Water Resources Engineering

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Lecture No. # 10

In the previous lecture, we had seen flow through confined aquifers in which there is an impervious layer at the bottom. It also bounded by an impervious layer at the top. It means the thickness of the aquifer remains constant, or in other words, it does not depend on the head. The piezometric head will vary if we take water out of the aquifer. It will then decrease towards the well but the thickness of the aquifer remains constant and therefore the flow area remains constant. Compared to this, the unconfined aquifer does not have any impervious confining layer at the top.

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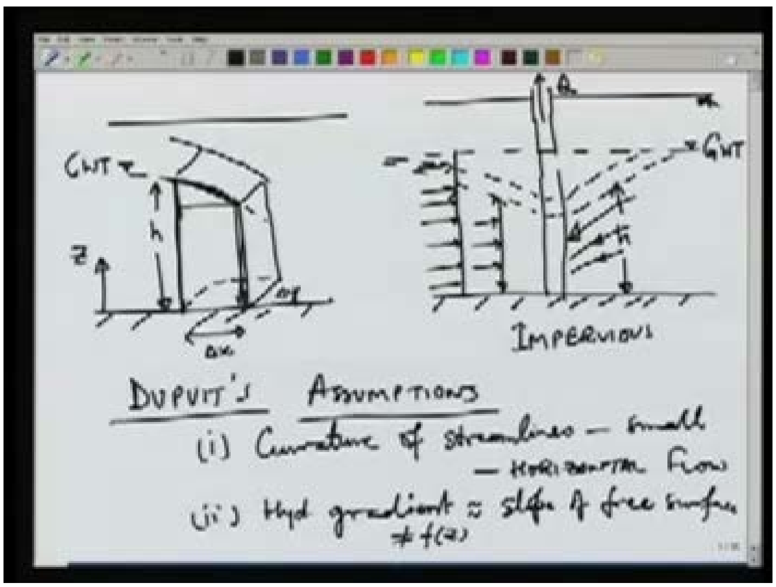

So if you look at an unconfined aquifer, there is a layer at the bottom which is impervious and then the ground level, the unconfined aquifer, the water table would be open and therefore the pressure on the top of this will be 0 or atmospheric. So once we start pumping an unconfined aquifer with some discharge Q, the water table will go down and it will keep on going down. We keep on pumping till it reaches some steady state conditions. We can notice the area of flow from here and earlier it did not have any pumping. The area of flow is the whole thickness of the aquifer. But as the cone of depression increases, the area of flow becomes smaller. The height h will represent the area of flow therefore the governing equations will be slightly different compared to the case of the confined aquifer. Let us look at the governing equation for an unconfined aquifer. Let us say that this is the ground water table which is having a certain slope.

The slope depends on the pumping rate. The higher the pumping rate, the more is the slope. This is the height h. We take an element which is delta x, in the x direction and delta y in the y direction. We then make some assumptions in order to solve this. First of all these assumptions were suggested by Dupuit's and therefore they are called Dupuit's assumptions. This enables us to solve unconfined aquifer flow problems. The assumptions are two. The first assumption is that the curvature of a stream lines is small. If you look at the stream line pattern in the unconfined aquifer case, they may look like this. But the first assumption says that the stream line curvature can be taken small and therefore the flow will essentially be horizontal. The second assumption is that the hydraulic gradient can be taken as the slope of the free surface. The hydraulic gradient will remain constant over height. So it will not be a function of z, where is the z is the vertical co ordinate. So if we take an element like this, the hydraulic gradient for this whole depth would be given by the slope of the water table and it will not change with depth. So using these two assumptions, we can now write the mass balance for this element.

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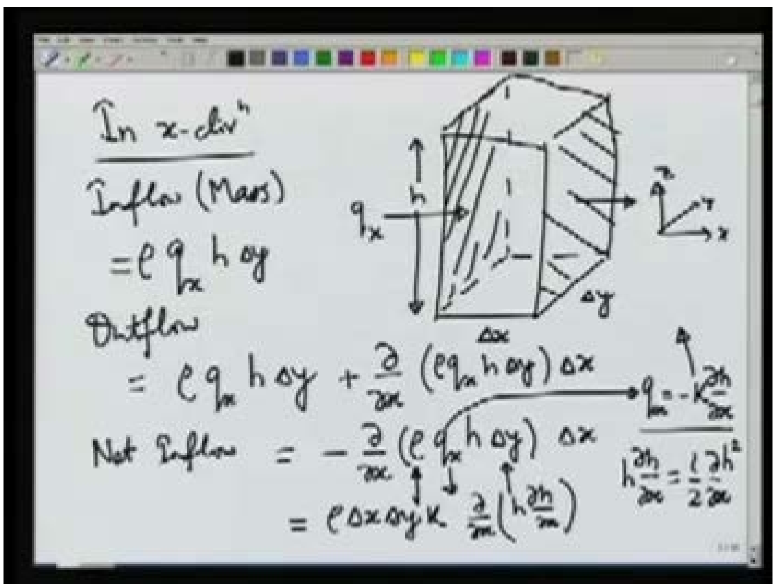

