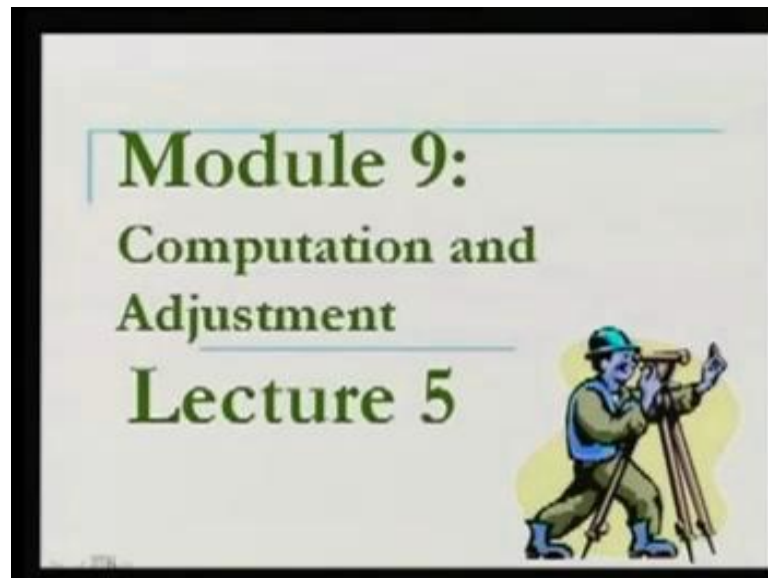


Basic Surveying
Prof. Bharat Lohani
Department of Civil Engineering
Indian Institute of Technology, Kanpur

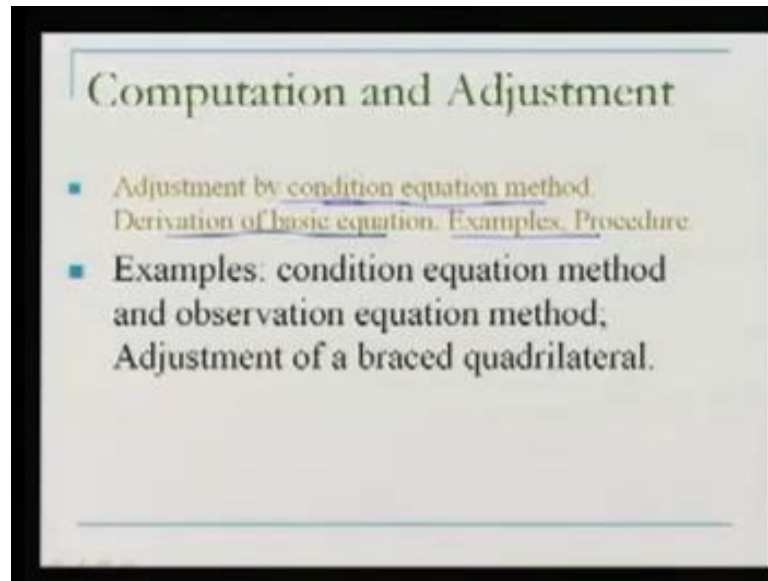
Module - 9
Lecture - 5
Computation and Adjustment

(Refer Slide Time: 00:23)



Welcome to this video lecture on basic surveying. Now, today in this lecture we are in module 9, which is on computation and adjustment. And we will be talking about the lecture number 5.

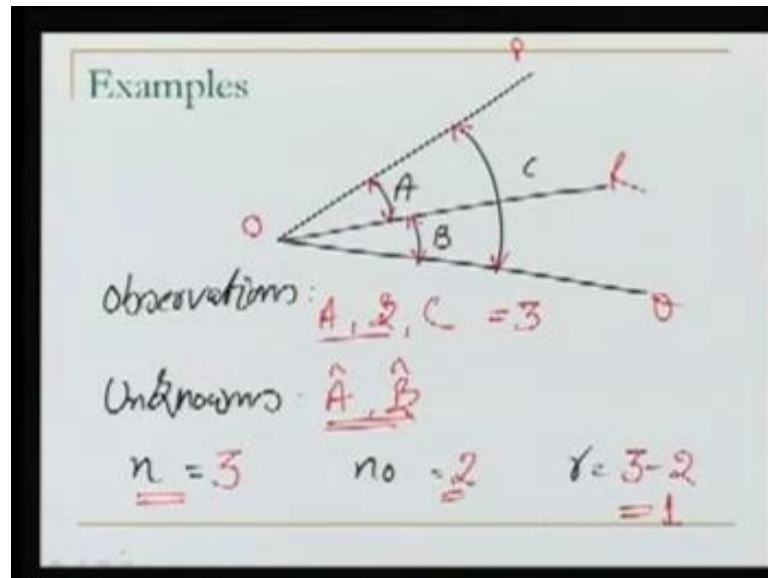
(Refer Slide Time: 00:33)



So, what we know right now, about the adjustment is that we can carry out the adjustment by observation equation method and the condition equation method. There are some more ways of doing it, but right now for this video lecture we will be restricted to only these 2 methods. Now, any problem can be solved by either of these 2 methods, but sometimes the problem is known we can solve easily 1 problem by a particular method from either of these methods. Now, we will see today some examples of these 2 methods, the condition equation method and observation equation method. Or rather what we will try to do? We will try to solve 1 problem by both the methods.

The important thing as I discussed in my last lecture also important thing is, how to frame the basic equations? How to say the conditions in our or the how to frame the basic model, the stochastic model and the functional model? That is the most important. Later on as we have seen the solution is just matrix manipulation. Then finally, towards the end of this lecture we will see one specific problem, which is the braced quadrilateral. Very often whenever we are in field we are doing the triangulation. We have to go for a base quadrilateral a very important figure. What is the procedure for adjustment of that? So, this also we will cover today.

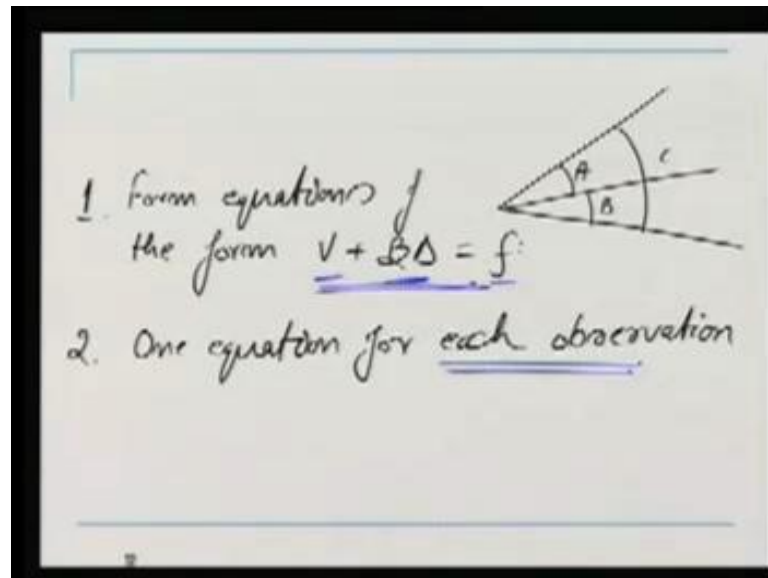
(Refer Slide Time: 01:59)



Fine, our first example is over here there is a observation point O and 2 points; P and Q are there and as well as R. And what we have done? We are observing the angle A by theodolite as well as angle B and also we are observing the angle C. Now, in this case the observations are A B and C. Well, what is the job what we desire from this? This particular problem we desired thing right now is only A and B. Someone wants to know the angles A and B, but while he is taking the observation in order to include redundancy he also takes that full angle C. So, he is taking 3 number observations. So, we can say n number of the observations is 3 notes.

