Water Resources Engineering
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Lecture No. 23

We have looked at some methods of converting a unit hydrograph which is for a given duration let's say 4 hour duration to some other unit hydrograph which is a duration multiple of 4 hours. We have seen that we can use the principle of superposition and add; let's say we want an 8 hour unit hydrograph. We can add two 4 hour rains find out the total hydrograph which is because of these two rains of 4 hours duration each and intensity of 1 by 4 centimetre per hour because that is the intensity of the 4 hour unit hydrograph. Now for the total 8 hour we have two centimetres of rain. After we add these two hydrographs up we have to divide the ordinates by 2 and then we will get the unit hydrograph for 8 hour duration. For durations which are not even multiples of 4 hours or the given hydrograph duration we have seen that an S curve can be developed and then any required duration hydrograph can be developed by shifting the S curve by that particular amount and then dividing the resulting hydrograph by appropriate amount.

We will look at the example where we are given a 4 hour unit hydrograph and we want to obtain a 2 hour unit hydrograph. Let's look at this table which shows the 4 hour unit hydrograph at different times. You can notice here that the intervals are not regular. Initially we have data at 2 hour intervals and then we have data at 4 hour intervals. It makes the computations a little bit trickier. If we have equally spaced data then the calculations are little easier but for this also we can use the method of S hydrograph. The S curve is nothing but addition of a large number of unit hydrographs. If we have this 4 hour unit hydrograph let's assume that this is due to the first rain of 4 hours then we shift it by 4 hours. All these ordinates get shifted by 4 hours. This zero corresponds to zero hour here but 4 hour for the second curve. Similarly for 6 we will take the value which corresponds to 2. Any value here for example this 24 will correspond to 20; 20 will correspond to 16 and 16 will correspond to 12; all the ordinates here are shifted by an amount of 4 hours. That means if we have the first 4 hour rain, intensity 1 by 4 centimetre per hour then the first column gives the ordinate because of that. Then we have another rain of 4 hour same intensity but now starting at t equal to 4. So we have shifted everything by 4 hours. The second column, this column, represents the hydrograph due to this rain and similarly we can have a third rain which represents this and so on.

We add a lot of these hydrographs together; for the first one 0 and 2 and then 5. These three ordinates remain the same as the 4 hour UH because no other rain has contributed yet but when you go to the 6 hours then we have an ordinate of 7.5 hours due to the first rain and 2 for the second rain. 7.5 plus 2 gives us 9.5 metre cube per second as the ordinate of the S curve. Ultimately it will reach a constant value 22.2.

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t (h)	4-h UH	Shifted	next	next	next	next	-	-	next	Q (m3/s)
0	0.0									0.0
2	2.0									2.0
4	5.0	0.0								5.0
6	7.5	2.0								9.5
8	8.8	5.0	0.0							10.5
10	4.4	7.5	2.0							13.9
12	3.5	5.5	5.0	0.0						14.0
14	2.8	4.4	7.5	2.0						16.8
16	2.4	3.5	5.5	5.0	0.0					16.4
20	1.9	2.4	3.5	5.5	5.0	0.0				18.1
24	1.3	1.8	2.4	3.5	5.5	5.0				19.5
28	1.1	1.3	1.8	2.4	3.5	5.5	-			20.5
32	0.9	1.1	1.3	1.9	2.4	3.5	-			21.3
36	0.5	0.9	1.1	1.3	1.9	2.4	-	-		21.9
40	0.3	0.5	0.8	1.1	1.3	1.8				22.1
44	0.2	0.3	0.5	0.8	1.1	1.3	-	-	0.0	22.2
48	0.0	0.2	0.3	0.5	0.9	1.1	-	-	5.0	22.2

The plot is shown here. This value as we have discussed already corresponds to a rain of 1 by 4 centimetre per hour occurring continuously over the catchment. The area is 8 kilometre square. Then we have seen that we can relate this Q as 2.78 A where A is the area in kilometre square and this 2.78 comes because of using the units of kilometre square and hours. So this will be in hours. This 2.78 comes because of conversion from kilometre square per hour or centimetre into kilometre square per hour to metre cube per second. Once we get this S curve you can notice here that this is not a very smooth curve and what we would then do is do some kind of a smoothing to get a smooth S curve hydrograph and we will use this smooth curve to obtain the unit hydrograph for any other duration. Here we have listed the ordinates which are after smoothing the curve and then we lag it by the required amount. In this case we want a 2 hour hydrograph, so we lag it by 2 hours. That means that once we get this smooth S curve from this figure we get the ordinates and then we shift everything by 2 hours. We can see here everything is shifted by 2 hours.

Initially we don't have any problem till 14 goes to 16. Once we go to 20 however if the data is unequally spaced we have a problem that we don't have any value corresponding to 18. In that case we can take the mean of these and put it here or when we are smoothing the curve we can take ordinates at every 2 hours or every 4 hours whatever we desire. In this case we could have taken ordinates at every 2 hours and written it in the table. But even if we don't have regular spacing of data we can do some kind of interpolation to obtain these ordinates. Once we lag it then we take the difference. What it means is that if we lag the S curve by 2 hours the difference of these ordinates as we have seen the first S curve is because of a rain which is of infinite duration, intensity 1 by 4 centimetre per hour. The second curve is due to a similar rain but starting at two hours and intensity is same 1 by 4 centimetre per hour.

This means that the difference of these two, the ordinates shown here, will be because of rain which is 2 hour duration and intensity of 1 by 4 centimetre per hour. Since we want unit hydrograph for 2 hours our rain should correspond to 2 hour duration and 1 by 2 centimetre per hour intensity. Comparing these two we see that whatever ordinate we get here has to be multiplied by 2 because the ordinate corresponds to 1 by 4 and we want intensity of 1/2 centimetre per hour.

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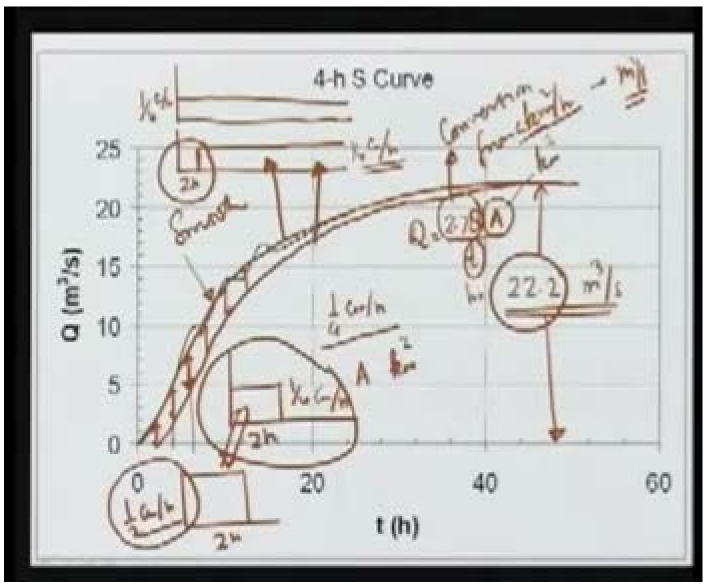

That's why when we take the difference we multiply it by 2 and this way we can get the ordinate of the 2 hour unit hydrograph. In this case it's quite straight forward; difference of these two into 2 will give us 8. Difference of these two is 4 times 2 which will give us 8. Any point we can take; difference of these two into 2 will give us 0.8.

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t (h)	Smoothed	Lagged	diff/2
0.00	0.00	0.00	0.00
2.00	1.50	0.00	3.00
4.00	5.50	1.50	2.00
6.00	9.50	5.50	0.00
8.00	12.00	6.50	5.00
10.00	13.60	12.00	3.20
12.00	14.90	13.00	2.40
14.00	15.00	14.00	2.00
16.00	16.70	15.80	1.80
20.00	17.90	17.30	1.20
24.00	18.00	18.35	0.90
28.00	18.70	18.25	0.00
32.00	20.50	20.10	0.80
36.00	21.20	20.05	0.70
40.00	21.80	21.50	0.60
44.00	22.30	22.00	0.40
48.00	22.20	22.20	0.00

This 2 hour unit hydrograph which we can obtain from the S curve can be plotted and in this case it looks like this. The peak occurs at about 8. In this case these two points are as can be seen from this table 8 and therefore when we draw the hydrograph it goes little more than 8. We can again draw a smooth curve. Some curve like this may be better suited for the hydrograph.

