Water Resources Engineering

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Lecture No. # 04

Today we are going to continue with our description of the unit hydrograph. We have already seen how to derive a unit hydrograph, how to apply it to predict the run off due to any given storm. Today we will see some of the limitations of this approach and how to go about getting other data to take care of some of these limitations. Let us look at some of the limitations. The first limitation is that the unit hydrograph approach is not valid for very large areas.

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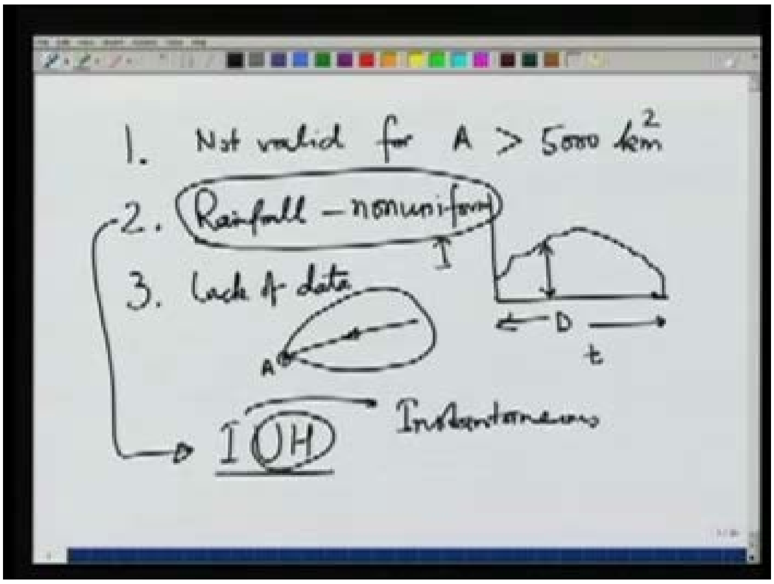

When we say large areas, areas more than 5,000 km square unit hydrograph approach will not be valid because we make a lot of assumptions in the UH analysis. For example uniform rain fall over the entire area will not be valid for a very large area. So we will say that upper limit of applying the unit hydrograph would be about 5,000 km square. The second limitation is that we have assumed that the rain fall occurs at uniform intensity for certain duration in practice. Actual rainfall event may not follow a uniform intensity for a large a large period of time. The actual rain fall pattern may look like this (Refer Slide Time: 02:09). So even though we have some storm of let us say D, our duration, the intensity will not be a constant and we may not be able to assume it to be a constant for certain duration.

In that case also the unit hydrograph approach will not work very well. So the rainfall is non uniform and also it cannot be approximated by a uniform distribution. In that case we cannot apply the unit hydrograph approach. The third limitation is that we must have data for deriving the unit hydrograph. So either if we have a catchment area and we want to derive unit hydrograph at point A, we should have a caging station at point A. If we do not have a caging station at point A, we will not know the run off at point A. Sometimes even if we have caging station at point A we may not have required storm occurring in that area, so lack of data would be another limitation which will prevent us from deriving a unit hydrograph. So today we will look at some ways of getting over these limitations or how to do our analysis. Let us say some of these limitations are not satisfied. For example the area may be very large, rainfall maybe very non uniform. So how do we go about it? We'll start with a non uniform rainfall and how to analyse that. We use a method which is known as IUH or instantaneous unit hydrograph. I stand for instantaneous and UH of course, is the unit hydrograph. So let us look at what an instantaneous unit hydrograph is and how we use it to generate direct run off for a non uniform rainfall.

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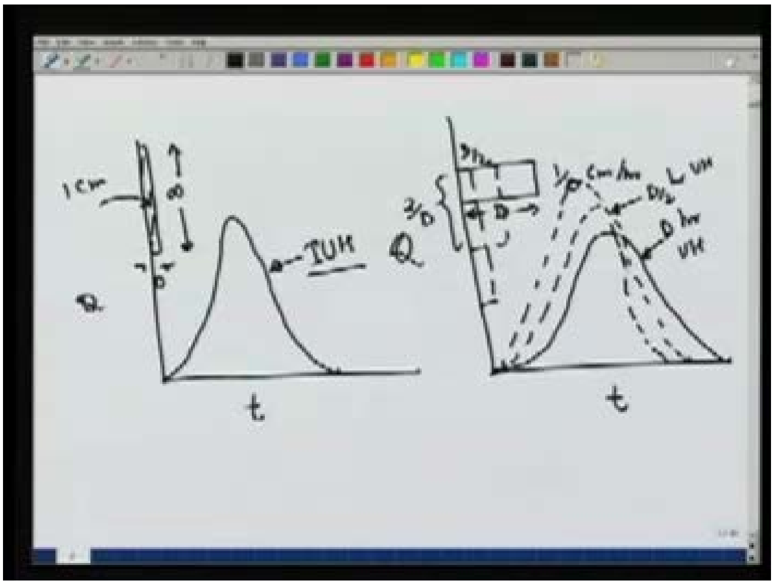

By now we are familiar with the unit hydrograph shape. So let us say that we have a unit hydrograph for an ERH of certain duration D and of course, the intensity would be 1/D cm per hour. Now if we reduce the duration of the rainfall and increase the intensity in such a way that the total amount of rain falling is still 1cm, we would get a different unit hydrograph, so this is a D hour UH. Let us reduce the duration to D/2 and increase the intensity to 2 over D. So in this case again we have a 1cm of rainfall but for a smaller duration and with a larger intensity, the unit hydrograph corresponding to this would have a smaller time base because the duration of rainfall has reduced but will have a higher peak because the area of the hydrograph must remain the same. So this would be a D/2 hour UH. Same argument can be extended further, if we further reduce the duration by 1/2 and increase the intensity into twice. We would get an ERH which is very narrow and the intensity is very large. In that case the UH alternate peak will again be larger, time base will become smaller, so we may get a curve which has a smaller time base but a higher peak. Carrying this argument further, we can reduce the duration to 0 and that is why this is called an instantaneous unit hydrograph.

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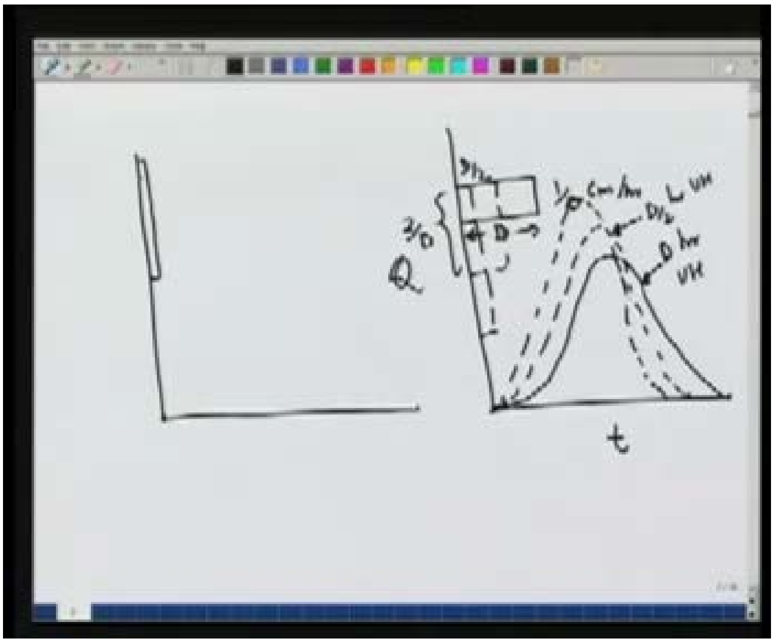

So this unit hydrograph is the result of rain which occurs instantaneously. The intensity is infinite of course, the duration is 0. I will not be able to show it on this figure, but the intensity and durations are such that the total amount of rain is 1cm and the DRH resulting from this is known as the IUH. As you can see that there is no duration attached with IUH, in unit hydrographs we had a D hour unit hydrograph. For example it may be a 6 hour unit hydrograph, 4 hour or 2 hours but IUH is for instantaneous precipitation with total depth of rainfall = 1cm or sometimes it will take 1 inch. So our aim is to derive this IUH and then use it to analyse the direct run off due to a non uniform storm. So let us first see how we can obtain the direct run off due to a non uniform intensity distribution.

