Surveying
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Lecture No. # 04
Module No. # 09

Computation and Adjustment

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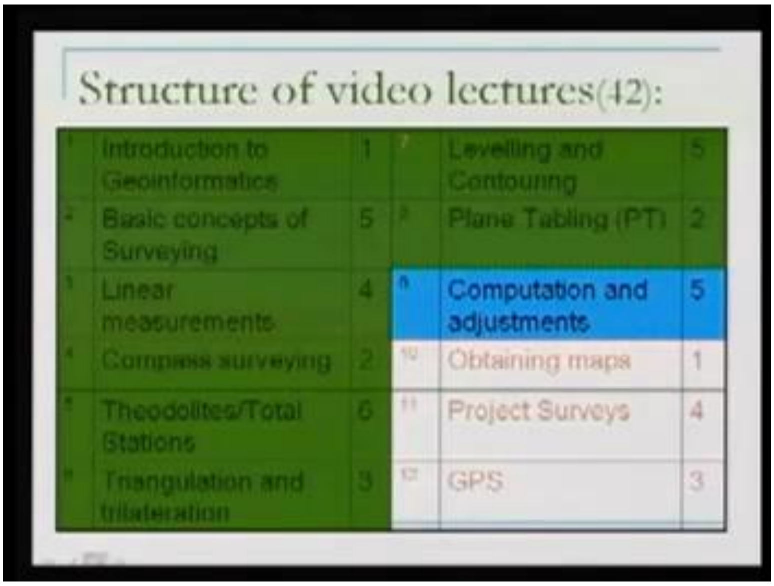

Welcome to this lecture, on basic surveying. And today, we are in module 9 lecture number 4. This is our entire schedule plan, the way we are preceding in this video lecture. So, right now, we are in computation and adjustment.

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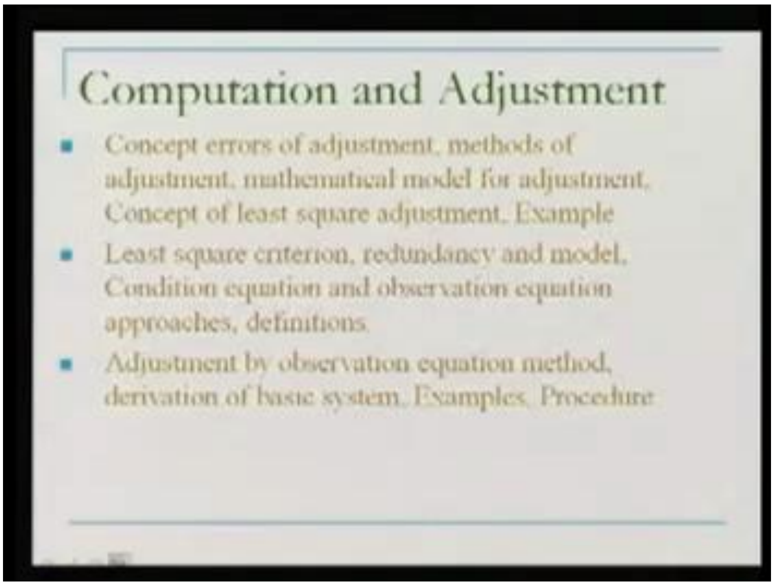

What we have done so far in our last lecture, we talked about adjustment by observation equation method. We saw what is the meaning of the observation equation? How we frame that, how we include the parameters as well as the observations in the adjustment process. And the adjustment is being carried out by least square technique. So, we saw the basic derivation, for this some examples. Finally I also gave the procedure; you know how to apply the steps. Now, today in this lecture, we are going to talk about same adjustment, but we are going to talk about the adjustment by condition equation method. So, what we will do today?

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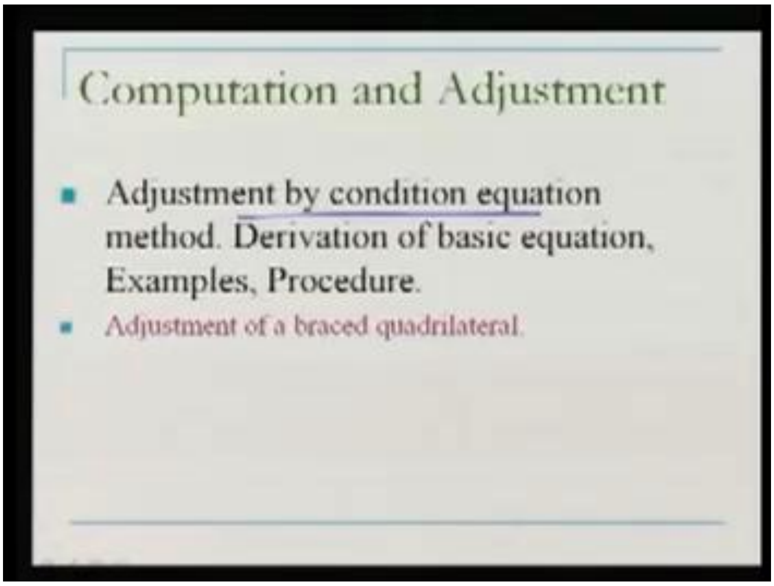

The condition equation method, we will go for the basic derivation. Then again we will see some examples and finally, the procedure.

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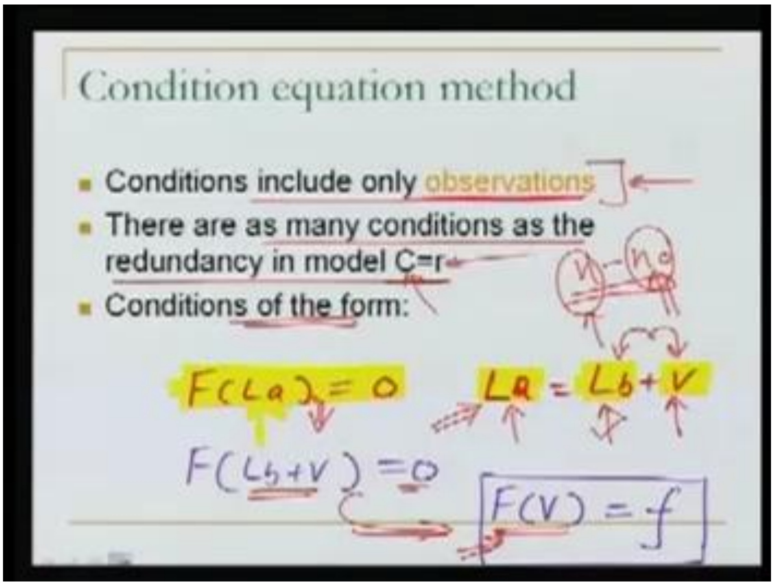

Well in the case of the condition equation method, we know now that why we carry out the adjustment. Because we have the observations from the field, observations have got the redundancy. Always we have something which is desired which we want to compute finally, from these observations. So, in this process those quantities for example, shape of the triangle, can be determined by some minimum number of observations, but we are taking more number of observations. And if I am taking more number of observations there are various ways in which the desired quantity can be computed. So, what we are looking for? We are looking for the best possible way, which is the least square technique.

So, in our adjustment process we are making use of the geometric conditions which we say. Now, the functional model and as well as the quality of the observations which we take into account the quality of the observations by weight matrix or the stochastic model. Now, in the condition equation method, what we will do? Well the conditions include, only observation. This is important; unlike observation equation method, where we are including the observations as well as the parameters the unknowns here. We include or rather we write the conditions using only observations.

So, we should keep this in mind then second there will be as many conditions as the redundancy in model. So, C is number of the equations, which we are going to write and r is the redundancy. We know r is n minus n naught, that is r where n naught is the minimum number of observations, which can defined or our desired thing n is actual number of observations. Now, the conditions we will write of the form, that there is a general form here as you can see here I highlight that is the general form. La stands for adjusted observations. So, basically, we are writing our conditions of this form in terms of adjusted observations La will be writing the conditions.

Now, La which is adjusted observation is Lb is observed. Something which we are observing in the field and v is the residual, because finally, our estimate La is our final estimate the adjusted observation. This is actually, something which is observed in the field, we know this which is observed in the field has some error. So, what we are saying now, error means the correction in something which we observed in the field we are applying this correction and by applying this correction we get La; that means, the estimate or adjusted observation. So, what I can do? I can write it as function of Lb plus v is equal to 0 and finally, this can be written as, a function of variable sorry the residuals v is stands here for residuals is equal to f. So, we will be trying to write our equations of conditions finally, in this form. Now, how we will do it?