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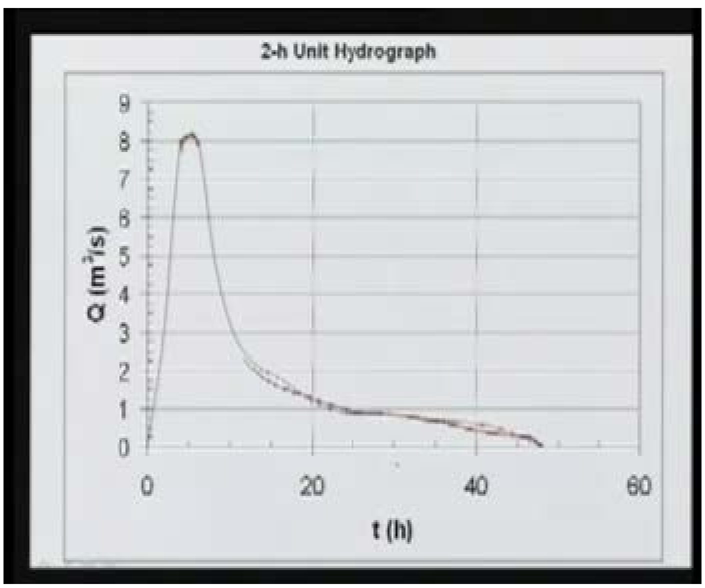

For any duration other than the multiple of the given hydrograph duration we can obtain the unit hydrograph using the S curve and this can be used to obtain even for higher durations. For example if you want 6 hour duration we can still do the same thing but suppose we want 6 hour duration we may again do it by first obtaining a 2 hour unit hydrograph and then superposing it three times to get 6 hour unit hydrograph. It depends on what we find more convenient but S curve can be used to derive unit hydrograph for any duration which is not a multiple of the given duration.

We have looked at if unit hydrograph for one duration is available how to get unit hydrograph for any other duration? But now the question is how to get unit hydrograph for a particular duration? We will look at some methods of obtaining let's say a 4 hour unit hydrograph. In this case suppose we have a catchment which is of area 60 kilometre square. This is our catchment and let's say that at this point A we have a gauging station at which the stream flow record is available to us and let's also say that there is a 4 hour rain which occurs in the catchment of some intensity which may not be equal to 1 by 4 centimetre per hour. In this case the rainfall which occurs on the catchment has an intensity of 5 millimetres per hour. This implies that total effective rain is 4 into 5 millimetre which is 2 centimetre. Again there are some assumptions which we have to make. We will assume that this 4 hour effective rain occurs uniformly over the entire catchment of 60 kilometre square area. If it occurs uniformly over the entire area and we measure the discharge at point A this is the table which gives the values of Q at different times. At zero hours we start. Now this is zero; this is the direct runoff hydrograph DRH. We have already separated the base flow. That's why it is starting from zero. The actual stream flow at A, at the beginning of the rainfall may be some other value which comes from the base flow. The DRH starts from zero and these are the ordinates at different times. In this case we have taken data at equal spacing of 2 hours. Every 2 hours we have measured the discharge at the point A and listed that in this table.

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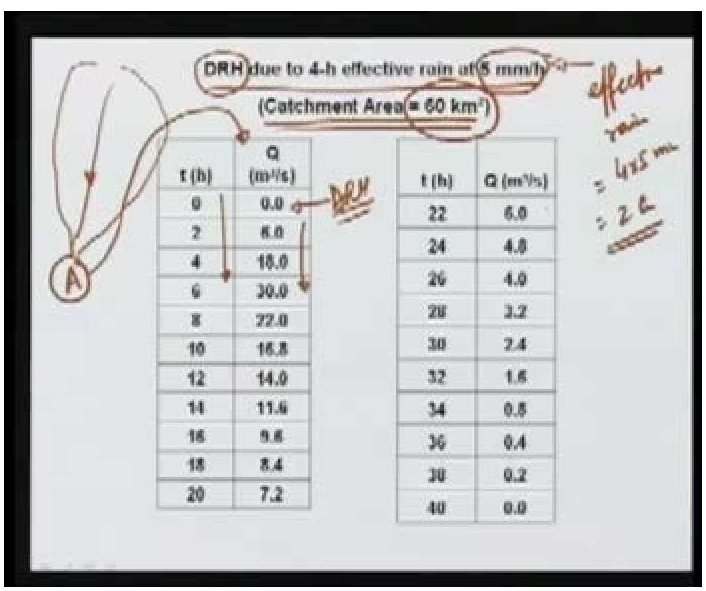

A plot of this DRH is given here. We can see that it starts from zero; 6, 18, 30. So 30 is the peak discharge. This is DRH due to 2 centimetre rain. Assumptions are that it is uniformly occurring over the entire area of 60 kilometre square. Once we have this DRH finding out the unit hydrograph is quite straight forward because this is due to 2 centimetres of rain and we want unit hydrograph which corresponds to 1 centimetre of rain; UH corresponding to 1 centimetre of rain.

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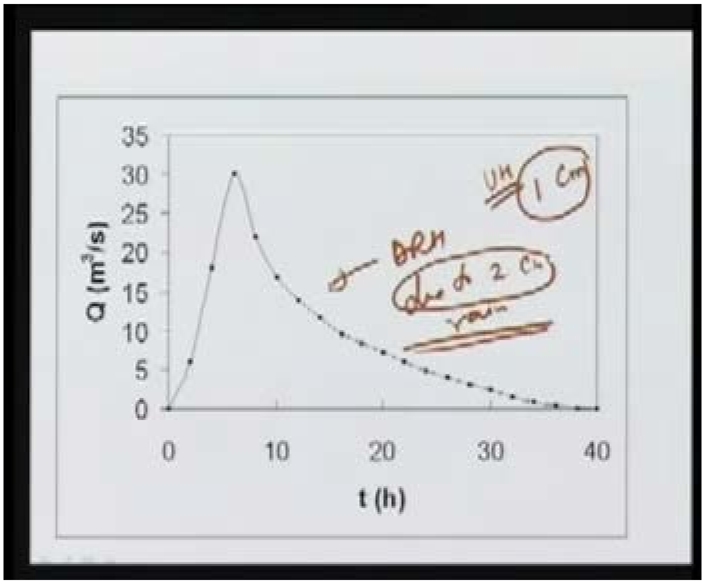

This is already available to us for 2 centimetre of rain. So we divide each ordinate by 2 and we will get the UH ordinate. This is nothing but DRH divided by 2. So the peak instead of 30 becomes 15. This is a very straight forward way of obtaining the unit hydrograph. But the condition is that a rainfall of almost uniform intensity and for that particular duration should be available. In this case we say that 4 hour unit hydrograph can be obtained if a 4 hour rain is available and a corresponding discharge is also available. The 4 hour unit hydrograph looks like this. This is similar to the DRH simply dividing by 2 all the ordinates we get this unit hydrograph.

Most of the times, constant intensity rainfall of 4 hours may not be available. We may have a rainfall which may be a different duration. It may be like 6 hours or 2 hours. In that case we can derive unit hydrograph for 2 hours or 6 hours and then get 4 hour unit hydrograph from that hydrograph. Sometimes uniform intensity rainfall may not be available. Sometimes intensity may be varying. We may have an ERH, effective rainfall hydrograph as let's say two different 4 hour rains of varying intensity. For example we may have intensity like this then we may have intensity like this.