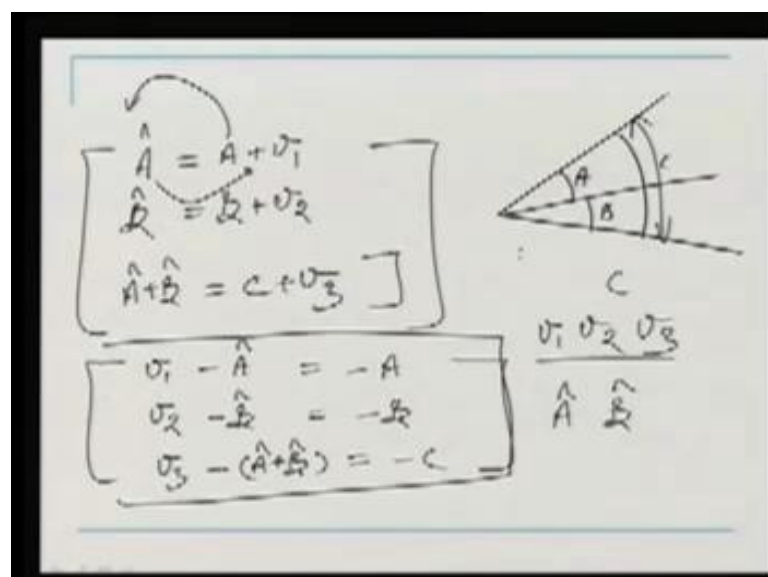
The desired thing is angle A and B. So, we can say A and B these are the desired I am writing A hat and B hat to differentiate them from the observations A and B. So, these are the unknowns. These are the parameters, which you want to determine for this particular model. So, we need to carry out minimum number of 2 observations in order to find a solution for this model or for this problem. So, the r is 3 minus 2 which is 1. Now, what we will we do? We will try to handle this problem of adjustment, because naturally we need to carry out the adjustment here. We have A B and as well as C. So, we need to carry out the adjustment. So, that we have the most probable values or the best estimates for A and B.

(Refer Slide Time: 04:01)



Now, what we will do? We will do it first by observation equation method. In this observation equation method, we know if you recall our last to last lecture, that we write the observation equations in which we include the observation as well as the parameters all are included here. Then for each observation we write one equation. And finally, we write it of this form v is sorry v plus B delta is equal to f . And one question for each observation. Now, what will do?

(Refer Slide Time: 04:41)



We will write the same thing over here, first of all for observation A. So, we can write \hat{A} as $A + v_1$. What is v_1 here? v_1 here is the residual or the error in observation A all right. Then we can write for \hat{B} , \hat{B} and \hat{B} these are the parameters here these are the unknowns here is $B + v_2$. Similarly, now, for this total angle can I write this total angle that is C in terms of my parameters or the unknowns? Well I can write as $\hat{A} + \hat{B}$ is equal to $C + v_3$. This important here, what we are doing here? What we have observed we have observed that full angle and that full angle is nothing, but $\hat{A} + \hat{B}$. And that full angle the value of that is C and in that C there is some error you know v_3 .

So, this is how we are writing it. Now, we will rearrange it. So, that we bring all the v terms on left hand side. So, we can write it as $v_1 - \hat{A}$ is equal to $-A$. Similarly, $v_2 - \hat{B}$ is equal to $-B$. What I am doing? This A is going here and \hat{A} is coming here. So, $v_1 - \hat{A}$ is $-A$ and $v_2 - \hat{B}$ is $-B$. Then similarly $v_3 - \hat{A} + \hat{B}$ is equal to $-C$. Now, we have to write it finally, as we saw in this form $v + B\Delta = f$. We have to write it in this form. We know that in our observation equation method we reduce our equations to this form. So, how we can write it here our residuals here are v_1 v_2 and v_3 . So, if I include if I now, write this in matrix form. So, what we will do our parameters are \hat{A} and \hat{B} ? So, we can write it as now this entire thing is being considered here.

(Refer Slide Time: 07:44)

The image shows a whiteboard with handwritten mathematical equations. At the top, the equation $V + B\Delta = f$ is written. Below this, the vector V is defined as $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ and the vector Δ is defined as $\Delta = \begin{bmatrix} \hat{A} \\ \hat{B} \end{bmatrix}$. Further down, the matrix B is defined as $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & -1 \end{bmatrix}$ and the vector f is defined as $f = \begin{bmatrix} -A \\ -B \\ -C \end{bmatrix}$.

So, I can write V as, $v_1 v_2 v_3$ and we have to write $V + B \Delta$ is equal to $f + v$ plus B delta is equal to f that is the form you have to write. What is delta? We know is a matrix of parameters which are A hat and B hat. Now, if it is, so what is B, what will be the B? To look at that B we have to go back? So, B is a matrix of coefficients of A hat and B hat. So, in this equation, in this very first equation the coefficient of A hat is minus 1 and the coefficient of B hat is 0. So, it is minus 1 0 over here coefficient of A hat is 0 and for B hat it is minus 1. So, 0 minus 1 then in the third 1 the coefficient of A hat and B hat both are minus 1. So, we can write minus 1 and minus 1. So, that is our B, what is f? f is the matrix of constants the constants here are minus A minus B and minus C. So, f is minus A minus B and minus C. Now, once we have determined this we can now straight away find the solution.

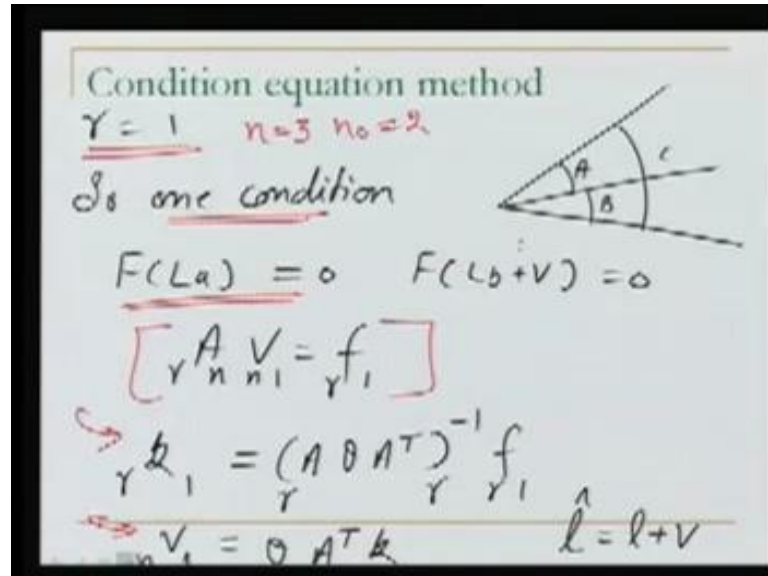
(Refer Slide Time: 09:29)

The image shows a handwritten derivation of the least squares solution. At the top, the equation $\Delta = (B^T W B)^{-1} (B^T W f)$ is written in blue ink. Arrows point from the terms in the equation to their definitions: B^T points to the top row of a matrix $\begin{bmatrix} A \\ B \end{bmatrix}$, W points to the identity matrix I , B points to the bottom row of the matrix, and f points to the vector f . Below this, the vector $\begin{bmatrix} A \\ B \end{bmatrix}$ is underlined and an arrow points to the word "Desired". At the bottom, the equation $\Delta = (B^T W B)^{-1} (B^T W f)$ is underlined in blue ink.

The solution for this, we know can be written as inverse and B transpose wf this what, I have written over here that is the solution. Because right now, we know the w well above this observation the observations, which we had. You know in this case we had 3 observations A B and C. Do we know anything of the weight of this observation, if we know the weight, so this is the weight matrix and then f this also known. So, from here we know the least square solution, so A hat and B hat both will be known to us. And this is what is desired these are the desired quantities. So, again in this problem, what is important? Important thing is the very first step when we are forming the equations as I can pointed out here. This step is important here and this know once we have we are able

to write this basic model then rest of the solution is easy rest of the solution is only a matrix manipulation as you can see here.

