Water Resources Engineering

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Lecture No. 18

In the previous few lectures we have seen about the floods, how to estimate the magnitude and how to obtain the value of discharge at different times at a particular location in catchment area. Once the flood is obtained at the outlet from a catchment area and we want to predict the flood at a downstream location we will have to find out how this input gets translated into the output at the downstream point and we use for this purpose what is known as routing. The flood routing is an important subject because it will translate any input at one point to the flood output at a downstream point by taking care of the storage resistance and other factors. Today we will discuss some aspects of flood routing. Flood routing simply means that we route the flood from one point to some point downstream. We translate the flood and obtain temporal distribution; how the discharge is changing with time of discharge flow, generally in meter cube per second, at downstream points. If we have a channel and we know the temporal distribution of discharge let's call it the inflow and at this point we want to obtain temporal distribution of the outflow which we can call Q. How will it will it be? Will it be like this or will it be like this? The purpose of flood routing is to obtain or estimate this distribution of Q. Hyetograph at one or may be more points; there may be a tributary coming in here and the hyetograph in this tributary may also be known. At few points upstream of the point at which we want to find out the hyetograph using these points we can estimate the outflow at this point using this technique of flood routing.

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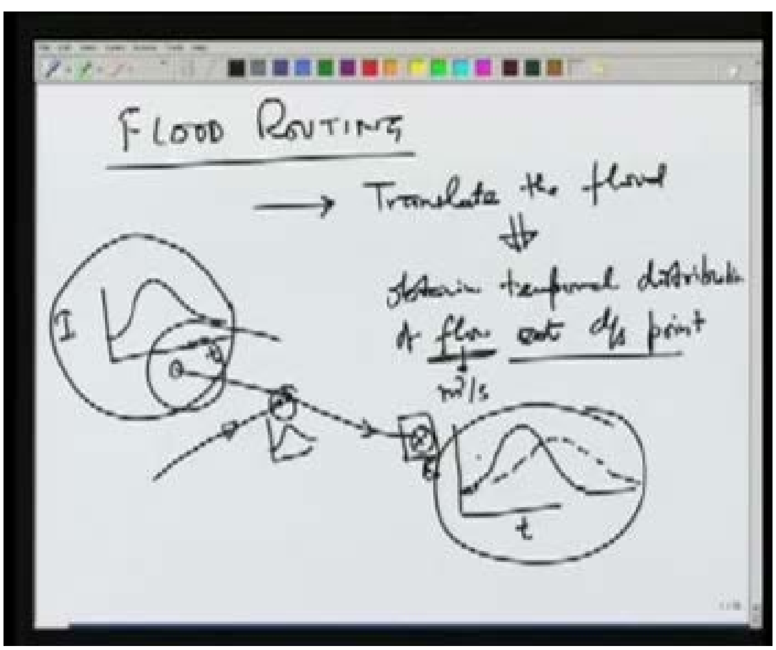

The basic techniques used in flood routing can be called either a lumped routing or distributed. In the lumped routing we obtain Q as a function of time at a given x. Again if we take some catchment area there may be tributaries entering this channel. For the tributaries suppose we know the hydrographs and all these hydrographs are routed to the outflow point. At this point if we find out outflow versus time then we are considering only one point at particular x. So given x would be this point and what we want is Q versus time. This could be a lumped routing. In distributed we find out Q as a function of x and t. At different times in a channel at different x values what will be Q? For this entire channel we can say that at any time what will be the Q at different locations?

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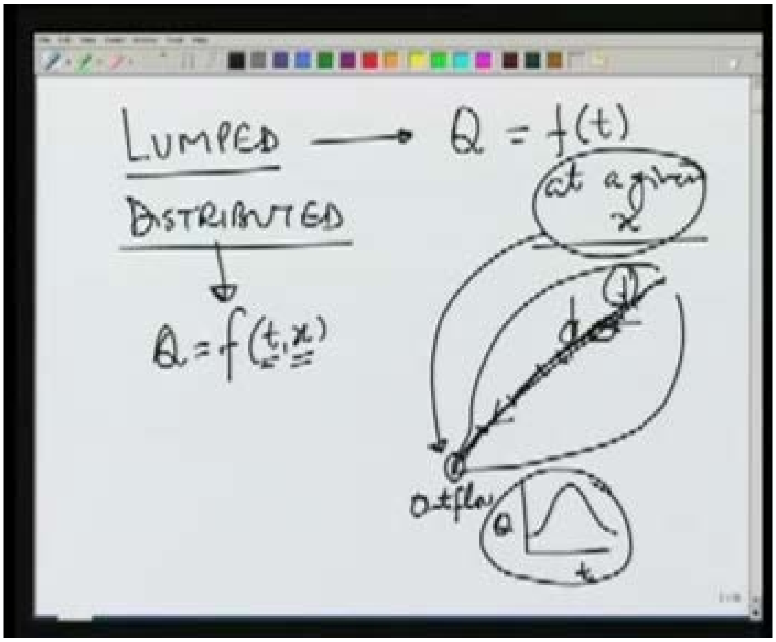

Lumped and distributed have the same purpose of routing the flood but they use different philosophies. In lumped routing which is also called hydrologic routing we use a continuity equation which is generally of the form inflow minus outflow equal to rate of change of storage. The lumped routing or the hydrologic routing uses the simple mass balance equation of inflow outflow and differentiating them with the change in storage. While in the distributed model which is also known as hydraulic routing we use a continuity equation and a momentum equation. If you are talking about a channel and the depth of flow is y at different times we can write a continuity equation which would be of the form and Q is a discharge; Q will be a function of x and t. The continuity equation can be written as ….. This is the change of discharge with distance; this denotes the net inflow or outflow into a particular control volume of very small (( )) (00:06:36) and then in the limit that the …… tends to zero we would get the differential equation. T is the top width. If a channel cross section is like this and water is flowing at this level then T would be the width at the top of this section y is the depth of flow and Q is the discharge.

This gives the continuity equation relating the net inflow with the change in control volume within this control volume. Momentum equation is obtained and written as del y by del x plus v by g del v by del x plus 1/g delv by del t is equal to S_0 minus S_f. S_0 is the bed slope of the channel. If the channel is here the slope of the bed will be S_0 and S_f is the friction slope.