We are drawing that element of the unconfined aquifer here which has a certain velocity q_x. This velocity is coming in and therefore we determine can write the mass which is coming in through this case. So let us first compare the x direction inflow and outflow. The axis of course are x, y and z. Inflow in the x direction would occur through this phase. The velocity is q_x multiplied by the area which will be h delta y. This gives us the amount of flow in terms of volume and since we are interested in the mass, we will multiply it with the mass density rho. Rho q_x h delta y will be the mass of flow coming in from the left hand phase. There is some mass which is going out in the x direction from the right hand phase. We can determine that outflow, using the same Darcy as we used earlier, plus change of this quantity multiplied by the length delta x. If we consider the x direction flow from both phases, the net inflow of mass would be = – (Refer Slide Time: 08:28). If we look at this equation, q_x can be obtained from the Darcy's law, rho can be assumed to be constant. Delta y of course is the element thickness by this constant. We can write this equation as rho delta x, delta y, q_x can be written as – k, using Darcy's law, we will again assume that the aquifer is homogeneous and isotropic. So k does not change with location as well as direction and therefore k can also be taken out of the differential. We can take k out of this and what we will be left with is del/del x of h partial h with x. We can further simplify this equation by noting that the h del h/del x term can be written as 1/2 partial of h square with respect to x.

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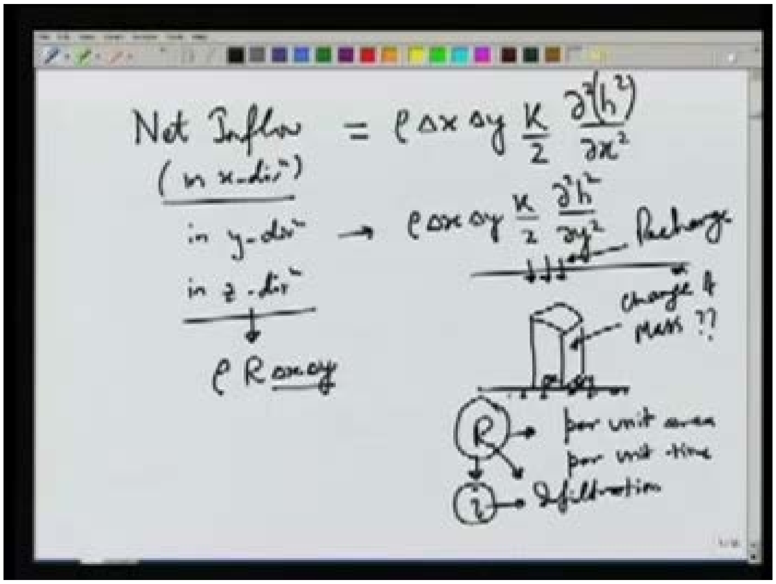

If we use this, we can write the net inflow. We get the second derivative of h square. So if we compare this with the unconfined aquifer and confined aquifer, we see that in confined aquifer, we had a second derivative of h, but here we are getting second derivative of h square. This was in x direction. We can write similar expression for flow in the y direction. The mass inflow in the y direction would be in the z direction due to the Dupuit's assumption. There is no flow component velocity component in the z direction. Therefore the only flow in the z direction for this element would occur if there is some infiltration from the ground surface. This is known as recharge, for example there may be a rainfall event which will cause some recharge or there may be a river which might be contributing to the ground water. So this recharge is generally expressed in terms of R, rate of recharge which tells us about the volume per unit area per unit time. So R has units of (for example) rainfall in density, so it will be centimeter per hour or millimeter per hour or centimeter per second depending on the units we are using. So this if there is a rainfall then typically it will be then intensity it will depend on the intensity of rain I and depending on how much infiltration we are getting that R will be = that infiltration. So let us say that there is some recharge rate R and a mass which will come in because of this recharge from the top. We are assuming the bottom to be impervious. There is no flow from the bottom and therefore the recharge which occurs would be per unit area. We have recharge rate of R, multiplied by the area of element which is delta x, delta y and multiplying by the mass density will give us the mass inflow in the z direction.

After knowing the net inflow of mass into the element, what we are left with is how to find out the change of mass within the element and for that we will be using the specific yield. How to find out the change of mass within this element using a specific yield? As we have seen already, specific yield is the amount of water released from storage for a unit drop of head from the prism of unit base area. This is the base area delta x, delta y that will have to multiply a specific yield with delta x, delta y.

