Water Resources Engineering
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Lecture No. 19

Today we will continue with our discussions of ground water. We have already seen the Darcy's law which relates the hydraulic gradient and the velocity with the constant of proportionality which is known as the permeability and we have also seen how when we pump a well the water level or the piezometric level goes down. Let's see what happens when we pump a well. This is the ground level and we have a confined aquifer being pumped through a well. As we have seen the piezometric surface will go down as we start pumping and the difference is known as the drawdown denoted by s. This s as we have seen is proportional to Q because we have assumed that the flow is laminar. This is generally true but when we are pumping from a well the velocity is very large near the well and near the well the flow will be turbulent. Similarly near the well because of the casings when the flow passes through the casing there will be additional head loss.

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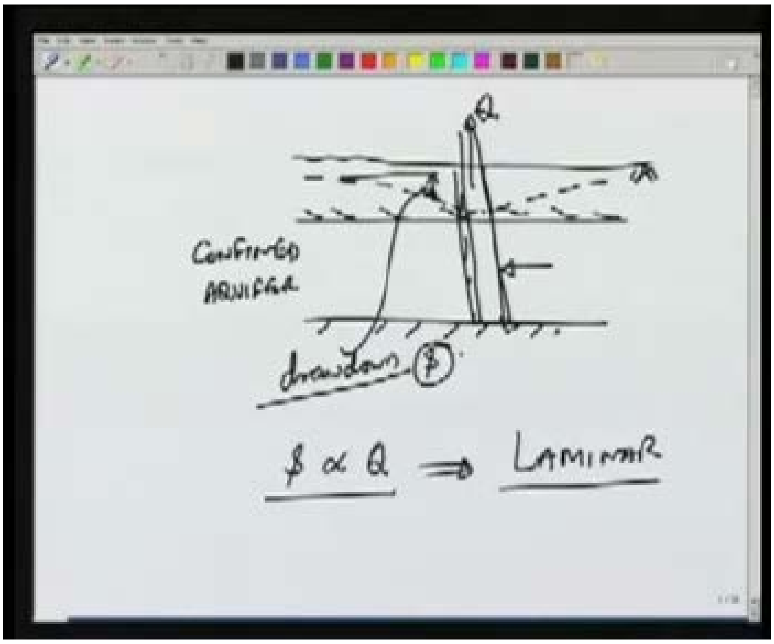

We combine these and call them the well loss and we write the total drawdown at the well as summation of the laminar loss, turbulent loss and the casing loss. So this will be total drawdown at the well and if we draw the same figure again this is the initial piezometric level before the pumping is started and then a final piezometric level after the steady state has been reached and this is the drawdown at the well s_w. If we say that there is only laminar flow then there will be head loss or a drawdown because of the laminar flow. This is because of turbulent flow near the well and this is the loss due to well casing. As we have seen the laminar loss generally is proportional to the discharge. So s_{wl} will be proportional to Q. The turbulent loss is normally Q to the power something and we can take it as Q square. So we can write s_{wt} is proportional to Q square. The casing loss has also been seen to be proportional to Q square. Both s_{wt} as well as s_{wc} are proportional to Q square. These are known as the well losses and since both are proportional to Q square we can write that the well loss is proportional to Q square or we can write s_w some constant into Q plus another constant into Q square where this C_2 Q square represents the well loss and we want the well loss to be as small as possible so that the well is considered more productive.

To find out these constants C_1 and C_2 we can pump the well at different rates and note down the drawdown at the well and we can write this also as which indicates that if you plot s_w over Q versus Q we would get a straight line and the intercept will us give C_1, the slope will give us C_2. If we pump the well at different rates, note down s_w for different Q values we can compute s_w over Q and plot it against Q and get the values of C_1 and C_2. We want C_2 to be small so that the losses are minimum.

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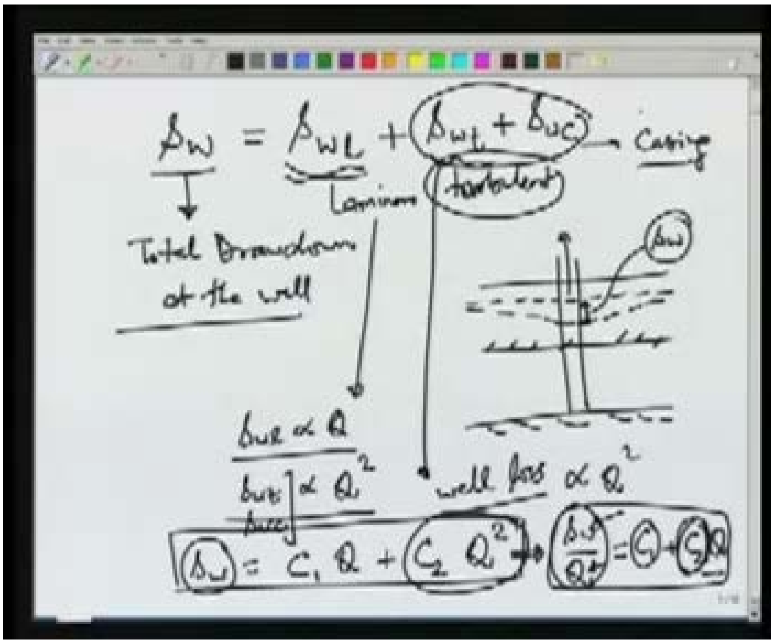

If well losses are large for example if C_2 is large, large C_2 will indicate inefficient well and if we plot s_w versus Q we would get a straight line with the coefficient C_1 and the slope as C_2. This method gives us a way of finding C_1, C_2 and we see that if C_2 is large the well is inefficient. So we want to have a small C_2 for the well.