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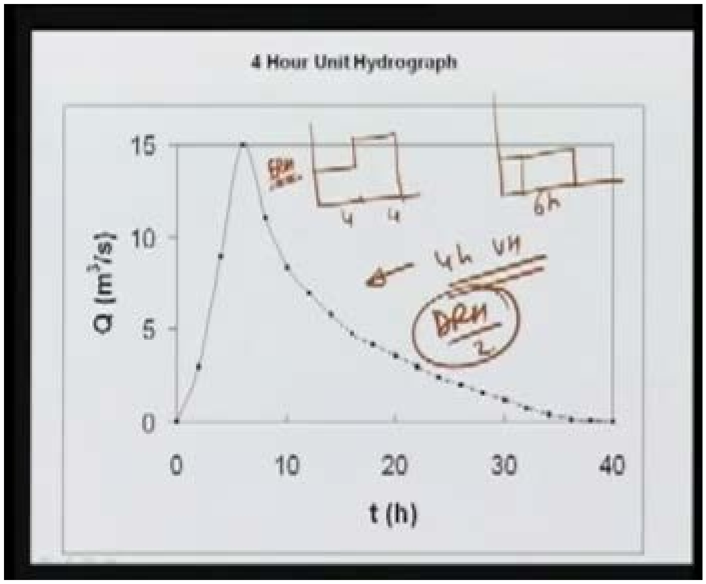

In this case how to obtain the 4 hour UH is the case which we will discuss now. Suppose we are given that a rainfall of 5 millimetre per hour occurs for 4 hour and then it is followed by another rainfall which is of uniform intensity but a different intensity 10 millimetre per hour again for 4 hours. The problem is that if we know the direct runoff hydrograph, DRH which corresponds to a combination of these two we have 5 millimetre per hour and then 10 millimetre per hour for 4 hours, 4 hours. This is the ERH. That means we have 2 centimetre rainfall here; 4 into 5 millimetre and then they have another 4 centimetre of rainfall here because this is 1 centimetre per hour for 4 hours. In total we have a 6 centimetre of rainfall over the area which is 60 kilometre square and the DRH which we have measured at A is given by these values where the peak value as can be seen here occurs at 10 hours and is equal to about 77 metre cube per second.

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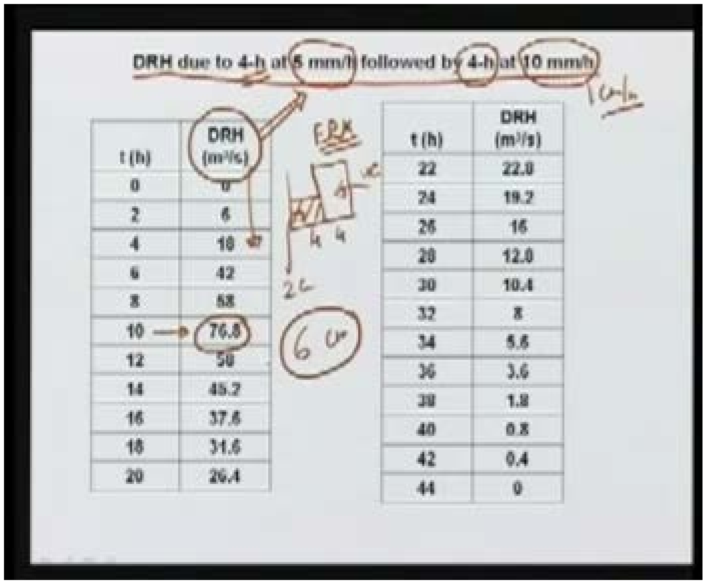

t (h)	DRH (m^3/s)
0	0
2	6
4	18
6	42
8	68
10	76.8
12	58
14	45.2
16	37.6
18	31.6
20	26.4

t (h)	DRH (m^3/s)
22	22.0
24	19.2
26	16
28	12.0
30	10.4
32	8
34	5.6
36	3.6
38	1.8
40	0.8
42	0.4
44	0

This DRH is plotted and can be seen in this figure. If you see this curve up to 4 hours there will be no change compared to the 4 hour UH but then since another rainfall occurs we will have this variation which means that we'll have a sum of rainfall ordinate or the DRH ordinate. Suppose we are looking at 6 hours. This 42 corresponds to the 6 hour value for the first rain and the 2 hour value for the second rain because the second rain has started after 4 hours. That's why in this curve, ordinate up to 4 hours there is no problem; it’s only because of the single rain. We can use the 4 hour UH ordinate directly but after that we will have a combination of the first 4 hour 5 millimetre per hour rain and then we have some ordinate which comes from the second 4 hour 1 centimetre per hour rain. These ordinates are not at the same time but they are shifted.

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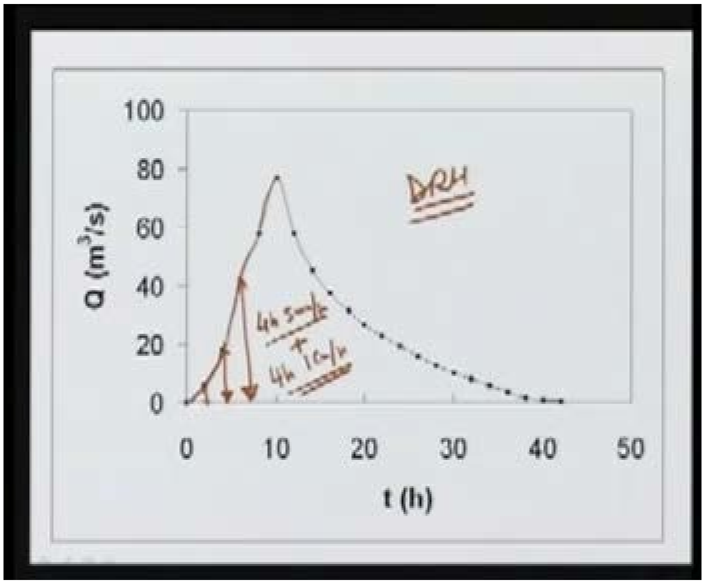

Using this DRH if we want to evaluate the unit hydrograph we will have to do an analysis like this. Suppose the ordinate of the 4 hour unit hydrographs are u_1, u_2 and u_3; t = 0 then 2, 4, 6, 8 like this and suppose the 4 hour UH ordinates are u_1, u_2, u_3 like this. The DRH ordinate since the first rainfall is of total 2 centimetre effective rain $2u_1$ should give us the direct runoff because of this rain. This represents the DRH ordinates d_1, d_2, d_3 and so on. What it tells is that if we have unit hydrograph for 4 hour with ordinate u_1 the direct runoff will be 2 times u_1 because the rainfall is 2 centimetre compared to 1 centimetre for the UH. d_1 is given to us. d_1 is 6 as can be seen from this table. d which is the direct runoff hydrograph ordinate at 2 hours is equal to 6 and it gives us a very straight forward way of computing u_1 and u_1 will come out to be 3 from this equation. Similarly at 4 hours since the second rain has not yet started we have a similar equation. This tells us that $2u_2$ will be equal to d_2. Since d_2 is given as 18, u_2 will be obtained as 9.

When we go to the next time which is 6 hours then we have a combination of ordinates coming from both the first and the second rain. For the first rain the ordinate will be u_3 which will be at 6 hours and multiplied by 2. For the second rain the ordinate will be u_1 which corresponds to 2 hours because we are computing at 6 hours means the second rain has started 2 hours before that time. Therefore we have to take u_1 and multiply it by 4 because in the second rain total rainfall is 4 centimetre; $2u_3$ plus $4u_1$ this should give us the direct runoff ordinate at the third point which is 6 hours and this is known to us as 42 from the table of DRH. This 42 will be equal to the sum of $2u_3$ and $4u_1$. u_1 is already known to us because of the first equation. u_1 is known to us as 3 and therefore we can solve this equation and get this 15 because $4u_1$ will be 12. 42 minus 12 divided by 2 will give us u_3. We get an ordinate u_3 equal to 15. Same way we can write $2u_4$ plus $4u_2$ equal to d_4 which is 58 giving us u_4 of 11.