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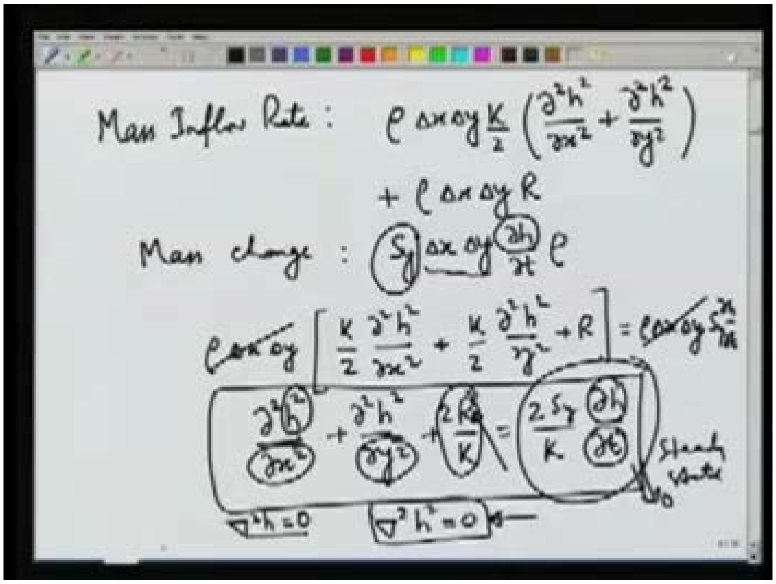

So if we write the total mass inflow combing all the x, y and z direction, in the x direction this is the flow in the y direction, this is the flow and then as we have seen here in the z direction, rho R. This should be = the change of storage within the control volume which we are considering. Mass released or mass change would be = the specific yield, S_y area and a change of head. This is because a specific yield is defined as the change per unit surface, per unit area of cross section for a unit change in head. So we have to multiply with the area and multiply with the change in head. This will give us the rate of change of mass within the control volume. Continuity equation says that this two should be equal. We can write the equation and since there is volume, it has to be multiplied by the mass density. So we can combine these and write it as k/2 rho del x del y. I will write here and then cancel it out, so these will cancel out and the final equation which we did plus 2R over k would be = 2 S_y over k.

This is the equation which governs the change of head with time and space change with time is here and change in space will be given by these two terms when it is subjected to some recharge R. In most cases, which we deal with, we can assume the recharge to be 0. In a confined aquifer, of course since there is an impervious layer at the top, unless the layer is slightly pervious there will be no recharge. Therefore we have not considered recharge for a confined aquifer, but sometimes confined aquifers, the over line layer may not be completely impermeable. It may be an acquitted which will allow some leakage into the confined aquifer. In that case in confined aquifers also we should consider the recharge. This equation needs to be solved with different boundary conditions and initial conditions to get the variation of h in an unconfined aquifer. We will take some simple cases. For example if we take steady state flow, then this term will be 0. Similarly if we say that there is no recharge, then this term will drop off. So in the absence of recharge and for steady state conditions, an equation is obtained which is similar to the equation which we had for confined flow. We had a laplacian of h = 0 there. So the only difference is that instead of h, now we have laplacian of h square as 0 which we can solve to obtain the value of h at different locations.

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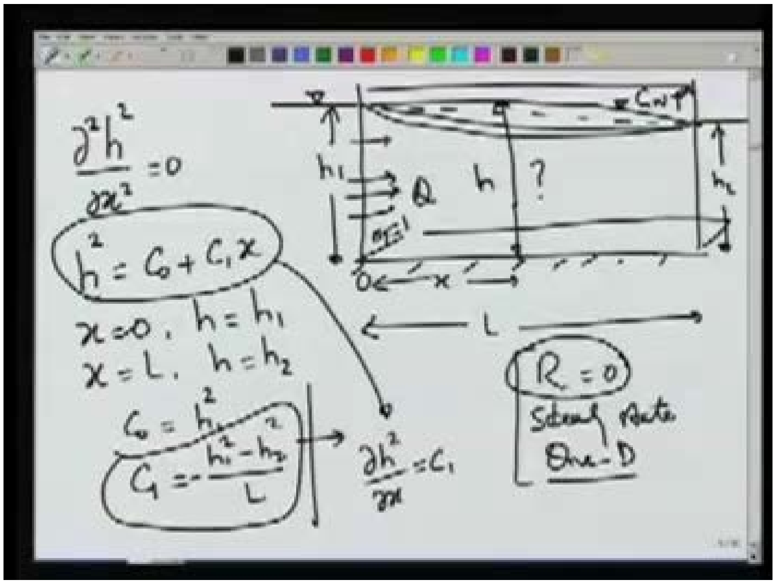