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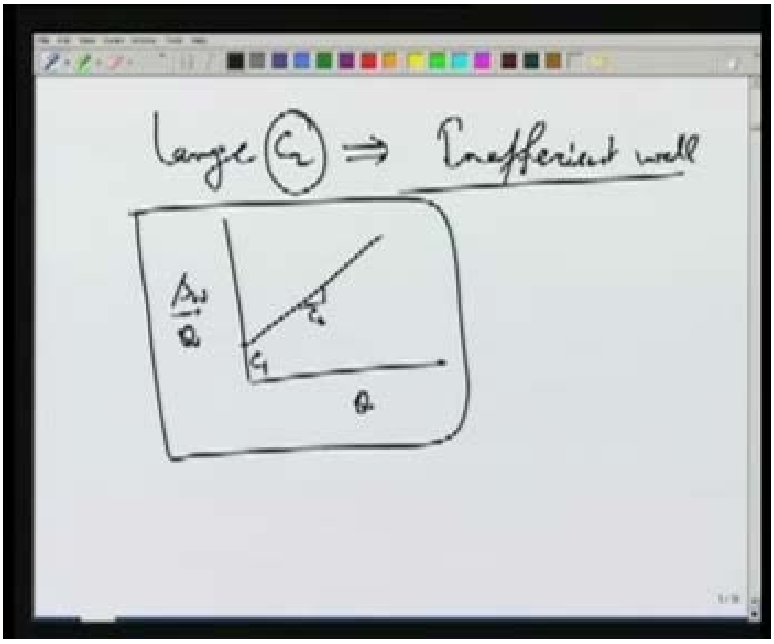

If the C_2 is very large then the well may be inefficient and we may have to look for replacements. The other concept which is related to the well loss is known as the specific capacity and this specific capacity is defined as the discharge per unit drawdown. We want large discharge and small drawdown. The specific capacity of the well should be high so that it is efficient. If we will look at the equations we have already seen that we can express this drawdown as Q over 2 pi T log of R over r_w where R is the radius of the influence, r_w is the radius of the well and T is the transmissivity. We can see that specific capacity is proportional to the transmissivity from this equation.

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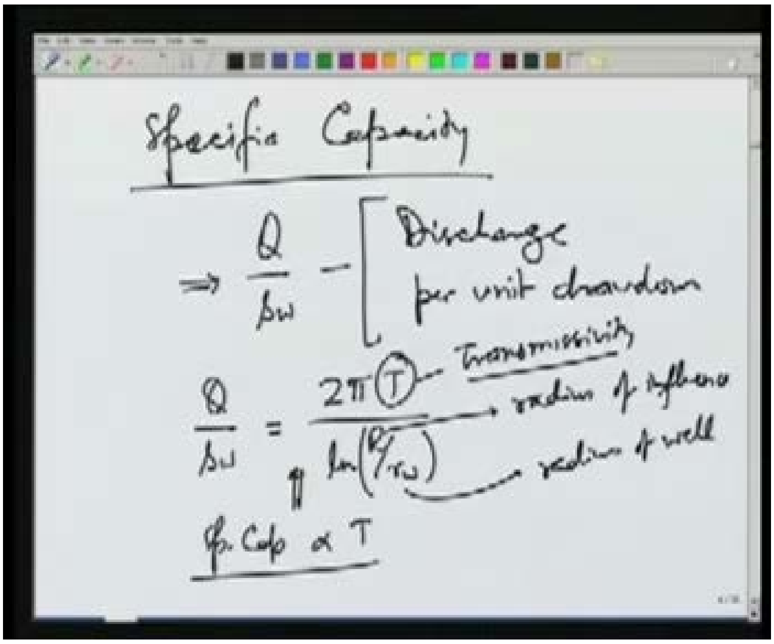

If the flow is transient then we have this equation using the Theis equation and the Cooper Jacob approximation including the well loss. If you look at the previous equation there was no well loss term included here. We have assumed ideal conditions here without any well loss, steady state conditions. But if the flow is transient and there is well loss then we have to include this C_2 Q term also and this is using the Jacob approximation for well function. Under transient conditions the specific capacity which is Q over s_w will be given by this equation and we can see that specific capacity decreases with increase in Q and t because if you increase Q this term will increase if you increase T this term will increase and specific capacity will decrease. Any well if we pump at higher rate specific capacity will be smaller. If we pump it for a larger time the specific capacity will decrease.

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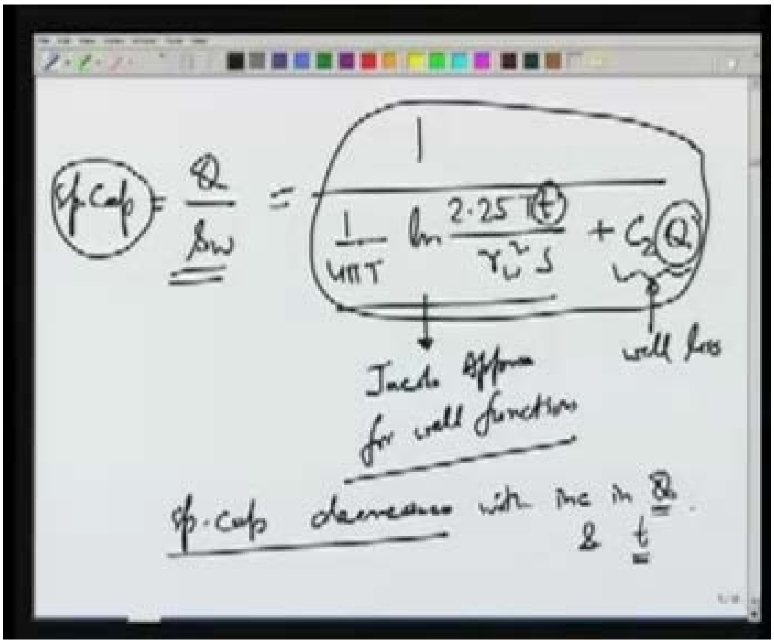

Now let’s look at some of the conditions which occur in the field. Till now we have discussed most of the things assuming that prefer to be homogeneous. That means there is no variation of conductivity with distance. But most of the times when we see natural porous medium there will be layering. If this is the bed rock then there may be a layer of material here. Depending on how the porous medium is formed there would be layers of different materials which will have different conductivities. This may be a thickness B_1 and conductivity K_1; similarly there will be another layer which may be of thickness B_2 and conductivity K_2.

Under these conditions we want to look at how to obtain an effective conductivity for this area. We can assume or we can replace this by an equivalent porous medium?. Let’s say that there is an impermeable layer here also. Then what we would like to do is we would like to replace this porous medium by an equivalent homogeneous porous medium of height B. To find out this K equivalent it depends on whether the flow is taking place along the layers or perpendicular to the layers. There may be some recharge from the top. Then the flow will be perpendicular to the layers and if there is natural ground water flow, horizontal direction, it will be along the layers.

Under both these conditions the equivalent conductivities are different. In this case the discharge Q_1, Q_2 and Q_n they would be added up to get the total discharge. But the head loss from one point to the other point would be same for all the layers. In the case of flow being perpendicular to the layers the discharge is same through all the layers Q but the head loss in each layer will be different and has to be added to get the total head loss. Doing these computations we can obtain the equivalent conductivity for this case. In this case the equivalent conductivity can be given by the arithmetic mean and then the harmonic mean. These two can be used to obtain the equivalent conductivity and then we can treat the porous medium as homogeneous with that equivalent conductivity and the thickness B, B is nothing but sigma B_i and i will go from 1 to n where n is the number of layers.