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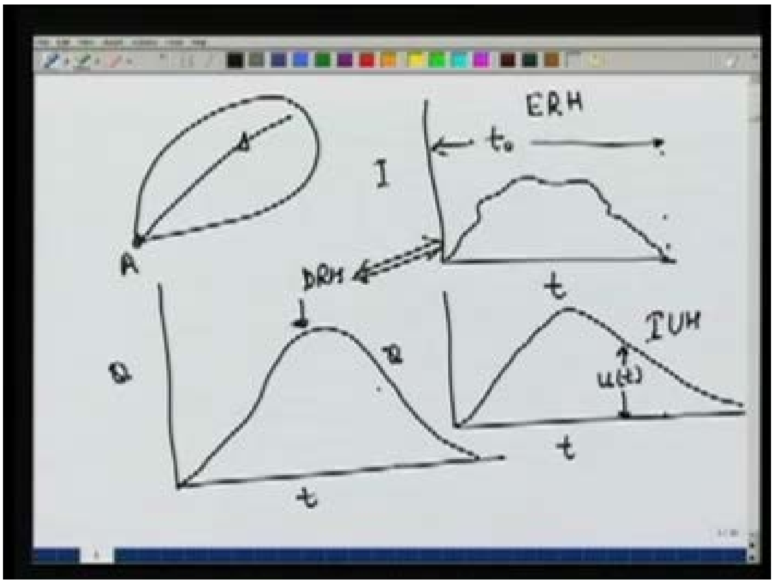

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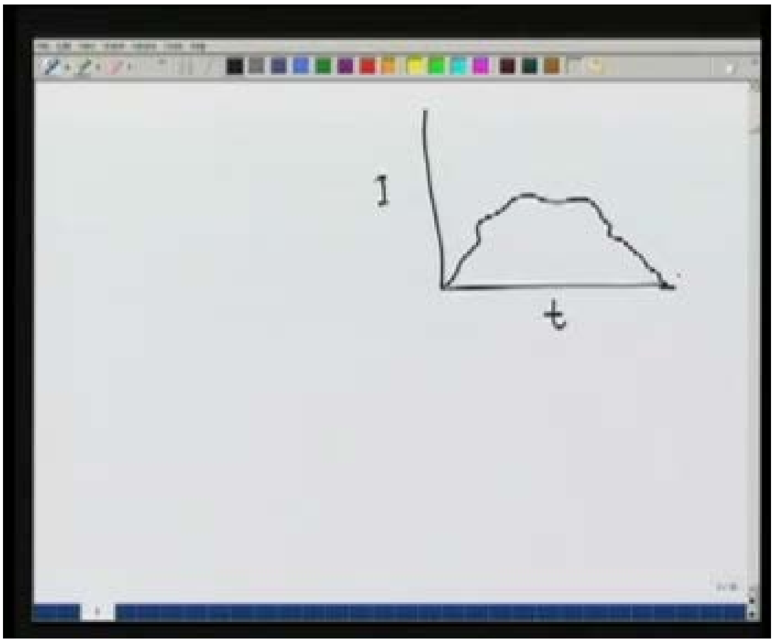

Depth diverse, the ERH and the ERH may look like this (Refer Slide Time: 08:47). The total duration of rain, assuming t_0 and suppose we know the IUH for the catchment and we want to find out the direct run off at point A. Due to this ERH over the entire catchment area, let us say that the IUH looks like this. The ordinates of IUH are denoted by U. Small ut represents the ordinate of the IUH at time t. Now in order to analyse this ERH and obtain the DRH, our aim is to obtain a DRH which may look like this. So this DRH is caused by this ERH subjected to this IUH and we will have to develop an equation to relate this DRH with the ERH and IUH. So let us do it now.

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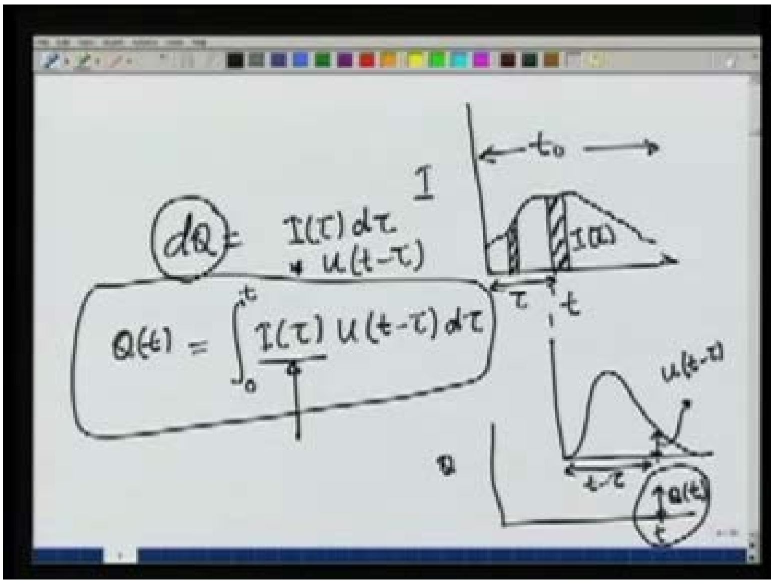

Take a small ERH element. Let us say this is the ERH showing the time verses intensity of the rainfall. Now this we can write as I tau where tau is the time from the starting of the rainfall. So at any time tau, the intensity of rain fall is given by I tau. Now if we draw the IUH is starting from the point tau and want to find out the DRH at any particular time t, our aim is to find out the DRH ordinate at the point t which we will call qt. Now this qt is because of this rainfall intensity and this IUH and if you draw the IUH starting from this point as origin, it means the direct run off due to this strip of rain can be given as the depth of rain fall here, which is I, the tau multiplied by the ordinate of the IUH at that point. So this ordinate is nothing but U t – tau. So if you look at this dq, dq is the direct run off due to this small strip of rainfall occurring over the entire catchment. We say that this is the depth of rain fall, I tau d tau occurring over the catchment instantaneously. It will cause a direct run off of Dq. If you want to find out the total ordinate Qt, we have to integrate it from 0 to t. So this equation gives us a way of finding the direct run off ordinate at any time t.

Now if you look at this expression, I tau will of course be 0, beyond t_0. So the is the duration of rainfall is t_0. So if t is more than t_0 then we have to perform the integral only from 0 to t_0 and this way, we see another strip here. What we are doing by the integral is taking all the small strips and finding out their contribution to the direct run off at the time t. So every time we have to take a strip I tau, d tau will be the depth of rainfall and ut – tau would tell us the contribution at time t due to that rain occurring at time tau. So T – tau will be the ordinate which we have to consider for the IUH. Now we should look at how to derive the IUH. As we have already discussed, it is a very small duration rain with total depth of 1 cm, so one way of deriving the IUH is using the S curve.

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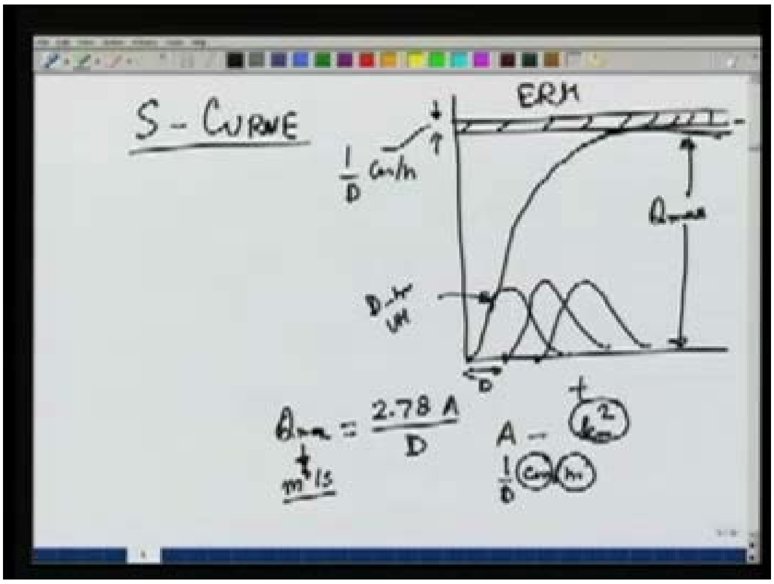

Now we have seen in the previous lecture, this S curve is the direct run of hydrograph because of an infinite duration of rainfall. So the ERH looks like (Refer Slide Time: 15:37) this and continuous to infinite the intensity would be 1/D cm per hour. If we have taken a D hour unit hydrograph and as we have seen earlier, what we do is we take a number of D hour hydrographs and shift it by D hours and this is a D hour UH. If we take a number of these and shift them, add them, we would get what is known as the S curve and let it reach a maximum ordinate here Q max, which can be obtained by the rainfall intensity and the catchment area. So if you have a catchment area A in Km square and rainfall intensity of 1/D cm per hour, then Q max can be given by a simple expression in meter cube per second. So here we have taken care of the units. Since this is per hour, we have factor of 3600, since this is Km square, we have a factor of 1000 1000 and since we have D in cms, we have another factor of 0.01. So if you put all these factors and here, you will get the maximum alternate of the S curve as 2.78 A over D.