Let us take an example in which similar to the confined aquifer case, which we had considered in ground level, let us say there are 2 water bodies in which have elevations of h_1 and let us say that there is a length l. x is 0 here and then at certain x, we want the location of the ground water table, so till the confined aquifer case, we have seen that the water table was a straight line because h was linear. Here it will not be a straight line, so we should find out whether it is like this (Refer Slide Time: 21:02) or it is like this or like this. So we are interested in finding out the value of h at any x and to solve this problem, we can assume that there is steady state condition - one dimensional flow. If we assume one dimensional flow, then the equation becomes straight forward. What we assume is no recharge and steady state one dimensional flow. If we make these assumptions, the governing equation will be reduced to the following, this term will be 0, R is 0, this term will be 0, steady state and this term will be 0, because the flow is one dimensional. So we are left with this very simple equation which we can solve and obtain h square to be linear. These constants can be obtained from the boundary condition and in this case, the boundary conditions are x = 0 h is = h_1 and x = l, h = h_2. So you can see that C_0 will be = h_1 square and C_1 will be = h_1 square – h_2 square over L. Using these two values of C_0 and C_1, we can obtain the value of h and since h square is linear, the actual ground water table will not be linear and we can see the discharge Q.

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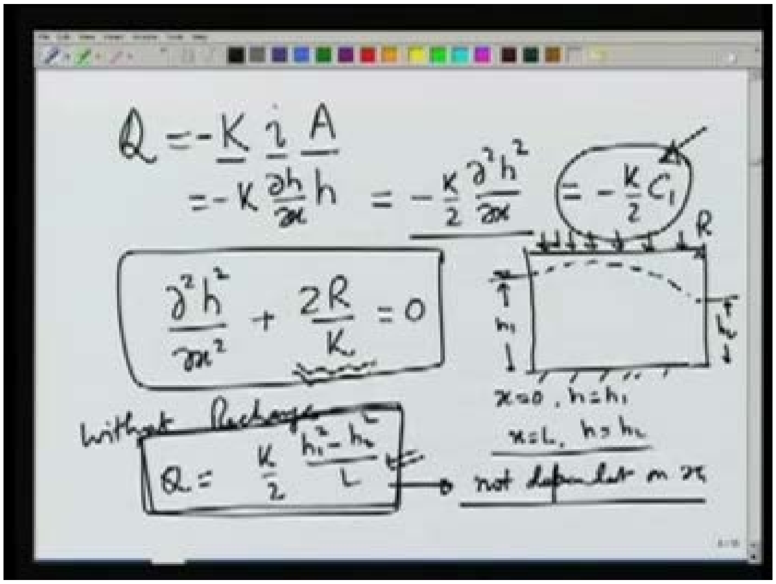

At any location x, Q can be given as KiA, where K is the conductivity, hydraulic gradient and the area of flow, i in this case is del h/del x and this negative sign will be present because we are saying that the flow will be in the direction of decreasing head and the area of flow at any location, where the he height is h, if we consider unit width perpendicular to this, the width is perpendicular to the plane of the paper. Let us say delta y is = 1, so per unit width, the discharge can be given as – k del h/del x into h and this can also be written the same way we have done earlier for deriving the equation. We had seen that del h/del x del h/del x could be written as 1/2 partial 1/2 h square with respect to x. Using that we can write this in terms of partial of x square with respect to x square and we know that from this equation, this is partial of h square with respect to x. We know that partial of h with respect to x is = C_1 and therefore Q will be given by – K/2C_1. If there is some recharge, here, we had assumed that recharge is = 0 and if there is some recharge, then we will have to add the recharge term and from this equation, this 2R/K term will also arrive.

The equation derivation is similar but now we have to account for the recharge and in this case we will see that the ground water table may rise above the level h_1 because there is recharge coming in here at the rate of R. Because of this recharge, ground water table may rise and we can find out the equation of this line by solving this equation with the boundary condition which are same x = 0. Using these 2 boundary conditions, we can obtain the value of h and if there is no recharge then of course – k/2C_1 and C_1 have already been obtained from here. 1 square – h_2 square over l, so without recharge Q will be Q/2 into C_1. k/2 into h_1 square – h_2 the square over L and there is a negative sign here. C_1 is – h_1 square – h_2 square over L and then this is the value of Q. With recharge, we have to add this 2R/k term and then we can solve the equation in similar form.

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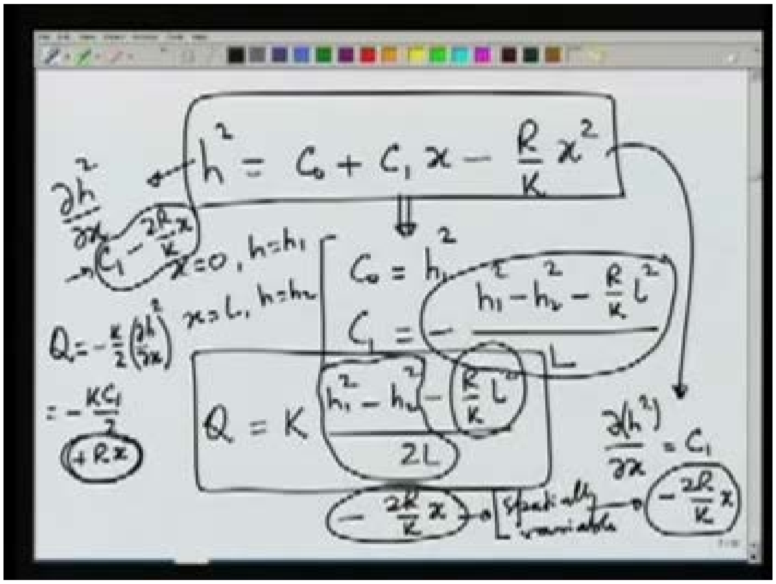