After looking at these flow situations where the flow may be steady, unsteady we may be having a one dimensional flow or an ideal flow towards the well. We may have layering and non-homogeneous conductivity. We have looked at all these true situations. Now we will take some examples to explain some of these concepts. We will take the example of a confined aquifer and unsteady flow through the confined aquifer. The data which is assumed is that there is a discharge of 2 meter cube per minute. So we have a confined aquifer being pumped at this rate Q. The transmissivity of the aquifer is given as 1 meter square per minute storativity as 0.0001. There is an observation well which is 50 meters from the pumping well and the water level in this observation well is being monitored. So r is 50 meters. Using this data we want to find out the water level in this piezometer. Initially there is some water level. So the piezometer will show the same water level. When we start pumping with time this water level will be going down. We want to see how this water level goes down and that we can do using the Theis method.

Suppose we want to find out what will be the drawdown at 1 minute after the pumping starts. If we take T equal to 1 minute we know that the equation for the drawdown is the drawdown Q over 4 pi T times the W u, well function and u is r square S over 4 T t. In this case suppose we take r equal to 50 meter; that is what we want. r square will be 2500 S is 10 to the power -4, 4T is 1 and t is also 1 minute. We could have used seconds also but in the drawdown conditions minutes are preferred. We will use the minutes units here and for discharge meter cube, for transmissivity meter square but the time units we will be using as minutes. If we do this u which is a dimensionless quantity will come out to be 0.0625 and corresponding to this u we can look at the well function tables or there are some formula which give us the well function for any given value of u and using that we can see that W u is equal to 2.26. Putting the value of W u here and the value of Q as 2, T as 1 we can get S equal to 0.36 meters which is what is shown here. For 1 minute we have 0.36 meter of drawdown. Similarly you can see that at the end of the measurement which is 240 minutes or 4 hours there is a 1.22 meters drawdown in the observation well which is 50 meters away from the pumping well.

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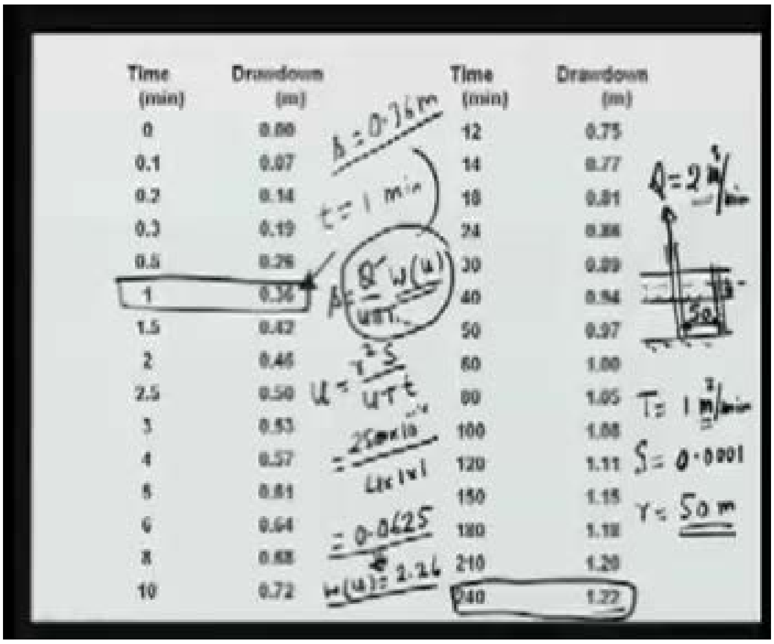

Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
0	0.00	12	0.75
0.1	0.07	14	0.77
0.2	0.14	18	0.81
0.3	0.19	24	0.86
0.5	0.26	30	0.89
1	0.36	40	0.94
1.5	0.42	50	0.97
2	0.46	60	1.00
2.5	0.50	80	1.05
3	0.53	100	1.08
4	0.57	120	1.11
5	0.61	150	1.15
6	0.64	180	1.18
8	0.68	210	1.20
10	0.72	240	1.22

This data is shown in the plot here. We have the time here in minutes and S is in meters. This shows the time Vs the drawdown curve at the observation well which is 50 meters away from the pumping well and we can see that initially the drawdown is increasing very fast but then as time progresses the rate of increase becomes slow. It will not reach a steady state because the Theis equation assumes that the aquifer is infinite. Slowly the ….. (17:52) will increase and there is no limit to the increase of the ….. (17:56). This will not achieve a steady state. It will continue to increase but at a very slow rate. This data which we have computed based on the Theis curve, most of the times what we will not have is T and S. We will not know the values of t and S but what we will have in the field is the time versus drawdown data or the time versus drawdown curve which is this curve.

The second question which comes to the mind is if this curve is given to us given S versus t curve how to find t and S? This is an inverse problem or a parameter estimation problem in which from a given drawdown data we want to estimate the transmissivity and the storage coefficient.

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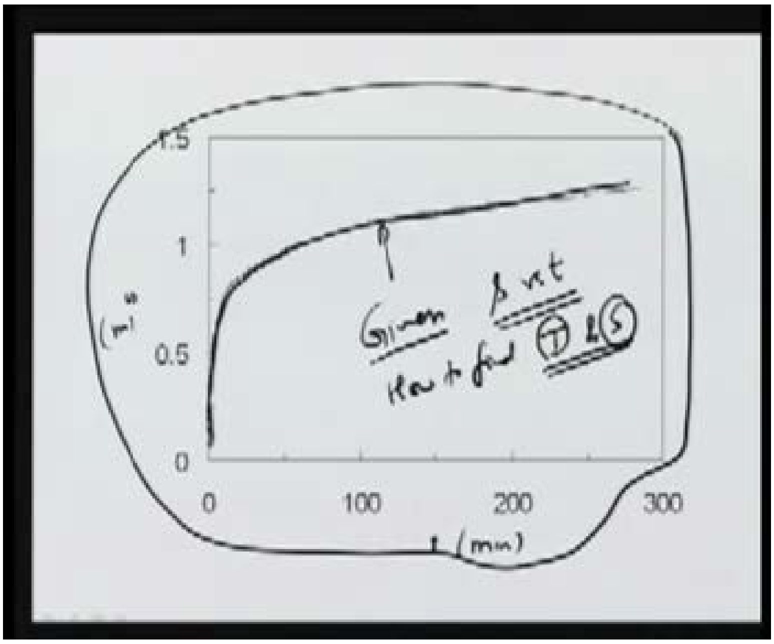

One of the methods which we have seen is known as Jacob's method or Cooper Jacob method in which we say that for very large value of time, large t, the well function can be approximated by the first two terms and then what we do is on a log plot we fit a straight line through the data corresponding to large time and extend that straight line up to the zero point, the drawdown equal to zero point and we call this t_0. In this case we see that t_0 comes out to be about 0.11. The other thing which we need in Jacob's method is the drawdown per unit log cycle of time. So we can look at 10 and 100. We can take any log cycle but let's take this and denote this by delta s. Delta s is about 0.36 meters. That is the difference between this point and this point which has one cycle of log here. We could have taken 1 and 10 also and get the same value here or here. Using the data on delta s and t_0 the Cooper Jacob's method has the equations 4 pi delta s and S …. (21:17). as we have seen Q is 2; delta s is 0.36 and this gives us a transmissivity of 1.02 meter square per minute which is very close to the transmissivity 1 meter square per minute which we had assumed to generate this data. This method works quite well for estimating the value of T.