(Refer Slide Time: 10:54)



Fine what we will do will solve the same problem now, by condition equation. And in this condition equation method, we know at the moment we have n is equal to 3 and n_0 is equal to 2. So, our redundancy is equal to 1 r is equal to 1. And as we have seen in the case of condition equation method we need to write one condition here. Now, what that condition will be? Well we need to write one condition and the solution as we know solution follows like this. You know our basic condition, we have to write in this way and then we have to form the equation in this form AV is equal f . And then we find the value of K , and then finally we find the value of v well we come back to this later on. Let us write the condition.

(Refer Slide Time: 11:48)

$$F(La) = 0$$

$$\hat{C} - (\hat{A} + \hat{B}) = 0 \quad \text{--- (I)}$$

$$L_a = L_b + v$$

$$C + v_3 - (A + v_1 + B + v_2) = 0$$

$$-v_1 - v_2 + v_3 + C - A - B = 0$$

$$-v_1 - v_2 + v_3 = A + B - C = f$$

So, to write the condition our observations are A and B and that is the C third observation. Now, what is the condition here? The condition here I can write as C hat minus A hat plus B hat should be equal to 0 is not, because what these are? These are the adjusted observations or the estimates of the observations if these are adjusted observations. Then they should satisfy this condition that is the condition and what is this? This is in the form of FLA is equal to 0 and this is 1 equation, because we need to write only 1 here.

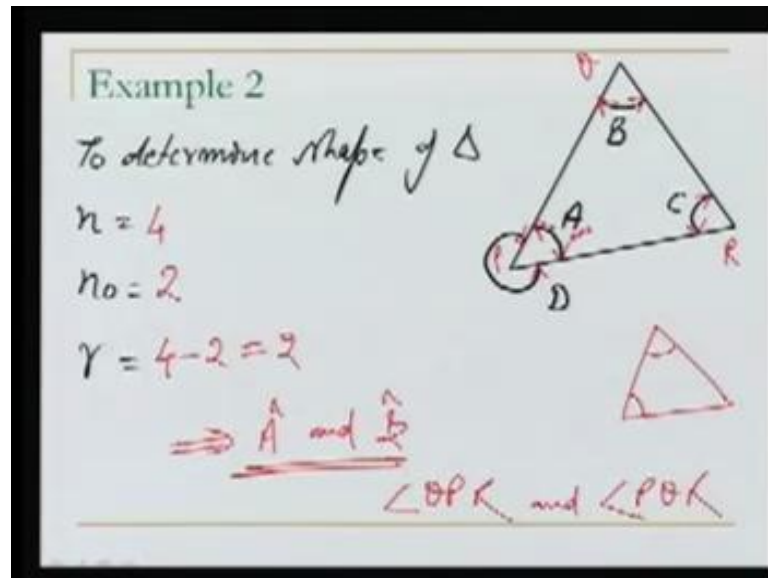
So, this is the one and only independent condition which is ((Refer Time: 12:34)). So, once we have written this now, we can write or rather we can replace L_a by L_b plus v . We have seen this before also. So, we can write it as C plus, v_3 minus; A plus, v_1 plus, B plus v_2 this is equal to 0. And finally, as we know what we have to do? We have to write it in this form AV is equal to f. So, we will try to write it in this form minus v_1 , minus v_2 , plus v_3 this is plus C, minus A, minus B is equal to 0. So, minus v_1 , minus v_2 , plus v_3 this all equal to A plus, B minus, C and this is the constant f if I write this entire thing again over here.

(Refer Slide Time: 13:46)

$$\begin{aligned} -v_1 - v_2 + v_3 &= f \\ AV &= f \\ v &= \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad A = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix} \\ f &= [w] = (A+B-C) \\ A, V, f &\Rightarrow K \quad \begin{aligned} \hat{A} &= A + v_1 \\ \hat{B} &= B + v_2 \\ \hat{C} &= C + v_3 \end{aligned} \\ [w] &\Rightarrow V \end{aligned}$$

Minus v_1 , minus v_2 , plus v_3 this is equal to f . We want to write it as AV is equal to f . So, our v is v_1 , v_2 and v_3 . So, our A becomes minus 1, minus 1, and 1. And f is that particular constant which was A plus B minus C . So, once we have determined A and V and f now, the rest of the solution is easy. First of all we will find for K of course, we need to know what is the weight matrix also. Once we know the weight matrix we can solve for K now. Once we have found for K we also solve for v . So, this is what the general form is shown over here. We know now, A from the weight matrix or from the precision of the observations, we know θ we know f . So, K is known once K is known we can also find the v the residuals. Once the residuals are known, we can find our \hat{A} , \hat{B} and \hat{C} as A plus v_1 , B plus v_2 , C plus v_3 . Now we will see one more example. Now, in this example what is there as you can see?

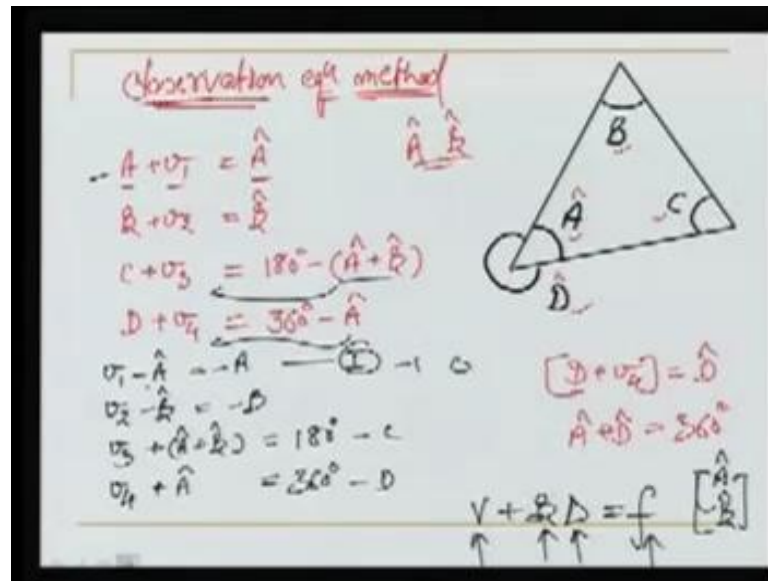
(Refer Slide Time: 15:30)



There are 3 points P Q and R. And the job is someone wants to determine the shape of the triangle. In order to determine the shape of the triangle, what all we need to observe if our observations are true observations. I need to observe only 2 angles, because the third angle can be determined by the geometric condition. So, minimum number of observations which we need to fix the shape of the triangle is 2, but someone were working in the field what he did he observed angle A; angle B also angle C also. So, basically A is the observation, the value for angle Q P R. At the same time we also introduced more redundancy into it.

So, what he also observed? He observed angle D he closed the horizon there. So, number of the observations which he has taken is 4. So, in this case the redundancy in our model is 2 nothing important here. What is desired here? In order to know the shape of the triangle the desired thing is \hat{A} and \hat{B} . If we know these 2 or we can say know angle QPR and angle PQR. If you know these 2 the third 1 can be determined know means you know the estimate of that. So, these are our parameters here 2 parameters now, what we will try to do? We will try to adjust this model this column by again 2 methods the observation equation method and the condition equation method. So, first of all we will try to do it by observation equation method.

(Refer Slide Time: 17:38)



In observation equation method, we know that we have to write the equations number of equations should be same as number of observations. So, for each observation we are going to write 1 equation. All right, so this is what we are going to do, what are the observations here? The observations are A B C and D. So, let us write those equations we can write number 1 A plus v 1 is A hat. And in these equations as we have seen earlier also in this observation equation that we are writing here we write the equations including the observation as well as the parameter we relate them. So, how we are relating them here? We are relating the observation is A v 1 is the residual and this is the parameter. That is ((Refer Time: 18:42)) we can write for B plus v 2 is B hat.