h square will be C_0 + C_1x – R/k x square and now this comes again by solving the differential equation which we had written here and applying the boundary conditions, we can obtain the value of C_0. C_0 will again be = h_1 square, but C_1 in this case will be different from the C_1 which we had derived earlier when R was 0 in this case the C_1 turns out to be – h_1 square – h_2 square till using the boundary conditions x = 0 h = h_1 x = l, so Q again will be given by the same equation – k by $2C_1$ but now C_1 will be a function of R. So we have k can we have have one square while in this case it was only h_1 square – h_2 square over L, here it would be h_1 square – h_2 square – R over k l square over 2L.

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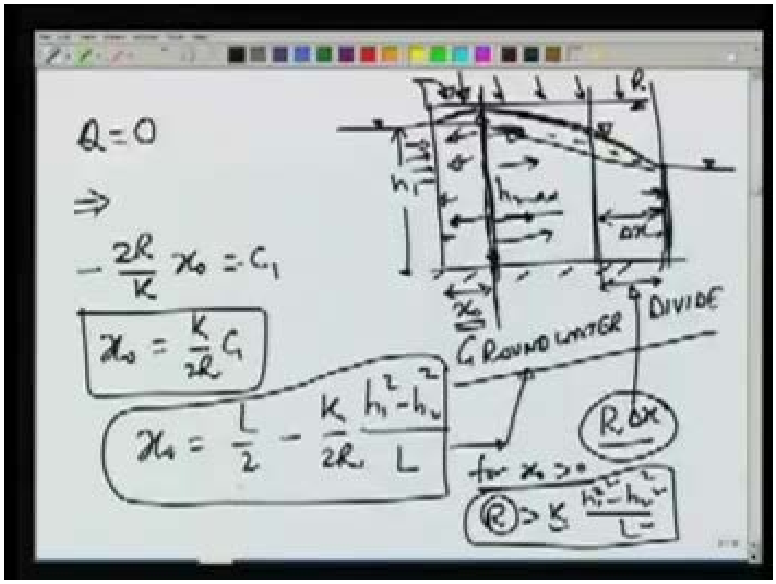

If we draw the profile of the water surface, this is the ground level. There may be a water body here. In practice, this may denote a river or a canal. So if we have two rivers or two canals which are running parallel in the unconfined aquifer, the water level in absence of recharge would look like this by plotting the equation and when we have recharged, the water level would look like this. In order to find out this equation and just by looking at this, we can see that there is a point here where h will be maximum and if you look at this, whatever flow is occurring on this side will move towards the water body on the left and similarly the ground water flow from this side will move towards the water body on the right, so this portion can be called a ground water divide. Because of the presence of a gradient in this direction, here and the gradient in this direction here, the ground water will flow towards one side, towards this side and towards this side. Therefore, this line can be thought of as dividing the ground water such that one thought flows in to one water body and the other in the other body. We can find out the location of this point also, for example the Q value is known to us. Now this may not happen in all cases. It depends on the rate of recharge. If the rate of recharge is small, it may not happen. It may be below h_1 all the time.

It will depend on the rate of recharge and we can find out the critical rate of recharge at which the ground water divide will occur. For this we need to write the equation for Q as we done earlier. You can see that at the ground water divide Q will be = 0 because before that Q will be in the negative direction after that Q will be in the positive direction and just at the ground water divide Q will be = 0 and this will give us a relation between the recharge rate and the location x. For example when there is no recharge, we know that Q is k/2 h_1 square – h_2 square over l which is not dependent on x. If we have the recharge then Q will depend on x. There is a constant term Q which is the same as without the recharge. If you look at this term, this gives us the constant term. The second term will be with recharge, so R by k_1 square and this Q will be dependent on x, also, 2 R / k x. If you look at this point, this line, the Q term has to be 0 on this line and therefore what we get is 2 R/k. Let us call this point x_0. So 2R/ kx not will be space dependent or, a spatially variable value of Q changes with space because of the recharge. There is some recharge coming in from the top and there is some flow. Let us say it is coming in from this water body or going into this water body. If we take any section here, the amount of flow which is coming in because of recharge is R into whatever delta x we have considered, so if you take Q at any section here and Q at a section here, the difference between these two Q's will be the amount of recharge coming into this from the top which is R delta x. When we look at this situation where we have some values of x_0 – 2 R/k x_0 should be = – C_1. So x_0 will be = k/2 R C_1 and C_1 that has already been obtained is h_1 square – h_2 square – R/k_1 square over L. We can write x_0 as L/2, so this gives us the position of the ground water divide and for ground water divide to occur, x_0 must be positive and therefore we can find out the critical recharge rate which will cause a positive x_0. So for x_0 to be positive, we can find out the critical recharge rate R should be greater than kh_1 square – h_2 square over L. What we have seen here is that if recharge is greater than this value h_1 h_2 l and k, they are the properties of the medium and the boundary conditions. If recharge rate is greater than this, then we will have a ground water divide otherwise we will not have a ground water divide.