Similarly once the T is known we can put back T here. t_0 is obtained as 0.11 minute and r is 50. So S will come out to be 1.01again very close to the value 1 into 10 to the power of -4 which we had taken to generate the data. The Jacob's method works very well for cases where we have a straight line portion available to us or where the measurements have been carried for large time. If we don't have this large time data available suppose we had only this data available to us then we would not be able to fit a line here or even if we fit the line it will not be a correct straight line. Jacob's method will not work if the the experiment is not carried out for a long time and data is not available for very large time. This straight line portion has to be available in order for Jacob's method to be applicable.

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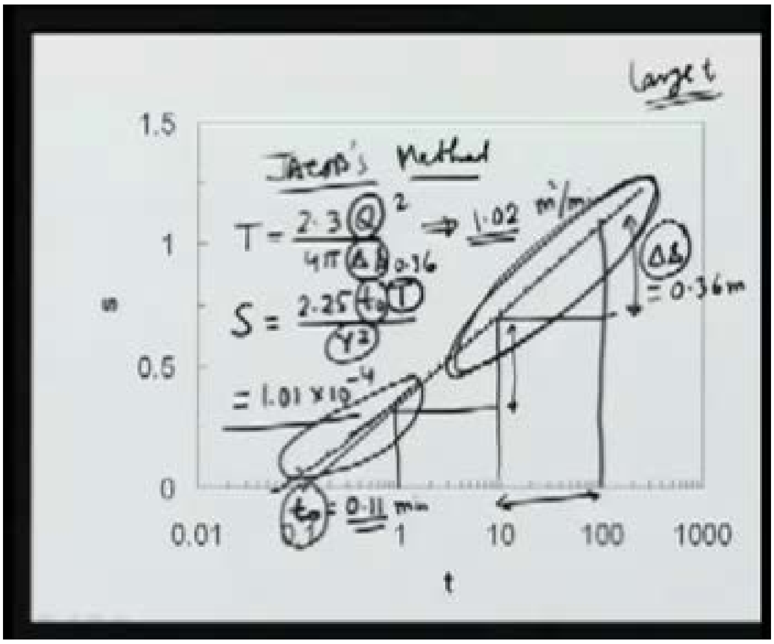

What we do mostly is use the Theis curve fitting method or the type curve matching method in which we prepare a generic plot. This is a generic plot, generic curve or the type curve which plots 1 over u versus W u. 1 over u is equivalent to t and W u as we have also seen is equivalent to s. On a log log scale if we plot 1 over u versus W u it should be similar to the behavior of the drawdown curve. This should be similar to s versus t curve. The idea of Theis curve matching as we have discussed is trying to match this curve, the type curve with the actual data.

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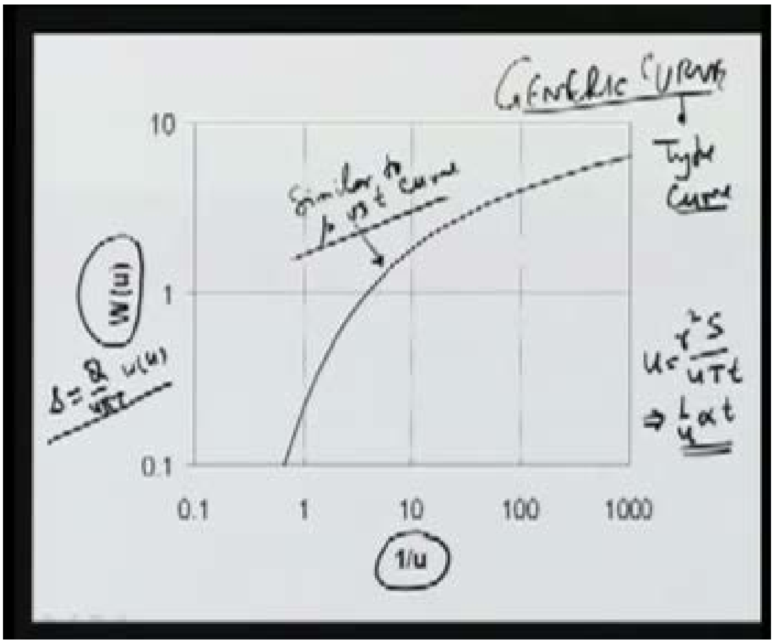

The actual data which we have plotted on log-log scale is the same data which we had earlier but earlier we had plotted on simple scale. Now we plot it on log-log scale. This s is again in meters, t is in minutes and now this curve should match with this curve.

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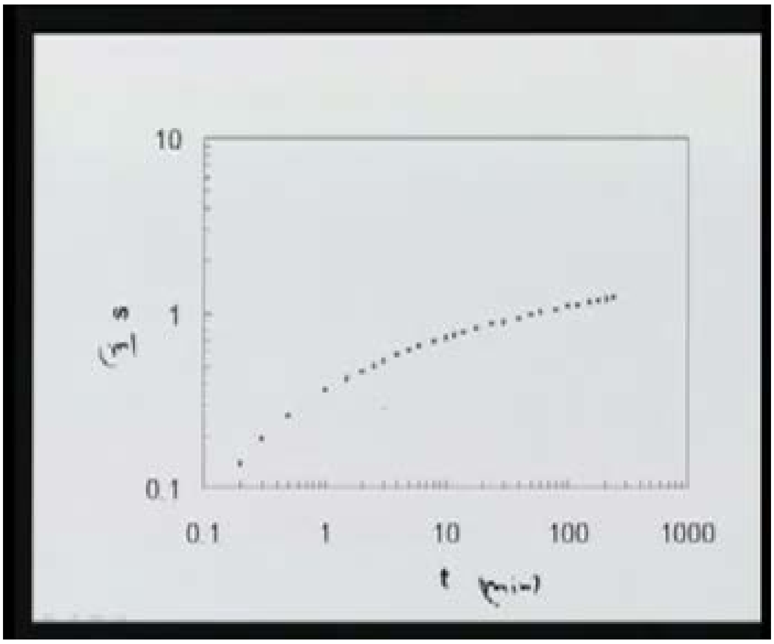

As you can see both curves will not match if we put it as it is. We have to shift the axis in order to make a match and that is what is done here. This is the drawdown curve, the symbols, and the line shows the type curve. We shift the drawdown curve or the type curve. The axis should be parallel. We shift in such a way that all the drawdown curve data matches with the type curve and by this shifting when we get the perfect match then we note the ordinates which match. For some W u we find out corresponding s and for some value of the 1 over u we find corresponding t and then we use these values to estimate the transmissivity and the storage coefficient. If we match it suppose we take 1 by u as 10 the corresponding t star which is the point which corresponds to 1 over u equal to 10 is about 0.6.