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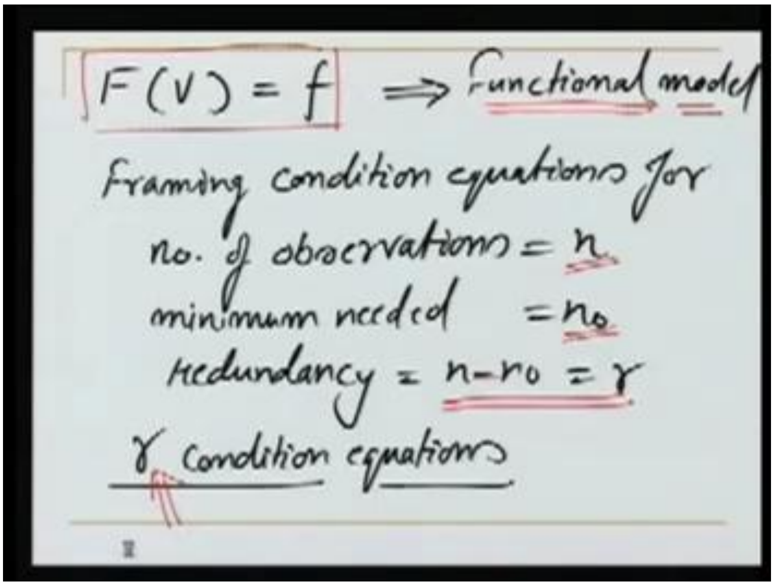

This is what we are going to see now, and this form FV is equal to f is our functional model. Somewhere, in the process of adjustment we will also include our stochastic model well let us do it for 1 case. Right now it is that we have n number of observations, and for that particular problem n naught is the number of observation, which can define the problem or we can which can find the answer for us, the desired quantity can be determined by only n naught. So, number of observations are n, minimum needed n naught, the redundancy in our model is r. Which is n minus n naught? So; obviously, we are going to write r number of condition equations. Now, how we write this a general form? Is a general form.

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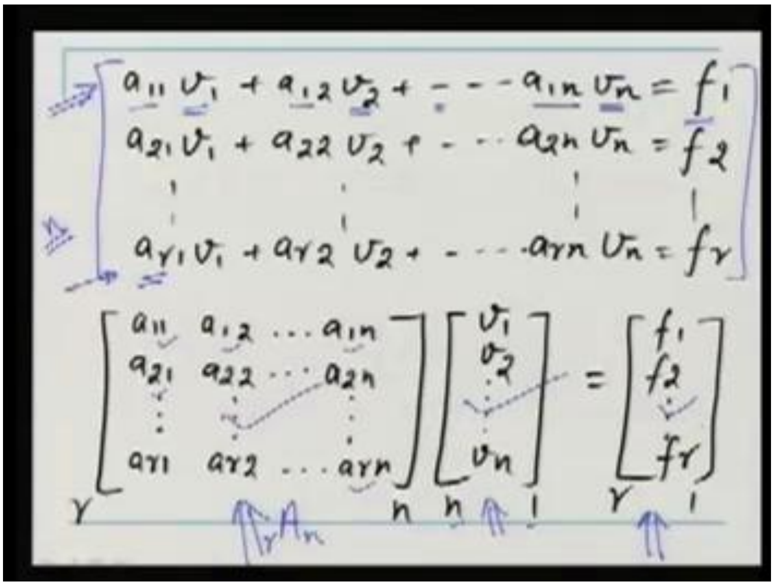

We are writing the equations, in the form of we will we start with number 1 here you can see please notice here v 1 v 2 v 3 and so on vn. What is the meaning of this? They are n number of observations, because they are n number of observations. So, each observation will have its residual. So, we had the residuals from v 1 to vn and as we saw we want to write our model of this form or also I can write this model as AV is f. Where A is a matrix of coefficients, V is the matrix of residuals; f is the matrix of constants. So, this is how I want to write my final model, well the same I am trying to do here. Now, a 1 1 a 1 2 a 1 n, these are coefficients and f 1 is the constant part.

Then we have to write r number of such equations. So, we are writing this as the rth equation now, if this equations we write them in matrix form. So, we get our first matrix here, which is of coefficients a 1 1 a 1 2 a 1 1 1 n a 2 1 and so, on up to a r n. So, this matrix a, is of size rn then we have a matrix for v the residuals, which is of size n 1 and these are the coefficients r 1. Once we look at 1 example then it becomes more clear what we are trying to do, but this is the general form. So, we are starting to learn it from the general form.

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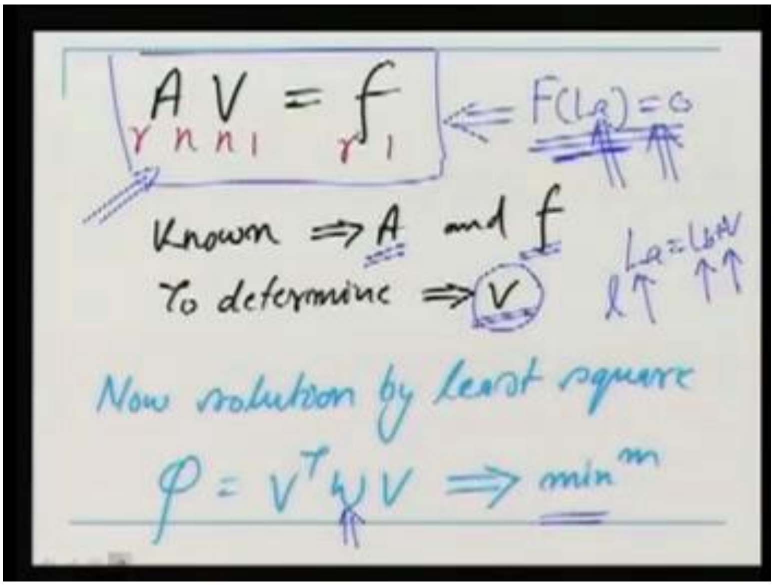

So, general form we can write our basic functional model as AV is equal to f. And how they are arriving? At this we arrive at that from F L a is equal to 0 from they only. What is the meaning? From here we are arriving at this the meaning is first of all we are looking for the conditions in our system. How many conditions are there? We are looking for those independent conditions r number of conditions will be there. And ensure that those conditions would be independent once we have found those conditions, then for those conditions we are writing the equation in the form F L a is equal to 0 in this form.

So, basically, condition number 1 1 set of equations second third fourth depending how many what is the redundancy the total number of equations which are required. Now, once we are writing these equations using the conditions, F L a here it stands for estimate or the adjusted observation. So, once we are writing the equations that way you know we are writing those conditions in terms of this is important here. We are writing the conditions in terms of estimates that is why F L a this is important is equal to 0. This is our starting point and then, because we know La is Lb plus v observation plus the residual. So, we are putting in that condition equation or the condition, which we have just written Lb plus v. And then we explained it and from there, we get our final model of the form AV is equal to f. Where V as we have seen? Is a matrix of residuals A is the matrix of coefficients and f is the matrix of constants. Well once we have formed this basic model.

Now, what we want to do? In this A and f are known and we have to determine the V if we can determine the V we can determine the parameters. Because the parameters, as we know La we are writing La is Lb plus v Lb we know V we will know after adjustment. So, we can determine La. So, our job here is to find V the residuals now, how do we find it? We make use of the principle of least square. And the principle of least square says over here I am also including the stochastic model. So, principle least square says that V transpose wv should be minimum isn't it. So, this is what we will try to do now, but there is a problem here.

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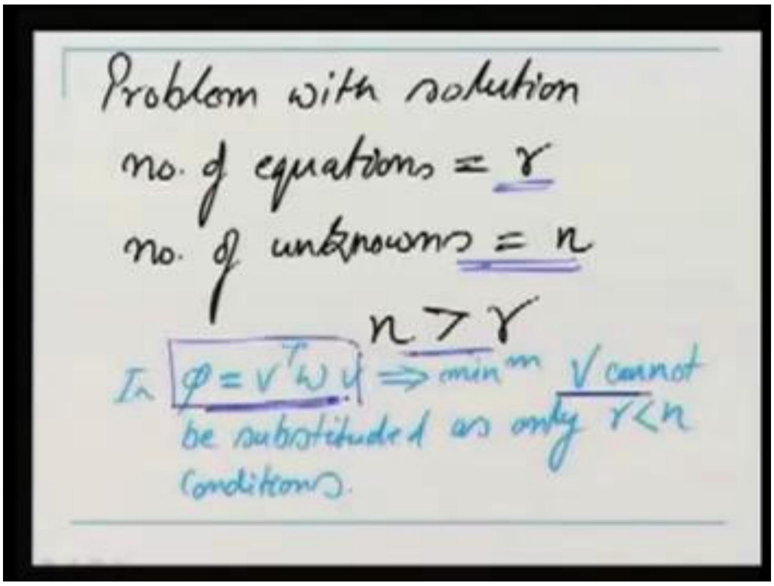

Now, problem in this case is we have only r number of equations, while numbers of unknowns are n, number of the residuals is n, is number of the observations. For each observation the residuals here, and r is smaller than n. So, we are in a situation where we have more number of unknowns than the available equations. So, how to solve it? What we will do? If we go for the least square then in this least square, we cannot substitute for each V or each residual. Because we have less number of equations, because what will be doing as we did in the observation equation method?

In observation equation method, we had sufficient number of equations. And for each V we substituted its value, from the equations that we had formed and then we put those in the least square solution. And the rest of the solution follows, but here in this case, because we have less number of condition equations more number of unknowns. So, we cannot substitute for V in our least square solution. So, what to do?