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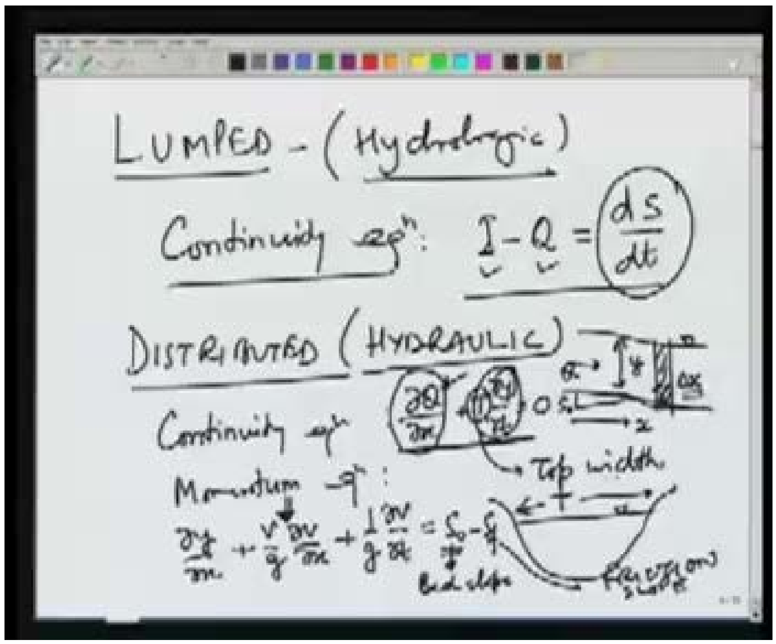

The friction slope represents the energy loss. If the channel bed is here water depth may be like this. The energy line may be going like this. This is the water level and this represents the total energy or energy line. Width is obtained by adding the velocity head.

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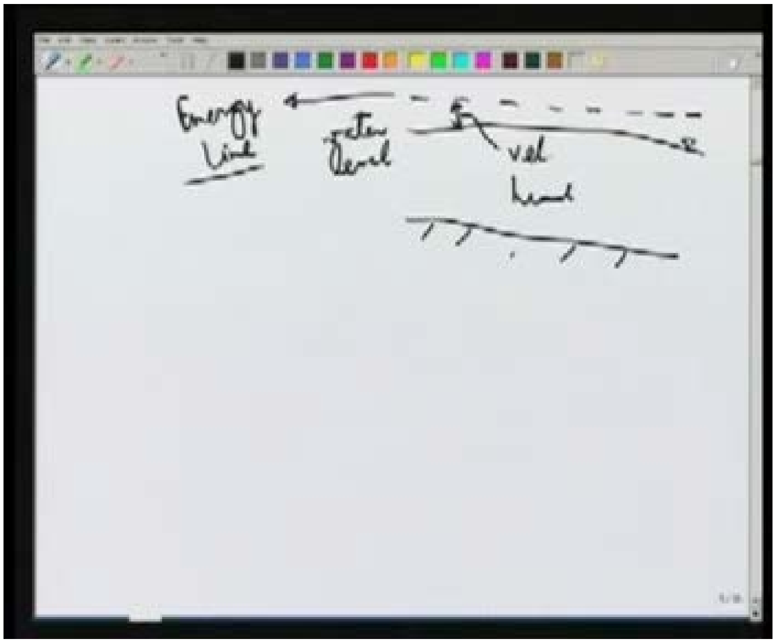

The slope of this line at any point is divided by S_f and this value of S_f is obtained from using equations like Manning's equation or Chezy's equation for open channel flow. In this equation S_0 and S_f are obtained. Bed slope is obtained from geometry of a channel. S_f is obtained from Manning's or Chezy's equation and then we can solve these two equations together to obtain the value of discharge at all values of x and at different times.

We will start with the lumped routing because it's easier. All the routings can be thought of in terms of two different kinds either storage routing or a channel routing. Storage routing is also called reservoir routing and as the name implies it denotes how a flood wave gets modified when it passes through some storage structure or a reservoir. Let's look at a case like this where we have some storage reservoir here. Water is stored up to certain level and there is some flow coming in. I(t) is the input and then the outflow can be written as say Qt and we want to find out how the outflow is changing with time. Storage in the reservoir lets denote it by S. S will depend on the height of flow in the reservoir. Let's call h from the top of this …… so that when h is zero there is no outflow. But we can relate this h. We can take the h from the bed of the channel also but for our purpose we will take h as the variable. So S will be a function of h. As we go higher the storage volume within the reservoir increases. So S will be a function of h like this where h equal to zero indicates the storage below this level and then it increases as we increase the h.

Similarly the outflow from the reservoir will also depend on h and generally …. uncontrolled …… here then Q is proportional to the three by two power of h and we can plot a curve like this showing the variation of Q with h. The equation which is used for lumped routing I-O, inflow minus outflow equal to dS by dt. This S will also be a function of h and O which we have denoted by Q here will also be a function of h. What we need to do is to solve this equation and see how outflow is changing with time.

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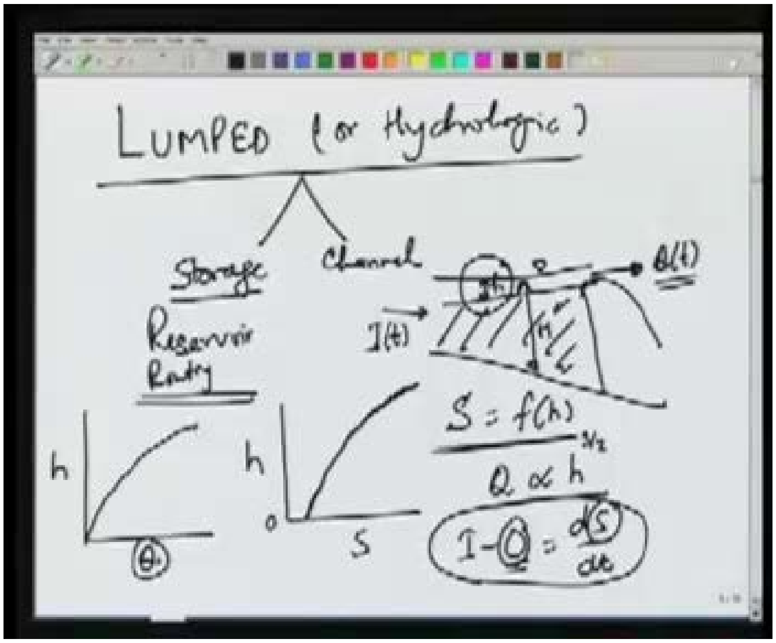

There are many methods of solving this. We start with a method which is known as level pool routing; level pool routing because we assume that reservoir level is horizontal. We will ignore any changes in the water level; any profile occurring like this we assume that the water level at all the times in the reservoir remains horizontal and this assumption will be true or approximately true most of the time because the slope of the water surface here will generally be small. Assuming this the reservoir routing or storage routing or this level pool routing we write the equation, the familiar mass balance equation as I-O equal to dS by dt and then we consider at small time interval delta t. Small interval is denoted by delta t and let's see what happens during this small interval, finite time interval of delta t. The inflow which comes in can be written as average inflow because I changes with time and during the finite time interval of delta t we can write I bar as the average inflow minus similarly we can write average outflow. Let's use the symbol Q for the outflow. Q bar multiplied by delta t will give us the change in storage delta S. In this equation I bar represents the average inflow during time period of delta t, Q bar average outflow during the time period and delta S is the change in storage during that finite time interval of delta t. We can write this I as I_1 plus I_2 by 2; similarly Q_1 plus Q_2 by 2 where I_1 and I_2 represent the inflow before the time period of delta t and after the time period of delta t.