In this way from a complex storm the condition is that all the storms should be of the same duration, 4 hour in this case. There can be multiple storms not only 2; there can be

3, 4 any number of storms; provided they are all of the same duration we can use this. The only thing is that after a few hours we will have sum of three different ordinates. Here we had two ordinates because there are only two rains of 4 hour duration. If we have 3 then after 8 hours; so from 0 to 4 we will have a single ordinate 6 to 8 then we will have 2 and then after that they will have 3 ordinates, sum of 3 ordinates. But again sequentially we can obtain all the values in this way. The problem with this is that if there is some small error in the data it gets magnified because we are doing it sequentially. Any small error in one of the ordinates will get magnified as we move away and sometimes it may happen that the values here get negative or some oscillations may be there.

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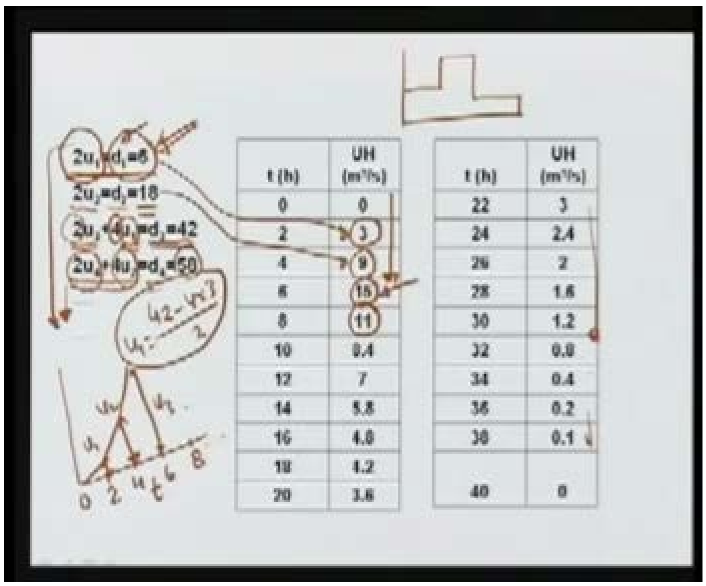

In this case since we have taken exact data this resulting unit hydrograph which we have obtained by computing this u_1, u_2 and u_3 turns out to be quite smooth.

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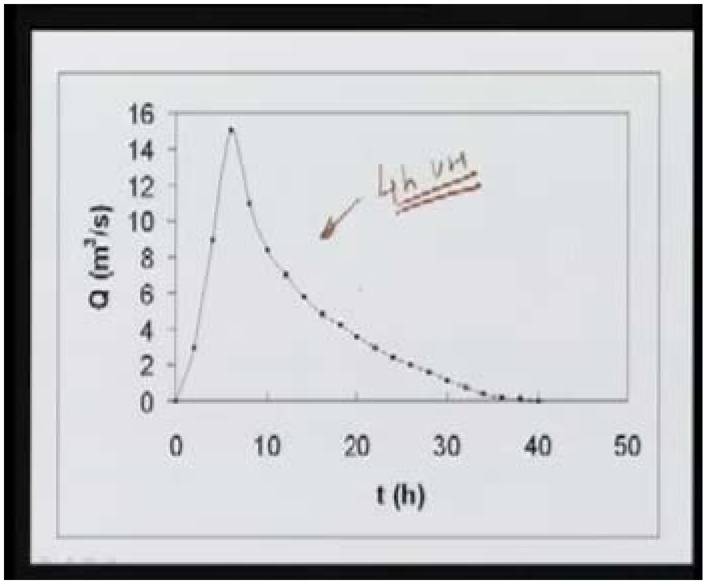

We say that this is a 4 hour UH which we have derived based on the direct runoff hydrograph corresponding to two different storms in this case of same duration 4 hours but different intensities. Obtaining the unit hydrograph for a gauge catchment is therefore not very difficult if we have the required rainfall available and corresponding runoff also available. We can first separate the base flow and obtain the DRH. From the DRH we can obtain the UH depending on whether we have a single storm or whether we have complex storms and then using that 4 hour unit hydrograph or whatever duration we want using that duration then we can obtain unit hydrograph for any other duration by using S curve or superposition. But suppose we don't have a gauging station available that means we don't have a DRH available to us for any given ERH. In that case we go for synthetic unit hydrographs in which we use the geometric parameters of the catchment. For example the length or length to centroid, the slope and so on and using those geometric parameters we can obtain a unit hydrograph which is not based on any observed runoff but it is just a synthetic hydrograph.

We look at the Snyder's method. Synthetic UH means we don't have any data available for the catchment. Snyder's method is the one which Snyder has proposed and there are some equations which are given by Snyder. He said that if we have a catchment the length of the catchment is L along the stream, centroid of the catchment is somewhere here and the length to the centroid is L_c. Area is A in kilometre square, lengths are although in the original Snyder method they were used in miles we will be using kilometres. So L and L_c would be in kilometres, A will be in kilometre square. What Snyder has in his synthetic hydrograph is he has related the characteristics of the hydrographs. Suppose this is the duration of the rainfall t_r effective duration from the centre of this rain to the peak he got the lag as t_p. Snyder related this tp with the catchment characteristics and he said that this t_p is standard value equal to some constant into L, L_c to the power 0.3 and for this standard lag the duration t_r is t_p divided by 5.5.

If we have the data available for let's say a gauge catchment, c_t is a constant which depends on the basin properties. In this case we will assume that there are two catchments. One is gauged catchment and gauged catchment means that we can obtain the unit hydrograph for this easily. Catchment two is ungauged but they are very similar. The assumption has to be made that they are very similar. That means whatever the value of c_t is for this catchment same value of c_t can be used for catchment 2. For the catchment A the data is given as area 60 kilometre square, length 15 and up to centroid 9. This L_c is 9, L is 15 and the area is 60 kilometre square. A 4 hour unit hydrograph for the first catchment has been obtained because it is gauged. We have obtained the unit hydrograph. The peak is given as 15 metre cube per second and time to peak is 4 hours from the centre of the rainfall. This t_r is 4 hours. Centre means that we have 2 hours and then further 4 hours from that. t_p is 4 hours from here and 6 hours from the time zero of starting of the rainfall. So t_p in this case is 4 and the duration of the rain is 4 hours. We can see that t_r and t_p do not satisfy this relationship.

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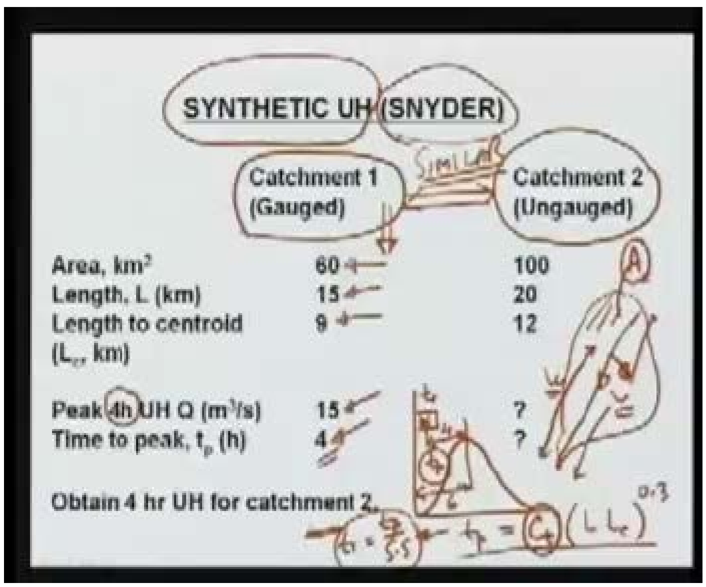

This 4 hour is therefore not a standard duration as per as Snyder's definition is concerned. We would have to find out the coefficient c_t and c_p knowing that this t, t_p and t_r are not standard values and in that case Snyder has given a relationship. If t_r is not a standard then for any other rainfall duration of t_R, Snyder has given this relationship which can also be written as this because this t_r we have seen is t_p divided by 5.5. Let's use this relationship to obtain the value of c_t for the catchment which is gauged and for which unit hydrograph is available to us. The other relationship which has been used to obtain the peak discharge is given here in which the peak discharge Q_p is given as C_p into 2.78A over t_p. Cp is also another constant which we can obtain for the catchment 1. Our aim would be to predict the value of C_t and C_p from this catchment and since they are similar we can use the same C_t and C_p for this catchment and predict the value of peak and time to peak for the second catchment.