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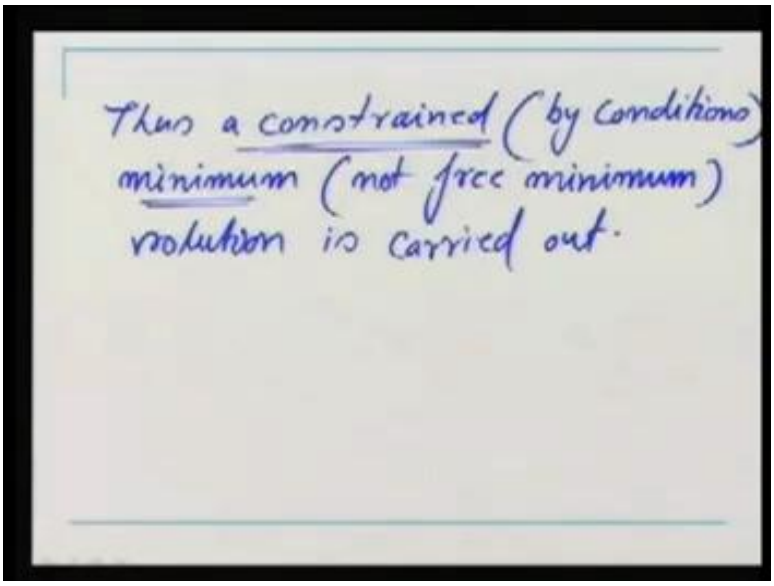

Well in this case we know, we will like to go for a constrained minimum. It is not a free minimum as in case of observation equation method. So, this constrained minimum will get those constrained again by where from the conditions which we have. Well we will see this by an example, this entire process because. So, far we are looking into a general case in general case, starting from F L a is equal to 0. You know the conditions ((written)) in basic you know adjusted observations or the estimates; these are the basic conditions which we are writing. Then we are input, we know we are replacing this La by Lb plus V. And then we are performing our model of the form AV is equal to f all right then we need the least square solution.

Now, we have seen just now that the least square solution is not possible directly, rather we have to go for constrained minimum. How to do that? This process we are going to see now, by 1 example, because it will be easier to understand this in the example. Well the example, in our case is again the 1 which we have taken in our previous classes or the previous lectures. What is the example? We are starting from the benchmark, and going to a point X 1 the benchmark in known, its RL is known. And this is the point for which we want to determine, the RL what we are doing? By levelling we are determining the difference in elevation. Similarly, we have another point X 2 X 3. So, what the job? Is we want to determine RL of X 1 X 2 and X 3 as we have done over here.

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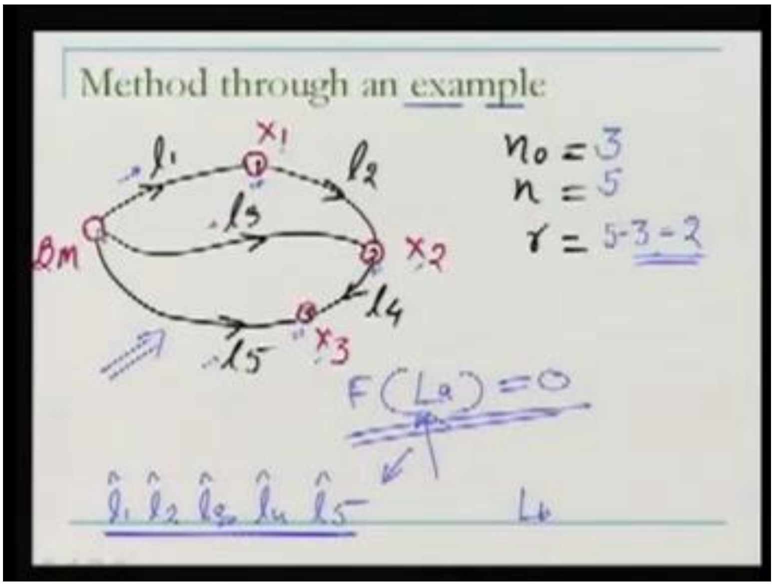

Now, in this slide X 1, point number 1, point number 2, point number 3 and their RLs are X 1, X 2 and X 3. Now, in order to determine these RLs, we had seen the minimum of observations that we need to carry out is we need to observe l 1 l 3 and l 5 only 3. So, by taking 3 observations this problem can be solved, but this solution will be a unique solution, no redundancy. So, actually, what we do? We rather go for more observations. So, how the observations have been taken? Here starting from benchmark. We have reached point number 1 by observing l 1, then from point number 1 find the difference between point number 2 and point number 1 which is l 2.

Again starting from benchmark, we have gone to point number two. So, we found the difference between benchmark and point number 2 which is l 3. Point again from benchmark, we have reached point number 3 by finding the difference between these 2 heights or their heights which is l 5. So, l 5 is the difference in height, between point number 3 and the benchmark. Again from point number 2, we have walked to point number 3 and found the difference between these 2 heights which is l 4. So, what we have done? We have taken 5 number of observations. So, the redundancy in this case is 5 minus 3 is 2. So, if we have redundancy of 2 we will need to write 2 equations here.

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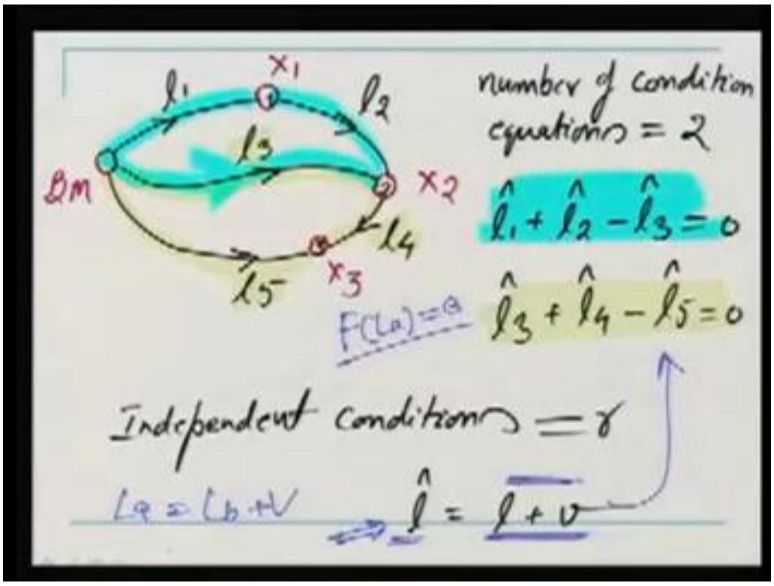

Now, what these 2 equations will be? So, we are writing the 2 equations and as we know the equations should be of the form, the starting point is F L a is 0 this is the form of an equation in adjusted observations. Now, observations here are l 1 l 2 l 3 l 4 and l 5 these are the actual observations, which we represent as Lb. And they adjusted 1 or estimates we write as l 1 hat, l 2 hat, l 3 hat, l 4 hat and l 5 hat which is La. Well in terms of these adjusted observations now, we will try to write the conditions. Now, can we say some conditions over here? If you follow the arrow, you start from this point. You go to X 1 point number 1 point number 2 and go back to benchmark again.

So, basically, what we are doing? We are finding the difference between these 2 points l 1 difference between point number 1 and point number 2 l 2 and point number 2 and point number 3. So, we are starting from a point going to this place, coming again here and then going back. So, if we are finding these differences in elevation and if I add those differences in elevations, their sum should be 0. Because we are starting from a point and we are reaching the same point again. So, their sum should be 0. So, this is how we can form the conditions.

Well, the first condition over here. The very first condition is along this l 1 plus l 2 minus l 3, because the sign is important here the l 3 has been observed in this direction. So, that is why I am using minus l 3. So, that is our first condition. How about the second? 1 well the second is again in another loop starting from l 3, let us do it with some other colour. Yes starting from now, l 3 plus l 4 minus l 5. So, l 3, l 4 minus l 5, this is the second condition. So, this is what we are saying? is of the form F L a is 0. Now, we have written our basic conditions. Now, we know that l hat is l plus v and this is what we are going to replace or otherwise also we are writing as. So, we are going to replace now, in place of l hat l plus v. So, by we are going to replace this in these 2 equations well let us do it.

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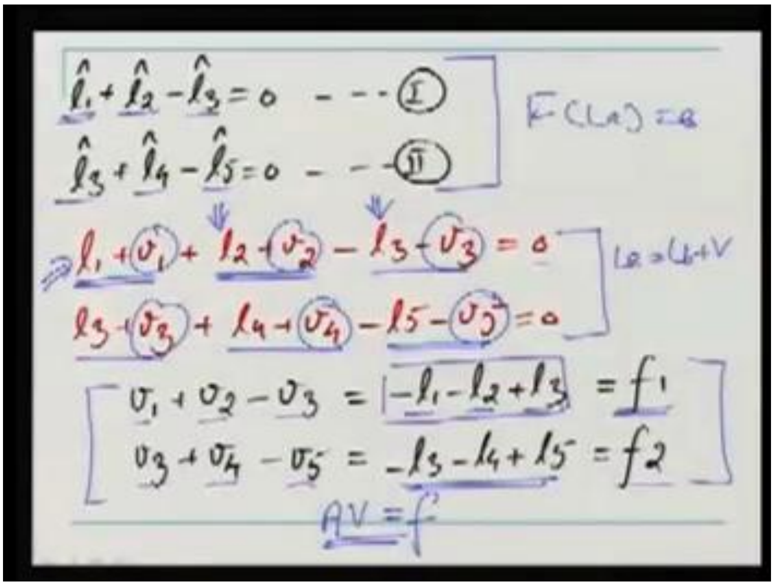

So, these are the basic 2 condition equations of this form, by replacing means for l 1 hat you write 1 plus v 1 l 2 hat is l 2 plus v 2 minus l 3 hat is l 3 and v 3. So, because minus sign here for minus l 3 minus v 3 this equal to 0 similarly, l 3 plus v 3 where l 3 here l 4 hat l 4 plus v 4 minus l 5 hat minus l 5 minus v 5 this equal to 0. So, basically in place of La we have replaced it by Lb plus v, this is what we have done now. Now, we are what we are doing here rearranging it, just rearrange. Because we know finally, we want to reach in this format as we saw in our general case also so, by rearranging it bringing all the v terms on left hand side.