If t versus I is known to us and suppose we have taken this delta t then this will be I_1 and this would be I_2. Delta t has to be small enough so that we can assume that this portion is more or less a straight line and this assumption of average flow being equal to I_1 plus I_2 by 2 will be valid and delta S will be the change in storage S_2 minus S_1.

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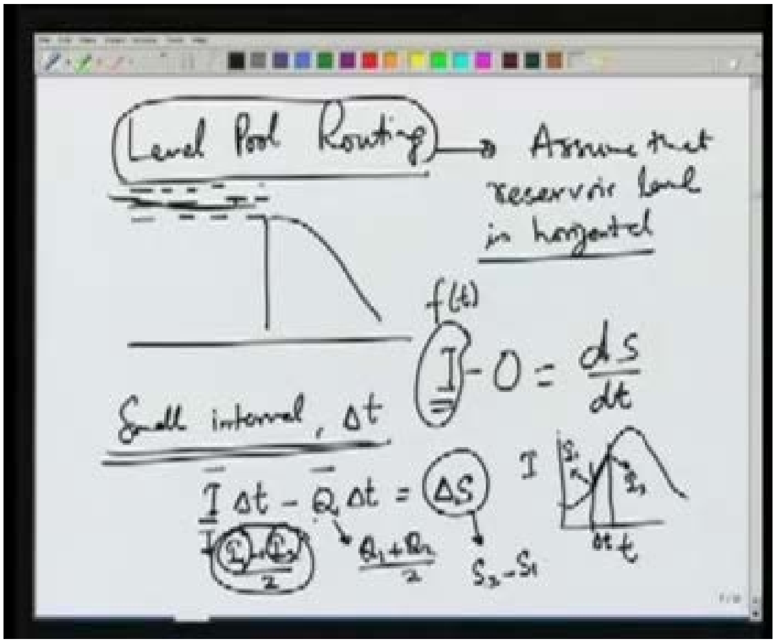

Using this equation now we would like to route the flow through the reservoir and see what kind of outflow we get for a given inflow? The information available to us would be the inflow curve. We also should know how the storage changes with h and how the outflow changes with h and let's plot the storage and similarly Q as function of h. If we have a storage reservoir here initially we should know what is the level? Let's call it h_0 and then the inflow comes and we want to route it through the reservoir. The inflow hydrograph is given as this. At time t equal to zero we know what is the inflow I_0? In fact the inflow hydrograph is known for all times. At any time we can find out I from this curve. If we take this time step we can use different methods to solve this equation. Let's write this equation again here but now in terms of unknown quantities on one side and the known quantities on the other side.

The same mass balance equation is now written in a slightly different form by putting all the unknown quantities; S_2 and Q_2 both of these are unknowns. I_1, S_1 and Q_1 are at the beginning of the time step. So these are known and I_2 will be known because this entire curve is given to us. The inflow hydrograph is completely known. Therefore I_2 will also be known to us. The problem now is that this S_2 and Q_2 both are functions of h. We cannot directly solve for this in an analytical form.

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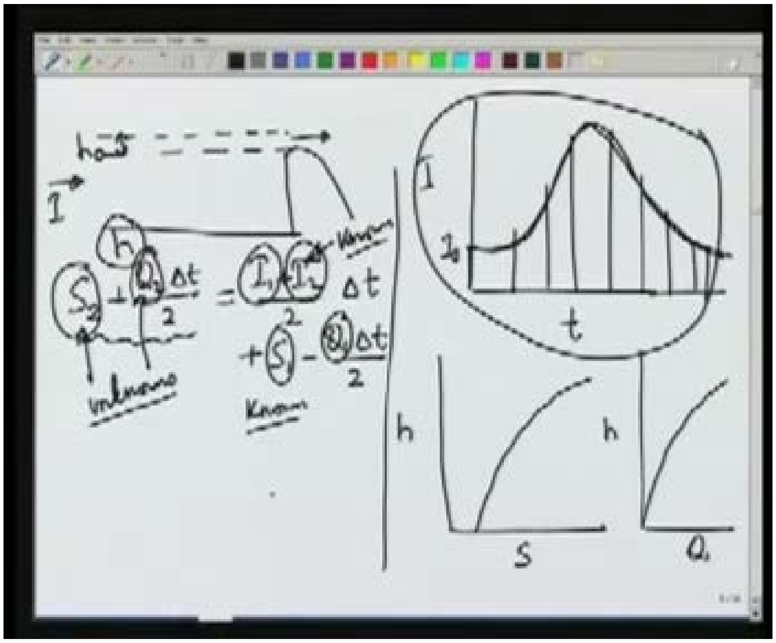

There are some graphical methods which are available for solving this equation. The commonly used method called the modified Pul's method looks at plotting the curve in a little different form so that we are plotting S plus Q delta t by 2 versus h. If you look at this equation the term S + Q delta t by 2 will be a function of h because both S and Q depend on h. The plot of S + Q delta t by 2 would look like this. On the same graph we can plot Q also. This will be on different scale; S + Q delta t by 2. Delta t we have to decide. For deciding delta t normally it is selected in such a way that the time to peak consist of about 3-5 delta t's. This time to peak the delta t should be roughly about 0.2 to 0.4 t. That is the general criteria for choosing delta t. Once we choose delta t then since we know both S and Q as functions of h plotting this curve is straight forward for a given value of delta t.

If you look at the equation all these quantities are known at the beginning of the time step. With these quantities we can find the sum of S and Q delta t by 2. We cannot find individually S_2 and Q_2 but the sum is known to us because of this equation. From that known value and from this curve we can directly find out h which gives us the head or the water depth at the end of interval delta t. On the same figure we have this plot of Q on this axis. This is Q and this is S plus Q delta t by 2. On the same graph we have this plot of Q and for this h we will know the value of Q.