The computations are shown here. For the gauged basin t_p star is 4 hours and t_r star is 4 hours. Star denotes the value for the gauged catchment. The standard lag, as we have seen, t_p can be obtained using this equation as 22/21 t_p star minus t_r star by 4 and comes out to be 3.14 hours. This means that for the given basin the standard lag should be 3.14 hours and since we know the equation for standard lag we can obtain the value of C_t as standard lag divided by L star L_c star to the power 0.3. For the catchment 1, L is15, L_c is 9. So we have 15 into 9 to the power 0.3 and t_p is 3.14 hours. This gives us a C_t of 0.72. Similarly since we know the peak discharge for the catchment we also know the peak lag, the basin lag. We also know the area. We can find out the C_p value as 0.36. These two values are now known to us C_t and C_p and using these values we can derive a synthetic value of the unit hydrograph for the catchment 2 which is not gauged and for that let's assume that we want a 4 hour unit hydrograph.

Given a 4 hour unit hydrograph for catchment 1 we want a 4 hour UH for catchment 2. This 4 hour again is not ……. duration as far as Snyder definition is concerned. So we can write this equation where C_t into L L_c 0.3. This is the standard lag 21 by 22 plus t_r by 4 should be equal to 1, the basin lag for this corresponding rainfall of 4 hours. This gives us a lag of 4.56 hours. We can see that the time to peak or the lag in this case was 4 hours. The area was 60 kilometre square. Since this is a larger basin 100 kilometre square area, L and L_c are also larger; we expect the lag will also be larger. In this case compared to 4 we get a lag of 4.56 hours. The peak discharge is obtained; C_p we have already seen from the previous basin as 0.36. C_t we have already seen as 0.72. Using these values of C_p and C_t we get the peak discharge as 21.95 metre cube per second. Area of the catchment is 100 and tpR of 4.56. Since the area is 100, the peak discharge per unit area can be written as 21.95 divided by 100 metre cube per second per kilometre square. Then to help draw the hydrograph there are some equations which are proposed. For example the time base of the hydrograph can be obtained by approximating it by a triangle and equating the area to be unit. So t_b can be given by 5.56 divided by qpR and it comes out to be 25 hours. In this case 25 hours seems to be a little small because we have earlier seen that the unit hydrograph for the catchment 1 which is gauged and is smaller has a time base of about 40 hours. So this 25 hours seems to be little small and we will increase it later when we plot the curve.

There are two other significant values one is what is the width at 50% of the peak discharge and what is the width at 75% of a discharge? The equations which are given are W_{50} which is the width at 50% of the peak discharge is given by 2.14 divided by the peak discharge to the power of 1.08. It comes out to be 11 hours. W_{75} comes out to be 6.3 hours. It has also been proposed that 1/3rd of this should be before the peak and 2/3rd after the peak.

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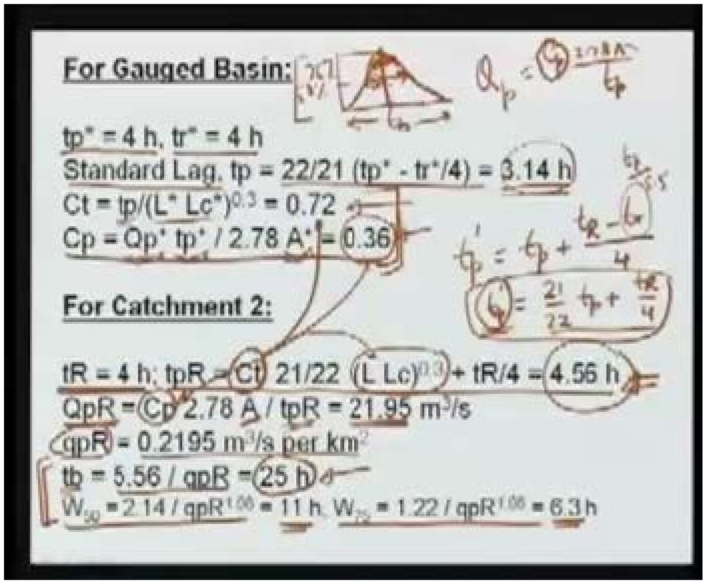

So out of this 11 hours, 1/3rd should go before the peak and 2/3rd after the peak; similarly for this. Using the time base, W_{50}, W_{75}, peak discharge, the lag 4.6 hours all this data we can plot a few points and then join them by a smooth curve to obtain the synthetic unit graph. In this case 0 and 25 these two are based on the time base of 25 hours. Then we have the peak. This should be 4.6 hours from the centre of the rain. Centre of the rain is at 2 hours because the rain actually is occurring up to 4 hours and from the centre this is 4.56 hours based on the value computed here. So this total should be 6.56 hours. The peak is also known to us. Let's say that this is 22 roughly because the value which came out from the calculation is 21.95. We can say that peak is 22; at 11 which is 50% of peak and then 16.5 which is 75% of the peak. We know that this total width is 11 hours and this is 6.3 hours. 1/3rd of 6.3, 2.1 goes here and 4.2 goes here. Knowing the peak we can obtain these two points. Similarly from the peak 11 by 3 here, 22 by 3 here; so we can obtain these. So 1, 2, 3, 4, 5, 6, 7 points are known on the Snyder's curve.

The area under the curve will be 1 centimetre of rain over 100 kilometre square area. But if you look at this curve it's not smooth. What we will probably do in most cases is join it by a smooth curve and also we have seen that 25 seems to be little small. So we would like to have it increased up to 40. We will try to make it such that the area remains similar. Whatever area we have excluded here can be included here and we can get a smooth curve based on neighbouring catchment, a similar catchment and we can obtain this synthetic unit hydrograph.

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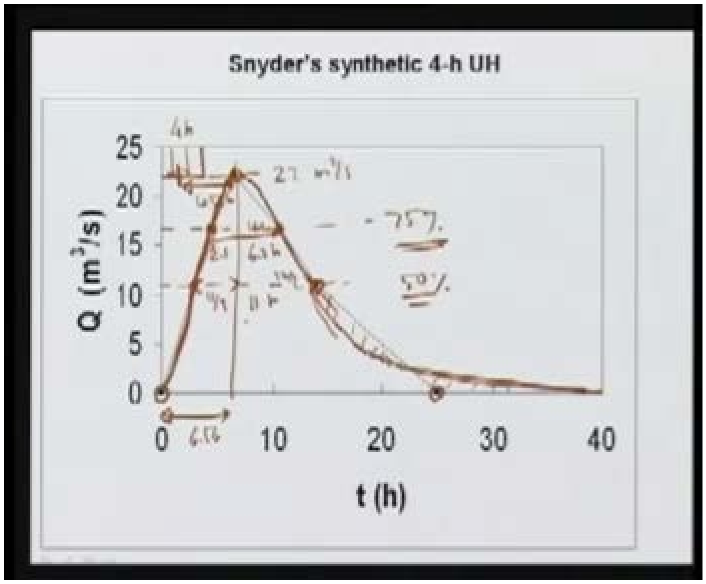

The synthetic unit hydrograph can be derived for a catchment for which we have geometric or geomorphological data available and we can estimate the values of the coefficients C_t and C_p for a similar catchment which is gauged and we can obtain those values from unit hydrograph derived for that other gauged catchment. Using those values of C_t and C_p we can estimate the synthetic unit graph for the basin which is not gauged. But this synthetic unit hydrograph or whatever hydrographs we have discussed till now they are of a particular duration. That means that we can use them to predict the DRH for a rainfall of the same duration and this duration can be changed. We can derive rainfall runoff relationship for a different duration of rain by using a unit hydrograph for that particular duration and we have all ready seen that given any duration of unit hydrograph we can derive any other duration hydrograph easily using S-curve or using superposition. But sometimes the rainfall which occurs may not have a uniform intensity. The intensity may be varying quite a bit. For example we may have a rainfall which has intensity like this. In this case making the assumption of a constant intensity over finite intervals may be valid but it may not be very accurate.