So, v 1 v 2 minus v 3 v 1 v 2 minus v 3 is equal to, we are taking l 1 l 2 and l 3 on the right hand side. So, this minus l 1 minus l 2 plus l 3 and mind it all these 3 are known, why because they are the observations, this is something which we observed in the field. So, we write these 3 as a constant f 1 similarly for the second equation v 3 plus v 4 minus v 5 v 3 v 4 minus v 5 is again another constant here.

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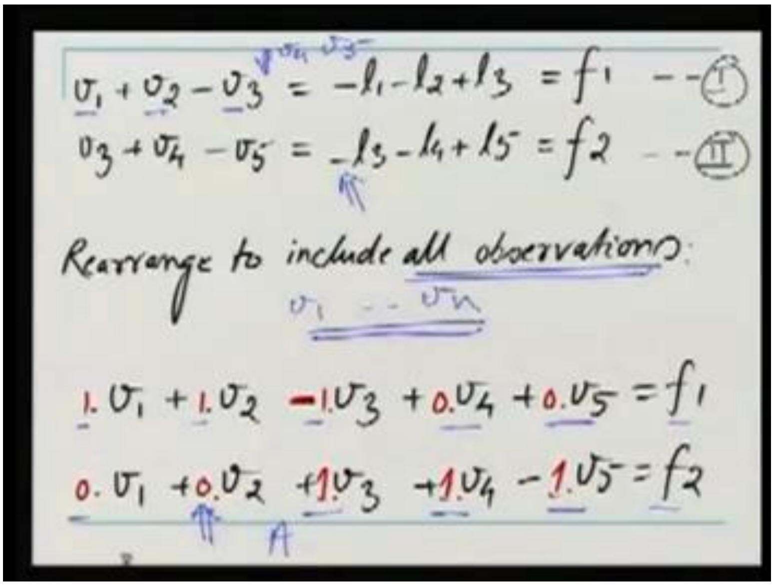

Now, this equation, which we have just written over there we are rearranging it. So, in rearranging to include all observations, in the general form we had written you know v 1 to vn all were included. So, we are trying to do the same thing over here. So, it is v 1 v 2 minus v 3 v 1 coefficient is 1 1 minus 1 there is no term of v 4 and v 5 here so, 0 0 coefficients and f 1. So, I am just trying to rearrange it similarly, over here. Well, all these are the coefficients, which are written in blue here and they will form my matrix A.

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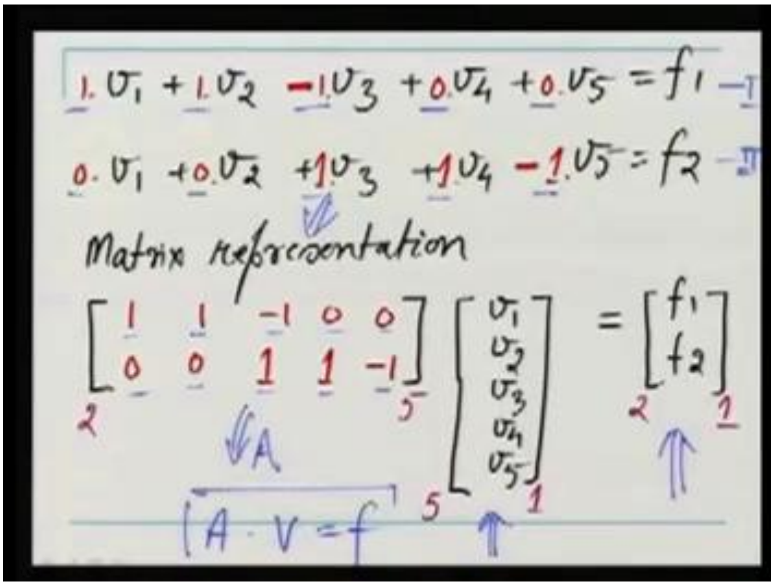

So, if I write it in the matrix form, the matrix A becomes 1 1 minus 1 0 0 so, 1 1 minus 1 0 0 and 0 0 1 1 minus 1 0 0 1 1 minus 1. So, that is my matrix A and the v matrix is v 1 to v 5 and this is the coefficients the f 1 and f 2. So, what we have been able to do? We have been able to generate or write our basic conditions in this form. Well next next is the step which we left they out you know we have said. Now, you want to go for the least square in order to go to the least square, we would like to have number 1 the stochastic model that is the quality of the observations. So, we will talk about that and then how this problem can be solved by least square? Because the problem here is we have more number of unknowns than the equations. So, we are going for the constant solution.

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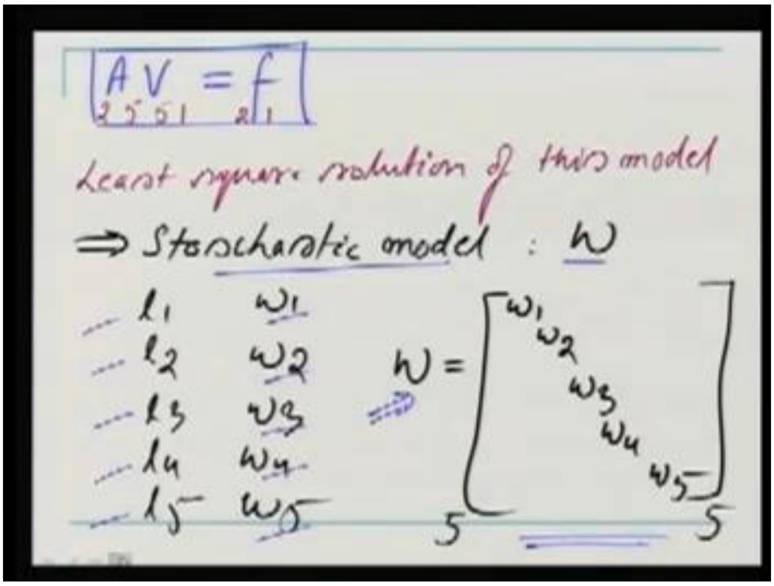

Well about the stochastic model, for each observation l 1 l 2 l 3 l 4 l 5 we have some weight. We know how to assign these weights, the quality of the observation. So, the weight matrix for uncorrelated observation is this a diagonal matrix.

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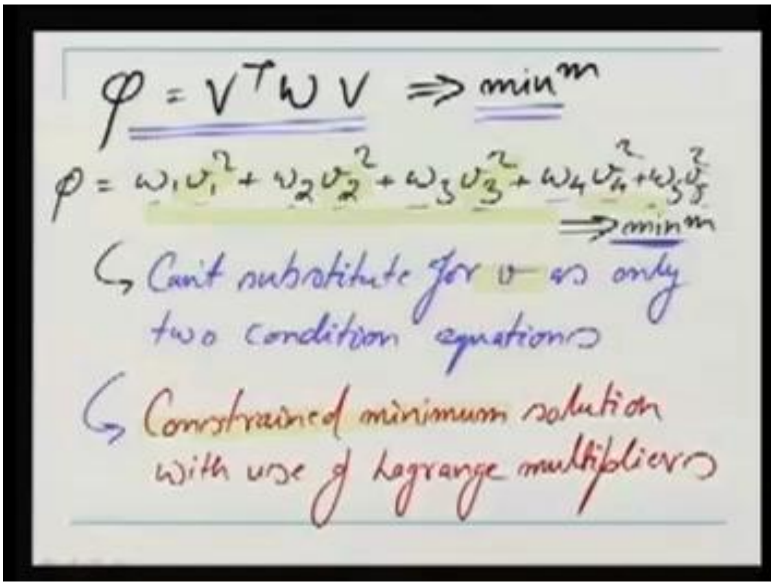

Well about the solution, because that is the least square and we want to minimise this for the least square solution. v transpose wv. And if I expand this it is w 1 v 1 square, w 2 v 2 square, w 3 v 3 square, w 4 v 4 square and w 5 v 5 square and this sum should be minimised. Now again this same problem which we are talking earlier that we cannot replace for v's here, because if I want to replace for v 1 and v 2 v 3, that is not possible. Because the way our condition equations are if you look at our basic condition equations here. These equations from here I can because we have only 2 equations. So, we cannot replace for v 1 v 2 and v 3 in our, this 5 which you want to make minimum. So, how we are going for the constrained minimum? Now, so, we will making use of Lagrange multiplier in order to solve this.