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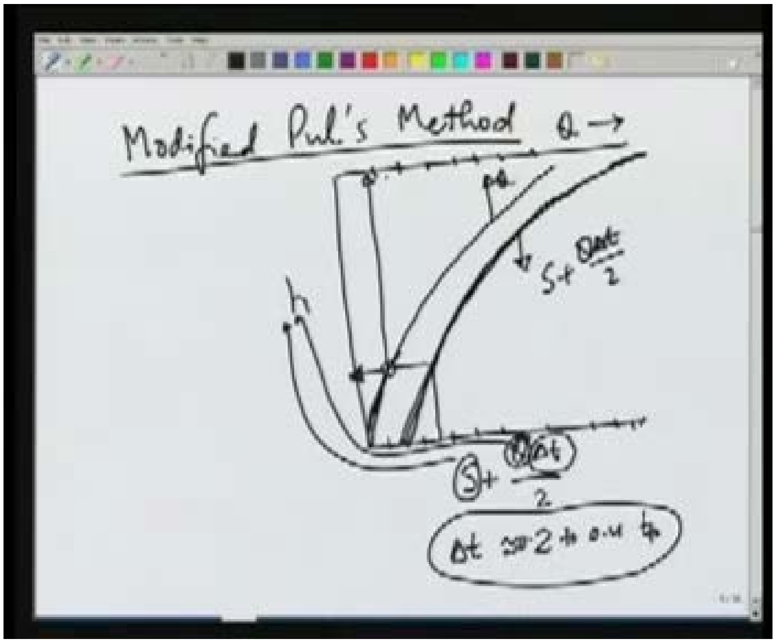

For the next time step where we need S minus Q delta t by 2 we can write this in terms of the known variable minus Q delta t. Knowing the value of Q from here we can find out Q delta t also at the end of the time step delta t and since S plus Q delta t by 2 is also known at the end from the graph we can subtract it and get the value of this variable at the beginning of the next time step. We solve for one step and then for the next step this will become the known value here and knowing the inflow we can again get the parameter S plus Q delta t by 2 at the end of time step and repeat this process till we route the whole inflow hydrograph through the reservoir. This modified Pul's method using this graphical procedure gives us the outflow.

If we have this as the inflow, outflow will look like this. This is t this is inflow and outflow where this shows the inflow and this shows the outflow. The thing to notice here is that till the intersection point the inflow is more than outflow and what it means is that the water is going into the storage or the storage is increasing and then beyond this point the outflow is more than inflow which means that water is being taken out of the storage and the volume of these two should be same because whatever additional water we have deposited into the storage here should come out during this time and after this the inflow and outflow will be same at the initial level. If they reach the initial level again then they will maintain at that initial level, their storage will return back to its original position. The two things which should be noted here is one is the change in peak discharge. This is called attenuation or sometimes peak attenuation means what is the reduction in the peak as the inflow passes through the reservoir and the other thing which can be seen from here is the lag. This is by how much time the peak has been delayed. These are the important implications in the sense that the flood at the inflow point is occurring at some point in time but it will have some lag as it passes through the storage reservoir. So the peak will be reduced and there will be lag also. There will be an additional time to peak which downstream locations will get. These two are important in the sense that how much peak is reduced and how much extra time we get will affect our preparation for protecting against flood on the downstream side. Storage reservoirs thus serve these two important purposes that they reduce the magnitude of the maximum flow or the peak and they also lag the peak discharge by some time.

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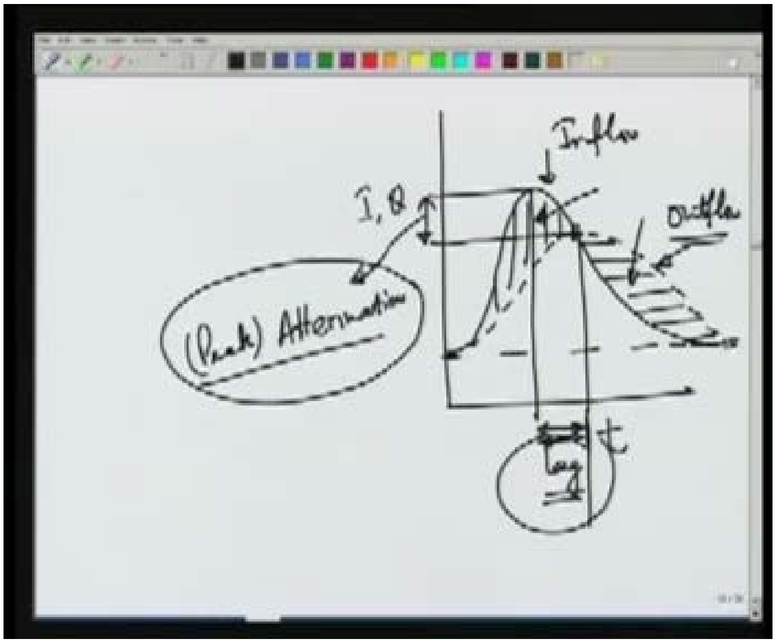

There is another method or sometimes what we do is we prepare another curve S minus Q delta t by 2. If you look at this equation here what we did was we related S minus Q delta t by 2 with S plus Q delta t by 2 and minus Q delta t. Every step we had to find this Q multiply by delta t and then subtract from S plus Q delta t by 2. Sometimes we can directly prepare this plot. Similar to S plus Q delta t by 2 we can prepare another plot which will have S minus delta Q by Q delta t by 2 and using this graph we can directly obtain the value without going through that additional step of computation. If we plot the inflow and outflow assuming that it returns back to its original position there will be a peak attenuation and lag. This point where the inflow and outflow intersect will have the largest outflow or the outflow will be maximum and the storage also.