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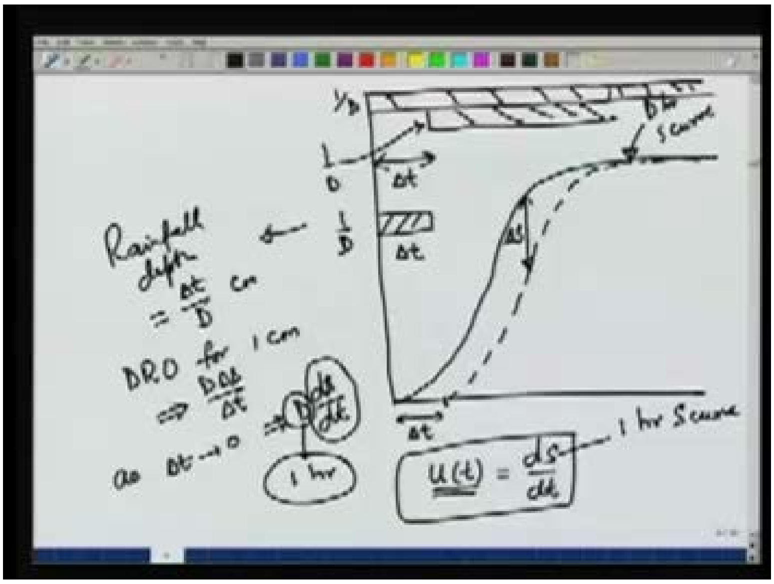

Now once we get the S curve, suppose we now shift this S curve by some amount, let us call it delta t. So this is an S curve obtained from a D hour UH.

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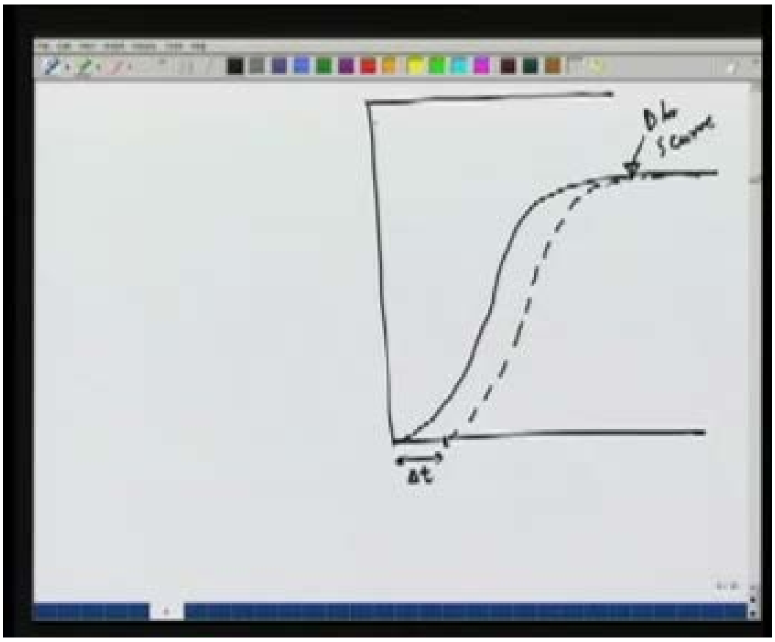

So let us call it a D hour S curve. Then we shift it by an amount delta t, draw the ERH for the first S curve again, ERH will go on to the infinity and then the ERH for the second curve will start from delta t, will have the same intensity 1 over D but a difference of delta t in the starting point. So if we take the difference delta s, this delta s indicates the direct run off due to the rain fall which is the difference of these two ERH. This is nothing but an ERH of duration delta t and intensity 1 over D. Now in the limit that delta t turns to 0, we will get the IUH but we have to maintain 1 cm of rainfall depth.

In this case the rain fall depth is delta t over D cm because the intensity is 1 over D and the time duration is delta t. So if we have delta s, due to delta t over D cm rain, we can find out what will be the ordinate due to 1 cm of rain by using the principle of linearity. The ordinate DRO, the net run of ordinate due to 1 cm of rain can be obtained from delta s, delta t and D. So let us write it in terms of first finite values D delta s over delta t, and then we will take the limit, as delta t turns to 0. We can get ds over dt. ds over dt is nothing but the slope of the S curve at a particular time and D as we have seen is the duration of the rainfall for the unit hydrograph. If we keep D = 1 then what we get is the alternate of the instantaneous unit hydrograph at any time t will be ds/dt.

So we have a very simple method of deriving the instantaneous unit hydrograph ordinate Ut by drawing an S curve which is for duration of 1 hour rain fall. So if we have developed a 1 hour Uh, we can derive the 1 hour S curve and then we can find out the slope of that S curve at different times and that slope will be equal to the ordinate of the instantaneous unit hydrograph. So this is 1 of the ways of finding out or deriving the unit hydrograph. There is another method which is commonly known as the Nash method because he was the first 1 to propose this concept and derive the IUH.

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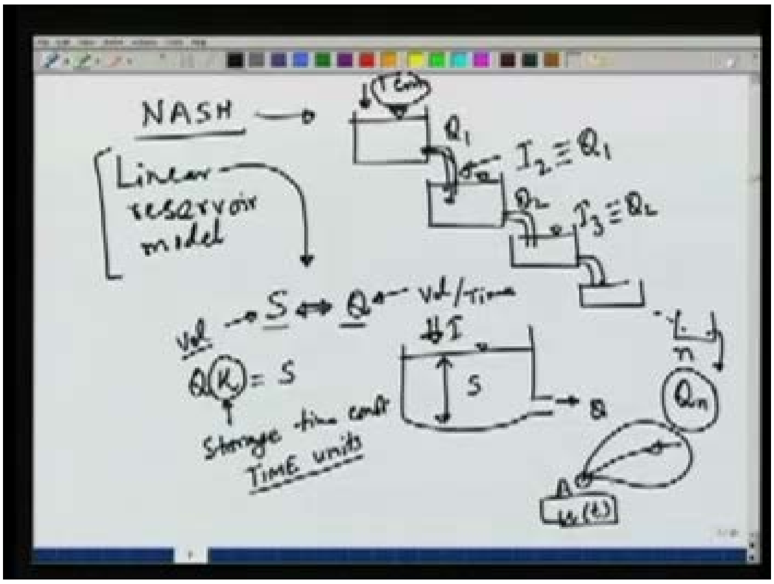

In Nash method which is also known as a linear reservoir model, he conceptualized the basin as a series of reservoirs such that the outflow from 1 reservoir goes to the next reservoir as inflow.

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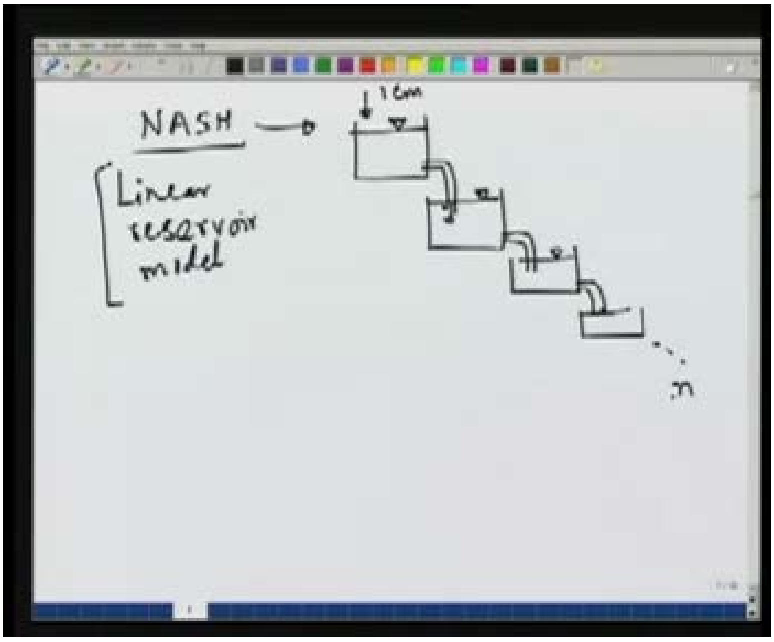

Like this there is n number of reservoirs and if we put the inflow as 1 cm instantaneously whatever outflow we get from the nth reservoir would indicate the IUH. This is the basic concept of the Nash model that the catchment can be thought of as a series of reservoirs which are linear. Now linear indicates that storage and discharge S and Q have a linear relationship. In a reservoir if you look at a reservoir which has an outlet here (Refer Slide Time: 23:46), the storage will depend on what the height in the reservoir was and the outflow will also depend on the height in the reservoirs. So both storage and the outflow are functions of the height in the reservoirs and therefore the concept of linear relationship between S and Q has been used by number of investigators. They say that Q and S can be linearly related. If you look at the dimensions of Q, this is volume over time meter cube per second.