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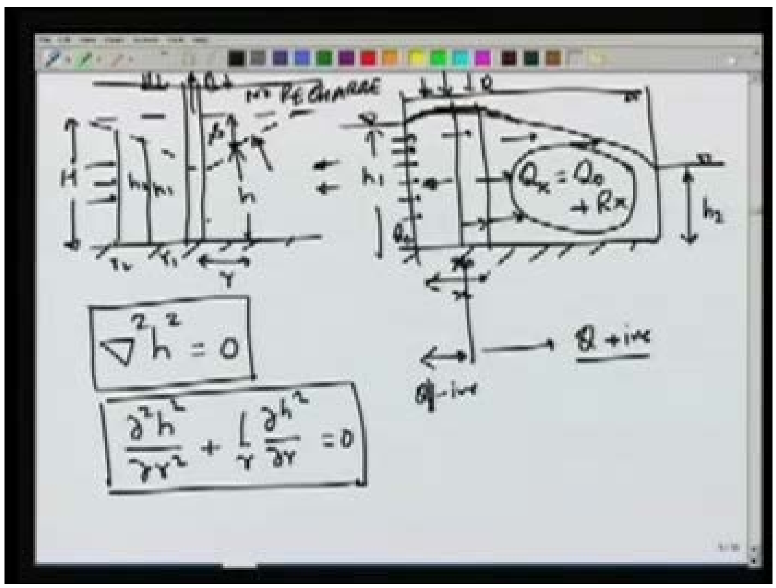

So for unconfined aquifer, flow between 2 water bodies, one dimensional steady state condition, we have derived some expressions for example, this h square using this h square, we have obtained as a constant value C_1 and a time space dependent value 2 R/k x. When we find out the Q as – k/2, it will have a constant value and a time dependent value which is Rx. This is the contribution of the recharge up to a certain distance x. So for example, if we have this recharge R occurring here and there is some Q here, I will show it in the positive direction. It maybe towards the water body, then at any other x, it says Q here would be Q at 0 + Rx and this term Rx therefore represents the contribution from the recharge to the discharge. We have seen that there will be some point x_0 at which the water table will reach a maximum value and therefore it can be taken as the ground water divide. The location of this point x_0 can be obtained by putting Q = 0 and since Q is = – k C_1/ 2 + R x, x_0 will be given by k/2 RC_1 which can be obtained as this. For ground water divide to occur, x_0 should be positive and therefore R has to be greater than k h_1 square – h_2 square over L. L will be large. h_1 square – h_2 square, k will all be small. R is larger than this means, if L is very large then, this value will be very small and a very small value of R will be sufficient to cause the water divide. This Q of course can be found out at any location. In the portion before x_0, Q will come out to be negative and in the portion after x_0, Q comes out to be positive. It means it is going in this direction before x_0. This is a one dimensional flow situation which may not occur in practice very often. The most common situation occurring in practice is flow towards the well. We will next look at this case of an aquifer, unconfined aquifer being pumped at a rate of Q and we will try to find out the steady state draw down cone. This cone of depression at any distance (we will be using R because we want to use radial coordinates) at any distance R, the height of water table H or in other words, we can also think of in terms of draw down s. Given that initial thickness of the aquifer is capital H, the equation will remain the same. Let us assume that there is no recharge. So R is = 0, no recharge and then we also assume that there is some water body applying, there is enough water, and that it had reached a steady state and there is no further depression of the cone.

In that case we have our governing equation as Laplacian of h square = 0, because there is no recharge steady state conditions and as we have seen earlier for radial coordinate system, we can write the laplacian of h square as this. We need two boundary conditions to solve this. Generally there will be 2 draw down values available to us or we must have two draw down values available to us. Let us say at r_1 h_1 and r_2 h_2. Now these r_1 and h_1 and r_2 and h_2 can be taken any where but a commonly used method is to take r_1 as the radius of the well.