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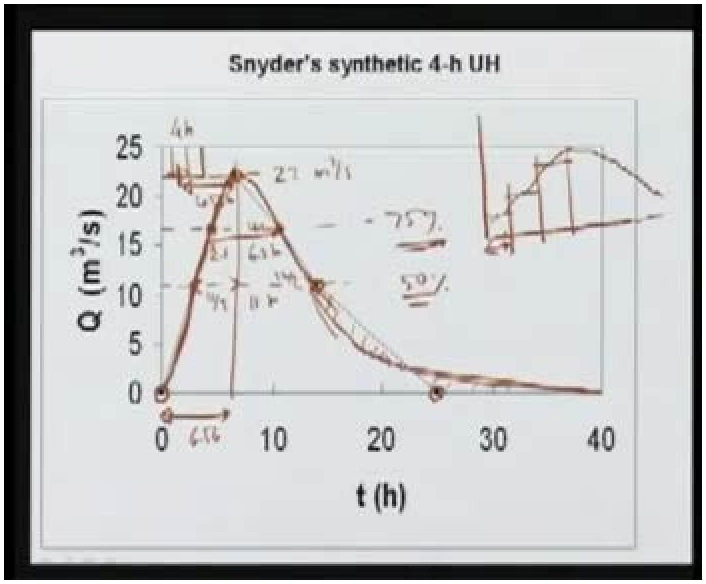

In these cases where the intensity varies continuously with time and uniform intensity assumption cannot be made we should look for a unit graph which corresponds to instantaneous rainfall and this is known as the IUH or instantaneous unit hydrograph. We have seen some of the methods. For example we have seen the Nash model which is defined as the instantaneous unit hydrograph, t versus Q. This is the UH. The effective rainfall which causes this instantaneous unit hydrograph is a rainfall of very high intensity and so very small. Suppose this is delta t and this intensity is let's call it 1 over delta t. The volume of rain which occurs is 1 or the depth of the rain into the catchment area. Depth of rain is 1 centimetre but it occurs at almost zero time as the limit delta t tends to zero it becomes instantaneous rainfall of very large intensity, very small duration in such a way that the depth is 1 centimetre. In other words the entire 1 centimetre of rainfall occurs instantaneously at the catchment and because of that it causes a runoff like this. We have seen that Nash model is one of the models which is used for instantaneous unit graph calculation.

The equation which is used for Nash model comes from linear combination in series of reservoirs. We have a reservoir in which there is some rain here; some outflow goes into second reservoir. There is some outflow which goes into third reservoir and so on. So n is the number of reservoirs and K is the time constant. In the case we have taken to illustrate the Nash hydrograph n is equal to 4 and K equal to 3 hours for a catchment of area 60 kilometre square. This u value is an ordinate which is expressed in per hour and what it really means is centimetre per hour per centimetre rain. Since we are saying that this is unit hydrograph it will have 1 centimetre of rain. Therefore u can be written in terms of centimetre per hour and that is what we have done in this table which shows the value of u computed from this equation with K equal to 3 and n equal to 4. The ordinates are expressed in centimetre per hour here and then based on the catchment area we can convert into metre cube per second.

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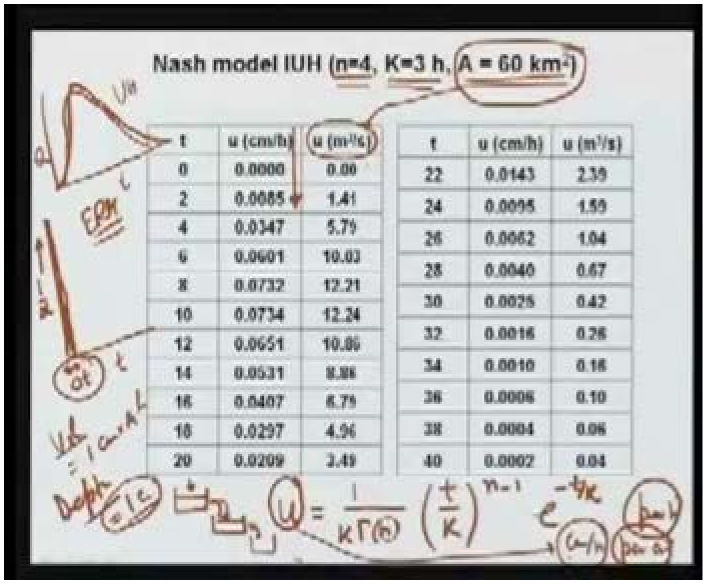

These ordinates can be seen here with the peak occurring at about 10 hours of almost 12 metre cube per second and the plot of the IUH indicates that over a catchment of 60 kilometre square if a 1 centimetre rain occurs at time t is equal to zero instantaneously then this would be the resulting hydrograph with the peak of almost 12 metre cube per second. We have already seen that this IUH can be used to obtain the DRH because of rainfall of varying intensity by using the convolution theorem. Here we would look at some uses of this IUH. Suppose we are given the IUH and we want a 2 hour unit hydrograph derived from the IUH. Since the IUH represents instantaneous rain if we take any time interval delta t the average of these two ordinates will give us the ordinate of the unit hydrograph.

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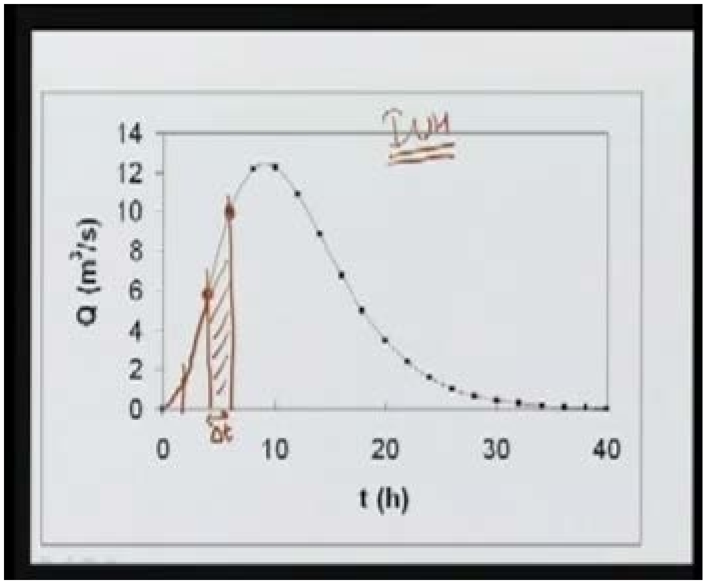

Suppose this is 2 hours then if you want the 2 hour unit hydrograph the value of 0.7 here is the average of these two; mean of zero and 1.4 will give us 0.7. Similarly the ordinate of 2 hour unit hydrograph at 4 hours would be the mean of 1.4 and 5.8 and so on. All the ordinates can be obtained in such a way. This 0.13 is the mean of 0.16 and 0.1. The 2 hour unit hydrograph can be derived. This is the 2 hour because of the difference of these times is 2 hours.