S is in terms of volume which is typically meter cube. So if we use the time constant, Q into K = S, where this K is the storage time constant and it has units of time, this K would be a property of the reservoir or in this case the basin. So in Nash model we say that Q and S are linearly related. S can be written as Q into K where K is the time constant and then we can write the continuity equation for N is a reservoir. If there is some inflow I and some outflow Q, we can write the continuity equation as the net inflow =change of storage. So let us take a particular reservoir.

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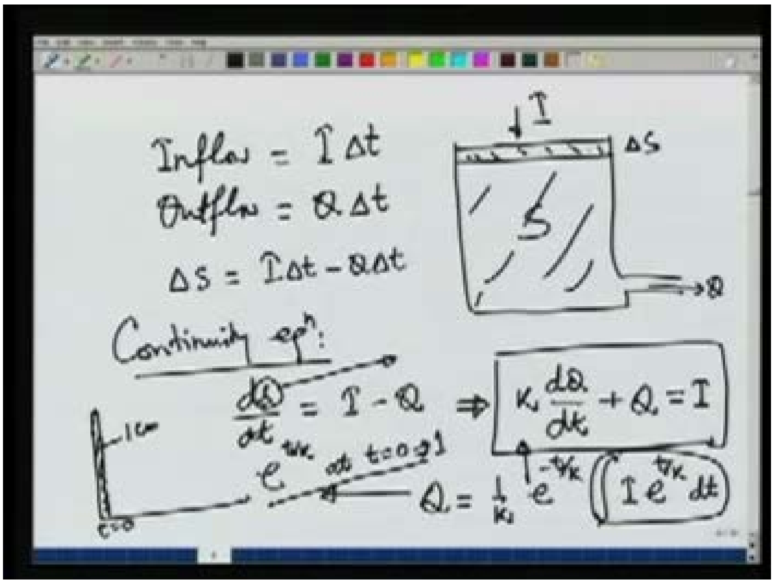

So let us take a particular reservoir.

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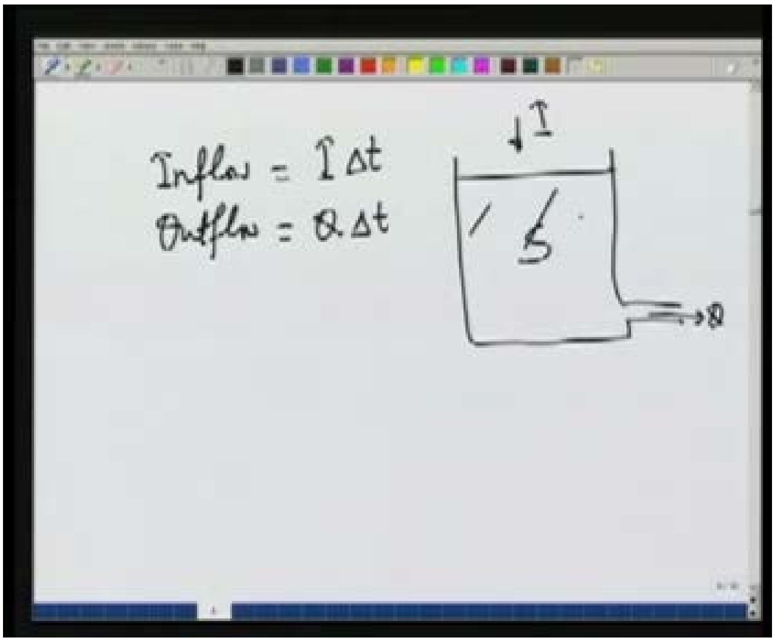

Consider a time period of delta t, the inflow in time period delta t will be I into delta t and the outflow will be Q delta t. Now S is the storage within the reservoir and supposing in time delta t, S changes by an amount delta S, then we can say that this change in storage is because of the net inflow and it should be equal to the net inflow. So we can write and then if we take the limit as delta t turns to 0, we get the differential equation of continuity. What is commonly known as the continuity equation or the mass balance equation tells us that ds/dt is = I – Q. Now our aim is for any given input we want to find out the output Q. Therefore assuming the linear reservoir, we can write this S in terms of K and Q and therefore we will get an equation describing the variation of Q with variation in I and this differential equation can be solved. This is a linear first order equation therefore it can be easily solved to get the value of Q in terms of I. Once we get the outflow from one reservoir let us call it Q_1. It becomes the inflow for the second reservoir. So I_2 is equivalent to Q_1 similarly for other cases, once we get Q_2, it will become the inflow for the third reservoir. So I_3 will be equivalent to Q_2 and so on. So ultimately our aim is to get the outflow from the nth reservoir Q_n and following this equation in order to solve this equation, for each reservoir we can get Q_n which will give us the IUH ordinate because 1 cm of rain falls instantaneously on the first reservoir and then is carried through to the nth reservoir. This Q_n represents the outflow at point A and therefore since this is the outflow, because of 1 cm of rain falling instantaneously, the outflow will give us the ordinate of the IUH. So this is the basic theory of the Nash conceptual model. If we solve these differential equations, the solution of this differential equation can be written easily in terms of I and K. So let us look at the function of this equation. For the first reservoir in which I is an instantaneous, the inflow is instantaneous at time t = 0, we have a rain which is 1 cm in terms t of course is infinite, duration is 0 but the total volume is 1 cm. So this I for the first reservoir would be a delta function with occurs at t = 0 and therefore the integral of this term would be nothing but the value of exponential t over k at t = 0. Using the property of delta function, the integral gives the function value at that particular point and therefore this will be = 1.

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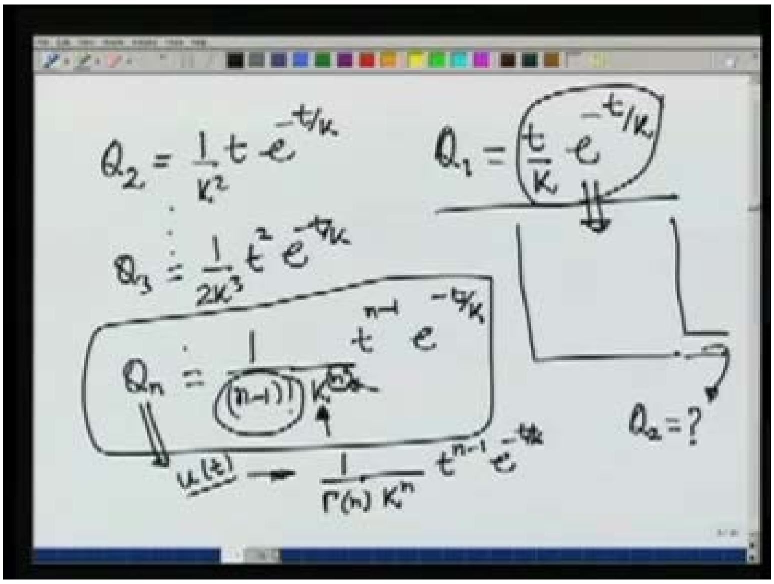

So the outflow from the first reservoir Q_1 is quiet straight. It has t over k, exponential – t over k and then we can apply this outflow as the inflow to the second reservoir and now we want to find out what Q_2 is. Equation is of course the same solution, we have already described is 1 over k e – tk integral of I exponential t over k dt. But this I now for the second reservoir will be replaced by t over k, e – t over k. So if we perform this integration, we get a value of Q which is given by Q_2 = 1 over K square, e – t over k and as we keep on doing this, for example Q_3 is (Refer Slide Time: 32:52) and so on, until we reach Q_n which comes out to be factorial (n – 1) k to the power n t to the power n – 1. So the outflow from the nth reservoir is given by this equation Q_n and since we say that this outflow is nothing but the IUH ordinate, we can write this as u. So the ordinate of IUH at any time t will be given by this, k and n are some properties of the catchment. So a number of reservoirs maybe there, the time constant of both of these will depend on the catchment area and there are various methods of estimating these k and n. We will discuss that little later. But the idea is that this ordinate depends on only two factors k and n. If we look at the values of let us say n, it does not have to be an interior. Although the way we have conceptualized it, n is the number of reservoirs but it does not have to integer your conceptual model. You may say that you have 2 and a half reservoir in the basin or 3.5 reservoirs in the basin. So in order to generalise this equation for non integer values of n, we use the gamma function and the definition of the gamma function is known. This n – 1 factorial is nothing but gamma n, k to the power n. So in order to make it valid for non integer, n values we replace factorial n – 1 by gamma n and say that this would be valid now for any non integer value of n. Also there are relations between k n and the moments of the IUH.