Now, we have only 2 parameters here; A hat and B hat. And we have to write further equations using only these 2 parameters. How we can do that? Well we have one more observation here that is C in C the residual is v 3 C plus v 3 this should be equal to 180 minus A hat plus B hat. Well, again we can write, because we have one more observations that is D, D for the observation D and some error in that that is v 4 this is equal to 360 minus A hat. You know the moment, I am writing D plus v 4. What is this? This is nothing but the adjusted values of this angle D and of course, A hat plus D hat they should be 360, because over here horizon is been closed. So, A hat and D hat they form 360 angle.

This is why I am writing that D plus v_4 is 360 minus A hat, because I have to write we have only 2 parameters here A and B hat. So, all these equations are being returned in the form. Next v D adjustment in the adjusting them, what we do? We take all the parameters as well as the residuals on left hand side and the constant on the right hand side. So, we can write it as v_1 minus A hat is minus A . Let me use a different colour here in order to differentiate from the equations here. So, for the number 1 it is v_1 minus A hat is minus A . Similarly v_2 minus B hat is minus B and v_3 plus A hat plus B hat is 180 minus C , because I have taken this their parameter terms.

Then v_4 plus A hat this A hat comes here v_4 plus A hat is 360 minus D . Well these equations we will again rearrange them, because what we know finally, we have to form this. In this form v plus B delta is equal to f . We have to write it in this form where delta is a matrix of parameters. Parameters here are A hat and B hat. This is delta and B is the matrix of coefficients. And v is the matrix of residuals; f is the matrix of constants. So, in order to do it you can see over here also in this first equation the coefficient of A is minus 1 coefficient of A is minus 1; coefficient of B hat is 0. So, what will do? We will rearrange this.

(Refer Slide Time: 22:56)

$$v_1 - 1A + 0B = f_1$$

$$v_2 - 0A - 1B = f_2$$

$$v_3 + 1A + 1B = f_3$$

$$v_4 + 1A + 0B = f_4$$

$$V + B \Delta = f$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

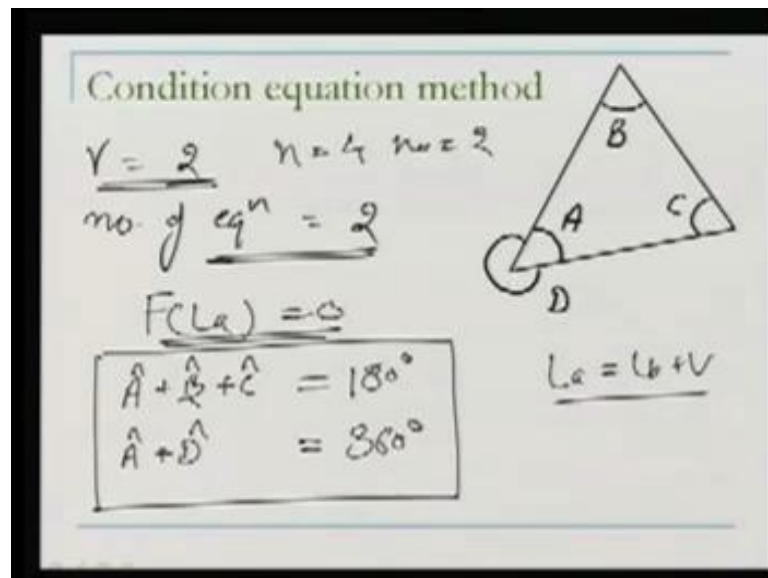
$$\Delta = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

So, v_1 minus 1 into A hat plus 0 into B hat this is equal to let me write as f_1 the first coefficient, because the terms here are all constants. So, I am writing f_1 f_2 f_3 and all that. Then v_2 minus 0 0 plus 0 0 into A hat minus 1 into B hat is f_2 then v_3 . And we

see with the v_3 we have A hat and B hat 1 into A hat plus 1 into B hat f_3 and then finally, we have A hat here v_4 plus 1 into A hat plus 0 into B hat this is f_4 . Now, we write it in matrix form now. So, our matrix v is v_1 v_2 v_3 and v_4 . The matrix B based the matrix of coefficients we as we can see here is $\begin{bmatrix} -1 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$ and f is f_1 f_2 f_3 and f_4 . So, once we have determined B , f we can solve for Δ . That solution we know, how to do it? The important thing is here is to form these basic matrices or to write that basic model. So, once we know that you can compute now, or you can determine A hat and B hat. Now, we will solve the same problem by condition equation method, for the condition equation method.

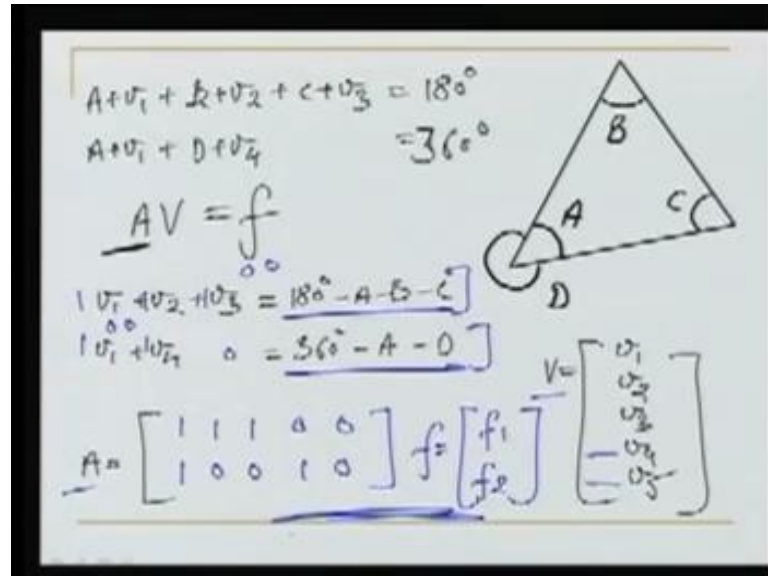
(Refer Slide Time: 25:20)



We know there is a redundancy, which is equal to 2 here; because number of observations was 4 n naught was 2. So, redundancy is 2. So, what we need? We need to form 2 conditions here; now, what these conditions are? So, what we have to do? Now we have to look into our system of observations in our problem. And we have to find those conditions. So, the 2 conditions which we can write here and the conditions have been written in terms of please remember that we have to write it this way in terms of adjusted observations or estimates of the observations. How we can write that? Well, first A hat plus B hat plus C hat these are the adjusted observations of these 3 angles. It should be 180 degrees. And the second one we can write as A hat plus D hat is 360 degrees. So, these are the 2 basic conditions we cannot write any other independent

condition in this case. Now, once you have done it we replace L_a by L_b plus v let us do it.

(Refer Slide Time: 26:46)



We can write it as $A + v_1 + B + v_2 + C + v_3$ this equal to 180 degrees and then $A + v_1 + D + v_4$ this equal to 360 degrees. So, what I am doing for A. I have replacing that by $A + v_1$. v_1 is the error in observation A. And for D, D hat is observation D plus v_4 . v_4 is the error in observation D. So, this is what we get now. Now, as we know finally, we have to reduce it to this form AV is equal to f . To reduce it to this form all the constant terms are to be taken to the right hand side all right. So, we will we do the same thing.

Well I am writing it as now, $v_1 + v_2 + v_3$ is 180 minus A minus B minus C as well as $v_1 + v_4$ is 360 minus A minus D. So, all the constant terms are now on right hand side. Next what is A? We know now, we know V here is $v_1 v_2 v_3 v_4 v_5$. This is V the matrix of residuals. So, what is A? A is the matrix of coefficients of residuals. So, what the A will be A here will be if you look at the coefficients let me change the colour for these coefficients. Well over here the coefficient is 1 1 1 then for v_4 and v_5 it is 0. And 0 again 0 0 1 and for rest for v_5 it is 0 for v_2 and v_3 also it is 0.

