow duration curves which help us in analysing the flow in a river in terms of what is the dependability of various discharges, what is the storage required in order to satisfy some demand and what is the maximum demand which can be met based on the given flow pattern.