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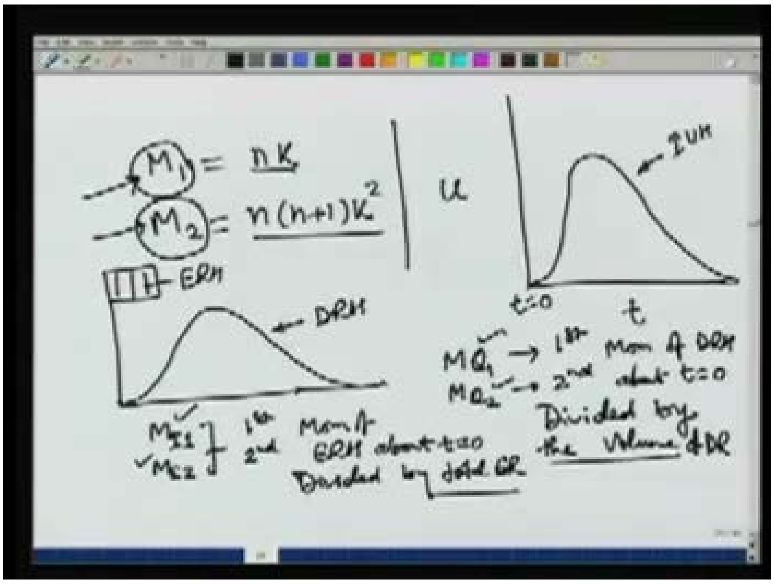

So if this is the unit hydrograph, if we take the moment of this curve about the origin, those moments can be shown to be functions of n and k.

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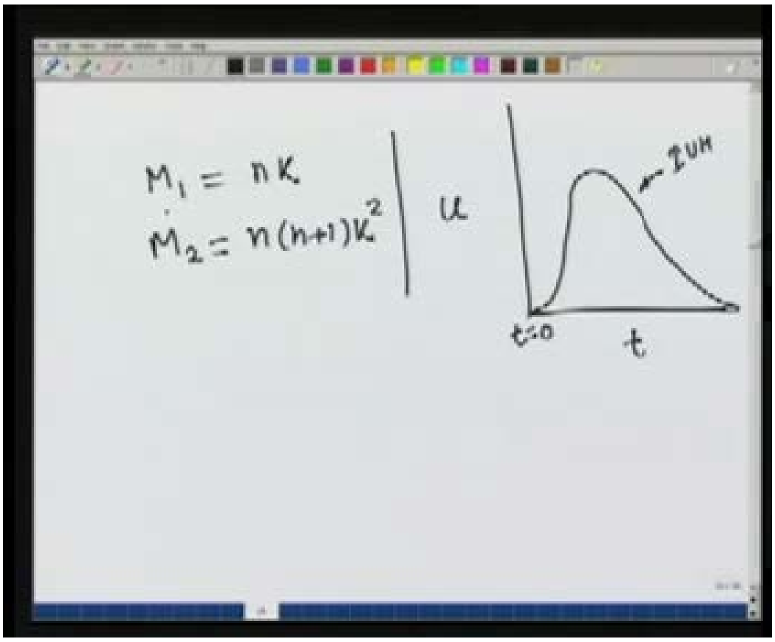

For example if we take the first moment, it is = n into k and if you take the second moment it is = n, n + 1k square. So if we have first and second moments of the IUH, we can write these two equations in terms of n and k and find out the values of n and k for a given basin. Now we will not have the IUH in general available to us because our aim is to derive the IUH. But what we may have is some ERH and corresponding DRH. If we have these ERH and corresponding DRH values, we may find out the moments of these ERH and DRH and those can be correlated with the moments M_1 and M_2 of the IUH and the formula which I gave for this case, use 4 values MQ_1 and MQ_2. They are the first and second moments of the DRH about t = 0 and then divided by the area under DRH which would be the volume of DRH.

So we said it should be divided by a total direct run off, so we normalise the DRH and take the first and second moments. We shall call them MQ_1 MQ_2. Similarly for the ERH we define two moments MI_1 and MI_2. These are the first and the second moments of ERH about t = 0 and again divided by total effective run off. Once we get these four moments MQ_1, MQ_2, MI_1, MI_2, we can obtain $M_1$and M_2 which are the first and second moments of the IUH and these we can correlate with nk and n, (n + 1) k square. We have 2 equations and 2 unknowns which we can solve to get the values of n and k.

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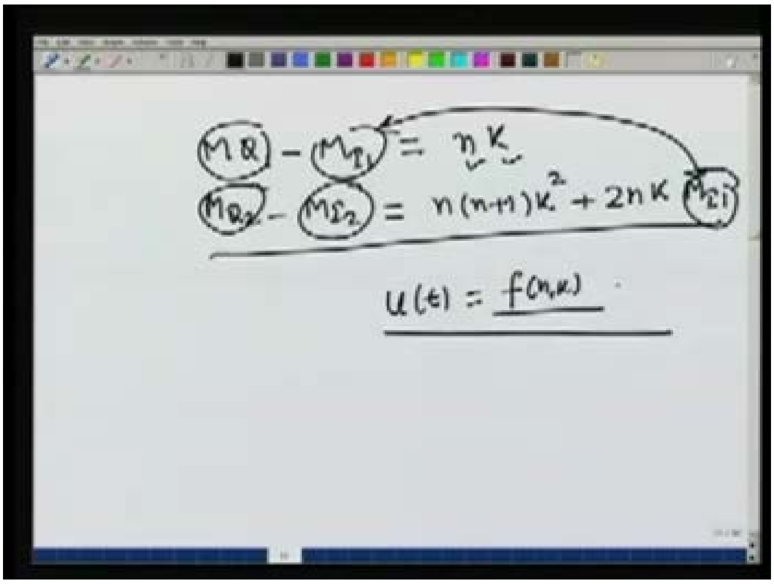

$MQ_1 – MI_1$ which is the difference in the first moment of the DRH and ERH. This is equal to the first moment of the IUH and therefore is = nk. The second moment difference $MQ_2 – MI_2$ can be correlated with the second moment of IUH which is n, (n + 1) k square + the ERH, first moment values. If we have known values for these four moments, then this MI_1 of course will be used here too.

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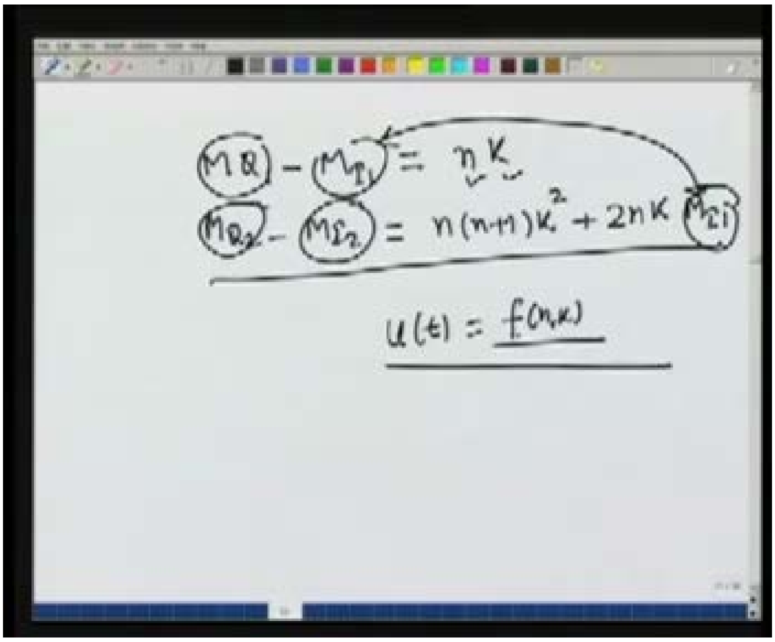

If we have these four values known from an ERH and corresponding DRH, we can estimate the values of n and k and once we have n and k then we know that IUH ordinate is given by the gamma function exponential and all these. So these two n and k are the parameters which will depend on the catchment and may be obtained from an available ERH and DRH values.