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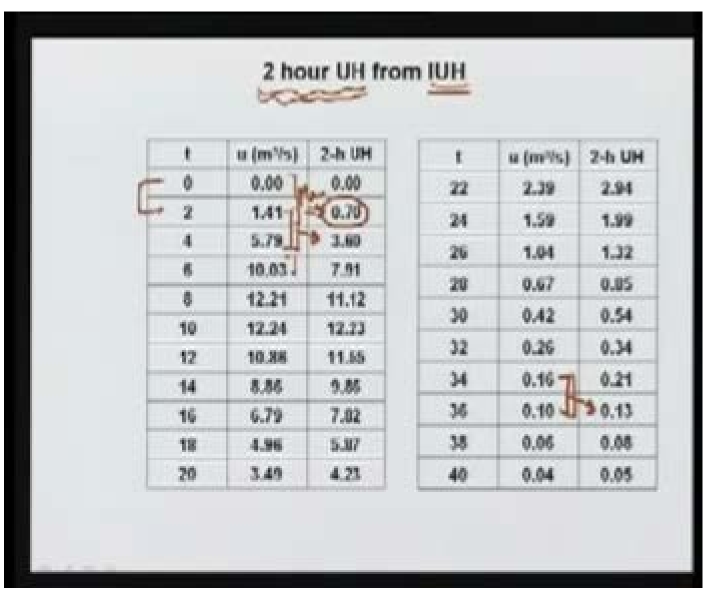
2 hour UH from IUH

t	u (m³/s)	2-h UH
0	0.00	0.00
2	1.41	0.70
4	5.79	3.60
6	10.03	7.91
8	12.21	11.12
10	12.24	12.23
12	10.86	11.55
14	8.86	9.85
16	6.79	7.82
18	4.96	5.87
20	3.49	4.23

t	u (m³/s)	2-h UH
22	2.39	2.94
24	1.59	1.99
26	1.04	1.32
28	0.67	0.85
30	0.42	0.54
32	0.26	0.34
34	0.16	0.21
36	0.10	0.13
38	0.06	0.08
40	0.04	0.05

It can be plotted like this with the peak of about 12 metre cube per second. Once 2 hour unit hydrograph is available to us we can use this to obtain S curve. Suppose this 2 hour unit hydrograph is available to us. This can be obtained from an IUH or it can be obtained directly if we have a gauged basin, suppose we have a 2 hour rain we can get the corresponding DRH and get that 2 hour UH from there. From IUH also we can obtain the unit hydrograph for a given duration. But what we would look at is for a given hydrograph how to obtain the IUH.

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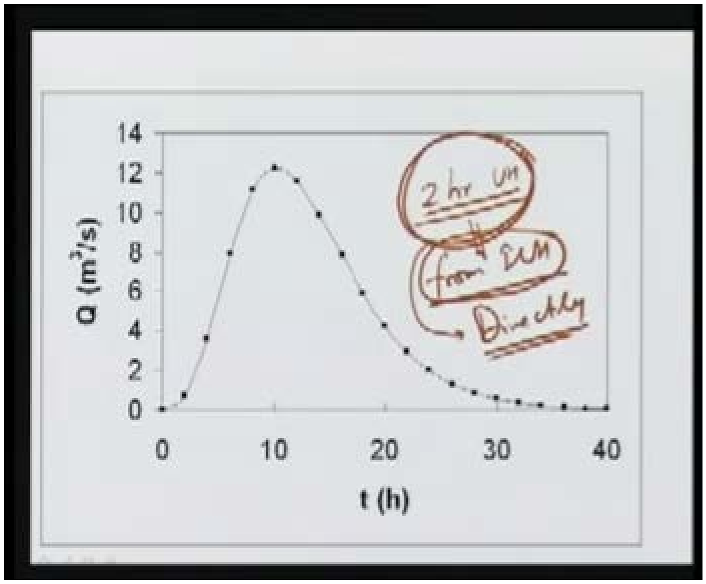

Suppose directly we know the 2 hour UH; based on that 2 hour unit hydrograph we can obtain the 2 hour S curve. Those values are shown here and as we have seen the relationship between the ordinate of the IUH and this is the S curve ordinate. The S curve looks like this. What this equation tells us is that the slope of the S curve at any point ds by dt divided by the intensity of rain will give us the ordinate of the instantaneous unit hydrograph. S curve represents the rainfall due to infinite value. Suppose we shift it by delta t and suppose this is delta S. When we shift it by an amount delta t, delta S represents the Q value due to a rain of intensity i for period delta t. This is what delta S represents. Since we have shifted it by an amount delta t it represents the amount of runoff which occurs because of two rains. One of intensity i continuously, the other of same intensity but shifted by delta t. Any change here delta S represents the Q due to intensity of I, period of shifting of delta t and as delta t becomes zero delta S by delta t will tend to ds by dt.

This i rainfall for a period delta t means the volume is i delta t. Since we want a volume of unity delta S over i delta t will give us the unit hydrograph, instantaneous unit hydrograph ordinate U. So U will be equal to delta S divided by the volume of rain which is i delta t and therefore in the limit delta t tending to zero delta S by delta t will tend to dS by dt and therefore U will be 1 over i dS by dt which is what is written here. Once we derive 2 hour S curve we can obtain the slope of the S curve at any point divided by the intensity. In this case since it is a 2 hour S curve i will be 1 by 2 centimetre per hour.

The procedure is obtaining this S curve first. S curve is very easy to obtain because once you have the 2 hour UH we have already seen how to lag them by 2 hours, add them up and then get the 2 hour S curve. In this case by doing that we have obtained the two hour S curve ordinates maximum value of 83.35 which corresponds to intensity 1/2 centimetre per hour over an area which in this case is given as 60 kilometre square; area is given as 60 kilometre square. If the intensity is taken as 1/2 centimetre per hour it was ascertain an ordinate of 83.35 at the end or the asymptotic value.

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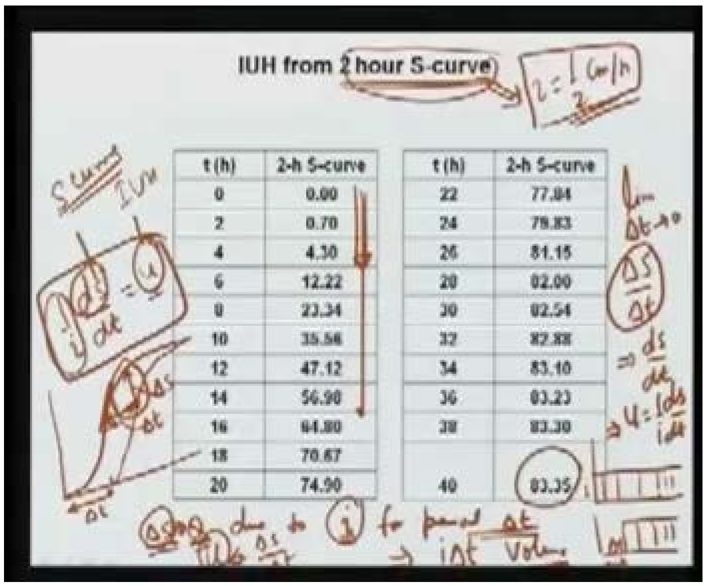

t (h)	2-h S-curve	t (h)	2-h S-curve
0	0.00	22	77.84
2	0.70	24	79.83
4	4.30	26	81.15
6	12.22	28	82.00
8	23.34	30	82.54
10	35.56	32	82.88
12	47.12	34	83.10
14	56.90	36	83.23
16	64.80	38	83.30
18	70.67		
20	74.90	40	83.35

You can plot this S curve and as it is shown here this is the 2 hour S curve. If you look at the slope of this S curve initially the slope is almost zero. At the end also it is almost zero because it attains a constant value. The slope is zero here; it increases and then there is a point of inflection. After that it will start decreasing and then ultimately the slope will become zero and the instantaneous unit hydrograph can be obtained as 1 over i which in this case will come out to be 2, since i is 1/2 into dS by dt. The slope of this we can obtain numerically. We can use central difference or we can try to fit a curve and then take the slope, a linear interpolation or quadratic interpolation. What we have done is suppose we want the slope here we have used a central difference approximation. The slope will be obtained by joining the two surrounding points and divide by delta t. In this case since we have every 2 hour intervals delta t will be 4 hours and delta Q will be the difference in the next Q and the previous Q.