Similar to this Q max we can plot a curve which shows t versus the level in the reservoir h. It will follow a curve similar to the outflow curve and it will reach a maximum. Since we say that Q is proportional to h to the power 3/2, the h will be maximum wherever Q is maximum. So these two times will be the same if we say that Q is proportional to h to the power 3/2 which normally happens when we have ungated spillway. In that case the spillway would be like this. The discharge through the spillway will be proportional to h to the power of 3/2. The discharge will be maximum when the head is maximum. If we look at the equation we can see that ds by dt will be zero when I is equal to O or if we use the symbol Q then when inflow is equal to outflow ds by dt will be equal to zero and therefore S will be maximum and since we say that S is also related with h may be some power m or may be a non-linear function of h S will be maximum at some point means that h will also be maximum at that point and Q will also therefore be maximum at that point. This shows that the intersection point where I is equal to O represents maximum outflow and maximum storage in the reservoir

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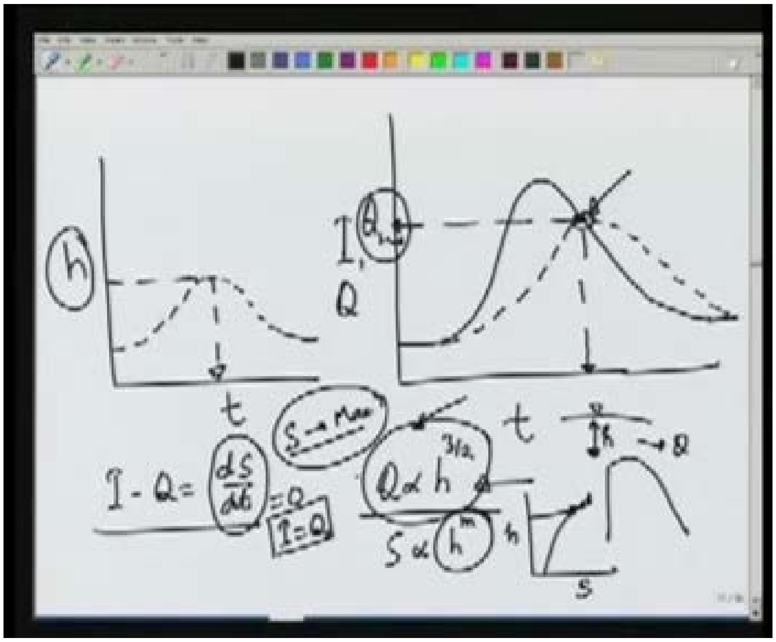

In addition to this modified Pul's method there is another method which is commonly used known as the Goodrich method. In the Goodrich method the same equations are written in slightly different form and the form which is used in this case is $2S_2$ by delta t plus Q_2. It is almost same as the modified Pul's method but a little bit different in the coefficients and the way we modify the equation. We can see that if you plot this term with h again we get a curve similar to this. We can plot Q also as we had done earlier but here the computations become a little easier because now we don't have this Q delta t term. We have only this Q term here. This $2S_1$ term is written as this term which is the same as this. These are obtained in a manner similar to that modified Pul's method in which we find out the value, find out h. Corresponding to this h we know the Q and from this value which is known at the end of the time step we just subtract minus 2Q and we will get the value of 2S over delta t minus Q at the beginning of the next time step. This makes a little bit easier the computations of the water level and the discharge at different times.

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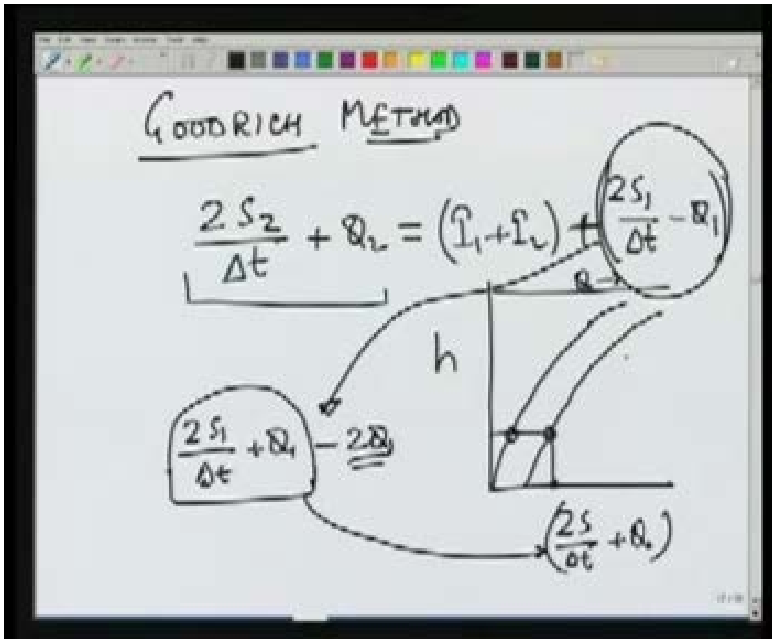

The other method in addition to these graphical methods may be the use of a numerical method and these days with the easy availability of computers and availability of various algorithms to numerically integrate a differential equation, it's quite common to use numerical methods. We can take the equation in its differential form the inflow minus outflow. dS is the change in storage in time dt and what we can do is we can prepare graphs which relate the area of the reservoir. This is the plan area of the reservoir and we can write the change in storage for a little change in h. If we have a little change here dh for very small change in dh we can assume that the area is almost constant. So dS will be given by Adh and this A will be a function of h. We can denote this A as function of h. O is the outflow which we have already seen will be a function of h and I is the inflow which is a function of time. So we get a differential equation in which the variables A are functions of h; outflow again a function of h, the inflow is a function of time. So the differential equation can be written as dh by dt because dS we have seen being equal to Adh. We can write dh by dt is equal to I as a function of t minus outflow which again let's denote by Q and divided by the area which is the function of h. We can write this as some function of time and h where this is a known function because area is a known function of height, inflow is known function of time and Q is a known function of the head over the spillway.

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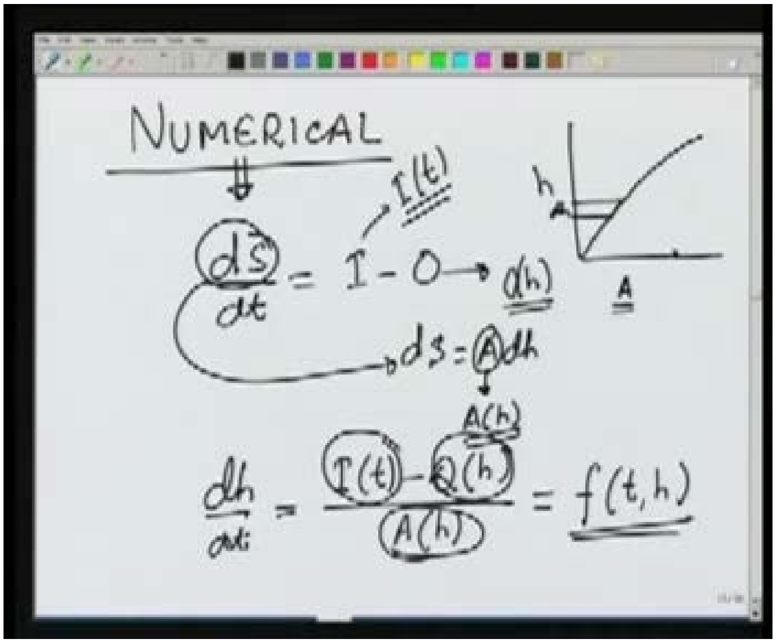