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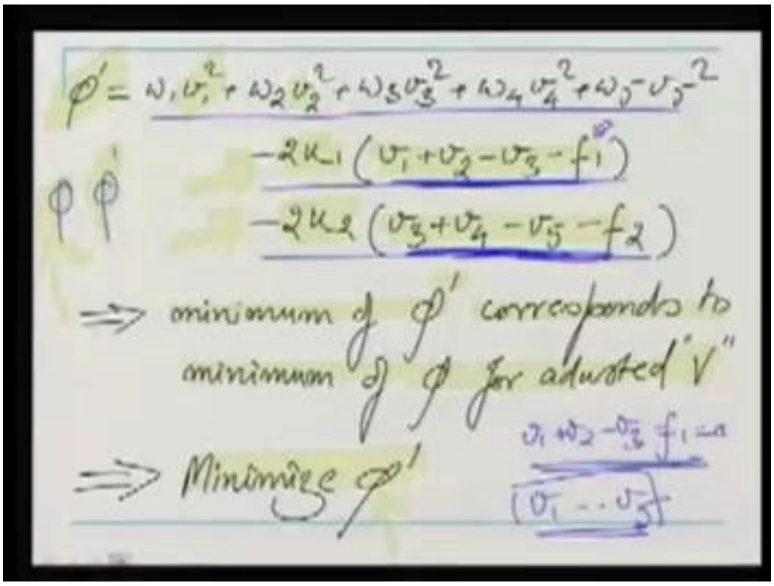

Well our conditions, what I am doing now? Instead of phi earlier we was we are using phi we are introducing now, phi dash phi dash is nothing but same phi less phi plus. In addition we have now, 2 more terms 1 term is minus 2 K 1 v 1 plus v 2 minus v 3 minus f 1 I will explain, how it is coming? Here and minus 2 K 2 v 3 plus v 4 minus v 5 and f two. Now, where from these are coming? If you look at this carefully it is v 1 plus v 2minus v 3 minus f 1 this is nothing, but our first condition equation. The condition equations, which we had formed is this, v 1 plus v 2 minus v 3 is the constant f 1.

So, what we are, what I have done? In this condition equation only the condition equation number 1 I have taken this f 1 on the left hand side and I have replaced it here. Well that is the first condition and this is the second condition here, this is the second condition. Now, so, what we have done? Instead of phi I have introduce the phi dash by introducing these 2 new terms. Well if my observations are adjusted or if we have determined v 1 to v 5, we know that and we know them for adjusted observations then in that case this v 1 plus v 2 minus v 3 minus f 1 this should be equal to 0. For the adjusted observations these should be this should be equal to 0 similarly, here also this value should be also equal to 0 v 3 plus v 4 minus v 5 minus f 2 this should be also equal to 0.

So, because this particular term is 0, and this term is also 0 both these terms are 0 for adjusted observations. So, including these 2 terms first here and the second here in this phi dash it does not have any actually. Because for the adjusted observations or or we can say the other (( )) if phi dash is minimum. If phi dash is minimum that is equivalent to say that is phi is also minimum, because for adjusted observation this term and this term is already 0. So, what will we try to do? Because we know now that if we are saying that minimum of phi dash correspondence to minimum of phi for the adjusted residuals.

So, instead of minimizing, phi we will try to minimise phi dash. Well why we were writing? It 2 K 1 and 2 K 2 these are the Lagrange multipliers and the value 2 K 1 2 K 2, because it helps latter in the solution. We could have written lambda 1 lambda 2 also, well now we have taken a decision well we are going to minimise phi dash. So, if we are going to minimise phi dash how to solve for that? We know we will be finding the partial derivative of phi with each of these variables, v 1 v 2 v 3 and we will be equating that to 0. Well let us do it if I do it for with respect to let us say v 1.

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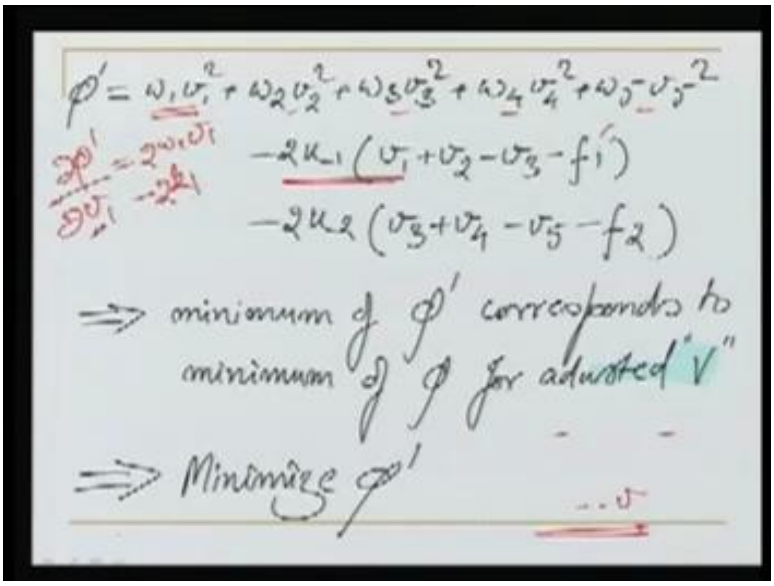

If we partially, differentiate this with v 1, what we get? Del phi dash del v 1 is how many terms of v 1 are there? 1 term is this. So, 2 w 1 v 1 no term no term here no term here and then 1 more term over here, we say minus 2 K 1 v 1 so, minus 2 K 1. So, the first partial derivative the partial derivative, against this variable v 1 is 2 w 1 v 1 minus 2 K 1

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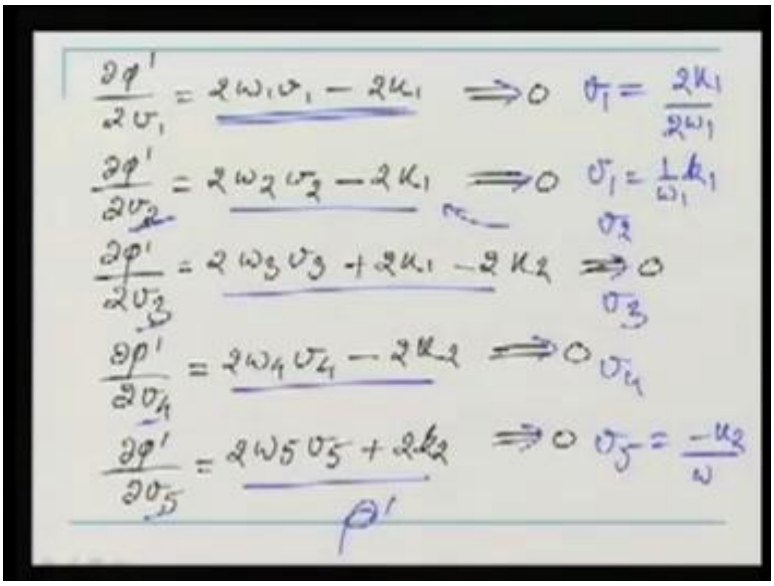

So, we write it here, 2 w 1 v 1 minus 2 K 1 similarly, with respect to this variable v 2 v 3 v 4 and v 5 and we are equating them all to 0. Because this is what we need to do in order to minimise phi dash. Well once we have equated them to 0. Next we can form these find the value of v 1 v 2 v 3 for example, here the v 1 will be 2 K 1 divided by 2 w. So, v 1 will be 1 by w 1 into K 1 all right. Similarly, we can do it for here for v 2 also and for v 3 for v 4 for v 5. So, the v 5 will be minus K 2 divided by w 5. So, we are going to write these for v 1 v 2 and others.

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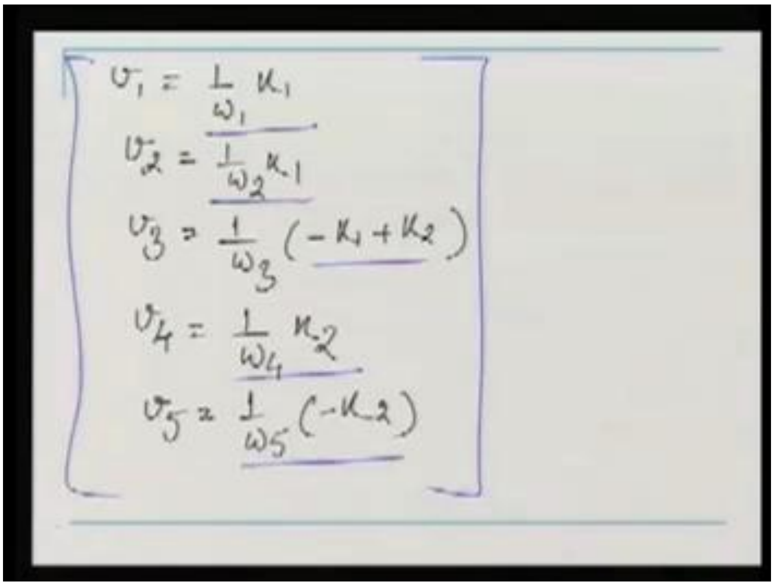

So, v 1 is K 1 by w 1 K K 1 v 2 is 1 by w 2 K 1 and this is for v 3 1 by w 3 minus K 1 plus K 2 v 4 is 1 by w 4 K 2 and v 5 is 1 by w 5 K 2. Next, we will be now rearranging these terms, the same thing what this I am writing now in this slide here.