So the IUH which we have discussed just now takes care of any non uniformity in the rainfall where we cannot approximate it by constant intensity over certain duration.

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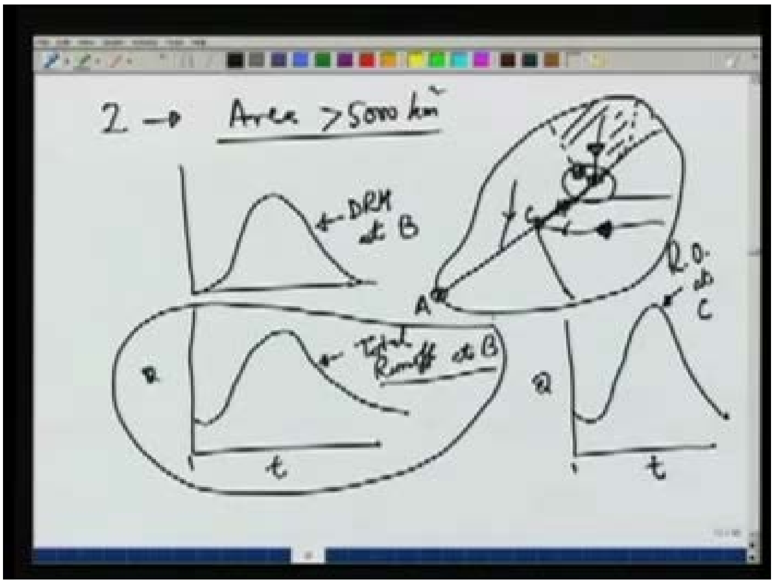

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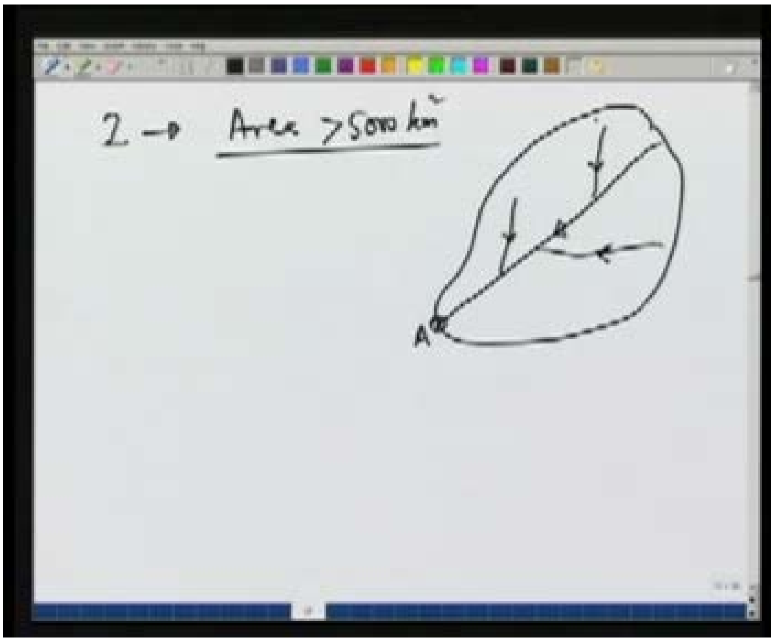

Now second problem which we will look at is if we have an area which is quiet large, there may be a river which is joined by number of tributaries and this area being very large, let us say it has more than 5,000 km square. We will not be able to use the unit hydrograph to predict the direct run off at A due to any storm occurring over this area because typically for a larger area the assumptions made in the unit hydrograph theory will not be valid. So if we look at this area we can sub divide this into smaller areas and then apply the unit hydrograph theory over the smaller areas. For example if the catchment area of this tributary is like this, then this area being small, we may be able to apply the unit hydrograph theory for this particular tributary to find out the flow at this point. Let us call it point B so once we apply the UH theory over this shaded area we can get the outflow at point B. But now this outflow at point B will go into the channel till it needs the outflow from the tributary. Let us say point at case C, the direct run off from the tributary will come at point B. It will travel to the stream and rigid point C. There it will again meet the runoff from this tributary.

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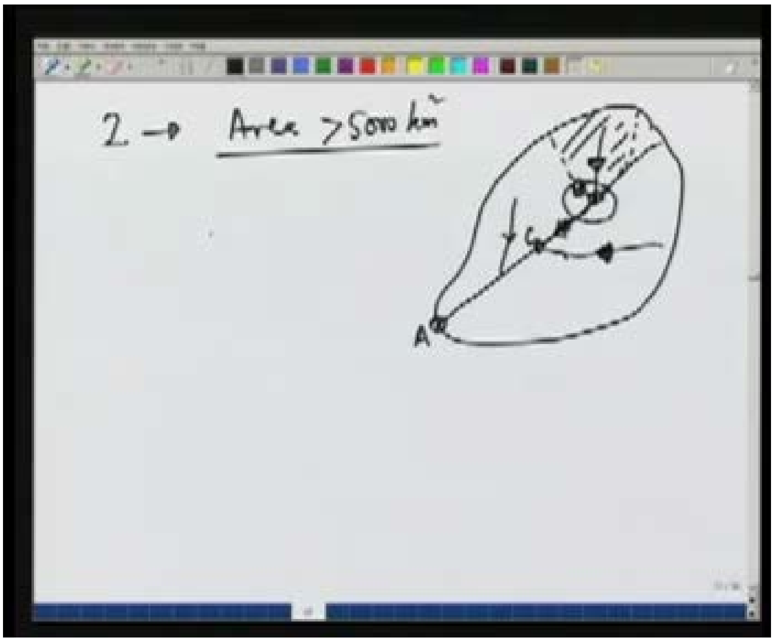

So if we find out at this point B, the hydrograph of course - the DRH at B. If we find out direct run off hydrograph at point B, we will then have to add the base flow to it to get the total run off.

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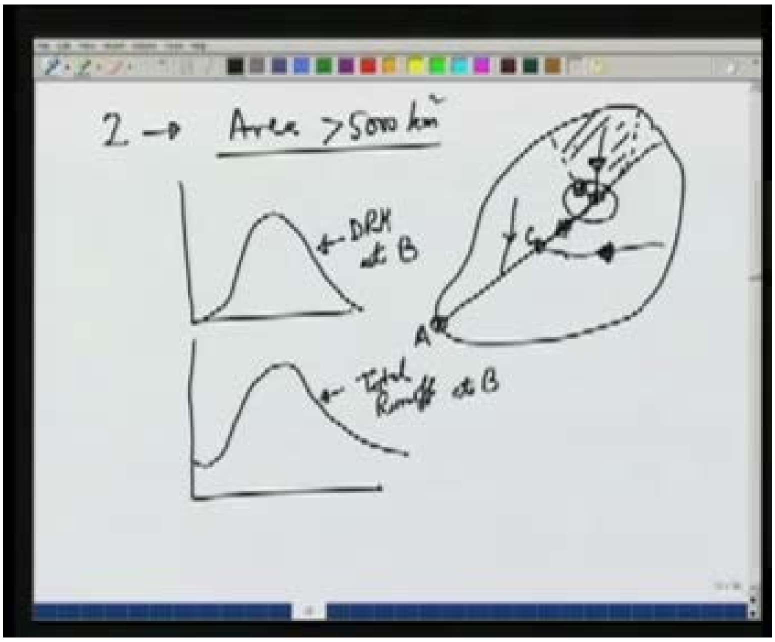

So these two steps can be done using the unit hydrograph theory because at B the catchment is small enough for us to apply the unit hydrograph theory. So this DRH can be obtained knowing the intensity of a storm in this area B and then total run off can be computed at B. Similarly we can apply the unit hydrograph theory for the catchment area at point C and obtain total run off at C which of course will be quite different from that at point B because the catchment area may be larger. It may have different slope, different properties. So this is runoff at C. Now the problem is that there is a time lag. So the time origins at which run off occur at point B and the time at which there is no focus at point C are not the same. Therefore we have to do what is known as routing. In this case we have to route the flow through the channel therefore it called channel routing. In some cases when we route the flow through the reservoir we call it reservoir or storage routing. For example in the Nash model we have done some kind of storage or reservoir routing. Here let us look at what we do, once the flow reaches the point B. So we know the inflow at point B. The equation which we use now is same as what we had derived earlier. Net inflow should be equal to the rate of change of storage.