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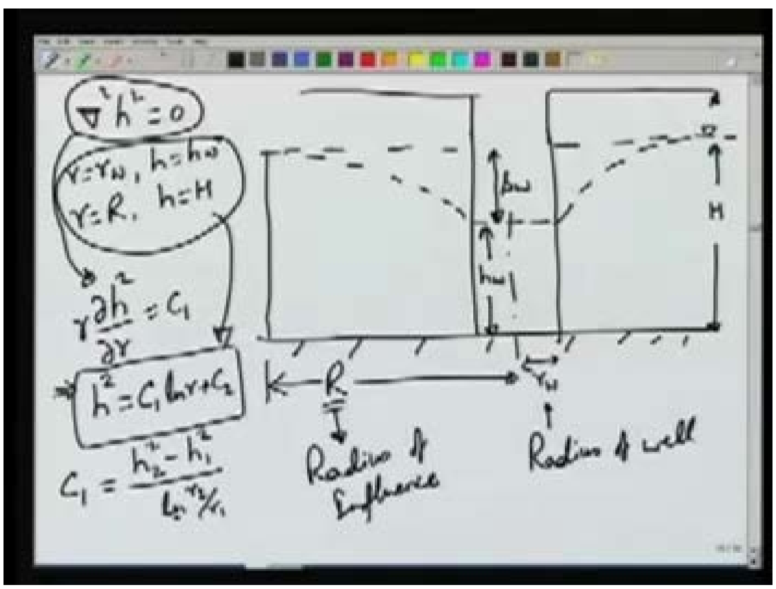

On a very large scale if I show the well radius, this is the ground level initial water table at a depth of H and then a steady state draw down cone may look like this from the centre of the well. We can write R_w at the radius of well and we have already defined a radius of influence, which is the distance beyond which there is no draw down and we denote it by capital R, although we have used for recharge rate also but there should not be any confusion. This R is the radius of influence so by measuring the draw down in the well s_w, we can say the height of water in the well is h_w. So the equation which we are solving is the Laplacian. So they have two boundary conditions, r = r_w, h = h_w and r = r, h = capital H. The solution of this equation can be obtained as we have done for confined flow case where C_1 is some constant and therefore it will give us identical to the confined aquifer. The only difference is that instead of h, we now have h square. This tells us how the head varies in the aquifer and we can also find using the boundary conditions the values of C_1 and C_2. C_1 is the one which is really important for us, so we can write C_1 as h_2 square for any r_1 and r_2 and we can write h_2 square – h_1 square or if we take these $r_{w,}$ h_2 can be taken as capital H.

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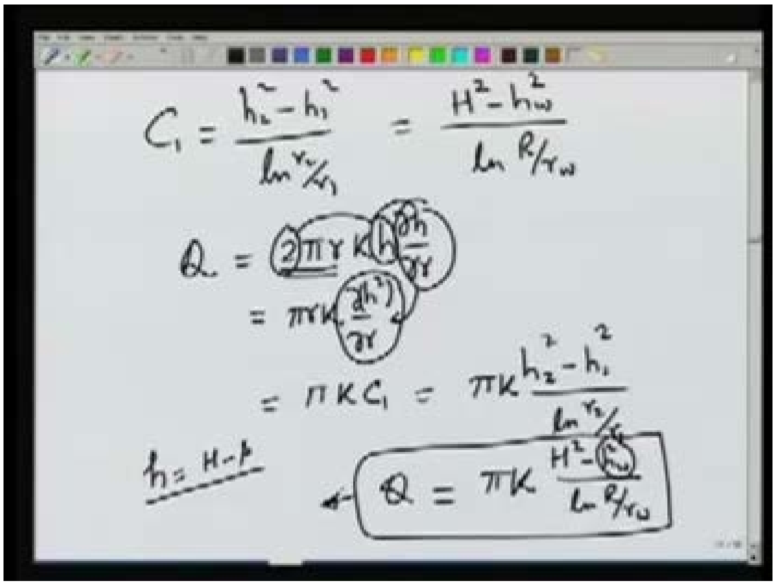

So if we take r_w and r values, then this will be equivalent to R/r_w. The discharge at any point Q can be written as 2 pi rkh and I am not putting negative sign here because this discharge is in the negative r direction. Therefore I am not putting this negative sign. Now in this h del h/del r, this 2 pi r is the circumference at any distance r and h is the height of aquifer at that point. 2 pi rh is the area and k del h/del r is the velocity. Now h del h/del r can be combined as pi rk and del/h square. The term 2 h and del h/del r can be combined to form this term. Q becomes pi rk del h square/del r and as we had seen from here, or here del h square/del r will be C_1 over r. It will become pi k C_1 which we have already derived in terms of any h_1 and h_2 or if we take the radius of the well and the radius of influence. As we have seen for the confined aquifer case, measurement of draw down is much easier therefore we want to express this equation in terms of draw downs. The only problem here is that this h is a square and we know that the h is capital H – draw down s. So when we do h square because of the presence of the square term, we cannot say that the difference of these two squares is the same as difference of the draw down square because it is not linear.