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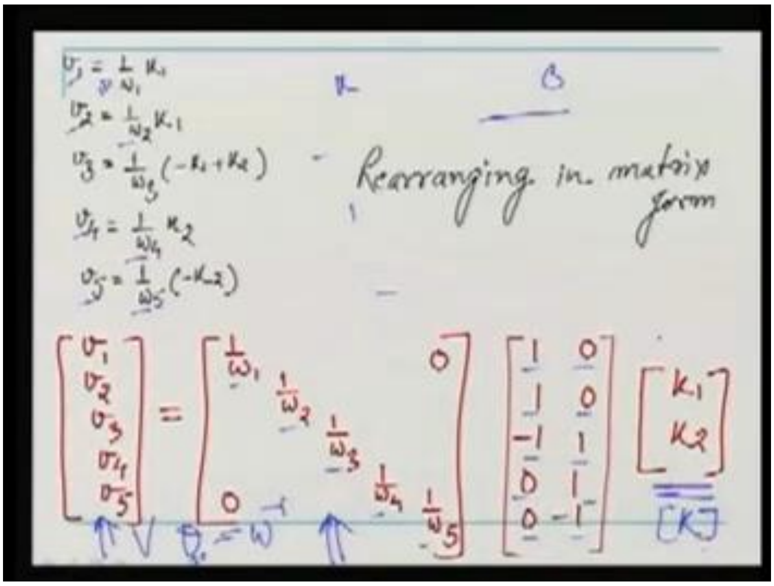

Now, in rearranging it how to be rearrange? v 1 v 2 v 3 v 4 v 5 in matrix form we are writing it here, because all these equal to K 1 by w 1 look at the you see the way I have written over here. I have written the w terms separately here, w 1 by w 1, 1 by w 2, 1 by w 3, 1 by w 4 and 1 by w 5 and all of these I am putting here in a diagonal matrix. Then the terms for see we have K 1 and K 2 here and then the coefficients. So, K 1 and K 2 the coefficients of K 1 and K 2 over here for example, in the very first 1 the coefficients of K 1 the coefficient of K 1 is 1 and K 2 is 0. So, I am writing 1 and 0. Similarly, over here in the second 1 again 1 and 0 for K 1 it is 1 for K 2 it is 0 1 and 0. Now, in this equation it is minus 1 and plus 1.

So, minus 1 and plus 1 and similarly, over here 0 for K 1 and 1 for K 2 0 and 1 and over here 0 and minus 1 0 and minus 1. So, what we have done? We have rearranged our these equations in matrix form. Now, this is very significant, this important will make an important observation here. Now, what is that observation? This is V capital V which is the residual matrix and 1 by w 1, 1 by w 2, 1 by w 3, 1 by w 4 and 1 by w 5. This is inverse of weight matrix, inverse of weight matrix is theta we represented by theta this is called the cofactor matrix and this is K matrix we say. Well this is important here what is this? If you look back when we are forming the equations.

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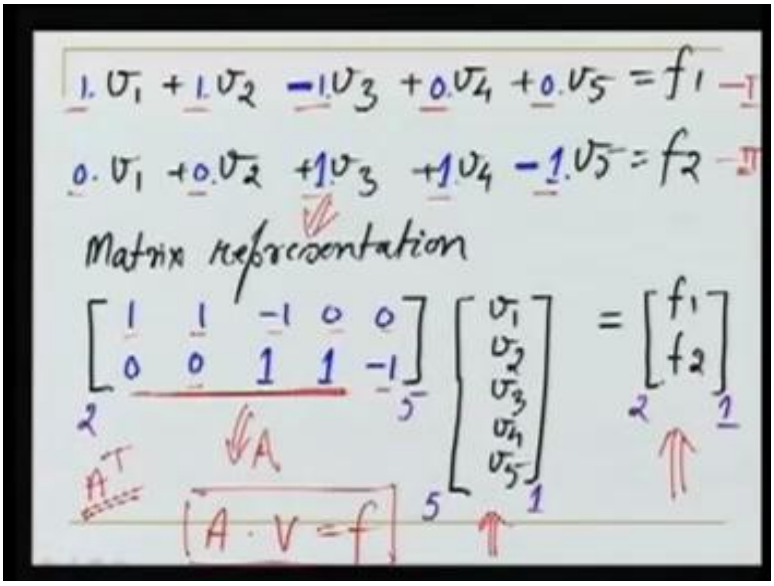

Now, here this matrix is A, if you find A transpose you get the matrix which you have just seen. This is A transpose so, how we can write it?

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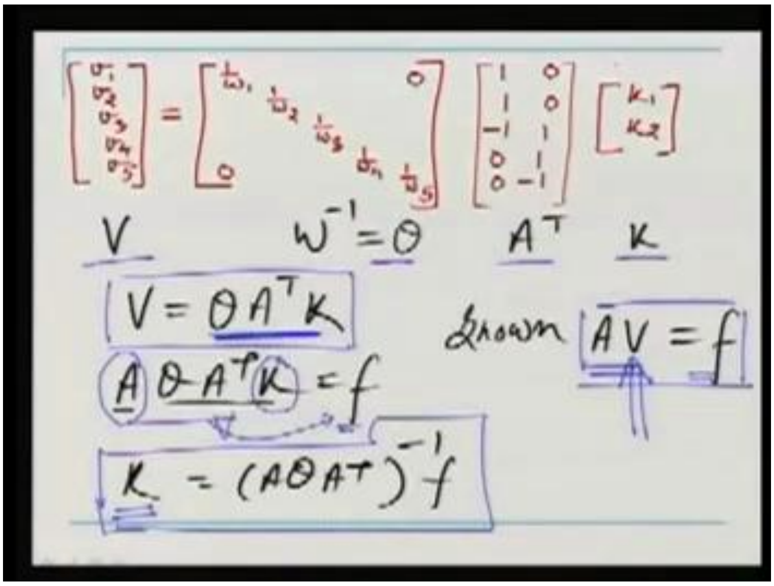

We can write it this is V cofactor matrix, A transpose and K. So, we are writing it now, as V is theta A transpose K. Now, over here in this we know that AV is f we know it, because this is how we had formed? Our initial equations AV is equal to f. Now, in this we are replacing this V by theta A transpose K. So, this is what we are doing? A and this is B theta A transpose K is equal to f. Well from here can we found K. So, to find K we will take A theta A transpose on right hand side. So, you find K is A theta A transpose inverse f.

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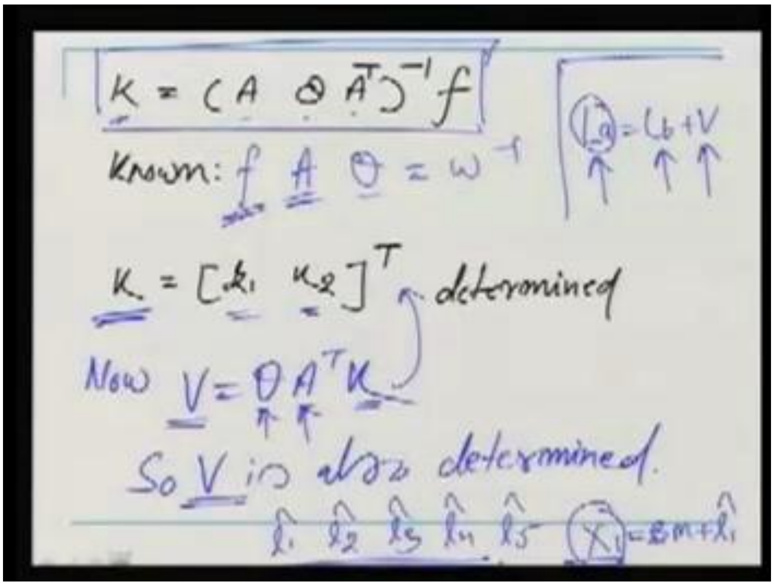

Well, if you write the same thing again here, K is A theta A transpose inverse f now, in this what all is known to us? Well f is known to us, because f is the matrix of constants. Now, where from these constants came? You remember when we are forming the equations; we took all the observations something which we had observed on the right hand side. As well as some times for example, in a triangle the sum of 3 angles is 180 that is also a constant part that is already on the right hand side.

So, all the observations have gone to the right hand side and those they form the f. So, we know this f the constant part, how about A? A is also known, because A the way we form the equations A is the matrix of the coefficients of residuals. So, A is also known to us and how about theta? Co-factor matrix which is inverse of weight matrix. Yes this is this is also known to us, because this means the quality of the observations the stochastic model. So, over here we know all f A and theta. So, if you know then we can determine the value for K. So, the K matrix can be determined over here K 1 and K 2, but these are known to us.

Now, once we know K 1 and K 2, we know could we had arrived earlier that V is theta A transpose K and the K is known to us now. If K is known to us A is known theta is known we can also find V. So, we can find V, because by theta A transpose K. K is already determined A is known theta is known. So, V is also determined, well if the V is determined what next the La is Lb plus V. Because V the residuals unknown we will add these residuals in our observations and we can find the adjusted observations.

So, finally, we are able to determine the adjusted observation. So, in this process what we did? Starting from our, you know basic conditions. We could now find the adjusted observation or we see the estimates of the observations. Once we have these adjusted observations in this particular problem l 1, l 2, l 3 and l 4. Or we can say you know we know now, l 1 hat, l 2 hat, l 3 hat l 4 hat l 5 hat all these are known to us now. If these are known to us, I can simply find the value of X 1 as benchmark plus l 1 hat.

Now, whichever way I find whichever route I take? I can find this action by different routes in that network of the level. I will get the same value of X 1 and this is the X 1 is my parameter the unknown the desired quantity, which you want to determine? why these observations. So, this is the procedure of adjustment by condition equation method.

Now, the general form, the general form is I am going to give you the procedure how to solve the problem by adjustment, but condition equation method.