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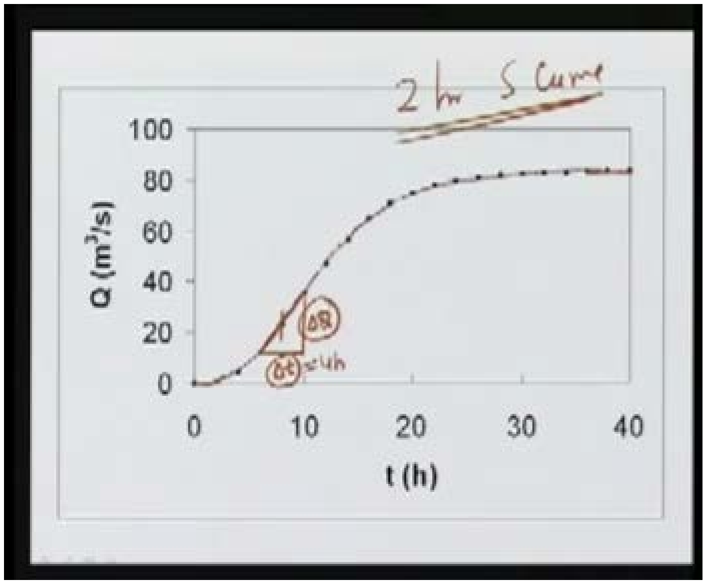

Using that, we have here the value of 0, 0.7 and 4.3. We want to find out the slope at this point at 2 hours. What we will do is take the difference of the two neighbouring points; the next point 4.3 and previous point 0. So 4.3 divided by delta t which is 4 gives us a slope of 1.08 intensity as we have seen. The IUH ordinate is 1 by i dS by dt. i is 0.5 centimetre per hour; dS by dt we have obtained. So 1.08 is 4.3 minus 0 divided by 4. Similarly 2.88 is the difference of 12.22 and 0.7 divided by 4 and then IUH will be multiplied by 2. So this into 2 will give us 2.15. Similarly all these values are multiplied by 2.

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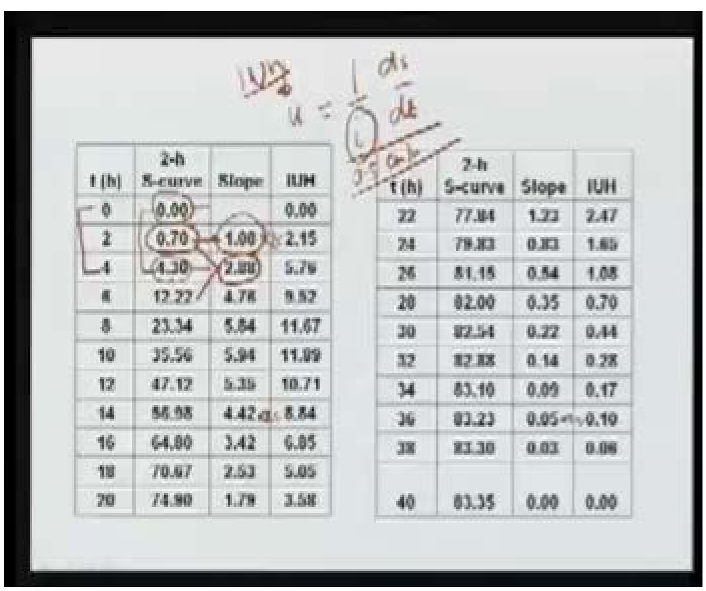

t (h)	2-h S-curve	Slope	IUH
0	0.00		0.00
2	0.70	1.08	2.15
4	4.30	2.88	5.76
6	12.22	4.76	9.52
8	23.34	5.84	11.67
10	35.56	5.94	11.89
12	47.12	5.36	10.71
14	56.98	4.42	8.84
16	64.80	3.42	6.85
18	70.67	2.53	5.05
20	74.90	1.79	3.58

t (h)	2-h S-curve	Slope	IUH
22	77.84	1.23	2.47
24	79.83	0.83	1.65
26	81.15	0.54	1.08
28	82.00	0.35	0.70
30	82.54	0.22	0.44
32	82.88	0.14	0.28
34	83.10	0.09	0.17
36	83.23	0.05	0.10
38	83.30	0.03	0.06
40	83.35	0.00	0.00

In this way given a 2 hour unit hydrograph we can derive an IUH for that particular basin and in this case the IUH turns out to be like this. We can join them smooth with a smooth curve and this IUH comes out to be very similar to the IUH which we had used to derive the 2 hour UH. This was the IUH which we had used to derive the 2 hour UH. Then we use the 2 hour UH to derive the S curve for 2 hours. The 2 hour UH is given by this curve. Using this 2 hour UH we have derived the 2 hour S curve and then from this 2 hour S curve we can obtain the IUH which should be similar to what we had started with.

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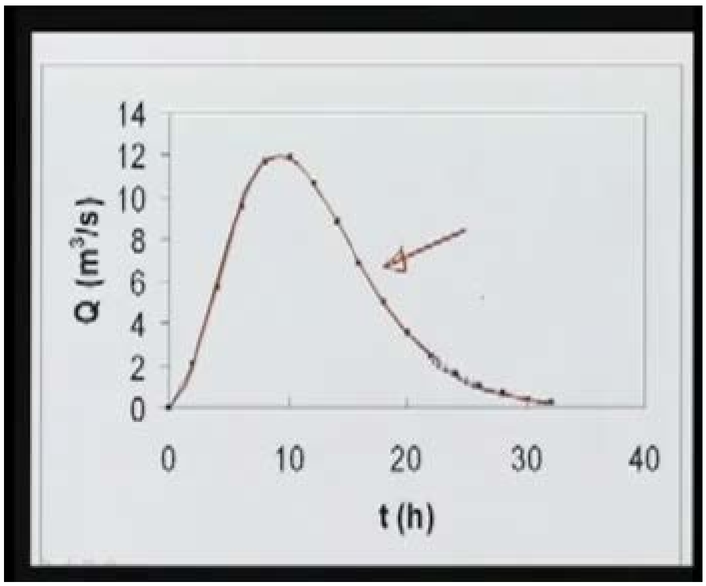

But this just gives an idea of how to obtain an IUH for a given unit hydrograph. Typically a duration of 2 hours we have taken but 1 hour duration would be better. More than 1 hour, taking the slope by this finite difference approximation may not work very well. But for 1 hour it should be quite good; less than 1 hour is okay but 2 hours that we have taken may not be very good from accuracy point of view.

We have looked at some of the techniques in this case to obtain the unit hydrograph for a duration which is not a multiple of the given hydrograph duration. We have also looked at how to obtain the unit hydrograph synthetically using Snyder's method if a catchment is not gauged but similar gauged catchment is available near by. Then we have seen that if the rainfall is not of uniform intensity over the whole period if it is varying widely then we may not be able to use the unit hydrograph theory because that assumes rainfall of constant intensity for a given duration. In that case we can go for instantaneous unit graph which can be used to obtain DRH for any varying rainfall by using the convolution theorem. The IUH can be derived based on Nash model which we have seen is based on a gamma function and it's a combination of …… the reservoirs in series.

There are other methods also of obtaining IUH. We have seen one method which is based on obtaining the S curve for a given unit graph and then taking the derivative of the S curve to obtain the unit hydrograph, IUH ordinates. There are some other methods also.

For example there is a Clarke's model in which we use what is known as time, area diagram. We divide the catchment area into sub areas all of which have almost same time of travel for reaching the outlet point and then these different areas can be plotted with time and we get a time area diagram and that time area diagram we can use to obtain the IUH because we assume that instantaneously the water is falling over the entire area. It will have some lag time for reaching the outlet and those time area diagrams can be prepared and they can be used for IUH but we will not discuss them here. We have looked at the two methods which are Nash model and then using any other UH of preferably a short duration for example 1 hour duration or 2 hour duration, from the slope of the S curve we can obtain the IUH.