So, I can write A as 1 1 1 0 0 1 0 0 1 0 That is A and what is. So, this AV , and what is f ? f is we can say f_1 and f_2 where f_1 is this term and f_2 is this term. So, this is the constant here with another constant. So, this f_1 and f_2 , so what we know now at this

stage? We know A, we know f and we know in our method of condition equations. If we know A and f we can find K once we find K we can also find the V and once we have found the V we can find our parameters. So, the important thing was forming these basic matrices then rest it is simple matrix manipulation. So, we can solve this particular problem by condition equation method in this way. Now, we will solve the same problem by condition equation method. Now, in this case we know.

(Refer Slide Time: 30:23)

The slide contains the following content:

- Title:** Condition equation method
- Equations:**
 - $V = 2$ $n = 4$ $n_0 = 2$
 - no. of eqⁿ = 2
 - $$\left[\begin{array}{l} \hat{A} + \hat{B} + \hat{C} = 180^\circ \\ \hat{A} + \hat{D} = 360^\circ \end{array} \right]$$
 - $$\begin{array}{l} \underline{A + v_1} + \underline{B + v_2} + \underline{C + v_3} = 180^\circ \\ \underline{A + v_1} + \underline{D + v_4} = 360^\circ \end{array}$$
- Diagram:** A triangle with interior angles A, B, and C. Angle B has a question mark above it. An exterior angle D is formed by extending the side AC.
- Formulas:**
 - $F(L_2) = 0$
 - $\uparrow L_2 + v$
 - $AV = f$

That we had n is equal to 4 while n naught was equal to 2. So, redundancy is 2 we need to form 2 conditions here now, what are those conditions. So, we will look into our system of observations and what is desired is there any geometric condition is there any observation condition. So, we need look into that. And we need to find those conditions well the very first condition we can write as A hat plus B hat plus C hat this should be equal to 180. Well A plus B plus C this is not 180, because A B and C these are the observations. And these observations have got some error, but A hat B hat and C hat they are the estimates of the ((Refer Time: 31:19)) adjusted values. So, they do not have any error. So, that is why that is the first condition and the second condition can be written as A hat plus D hat is 360 degrees. So, these are the conditions 2 conditions.

Now, what is this of form? This is of the form FL is equal to 0. Now, we need to replace for this adjusted observation as Lb plus v. Now let us do it, so I can convert this as now, A plus v 1 plus B plus v 2 plus C plus v 3 this is equal to 180. What I am doing I am

replacing A hat with A plus v_1 . Similarly, A for the second equation A plus v_1 plus D plus v_4 is 360 degree. So, v_4 is the error in observation D all right or the residual for that observation D . Once we have done it because finally, we need to write this in a form AV is equal to f . We need to write in this form, and where f is the constant term. So, all the constant needs to be taken on the right hand side. So, I will take these constants you know $A B C$ to the right hand side now, in taking these to right hand side we can write it as now.

(Refer Slide Time: 33:06)

The image shows a handwritten derivation on a slide. At the top, a triangle is drawn with interior angles labeled A , B , and C . An exterior angle at vertex A is labeled D . Below the triangle, the following equations are written:

$$A + v_1 + B + v_2 + C + v_3 = 180^\circ$$

$$A + v_1 + D + v_4 = 360^\circ$$

These are then rearranged into the form $AV = f$. The matrix A is shown as:

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

The constant vector f is shown as:

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

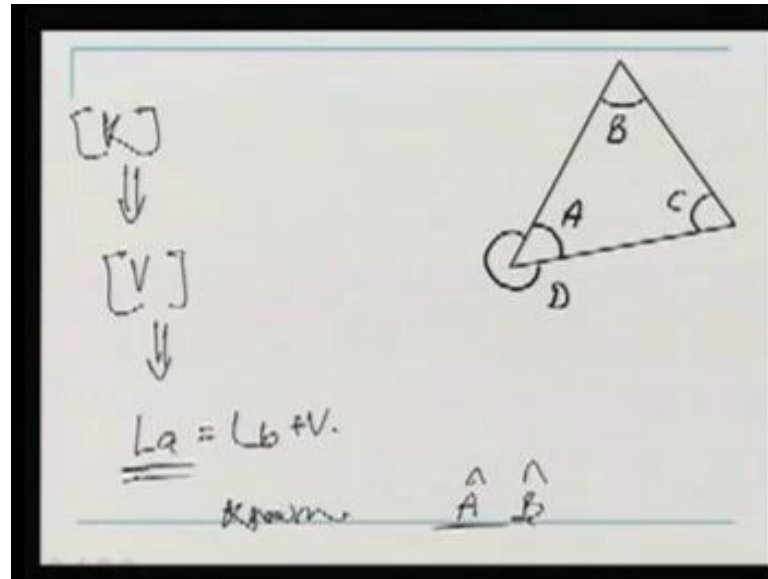
The residual vector v is shown as:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

Arrows point from the equations to the corresponding rows and columns in the matrices.

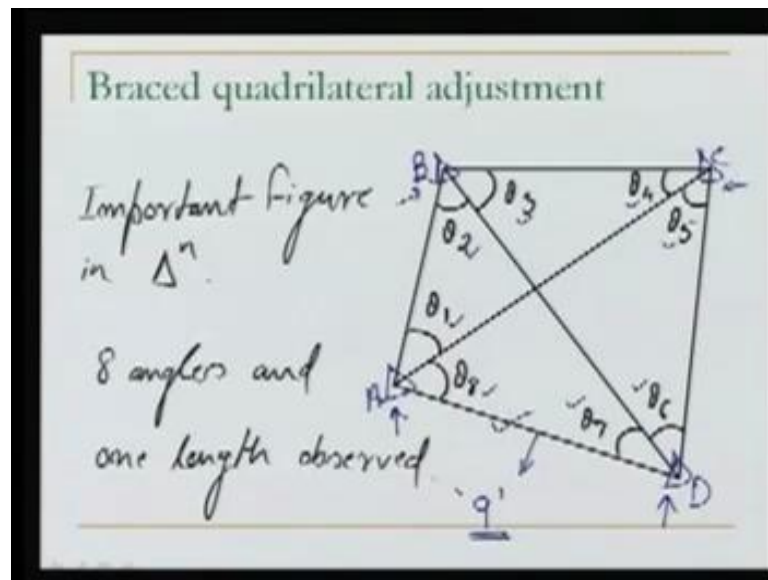
These are the basic equation which we are writing here same and by taking these constants to the right hand side we can write v_1 plus v_2 plus v_3 this is 180 minus A minus B minus C and v_1 plus v_4 this is equal to 360 minus A minus D . Let us say this is f_1 and this f_2 the constant terms. Further, because finally, we have to have in the form of AV is equal to f . Now, v what is v ? v is a matrix of residuals, which is v_1 v_2 v_3 and v_4 , because we have 4 observations. So, we will have 4 residuals and this matrix B . Now, what is matrix A ? A is the matrix of coefficients of these residuals. So, over here it was C for matrix A the matrix A can be written now, as if I write the coefficient the coefficient is 1 1 and 0 and 0 for sorry 0 for v_4 . Over here it is 1 for v_1 1 for v_4 0 and 0 for v_2 and v_3 . So, I can write it as 1 1 1 0 1 0 0 and 1 . So, that is A and so we know f is f_1 and f_2 . So, at this stage we know, A we know f we can find now K .