Similarly we can match W u equal to 1; so W star equal to 1 and corresponding value of s comes out to be 0.16 from this figure. This is s star, this is t star, this is one over u star and this is W star. Using these four values of the star variables we can use the Theis curve matching equations to obtain the transmissivity. The equations which are given T equal to Q over 4 pi W star over s star and S equal to 4 pi t star u star over r square. W star here is 1 s star is 0.16. Putting these values we get T equal to 0.995. Again this value is very close to 1 and s comes out to be 9.6 into 10 to the power of -5. This is also very close to 1 into 10 to the power of -4 which we had obtained or which we had used to generate this data. Type curve matching method involves some subjectivity. In this case you can see that the fit is very good here. The data points all lie exactly on the type curve. Because the data which we have generated is synthetic there are no errors. In general the data would not lie exactly on the line. It may have some values like this. So we have to adjust the best fit. Once we get the best fit W star, s star, u star and t star can be obtained and they will give us the value of the transmissivity and the storage coefficient using this method.

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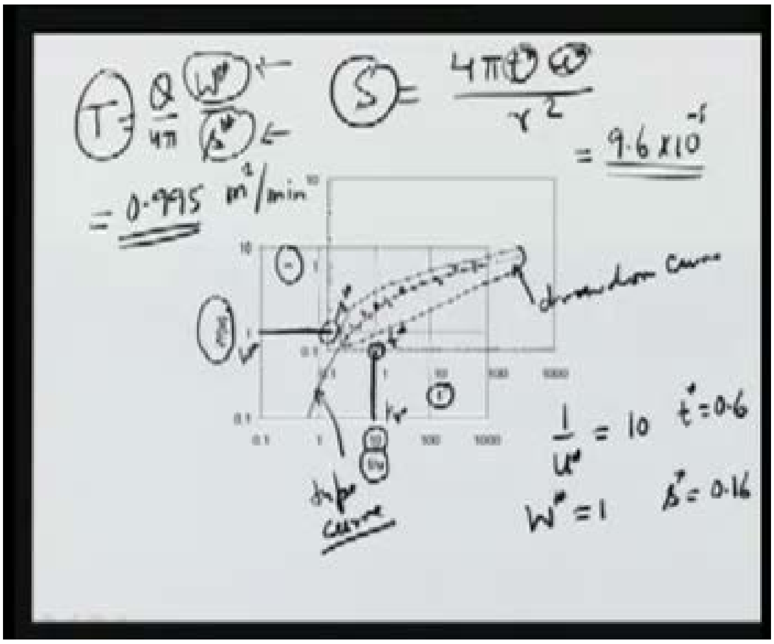

Now let's look at the first slide which showed us time versus drawdown for values of Q equal to 2, T equal to 1, S equal to 0.0001 and r equal to 50. Now we can see what will be the effect of changing T and S? We can generate a new data set which includes T value which is one tenth and s value which is 10 times. So the value which we use for T and S now our T equal to 0.1 and S equal to 0.001. Using these values we can regenerate the data using the Theis curve and you can see that this figure shows time versus drawdown. Earlier for a time of 1 minute we had a drawdown of 0.36 but here you can see that there is almost no drawdown till 1 minute. But at the end we had a drawdown of about 1.22 meter. Here we have a drawdown about 4.93 meters.

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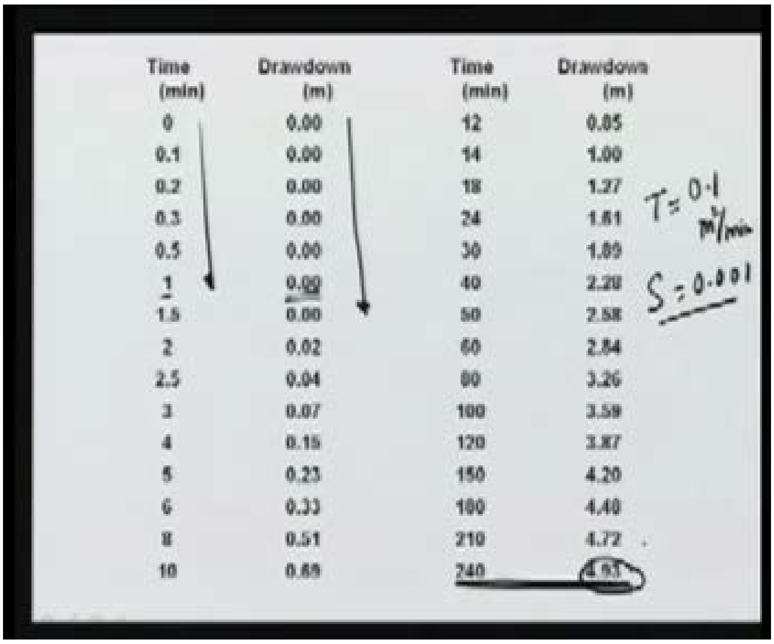

Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
0	0.00	12	0.85
0.1	0.00	14	1.00
0.2	0.00	18	1.27
0.3	0.00	24	1.61
0.5	0.00	30	1.89
1	0.00	40	2.20
1.5	0.00	50	2.58
2	0.02	60	2.84
2.5	0.04	80	3.26
3	0.07	100	3.59
4	0.15	120	3.87
5	0.23	150	4.20
6	0.33	180	4.40
8	0.51	210	4.72
10	0.69	240	4.93

So changing T and S will affect the rate of drawdown or the shape of the drawdown curve and if we plot the drawdown curve it looks like this. We can see that the earlier one had a shape like this. This one has the shape which is much more steeper. So again we can use the Theis curve matching technique to match this observed drawdown curve with the type curve and in this case the shifting will be different than the previous case where the curve was like this