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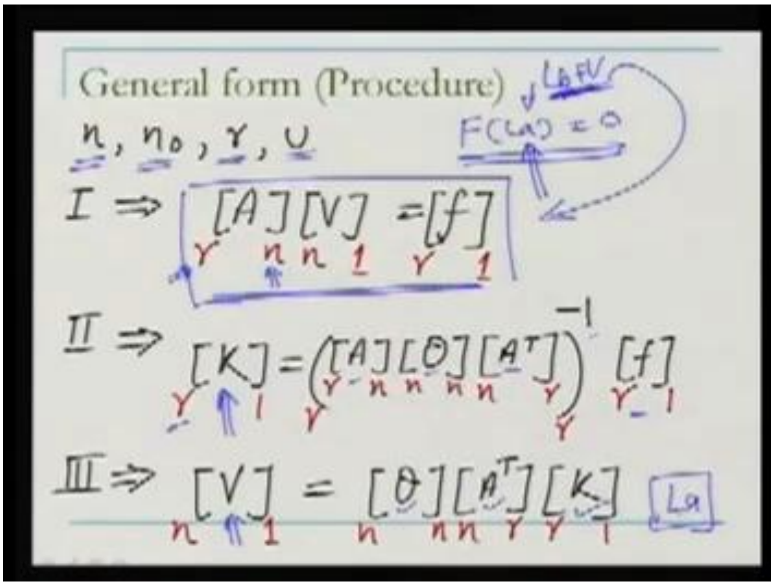

Well, in this first of all in that particular problem, whichever is the problem look for the number of the observations? How what is the minimum number of observations? which can define the problem then find the redundancy. And how many unknowns are there? First of all, write the conditions of this problem, These conditions once you replaced for, La as Lb plus V and rearrange the term. So, you need to form,our functional model of this form. So, this is the functional model which we have arrived from here that is the very first step.

So, the important thing is to look in that particular problem. Where the independent conditions and those in independent conditions are should be equal to the redundancy. And we have to write those condition equations, this is very important starting of the problem. You know the very first step; we have to look at the problem. We are solving here for the simple problems, but sometimes the problems could be very complex, but still we have to find. Well where are the conditions and we need to write those conditions in this form F L a is equal to 0 and then replacing this La by Lb plus V finally, we need to write it in this form.

So, over here r is the redundancy, n is number of observation. once we have done it this way, we need to compute for K the K term we know. Now, where from this K term is coming? And we know it is A theta A transpose inverse f that is the K once this K is known. Once this K is known we can find because the K is known A theta A the A is also known theta is known. We can find the V and once the V is known we can also find the La and our problem is solved. So, this is a general procedure which we have to adopt. What we do now? We will take 1 example a very simple example, but interesting example,

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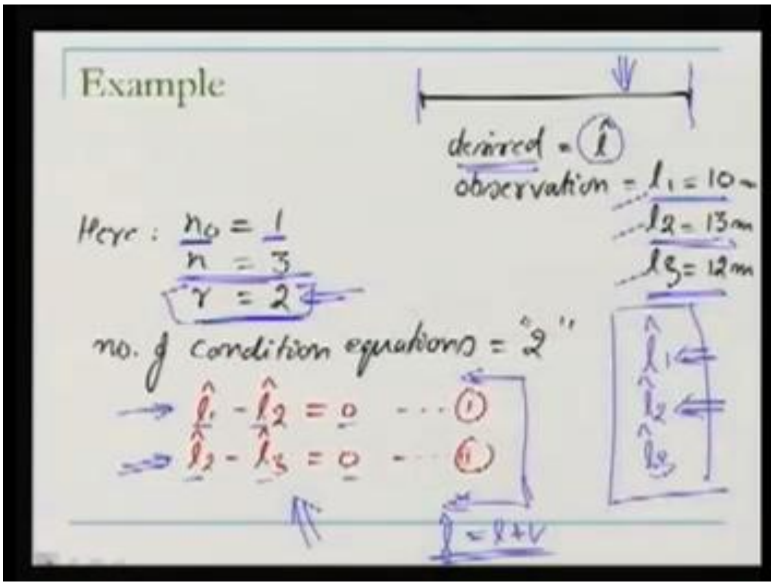

The example is we have a line; we are taking the observations of the length. And we are taking multiple observations let us say, we have measured this line 3 times. If we have measured the line 3 times this observations are first observation is 10 meter, second is 13 meter, third is 12 meter, this is possible 3 observations. What is desired here? The desired is l hat and; that means, what is the length of this line? What is the estimate estimated length of this line? That is the desired. So, n naught if you are looking for the n naught, n naught means we are writing it 1 know. What is the minimum number of observations? Which are required in order to find the answer here well 1 observation? You measured the lien once you have the estimate of the line or other you have the, if that observation is true observation you have the true value of the line.

So, basically, we need only 1 observation, but we are taking 3 observations, because in 1 observation we do not have any check on the error. So, we are taking 3 observations here. So, redundancy in our case is 2 now, we know we will be writing the number of the equations equal to redundancy. So, we need to write 2 condition equations here. So, where are the condition equations here? This is what we need to see now; the conditions and the conditions are to be returned in terms of adjusted observations. The observations are l 1, l 2, l 3 and the adjusted observations will be l 1 hat, l 2 hat, l 3 hat these are the adjusted observations. So, we need to write the conditions in terms of these adjusted observations. Fine, if this is adjusted observation for the line, for the length, which we are talking about and this is also the adjusted observations.

Of course, both are adjusted both the observations have been adjusted; that means, both should be same. So, this is why we are writing this first condition equation l 1 hat minus l 2 hat is equal to 0. Similarly, the second 1 we can write as l 2 hat minus l 3 hat is equal to 0. So, these are the only 2 possible independent conditions which we can write here. And this is important the very first step of writing the conditions is very important, you are writing here, because you know l 1 hat minus l 2 hat is equal to 0. Both the observations are adjusted if they are adjusted these should be same. Well have been written it next step we know we need to replace for l hat as l plus v, this is what we are going to do now.

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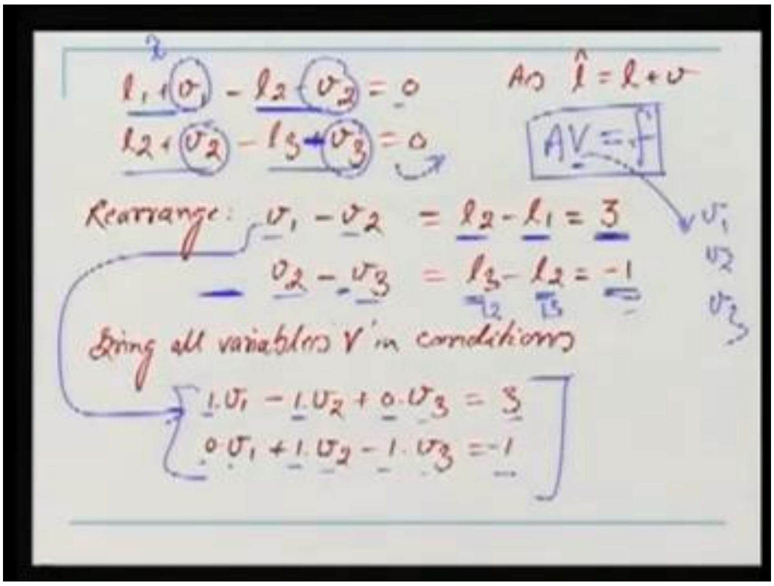

If you replace so, l 1 hat is l 1 plus v 1 minus l 2 hat minus l 2 minus v 2 is equal to 0, this is l 2 hat minus l 3 hat. Now, minus l 3 hat over here please check this should be minus, this is not plus this is minus. We will having done that we want to rearrange these terms, rearranging means we want to keep because finally, we want to write it in form of AV is f. So, we are taking all the constants on right hand side, we are putting the v terms the residual terms on the left hand side. So, I take the residual v 1 minus v 2. So, v 1 minus v 2 is l 2 minus l 1 is equal to 3, because l 2 is 13 l 1 is ten. So, l 2 minus l 1 is 3. So, v 1 minus v 2 is equal to 3 similarly, v 2 minus v 3 v 2 minus v 3 is equal to minus 1.

Because v 2 is this is l means l 2 minus l 3; that means, third sorry 12 minus 13 is l 3 minus l 2 l 2 is 13 this is 12 that is why it is minus 1. Now, in AV is equal to f, in this v we need to bring all the v terms, v 1 v 2 and v 3 in all the equations. So, what we are doing? We are bringing them here and where I am writing the coefficients also. So, 1 into v 1 minus 1 into v 2 and 0 into v 3 this is the equation, which I am writing here again by rearranging by bringing all v 1 v 2 and v 3 equal to 3. Similarly, here no term of v 1 here so, 0 into v 1 1 into v 2 minus 1 here. So, minus 1 into v 3 is equal to minus 1. Once we have written it in this form now, we will write it in the matrix form.

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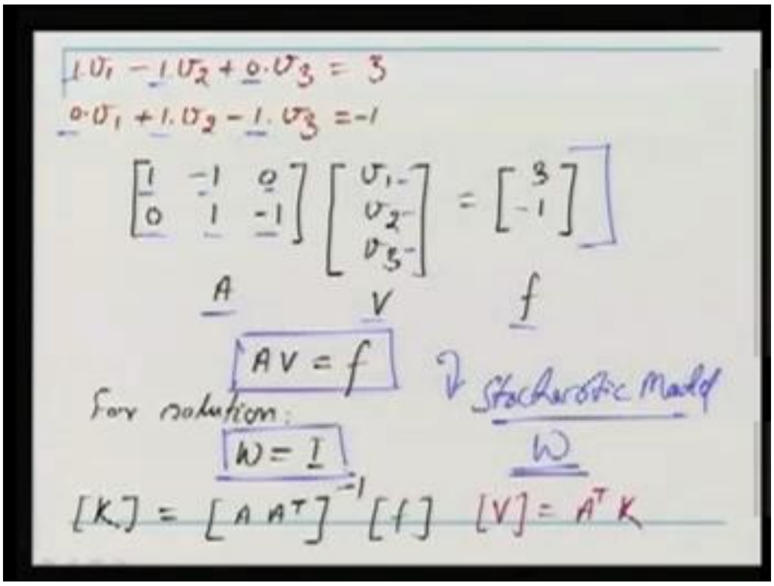

So, in the matrix form as AV is equal to f, you can make this A from these coefficients. Because we know the A is the matrix of coefficients, 1 minus 1 0, 1 minus 1 0, 0 1 minus 1 0, 1 minus 1 and v is v 1 v 2 and v 3 this is equal to 3 and minus 1. So, what we have been able to do right now? We have been able to write it in this form AV is equal to f. Now, we need to look for at this stage the stochastic model means what is the weight matrix? If we know about weight of these observations we will consider the both. Over here we are considering the weight to be the same the identity matrix.