(Refer Slide Time: 35:09)



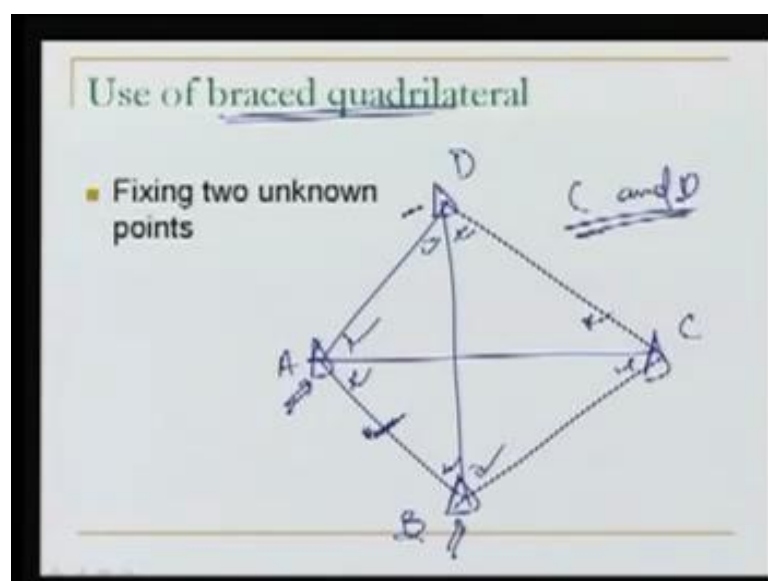
So, K can be determined once, the K is determined we can find v was the v v is determined we can also find the La. La is Lb plus v all right. So, our adjusted observations are known once. You know the adjusted observation you know we also know now; that means, A hat and B hat our basic thing you know basic parameters which define the problem. So, we know now, the shape of the triangle. So, this particular problem has now been adjusted by condition equation method. Now, what we will do? We will see the adjustment of braced quadrilateral we know what is the braced quadrilateral. We have seen that when we are talking about the triangulation and we will do this adjustment by condition equation method.

(Refer Slide Time: 36:00)



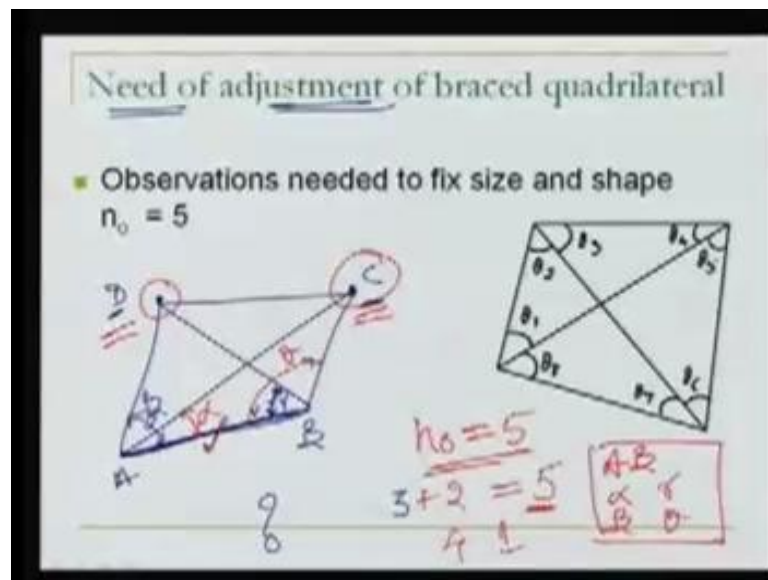
Now, a braced quadrilateral, as you can see here as A B C and D these are the triangulation stations. And from this triangulation stations the thing is here we know one of the line. This line is known from previous triangulation or you know it has been measured or may be known from the previous triangulations. So, this is known to us. What you want to do? This point is fixed this point is fixed and you want to fix B and C. Now, in doing this we observed 8 angles theta 1 theta 2 theta 3 theta 4 theta 5 theta 6 theta 7 and theta 8. And this length is already known. So, total number of known quantities or observed quantities we can say is 9 here.

(Refer Slide Time: 37:06)



Now, a little bit about why we need the braced quadrilateral basically what we are doing here, because we know the point A let us say and point B these 2 points are known. So, this length is also known and by observing these angles we are aiming to establish point C and D also. We want to find the coordinates of C and D and this is very important you know when we are doing the control survey we are extending the control. So, by knowing this length A B or maybe you know we know the position of A we know the position of B. We can now, by taking these angles into account we can compute the position of D and the position of C. Now, in a braced quadrilateral why there is a need of adjustment?

(Refer Slide Time: 38:05)



Why we need to adjusted? Let us see our aim is as we are looking at you know braced quadrilateral our, because this line is known. What are the way? And in order to fix point D and C; this is A B. In order to fix point C and D, what is the minimum number observation that we need to know? The minimum number well, I can fix D. For this figure by knowing this total angle and by knowing this angle I can fix point D. So, I need to know this length; this angle and this angle. So, 3 observations plus in order to fix C, I need to know the angle here and the angle total 1. So, I can draw this by a different colour. So, fix it, I need this total angle and this angle. So, the length is already known.

So, plus 2 observations, so what we need to know in order to fix D and C from known length we need to have a total of 5 observations. 5 means 4 angles and 1 length this

length and 4 and 4 angles. So, right now I am saying well I need to know the angle here alpha and the angle theta. This full angle is beta for write it in the blue colour. This angle is the beta the full angle and here this small angle is let us saying gamma and this big angle is theta. So, basically what we need to know in order to fix D and C, I need to know 1 length A B and angle alpha beta gamma and theta. So, we should have these 5 basic observations. So, we say n naught is equal to 5. Well, what actually we do as you seen in this particular figure, because only 5 observations can solve our job. But in that will be a unique solution and we will not have any control on the error.

(Refer Slide Time: 41:04)

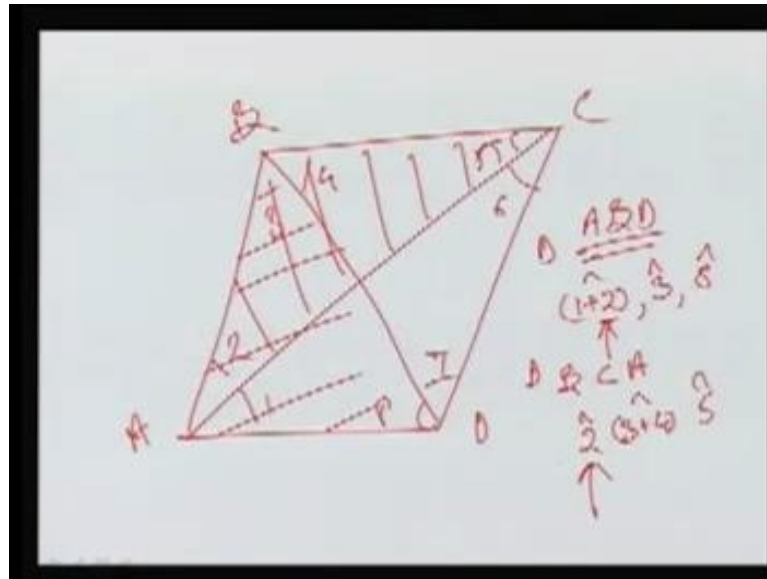
Need of adjustment of braced quadrilateral

- Observations needed to fix size and shape
 $n_0 = 5$

$\left. \begin{array}{l} 1 \text{ length} \\ 8 \text{ angles} \end{array} \right\} \underline{9 \text{ obs.}}$
 $n = 9$
 $r = n - n_0 = 9 - 5$
 $\underline{= 4}$

So, what exactly we do this length is known to us. We also take now several angles we take all these angles. So, rather we are observing 1 length A B plus 8 angles. So, how many observations we are taken here? In order to do this braced quadrilateral we have taken 9 observations. So, from here we know the redundancy because n is 9 here. So, the redundancy here is n minus n naught that is 9 minus 5 and that is 4. Why this redundancy? Because we took extra angle we took these are the extra angles. Even if you do not observe these angles just by knowing the length here and these angles the problem can be solved. So, we have a redundancy of 4 here in this particular problem, because you have the redundancy. So, we would like to go for the best possible solution one more thing here.