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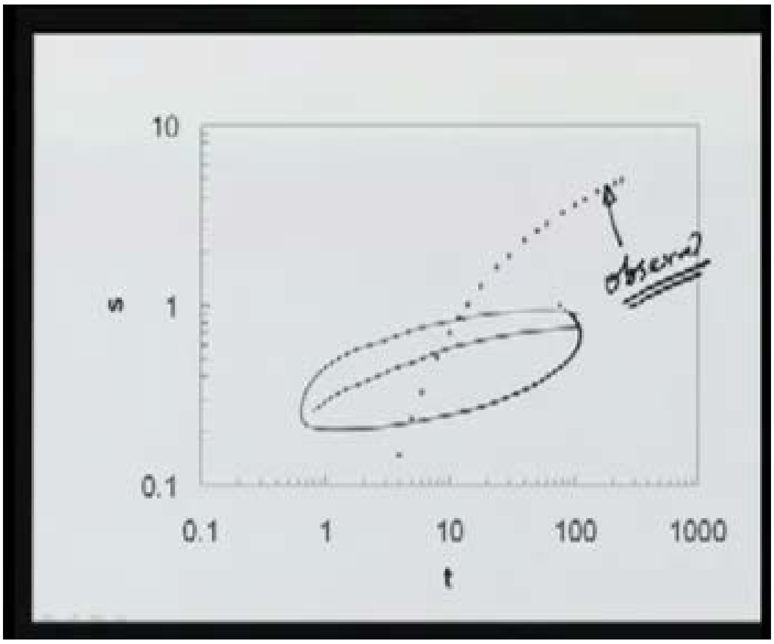

Here you can see again that the data now plots in this portion. Again a very good match is being observed. We can again obtain the same. This 1 over u is equal to 1 and it corresponds to t of t star of 7. Similarly W also we can take as 1 and it will correspond to s star. Using this data we can again obtain T. In this case T is obtained as 0.094 meter square per minute compared to the value which we had used as 0.1 and S is obtained as 1.05 into 10 to the power -3 compared to the 1 into 10 to the power of -3 which we had used to generate this data. Different values of T and S can be obtained using the type curve matching. Only thing is that the axis will have to be shifted little different for different kinds of data and the match depends on our judgment little bit where to match this. Similarly since it is a graphical method the reading of t star and s star would also be a little approximate. For example the value here and the value here there may be some approximation involved in reading these values which may cause some deviation of the values.

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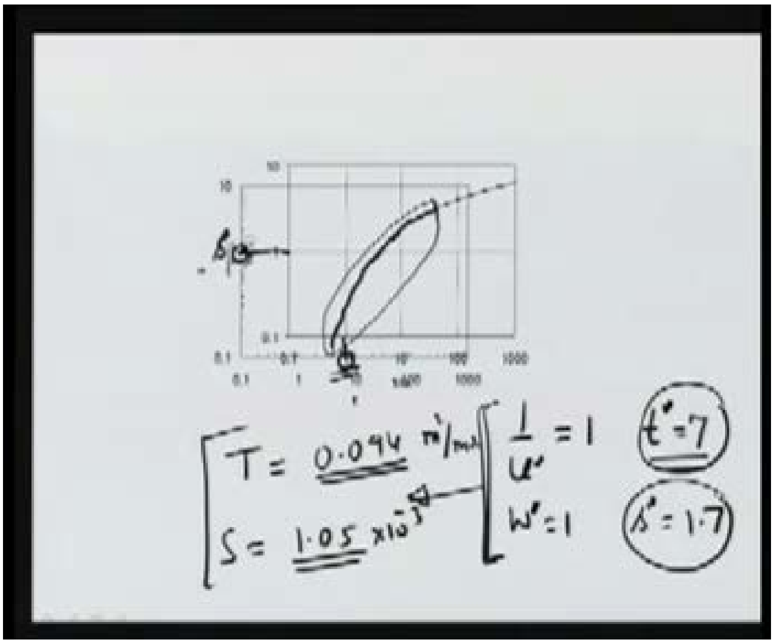

For example this shows about 6% error, this shows about 5% error but for most ground water practical cases 5%, 6% error is tolerable. So this method can be used. These days there are a lot of other methods which are proposed. For example we can use computational methods to try different values of S and T and generate the drawdown data try to match it with the observed drawdown data and then try to minimize the error by choosing optimum values of S and T. Computational methods are becoming more popular these days but graphical methods are still very good for first approximate guess.

Now let’s look at another example. This data we had generated for r equal to 50 meters or the first slide showed the data which had T equal to 1, S equal to 0.0001 but again at r equal to 50 meters. Now what happens if we have two different wells? Suppose we have the confined aquifer being pumped and then we take the observations in two different observation wells. One may be let’s say r equal to 50 meters the other may be let’s say 100 meters. The 50 meter well will show a larger drawdown; 100 meter well will show a smaller drawdown. This curve shows T again in minutes and S in meters. The drawdown corresponds to 100 meter well and 50 meter well. We can see from this figure that these open circles which represent the 50 meter drawdown are higher than 100 meter. The drawdown at the 50 meter well would be higher than that at 100 meter well and these two are shown here.

Since these drawdown curves are different we can estimate the T and S values either separately for both of them or there is another method which uses combined data and if you look at the definition of u then we can see that this r square over t term occurs in the definition of u. Instead of plotting t versus s if we plot t over r square versus s then both of these curves may come at the same location. That is what is shown here; again the filled circles are 100 meters drawdown and the open circles are 50 meters drawdown. We can see also that at the end of 4 hours which is 240 minutes the drawdown in the well at 100 meter is about 1 meter and the drawdown in the well at 50 meters is about 1.2 meters.

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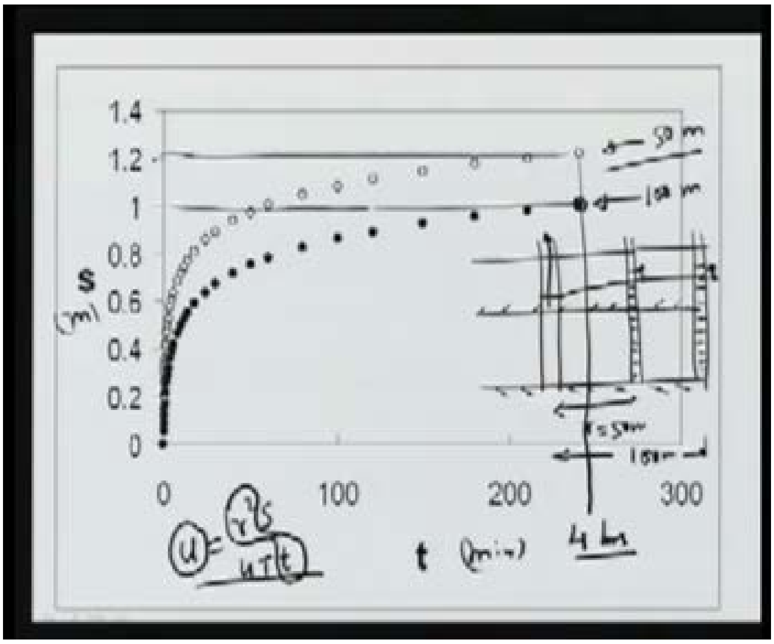

The combined curve shows the same thing. The data for t over r square for 100 meter well stops here which is 1 meter drawdown. The data for the 50 meter curve goes up to 1.2 meter drawdown. t over r square what we are doing is kind of non-dimensionalizing the time with respect to the distance. In t over r square versus S curve S is in meters; t over r square here is minute per meter square.