This gives us the differential equation of the form dh by dt h and t or t and h. This can be solved by using various available numerical techniques. One of the most popular numerical techniques is known as the fourth order Runge Kutta method and this is normally taught in normal numerical method courses. The equation which we use in this case is Knowing the values at the Ith interval we can write the values at the I plus first interval as a weighted mean of some dh by dt values which are denoted by k multiplied by delta t and adding to the h at the previous time step. The values of k_1, k_2, etc are based on the function value h t. dh by dt denotes the slope of h versus t curve. dh by dt really indicates the slope at any point t. k_1 is also a slope but evaluated at some point which is t_i and h_i. This is known to us because at t equal to t_i we know h equal to h_i. Our aim is to find out the value of h at t_i plus 1. So what will be this value h_i plus 1? Whether the h curve will go like this or like this or like this? This is our aim finding out the value at the end of this time interval h_i + 1.

What we do here is since t_i and h_i are known we can find the slope dh by dt at the point t_i h_i. Using this slope we can go to a point which is midway between t_i and t_i + 1. Let's call this point t_i+1/2 and the value of h we assume at this point will be based on h_i and then the slope which we have used are obtained. This is based on the known value. k_1 is the value known at x_i or t_i h_i multiplied by delta t by 2 which is the half way up to here. Let's call this delta t. This will give us a value of k_2 at half point corresponding to a depth which denotes the slope at the beginning moving up to the midway point. This is some h value which we predict based on the original slope of the line at h_i t_i and the time step delta t by 2. This k_1 is known from the initial conditions or at the beginning of the time step. k_2 will be known at t_i+1/2 and using the value of k_1 we predict the value at the mid point. So k_2 will also be known. Similarly we can find out k_3 again at the midpoint but using the new slope k_2. So delta t by 2 here and here they are because we are estimating the value at the midpoint and k_4 is evaluated at the end of the time intervals. So t_i plus 1 h_i but now using a slope of k_3. k_3 delta t and now we use delta t because we want the value at the end of the time interval. What these given k_2, k_3 and k_4 represent are estimate of slope of the head versus time curve at different times. k_1 at the time t_i, k_2 and k_3 at the time t_i+1/2 and k_4 at t_i+1 and using these slopes and giving them a weight 1/6 for k_1, 1/3 for k_2 and k_3 and 1/6 for k_4 we get some kind of an average slopes.

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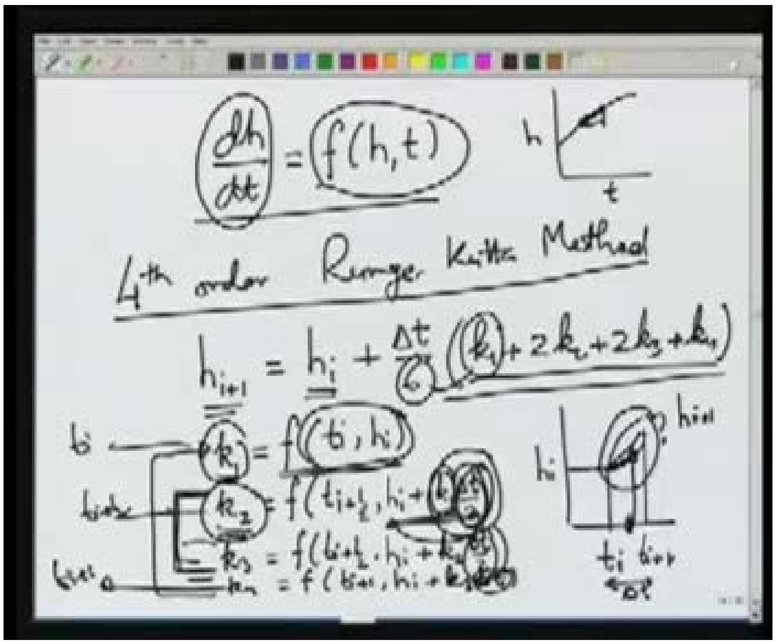

We can see that this equation can be written as h_{i+1} equal to h_i plus some average slope which we can call let's say dh by dt bar into delta t and this dh by dt is a weighted average of the values of dh by dt at beginning and end midpoints. This is nothing but 1/6 k_1 plus 1/3 k_2. We assign a weight of 1/6 to the first point and the last point and $1/3^{rd}$, $1/3^{rd}$ to the two estimates which we have obtained for the midpoint. Using this equation starting from a known value of h_i we can estimate what will be h_i plus one at the end of that time period delta t and then we can proceed once we know from a known value of t_i and h_i taking this delta t, t_i plus 1. Using fourth order ….. (00:42:29) we estimate the value of t_i plus 1 and then we proceed to the next step ti plus 2 now using this as known and this as the unknown value. These numerical methods have become quite common recently because of the wide use of computers and easy availability of programs for numerical integration of or or? differential equations. These are some methods which can be used to do the storage routing.

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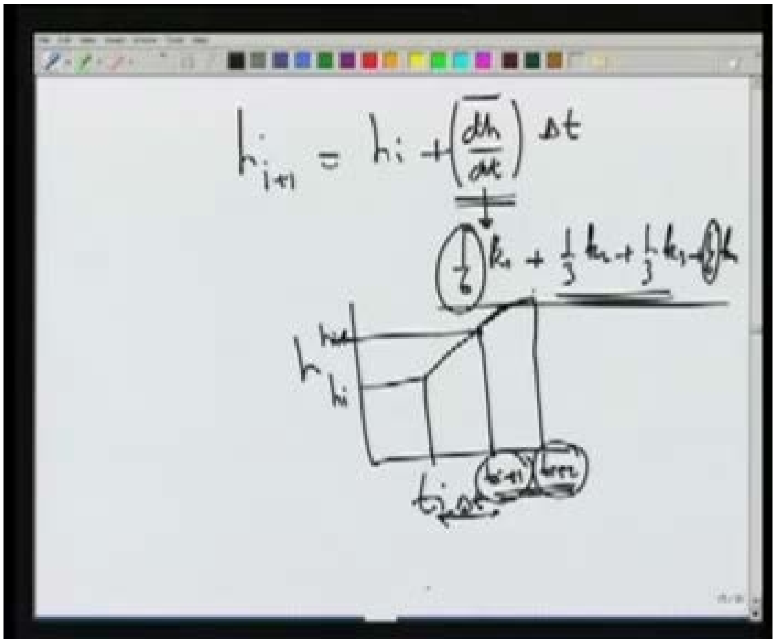