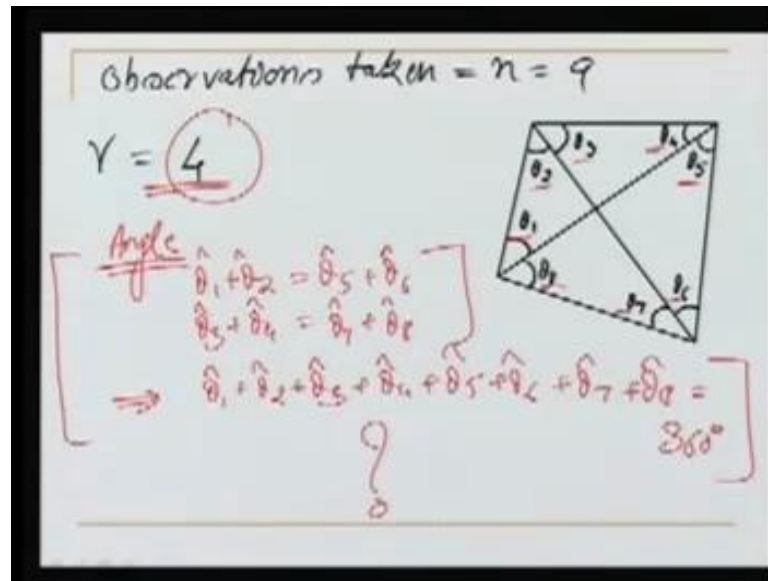
(Refer Slide Time: 42:13)



In a braced quadrilateral as we have observed in angle 1 2 3 4 5 6 7 8 and all these angles have some error in them. Well if I adjust for example, let us say a triangle I give them name A B C D. If I adjust only the triangle A B and D if I adjust them well I will have 1 plus 2 hat 3 hat and 8 hat the adjusted values. If I adjust triangle, which is B C and A then I will have again 2 hat 3 plus 4 hat and 5 hat. So, what we are seeing here? If I adjust this braced quadrilateral in parts I am adjusting only this triangle first I have the adjusted values of the observation inside. Then I can also adjust this particular triangle.

Now, the angle for example, the angle number 2 it has a 1 value here and 1 value here. So, angle number 2 is being adjusted in 2 different ways. So, it will have 2 different values which not possible. What I mean to say we cannot adjust this kind of problem in parts rather we have to take all the observations together we have to form one system. In that system all the observations should participate. So, whatever the conditions are there the condition should be satisfied it is not that we are adjusting them in parts well.

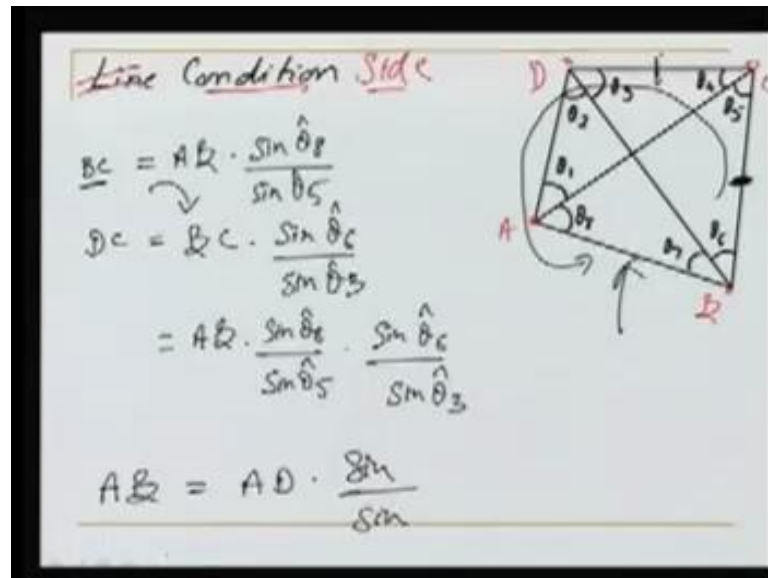
(Refer Slide Time: 44:19)



So, we know now in this case n naught was 5 n was 9, r was 4. So, we need to form 4 equations or the 4 condition equations. Now, what this condition equation will be here? The condition equations will be as you can see over here; can we form some angle conditions? Well, let us say and again these 4 conditions should be independent. So, first condition equation we can form as θ_1 plus θ_2 is equal to θ_5 plus θ_6 . So, θ_1 plus θ_2 is θ_5 plus θ_6 of course, all these are the adjusted values. So, I am putting $\hat{\theta}_1$ plus $\hat{\theta}_2$ is equal to $\hat{\theta}_5$ plus $\hat{\theta}_6$.

Similarly, θ_3 plus θ_4 is θ_7 plus θ_8 hat, what I am doing this plus? This is this angle plus this angle. Well I can write 1 more equation which is independent of these that is θ_1 plus θ_2 plus θ_3 plus θ_4 plus θ_5 plus θ_6 plus θ_7 plus θ_8 this is equal to 360. So, this also one more condition that I can write, so using the angles I can write only 3 independent conditions. Now, what is the fourth 1, because we need 4 conditions, what is the fourth condition now? So, these 3 conditions, which we say angle conditions these are the angle conditions, which we have just written here these are the angle conditions. Well, we need to write one more condition now and that condition we say as side condition.

(Refer Slide Time: 46:45)



Not line condition it is called side condition. Now, what that side condition is? The side condition takes into account the length or rather the figure is closed. Because you know this figure needs to be a closed figure as it is in the field. In the field we have 4 stations and those 4 stations they form a closed figure. So, we have to ensure that our angles are such that they form a closed figure. Now, how to be ensured that? Let us say it is A B C and D that is our quadrilateral I can do one thing. We can write here BC in terms of AB as using the sin rule sin of BC. So, sin theta 8 divided by sin of the angle in front of AB is theta 5 is not.

And of course, these are the theta 8 hat and theta 5 hat the figure will be closed not for the observations, but for the adjusted observations that is why I am using theta hat. Well now I can write DC DC in terms of BC. So, I can write it as BC BC sin theta 6 by sin theta BC 3 well I replace BC over here. So, I get AB sin theta 8 of course, hat everywhere sin theta 8 and sin theta 5 hat sin theta 6 hat sin theta 3 hat. If we continue this process and, because right now, you know we started we are writing BC in terms of AC then we wrote DC in terms of BC and replace this BC by AB. And then if you continue doing it and finally, we reach again at AB. So, what we can do we can write AB again in terms of AD and sin of the respective angles. So, what we get by doing this?

(Refer Slide Time: 50:00)

The image shows a whiteboard with handwritten mathematical derivations. At the top, the equation $AB = AB \cdot \frac{\sin \hat{\theta}_8 \cdot \sin \hat{\theta}_6 \cdot \sin \hat{\theta}_4 \cdot \sin \hat{\theta}_2}{\sin \hat{\theta}_7 \cdot \sin \hat{\theta}_5 \cdot \sin \hat{\theta}_3 \cdot \sin \hat{\theta}_1}$ is written. A curved arrow points from the AB on the right to the AB on the left. Below this, two brackets are shown: $[\sin \hat{\theta}_7 \cdot \sin \hat{\theta}_5 \cdot \sin \hat{\theta}_3 \cdot \sin \hat{\theta}_1]$ and $[\sin \hat{\theta}_8 \cdot \sin \hat{\theta}_6 \cdot \sin \hat{\theta}_4 \cdot \sin \hat{\theta}_2]$, with an equals sign between them. The next line is $\sum \log \sin \hat{L} = \sum \log \sin \hat{R}$. The final line, labeled with a circled 'IV', is $\sum \log \sin \hat{L} - \sum \log \sin \hat{R} = 0$.