So, all the ways are same, all the observations have been taken with same precision. So, this is for the weight matrices identity here. Well if we know that weight matrix identity. So, we are going to find the solution, how do we find the solution? We have already seen the general form. This way we were written the general form after knowing A theta and f I can find A and once we know K we can find for v. So, this is what we are doing here now? We know A we know f. So, the K can be determined now. K is A A transpose inverse f let us do it and then we will find once we know the K we will find v.

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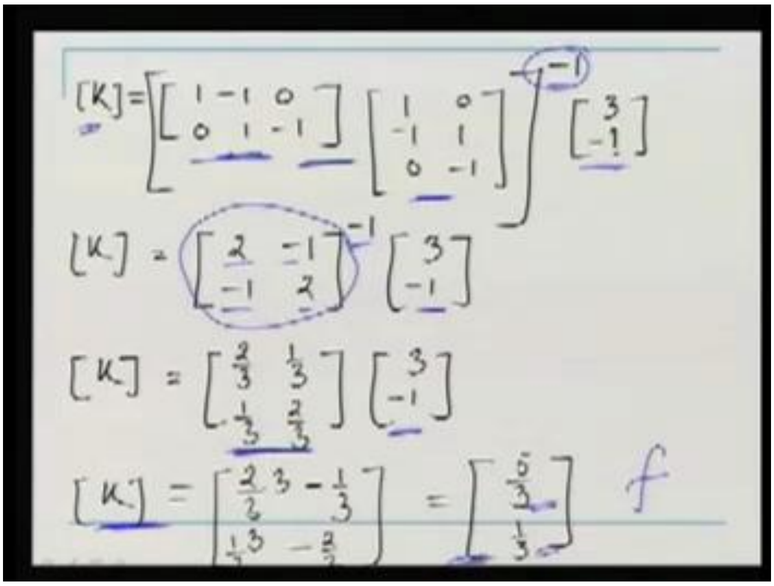

So, K is A A transpose, inverse and f. So, we can solve it now, if you multiply these 2 we get 2 minus 1 minus 1 2 by inverse. So, inverse of this f matrix here and again further the inverse of this inverse of this, you can determine this is here and this is the f. So, the K finally, we can solve it comes out to be 5 by 3 and 1 by 3. So, we know K once we know K we can using v is A transpose K, because this weight matrix is identity. So, I am not including the theta term here. So, once we know K we also know f we can find v.

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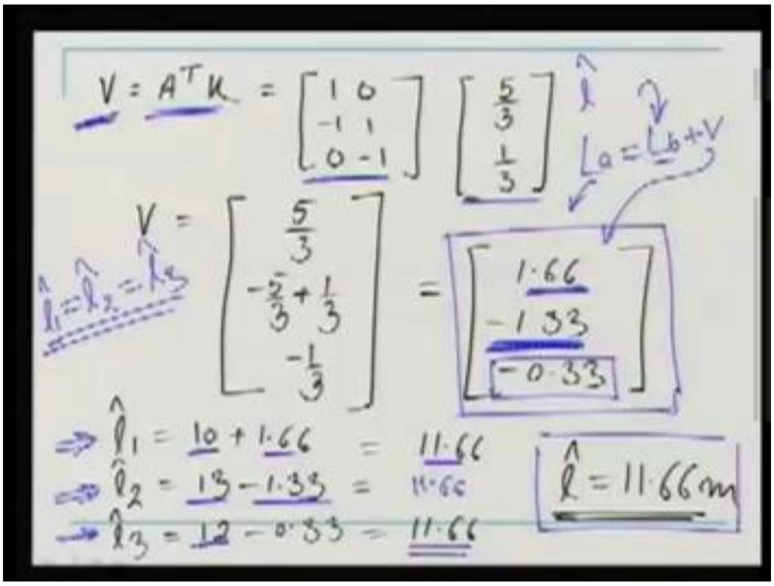

So, v is A transpose K, where A transpose is written over here and this is K. So, by solving for this; this v can be computed. Please do these computations for this particular problem and other problems also. So, this is what is v now, what these v's are, these are the, because there is a basic model. What are those basic model? La is Lb plus v means, we have determined the v if we add these v to observations Lb we will find La means l hat. So, l 1 hat can be computed by l 1 plus v 1 and this particular value is 11 point 6 6 similarly, for l 2 also l 2 hat can be computed by l 2 the observation. This is plus v 2 plus v 2 means minus 1 point 3 3 and this is also 11 point 6 6. Similarly, l 3 also l 3 hat can be computed, because this is l 3 minus sorry plus v 3 what is v 3? Is minus 0 point 3 3. So, again 11 point 6 6. So, what we observe here?

L hat something which was desired is 11 point 6 6. As well as, we observing the l 1 hat, l 2, hat and l 3 hat are now same. And this is what is our basic assumption, all these 3 adjusted observations have to be same finally, something which we observed in the field. In each observation the amount of the error is different, that is why the 3 observations are different, but we know that our adjusted observations should be same. And this is how we wrote the conditions, our 2 conditions here and by this procedure of adjustment the way we have done. Just now, what we saw that finally, we get for each observation, we are getting the corresponding residual or the correction.

So, by applying that correction to that observation, we are getting the estimate of that observation or we can say adjusted observation. So, l 1 hat, l 2 hat and l 3 hat these are 3 adjusted observations for observation number 1, 2 and 3. And as we you thought you know assumed earlier or as our condition was earlier these 3 are same. So, this is what we have done by this particular, you know exercise? We have now; found the desired thing, the desired thing was length of the line the estimate of the length of the line, which we have found as 11.66 by following the procedure. So, that all our observations were, you know we included all the observations and followed the condition equation method and adjusted the observations.

One in interesting thing, you know we have proved this before also generally, what we do? When we have the observations like this for 1 line we have 3 observations. And what we do? Without realising the concept of adjustment or the least square, approach. What we do? We take these 3 observations and finally, arithmetic mean. And we had proved also when we are talking about this adjustment that arithmetic mean is also the least square solution is the best estimate, we have proved that also earlier.

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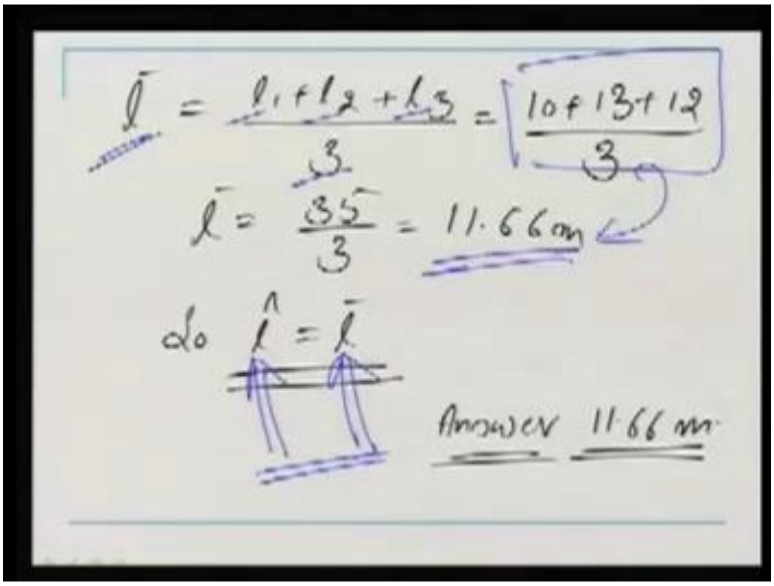

So, over here also if I do the same thing, the arithmetic mean is l 1 plus observation l 2 and l 3 divided by 3. So, this particular value also comes out to be the same. So, this is also again now, we are proving here the, what we found from the least square adjustment is same, as the arithmetic mean. So, what we have seen in this lecture? In this video lecture, we saw the condition equation method. How do we start from the beginning? You know we have to look into the problem, where the conditions are; we have to see how many observations have been taken? What minimum number of observations are required? In order to define the problem, find the redundancy. Once the redundancy is known, then we write equal number of conditions. So, we have to look into the problem that where this conditions are.

So, after writing those conditions and this is really the important step, the most important step we you know replace, because we are writing the conditions in terms of estimates. So, we replace those estimates by observation and the residual, and then we write finally, in the form of AV is equal to f and next once we have written this. We include stochastic model the quality of the observations and then the rest of the solution is simpler is very very simple. You know just we have to do some matrix manipulation and finally, we get the v, v is the residual values. Once we have the v we can find the adjusted observation and then finally, the parameters you know something which was desired. So, this is what we discussed in this lecture.

Thank you.