Similar to this we have channel routing also in which lots of different methods are available. In the channel routing the basic methodology is knowing the inflow at certain point we want to predict what will be the outflow at some other point and in this case as compared to the reservoir routing where we had storage in the reservoir now we have to think of the storage in the channel. Storage in this area we can represent by let's say S. The channel storage can be divided into two parts. If we have water level here and the bed level and the water level like this then we could divide this into two parts one part which has the same depth throughout and this will be in the shape of a prism. If you draw the cross section then this part is a prism of this section and of this length and the storage within this area where the depth is taken same as the downstream depth is called the prism storage and we can write this as S_p for prism storage. The other portion which is above the prism and below the water surface up to the water level is in the shape of a wedge and is called wedge storage or S_w. Total storage in the channel will be a sum of the prism storage and the wedge storage.

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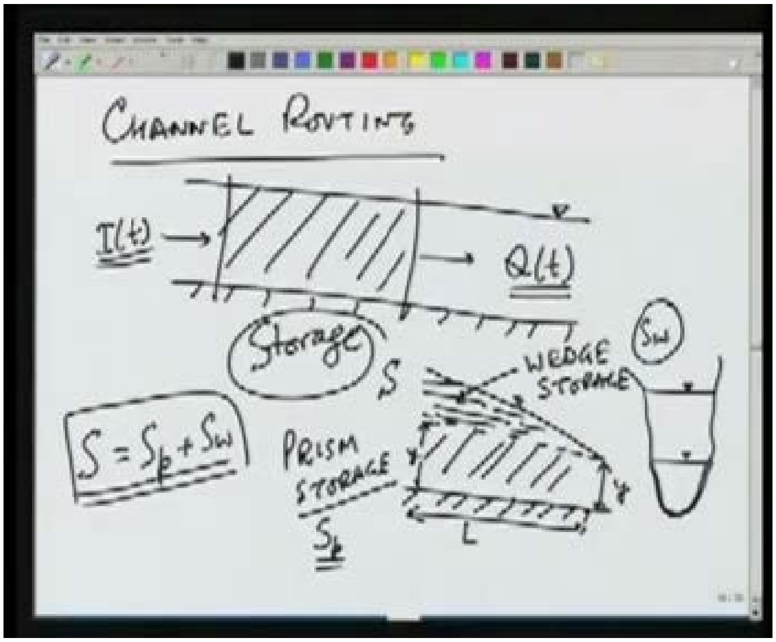

This storage is used for the routing and the equation which we use is that prism storage will be a function of Q because the prism storage is taken based on the downstream depth and the downstream depth will directly affect the outflow. Therefore the prism storage will be a function of Q the wedge storage S_w is a function of the inflow. The total storage S can be written normally for channel routing. We assign some weight to the inflow some weight to the outflow and arrive at the total storage as the wedge storage which is the function of I and prism storage which is a function of Q. In this k is a coefficient which is sometimes called storage time constant 'm' would be an exponent which varies from about 0.6 for rectangular channels and it's about 1 for natural cross sections. m may vary from 0.6 to 1. x is the weight which decides how much weight we are giving to the inflow for this storage computations and how much weight to the outflow. S is the storage in the channel. Using this equation we can obtain various methods of routing the flood through the channel.

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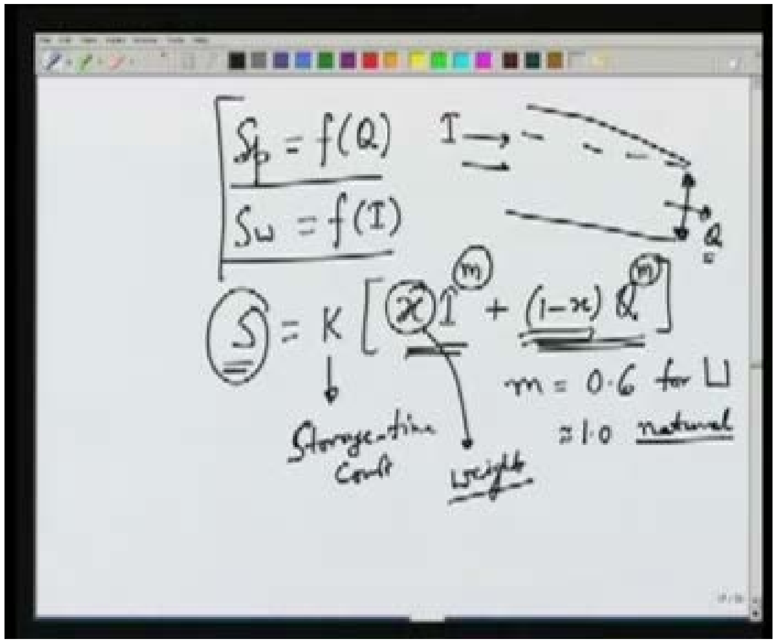

Knowing the inflow we again want to compute the outflow Q for the channel and one of the methods which is quiet commonly used is known as the Muskingum method. In the Muskingum method we take m equal to 1. So the equations which are written for Q in terms of I and O are some function k xI plus 1 minus x Q; as we can compare with the previous equation this m value is taken as 1 in the Muskingum method. Therefore we will have only I here and Q here. x is the weight which for the Muskingum method is taken to vary from 0 to 0.5. 0.5 means we are getting equal weight to the inflow and the outflow and zero means we are giving no weight to the inflow and weight of 1 to the outflow. If we put x equal to zero then we get S equal to kQ which is known as the linear reservoir and this is used sometimes in developing the continuous unit hydrographs which we have talked about already. But x value less than 0.5 will be used for Muskingum method about 0.3, 0.4 we can use.

We can write this equation for the mass balance or the continuity using the Muskingum method. We can find the change in storage between two points which is one before the interval and one after the end of the interval. With the time step of delta t, S_2 is at the end of the time interval S_1 is in the beginning. This S can be related with I and Q using the Muskingum equation with m equal to 1 and we can get the change in storage by this equation. This change S_2 minus S_1 from the continuity equation is also equal to I_2 plus I_1 by 2 delta t minus Q_2 plus Q_1 by 2 again multiplied by delta t.