We get AB is AB sin of theta hat sin theta 8 hat and sin theta 6 hat sin theta 4 hat sin theta 2 hat sin theta 7 hat sin theta 5 hat sin theta 3 hat sin theta 1 hat. This we get and AB and AB they will cancel out. So, we can write this as now, if I take it here. So, sin theta 7 hat sin theta 5 hat sin theta 3 hat sin theta 1 hat is equal to sin theta 8 hat sin theta 6 hat sin theta 4 hat and sin theta 2 hat. Well I take log of both the sides. So, taking log of both the sides I can write this as now, if you look at that this is 7 5 3 1 what kind of angles are? These look at the figure here 7 5 3 1 if I stand at this point B this angle 7 is towards my left hand side.

If I stand at 7 5 is towards my left hand side. So, what I do sigma log sin of left hand angle this is equal to sigma log of sin right hand angle, because these are right hand angles and these are the left hand angles I can write it like that. And of course, the adjusted value I can write it further as sigma log sin L hat minus sigma log sin R hat that equal to 0. This is the fourth condition, because we had 3 conditions which are coming from the angles and this is the fourth condition which is coming from the side. So, we say this as a side condition and this ensures that the figure is closed figure these are our 4 conditions.

(Refer Slide Time: 52:45)

Four Conditions

$$\begin{aligned} \hat{\theta}_1 + \hat{\theta}_2 &= \hat{\theta}_5 + \hat{\theta}_6 \\ \hat{\theta}_3 + \hat{\theta}_4 &= \hat{\theta}_7 + \hat{\theta}_8 \\ \sum_{i=1}^8 \hat{\theta}_i &= 360 \end{aligned} \quad \left. \begin{array}{l} \text{Angle} \\ \text{Angle} \\ \text{Angle} \end{array} \right\} L_a = 6\theta$$

$$\sum \log \sin \hat{L} = \sum \log \sin \hat{R}$$

Side ↑

Three angle conditions all right, and one side condition. So, there the 4 conditions which we will use for adjustment. Now, we know in the condition equation method that we have to replace L_a by $L_b + v$ and this is what we will do?

(Refer Slide Time: 53:04)

We have to write in $AV = f$ format

$$\hat{L} = L + v$$

$$L_a = 6\theta + v$$

Angle conditions

$$\begin{aligned} \text{I } \theta_1 + v_1 + \theta_2 + v_2 &= \theta_5 + v_5 + \theta_6 + v_6 \\ \text{II } \theta_3 + v_3 + \theta_4 + v_4 &= \theta_7 + v_7 + \theta_8 + v_8 \\ \text{III } \sum_{i=1}^8 \theta_i + \sum_{i=1}^8 v_i &= 360^\circ \end{aligned}$$

Now, to replace this, so we can write for theta 1 hat theta 1 plus v 1 for theta 2 hat theta 5 hat theta 6 hat theta 3 hat theta 4 hat theta 7 hat and theta 8 hat. And this what we have to write you know theta 3 hat theta 4 hat theta 7 hat theta 8 hat. So, this is being reduced to the equation here similarly this is sum of all the angles is equal to 360. So, for all the

angles sum of all the angles plus sum of all the residuals these are the observations now, this should be equal to 360. So, what we are doing? We are replacing L_a by L_b plus v this is what we are doing and the same thing can be done also in the side equation, we will have to do that and the side equation.

(Refer Slide Time: 54:03)

The slide shows the following equation:

$$\log \sin \theta_1 + \cot \theta_1 \cdot v_1 - \log \sin \theta_2 - \cot \theta_2 \cdot v_2 + \log \sin \theta_3 + \cot \theta_3 \cdot v_3 - \log \sin \theta_4 - \cot \theta_4 \cdot v_4 + \log \sin \theta_5 + \cot \theta_5 \cdot v_5 - \log \sin \theta_6 - \cot \theta_6 \cdot v_6 + \log \sin \theta_7 + \cot \theta_7 \cdot v_7 - \log \sin \theta_8 - \cot \theta_8 \cdot v_8 = 0$$

Below the equation, there is a diagram consisting of a horizontal line with a vertical line segment labeled 'IV' on the left. An upward-pointing arrow is drawn from the horizontal line to the equation above it. Below the arrow, the equation $L_a = L_b + v$ is written.

Takes the form of, as I have written over here, the various terms are $\log \sin \theta_1$ θ_1 is the observation $\cot \theta_1 v_1$. We are not getting into this that how we have arrived at that, but this is what once we replace observation. Sorry, the adjusted observation with actual observation in the field plus the residual this side equation reduces to this. Similar the second term minus $\log \sin \theta_2$ minus $\cot \theta_2 v_2$ and so on, this is equal to 0. So, please write it down yourself then only you will understand it and once we have reduced this way.

(Refer Slide Time: 54:54)

In form of $\| \underline{A} \underline{v} = \underline{f} \|$

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \cot \theta_1 & -\cot \theta_2 & \cot \theta_3 & -\cot \theta_4 & \cot \theta_5 & -\cot \theta_6 & \cot \theta_7 & -\cot \theta_8 \end{bmatrix}$$

$[A] \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} \quad f =$

The next step is we need to have our basic model in this form we know it. So, we need to know A, we need to we know v and we need to know f. Now what will be A here if you look at A this A will come from the coefficients of v 1 v 2, so on upto v 8 the coefficients of these from these equations. So, from the first very first equation the coefficient here is 1 1 and minus 1 minus 1 for and for theta 3 theta 4 it is 0 0 for theta 7 and theta 8 again 0 0. So, how we can write it 1 1 0 0 this what theta 1 sorry v 1 v 2 v 3 v 4 v 5 v 6 v 7 and v 8. So, these are the coefficients of v 1 to v 8. Similarly, you can see for this third equation in the case of a third equation, the third condition equation is these are the terms v 1 to v 8 are the terms here and the coefficients for all of these is 1. So, it is 1 1 1 1 1 1 1 and 1 1.

Similarly, for the side equation or the side condition, the coefficients of v 1 the coefficient of v 1 is cot theta 1 coefficient of v 2 is minus cot theta 2. So, this is what we are writing here cot theta 1 minus cot theta 2 cot theta 3 minus cot theta 4 cot theta 5 minus cot theta 6 and so on. So, we know A here we also know v. v is v 1 v 2 and upto v 8 that is v. And what is f? f again it can be determined from here, because all these constant terms you know theta 1 theta 2 all these are coming to the right hand side. So, we can determine the value of the constant. For example here the constant will be 360 minus sigma theta i. So, this is the constant term.

(Refer Slide Time: 57:35)

In form of $AV = f$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix}$$

$$f = \begin{bmatrix} \theta_5 + \theta_2 - \theta_1 - \theta_2 \\ \theta_7 + \theta_8 - \theta_3 - \theta_4 \\ 360^\circ - \sum \theta_i \\ \sum \log \sin R - \sum \log \sin L \end{bmatrix}$$

So, we know the constant term also. So, that is v and these are the constant terms theta 5 plus theta 6 minus theta 1 minus theta 1 theta 2 this and this. So, we know this we know A. So, A and f are known once A and f are known we know, how to solve for it?

(Refer Slide Time: 57:56)

Having known A and f solve in normal fashion for $[K]$

$$\hat{\theta} = \theta + v$$

$\hat{\theta}$ Answer

We can find for K then from the once we know K we can find for v and once the v is determined we can find the values of theta 1 hat theta 2 hat and so on theta 8 hat. All these adjusted observations can be determined. So, what we have seen know the braced quadrilateral can be also adjusted by this condition equation method. So, in this lecture

we saw some examples we solved one you know 1 simple problem or 2 simple problems by both the methods observation equation method as well as the condition equation method. Then later we saw about the braced quadrilateral, we also discussed this question, why it is required to adjust it? What is the use of the braced quadrilateral? And then the procedure of adjusting it by condition equation method. So, basically the important thing is to form that basic model. Once the model is formed, we have the matrices and it is simple matrix manipulation.

Thank you.