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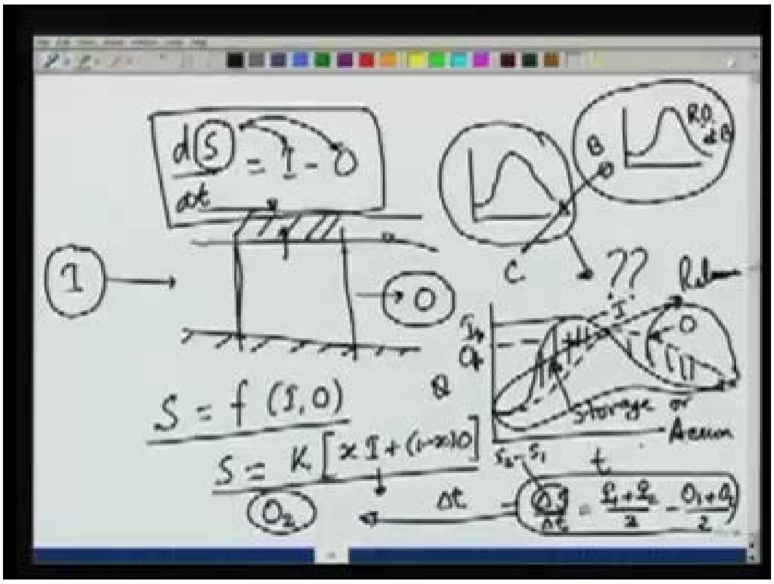

So ds/dt =I – O.

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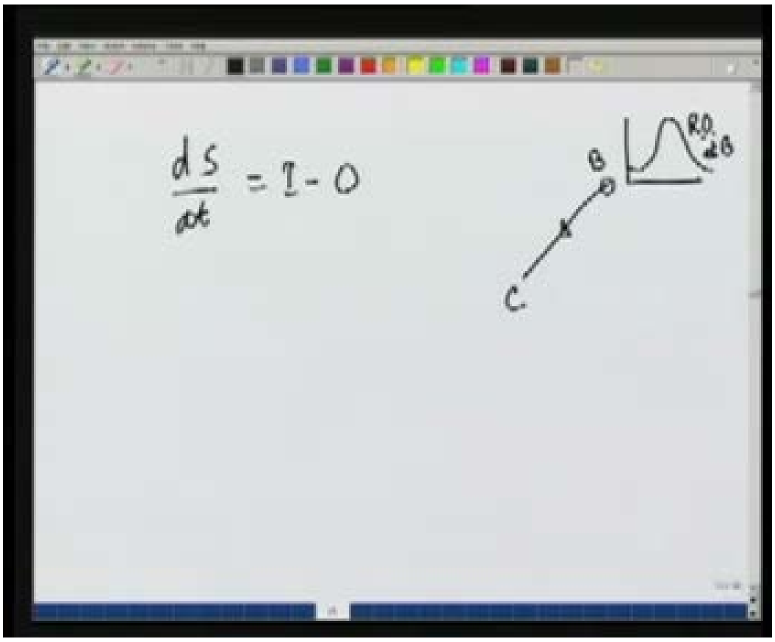

So I will draw only the stream here. Now point B, point C and at B we know the inflow because whatever the unit hydrograph theory gives us, we had the base flow and we get the total run off and that run off at point B becomes the inflow by stream at point B. So if you look at the stream there is water level in the stream and there is some inflow. If we consider section like this there will be some outflow and the continuity equation tells us that ds/dt would be = I – O. So when we are routing the flow through the channel what we need is the outflow for any given inflow. So basically given an inflow hydrograph like this at point B, what would be the outflow hydrograph at point C? It maybe something like this.

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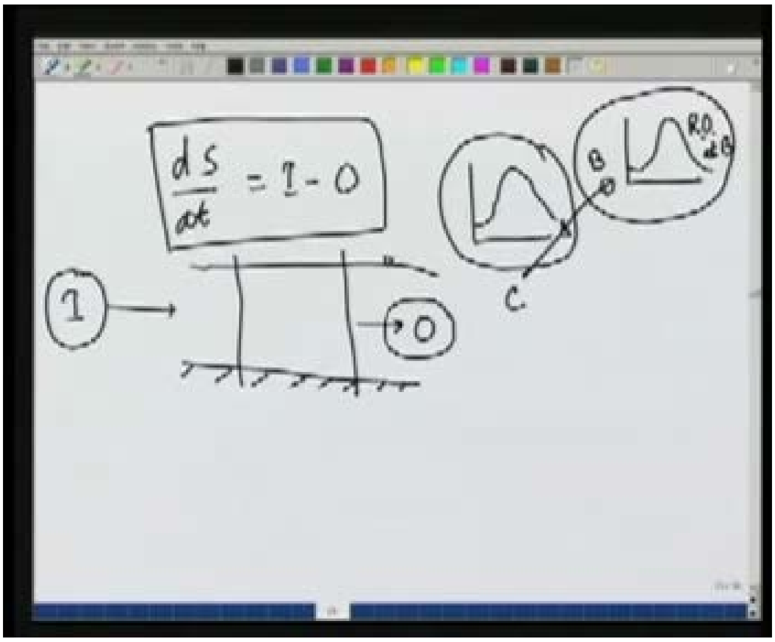

This is what we need to find out. How will the outflow at point C look? Because of this flow at B moving through the channel and reaching point C, because when the flow is routed through the channel it will not regain its shape. If the inflow hydrograph is like this, the outflow hydrograph may look like this I O.

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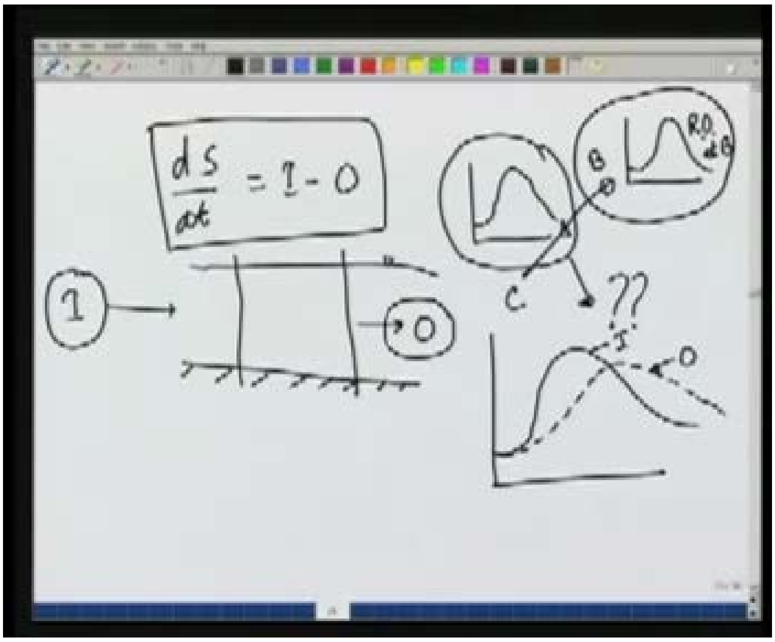

So here we are indicating O for outflow, I for inflow. Inflow will have some peak and the outflow will generally have a smaller peak. OP if you look at this curve the area before these two curves intersect represents that outflow is less than inflow and therefore water is going into the storage and what is happening is that this water level is raising and therefore the storage is increasing. So water is going into the storage up to this point and beyond this point, water is being released from the storage, so we call this as the storage or accumulation and this as released. So inflow more than outflow means water is going into the storage. Outflow more than inflow means water is coming from the storage or being released from the storage. The aim of all these routing flood, routing or storage routing is to obtain this curve for O.

Given the curve for I and for that we need some kind of relation between S and I and O. For example in Nash model we have assumed a linear relationship between S and O. But in most cases S will depend on both I and O and therefore typically we write S as a function of both I and O. There are different methods for routing a flood through a reservoir or a channel which we will discuss later. Right now we will not go further into this discussion. But this S has to be described as a function of I and O. For example there is a method known as Muskingum method where S is written as K into some factor X into I + 1 – X into O. So if we assume some kind of relationship like this because the storage will be a function of I, the amount of flow coming in will decide what the water level at this point is, similarly the amount of flow going out will decide what the water level at this point is.