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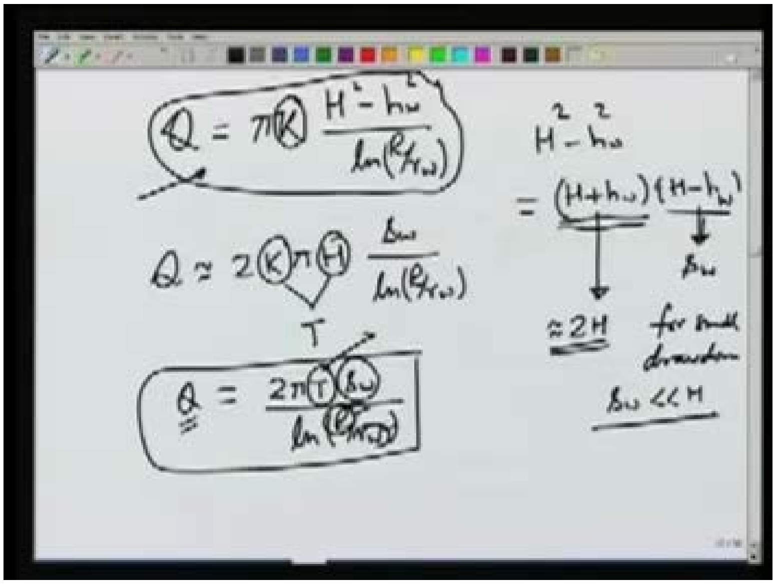

Therefore sometimes we approximate it and write this equation in a bit of a modified form and by using some approximation, we can write this term h square – hw square as H + hw, H – hw. Once we separate them out, then this H – hw term is nothing but the draw down in the well s_w. This is very easy to measure in the field. The other term which we have is H + h w and sometimes we have to make some approximation. For example here, we can say that H + hw will be very nearly = 2 H if s_w is small. For a small draw down (Refer Slide Time: 54:23). The logic is that the draw down is very small compared to the thickness of the aquifer H. If s_w is quite small then we can assume that H + h w will almost be equal to 2H and therefore Q can be written as 2 pi 2k pi H and H – hw is s_w k into H is the transmissivity of the aquifer as we have seen k into the thickness of the transmissivity. So we can write this as 2 pi Ts_w.

This gives us an equation which can be used to estimate the transmissivity of the aquifer. This equation is exact but cannot be used to estimate the value of k, typically because s_w and H values may not be known very exactly but the draw down in the well, s_w can be obtained easily. R and r_w are also easy to obtain and therefore for a known value of T, we can find out known value of Q and we can find out transmissivity using this equation. This is useful for estimating the parameter values. After looking at the confined and unconfined aquifers in all cases, we have assumed that Darcy's law is valid. Sometimes Darcy's law may not be valid, the flow may not be laminar and

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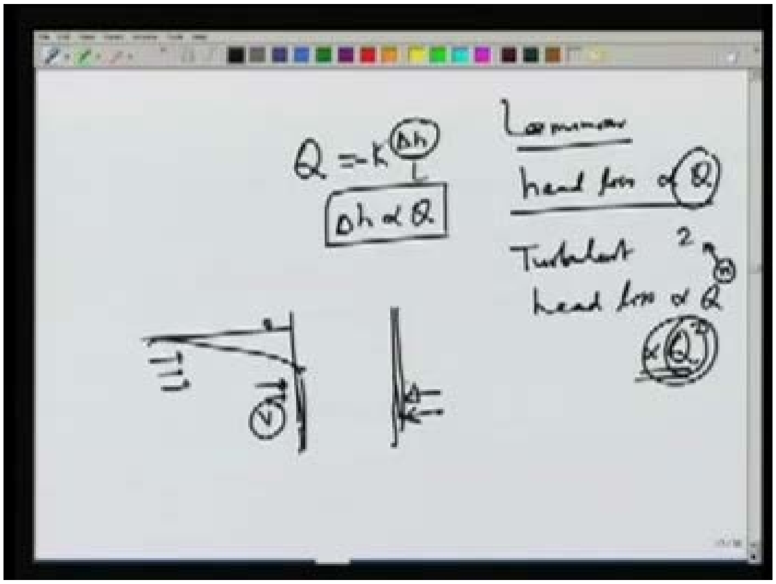

where we know that the head loss is proportional to (Refer Slide Time: 57:01), when the flow is turbulent, the head loss is typically proportional to some power of Q. Generally we can take it to be a square. Sometimes it is 1.7 – 1.8 but we can say that head loss is proportional to Q square. All the analysis which we have done till now does not account for variation of head loss as Q square. It only says that since we are using Darcy's law, the head loss or we say that Q = – K delta H over L, which says that delta h is proportional to Q. We will look at cases where the flow is not laminar but turbulent especially near the well. So here the velocity may be small but as we go near the well, the velocity becomes very large and the flow may not be laminar. It may become turbulent and then there will be an additional loss because of this proportionality Q square. Similarly near a well, we have a screen and there will be some head loss when the flow passes through the screen. The total draw down in the well will be a combination of these three terms, laminar loss, turbulent loss and the screen loss. We will look at a way to combine these in order to obtain the total well loss which will define some kind of efficiency of the well.