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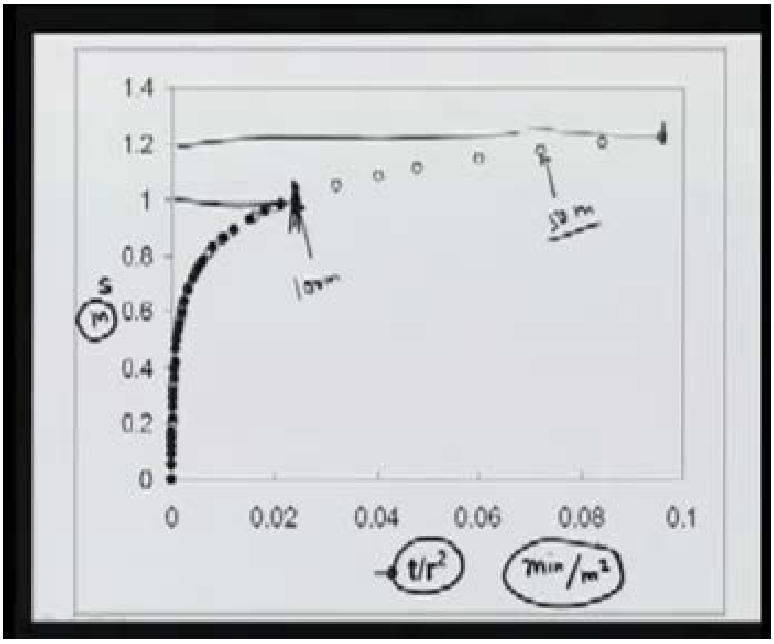

The two curves are quite different here. These two are quite different here but when we make it non-dimensional with respect to r square then we get the same curve and now we can use this single curve for curve matching or the type curve fitting and we can see that the data fits again very well. Here these again are open circles; these are the 50 meter well. The filled circles represent the 100 meter well. This is a type curve and this match again shows a very good matching within the data and the type curve. The procedure is exactly the same. Let’s say that we take 1 over u star equal to 1 which is this line and it gives us t by r square equal to 2.4 into 10 to the power -5 which is corresponding to this point. The scales are shown on this line. So I have taken it here. Similarly I will write this also as star; t over r square star. For W star we can take again 1 which is this and it corresponds to s star of 0.16. This is 0.1; this is 1. So this point is 0.16 and using these now we have T=0.995 and S. Again these two are very close to the values which we had used to generate the curve for t over r square versus S. This was used as 1 again this was used as 1 into 10 to the power -4. So this has 2% error. This has 0.5% error which is quite good. The only thing which needs to be mentioned here is that S is given as 4 pi t star u star over r square. Now instead of t star we will have t star over r square directly substituted from this value.

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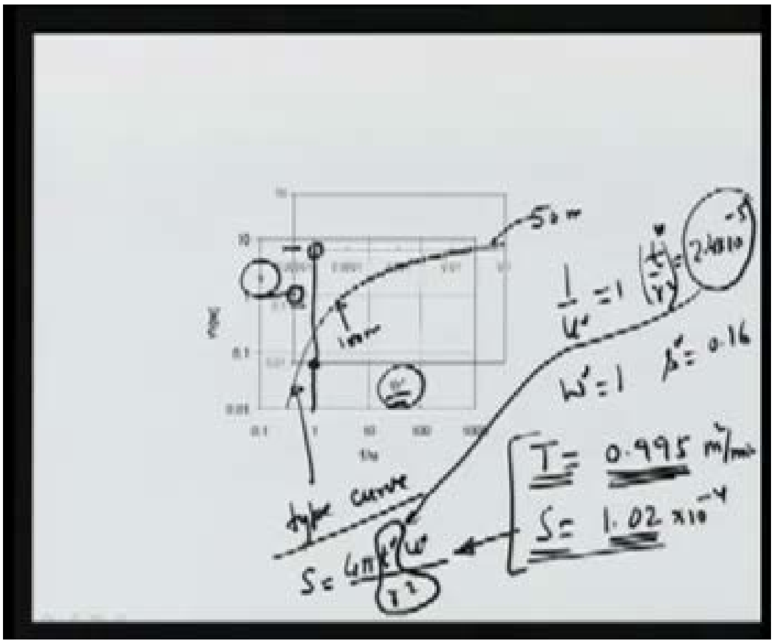

Using the Theis curve we can estimate the value of S and T for any confined aquifer. Most of the times we will have data for one well or some times more than one well and we can combine them using the t over r square to get all the data on a single curve and then match with the type curve. Cooper Jacob’s method can be used if the data is available for a longer time or as the dimensionless parameter u should be small. If it’s not available then we will have to look for other methods; for example the graphical method of curve matching or numerical methods of parameter estimation.

There are some other advanced methods also. For example slope matching or the revertive matching methods but we will not discuss them here. This method tells us how to estimate the ….. (41:15) parameters if we have data available for a well. Sometimes data from wells may not be available or there may not be a pumping well. Sometimes we may have just a few observation wells in aquifer. We can look at a technique to estimate the gradient and then knowing the conductivity how to estimate the ground water velocity or if we know the ground water velocity we can estimate the hydraulic conductivity. Suppose we have the ground level, impermeable bed and then there is this water table which is at a certain gradient so that the flow occurs because of this gradient. If we have observation wells here we can note down the ground level here and the ground water table at this point. This will give us an idea about the ground water level at this point. What we will have is elevation at the ground level. This data is easily available and the other measurement will be depth to ground water.

If we have another well here and we also have the same data available here like ground level as well as depth of water table this will give us some idea about the gradient here if we know that the flow is in this direction. But a number of times it is not possible for example if you look at the top view there may be some area here where there are wells here, here and here and we don't have any idea about the direction of flow of the ground water. It may be flowing like this, it may be flowing like this or it may be flowing like this. Some idea about this movement may be obtained by noting down the ground water elevation here. If this is higher elevation and these are lower then we know that the flow will be towards this side but exact direction of flow will not be known.