So the storage typically will be a function of both the inflow and the out flow. The factor X can be taken as 0.5. This means we are giving equal weight to inflow and out flow. If we take the factor X as 0, then we get a linear reservoir as in Nash model which tells S = K into out flow. This is called the Muskingum equation and when we route the flow through the channel, what we do is we take small time steps. Let us say delta t, we assume some inflow to be average during the time the delta t. We then write delta S over delta t, average of I_1+ I_2 – average of $O_1$and O_2. So in the finite difference form, we write an equation like this. This S delta, S will be nothing but S_2 – S_1 and these S_2 and S_1 again is correlated with I_1 I_2 and O_1 O_2. Finally in this equation the only unknown will be O_2 because the conditions at the beginning of the time steps are known. So $O_1 I_1$will be known. Condition at the end of the time step, the input would be known because the inflow hydrograph is known to us. So I would be known at both I_1 and I_2, so O_2 will be the only unknown which we can solve from this equation.

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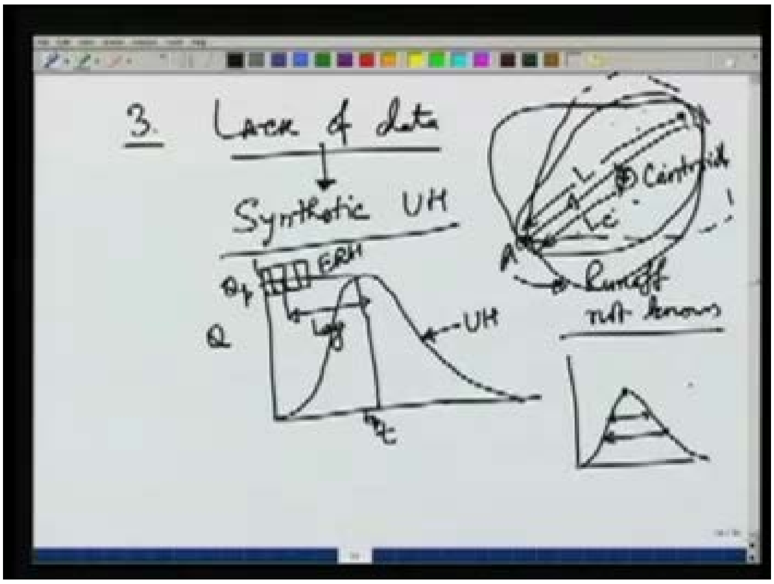

Now let us come to the third limitation of the unit hydrograph method which we say, is lack of data. What we mean by lack of data is there may be a stream or a catchment where there is no caging station.

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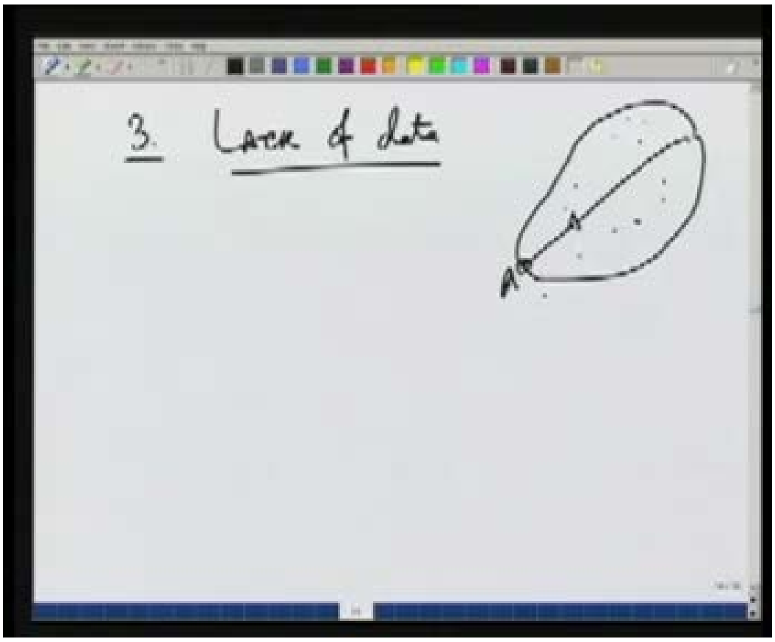

We choose this point A as the point where there are some caging stations, so we can measure the flow at that point and correlate it with rainfall occurring in the catchment area. We use that information to derive the UH at point A but if there is no caging station at point A then we do not know what the run off is. Even if we have a caging station at point A, it is possible that the required data of given intensity at a constant duration of this storms may not be available to us. So in that case we use what is known as the synthetic unit hydrograph. The idea of synthetic unit hydrograph is that the shape of the hydrograph will depend on some basin properties. So if this is the ERH which has unit volume 1 cm depth and this is the unit hydrograph then this unit hydrograph has some properties or some parameters which will depend on the basin properties. It can be correlated with the basin properties. One of the important basin properties for example is the length.

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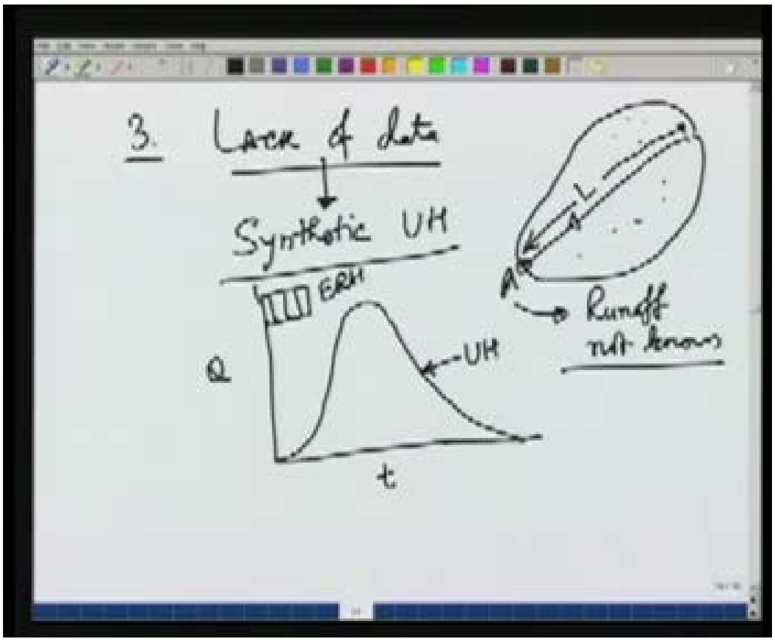

This will depend on the size of the basin. So larger the length, larger will be the size of the basin. Another important property which represents the shape of the basin is called Lc, which is the length to the centroid where this point is the centroid of the catchment area. We take a point which is just opposite the centroid on the stream and find out the Lc along the stream up to the centroid point. This Lc will characterise the shape of the catchment area. For example if you have catchment area like this, its centroid will be closer to the stream and therefore Lc will be smaller but if you have a catchment area like this (Refer Slide Time: 55:59), the centroid will be farther from the point A and therefore Lc will be larger. So size and shape of the basin can be characterised by these two lengths length L and Lc. What is generally done is the lag between the peak of the unit hydrograph and the centroid of the rainfall which is taken as the function of the basin size and shape. There are lot of imperial relationships based on observed values of these lags and the corresponding length an Lc.

So the time to peak, the peak discharge and all these factors are dependent on the basin characteristics and are based on available data. Lot of these synthetic hydrographs have been developed which help us in plotting this curve by defining the peak time to peak and sometimes the width of hydrograph at different locations and so on. We would look at all these synthetic hydrographs in detail in the next lecture. So in today’s lecture we looked at the limitations of the unit hydrograph theory. Mostly we looked at the three limitations which is lack of data, which can be taken care of by synthetic unit hydrograph, Synthetic unit hydrograph, the area being large for which unit hydrograph cannot be applied, that we can take care of by routing the flow through the channel, and then if the rain fall is quite non uniform, we can take care of that by applying the IUH or instantaneous unit hydrograph theory. So in the next lecture we would go into details of the synthetic unit hydrograph. What are various basin properties which are affected? How to obtain the peak flow and time to peak? We would also look at some methods of routing the flow to obtain the discharge at the outlet.