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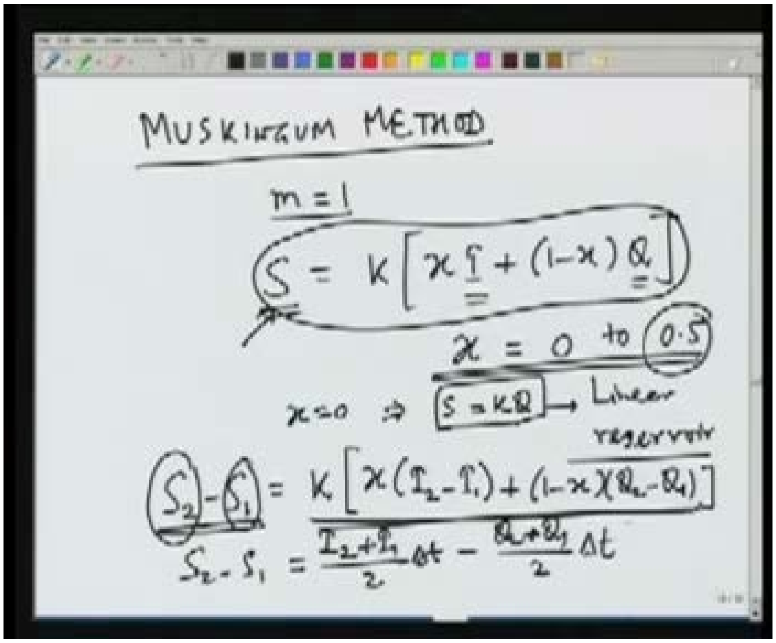

We get two equations one for S_2 minus S_1 from the continuity, one from the Muskingum relationship here between S, I and Q and using these two equations we can write Q_2 which is the unknown in terms of the known quantities I_2, I_1 and Q_1. I_2 and I_1 are known because I is known for all times. Q_1 is known because at the beginning of the time step we assume that Q is known. This constant C_0, C_1 and C_2 are given as minus kx. So C_0 is given by this. C_1 is expressed as kx and C_2 is given as k 1 minus x minus delta t by 2 by k 1 minus x plus delta t by 2. Using these values of C_0, C_1 and C_2 which can be obtained based on k and x and delta t we can write an equation which will be applied repeatedly for different time steps to obtain the unknown Q_2 at the end of time step in terms of the known values of I_2, I_1 and Q_1. It is done in tabular form to make the computations easier.

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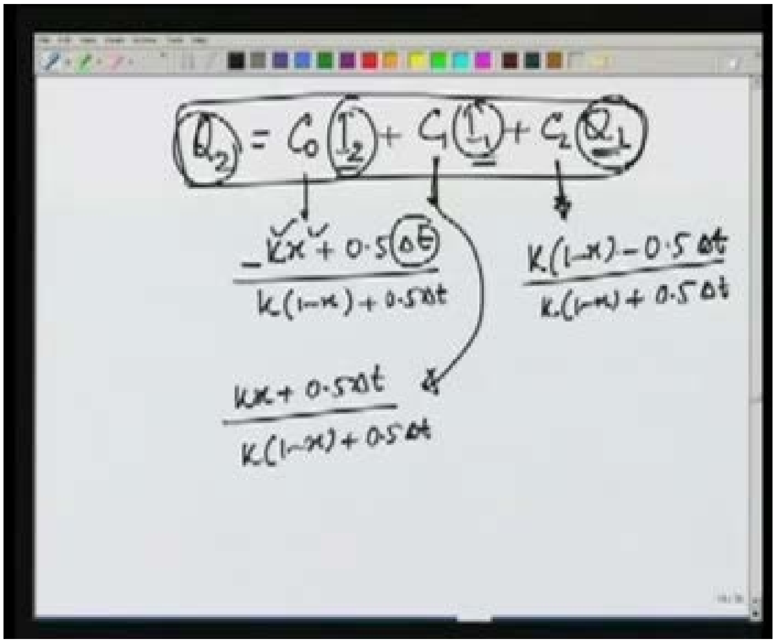

Sometimes it is also written in a different form as Q at any step I …. with previous time step value some constant and delta net inflow before the time step and net inflow after the time step.

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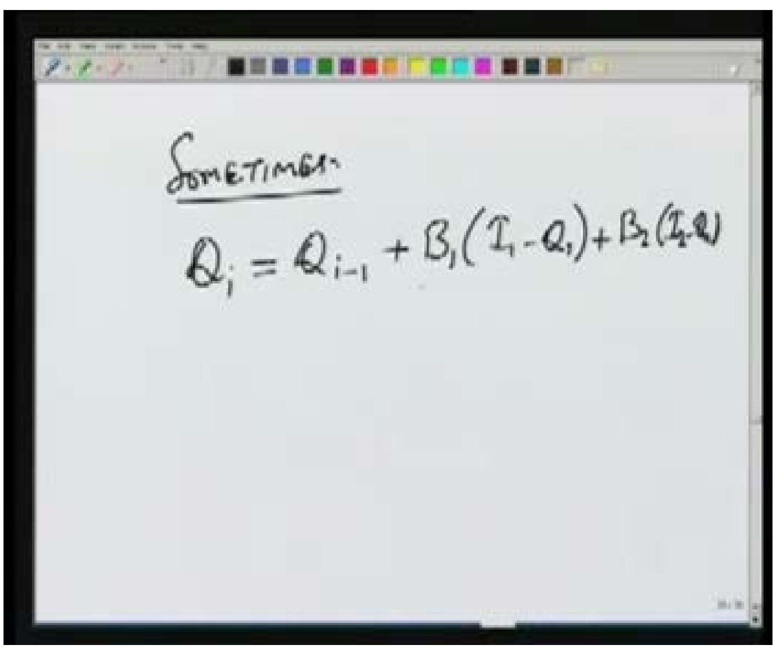

We can write an equation in this form also but the Muskingum method using this form is preferred.

In today's lecture we have seen the flood routing. There are different methods of routing the flow either through a storage reservoir or through a channel. The storage reservoir routing is also called level pool routing because we assume that the reservoir level is horizontal and we don't account for water level variation inside the reservoir and there are methods which can be used for the reservoir routing. For example the modified Pul's method or Goodrich method and there are some methods like Muskingum method which can be used for channel routing. We have looked at the techniques used for different kinds of routing and in the next lecture we will look at some examples which will explain various aspects of designing for a flood, computing the maximum flood discharge, routing the floods through the storage reservoirs and channels.