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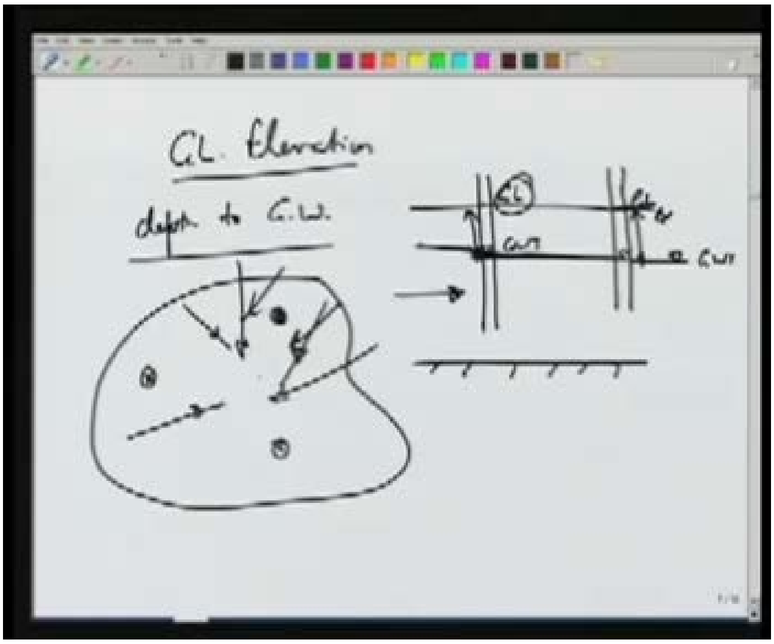

For this case if we have three different observation wells and we have the measurement of ground water elevation and depth to ground water at the three wells we can estimate roughly what is the direction of flow and if we know the hydraulic conductivity, K we can also find out or estimate what is the velocity of ground water movement or if we some how measure the ground water velocity then we can estimate the hydraulic conductivity.

In this case what we do is let's say that we have one well here and other well here and other well here and let's say we have measured at the wells A, B and C the ground water elevation by measurement of the ground level and the depth of water. Let's say that at A, ground level is 105 meters and depth to ground water let's call it 4.6 meters. If we have these two data available to us we know that the ground water elevation, ground water level at A would be nothing but the difference of these two and it will be 100.4 meter. So at A we have ground water elevation which is of 100.4 meter and similarly at B we have all the data available and at C also and let's say that at B we have an elevation of 101 meter. This is all ground water elevation; so these are all ground water levels and at C let's say that the elevation is 100 meters. If we look at these three elevations 101 here, 100 here, 101.4 here we know that the ground water flow will be from the higher head to the lower head. But it may be in this direction, it may be in this direction or it may be in this direction.

In order to find out or estimate roughly what is the direction we can do a linear interpolation. For example this is from 100 to 101 so we can estimate where on this line would be 100.4. If we divide this into 5 parts then this would roughly be 100.4 and that means the line joining A with this point will roughly be a line where the ground water elevation is 100.4 and then we can draw lines parallel to this from all the points. They will represent ground water elevation of 100, 100.2, 100.4, 100.6 and 100.8. Knowing these ground water contours we can say that the ground water flow direction would be perpendicular to this and we can make a rough estimate of the gradient by measuring this length and saying that the head loss delta h is 0.2 meter in a length of L. So delta h over L will be 0.2 over L. Knowing the conductivity we can say the velocity will be Ki and knowing the conductivity and knowing the i we can estimate what will be the velocity of ground water or if somehow we have obtained the velocity of the ground water, if we know this we can estimate the hydraulic conductivity.

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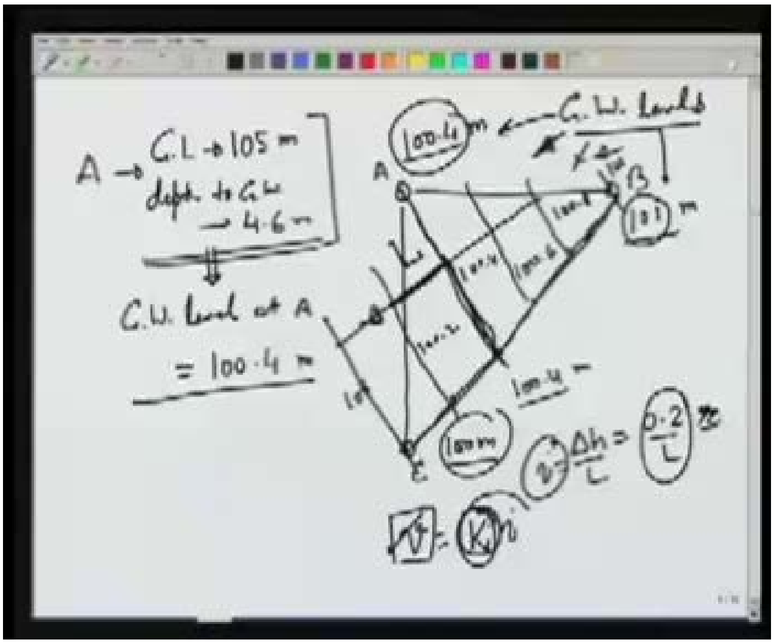

This will give us an estimate of the conductivity if we don't have a pumping well and we have some three observation wells which measure the ground water elevation.

In the ground water chapter we have looked at various flow situations. We have looked at Darcy's Law which relates the conductivity gradient and the velocity. We have also seen what happens when flow occurs between two parallel bodies of water one dimensional flow or sometimes we have a pumping well which will cause radial flow. We have looked at confined aquifers and unconfined aquifers in which the behavior of or the distribution of the head is very different from each other. In the confined aquifer it's the piezometric head which changes but the area of flow remains constant. In the unconfined aquifer the area of flow also depends on the head in the aquifer and therefore it becomes more complicated. We have looked at unsteady state flow towards the well in confined aquifers and we have looked at steady state flow in both confined and unconfined aquifers. We have seen some methods which are used to predict the drawdown for known aquifer properties and pumping conditions and we have also looked at inverse problem in which we estimate the aquifer parameters from the measured data of drawdown versus distance or time and for a given pumping rate. For any aquifer we can design an experiment with different pumping rates to find out the aquifer properties. Similarly we have looked at the well loss, how to estimate the specific capacity of a well, how to estimate the C_1 and C_2 constants in the well loss term and how to obtain or how to estimate whether the well is efficient or not and when it should be replaced.